Optimal Simple Monetary Policy Rules in a Small Open Economy with Exchange Rate Imperfections

by

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April 2008

Abstract

The paper addresses whether or not the exchange rate or some other dimension of the external side of the economy should form an integral part of the monetary rule for a small open economy (SOE) in which the central bank faces data deficiencies. Under a number of information scenarios, the model's simulations suggest that some reflection of the external environment facing the SOE—either the real exchange rate gap and/or the law of one price gap—is needed to improve monetary policy performance. When the money rule includes both interest rate smoothing and the real exchange rate (or law of one price gap), the relative welfare gain from their inclusion increases as the monetary authorities loses access to more current and reliable information.

Key words: New Keynesian small open economy model, exchange rate pass through, optimal simple money rules, stochastic general equilibrium model.

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* We would like to thank Ehsan Choudhri, Lilia Karnizova, Fanny Demers, Michel Demers, Hashmat Khan, Charles Freedman, Jiankang Zhang for providing advise and constructive criticism. Comments are welcomed.
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1. Introduction

A large number of closed economy studies show that simple feedback money rules, especially of the Taylor type, work well in simulated economic models to deliver low inflation rates and output stability in the presence of monetary and real shocks. Similarly, many empirical studies report that Taylor policy specifications can be used to describe the actual behavior of central banks in several countries.[1]

In the small open economy literature, a number of similar studies of monetary policy rules have been undertaken, but a consensus conclusion has not yet been reached. That is, the literature has not yet resolved whether the Taylor rule is sufficient or whether the exchange rate should form part of the policy rule. Authors advocating external considerations include Ball (1999) who uses a backward-looking open economy model with sticky prices to argue that a money rule that includes the real exchange rate will perform better than either simple inflation targeting or the use of the Taylor rule, unless the latter is modified in important ways.[2] In a dynamic stochastic forward-looking general equilibrium model with the incomplete exchange rate pass-through, Monacelli (2003) finds that optimal commitment entails the smoothing of deviation from the law of one price and hence requires more stable nominal and real exchange rates. Using a two-country model, Benigno and Benigno (2000) and Weerapana (2000) report substantial (world) welfare improvement when an exchange rate term is incorporated into the policy rule. Similarly, Smets and Wouters (2002) argue that an exclusive focus on the stabilization of domestic price inflation is no longer optimal when import prices are sticky and exchange rate pass-through is gradual and Malik (2005) demonstrates that the welfare-enhancing policy implies a "dirty floating" under domestic inflation targeting when incomplete and imperfect asset markets are introduced to the model.

On the other hand, writers such as Clarida, Gali, and Gertler (2001), Aoki (2001), Gali and Monacelli (2002) find that the policy problem in a small open economy in which the households can share consumption risk internationally and exchange rate pass-through is complete is essentially identical to the policy problem faced by a closed economy. Hence the closed economy rule remains optimal if combined with flexible exchange rates. A similar result is obtained by Clarida, Gali, and Gertler (2002) when the analysis is extended to a two-

[1] See, for example, Rotemberg and Woodford (1999), Rudebusch and Svensson (1999), Clarida, Gali and Gertler (1998, 2000).
[2] For robustness, Ball (1999) considers the six rules, distinguished mainly by the size of their reaction coefficients and lagged interest rate terms.

country setting. When exchange rate pass-through is incomplete, Dib (2003) shows that the optimal policy problem for a small open economy is isomorphic to that in a closed economy. Kollmann (2002), Batini, Harrison, and Millard (2003), Leitemo and Soderstrom (2005), and Adolfson (2007) all find that welfare performance is improved by incorporating an exchange rate term into the policy rule but only marginally so.

Standing somewhat in the middle, Taylor (1999) uses a multi-country model with complete exchange rate pass-through to simulate a policy rule for the European Central Bank that includes in the interest rate rule the exchange rate as well as output and inflation, and finds that policy outcomes are mixed. In some countries (such as France and Italy), including the exchange rate leads to better performance, but in others (such as Germany) the result is poorer. In closely related work, Devereux and Engel (2003) show that the impact of alternative exchange rate regimes on the policy performance is related to the pricing strategy used to set the prices of import goods. In particular, when prices are pre-set in the producer's currency (denoted producer currency pricing, PCP), floating exchange rates are optimal in the presence of country specific real shocks. However, when prices are pre-set in the consumer's currency (denoted local currency pricing, LCP), there is no benefit to having exchange rate flexibility. In a small emerging economy, Choudhri (2005) finds that the relative performance of the alternative rules depends on the social welfare criterion used for assessing different policy rules and on the type of shock that the economy experiences.[3]

In this paper we re-examine the role of external factors in simple money rules by evaluating the relative performance of a series of different simple monetary policy rules applied to a small open economy when the monetary authority lacks current information. The objective is to discover whether there exists a set of regularities that describe the optimal properties of a simple optimal money rule. The small open economy developed here is distinctive in that as well as incorporating Calvo pricing with indexation (to induce sticky domestic prices), the model incorporates incomplete exchange rate pass-through (because of quadratic costs of import price adjustment) and also assumes that asset markets are incomplete. With a domestic economy of this type and facing a number of domestic and external shocks, the economy is assumed to have a welfare maximizing central bank that can choose across a number of potential money rules for interest rate setting. We examine whether the ranking of these choices change as the central bank loses information.

