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Richard L. Carson

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Carleton UNIVERSITY

Department of Economics

1125 Colonel By Drive
Ottawa, Ontario, Canada
K1S 5B6

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Richard L. Carson
Carleton University

ABSTRACT

This note gives a method of determining the long-run equilibrium output of a firm operating under imperfect competition that differs from standard methods of output determination and reveals properties of the equilibrium that standard methods conceal. This method requires a basic cost-effectiveness condition to hold, which leads to a highly elastic demand. If we strengthen that condition a bit, long-run equilibrium under free entry and exit becomes indistinguishable from a long-run equilibrium under perfect competition, even though products are differentiated and the condition given by Rosen for perfect competition with product differentiation fails to hold.

JEL Classifications: D24, D43, D61.
Key Words: Imperfect Competition, Substitutability, Cost Effectiveness, Firm Size, Excess Capacity.

A COMPETITIVE EQUILIBRIUM WITH PRODUCT DIFFERENTIATION

I. Introduction

This note gives a method of finding the long-run general equilibrium output of a firm operating under imperfect competition that differs from standard methods of output determination and reveals properties of the equilibrium that standard methods conceal.[1] Let X be a product supplied by a firm under oligopoly or monopolistic competition, with long-run general equilibrium output, x_e. Suppose there is a related good, Y, sold under perfect competition or at least under competitive pricing, with price equal to minimum average cost in long-run equilibrium.[2] For example, X might be a differentiated version of a good or service and Y a generic version. When a cost-effectiveness condition is met, x_e will be where the average cost of Y reaches its minimum or where the production function for Y displays constant returns to scale. This is true as long as X and Y are viable and the cost-effectiveness condition holds. Differentiating a product does not then change its long-run equilibrium output at the level of the firm, which is entirely determined by technical properties of the production function for Y.

Moreover, the production function for X displays decreasing returns to scale at x_e, even though this is not true of the rent-inclusive average cost of X. Thus firm size is not too small nor is there excess capacity, and product demand is highly elastic. If we strengthen the cost-effectiveness condition a bit, long-run equilibrium under free entry and exit becomes indistinguishable from a perfectly competitive equilibrium, even though products are differentiated, and the basic condition given by Rosen [1974] for product differentiation to be compatible with perfect competition fails to hold. If we think of a product as a bundle of attributes in the manner of Lancaster [1966, 1971], Rosen's condition is that each product be a different combination of the *same* attributes. Perfect competition in the supply of each attribute could then result in perfect competition in the supply of products. Firms would be price takers even though no two supply exactly the same good or service.

Here each firm's differentiated product is assumed to have a unique attribute, and Rosen's argument therefore fails to apply.

II. Basic Assumptions and Results

Let P_x and P_y be the demand prices of X and Y and x be the output of X by the supplier of this good. X and Y are assumed to be measured in the same well-defined output units, and with x plotted on the horizontal axis, AC_y is the long-run average cost of Y that a supplier of Y would face. It is assumed to take the conventional U shape with a minimum at $x = x^*$. Let AC^R_x be the "rent-inclusive" long-run average cost of x. This is long-run average cost inclusive of all rents earned in the supply of X that persist over the long run. These include the rent to the supplier's market position—when this is protected by entry barriers—as well as the earnings of product-specialized inputs over and above the opportunity costs of these inputs in supplying other goods. If all inputs are mobile between firms—including alternative possible suppliers of X—the latter rents are part of the opportunity cost of a firm supplying X. Thus when inputs are mobile between firms, and market positions can be freely bought and sold, rent-inclusive and opportunity costs will be the same for each firm.

