Addition Strategies: An Algorithm for Solving Multi-digit Problems

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Can people's self-reports shed light on the cognitive processes involved in multi-digit mental arithmetic?

The study of the cognitive processes underlying the solution of simple and multi-digit mental arithmetic problems has largely relied on measures of speed and accuracy. Variation in reaction times and error rates with variation in task features such as presentation format and problem complexity have inspired theoretical hypotheses regarding how people solve mental arithmetic problems. Dual-task studies have provided insight into the involvement of working memory in mental arithmetic. The self-reports of participants doing simple arithmetic have revealed the use of non-retrieval as well as retrieval strategies.

The present study further evaluated the mental processes underlying adults' performance on multi-digit mental arithmetic. In addition to RT and accuracy measures, we examined people's self-reports of what they did to solve multi-digit mental addition problems.

Method

Participants solved 48 different 1-digit plus 2-digit mental addition problems with sums less than 100. Twenty-four involved carrying from the ones to the tens and 24 were no-carry problems. Each problem was presented in two orders, double-digit first and single-digit first, and two formats, vertical and horizontal, for a total of 192 problems.

Carry		
Vertical	34 +7	7 +34
Horizontal	34 + 7	7 + 34

No Carry		
Vertical	23 +2	2 +23
Horizontal	23 + 2	2 + 23

Problems were presented continuously on a computer screen until participants responded vocally. After solving each problem, participants reported their solution procedures. Self-reports were recorded verbatim. RT and accuracy (or percent error) were also measured.

Results

Reaction Times and Errors: Participants solved double-digit first problems more quickly and accurately than single-digit first problems and no-carry problems more quickly and accurately than carry problems. These results are consistent with previous findings.

Order of Processing (Figure 1.) Participants' descriptions of the steps they followed in solving these multi-digit addition problems were coded as consistent with beginning the solution process with either the 1-digit number or the 2-digit number. For example, for the problem 5 + 68, "8 plus 2 plus 3…" or "68 plus 2 plus 3" was considered consistent with beginning with the double-digit operand and then using a decomposition strategy, whereas "5 plus 5 plus 3…" suggested that the single-digit operand was processed first. Participants began with the double-digit operand significantly more often than the single-digit operand regardless of which one was presented to them first.

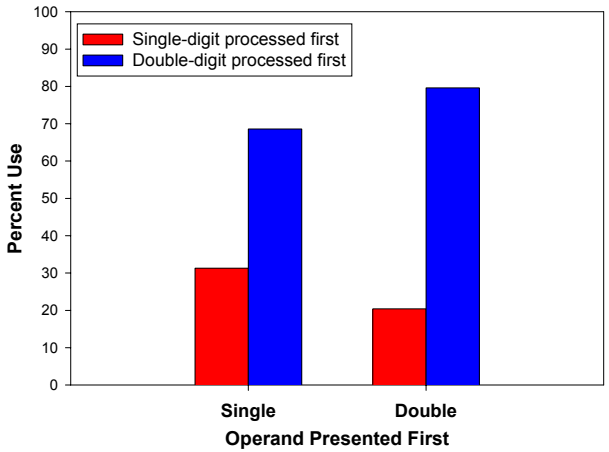

Figure 1. Percentage of trials with double-digit vs. single-digit operand processed first when problems were presented with double-digit or single-digit operand first.

Strategy Choice: A Plan of Action? Participants' reports frequently suggested that a preliminary decision was made regarding whether or not carrying would be required.

Examples:

"I knew it would go into the next decade…"

"Both ones digits were less than 5, so I knew it wouldn't carry…"

Solution Strategies and Carry Problems (Figure 2.) The percent use of various solution strategies was recorded for carry and no-carry problems. The two predominant strategies were retrieval from memory (of the sum of the units digit; e.g., "I just knew it") and decomposition (e.g., "I took 45 plus 5 plus 2"). Decomposition was the strategy of choice for carry problems while there was a clear preference for retrieval use on no-carry problems.

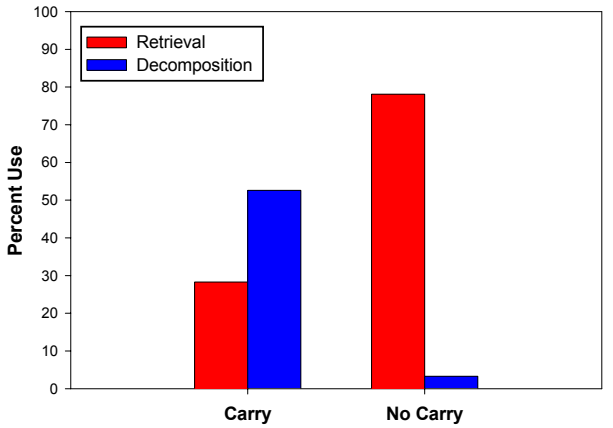

Figure 2. Percent use of retrieval vs. decomposition to solve multi-digit addition problems with and without a carry operation.

Conclusion

Self-reports reveal a common algorithm for solving multi-digit mental addition problems. When solving double-digit plus single-digit problems, adults show a preference for processing the 2-digit operand first, whether it is presented first or second in the problem. The extra step of 'locating', or focusing on, the larger operand may contribute to the longer solution times found for problems presented with the single-digit operand first. Further, there is evidence that adults make a preliminary decision about whether the addition of the single digit will entail a carry and then they choose an addition strategy. The greater use of decomposition may help to explain why it takes longer to solve carry problems. Results such as these from people's self-reports of solution strategies contribute to our understanding of how people approach multi-digit arithmetic.