Social Welfare and Collective Goods Coercion in Public Economics

by

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Abstract

This paper develops an expanded framework for social planning in which coercion stemming from the provision of public goods is explicitly acknowledged. Key issues concern the precise definition of coercion, its difference from redistribution, and its incorporation into social welfare optimization. The paper examines the implications for optimal policy, showing how the Samuelson condition, rules for optimal linear income taxation and commodity taxation, and for the marginal cost of public funds must be modified. In addition, the trade-off between social welfare and coercion is mapped under specific conditions and the implications of this trade-off for normative policy choice are considered.

Key words: Coercion, optimal linear income taxation, optimal commodity taxation, marginal cost of public funds, public goods, collective choice
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The essential feature that defines a democratic government is voluntary agreement by members of the public to subject themselves to its coercion.

William Baumol (2003, 613)

From the point of view of general solidarity...parties and social classes should...share an expense from which they receive no great or direct benefit. Give and take is a firm foundation of lasting friendship... It is quite a different matter, however, to be forced so to contribute. Coercion is always an evil in itself and its exercise, in my opinion, can be justified only in cases of clear necessity.

Knut Wicksell (1896/1958, 90)

1. Introduction

Coercion is a fundamental and unavoidable aspect of public life. Although philosophers and constitutional experts have examined its nature at length, economists have not offered a comprehensive analysis of its role in traditional normative theory.

The planner model, the main theoretical engine of normative public economics, does deal with redistribution, which may indeed be coercive, but it does not assign any normative weight to the coercive aspects of redistributive policies per se, acknowledging only the social welfare generated by redistribution, whatever its coercive impact. Nor does it deal with other aspects of coercion that limit the freedom of individuals to make important marginal adjustments.

Coercion in collective decision making occurs in many situations. Consider, for example, a group of people who have come together in a room for a common purpose and who must set the temperature on a thermostat and then pay for the resulting use of energy. Since Lindahl pricing is not feasible, some will be too hot and some too cold, and even those for whom the temperature is just right may be unhappy with the resulting balance between what they pay and what they get. Individuals can escape the situation if they move to another room or out of the building that represents the collectivity in this example. But if they stay, they must cope with the coercion implied by their assent to the collective decision.

As Baumol (2003) points out, people participate in a collectivity despite its coercive aspects because joining makes them better off. In other words, they will accept some coercion as a necessary cost of having a community. But this is a keenly felt cost as Wicksell (1896), Buchanan (1967), Breton (1974), Usher (1981) and others have stressed. If it becomes too great for too many, unrest, emigration and eventually failure of the state as a productive enterprise may occur. Too much coercion endangers the operation of the collective choice process and the production of public services.

There are two well-known normative approaches falling outside the social planning literature that deal formally with coercion inherent in public policies. The first is represented by the work of Wicksell (1896), who proposed that all policies be implemented using a qualified unanimity rule, and by Lindahl (1919), who devised a method of voluntary exchange that would have the same result.[1] Unanimity and Lindahl's voluntary exchange both result in efficiency and the absence of coercion. As has been pointed out repeatedly however, such processes lead to high decision costs and are not descriptive of the real world.

A second approach, developed by Buchanan and Tullock (1962), aims at finding an optimal policy rule in the face of decision costs and coercion. Like the two Scandinavian economists, they focus primarily on process and do not link their analysis to the welfare implications of particular policies. None of these contributions make clear how coercion is to be defined or how it is to be measured.

The power of the social planning literature derives in large part from its ability to consider the implications of specific policy choices. This allows one to evaluate and compare

[1] The relationship between Lindahl equilibrium and collective choice, on the one hand, and coercion, on the other, has been discussed extensively in the literature, although not in a context reflecting recent normative analysis in public economics. See, for example, Escarrez (1967), and for a review of the relevant literature, Mueller (2003). Buchanan and Congleton (1998) have considered how the design of fiscal systems can limit the coercive aspects of taxation.

different outcomes according to the social welfare that they imply. A purely process-oriented approach cannot accomplish this, limiting the analyst to giving advice on institutions and their design rather than on specific policies. Yet, the literature focusing on process has made an important contribution by drawing attention to the central role of coercion in public life, by linking it to different collective choice processes, and by pointing out the need to formally acknowledge it as a significant factor in policy analysis.

This paper develops an ecumenical approach to normative policy choice by formally introducing coercion into social planning.[2] We examine maximization of social welfare when the planner is bound by constraints on the degree of coercion, as distinct from redistribution, that may be generated in order to supply public goods financed by compulsory taxation.

The necessity of coercion in a democratic society with public goods, the difference between what we shall call "collective goods coercion" and redistribution, and the incorporation of coercion into a broader approach to social welfare optimization are the key issues addressed in section 2 of the paper.

The precise definition of collective goods coercion, a significant problem in itself, is developed in section 3. One should note in this connection that coercion also plays a role in the origins of the state (Usher 1993, Baumol 2003, Perroni and Scharf 2003), and that it can be imposed in many other ways including, for example, though tax administration (Alm, McClelland and Schulze 1992), conscription (Levy 1997) or by regulating access to private markets and limiting the scope of markets (Wiseman 1989). The term 'collective goods coercion' acknowledges that in this paper, we confine the analysis to coercion arising from the collective provision of public goods. (We note here that for simplicity of expression, we shall generally use

[2] Breton (1974), Dalton (1977) and Breton (1996) also analyze coercion implicit in public finance using a defintion of coercion that depends exclusively on the level of the public good, as discussed below.

the term coercion in the rest of the paper, rather than to the full term 'collective goods coercion'.)

In section 4 we investigate the implications of the expanded social planning framework for the nature of optimal policy, showing how the Samuelson rule and the optimal structure of taxation must be substantially modified. (In an Appendix, we use the same approach to consider how the Ramsey rule (1927) for optimal commodity taxation must be adjusted to account for coercion.) In section 5 we map the trade-off between social welfare and coercion under specific conditions and consider the implications of this trade-off for normative policy analysis.

We realize that imposing constraints on a planner derived from a concern with the quality of collective choice extends the analysis beyond criteria generally accepted in the social planning or optimal tax literature. We believe that our approach is justified because it allows us to address significant questions about the role of coercion and its relationship to social welfare, and because we think that a concern with coercion must be an important and legitimate focus in the analysis of the public sector.

2. Coercion Versus Redistribution In Policy Analysis

Coercion is inevitable in a democratic society as individuals interpret and react to the nature and outcome of collective decision making. Jointness in supply, problems with preference revelation, and economies of scale in consumption create a situation where public goods and services cannot be efficiently provided in private markets. Since unanimity does not work as a decision mechanism because of possible strategic behavior, some type of majority rule is required, as long as we are interested in a democratic society. But such rule in whatever form leads to situations with an imperfect matching of what people pay in taxes and what they receive in public services.

Wicksell (1896) reminds us that this imperfect matching differs from voluntary redistribution. Nor is it the same as redistribution instituted by a planner as part of a program aimed at increasing social welfare. To see in general terms that coercion in public life is distinct from income redistribution, it is instructive to consider the Bill of Rights in the United States and similar documents or unwritten constitutional rules in other countries. The rights afforded by these documents are intended to apply equally to the poor and rich; they were not created with reference to income levels, but with reference to individual lives. There may, of course, be an interaction of redistribution and coercion, an issue to be dealt with later in the paper, but this only reinforces the insight that redistribution is not the sole origin of coercion. [3]

One may ask more directly why maximization of social welfare, defined as the weighted or unweighted sum of utilities, does not deal with coercion, even though the difference between benefits and costs for each individual is reflected in individual utility and therefore in the social objective. The reason lies in the fact that the approach posits no limits on the loss or gain in utility for particular individuals occurring as part of a social plan. (This will be shown formally below when the coercion-constrained social planner's optimization problem is explicitly stated and compared to a traditional planning problem.)

It might be argued that application of the Pareto criterion - that only reallocations leaving every one better off are permissible - can attenuate concern with coercion. Strict application of the Pareto criterion limits the degree of individual coercion for alternative moves from the status

[3] In an interesting paper that is complementary to the current one, Perroni and Scharf (2003) develop a positive theory of the self-enforcing fiscal system. The problem they begin with is that there is no external power to enforce the power to tax so that ultimately, in their view, all fiscal systems must be self-enforcing equilibria where the continual consent of the public (to the degree of taxing power, or power of the state to coerce) is sought. They search for efficient, self-enforcing equilibria that are robust to renegotiation among groups of citizens in future periods. As a consequence, they claim (result 4) that when citizens have identical preferences, efficiency and renegotiation proofness requires horizontal equity in taxation. But, as they explicitly state, this is "fully unrelated to any distributional goal" (p. 9). Rather, in their approach it is a matter of insuring the stability or viability of society as a whole.

quo. It does not, however, alleviate any mismatch between benefits received and taxes paid that is embedded in the status quo itself. Moreover, much applied work using social welfare analysis goes beyond the strict Pareto criterion, which is too weak to allow for most social action, using the Hicks-Kaldor criterion of potential compensation instead. In that case, reallocations are considered desirable and permitted even if some people become worse off, as long as gainers could *in principle* more than compensate losers. For this reason, an explicit concern with coercion is justified and needed in most practical instances.

2.1 Our approach to modeling the role of coercion

To develop a normative approach that allows us to compare and evaluate specific policies, we proceed by imposing specific coercion constraints on maximization of social welfare as usually defined. We then investigate the nature of public policies that emerge in such a context and compare them to policies that are consistent with the traditional social planning approach. In this way we use an expanded approach to social planning that still falls into the neo-classical tradition in public economics.

Is it reasonable to use coercion constraints to acknowledge the importance of collective goods coercion rather than to reformulate the social welfare function that is to be maximized? Consider an analogy to modeling the social role of money. Macroeconomists have tried to come to grips with the role of money in society either by putting money into the utility function following Patinkin (1965), an obvious approximation to the social role of money, or by adding constraints to the specification of the economy (e.g. the cash-in-advance constraints of Robert Clower 1967). Our approach is analogous to the second method. We add coercion constraints to a planning problem in order to incorporate an important aspect of collective choice in a simple

manner.

Using a constraint to deal with coercion also has a second justification. The most important way in which limits on coercion have been introduced into political arrangements is by constitutional provisions restricting the power of government to abridge certain rights. Such provisions do not in principle allow for a trade-off between the rights that are given and other policy objectives. They may, of course, be subject to interpretation by the courts, but always with the understanding that the rights take precedence over other public aims. Setting of boundaries or constraints on public action thus represents a well-known and tested approach to dealing with coercion in public life.

In the analysis that follows, we distinguish between definitions of coercion on an individual basis, and those that are analogous to the use of the Hicks-Kaldor potential compensation criterion, since collective goods coercion will be of heightened concern when the planner is allowed to make trade-offs among individuals without explicit compensation.

We also assume that the state will coerce individuals up to the maximum possible, so that coercion constraints are generally introduced as equalities. One may want to acknowledge that in some contexts coercion may serve as a method of reducing the excess burden of taxation, thus having a productive as well as a harmful social role. We do not incorporate this possibility explicitly into the analysis. Excess burdens are defined in the usual manner, independently of the degree of coercion.[4] The emphasis here is on defining coercion arising from the collective provision and financing of public goods and on investigating how an acknowledgment of limits to such coercion alters the structure of the fiscal system in a broader social welfare framework.