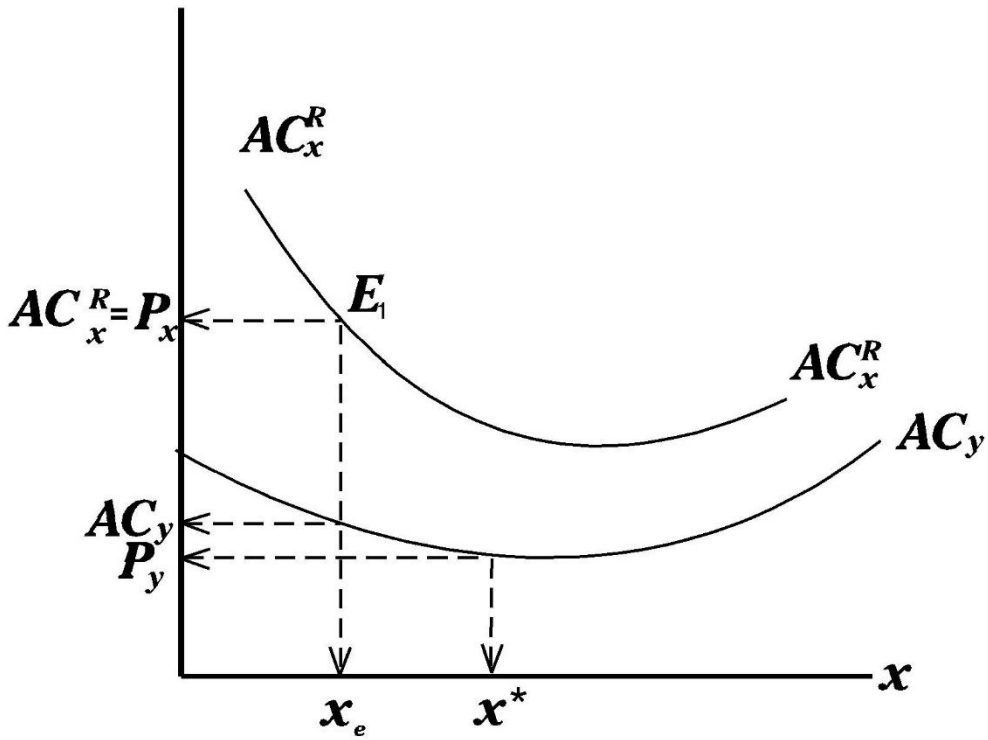
This is the assumption made here. A firm will then supply X over the long run if and only if it expects to be able to produce where $P_x \geq AC^R_x$, but long-run competitive pressures will also force $P_x \leq AC^R_x$ to hold at each x. If it survives in long-run equilibrium, the firm supplying X will maximize profit where $P_x = AC^R_x$—and thus where its demand is tangent to AC^R_x—and the firm supplying Y will maximize profit where $P_y = AC_y = MC_y$, where MC_y is the marginal cost of Y. The outcomes, $P_x > AC^R_x$ or $P_y > AC_y$, can occur only in the short run when sellers are able to earn disequilibrium or entrepreneurial profits (quasi-rents) that attract entry and are competed away over the long run via expansion of supply by competitors.

Production of X is assumed to use product-specialized inputs, which add value to X, but are not used to produce Y. Let F_x be a fixed cost of X in the form of rent to the supplier's market position and to its product-specialized inputs. Then $AC^R_x = AC_x + F_x/x$, where AC_x is the average cost of X, exclusive of this rent. (In general, AC_x and AC_y are not the same.) A differentiated product is assumed to possess unique attributes,

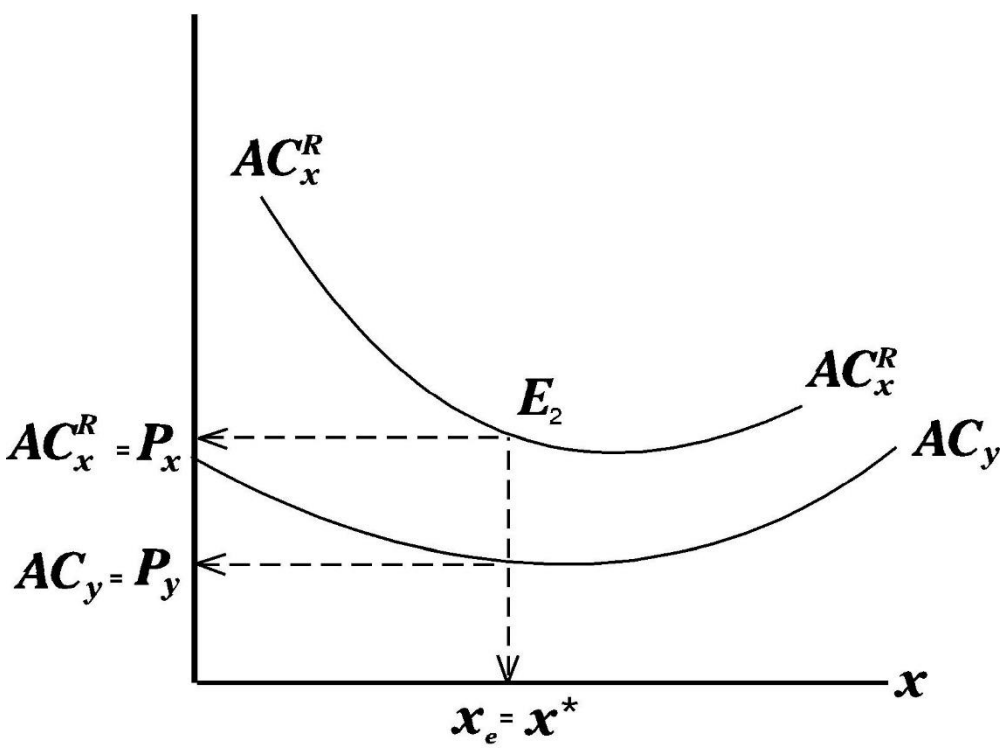
features, or properties whose cost-effective supply requires inputs specialized to these unique elements and thus to the product. For example, suppose a restaurant supplies cuisine with a unique flavor and ambience. Its product-specialized inputs are its recipes, together with the tacit knowledge and the talent—embodied in a master chef and his/her team—used to create the cuisine from the recipes. These inputs could be bought by another owner who would acquire the ability to supply the unique cuisine in question. Thus the rent earned by these product-specialized inputs is part of the opportunity cost of the restaurant supplying the cuisine that they produce. The recipes, tacit knowledge, and talent in question are fixed stocks, which can be bought and sold, and their remuneration is therefore a cost that is fixed or "quasi-fixed", in the sense that it does not vary with output once a decision has been made to produce a product.