Five simple rules are evaluated with and without interest rate smoothing. Rule 1 is our most comprehensive money rule, corresponding to the case where the monetary authority

[3] Choudhri argues that the loss function used to evaluate different monetary policy rules should also include exchange rate variability if there is fear of a floating exchange rate within the emerging economy. Variability of the interest rate may also be of concern, especially if financial institutions are vulnerable. Traditional loss functions typically do not include these terms.

sets the short-term interest rate in response to changes in the inflation rate, the real GDP output gap, the real exchange rate gap, and the law of one price gap. This we call the benchmark rule (B). Dropping the law of one price gap from Rule 1 gives us Rule 2, called the real exchange rate rule (ER). For comparative purposes, we replace the real exchange rate term in Rule 2 with the law of one price gap. This becomes Rule 3, the exchange rate pass-through rule (PT). Rule 4 has no openness component and corresponds to the Taylor rule (TR). Rule 5 further supposes that the monetary policy authority chooses to focus exclusively on inflation. This we call inflation targeting (IT). [4]

Four information scenarios are considered for the central bank. The first assumes that the central bank has access to "full information". In particular it is assumed that the central bank can observe all relevant contemporaneous variables (such as GDP and the price index), can calculate all necessary unobservable variables (such as the Wicksellian natural interest rate and the natural level of output). This serves as our benchmark full information case. Next we assume that while the central bank can observe the current value of all observable variables, it cannot acquire enough information to compute accurately the important unobservable variables and hence must use steady state values instead. In the third case, the central bank can acquire information on all variables but only with a one period lag. The final case also assumes that all observable variables can be known accurately only with a lag, but that again this information is insufficient to allow the monetary authority to calculate the relevant unobservable variables. Instead the monetary authority adopts the steady state values. These different information cases are used to analyze if and how the optimal simple monetary policy changes with the withdrawal of accurate information.

The remainder of the paper is organized as follows. Section 2 sets up the small open economy model. The loss function employed to evaluate the alternative simple policy rules is derived in Section 3 and Section 4 explains the calibration of the structural model. Optimal simple policy rules are investigated in section 5 and the conclusions are summarized in the final section.

2. A small open economy model

We suppose that the world economy consists of two parts: a home economy and a foreign one, the latter called the rest of the world. Compared to the rest of the world, the home economy is small so that developments there have little impact on the rest of the world. This implies that policy makers deciding on domestic monetary policy can take foreign variables,

[4] The terminology used here differs somewhat from that used by Svensson (2000). Svensson links "targeting" to the monetary authority's objective. Hence when stabilizing inflation around the inflation rate target is the only objective for monetary policy, Svensson calls this strict inflation targeting. When there are additional objectives for monetary policy, for instance stabilizing output as well, monetary policy is called flexible inflation targeting.

such as foreign output, prices, inflation rates, and interest rates as given or as exogenous stochastic variables.

The model used to analyze monetary policy in this context is a variant of the dynamic New Keynesian model applied to a small open economy such as Canada (see Gali and Monacelli, 2002). The model consists of a representative household who purchases both home produced and imported goods for consumption from its supply of specialized labor and the profits from its ownership share of domestic firms. Households are also assumed to be able to reallocate consumption over time by accessing a one-period risk-free (non-contingent) domestic bond and a risky foreign bond. The foreign bond is assumed to be denominated in the foreign currency and traded freely internationally subject to a risk premium which depends on both the net foreign asset position of domestic economy and a time-varying shock to the risk premium (Adolfson, 2005).

The model assumes a large number of identical firms that hire specialized labor to produce differentiated domestic goods that are used for both domestic consumption and export. For simplicity we assume that there is no capital accumulation and there are no intermediate goods. The labor market is assumed to be competitive with flexible wages but each domestic producer is assumed to be a monopolistic competitor setting prices by using Calvo's (1983) pricing specification. We assume that there are no transportation costs or trade barriers between the small open economy and the rest of the world so that the law of one price holds when imported goods cross the domestic border. However, in setting the domestic currency price for these differentiated goods, importers face quadratic adjustment costs that will generate deviations from the law of one price in the short run. In the literature this is described as incomplete exchange rate pass-through. For simplicity we assume that the law of one price holds for domestic exporters.

In our setup, the monetary authority is assumed to maximize community welfare by choosing as its policy instrument the short-term interest rate. The policy instrument is used to correct distortions arising in the model from stochastic real shocks impacting on the model in the presence of sticky prices. The secondary distortion that results from the presence of monopolistic competition in the model is dealt with separately through an optimal output subsidy. Aside from these tax/transfer activities, government behavior is not modeled explicitly.

2.1 The Representative Household

The small open economy consists of a continuum of households indexed by $i \in [0,1]$. The representative household seeks to maximize the expected value of the discounted sum of

time separable utilities subject to an intertemporal budget constraint. Suppose then that total expected utility can be denoted as

$$U_0 = E_0 \sum_{t=0}^{\infty} \beta^t u\left(C_{t,} \frac{M_t}{P_t}, N_t, \chi_{1t} \right), \qquad (1)$$

where $\beta \in (0,1)$ is the subjective discount factor, C_t, denotes a composite bundle of consumption goods, M_t / P_t is the level of real money balances held by the household, and N_t is the proportion of household time devoted to the labor market (so that $1 - N_t$ is the proportion of time enjoyed as leisure by the representative household). Following Woodford (2003), we assume that each of the differentiated goods is produced with a specialized type of labor and that the representative household supplies each type of specialized labor. In this case, $N_t = \int_0^1 N_t(i)di$, where $N_t(i)$ is the quantity of labor of type i supplied by the household.[5]

While the household derives utility from consumption, holding real money balances, and taking leisure, the presence of an exogenous disturbance χ_{1t} in the utility function implies that household utility is subject to a stochastic taste shock. This is given more specific form through the per-period utility function

$$u\left(C_{t,} \frac{M_t}{P_t}, N_t, \chi_{1t} \right) = \frac{\chi_{1t}}{1-\sigma} C_t^{1-\sigma} + \frac{1}{1-\varsigma}\left(\frac{M_t}{P_t}\right)^{1-\varsigma} - \frac{1}{1+\mu}\int_0^1 \left(N_t(i)\right)^{1+\mu} di . \qquad (2)$$

Here $1/\sigma$ denotes the intertemporal elasticity of substitution in consumption, $1/\varsigma$ is the interest elasticity of money demand. μ is the elasticity of marginal disutility with respect to labor supply[6]. The shock to the utility function makes the consumption weight in the utility function stochastic. Hence a change in χ_{1t} represents a preference shock to consumption, where χ_{1t} is assumed to follow the first-order autoregressive process given by

$$\ln \chi_{1t} = \rho_1 \ln \chi_{1,t-1} + \varepsilon_{1t},$$

and where $0 < \rho_1 < 1$ and the error term, ε_{1t}, is normally distributed with zero mean and standard deviation σ_1.