[4] For example, we do not explicitly allow the planner to force independent evaluations of ability on taxpayers, or to coercively uncover economic activity, thereby relaxing incentive compatibility constraints.

3. Defining Coercion

In the formal analysis, we shall define coercion for an individual as the difference between this person's utility under what he or she regards as *appropriate* treatment by the public sector and the utility that he or she actually enjoys as a result of social planning.[5]

To make this definition concrete, it is necessary to define what appropriate treatment means. There are two approaches to this issue, each corresponding to a particular view of the relationship between the individual and the state. Both are illustrated in Figure 1. One possibility is to think of individuals as judging social outcomes from a perspective in which they alone decide what is best for them and for others. In this *individual-as-dictator* approach, the counterfactual utility, denoted as V^D_j, is determined by maximizing the person's (indirect) utility subject to the government budget restraint $t(G)$ that shows all feasible combinations of income tax rates and actual public good levels. Coercion can then be defined as $[V^D_j - V_j]$, where V_j denotes the actual level of utility. The corresponding counterfactual level of the public good is G^D_j.

[Figure 1 here]

An alternative view sees the individual as a social being who does not desire dictatorial outcomes. In this '*individual-in-society*' approach, the counterfactual is determined by finding the maximum individual utility attainable if the individual could adjust the level of the public good at the tax-prices he or she actually faces. To illustrate this case, we let the individual's actual average tax-share be $\tau_j = (T_j/G)$, where T_j is his or her total tax payment, and assume, as in Buchanan (1968) and Breton (1974), that the individual believes he or she would pay this tax share if quantity adjustment were possible. The relevant tax rate for defining the counterfactual

[5] Breton (1974) defines coercion as depending on the deviation of marginal evaluations of public services from tax-prices. While (the total amount of) coercion as defined below varies with this difference, as we show below, it is not coercion itself.

as a fraction of income is $t_j = (\tau_j P/Y_j)G$, where P is the supply price of the public good - so $\tau_j P$ equals the tax-price - and Y_j is income. The counterfactual level of utility then can be shown in Figure 1 as V^*_j where utility is maximized subject to the tax-share line with slope $(\tau_j P/Y_j)$. Coercion is given by $[V^*_j - V_j]$, with G_j^* equal to the corresponding counterfactual level of the public good.

In the individual-in-society approach, the formal expression for an individual's coercion thus equals

$$[V^*_j(G^*_j, W_j, \tau_j P) - V_j] \quad \text{where} \quad G^*_j = \underset{\{G\}}{arg\ max}\ V_j(G, W_j\ \tau_j P), \tag{1}$$

where, in addition to the definitions above, V_j = actual utility $V(G, W_j, \tau_j P)$; and W_j = the wage or ability of individual j. We shall use primarily this approach to defining the counterfactual in our analysis. It is the one implicit in the work of Wicksell, Lindahl, Buchanan and Breton. It assumes that the individual accepts some coercion by society, along with a socially determined tax-price. We shall note, however, at various points how the *individual-as-dictator* approach would alter results. It turns out that both definitions have similar implications for optimal fiscal structure and the coercion-welfare trade-off.

Before we can specify coercion constraints imposed on the social planner, there are two additional distinctions to consider: First, coercion can be determined on an individual basis as implied by the preceding discussion, or it can also be defined for a group. While applying constraints to each individual is consistent with the tradition initiated by Wicksell and Lindahl, we also want to explore a group approach that allows for stronger policy judgments and a greater degree of coercion. Although there isn't a complete parallel, defining coercion over a group of individuals is similar to the use of the Hicks-Kaldor potential compensation criterion.[6]

[6] It is interesting to note that Becker (1983) has proposed a *positive* theory of political outcomes in a democratic

As regards the second distinction, coercion can be defined either by using utility or by approximating changes in utility levels by using levels of the public good, a method that follows Buchanan (1967) and Breton (1974, 1996) and that proves useful for working out examples.

The resulting four possibilities, each of which corresponds to the choice of a particular counterfactual, are summarized as coercion constraints in Table 1, where only the *individual-in-society* approach to the counterfactual is employed for illustrative purposes. Here we use K_j to denote the 'degree' of coercion applied to individual j (and, later, κ for the associated Lagrange multiplier) because the Greek word for coercion is *katanagasmos.* We note that in our formal analysis we assume that equalities apply and that solutions are interior with respect to all constraints, even though inequalities are used in the specification of the constraints given in the table.

[Table 1 here]

One should note that for all cases in the Table, the counterfactual level of utility and of the public good, as well as individualized tax-prices and the degree of coercion, must be simultaneously determined. This holds because tax-prices partly determine coercion while, in turn, the degree of coercion must be taken into account in deciding upon the tax system and its implied tax-prices.

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state that combines Hicks-Kaldor potential compensation with an understanding of *actual or existing* inequalities in political influence. If gainers from a policy action gain more than the loss to losers, they will, according to his argument, spend more and be more influential in the political process, unless there is some inequality in the distribution of political influence that favours the losers. Here we explore a normative theory that links the Hicks-Kaldor criterion with normatively desirable constraints on the extent to which gainers or losers *should be* coerced by the public sector.

3.1 Coercion defined by levels of the public good

Table 1 indicates that coercion can be approximated using the level of the public good, either on an individual or on an aggregate basis. The argument is illustrated in Figure 2. Because the marginal evaluation of the public good declines with the size of the public sector, the difference in utility in (1) above is monotonically related to the difference between the level of the public good in the counterfactual and that provided by the planner, $(G^* - G)$. Thus we can use the difference in public good levels as an index of coercion. This will be so regardless of whether the individual prefers a level of the public good (given his or her tax-price $\tau_j P$) that is lower or higher than that determined by the planner.

[Figure 2 here]

We shall treat people who want less of the public good in the counterfactual symmetrically with those who would like more. Citizens of the first type, illustrated in Figure 2, are losing utility because they would like a lower quantity of the good at the tax-price they face. Those of the second type also fail to get the desired amount, but gain from the fact that they would be willing to pay more for what they receive than what they are required to pay. Although one may argue that those of the first type are being 'coerced' in the popular sense of the word, we adopt a broader perspective that relates to the overall functioning of the system. All differences between what people would like and what they get are perceived as damaging and shall be recognized in choosing a fiscal system. If deviations in either direction become too large, the collective choice that is being represented here by the incorporation of coercion constraints loses its legitimacy. To allow for both cases, we may use the absolute value of the difference between hypothetical levels of G and planned levels, as shown in case 3 of Table 1. Case 4 in Table 1 is the analogue to case 2 where an aggregate definition of coercion is used.

4. Coercion-Constrained Optimal Linear Income Taxation with a Public Good

We now show how the introduction of coercion constraints alters the welfare analysis of a fiscal system in which a pure public good is financed with a linear income tax.[7] In the Appendix we demonstrate how the analysis by Ramsey (1927) of the structure of commodity taxation is amended by incorporation of a concern with coercion.

Assume that there are N individuals indexed by j, each maximizing utility defined over a private good X_j, leisure L_j and a public good G:

$$U_j = U_j(X_j, L_j, G), \qquad j = 1, \ldots, N. \tag{1}$$

Utility is maximized subject to the budget constraint

$$X_j = (1-t)W_j(1-L_j) + a \tag{2}$$

where W_j denotes the wage rate or ability and H_j denotes the supply of labour, with $L_j + H_j = 1$. Because the lump sum component 'a' does not vary across individuals, it provides a simple way of introducing the excess burden of taxation, and also of ruling out a Lindahl voluntary exchange equilibrium in which taxes are raised without any welfare loss.

Maximization of utility subject to the individual's budget constraint yields the usual condition $U_{jL}/U_{jX} = (1-t)W_j$, the final demand for the private good $X_j = X_j[(1-t)W_j, a, G]$, the labor supply $H_j = H_j[(1-t)W_j, a, G]$, and the indirect utility function $V_j = V_j[(1-t)W_j, a, G]$.[8]

4.1 Establishing the counterfactual

To establish the counterfactual, we consider the individual when he is free to choose the level of the public good G_j for a given tax share, assumed to be constant with respect to the level of

[7] With $a = 0$ the tax is proportional to income, with $a > 0$ it is regressive, and with $a < 0$ progressive. For comparability with the literature, we follow Sandmo's (1998) notation as far as possible here and below.

[8] Denoting the marginal utility of income by λ_j, the partial derivatives of utility with respect to the fiscal variables for person j are: $V_{jt} = -\lambda_j Y_j$, *with* $Y_j = W_j(1-L_j)$; $V_{ja} = -\lambda_j$ and $V_{jG} = U_{jG}$, and the marginal willingness to pay for the public good is $m_j = U_{jG} / U_{jX} = V_{jG}/\lambda_j$.

the public good. His optimization problem is to maximize

$$U_j = U_j(X_j\,,\, L_j,\, G^*_j\,), \tag{1'}$$

subject to the budget

$$X_j + \tau_j PG_j = W_j(1-L_j), \tag{2'}$$

where $\tau_j P$ is the tax-price per unit, with P the unit cost of the public good, and τ_j the tax share of person j, defined as the ratio of the tax paid by j to the total tax revenue, $\tau_j = T_j / \Sigma_j T_j$.

We note for later use that with the linear tax system $T_j = tY_j - a$, this tax share can be written as

$$\tau_j = \frac{t\,Y_j - a}{t\,\Sigma_j Y_j - Na}\,. \tag{3}$$

It should be pointed out that there are several ways to translate t into τ besides using (3). In our formulation, we assume that the average tax price implied by the tax system is also the one that applies to marginal changes in public services when viewed from the perspective of each individual.

Maximization of (1') subject to (2') yields the usual first order conditions $U_{jX} = \lambda^*_j$, $U_{jL} = \lambda^*_j W_j$, and $U_{jG} = \lambda^*_j \tau_j P$, where subscript X, L or G indicates a partial derivative with respect to that variable for person j. It also gives the indirect or *counterfactual* level of utility $V^*_j = V^*_j[W_j,\ \tau_j P]$, and the effect of a change in the tax share on utility $V^*_{j\tau} = -\lambda^*_j PG^*_j$. Again the (*) reflects the fact that the individual is considered to be choosing his or her most preferred level of G at the *given* tax-price.

4.2 Social welfare maximization under aggregate coercion

In choosing fiscal policy instruments, the coercion-constrained planner is assumed to maximize the sum of individual utility functions,

$$S = \sum_j V_j \tag{4}$$

subject to the budget constraint of the government

$$t\sum_j W_j H_j - Na = PG. \tag{5}$$

In addition, the planner faces one or more coercion constraints. To begin, we consider case 2 in Table 1 where coercion is defined using utility levels and aggregated across individuals. As we have pointed out earlier, this case is analogous to the use of the Hicks-Kaldor criterion in cost-benefit analysis.