Because production of X uses product-specialized inputs that are not used to produce Y and because the supplier of X may be able to exercise market power, $P_x > P_y$ will hold. In long-run general equilibrium, $(P_x - P_y) \geq (AC^R_x - AC_y)$ must also hold for the firm supplying X if AC^R_x and AC_y are evaluated at $x = x_e$. Since $P_x = AC^R_x$ in equilibrium, $(P_x - P_y) < (AC^R_x - AC_y)$ would imply that $P_y > AC_y$ is possible. In general, either $(P_x - P_y) > (AC^R_x - AC_y)$ or $(P_x - P_y) = (AC^R_x - AC_y)$ can hold, equality implying that $x_e = x^*$. Figures (a) and (b) illustrate these possibilities. In Figure (a), the equilibrium price and output of X are assumed to be at E_1 where $x_e < x^*$ and $(P_x - P_y) > (AC^R_x - AC_y)$. In Figure (b), equilibrium price and output are at E_2, where $x_e = x^*$, and thus $(P_x - P_y) = (AC^R_x - AC_y)$. In traditional monopolistic competition [Chamberlin 1933], E_1 and E_2 are two possible points of tangency between demand and AC^R_x.

E_2 is the outcome under the cost-effectiveness condition below and is the more likely outcome under free entry and exit because only one product in the economy needs to satisfy that condition. Suppose there is a third product, Z, that is viable in long-run equilibrium with equilibrium output, $z = z_e > 0$ and rent-inclusive average cost, AC^R_z. When Z is scaled in such a way that $P_z = (P_x - P_y)$, suppose that AC^R_z is no greater than $(AC^R_x - AC_y)$. Roughly speaking, this means that one can add value to Y externally, by supplying a separate product, Z, as efficiently as one can add value internally, by producing a unit of X instead of a unit of Y.


AC_x^R
E_1
$AC_x^R = P_x$
AC_x^R
AC_y
AC_y
P_y
x
x_e
x^*

(a)


AC_x^R
E_2
AC_x^R
$AC_x^R = P_x$
AC_y
$AC_y = P_y$
x
$x_e = x^*$

(b)

More precisely, scale Z as above and assume that whenever x takes on a value at which AC^R_x is downward-sloping and demand is such that $P_x = AC^R_x$, there is a value of z at which:

$$AC^R_z \leq (AC^R_x - AC_y) \qquad (1).$$

for the indicated values of x and z. Given this, suppose that long-run general equilibrium prevails. Then $P_x = AC^R_x$ and $P_z = AC^R_z$. If (1) holds as well, $(P_x - P_y) = P_z = AC^R_z \leq (AC^R_x - AC_y)$. Since $(P_x - P_y) < (AC^R_x - AC_y)$ implies $P_y > AC_y$, $(P_x - P_y) = (AC^R_x - AC_y)$ must hold. By equating prices with long-run rent-inclusive average costs, the market brings about $x_e = x^*$ when (1) holds, where x^* is the output at which AC_y reaches its minimum value and at which:

$$P_y = MC_y = AC_y, \qquad (2).$$

where MC_y is the marginal cost of Y.

