Procedure Use in Basic Arithmetic: Developing an Objective Indicator

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Can evidence of non-retrieval procedure use be found in the shape of response time distributions?

Distributional analyses were performed using the *ex-Gaussian* model, which consists of a normally distributed leading edge component and an exponentially distributed tail. The ex-Gaussian model provides a good fit to response time data and allows three quantitative summary measures of distributional shape to be obtained:

- *Mu* (μ) the mean of the normal component
- *Sigma* (σ) the standard deviation of the normal component, and
- *Tau* (τ) the mean of the exponential tail.

We evaluated the position of Penner-Wilger, Leth-Steensen & LeFevre (2002) that non-retrieval procedure use is reflected in tau. Self-reports were collected in order to provide a crucial check against an existing measure of procedure use. If tau does index procedure use, then individuals using a greater proportion of procedures should have more positively skewed response time distributions, reflected in larger tau values.

Method

Adult participants solved 288 single-digit addition problems ranging from 2 + 2 to 9 + 9. Problems were presented on a computer screen and participants responded vocally. After each problem participants gave self-reports of their solution method from the following list: Transform (if knowledge of another problem was used), Count (if a strategy based on counting was used), Remember (i.e. *retrieval*, if the answer came to them without any steps or calculations), and Other (Campbell & Timm, 2000).

Analyses

Large problems were defined as products greater than 25, small problems as products less than or equal to 25. Participants were divided into groups based on percentage of retrieval use reported. *Retrievers* (9 participants) reported retrieval on 99% of small problems and 98% of large problems. *Procedure users* (8 participants) reported retrieval on 40% of small problems and 30% of large problems.

Traditional measures including mean response time (RT), standard deviation (SD), percent error, and the three ex-Gaussian parameters, obtained by fitting the ex-Gaussian distribution to the individual participant data in each of the four conditions, were analyzed in a series of 2 (problem size: small, large) x 2 (group: retrievers, procedure users) repeated-measure ANOVAs.

Results

Traditional analysis (Figure 1, left panel).
Participants were slower, their response times more variable, and more errors were made on large problems than on small problems. Procedure users were slower to solve problems than retrievers and their response times were more variable. Mean RT and SD showed a Problem size x Group interaction such that retrievers did not show a problem-size effect whereas procedure users did.

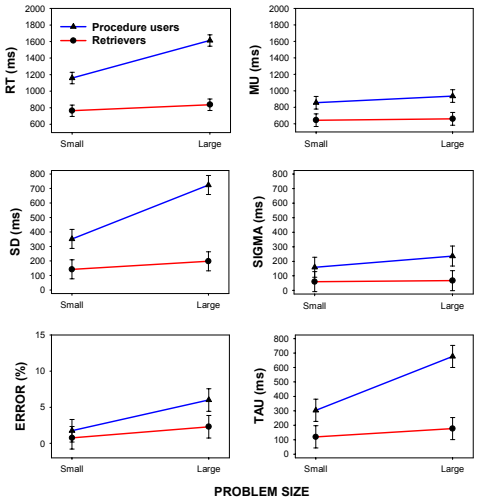

Figure 1. **Traditional (left) and ex-Gaussian (right) measures as a function of group and problem size, showing that the interaction present in Mean RT and SD is a result of an interaction in tau such that procedure users show a problem-size effect whereas retrievers do not. Tau is larger for procedure users than retrievers, and larger for large problems than small problems for procedure users.**

Ex-Gaussian analysis (Figure 1, right panel).
Tau was larger for large problems than small problems. Mu and tau were both larger for procedure users than retrievers. The Problem size x Group interaction evident in RT and SD is evident only in tau such that retrievers did not show a problem-size effect whereas procedure users did. As mean RT = mu + tau, and variance = sigma^2 + tau^2, the effects evident in mean RT and SD result from the effect in tau.

But mean RT and SD show the same pattern as tau, so why not just use mean RT and forget about tau? Increases in RT (or SD) can result either from an increase in tau, or mu (or sigma), or both. It is also possible to get a tradeoff between tau and mu (or sigma) such that no effect is evident in traditional measures. Tau isolates the effect of interest and ensures it is not obscured.

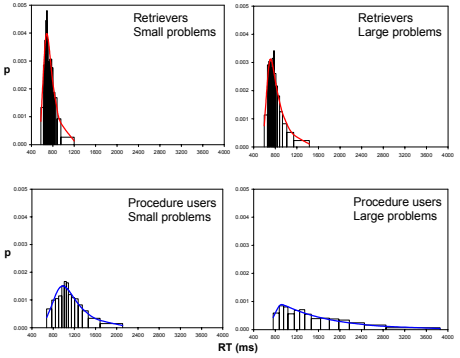

Figure 2. **Group probability density histograms for distributions of response times to small and large problems for retrievers and procedure users, with solid lines representing the fitted ex-Gaussian distribution function, showing increase in positive skew as procedure use increases both across groups and across problem size within the procedure use group.**

Group histograms (Figure 2).
RT distributions display striking differences in shape both between groups and within the procedure users group with increases in problem size. The tails of the distributions are larger for procedure users than retrievers. Moreover, for procedure users, the tails of the distribution are larger for large problems than small problems.

These results indicate that tau increases with increases in procedure use, both between groups and within the procedure users group. Thus, tau is serving as an indicator of procedure use in single-digit addition.

Conclusion

Evidence of procedure use can be found in tau, the mean of the exponential tail of response time distributions, as posited by Penner-Wilger et al. (2002). Tau successfully distinguishes between groups of retrievers and procedure users for single-digit addition. Thus, tau shows promise as an additional, objective tool for exploring phenomena in mathematical cognition within the framework of individual differences in solution procedures.

References

Campbell, J.I.D., & Timm, J.C. (2000). Adults' procedure choices for simple addition: Effects of retrieval interference. *Psychonomic Bulletin and Review, 7,* 692-699.

Penner-Wilger, M., Leth-Steensen, C., & LeFevre, J.-A. (2002). Decomposing the problem-size effect: A comparison of response time distributions across cultures. *Memory and Cognition, 30,* 1160-1167.

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