Let κ denote the Lagrangean multiplier of the coercion constraint. Then incorporating the coercion constraint, the problem is to maximize

$$\mathcal{L} = \sum_j V_j + \mu [\, t\sum_j W_j H_j - Na - PG \,] + \kappa [\sum_j (V^*_j - V_j) - K]. \tag{6}$$

Before proceeding to explore the solution, it is useful to point out that this Lagrangean illustrates clearly that acknowledging coercion does not amount to simply placing added weight on the utility of some individuals in a social plan. In other words, the Lagrangean shows that redistribution and coercion are not equivalent concepts. In (6) it can be seen that a concern with coercion requires that weight be given to the *counterfactual* level of utility for each individual V^*_j. [9]

Differentiating (6) with respect to policy instruments t, a and G and using also the definition of $V^*_{j\tau}$ we have first order conditions:

$$(1-\kappa)\sum_j \lambda_j W_j H_j + \kappa \sum_j \lambda^*_j P G^*_j \, (\partial\tau_j/\partial t) = \mu [\sum_j W_j H_j + t\sum_j W_j (\partial H_j/\partial t)] \tag{7.1}$$

$$(1-\kappa)\sum_j \lambda_j - \kappa\sum_j \lambda^*_j PG^*_j \, (\partial\tau_j/\partial a) = \mu [\, N - t\sum_j W_j (\partial H_j/\partial a)] \tag{7.2}$$

$$(1-\kappa)\sum_j \lambda_j m_j = \mu [P - t\sum_j W_j (\partial H_j/\partial G)]. \tag{7.3}$$

where m_j is the marginal rate of substitution between public and private goods.

[9] One may also note that if there is only one person, or if everyone is identical, there will be no difference between V^* and V at an optimum, and any coercion constraint will be irrelevant. Coercion has no meaning in a single agent social planning model.

These equations feature two important new elements that are absent from traditional optimal taxation but that are always present in the analysis of collective goods coercion when one uses the *individual-in-society* approach to the counterfactual: (a) the translation of tax structure into the tax price - here shown as $(\partial\tau_j/\partial t)$ and $(\partial\tau_j/\partial a)$; and (b) the translation of the tax price into the demand for G - here shown as $\lambda^*_j P G^*_j$. These two elements must be present because coercion involves the difference between what people pay at given tax-prices and what they get in public services. These elements do *not* appear if one uses the *individual-as-dictator* approach, since in that formulation the individual picks a fiscal system directly from the set of solutions consistent with the government budget constraint.

Using the covariance formula $\sigma^2_{\lambda m} = (\sum_j \lambda_j m_j/N) - (\sum_j \lambda_j/N)(\sum_j m_j/N)$ and setting $\lambda = \sum_j \lambda_j/N$, the mean value of the marginal utility of income, (7.3) can be rewritten as

$$(1-\kappa)[N\sigma^2_{\lambda m} + \lambda\textstyle\sum_j m_j] = \mu\,[P - t\textstyle\sum_j W_j(\partial H_j/\partial G)].$$

Let $m = \sum_j m_j/N$ denote the mean value of the marginal rate of substitution between public and private goods, and let $\delta \equiv \sigma^2_{\lambda m}/\lambda m$ be the normalized covariance between the marginal rate of substitution and the marginal utility of income, reflecting the distributional characteristics of the public good. Using these definitions in the above equation and manipulating then yields

$$\left(\Sigma_j m_j\right)(1+\delta)(1-\kappa) = \frac{\mu}{\lambda}\left(P - t\Sigma_j W_j \frac{\partial H_j}{\partial G}\right). \qquad (8)$$

Equation (8) represents the condition for optimal provision of the public good when the planner cannot breach a coercion constraint. It is a generalization of the Samuelson condition as amended by Atkinson and Stern (1974) to allow for a concern with coercion. It turns out that the

form of this condition is unaffected by the choice of the counterfactual.[10]

The left-hand-side represents the social marginal benefit from public provision in the presence of the coercion constraint. It is the product of three terms: (a) the sum of the marginal rates of substitution between the private and the public good; (b) term $1+\delta$, which adjusts the sum of the marginal rates of substitution for the distributional characteristics of public provision – captured by the expression $\sigma^2_{\lambda m}/\lambda m$; and (c) the term $(1-\kappa)$, which reflects the effect of the coercion constraint.

The right-hand-side represents the social marginal cost of public provision. It is the product of two components, familiar from the standard social welfare maximization case: (a) the marginal valuation of government revenue, μ/λ (see Sandmo 1998 or Atkinson and Stern 1974); and (b) the net marginal rate of transformation of the public good, $P - t\sum_j W_j(\partial H_j/\partial G)$, which equals the unit cost of production of the public good adjusted for the effects of public provision on income and therefore on income taxation .

The standard rule of optimal public good provision can be derived as a special case when it is assumed that the planner does not observe a coercion constraint, so that $\kappa=0$. In this case

$$\left(\Sigma_j m_j\right)(1+\delta) = \frac{\mu}{\lambda}\left(P - t\Sigma_j W_j \frac{\partial H_j}{\partial G}\right). \qquad (8')$$

Comparing (8) with (8') we observe that since κ is positive, $\sum_j m_j$ must be larger to maintain the equality in (8), and thus G and t must be lower in the presence of a coercion constraint than in the standard planning solution. This holds true regardless of what definition of the counterfactual is adopted because, assuming that δ is positive, a declining marginal rate of substitution between

[10] To see that the *individual-as-dictator* counterfactual results in the same general formula (8), one solves for indirect utility under the assumption that the individual chooses t, a and G directly, and proceeds as before. Note that in this case, it is crucial to acknowledge that in the counterfactual, the partial derivatives of indirect utility with respect to fiscal parameters are zero since the individual chooses fiscal structure directly to maximize his or her welfare.

public and private goods for all individuals implies that those who are coerced less when G is increased gain less than do those who want a lower G when G is decreased.

The comparison also shows that the coercion-adjusted marginal cost of funds (MCF) appropriate for policy analysis is given by the term $\mu/[\lambda(1-\kappa)]$. It must be higher than when coercion is ignored as long as κ is positive, as we should expect. But note that the MCF remains relevant as an analytical concept.

To derive the optimal income tax rate in the presence of coercion, we proceed as follows. Multiplying (7.1) by (1/N) and (7.2) by $(\sum_j W_j H_j/N^2)$, subtracting the latter from the former[11], and using the Slutsky decomposition, $\partial H_j/\partial t = s_j - W_j H_j(\partial H_j/\partial a)$, yields

$$(1-\kappa)\sigma^2{}_{\lambda Y} = t\frac{\mu}{N}\left[\Sigma_j W_j s_j - \Sigma_j W_j H_j \frac{\partial H_j}{\partial a} + \left(\Sigma_j W_j \frac{\partial H_j}{\partial a}\right)\left(\frac{\Sigma_j W_j H_j}{N}\right)\right] - \frac{\kappa}{N}\Sigma_j \lambda^*{}_j PG^*{}_j\left(\frac{\partial \tau_j}{\partial t} + \frac{\partial \tau_j}{\partial a}\overline{Y}\right) \quad (9)$$

where $Y_j = H_j W_j$, and the negative covariance $\sigma^2{}_{Y\lambda}$ shows the relationship between the marginal utility of income and income from work and reflects the distributional effects of income taxation.[12] The term $\overline{WS} = \Sigma_j W_j s_j / N$ is the mean substitution effect of taxation on labor supply, which is also negative. The covariance term

$$\sigma^2{}_{Ya} = \frac{1}{N}\left[\Sigma_j W_j H_j \frac{\partial H_j}{\partial a} - \left(\Sigma_j W_j \frac{\partial H_j}{\partial a}\right)\left(\frac{\Sigma_j W_j H_j}{N}\right)\right]$$

shows the relationship between income and the income effect of taxation; it is non-negative when the effect of income on labor supply is small for those with high incomes.

The quantity $q_j = \partial \tau_j/\partial t + (\partial \tau_j/\partial a)\overline{Y}$ in (9) is the change in the tax share of j given by

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[11] Yielding as an intermediate step:

$$(1-\kappa)\left(\frac{\Sigma_j \lambda_j W_j H_j}{N} - \frac{\Sigma_j \lambda_j}{N}\frac{\Sigma_j W_j H_j}{N}\right) = t\frac{\mu}{N}\left[\left(\Sigma_j W_j \frac{\partial H_j}{\partial t}\right) + \left(\Sigma_j W_j \frac{\partial H_j}{\partial a}\right)\left(\frac{\Sigma_j W_j H_j}{N}\right)\right] - \frac{\kappa}{N}\Sigma_j \lambda^*{}_j PG^*{}_j\left(\frac{\partial \tau_j}{\partial t} + \frac{\partial \tau_j}{\partial a}\overline{Y}\right).$$

[12] $\sigma^2{}_{Y\lambda}$ is negative since the higher the level of income the lower is the marginal utility.

(3) when the tax rate and the lump-sum transfer both change. If we let $\psi_j \equiv V^*_{j\tau} = -\lambda^*_j PG^*_j$ be the marginal utility of the tax share, we can write $\Sigma_j \lambda^*_j PG^*_j [\partial\tau_j/\partial t + (\partial\tau_j/\partial a)\overline{Y}] = \Sigma_j \psi_j q_j$. Using the covariance formula, the right side of this last expression is $\Sigma_j \psi_j q_j = N\sigma^2_{\psi q} + N\overline{\psi}\,\overline{q}$, where $\overline{\psi}$ and $\overline{q}$ denote the mean values of ψ_j and q_j respectively. Here $\sigma^2_{\psi q}$ shows the relationship between the marginal utility of the tax share and the marginal tax share[13].

Also by differentiating the tax share, we obtain[14]

$$\Sigma_j \frac{\partial \tau_j}{\partial t} = \Sigma_j \frac{\partial \tau_j}{\partial a}\overline{Y} = 0, \tag{10}$$

and thus $\overline{q} = 0$. Substituting (10) into (9) and using the definition of $\sigma^2_{\psi q}$ then leads to the coercion-constrained optimal income tax rate:

$$t = \frac{(1-\kappa)\sigma^2_{\lambda Y} - \kappa\sigma^2_{\psi q}}{\mu\left(\overline{WS} - \sigma^2_{Ya}\right)}. \tag{11}$$

We see that the optimal tax rate is decreasing in the marginal utility of the coercion constraint κ. That is, the less social solidarity or tolerance for coercion, the lower the optimal income tax rate. Given that both $\sigma^2_{Y\lambda}$ and the denominator are negative, the optimal tax rate will be lower the larger is the algebraic value of $\sigma^2_{\psi q}$.

If we were to adopt the *individual-as-dictator* approach to the counterfactual, the optimal tax rate would no longer depend on $\sigma^2_{\psi q}$, since the tax share is not relevant to that definition of the

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[13] The value of $\sigma^2_{\psi q}$ depends on the size of the parameters of the utility function and is therefore an empirical matter. If tax payers who experience a large increase in their tax shares will also experience a significant fall in utility, $\sigma^2_{\psi q}$ will be negative.

[14] Differentiating τ_j in (3) with respect to the tax rate t and the lump-sum transfer a, and recognizing that a change in t and a affects the level of income,

$\frac{\partial\tau_j}{\partial t} = \frac{t^2(\overline{Y}Y_{jt} - Y_j\overline{Y}_t) + a(\overline{Y} - Y_j) + at(\overline{Y}_t - Y_{jt})}{N(t\overline{Y} - a)^2}$ and $\frac{\partial\tau_j}{\partial a} = \frac{t[t(Y_{ja}\overline{Y} - Y_j\overline{Y}_{ja}) + Y_j - \overline{Y} + a(Y_{ja} - \overline{Y}_a)]}{N(t\overline{Y} - a)^2}$ where $\overline{Y} = \Sigma_j Y_j / N$ is mean income.