Here x^* is also the output at which the production function for Y exhibits constant returns to scale. Its value depends only on technical properties of that function, and the equilibrium outputs of X and Y are the same at the firm level. As a result, x_e will be invariant to shifts in demand as long as the supplier of X can cover its costs and at least one product is available that satisfies (1). Demand shifts will affect P_x and the rent earned from supplying X, as well as the number of firms in the X industry, but not x_e.

Let T be any product other than X, Y, or Z. If output units of T are scaled in such a way that its price, P_t, equals $(P_x - P_y)$, suppose that T does not satisfy (1). But then $P_t = (P_x - P_y) = (AC^R_x - AC_y) < AC^R_t$, and T cannot survive in long-run equilibrium. Thus if one product that is viable in long-run equilibrium meets the cost-effectiveness condition, *all* viable products other than X and Y must satisfy this condition. Moreover, suppose that no product meeting the cost-effectiveness condition exists initially. Then it will be profitable in the short run to introduce such a product, Z, since with the above scaling, $P_z = (P_x - P_y) > (AC^R_x - AC_y) \geq AC^R_z$ or $P_z > AC^R_z$. There is no reason why such a product should not also be viable over the long run.

III. More on the Nature of Equilibrium.

If we measure returns to scale using AC_x rather than AC^R_x, we remove the effect of F_x on economies of scale. At $x = x^* = x_e$, AC_x could therefore be constant or upward-sloping, even if AC^R_x is downward-sloping. Let $L_{ex} = (P_x - MC_x)/P_x$ be the Lerner index of market power for X. When the supplier of X faces fixed input prices and uses no indivisible inputs other than those that are specialized to X, measuring returns to scale from AC_x is the same as measuring them from the production function for X. If R_x denotes returns to scale in the production of X at any given output—or the percentage increase in x that results from increasing each input by one percent—$R_x = AC_x/MC_x$. At any given x, it would be reasonable to expect production returns to scale to be at least as great for a more homogeneous version of a product as for a more differentiated version. Returns to scale increase as a product becomes more standardized and as its production process becomes more routinized. Thus $AC_x/MC_x \leq AC_y/MC_y$ at any given x.

Suppose then that differentiating the firm's product (supplying X rather than Y) does not raise returns to scale at any output, when these returns are measured as AC_x/MC_x and AC_y/MC_y. As a result, $MC_x \geq AC_x$ at $x = x_e = x^*$. There is no excess capacity in production of X nor is firm size too small. If $f_x = F_x/P_x x$ is the share of product-specialized rent in product value, we have:

$$L_{ex} \leq f_x, \qquad (3).$$

when both L_{ex} and f_x are evaluated at $x_e = x^*$. Thus when competitive pressures on X are "strong", in the sense that they keep f_x low, these same pressures will ensure approximate marginal-cost pricing. The supplier of X will be producing where AC_x is constant or upward-sloping, reflecting constant or decreasing returns to scale in production. If AC^R_x then slopes downward at x^*, it does so solely because F_x is positive.

In the simplest case, L_{ex} equals one over the elasticity of demand for X at the profit maximum. Thus if f_x is no more than 10 percent or .1 (more generally α), the elasticity of demand will be at least 10 (more generally $1/\alpha$). This may seem like a high elasticity, and we therefore want to investigate the effect on equilibrium in the X industry of strengthening (1). To this end, let x^{**} be the value of x at which AC^R_x reaches its minimum, and

suppose as above that $x^* \neq x^{**}$. Since $x^* > x^{**}$ would require an equilibrium where AC^R_x is upward-sloping, $x^{**} > x^*$ must hold.

Suppose then that we replace (1) with a slightly stronger condition. Whenever x takes on a value at which AC^R_x is downward-sloping and demand is such that $P_x = AC^R_x$, there is a value of z at which:

$$AC^R_z \leq \text{min. } (AC^R_x - AC_y), \qquad (1a).$$

where min. $(AC^R_x - AC_y)$ is the minimum value of $(AC^R_x - AC_y)$ over all x at which AC^R_x is downward-sloping. In long-run equilibrium this includes all x between x^* and x^{**}. In (1a), Z is again scaled in such a way that $P_z = (P_x - P_y)$ at the above value of x where $P_x = AC^R_x$. In equilibrium, this is $x = x^*$.