Turning next to the household's budget constraint, the representative household is viewed as entering period t with a set of financial assets. These include its claim on the profits arising in domestic firms (including imports), its initial nominal money holdings, and a portfolio consisting of its initial holdings of a risk free domestic bond and a foreign bond

[5] This implies that the import sector uses no labor, merely re-pricing goods imported from abroad.
[6] Note that $1/\mu$ denotes the elasticity of labor supply with respect to real wage.

denominated in units of the foreign country's currency. We assume that only the foreign bond can be traded internationally. Because the household cannot insure himself from all risks financial asset markets are incomplete and the domestic household is assumed to have to pay a premium on its foreign borrowing that depends directly on the size of its net foreign asset position, NFA_t. It then follows that because the household can accumulate or decumulate foreign assets in any period, differences can arise between the income earned and the level of consumption expenditures in period t. Hence in real terms, the representative household's budget constraint can be written as,

$$\begin{aligned} &C_t + \frac{M_t}{P_t} + \frac{B_t}{P_t R_t} + \frac{\varepsilon_t B_t^*}{P_t R_t^* \exp(-bNFA_t + \ln \chi_{2t})} + \frac{T_t}{P_t} = \\ &\frac{1}{P_t}\int_0^1 N_t(i)W_t(i)di + \frac{1}{P_t}\int_0^1 D_t(i)di + \frac{1}{P_t}\int_0^1 D_t(z)dz + \frac{M_{t-1}}{P_t} + \frac{B_{t-1}}{P_t} + \frac{\varepsilon_t B_{t-1}^*}{P_t} \end{aligned} \tag{3}$$

where M_t is the level of money holdings chosen for the end of period t. B_t and B_t^* are the country specific values in period $t+1$ dollars of one period domestic and foreign bonds chosen to be held at the end of period t. R_t and R_t^* denote, respectively, the gross nominal domestic and the risk free foreign (world) interest rate arising between t and $t+1$. Following Benigno (2001) and Adolfson (2005), we define $\exp(-bNFA_t + \ln \chi_{2t})$ as the risk premium that the representative household must pay to borrow on world markets, a premium that increases in the size of the net foreign debt held by the representative household, and b is a constant that governs the size of the risk premium.[7] Because that premium is subject to a period specific shock, χ_{2t}, the time dependent risk premium will produce temporary departures from uncovered interest parity in the short run. Here $NFA_t = \varepsilon_t B_t^* / P_t$ is the domestic value of net foreign assets held by the household, ε_t is the nominal exchange rate, defined as the price of foreign currency (in terms of domestic currency), and χ_{2t} is the shock to the risk premium. Analogous to χ_{1t}, χ_{2t} is defined as

$$\ln \chi_{2t} = \rho_2 \ln \chi_{2,t-1} + \varepsilon_{2t},$$

where $\rho_2 < 0$ and ε_{2t} is white noise.

In (3) the representative household is modeled as receiving a nominal wage rate for each unit of type i labor provided, $W_t(i)$. The household also owns a representative share of all firms producing domestic goods so that the nominal value of dividend income (profits)

[7] Having the risk premium depend on the level of net foreign borrowing allows the model to equate differences that may arise between the household's subjective rate of time preference and the risk free world interest rate.

received from firm i is equal to $D_t(i)$ in period t. The representative household is also assumed to own a representative share of the profits (losses) made on each differentiated import good, z, profits consisting of $\int_0^1 D_t(z)dz / P_t$ in period t. Finally, the household is subject to a lump-sum tax levied by the government T_t at the start of period t just sufficient to over the output subsidies made by the government (to induce the efficient level of domestic and import output).

Finally, to simplify the analysis, we assume that there is no other government spending on final goods so that any seigniorage on money creation will be rebated to households in the form of a lump-sum transfer, T, and all output subsidies paid to offset firm markups are funded through lump-sum taxes. Hence in our economy, the government budget constraint is, $\tau P_t Y_t + \tau_M P_{F,t} C_{F,t} = M_t - M_{t-1} + T_t$, where τ and τ_M are subsidy rates motivated in greater detail later.[8] Thus $\tau P_t Y_t$ denotes the subsidy paid by the government to domestic producers in period t; $\tau_M P_{F,t} C_{F,t}$ denotes the subsidy paid by the government to importers; and $M_t - M_{t-1}$ represents the seigniorage revenue arising from money creation. Note that seigniorage need not be positive.

The representative household then chooses $C_t, N_t(i)$, M_t, B_t and B_t^* to maximize lifetime utility given in (1) subject to the flow budget constraints given in (3). Rewriting the first order conditions for an internal optimum for C_t, $N_t(i)$, M_t, B_t, B_t^* and ω_t, we then obtain

$$C_t^{-\sigma} = \beta R_t E_t \left(\frac{P_t}{P_{t+1}} \right) \left(\frac{\chi_{1t+1}}{\chi_{1t}} \right) C_{t+1}^{-\sigma} , \qquad (4)$$

$$C_t^{-\sigma} = \beta R_t^* \exp(-bNFA_t + \ln \chi_{2t}) E_t \left(\frac{\varepsilon_{t+1}}{\varepsilon_t} \right) \left(\frac{P_t}{P_{t+1}} \right) \left(\frac{\chi_{1t+1}}{\chi_{1t}} \right) C_{t+1}^{-\sigma} \qquad (5)$$

$$\left(\frac{M_t}{P_t} \right)^{-\varsigma} \Big/ \chi_{1t} C_t^{-\sigma} = \frac{R_t - 1}{R_t} , \qquad (6)$$

$$\frac{N_t(i)^{\mu}}{\chi_{1t} C_t^{-\sigma}} = \frac{W_t(i)}{P_t} , \qquad (7)$$

$$R_t = R_t^* \exp(-bNFA_t + \ln \chi_{2t}) \frac{\varepsilon_{t+1}}{\varepsilon_t} \qquad (8)$$

[8] In this model the two potentially different subsidy rates will end up being equal.