Equation (10) follows from these expressions.

counterfactual.[15]

The standard linear optimal tax rate can be obtained as a special case of (11) when $\kappa = 0$:

$$t = \frac{\sigma^2_{\lambda Y}}{\mu\left(\overline{WS} - \sigma^2_{Ya}\right)} \ . \qquad (11')$$

By comparing (11) and (11') we see that with the *individual-in-society* counterfactual, the more general formulation of the optimal income tax rate features two new terms in comparison to the standard formula: (i) the marginal utility of the coercion constraint κ, and (ii) the covariance of marginal utility of the tax share and the marginal tax share $\sigma^2_{\psi q}$.

4.3 Individual coercion constraints

When the planner is constrained by how much he or she can coerce each individual taxpayer separately (as in case 1 of Table 1), the Lagrangean for the planning problem becomes

$$\mathcal{L} = \sum_j V_j + \mu\,[t\sum_j W_j H_j - Na - P_G G] + \sum_j \kappa_j\,(V^*_j - V_j - K_j). \qquad (12)$$

The resulting first order conditions

$$\sum_j (1-\kappa_j)\lambda_j W_j H_j + \sum_j \kappa_j \lambda^*_j P G^*_j (\partial\tau_j/\partial t) = \mu\,[\sum_j W_j H_j + t\sum_j W_j (\partial H_j/\partial t)] \qquad (12.1)$$

$$\sum_j (1-\kappa_j)\lambda_j - \sum_j \kappa_j \lambda^*_j P G^*_j\ (\partial\tau_j/\partial a) = \mu\,[N - t\sum_j W_j (\partial H_j/\partial a)] \qquad (12.2)$$

$$\sum_j (1-\kappa_j)\lambda_j m_j = \mu\,[P - t\sum_j W_j (\partial H_j/\partial G)] \qquad (12.3)$$

are more complicated than the ones encountered before because they feature three additional distributions: those of κ_j, λ_j, and m_j.

Using the same approach as with $s_{\lambda m} \equiv \sigma^2_{\lambda m}/\lambda m\ (\equiv\delta)$, which denotes the normalized covariance between the marginal rate of substitution and the marginal utility of income, we may define $s_{\kappa\lambda} \equiv \sigma^2_{\kappa\lambda}/\lambda\kappa$ as the normalized covariance between coercion and the marginal utility of

[15] That is, the term $\kappa\sigma^2_{\psi q}$ would not appear. As a result, the level of G under the two counterfactuals will differ even though the general form of the solution for G, equation (8), is the same.

income; $s_{\kappa m} \equiv \sigma^2_{\kappa m}/\kappa m$ as the normalized covariance between coercion and the marginal rate of substitution; and $s_{\kappa\lambda m} \equiv \sigma^2_{\kappa\lambda m}/\kappa\lambda m$ as the normalized covariance between coercion, the marginal utility of income and the marginal rate of substitution. Letting the sum of these normalized covariance terms be

$$\phi \equiv s_{\lambda m} + s_{\kappa\lambda} + s_{\kappa m} + s_{\kappa\lambda m}\,, \tag{13}$$

substituting into (12.3) and manipulating,[16] gives

$$\left(\sum_j m_j\right)\left[(1+\delta)(1-\kappa)-\kappa\phi\right] = \frac{\mu}{\lambda}\left(P_G - t\sum_j W_j \frac{\partial H_j}{\partial G}\right). \tag{14}$$

The left-hand-side of (14) again shows the marginal benefit from the public good. But now it is the product of the sum of marginal rates of substitution multiplied by the adjustment for the combined effect of the distributional characteristics of the public good *and* the effects of coercion. In the present case of individual coercion constraints, the adjustment for coercion contains two new elements, (i) the average effect of coercion captured by the term $(1+\delta)(1-\kappa)$, a term that also appears in the previous case of aggregate coercion, and (ii) the distributional characteristics of coercion captured by the term $\kappa\phi$, which corrects the aggregate term for the distributional characteristics of coercion. The right-hand-side of equation (14) is already familiar: it is the product of the marginal valuation of government revenue times the net marginal rate of transformation of the public good.

The solution in (14) differs from the solution in (11) where coercion is defined on an aggregate basis by the term $\kappa\phi$, reflecting the fact that not just aggregate coercion matters, but also

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[16] The left-hand-side of (12.3) is written as $\sum_j(1-\kappa_j)\lambda_j m_j = \sum_j \lambda_j m_j - \sum_j \kappa_j\lambda_j m_j$. Recall that κ, λ, and m are the means of κ_j, λ_j, and m_j respectively, $cov(\kappa_j\lambda_j) = (1/N)\sum_j(\lambda_j-\lambda)(m_j-m)$ and $cov(\kappa_j\lambda_j m_j) = (1/N)\sum_j(\kappa_j-\kappa)(\lambda_j-\lambda)(m_j-m)$. Manipulating, these covariances and using (12.3) yields as intermediate steps: $\sum_j\lambda_j m_j = N\sigma^2_{\lambda m}+\lambda\sum_j m_j$ and $\sum_j \kappa_j\lambda_j m_j = [\kappa\lambda\sum_j m_j + N(\kappa\sigma^2_{\lambda m}+\lambda\sigma^2_{\kappa m}+m\sigma^2_{\kappa\lambda}+\sigma^2_{\kappa\lambda m})]$.

its distribution. Now the benefit from public provision falls in the following cases: (i) If the rich (low λ) have more social solidarity (low κ), so that $\sigma^2_{\kappa\lambda} > 0$ and $s_{\kappa\lambda} > 0$; (ii) If those who value public goods less (low m) have more social solidarity (low κ), then $\sigma^2_{\kappa m} > 0$ and $s_{\kappa m} > 0$; and (iii) If the rich (low λ) also value public goods less (low m) and have more social solidarity (low κ), then $\sigma^2_{\kappa\lambda m} > 0$ and $s_{\kappa\lambda m} > 0$, because the previous two effects are compounded.

In these cases, G in (14), and the corresponding tax rate, will tend to be lower than when coercion is defined in an aggregate sense. That is, if all these conditions apply, the Hicks-Kaldor-like solution for a coercion-constrained optimum (11) will involve more spending than when coercion is defined on an individual basis. However, unlike the aggregate coercion case with positive δ, it could in principle go the other way, and it will be interesting to determine in practice which case is likely to apply.

In order to derive the optimal income tax rate under individual coercion constraints, we work in a manner similar to the one before. Multiplying (12.1) by (1/N) and (12.2) by $(\sum_j W_j H_j / N^2)$ and subtracting the latter from the former we have

$$\left(\frac{\Sigma_j \lambda_j W_j H_j}{N} - \frac{\Sigma_j \lambda_j}{N}\frac{\Sigma_j W_j H_j}{N}\right) - \left(\frac{\Sigma_j \kappa_j \lambda_j W_j H_j}{N} - \frac{\Sigma_j \kappa_j \lambda_j}{N}\frac{\Sigma_j W_j H_j}{N}\right) =$$

$$t\frac{\mu}{N}\left[\left(\Sigma_j W_j \frac{\partial H_j}{\partial t}\right) + \left(\Sigma_j W_j \frac{\partial H_j}{\partial a}\right)\left(\frac{\Sigma_j W_j H_j}{N}\right)\right] + \frac{1}{N}\Sigma_j \kappa_j (-\lambda^*_j PG^*_j)\left(\frac{\partial \tau_j}{\partial t} + \frac{\partial \tau_j}{\partial a}\bar{Y}\right). \qquad (15)$$

The left-hand-side involves the frequency distributions of three variables, the individual coercion constraint κ_j, the marginal utility of income λ_j, and income Y_j. Similarly, the right-hand-side features the individual coercion constraint κ_j, the marginal utility of the tax share $\psi_j = -\lambda^*_j PG^*_j$, and the marginal tax share, $q_j = [(\partial\tau_j/\partial t) + (\partial\tau_j/\partial a)\ \bar{Y}]$, as well as the effect of income taxation on labour supply.

Let $\sigma^2_{\kappa\lambda Y}$ = covariance of κ_j, λ_j and Y_j; $\sigma^2_{\kappa\psi q}$ = covariance of κ_j, ψ_j and q_j; $\sigma^2_{\kappa Y}$ = covariance of κ_j and Y_j; $\sigma^2_{\psi q}$ = covariance of ψ_j and κ_j; $\sigma^2_{\kappa q}$ = covariance of κ_j; and q_j; and let κ, λ, ψ and q be the mean values of κ_j, λ_j, ψ_j and q_j respectively. Then using these definitions and the Slutsky equation, we obtain the formula for the coercion-constrained optimal income tax rate:

$$t = \frac{(1-\kappa)\sigma^2_{\lambda Y} - \kappa\sigma^2_{\psi q} - \lambda\sigma^2_{\kappa Y} - \psi\sigma^2_{\kappa q} - \sigma^2_{\kappa\lambda Y} - \sigma^2_{\kappa\psi q}}{\mu\left(\overline{WS} - \sigma^2_{Ya}\right)}. \tag{16}$$

The optimal income tax rate depends as usual on the income distribution effect of taxation, captured by $\sigma^2_{\lambda Y}$, and the efficiency effect of taxation on labour (shown again by the denominator). In common with the case of aggregate coercion, it also depends on the relationship between the marginal utility of the tax share and the marginal tax share $\sigma^2_{\psi q}$.

In addition, the optimal tax rate depends on the distributional effects of coercion, as the remaining four covariance terms make clear. While we can again specify conditions under which the tax rate will tend to be lower than when coercion is defined on an aggregate basis[17], it should be noted that little is presently known about the sign or size of these covariances.

5. The Trade-off between Social Welfare and Coercion in Income Taxation

In the linear income tax example, there is an optimal level of welfare corresponding to each assumed level of coercion. In this section, we explore the trade-off between welfare and coercion that is implied by this relationship, once again adopting the *individual-in-society* perspective on the counterfactual. To allow a closed form solution for the trade-off, we assume that utility is Cobb-

[17] The tax rate will tend to be lower than when coercion is defined on an aggregate basis: (i) If the rich (high Y) value social solidarity more (low κ) so that $\sigma^2_{\kappa Y} < 0$; (ii) If taxpayers who experience a large change in their tax share (high q) have low social solidarity (high κ), so that $\sigma^2_{\kappa q} > 0$ (since ψ is negative); (iii) If the rich (high Y and low λ) have more social solidarity (low κ), in which case $\sigma^2_{\kappa\lambda Y} < 0$; and (iv) If $\sigma^2_{\kappa\psi q} < 0$, which occurs when (a) those who experience a small change in the tax share and suffer a large utility loss also have less solidarity, or (b) when those who experience a large change in the tax share and suffer a large utility loss also have more solidarity.

Douglas and use an aggregate definition of coercion based on the level of the public good (case 4 in Table 1). Later we also simplify further by assuming that individuals can be separated into two homogeneous groups.