Since AC_y is upward-sloping over the range between x^* and x^{**}, it follows that $(AC^R_x - AC_y)$ is smaller at x^{**} than at x^*. The vertical distance between AC_y and AC^R_x falls as x increases from x^* to x^{**}. Thus (1) must hold with a strict inequality at x^* and as a result, the supplier of Z is earning a positive economic profit since $P_z = (P_x - P_y) = (AC^R_x - AC_y) > AC^R_z$. In fact, many or all of the products that are viable in long-run equilibrium—and which therefore satisfy (1)—are likely to be profitable when $x = x^*$ and $x^* < x^{**}$. Widespread further entry will therefore occur, and long-run equilibrium cannot prevail until this happens.

Thus if X survives in long-run equilibrium, this equilibrium must be where $x^* = x^{**}$. Entry will cause x^{**} to fall toward x^*, and in order for this to happen, some of the new entrants must supply products that are substitutes for X. Such entrants will lower the demand for X and make this demand more elastic. They will also reduce F_x, which lowers both AC^R_x and x^{**}. Likewise, entrants that supply substitutes for Z will lower P_z and F_z whenever F_z is initially positive. Let TC_x be the total cost of x exclusive of rent. As long as the demand for X is high enough to cover TC_x for an efficient supplier of X, this product will continue to be supplied over the long run.

Since the long-run equilibrium output, x^*, is now the output at which AC^R_x has a zero slope, the supplier of X is facing horizontal demand in equilibrium and is a price taker. Under free entry and exit, its equilibrium is indistinguishable from that of a perfect competitor, even though no other firm supplies a product that is a

perfect substitute for *X*, in the sense that the elasticity of substitution between *X* and any other firm's product is less than infinity. Moreover, the condition given by Rosen [1974] for perfect competition with product differentiation fails to hold. Here the effect of many substitutes that are imperfect duplicates the effect of many perfect substitutes, even when each product has a unique attribute. The existence of *Z* extends the competitive equilibrium in *Y* to differentiated substitutes for *Y*.

V. Conclusion

Given products *X* and *Y* plus a third product, *Z*, that meets the cost-effectiveness condition (1), the long-run equilibrium output of *X* at the firm level will be where the average cost of *Y* reaches its minimum and therefore the same as that of *Y*. The differentiated version of a product (*X*) has the same equilibrium output, $x_e = x^*$, as the non-differentiated version (*Y*), and the value of x^* is wholly determined by technical properties of the production function for *Y*. When free entry and exit prevail, moreover, and we replace (1) with the slightly stronger condition (1a), the equilibrium output, x_e, will be where rent-inclusive average cost (AC^R_x) reaches its minimum and is therefore indistinguishable from a long-run equilibrium under perfect competition when free entry and exit prevail. This is true even though products are differentiated and each product has a unique attribute so that the analysis of Rosen [1974] does not apply.

NOTES

1. For surveys of models of imperfect competition, see Mansfield and Yohe [2004], Nicholson and Snyder [2012], and Shapiro [1989]. See Chamberlin [1933] for the basic model of monopolistic competition.
2. The market for *Y* may be vulnerable to "hit-and-run" entry, which causes *Y* to be priced competitively, even though *Y* is not supplied under perfect competition. See Baumol [1982]. Also perfect competition is compatible with some product differentiation. See Rosen [1974].

REFERENCES

1. **Baumol, W.J.** Contestable Markets: An Uprising in the Theory of Industry Structure. *American Economic Review*, March 1982, 72(1), pp. 1-15.
2. **Chamberlin, E.** *The Theory of Monopolistic Competition.* Cambridge, MA: Harvard University Press, 1933.
3. **Lancaster, K.** A New Approach to Consumer Theory. *Journal of Political Economy*, April 1966, 74(2), pp. 132-159.
4. **_____.** *Consumer Demand: A New Approach.* New York: Columbia U. Press, 1971.
5. **Mansfield, E. and Yohe, G.** *Microeconomics.* New York: Norton, 11th ed., Chs. 10-13, pp. 356-493.
6. **Nicholson, W. and Snyder, C.** *Microeconomic Theory: Basic Principles and Extensions.* Mason, OH: South-Western, 2012, 11th ed., Chs. 14-15, pp. 501-569.
7. **Rosen, S.** Hedonic Prices and Implicit Markets: Product Differentiation in Pure Competition. *Journal of Political Economy*, January/February 1974, 82(1), pp. 34-55.
8. **Shapiro, C.** "Theories of Oligopoly Behavior." Ch. 6 of **Schmalensee, R. and Willig, R.D.** *Handbook of Industrial Organization.* Amsterdam: North-Holland, 1989, vol. I, pp. 329-414.