Equation (4) is the Euler condition, representing the household's optimal intertemporal allocation of consumption given domestic nominal interest rate and price levels arising in the two time periods. It can also be written as

$$\chi_{1t} C_t^{-\sigma} / \beta E_t \chi_{1t+1} C_{t+1}^{-\sigma} = R_t / (1 + E_t \pi_{t+1}),$$

which states that the marginal rate of substitution between consumption at t and $t+1$ is equal to the real marginal rate of transformation in "production"— that is, the real return available when using domestic bonds to transfer consumption between t and $t+1$. Here the inflation rate is written in terms of the consumer price index, π_{t+1}, and defined as $\pi_{t+1} = (P_{t+1} - P_t)/P_t$. Similarly (5) is another version of the Euler condition,

$$\chi_{1t} C_t^{-\sigma} / \beta E_t \chi_{1t+1} C_{t+1}^{-\sigma} = R_t^* \exp(-bNFA_t + \ln \chi_{2t}) / (1 + E_t \pi_{t+1}),$$

where optimal intertemporal consumption requires the marginal rate of substitution between consumption at t and $t+1$ is equal to the real marginal rate of transformation in "production" when using foreign bonds to transfer consumption between t and $t+1$. Note that the risk premium term $\exp(-bNFA_t + \ln \chi_{2t})$ appears here because of our assumption of incomplete asset markets.

Equation (8), the well-known uncovered interest parity condition, is the implication of (4) and (5) holding simultaneously. Compared to the usual uncovered interest parity condition, (8) has one more term--the risk premium--that guarantees that asset market returns converge when the asset markets are incomplete.

Equation (6) represents the household's optimal money holding condition. This sets the marginal rate of substitution between real money balances and consumption equal to the opportunity cost of holding money (the money rate of interest)[9]. Equation (7) represents the intratemporal optimality condition between leisure and consumption, requiring the marginal rate of substitution between leisure and consumption to be set equal to the real wage.

In addition to the these first order conditions and the model's initial conditions, the following transversality conditions must hold,

$$\lim_{t\to\infty} \beta^t \lambda_{3t} m_t = 0, \ \lim_{t\to\infty} \beta^t \lambda_{3t} B_t / P_t R_t^t = 0,$$

$$\text{and } \lim_{t\to\infty} \lambda_{3t} \varepsilon_t B_t^* / P_t \left(R_t^* \exp(-bNFA_t + \ln \chi_{2t})\right)^t = 0, \qquad (9)$$

where $m_t = M_t / P_t$ is the level of real money balances.

To solve the domestic firm's optimization problem, it is first necessary to derive the demand for domestically produced goods arising from those consumers located in the rest of

[9] Because the return on the bond, $i_{t+1} = R_t - 1$, is paid in period t+1 rather than in period t, the return needs to be discounted to the time of choice in period t, i.e. divided by R_t.

the world. To do this, we assume that the representative household in the rest of the world has the same preferences as domestic households.

2.2 Firm Behavior

To model the supply side of our economy we assume that there is a continuum of firms indexed by i∈[0,1] in the small open economy. All firms use the same technology to produce a similar but differentiated good. The common technology incorporates constant return to scale (for simplicity, capital is ignored) and is written as,

$$Y_t(i) = A_t N_t(i)\,, \qquad (10)$$

where $N_t(i)$ is the aggregate quantity of the labor employed by firm i and A_t is an exogenous technology shock common to all firms. The shock is assumed to follow an AR (1) process, $\ln A_t = \rho_A \ln A_{t-1} + \varepsilon_{At}$, where $0 < \rho_A < 1$, and ε_{At} is a white noise disturbance that is assumed to be normally distributed, serially uncorrelated, with zero mean and standard deviation σ_A.

Firm i is assumed to hire labor in a competitive labor market but to be the sole seller of its differentiated product. This makes each firm a price taker in the input market and a price setter in its output market. The existence of market power, however, complicates our analysis because we eventually wish to evaluate the welfare consequences of alternative monetary policy rules through their effects on the representative household's utility function. Doing so requires the use of a linear approximation (to the utility function) that is valid only in the neighborhood of the social optimum (see Woodford, 2003). Hence to offset the distortion arising under monopolistic competition, we assume that the government uses a production subsidy (τ) set in such a way that in the steady state the net markup is zero (see Rotemberg and Woodford, 1997 and 1999, and Smets and Wouters, 2002).[10]

The firm then chooses the employment level, $N_t(i)$, and the selling price, $P^n_{H,t}(i)$, that will maximize its expected profits subject to its production technology and demand function. With the ability to recontract in the labor market each period, firm i will choose the quantity of $N_t(i)$ each period that minimizes the input cost of producing any level of output. That is,

$$\min_{N_t(i)} \left(\frac{W_t(i)}{P_{H,t}} \right) N_t(i) + \varphi_{H,t}(i)\big(Y_t(i) - A_t N_t(i)\big), \qquad (11)$$

[10] Despite using a subsidy that is optimal from the point of view of the steady state subsidy rate, some overall distortion may arise period to period due to fact that the distortions arising from tax incidence and market power may be dependent on the size of the shocks realized each period. This period-specific distortion is assumed to be of second order.