Individuals are assumed to have different preferences for the private good, leisure and the public good so that $\alpha_j \neq \alpha_i$, $\beta_j \neq \beta_i$ and $\gamma_j \neq \gamma_i$, for $j \neq i$. The utility function of taxpayer j is

$$U_j = \alpha_j \log X_j + \beta_j \log L_j + \gamma_j \log G. \tag{17}$$

The tax system consists of a single proportional tax at rate t. The budget constraint of the *jth* taxpayer is then given by

$$X_j = (1-t)W_j(1-L_j). \tag{18}$$

Maximizing the utility subject to the budget constraint, we derive for later use the following expressions for the partial derivatives of income and indirect utility with respect to given fiscal instruments t and G:

$$Y_j = \alpha_j W_j / (\alpha_j + \beta_j); \quad Y_{jt} = 0; \quad Y_{jG} = 0; \; V_{jt} = -\alpha_j/(1-t); \; V_{jG} = \gamma_j/G; \quad j=1,\ldots,N. \tag{19}$$

To study coercion in what follows we must again determine what individuals would like to have at their given tax-prices in order to explicitly construct the counterfactual. To do so, we maximize utility subject to the budget constraint written as

$$W_j(1-L_j) = X_j + \tau_j P G_j\,, \tag{18'}$$

where the tax share is $\tau_j = tY_j/\Sigma_j tY_j$. Solving yields

$$X^*_j = \alpha_j W_j; \quad 1-L^*_j = 1-\beta_j; \quad Y^*_j = (1-\beta_j)W_j \quad \text{and} \quad G^*_j = \frac{\gamma_j \Sigma_j (1-\beta_j) W_j}{(1-\beta_j)P}. \tag{20}$$

Note that comparison of the equilibrium values of income implied by equations (19) and (20)

implies that $1-\beta_j > \alpha_j/(\alpha_j+\beta_j)$ and therefore that $Y^*_j > Y_j$; that is, income in the counterfactual would be higher than income when the individual does not receive his or her desired level of public services.

For later use it is also useful at this point to solve the traditional social planning problem of maximizing social welfare $\sum_j V_j$ subject to the government budget constraint $PG = t\sum_j W_j H_j$. To further simplify at this point, we divide the population into two homogeneous groups with taste parameters for the public good γ_1 and γ_2, and wage rates W_1 and W_2. The numbers of individuals in the two groups are N_1 and N_2, with $N=N_1+N_2$. Denoting the mean value of α_j and γ_j by $\bar{\alpha}$ and $\bar{\gamma}$, the socially optimal level (o) of the public good and tax rate are

$$G^o = \frac{N_1\gamma_1 + N_2\gamma_2}{N_1(\alpha_1+\gamma_1)+N_2(\alpha_2+\gamma_2)}\left(\frac{N_1\alpha_1 W_1}{\alpha_1+\beta_1}+\frac{N_2\alpha_2 W_2}{\alpha_2+\beta_2}\right)\frac{1}{P} \tag{21.1}$$

and

$$t^o = \frac{N_1\gamma_1 + N_2\gamma_2}{N_1(\alpha_1+\gamma_1)+N_2(\alpha_2+\gamma_2)} = \frac{\bar{\gamma}}{\bar{\alpha}+\bar{\gamma}} \quad . \tag{21.2}$$

5.1 The coercion implied by social planning, and the welfare-coercion trade-off

We now proceed by deriving the coercion-constrained social welfare optimum. The resulting expression for aggregate welfare as a function of coercion can then be used to derive the social welfare - coercion trade-off.

With the population divided into two groups, it follows from (20) that the corresponding demands for the public good in the counterfactual are

$$G^*_1 = \frac{\gamma_1[N_1W_1(1-\beta_1)+N_2W_2(1-\beta_2)]}{(1-\beta_1)P} \qquad \text{and}$$

$$G^*_2 = \frac{\gamma_2[N_1W_1(1-\beta_1)+N_2W_2(1-\beta_2)]}{(1-\beta_2)P}. \tag{20'}$$

Of course, only a single level of the public good G will be provided by the planner.

Assume that taxpayers in group 1 consume a quantity of the public good larger than the quantity that they would have chosen freely, that is, $G > G^*_1$, and that taxpayers in group 2 consume less than they would like, that is, $G < G^*_2$. Coercion must be limited, so $(G - G^*_1)P = K_1$ and $(G^*_2 - G)P = K_2$. Summing over the two groups, the aggregate coercion constraint is

$$N_1(G - G^*_1)P + N_2(G^*_2 - G)P = N_1K_1 + N_2K_2. \tag{22}$$

Setting $K = N_1K_1+N_2K_2$, substituting from (20'), and manipulating, implies that the coercion-optimal size of public output is[18]:

$$G^C = \frac{1}{(N_2 - N_1)P}\left[\left(\frac{N_2\gamma_2}{1-\beta_2} - \frac{N_1\gamma_1}{1-\beta_1}\right)\Sigma Y^* - K\right] \tag{23.1}$$

where $\Sigma Y^* = N_1Y^*_1+N_2Y^*_2$ and Y^*_1 and Y^*_2 are given from the relevant expression in (20)[19].

For G^C to be positive, it must be that $N_2 - N_1 > 0$ and $N_2\gamma_2/(1-\beta_2) - N_1\gamma_1/(1-\beta_1) > K/\Sigma Y^*$, or $N_2 - N_1 < 0$ and $N_2\gamma_2/(1-\beta_2) - N_1\gamma_1/(1-\beta_1) < K/\Sigma Y^*$. In what follows we assume that the former two inequalities hold. In addition we assume that $\gamma_2/\gamma_1 > (1-\beta_2)/(1-\beta_1)$, so that $(1-\beta_1)N_2\gamma_2 > (1-\beta_2)N_1\gamma_1$. Since we are exploring case 4 in Table 1, coercion only exists if the two groups are not balanced with respect to size and preference for the public good.

Then, substituting (23.1) into the budget constraint of the government (written to reflect the existence of the two groups),

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[18] In the present setting, the planner possesses only two instruments, the income tax rate, t, and the size of public provision, G. As the two depend on each other through the budget constraint, there is a single free instrument, whose value is found by solving the aggregate coercion constraint. In the more general case of more than two instruments, their values are found by maximizing the social welfare function subject to the budget constraint and the coercion constraint.

[19] In longer form, $G^C = \{[(1-\beta_1)N_2\gamma_2 - (1-\beta_2)N_1\gamma_1][N_1W_1(1-\beta_1)+N_2W_2(1-\beta_2)] - (1-\beta_1)(1-\beta_2)K\}/\{(1-\beta_1)(1-\beta_2)(N_2-N_1)P\}$

$$P\,G = t\,\Sigma Y, \text{ with } \Sigma Y = \left(\frac{N_1 \alpha_1 W_1}{\alpha_1 + \beta_1} + \frac{N_2 \alpha_2 W_2}{\alpha_2 + \beta_2} \right),$$

leads to the optimally coercive income tax rate

$$t^C = \left[\left(\frac{N_2 \gamma_2}{1-\beta_2} - \frac{N_1 \gamma_1}{1-\beta_1} \right) \frac{\Sigma Y^*}{\Sigma Y} - \frac{K}{\Sigma Y} \right] \frac{1}{N_2 - N_1} \tag{23.2}$$

It is immediately seen that the less coercion society tolerates, the smaller the income tax rate and the smaller the size of public provision. In addition, we see that the more intense are the preferences for the public good of group 2 (members of which all want more), the higher is the optimal tax rate, and that the opposite holds for group 1.[20]

It is now possible as an intermediate step to find the level of coercion implied by the standard social planning solution, K_{OT} , a matter of interest in its own right because *it is not zero*. Using (21.2) and (23.2), setting $t^C = t^O$ and solving for K yields the level of coercion for which the coercion-constrained, welfare maximizing planner would levy the same tax rate as the unconstrained planner:

$$K_{OT} = \left(\frac{N_2 \gamma_2}{1-\beta_2} - \frac{N_1 \gamma_1}{1-\beta_1} \right) \Sigma Y^* - (N_2 - N_1) \frac{\bar{\gamma}}{\bar{\alpha} + \bar{\gamma}} \Sigma Y\,, \tag{24}$$

where $\Sigma Y^* = N_1 W_1 (1-\beta_1) + N_2 W_2 (1-\beta_2)$ and $\Sigma Y = \left(\frac{N_1 \alpha_1 W_1}{\alpha_1 + \beta_1} + \frac{N_2 \alpha_2 W_2}{\alpha_2 + \beta_2} \right)$. Note that our assumptions that $N_2 > N_1$ and $\gamma_2/\gamma_1 > (1-\beta_2)/(1-\beta_1)$ along with the inequality $\Sigma Y^* > \Sigma Y$ ensure that $K_{OT} > 0$.

Putting equations (23) into the individual indirect utility function and summing over the

[20] For completeness, we also note that the level of public provision when coercion $K = 0$ is found using equations (23) with K = 0. We may call this a Lindahl-like solution, since taxation is still imposed by the planner rather than representing the result of voluntary exchange, implying that there will be an excess burden of taxation. Moreover, here coercion is zero in the aggregate (across groups) rather than zero for every person. Solving equations (23) when K = 0 is an example of the analysis of such Lindahl-like solutions first suggested by Buchanan (1964).

groups leads to what we shall call *'coercion-constrained social welfare', S(K),* the basis for drawing the welfare-coercion trade-off:

$$S(K) = A + (N_1\alpha_1 + N_2\alpha_2)\log(1 - t^C(K)) + N_1\alpha_1 \log W_1 + N_2\alpha_2 \log W_2 + (\gamma_1 N_1 + N_2\gamma_2)\log G^C \quad (25)$$

where $A = N_1[\alpha_1 \log \alpha_1 + \beta_1 \log \beta_1 - (\alpha_1 + \beta_1)\log(\alpha_1 + \beta_1)\ ] + N_2[\alpha_2 \log \alpha_2 + \beta_2 \log \beta_2 - (\alpha_2 + \beta_2)\log(\alpha_2 + \beta_2)]$.

Differentiating (25) with respect to K shows that the trade-off between S and K is concave with an inflection point at $K = K_{OT}$: [21]

$$\frac{dS}{dK} = \frac{N_1(\alpha_1 + \gamma_1) + N_2(\alpha_2 + \gamma_2)}{t^C(1 - t^C)(N_2 - N_1)^2(\Sigma Y)^2}(K_{OT} - K) > 0 \quad \text{for } K_{OT} > K \qquad (26.1)$$

and

$$\frac{d^2S}{dK^2} = -\frac{N_1(\alpha_1 + \gamma_1) + N_2(\alpha_2 + \gamma_2)}{t^C(1 - t^C)(N_2 - N_1)^2(\Sigma Y)^2} < 0 \qquad (26.2)$$

The trade-off is illustrated in Figure 3, where coercion-constrained welfare is shown on the vertical axis and the given degree of aggregate coercion is marked on the horizontal axis. The Appendix shows that the trade-off has the same general shape if the *individual-as-dictator* counterfactual is used to define coercion. Of course different assumptions about counterfactuals, group sizes and tastes will result in a different trade-off curve. But we stress that it is possible in principle to derive the trade-off using the methodology outlined above.

[Figure 3 here]

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[21] **Proof of equation (26.1):**

Differentiation of S(K) yields $\frac{dS}{dK} = \frac{-(N_1\alpha_1 + N_2\alpha_2)}{1 - t^C}\frac{dt^C}{dK} + \frac{N_1\gamma\alpha_1 + N_2\gamma_2}{G^C}\frac{dG^C}{dK}$.

Substituting from (23.1) and (23.2) for t^C and G^C and noting that upon differentiation,

$\frac{dt^C}{dK} = \frac{-1}{(N_1 - N_2)\Sigma Y}$ and $\frac{dG^C}{dK} = \frac{-1}{(N_1 - N_2)P}$, we have that

$$\frac{dS}{dK} = \frac{(N_1\alpha_1 + N_2\alpha_2)(1-\beta_1)(1-\beta_2)}{(1-\beta_1)(1-\beta_2)\Sigma Y(N_2 - N_1) - [(1-\beta_1)N_2\gamma_2 - (1-\beta_2)N_1\gamma_1)]\Sigma Y^* + (1-\beta_1)(1-\beta_2)K} - \frac{(N_1\gamma_1 + N_2\gamma_2)(1-\beta_1)(1-\beta_2)}{[(1-\beta_1)N_2\gamma_2 - (1-\beta_2)N_1\gamma_1)]\Sigma Y^* - (1-\beta_1)(1-\beta_2)K}.$$

Manipulating and using the definition of K^{OT} in (24), the numerator is written as $(1-\beta_1)^2(1-\beta_2)^2[N_1(\alpha_1 + \gamma_1) + N_2(\alpha_2 + \gamma_2)](K_{OT} - K)$. Using the definition of t^C in (23.1) and manipulating, the denominator is written as $t^C(1 - t^C)(N_2 - N_1)^2(1-\beta_1)^2(1-\beta_2)^2(\Sigma Y)^2$. Substituting back into *dS/dK* yields equation (26.1). **QED.**

The figure records that coercion-constrained social welfare is increasing in K for $K < K_{OT}$. This suggests that starting from low levels of coercion, the higher the coercion allowed, the easier it is for the planner to implement traditional social planning. The upward sloping part of the locus corresponds to what we call the 'consenting society'. Points in this region inside the curve represent combinations of coercion and welfare that are not Pareto-efficient. Coercion-constrained welfare reaches a maximum at $K = K_{OT}$, where the solutions for constrained and unconstrained planning are equal. Finally, coercion-constrained welfare is decreasing for values $K > K_{OT}$. For levels of coercion higher than K_{OT} the income tax rate falls below its Optimal Tax size, as seen from (23.2), depressing the level of social welfare. The downward sloping part of the trade-off may thus be appropriately labelled the 'masochistic society'.

If coercion is of vital concern, it is desirable for society to locate on the upward segment of the curve, and not at the peak of the trade-off. The analysis in this paper does not determine the degree of coercion that a society would consider optimal. Work on a framework to derive K represents a promising avenue for further research on public economics when coercion is acknowledged as an important element.[22]

The paper demonstrates that it is possible to conduct significant formal analysis of the structure of public policy even without knowing the value of K. In particular, one can delineate the trade-off and ask what policies are consistent with attainment of the coercion-welfare frontier. The previous analysis in section 4 of a fiscal system where a linear income tax is used to finance a public good provides an example of such work. A further illustration is presented in the Appendix, where we reformulate the Ramsey Rule (1927) for the structure of commodity

[22] The public choice literature contains possible suggestions about how to approach the choice of K. Although not worked out in a quantifiable manner, the analysis of Buchanan and Tullock (1962, chp. 6) of the optimal decision process may serve as a guide An alternative approach to determining K may rely on ideas from the contractarian literature.

taxation when coercion constraints are introduced. The work on the Ramsey rule demonstrates that tax rates in an extended framework depend not merely on own-price elasticities, but also on the distribution of tastes for the taxed goods, even when cross-elasticities are ignored. This holds true because the extent of coercion depends on the difference between what citizens receive and what they pay in taxes.

6. Conclusion

Although coercion is a central fact in the operation of the public sector, normative public economics based on the planning model has not made it an explicit element of the analysis. In this paper, we formally introduce coercion into normative analysis by adding constraints that limit allowable coercion caused by tax and expenditure programs. We focus on what we call "collective goods coercion", a problem that arises when citizens experience a mismatch between what they receive in public goods and services and what they pay in taxes.

We use a methodology similar to the one employed in optimal taxation, but the focus of the analysis is different. Optimal taxation attempts to determine the best public decision, given a consensus on the welfare function. We focus on the trade-off between social welfare and coercion given that a consensus is lacking about how the collectivity should conduct its affairs.

To make the concept of coercion operational, a counterfactual specifying what individuals regard as appropriate treatment by the public sector is required. We have formulated several alternative standards. One may define the ideal either in terms of individual or aggregate utility, or by using a convenient approximation that relies on a reference level of government expenditure. The aggregate definitions are analogous to the use of the Hicks-Kaldor criterion and impose a less severe constraint on decision making than those having an individual basis.

Coercion constraints have important and complex effects on a social plan. We work out these effects for a system that uses an optimal linear income tax to provide a public good and, in the Appendix, for the implementation of a simplified version of the Ramsey rule for commodity taxation. We also demonstrate significant implications for the marginal cost of funds (MCF), a central concept in public finance. In the case based on the aggregate definition of coercion, for example, the MCF must be amended to incorporate the value placed on relaxation of the coercion constraint. If this valuation is positive as expected, the adjusted MCF will be higher than the one determined in a framework that fails to account for public goods coercion.

A novel aspect of the analysis relates to the trade-off between social welfare as traditionally defined and coercion. Using a Cobb-Douglas formulation, we derive a trade-off function, as well as the degree of coercion implied by the unconstrained social plan. The analysis allows us to examine how to achieve the highest level of traditionally defined welfare for a given degree of coercion or, in other words, how to be coercion-efficient. The trade-off function between narrowly defined welfare and coercion is shown to be concave, at least in a simplified case, with the maximum representing the point of maximum welfare as traditionally defined. A society (or a planner acting on its behalf) will prefer to be on the upward-sloping part of the relationship, a locus of points corresponding to what we have termed “the consenting society”.

Extensions of the analysis are possible is several directions. One could, for example, explicitly account for the interaction of incentive compatibility and coercion constraints. Such interaction could occur in situations where the coercion of individuals in different income groups is relevant to decision making by those who may find it advantageous to mimic the behavior of others. In addition, coercion will have relevance for the structure of public expenditure, for the choice between income and commodity taxation and for comparative study of how alternative

tax structures affect the welfare-coercion trade-off.[23]

The trade-off analysis can also be used to investigate how coercion can be reduced at given levels of social welfare through institutional means. Work on the scope of the public sector suggests that the boundary between private and public sectors matters in this regard, and that the welfare-coercion frontier may be shifted favorably by removing certain types of economic activity from the public sphere. The trade-off function could be used to formalize this argument.

Public goods coercion also has relevance for the discussion of federalism. Following Tiebout (1956), the literature on optimal assignment in federations has been concerned with balancing the welfare gains from decentralization with the loss of efficiency from fiscal externalities that arise under decentralized decision making. One may expect decentralization to reduce coercion, but this relationship that has not yet been formally acknowledged or analyzed in the optimal assignment literature.[24]

Finally, we note that the welfare-coercion frontier also allows us to extend the analysis of collective choice in important ways. The concept provides a new basis for comparing political equilibria under alternative institutional arrangements or voting rules, and for the ranking of such equilibria with respect to the implied trade-off between welfare and coercion.

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[23] See, for example, Boadway and Marchand (1995) on incentive compatibility and public expenditure, and Hettich and Winer (1988, 1999) on the formation of tax structure. Compared to the existing literature, a new element in the work on tax structure will be preferences for public goods, because coercion depends in part on such preferences.

[24] For reviews of the literature, see Wildasin (2006) and Wilson (1999). Pennock (1959) analyzed the relationship between majority rule and federalism, arguing that decentralization increases the total number of citizens in a majority coalition. But while this suggests that decentralization reduces coercion, he did not measure coercion formally nor integrate efficiency into his argument.

Appendix

1. Optimally Coercive Commodity Taxation

A longstanding problem concerns the relationship between elasticities of demand and the structure of commodity taxation required to minimize the aggregate excess burden of taxation. We re-examine this problem, first solved by Ramsey (1927), when coercion matters.

To keep the analysis simple, we assume two homogeneous groups $j=1, 2$ consuming two commodities $i = 1, 2$, whose prices are P_1 and P_2. Individual consumption is denoted by X_{ji}. Each taxpayer pays ad-valorem commodity taxes t_1 and t_2 on each of two consumption goods. The tax revenue collected is returned lump sum to each individual in equal amounts, denoted by $R/2$. Taxpayer 1 is assumed to pay too much tax in comparison to what he receives and vice versa for taxpayer 2. This is a highly simplified version of case 4 in Table 1, where the *desired* size of tax payments is equal to $R/2$.

When coercion constraints bind, taxes must be set so that the overpayment made by taxpayer 1 does not exceed a given sum K_1 and the underpayment made by taxpayer 2 must not fall below a certain level K_2: thus,

$$t_1P_1X_{11} + t_2P_2X_{12} - (R/2) = K_1 \qquad \text{and} \qquad t_1P_1X_{21} + t_2P_2X_{22} - (R/2) = -K_2 .$$

The planner chooses t_1 and t_2 to minimize the excess burden of commodity taxation after securing revenue R, subject to the coercion constraints. Denoting the (absolute value of the) price elasticity of demand for i by e_i, assuming that it is identical across groups, and ignoring cross-price effects, the total excess burden of the tax is

$$B = (1/2)\ e_1t_1^2P_1(X_{11} + X_{21}) + (1/2)e_2t_2^2P_2(X_{21} + X_{22}),$$

and the budget constraint of the government is

$$t_1P_1(X_{11} + X_{21}) + t_2P_2(X_{12} + X_{22}) = R .$$

Using κ_1 and κ_2 to denote the relevant multipliers of the coercion constraints, the solution to the problem of minimizing B subject to the government budget restraint and the coercion constraints is in the usual way seen to be:

$$\frac{t_1}{t_2} = \frac{\mu + \kappa_1 w_{11} + \kappa_2 w_{21}}{\mu + \kappa_1 w_{12} + \kappa_2 w_{22}} \frac{e_2}{e_1}, \qquad \text{(A1)}$$

where $w_{ji}=X_{ji}/\Sigma_i X_{ji}$ is group j's share of the consumption of commodity j.

The standard inverse elasticity formula is obtained as a special case of (16) when $\kappa_1=\kappa_2=0$. Then $t_1/t_2 = e_2/e_1$ which implies that the more inelastic good must be taxed more heavily.

However, it is now clear from (A1) that this standard result is no longer valid when coercion is taken into account. The optimally coercive commodity tax rate now depends not only on the inverse of the demand elasticity, but also on the level of coercion tolerated by taxpayers, and this dependence may in turn reverse the standard conclusion and require that the more elastic good must be taxed more heavily. For example, assume $e_2 > e_1$ so that the Ramsey formula indicates lower taxation of the elastic good 2, but that the ratio

$$p = (\mu + \kappa_1 w_{11} + \kappa_2 w_{21}) / (\mu + \kappa_1 w_{12} + \kappa_2 w_{22}) \qquad \text{(A2)}$$

is lower than the elasticity ratio. Then for coercion-constrained efficiency it must be that $t_1 < t_2$ and the more elastic good 2 must now be taxed relatively more highly.