where the Lagrangian multiplier becomes, $\varphi_{H,t}$, the firm's real marginal cost of production (inclusive of the subsidy). The first order condition for $N_t(i)$ is then $\varphi_{H,t}(i) = W_t(i) / A_t P_{H,t}$. Notice that because all firms exhibit the same constant returns to scale technology, the real marginal cost for each firm is the same and is independent of its individual output level. This, together with equal availability of labor supplies and perfectly competitive labor markets, means that we can rewrite the first order condition without each firm specific index and derive the following representation of each firm's marginal cost,

$$\varphi_{H,t} = \frac{W_t}{A_t P_{H,t}}. \qquad (12)$$

Turning next to the firm's optimal pricing decision, we add price stickiness by following Calvo (1983) as later adapted for price indexation by Christiano et al. (2001)[11]. In Calvo's model, a random fraction, $1-\omega$, of all firms will receive each period a signal that allows them to adjust their price. The remaining fraction, ω, must keep their previously set price fixed. Christiano et al (2001) modify this to allow the fraction, ω, who cannot reset their price this period to index it to last period's inflation rate. We assume that the degree to which these firms can index their price is δ.

Under this price setting strategy, firm i with the opportunity to set a new price this period will choose the price that solves the following maximization problem,

$$\max_{P^n_{H,t}(i)} \sum_{k=0}^{\infty} \omega^k E_t(Q_{t,t+k}) \left[(1+\tau) P^n_{H,t} \left(X_{H,tk} \right)^{\delta} Y_{t+k}(i) - \varphi_{H,t+k} P_{H,t+k} Y_{t+k}(i) \right] \qquad (13)$$

subject to its demand function, $Y_{t+k}(i) = C_{H,t+k}(i) + C^*_{H,t+k}(i)$. Here the stochastic discount factor, $Q_{t,t+k}$, given in (4) as $\beta^k E_t \left(P_t / P_{t+k} \right) \left(\chi_{1t+k} / \chi_{1t} \right) \left(C_{t+k} / C_t \right)^{-\sigma}$, is used to compute the real value in period t of a unit of a good produced in period $t+k$.

The solution to (13) is the new price, $P^n_{H,t}(i)$, that will be chosen by firm i when it gets the opportunity to adjust its price. Since all firms face the same cost and demand conditions, each will pick the same new price so that we can drop the individual subscript i and write $P^n_{H,t}(i) = P^n_{H,t}$. In addition, we use $(X_{H,tk})^{\delta}$ to stand for the price indexation rule

[11] Christiano et al. (2001) consider two specifications for how indexation can affect the firm that cannot reset its price each period. The first specification has $P_{it} = \overline{\Pi} P_{i,t-1}$, where $\overline{\Pi}$ is the steady state value of the gross inflation rate (see also Erceg, Henderson and Levin, 2000 and Yun, 1996). Christiano et al refer to this case as static price updating. The second specification sets $P_{it} = \Pi_{t-1} P_{i,t-1}$ and is called dynamic price updating. The latter specification is motivated in part by claims that the former does not generate sufficient inertia in inflation (see Fuhrer and Moore, 1995, and Gali and Gertler, 1999). We adopt the second specification.

where $X_{H,tk} = \frac{P_{H,t}}{P_{H,t-1}} \frac{P_{H,t+1}}{P_{H,t}} \frac{P_{H,t+2}}{P_{H,t+1}} \cdots\cdots \frac{P_{H,t+k-1}}{P_{H,t+k-2}}$ for $k \geq 1$, otherwise, $X_{H,tk} = 1$ and where $\delta \in [0,1]$ denotes the degree of price indexation. Thus when $\delta = 1$, all firms' prices are fully indexed and when $\delta = 0$, no prices are indexed.

Differentiating (13) with respect to $P^n_{H,t}$ gives the following first order condition for an internal optimum,

$$P^n_{H,t} = \left(\frac{\theta}{(\theta-1)(1+\tau)} \right) \frac{E_t \sum_{k=t}^{\infty} (\omega\beta)^{k-t} \chi_{t+k} C_{t+k}^{-\sigma} Y_{t+k} P_{t+k}^{-1} P_{H,t+k}^{1+\theta} X_{H,tk}^{-\delta\theta} \varphi_{H,t+k}}{E_t \sum_{k=t}^{\infty} (\omega\beta)^{k-t} \chi_{t+k} C_{t+k}^{-\sigma} Y_{t+k} P_{t+k}^{-1} P_{H,t+k}^{\theta} X_{H,tk}^{\delta(1-\theta)}}. \quad (14)$$

Equation (14) implies that the new nominal price picked by firm i in period t will be a markup, $\theta/(\theta-1)$, over expected future nominal marginal cost. Because the firm is forward looking (given the previous pricing decisions made by the firm's rivals as embodied in $P_{H,t-1}$), optimal price setting requires an inflation forecast and a set of predetermined prices to inform the firm's current price decision.

In the special case where $\omega = 0$, all firms can adjust their price each period. This implies that prices are perfectly flexible so that (14) collapses to

$$\frac{P^n_{H,t}}{P_{H,t}} = \left(\frac{\theta}{(\theta-1)(1+\tau)} \right) \varphi_{H,t}.$$

In each period each firm sets its new money price so that its relative price is equal to a constant markup (inclusive of the subsidy) over its marginal cost. This is standard optimizing behavior under the monopolistic competition. Under the assumption that the optimal subsidy is chosen by the government, $\tau = [(\theta-1)/\theta] - 1$ so that $P^n_{H,t} / P_{H,t} = \varphi_{H,t}$, i.e., price equals marginal cost.

2.3 Import Pricing Behavior

In a small open economy when assets markets are incomplete and inhabitants cannot insure themselves perfectly from foreign shocks, the exchange rate (nominal, real or both) becomes an important endogenous variable and one that the monetary authority may need to take into account when setting policy. To understand the exchange rate's impact on the open economy one needs to know exactly how changes in the exchange rate will affect the domestic price of imports, that is, how exchange rate changes pass-through into domestic prices. In the literature there are two extreme hypotheses. First, foreign producers (exporters) could set their export good price optimally in terms of their own (foreign) currency and then simply translate set the small open economy price into domestic currency by the exchange

rate. Here the import good price in the domestic market is set as $P_{F,t}(z) = \varepsilon_t P_{F,t}^{*}(z)$. This is called producer currency pricing (PCP) and implies that the exchange rate elasticity is one and exchange rate pass-through is complete in the short run. On the other hand, foreign producers could set their export good price directly in terms domestic currency in relation to the demand conditions faced there. This is called local currency pricing, LCP, (or pricing to market, PTM,) and implies that in the short run, the elasticity of the foreign export good price with respect to the exchange rate will be zero.

Note that while the degree of exchange rate pass-through into domestic prices is related to pricing method used by the firm, it is not the only factor. Another key determinant of the degree of exchange rate pass through is the presence of costs of price readjustment. These lead directly to sticky prices in the destination market. To model incomplete pass-through, we consider a representative importer who imports a representative differentiated import good z at the cost $\varepsilon_t P_{F,t}^{*}(z)$. The importer is then allowed to change the domestic price if it wishes, but can do so only by incurring a quadratic adjustment cost. Following Rotemburg (1982) and Laxton and Pesenti (2003), the adjustment cost (per dollar sold) is defined as

$$AC_{M,t} = \frac{\omega_M}{2}\left(\frac{P_{F,t}(z)/P_{F,t-1}(z)}{(P_{F,t-1}/P_{F,t-2})^{\delta}} - 1\right)^2$$

where $\omega_M \geq 0$, scales the magnitude of the price adjustment cost as a function of its rate of change. In (21), the adjustment cost rises as the importer's specific inflation rate rises relative to some fraction, δ, of the past inflation rate experienced by the import good sector as a whole. Here the δ allows for some quasi-automatic price adjustment process equivalent to the feature of indexation allowed for under Calvo pricing. In terms of its contribution to the analysis, the δ allows the model to capture the nominal inertia appearing empirically in inflation dynamics.[12] Lastly, just as was the case for domestic goods, the fact that the importer has market power means that the government will set an optimal subsidy, τ_M, to induce the efficient level of output. Then because we have assumed that the elasticities of substitution among each variety of domestic and import goods are both equal to θ, the optimal subsidy rates will also be equal. That is, $\tau = \tau_M = 1/(\theta - 1)$.

[12] The adjustment cost can be specified differently. For example, a variant of this specification relates changes in the importer's price inflation rate to a steady state gross inflation rate, $\Pi \geq 1$ instead of the last period of inflation rate, $AC_{M,t} = \omega_{M1}\left(P_{F,t}(z)/\Pi P_{F,t-1}(z) - 1\right)^2/2$, and alternative versions add both the steady state Π and the lagged inflation rate to the quadratic adjustment cost term, $AC_{M,t} = \omega_{M1}\left(P_{F,t}(z)/\Pi P_{F,t-1}(z) - 1\right)^2/2 + \omega_{M2}\left([P_{F,t}(z)/P_{F,t-1}(z)]/(P_{F,t-1}/P_{F,t-2}) - 1\right)^2/2$. See Ireland 2001 and 2004 for details.

Hence facing this price adjustment cost, the representative importer will set its price $P_{F,t}{}^{n}(z)$ by solving the following maximization problem subject to the downward sloping demand function that it faces,

$$\max E_t \sum_{k=0}^{\infty} Q_{t,t+k}\left[(1+\tau_M)P_{F,t+k}{}^{n}(z) - \varepsilon_t P_{F,t+k}{}^{*}(z)\right]C_{F,t+k}(z) - AC_{M.t+k}P_{F,t+k}C_{F,t+k}$$

where

$$C_{F,t}(z) = \left(\frac{P_{F,t}(z)}{P_{F,t}}\right)^{-\theta} C_{F,t}$$

The first order condition for $P_{F,t}{}^{n}(z)$ is

$$\begin{aligned}&(1-\theta)(1+\tau_M) + \theta\frac{\varepsilon_t P_{F,t}{}^{*}(z)}{P_{F,t}} - \omega_M\left(\frac{\Pi_{F,t}}{(\Pi_{F,t-1})^{\delta}} - 1\right)\left(\frac{\Pi_{F,t}}{(\Pi_{F,t-1})^{\delta}}\right)\\ &- \omega_M\beta\frac{P_t}{P_{t+1}}\frac{\chi_{1,t+1}}{\chi_{1,t}}\left(\frac{C_{t+1}}{C_t}\right)^{-\sigma}\left(\frac{\Pi_{F,t+1}}{(\Pi_{F,t})^{\delta}} - 1\right)\left(\frac{(\Pi_{F,t+1})^2}{(\Pi_{F,t})^{\delta}}\right)\frac{C_{F,t+1}}{C_{F,t}} = 0\end{aligned} \tag{15}$$

where $\Pi_{F,t} = P_{F,t}/P_{F,t-1}$ denotes the gross inflation rate of import prices and where symmetry again produces $\Pi_{F,t}(z) = \Pi_{F,t}$. In the special case where $\omega_M = 0$, (15) collapses to $(1+\tau_M)P_{F,t}(z) = \frac{\theta}{\theta-1}\varepsilon_t P_{F,t}{}^{*}(z)$ *and implies* $P_{F,t}(z) = \varepsilon_t P_{F,t}{}^{*}(z)$, when the optimal subsidy is set.