Although the actual size of the p-ratio is an empirical issue, one may expect that the higher the budget shares w_{12} and w_{22} (the more both people consume good 2), the more likely that the p-ratio is lower than the ratio of elasticities, and therefore that commodity 2 should be taxed more heavily, contrary to the simple Ramsey formula. Thus even when cross-elasticities are ignored, tax rates in the amended analysis now depend not merely on own-price elasticities, but also on the distribution of tastes for the taxed goods, since the extent of coercion depends on the difference between what citizens receive and what they pay in taxes.

The structure of commodity taxation and the nature of social solidarity

In order to study more carefully how the nature of the coercion constraints affects the structure of commodity taxation, we can differentiate the first order conditions for excess burden minimization with respect to t_1, t_2, κ_1 and κ_2. Solving the resulting system of four equations, we find that[25]

$$\frac{dt_1}{dK_1} = \frac{1}{P_1}\frac{X_{22}}{X_{11}X_{22} - X_{21}X_{21}}, \qquad \frac{dt_1}{dK_2} = \frac{1}{P_1}\frac{X_{12}}{X_{11}X_{22} - X_{21}X_{21}}$$

and

$$\frac{dt_2}{dK_1} = \frac{-1}{P_2}\frac{X_{21}}{X_{11}X_{22} - X_{21}X_{21}}, \qquad \frac{dt_2}{dK_2} = \frac{-1}{P_2}\frac{X_{11}}{X_{11}X_{22} - X_{21}X_{21}} \ .$$

The sign of these derivatives depends on the sign of the expression $(X_{11}X_{22} - X_{21}X_{12})$. Let us assume that group 1 is the intensive user of commodity 1. Thus, $X_{11}/X_{21} > X_{12}/X_{22}$, which implies that $X_{11}X_{22} - X_{21}X_{12} > 0$. We then have the following two results:

---

[25] The first order conditions are: $e_1t_1P_1X_1 = \mu P_1X_1 + \kappa_1P_1X_{11} + \kappa_2P_2X_{21}$; $e_2t_2P_2X_2 = \mu P_2X_2 + \kappa_1P_1X_{12} + \kappa_2P_2X_{22}$; $t_1P_1X_{11} + t_2P_2X_{12} - (R/2) = K_1$; $t_1P_1X_{21} + t_2P_2X_{22} - (R/2) = K_2$. Here except for t_1, t_2, κ_1 and κ_2 all parameters are assumed fixed. Differentiating these equations with respect to t_1, t_2, κ_1 and κ_2 yields: $e_1P_1X_1dt_1 + 0\times dt_2 - P_1X_{11}d\kappa_1 - P_2X_{21}d\kappa_2 = 0$; $0\times dt_1 + e_2P_2X_2dt_2 - P_2X_{12}d\kappa_1 - P_2X_{22}d\kappa_2 = 0$; $P_1X_{11}dt_1 + P_2X_{12}dt_2 + 0\times d\kappa_1 + 0\times d\kappa_2 = dK_1$; and, $P_1X_{21}dt_1 + P_2X_{22}dt_2 + 0\times d\kappa_1 + 0\times d\kappa_2 = -dK_2$.

$$\text{(i)}\ \frac{dt_1}{dK_1} > 0, \frac{dt_1}{dK_2} > 0 \quad \text{and} \quad \text{(ii)}\ \frac{dt_2}{dK_1} < 0, \ \frac{dt_2}{dK_2} < 0 \qquad \text{(A3)}$$

We consider only the first set of inequalities in (A3) since the second set are analogous. The derivatives show that when K_1 rises, which means that group 1 is coerced more, the optimal coercive tax rate on the commodity which it uses more intensively (commodity 1) rises too. And when K_2 rises, the tax rate on the commodity which the other group uses more intensively rises. The intuition for this last result is as follows: recall that group 2 gets more than they pay, and that coercion is defined symmetrically for both groups. Hence, when K_2 rises, group 2 gets on balance even more than before, which requires that additional taxes be efficiently raised from group 1.

2. Demonstration that the Welfare - Coercion Trade-off is Essentially Unaffected by the Choice of the Counterfactual

Here we adopt the *individual-as-dictator* counterfactual rather than the *individual-in-society* perspective, and show that the coercion - welfare trade-off is concave with an inflection point at the unconstrained welfare optimum.

We must first derive what an individual would choose if he or she were allowed to decide for everyone the levels of t and G which maximizes his or her own utility.

We maximize the utility function $U_j = \alpha_j logX_j + \beta_j logL_j + \gamma_j logG$, with respect to t and G subject to the government budget constraint $t\sum_j W_j [\alpha_j W_j/(\alpha_j + \beta_j)] = PG$. Solving yields $t_j^D = \frac{\gamma_j}{\alpha_j + \gamma_j}$ and $G_j^D = \frac{\gamma_j}{\alpha_j + \gamma_j} \frac{1}{P} \sum_j \frac{\alpha_j W_j}{1-\gamma_j}$.

With the population divided into two groups, as in Section 5, the corresponding demands for the public good are

$$G_1^D = \frac{\gamma_1}{\alpha_1 + \gamma_1} \frac{1}{P} \left(\frac{N_1 \alpha_1 W_1}{1-\gamma_1} + \frac{N_2 \alpha_2 W_2}{1-\gamma_2} \right) \quad \text{and} \quad G_2^D = \frac{\gamma_2}{\alpha_2 + \gamma_2} \frac{1}{P} \left(\frac{N_1 \alpha_1 W_1}{1-\gamma_1} + \frac{N_2 \alpha_2 W_2}{1-\gamma_2} \right).$$

Summing the coercion constraints over the two groups, the aggregate coercion constraint when it just becomes binding is again $N_1(G - G^*_1)P + N_2(G^*_2 - G)P = N_1K_1 + N_2K_2$. Setting $K = N_1K_1 + N_2K_2$, substituting from above, and manipulating, implies that the coercion-optimal size of G is

$$G^D = \left[\left(\frac{N_2\gamma_2}{\alpha_2 + \gamma_2} - \frac{N_1\gamma_1}{\alpha_1 + \gamma_1} \right) \Sigma Y - K \right] \frac{1}{(N_2 - N_1)P}, \text{ where } \Sigma Y = \frac{N_1\alpha_1 W_1}{1-\gamma_1} + \frac{N_2\alpha_2 W_2}{1-\gamma_2} \text{ denotes total income.}$$

Using the government budget constraint then leads to the optimally coercive income tax rate

$$t^D = \frac{1}{N_2 - N_1}\left(\frac{N_2\gamma_2(\alpha_1+\gamma_1) - N_1\gamma_1(\alpha_2+\gamma_2)}{(\alpha_1+\gamma_1)(\alpha_2+\gamma_2)} - \frac{K}{\Sigma Y}\right).$$

As in the text, setting $t^D = t^O$, the optimal solution for t under unconstrained social planning, and solving for K, then yields the level of coercion for which the coercion-constrained, welfare maximizing planner would levy the same tax rate as the unconstrained planner:

$$K_D = \frac{N_2 N_1(\alpha_1+\gamma_1+\alpha_2+\gamma_2)(\alpha_1\gamma_2 - \alpha_2\gamma_1)}{(\alpha_1+\gamma_1)(\alpha_2+\gamma_2)[N_1(\alpha_1+\gamma_1)+N_2(\alpha_2+\gamma_2)]}\Sigma Y\,.$$

To derive the trade-off following the text, we substitute G^D and t^D into the welfare function consisting of the sum of individual utilities. Differentiating this expression with respect to K shows that the trade-off between S and K is concave with an inflection point at $K = K_D$:

$$\frac{dS}{dK} = \frac{B'}{\Pi'}(K_D - K) > 0 \quad \text{for } K_D > K\,, \quad \text{and} \quad \frac{d^2S}{dK^2} = -\frac{\mathrm{B}'}{\Pi'} < 0\,,$$

where $\mathrm{B}' = (\alpha_1+\gamma_1)^2(\alpha_2+\gamma_2)^2[N_1(\alpha_1+\gamma_1)+N_2(\alpha_2+\gamma_2)] > 0$ and $\Pi' = t^D(1-t^D)(N_2-N_1)^2(\alpha_1+\gamma_1)^2(\alpha_2+\gamma_2)^2(\Sigma Y)^2 > 0$.

This again leads to equations similar to (25) in the text with respect to the shape of the implied trade-off, though of course its exact position in the plane will be different.

References

Alm, James, Gary H. McClelland and William D. Schulze (1992). "Why Do People Pay Taxes?". *Journal of Public Economics* 48(1), 21-38

Atkinson, Anthony B. and Nicholas H. Stern (1974). "Pigou, Taxation and Public Goods". *Review of Economic Studies* 41, 119 - 128.

Atkinson, Anthony, B. and Joseph Stiglitz (1980). *Lectures on Public Economics.* McGraw-Hill.

Baumol, William J. (2003). "Welfare Economics and the Theory of the State". In Charles Rowley and Freidrich Schneider (eds.). *The Encyclopaedia of Public Choice.* Kluwer, 610 - 613.

Becker, Gary (1983). "A Theory of Competition Among Pressure Groups for Political Influence". *Quarterly Journal of Economics* 98, 371-400.

Boadway, Robin and Maurice Marchand (1995). "The Use of Public Expenditures for Redistributive Purposes". *Oxford Economic Papers*, New Series, 47 (1), 45-59.

Breton, Albert (1974). *The Economic Theory of Representative Government*. Chicago: Aldine.

Breton, Albert. (1996). *Competitive Governments: An Economic Theory of Politics and Public Finance*. Cambridge University Press.

Buchanan, James M. (1964). "Fiscal Institutions and Efficiency in Collective Outlay". *American Economic Review, Papers and Proceedings*, May, 227 - 235.

Buchanan, James M. (1968). *The Demand and Supply of Public Goods*. Rand McNally

Buchanan, James M. and Roger Congleton (1998). *Politics by Principle, Not Interest*. Cambridge University Press.

Buchanan, James M. and Gordon Tullock (1962). *The Calculus of Consent. Logical Foundations of Constitutional Democracy*. University of Michigan Press.

Clower, Robert (1967). "A Reconsideration of the Microfoundations of Monetary Theory", *Western Economic Journal* 6, 1 - 8.

Dalton, Thomas (1977). "Citizen Ignorance and Political Activity". *Public Choice, 85 - 99.*

Escarrez, D. R. (1967) "Wicksell and Lindahl: Theories of Public Expenditure and Tax Justice Reconsidered". *National Tax Journal* 20, 137-148

Hettich, Walter and Stanley L. Winer (1988). "Economic and Political Foundations of Tax Structure," *American Economic Review* 78(4), 1988, 701-712.

Hettich, Walter and Stanley L. Winer (1999). *Democratic Choice and Taxation: A Theoretical and Empirical Investigation*. Cambridge University Press.

Levy, Margaret (1997). *Consent, Dissent, and Patriotism*. Cambridge University Press.

Lindahl, Eric (1919). "Just Taxation, A Positive Solution". in Richard Musgrave and Alan Peacock (1958). *Classics in the Theory of Public Finance*. Macmillan, 168-176.

Mirrlees, James A. (1971). "An Exploration in the Theory of Optimum Income Taxation," *Review of Economic Studies*, 38, 175-208.