2.4 The Log-linearization of the model

Log-linearizing the relevant equations and then rewriting the equations of motion in terms of efficiency gaps, we obtain the following equations that describe the evolution of economy.

$$\begin{aligned}\hat{\Pi}_{H,t} &= \frac{\delta}{1+\beta\delta}\hat{\Pi}_{H,t-1} + \frac{\beta}{1+\beta\delta}E_t\hat{\Pi}_{H,t+1} + \frac{\gamma(\mu+\sigma-\mu\alpha)}{1-\alpha}x_t\\ &- \frac{\gamma(\phi+\alpha-1)}{(1-\alpha)^2}q_{g,t} + \frac{\gamma\alpha(\sigma\eta+\alpha-1)}{(1-\alpha)^2}\hat{\Psi}_t.\end{aligned} \tag{16}$$

$$\hat{\Pi}_{F,t} = \frac{\delta}{1+\beta\delta}\hat{\Pi}_{F,t-1} + \frac{\beta}{1+\beta\delta}E_t\hat{\Pi}_{F,t+1} + \frac{\theta-1}{\omega_M(1+\beta\delta)}\hat{\Psi}_t \tag{17}$$

$$\begin{aligned}x_t &= E_t x_{t+1} - \frac{1-\alpha}{\sigma}\left(\hat{R}_t - E_t\hat{\Pi}_{H,t+1} - \hat{r}_{H,t}{}^{f}\right)\\ &- \frac{\phi+\alpha-1}{\sigma(1-\alpha)}E_t\Delta q_{g,t+1} + \frac{\alpha(\sigma\eta+\alpha-1)}{\sigma(1-\alpha)}E_t\Delta\hat{\Psi}_{t+1}.\end{aligned} \tag{18}$$

$$q_{gt} = E_t q_{g,t+1} + \frac{\sigma(1-\alpha)}{\phi}(x_t - E_t x_{t+1})$$
$$+\frac{\alpha\sigma\eta}{\phi}(\hat{\Psi}_t - E_t\hat{\Psi}_{t+1}) - \frac{b(1-\alpha)^2}{\phi}NFA_t + \frac{(1-\alpha)^2}{\phi}\hat{\chi}_{2t}. \tag{19}$$

$$m_{g,t} = \frac{\sigma}{\varsigma(1-\alpha)}x_t - \frac{\beta}{\varsigma(1-\beta)}\left(\hat{R}_t - \hat{r}_{H,t}{}^f - \hat{\Pi}_{H,t}{}^f\right)$$
$$-\frac{\alpha\sigma\eta(2-\alpha)}{\varsigma(1-\alpha)^2}q_{g,t} + \frac{\alpha\sigma\eta}{\varsigma(1-\alpha)^2}\hat{\Psi}_t. \tag{20}$$

$$NFA_t = \frac{1}{\beta}NFA_{t-1} - \frac{\alpha}{\beta(1-\alpha)}x_t + \frac{\alpha[\eta(2-\alpha)+\alpha-1]}{\beta(1-\alpha)^2}\hat{q}_t$$
$$-\frac{\alpha(\eta+\alpha-1)}{\beta(1-\alpha)^2}\hat{\Psi}_t - \frac{\alpha}{\beta(1-\alpha)}\hat{Y}_t{}^f + \frac{\alpha}{\beta(1-\alpha)}\hat{Y}_t{}^* \tag{21}$$

$$\hat{\Psi}_t = \hat{\Psi}_{t-1} + (1-\alpha)\hat{\Pi}_{H,t} - (1-\alpha)\hat{\Pi}_{F,t} + \hat{q}_t - \hat{q}_{t-1} \tag{22}$$

$$\hat{Y}_t{}^f = \frac{\phi+\alpha-1}{(\mu+\sigma-\mu\alpha)(1-\alpha)}\hat{q}_t{}^f + \frac{\alpha\sigma}{\mu+\sigma-\mu\alpha}\hat{Y}_t{}^*$$
$$+\frac{(1+\mu)(1-\alpha)}{\mu+\sigma-\mu\alpha}\hat{A}_t + \frac{1-\alpha}{\mu+\sigma-\mu\alpha}\hat{\chi}_{1t}, \tag{23}$$

$$\hat{q}_t{}^f = \frac{\sigma(1-\alpha)}{\phi}(\hat{Y}_t{}^f - \hat{Y}_t{}^*) \tag{24}$$

$$\hat{r}_{H,t}{}^f = \frac{\sigma}{1-\alpha}E_t\Delta\hat{Y}_{t+1}{}^f - \frac{\phi+\alpha-1}{(1-\alpha)^2}E_t\Delta\hat{q}_{t+1}{}^f - \frac{\alpha\sigma}{1-\alpha}E_t\Delta\hat{Y}_{t+1}^* - E_t\Delta\hat{\chi}_{1t+1}. \tag{25}$$

where equations (16) to (22) are the Phillips curve, the import price inflation, the IS equation, the real exchange rate equation, the LM equation, the net foreign asset position equation, and the law of one price gap, respectively, $\gamma = \frac{(1-\omega)(1-\omega\beta)}{\omega(1+\beta\delta)}$, $\phi = [1+\alpha(2-\alpha)(\sigma\eta-1)]$, $x_t \equiv \hat{Y}_t - \hat{Y}_t{}^f$, $q_{g,t} = \hat{q}_t - \hat{q}_t{}^f$, and $m_{g,t} = \hat{m}_t - \hat{m}_t{}^f$ is our definitions of the output gap , the real exchange rate gap, and the real money balance gap.[13] If we define $\Psi_t = \varepsilon_t P_{F,t}{}^* / P_{F,t}$ so that $\hat{\Psi}_t = \hat{\varepsilon}_t + \hat{P}_{F,t}{}^* - \hat{P}_{F,t}$. Following Monacelli (2003), we call this the law of one price gap. $\hat{Y}_t{}^f$ is the log deviation of output from its steady state value when prices are

[13] Note that we are following Woodford (2003) in defining x_t rather than $\hat{Y}_t$ as the output gap. Since $\hat{Y}_t = \log Y_t - \log\overline{Y}$ and $\hat{Y}_t{}^f = \log Y_t{}^f - \log\overline{Y}$, $x_t = \log Y_t - \log Y_t{}^f$. This is the distortion in output arising only from the fact that domestic output prices and import good prices are sticky.

completely flexible. Similarly, $\hat{q}_t^{\,f}$ is the log deviation of the real exchange rate from its steady state value when prices are completely flexible. Here we define the Wicksellian rate $\hat{r}_{H,t}^{\,f}$ as the real rate of interest that would arise under perfectly flexible domestic and import price (so that the law of one price holds for import goods).