Mueller, Dennis (2003). *Public Choice III*. Cambridge University Press.

Patinkin, Don (1965). *Money, Interest and Prices: An Integration of Monetary and Value Theory*. 2nd Edition. Harper and Row.

Pennock, Roland (1959). "Federal and Unitary Government - Disharmony and Frustration". *Behavioral Science* 4, 147-57

Perroni, Carlo and Kimberley Scharf (2003). "Viable Tax Constitutions". Unpublished, CEPR Working Paper DP 4210, University of Warwick.

Ramsey, Frank P. (1927)."A Contribution to the Theory of Taxation". *Economic Journal* 37,47-61.

Sandmo, Agnar (1984). "Some Insights from the New Theory of Public Finance," *Empirica* 2, 111-124.

Sandmo, Agnar (1998). "Redistribution and the Marginal Cost of Public Funds" *Journal of Public Economics* 70, 365-382.

Tiebout, Charles (1956). "A Pure Theory of Local expenditures". *Journal of Political Economy* 64, 416 - 424.

Usher, Dan (1981). *The Economic Prerequisites of Democracy*. Routledge.

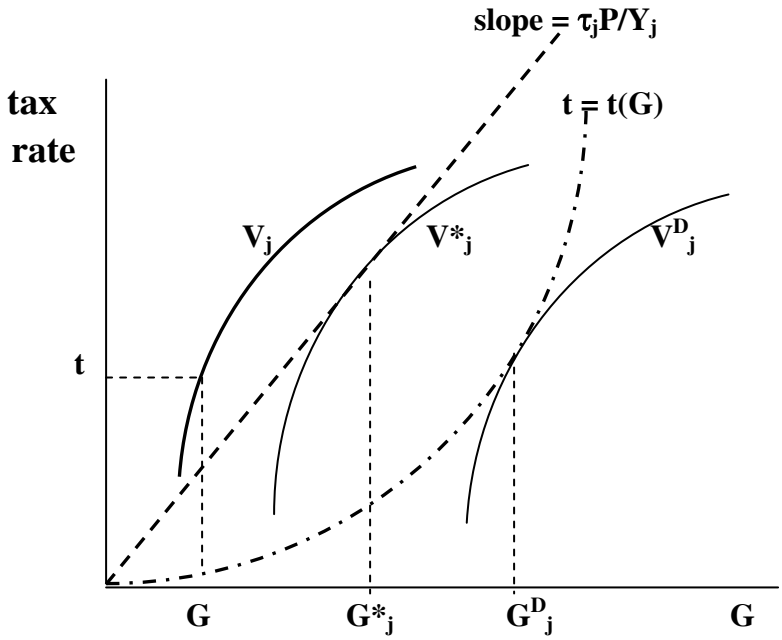
Usher, Dan (1993). *The Welfare Economics of Markets, Voting and Predation*. University of Michigan Press.

Wicksell, Knut (1896). "A New Principle of Just Taxation". in R. Musgrave and A. Peacock (1958). *Classics in the Theory of Public Finance*. Macmillan, 72-118.

Wildasin, David (2006). "Fiscal Competition", in Barry R. Weingast and Donald A. Wittman (eds.). *The Oxford Handbook of Political Economy*, 502 - 520.

Wilson, John (1999). "Theories of Tax Competition". *National Tax Journal* 52, 269 - 304.
Wiseman, Jack (1989). *Cost, Choice and Political Economy*. Edward Elgar.

Figure 1
Coercion and the Choice of Counterfactual



Legend:

t: actual tax rate paid

t = t(G): all feasible combinations of the income tax rate and the level of the public good

τ_j P G^*_j /Y_j : the implied income tax rate tax at which the *individual-in-society* is assumed to be able to quantity-adjust the level of the public good, given his tax-share τ_j, the price of the public good P and income Y_j

G: actual level of the public good

G^D_j : level of the public good which maximizes the utility of j when he chooses G and t to maximize his utility as if he were a dictator

V^D_j : level of utility attained when j consumes G^D; this is the counterfactual with the *individual-as-dictator*

V^D_j - V_j: coercion when the *individual-as-dictator* counterfactual is adopted

G^*_j : level of the public good that the individual would like the community to provide at his given tax price

V^*_j : maximum desired utility at the individual's given tax price if that person could quantity- adjust the level of the public good

V^*_j - V: coercion when the *individual-in-society* counterfactual is adopted.

Note: The indifference curves in the (*t, G*) space are constructed from the individual utility function $U = U(X,G)$, where X is the sole private good, and the individual budget constraint $X=Y(1- t)$. Substituting the latter into the utility function and totally differentiating we obtain the equation for the slope of the indifference curves in (t, G) space $\frac{dt}{dG} = \frac{\frac{\partial U}{\partial G}}{\frac{\partial U}{\partial X} Y} = (MRS_{GX}) \div Y > 0$

Table 1
Definitions of Coercion
(*Individual-in-society* counterfactual)

	Type of coercion constraint		
Coercion defined With respect to:	**Counterfactual**	***Individual (j = 1...N)***	***Aggregate***
Utility	*Utility when the public good level is what the individual wants at the tax-price determined by the social plan*	***Case 1:*** $(V^*_j - V_j) \leq K_j$	***Case 2:*** $\sum_j (V^*_j - V_j) \leq K$
Level of public good	*Desired level of the public good compared to its supply in the social plan*	***Case 3:*** $\lvert P(G^*_j - G)\rvert \leq K_j$	***Case 4:*** $n_1 P \sum_j (G - G^*_j) +$ $n_2 P \sum_j (G^*_j - G) \leq K$

Legend:

G^*_j = level of the public good that the individual would like the community to provide at his given tax price
G = actual level of the public good provided
K_j = the degree of coercion for citizen j. We note that the Greek word for coercion is *katanagasmos*.
K *(unsubscripted)* = an aggregate level of coercion
n_i = the number of taxpayer/citizens of type i
P = marginal cost of the public good (assumed constant)
V^*_j = maximum desired utility at the individual's given tax price if that person could determine the level of the public good.
V_j = actual level of utility

Figure 2
Coercion Measured by the Level of the Public Good, When $G^*_j < G$

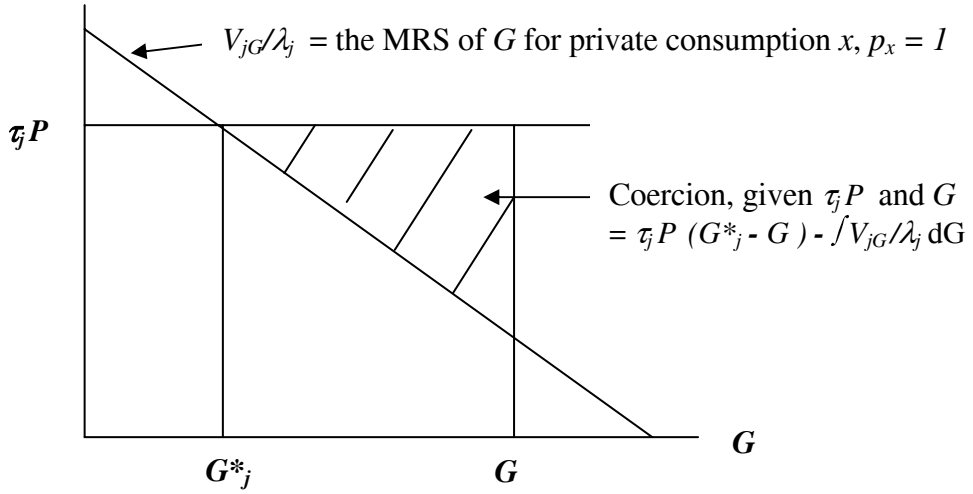
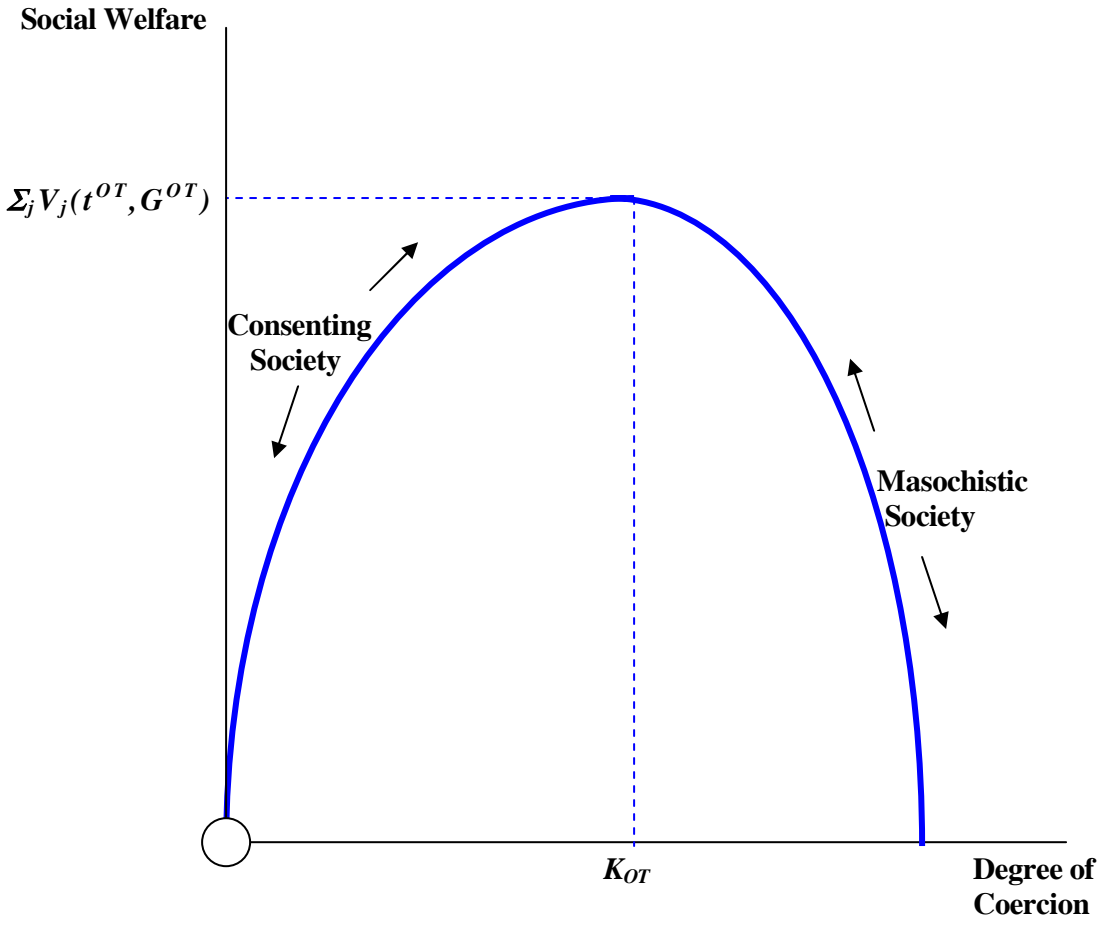


Figure 3
The Coercion - Welfare Frontier



$\Sigma_j V_j(t^{OT},G^{OT})$: Social welfare with the Optimal Tax solution.

Note: The circle at the origin indicates that if K=0 is feasible, it will not be a solution with zero social welfare.