In a small open economy, foreign output is exogenous. Here we specify it as incorporating a stochastic shock. In particular, foreign output is assumed to follow an AR (1) process, $\ln Y_t^{\,*} = \rho_y \ln Y_{t-1}^{\,*} + \varepsilon_{y,t}$, where ε_y is a white nose process with a zero mean and a constant standard error σ_y.

We now close the model by defining a general form for the monetary policy rule. In our small open economy model, inefficiency arises and welfare falls because contemporary shocks are not incorporated into the price setting behavior adopted by monopolistically competitive domestic producers and foreign good importers in the presence of incomplete asset markets. Hence because of incomplete asset markets and sticky domestic and import prices, actual outcomes differ from the efficient levels that would arise if prices were fully flexible. Monetary policy is then designed to use the information available on the gaps between actual and efficient levels to set the interest rate to counter the effects of these market imperfections. Hence monetary policy can be described as having the monetary authority set an interest rate as a function of the gaps arising between the interest rate, level of output, inflation rate, the real exchange rate, and the law of one price gap and the levels these values that would take under complete asset markets and perfectly flexible prices.

3. The loss function

Following the method set out in Rotemberg and Woodford (1999) we derive the loss function for the small open economy incorporating all the special model features described above. This can be written as

$$L = \operatorname{var}\hat{\Pi}_{H,t} + \phi_1 \operatorname{var} x_t + \phi_2 \operatorname{var} q_{g,t} + \phi_3 \operatorname{var}\hat{\Psi}_t + \phi_4 \operatorname{var} Rg_t \quad (26)$$

where

$$\phi_1 = \left(\mu + \sigma + \frac{\varpi\sigma^2(\varsigma - 1)}{\varsigma^2}\right)\frac{(1-\omega)(1-\omega\beta)}{\omega\theta^2(1+\mu)}$$

$$\phi_2 = \left((\sigma - 1)\Theta^2 + \frac{\varpi\sigma^2\Theta^2(\varsigma - 1)}{\varsigma^2} \right) \frac{(1-\omega)(1-\omega\beta)}{\omega\theta^2(1+\mu)}$$

$$\phi_3 = \left[(\sigma - 1)\left(\frac{\alpha\eta}{1-\alpha} \right)^2 + \varpi(\varsigma - 1)\left(\frac{\alpha\sigma\eta}{\varsigma(1-\alpha)} \right)^2 \right] \frac{(1-\omega)(1-\omega\beta)}{\omega\theta^2(1+\mu)}$$

$$\phi_4 = \varpi(\varsigma - 1)\left(\frac{\beta}{\varsigma(1-\beta)} \right)^2 \frac{(1-\omega)(1-\omega\beta)}{\omega\theta^2(1+\mu)}$$

$$Rg_t = \hat{R}_t - \hat{r}_{H,t}{}^f \text{, and } \Theta = (\phi + \alpha - 1)/\sigma(1-\alpha) .$$

Before leaving this topic it is important to note that the period loss function that appears standard in the literature is usually the weighted sum of only two arguments: the squared variances of inflation around its targeted level $\hat{\Pi}^*$ and the output gap. That is,

$$L = \text{var}(\hat{\Pi}_{H,t} - \hat{\Pi}^*) + a \,\text{var}\, x_t ,$$

where $\hat{\Pi}^*$ is often assumed to be zero, and a takes a value somewhere between 0 and 1. One typical value given to a is 0.5 (see Svensson, 2000). Hence the standard loss function typically ignores important efficiency gaps that arise in even slightly more general models. In addition, when we use the calibrated parameter values that appear appropriate for simulation purposes (in the following section), the calibrated weight placed on the output gap from our utility function is approximately 0.0025. While this weight may seem small, it is worth noting that such a small weight is not specific to our model, but rather typical of the weights found in other calibrated studies, where the loss function has been derived from the underlying representative household utility function. For example, using the calibrated parameter values adopted by Rotemberg and Woodford (1999), the weight that is placed on the output gap in their loss function is 0.00298. Similarly, Gali and Monacelli (2002) find that their value for a is equal to 0.0027.

4. Calibration

The solution to the model represented by the set of simultaneous equations is too complicated to work with analytically in a tractable way. Hence we follow the literature in setting up a quantitative dynamic stochastic general equilibrium model to evaluate alternative monetary policy rules based on the model's simulated results. To that end we calibrate the structural parameters of the model. These parameter values were chosen from the set of empirical estimates reported in the literature. In most cases, the parameter values used in our calibration are close to the mean of the available set of estimates. The final parameter values used in our simulations appear in summary form in Table 1.

Table 1 The Parameter Values Chosen to Calibrate the Model

Parameters	Values	Parameters	Values
Utility		Shocks	
β	0.99	ρ_A	0.85
σ	1.2	ρ_1	0.90
ς	2.5	ρ_2	0.50
μ	3.0	ρ_{y*}	0.85
		σ_A	0.0098
Shares		σ_1	0.0351
α	0.4	σ_2	0.0050
b	0.01	σ_{y*}	0.0070
δ	0.1	Substitution Elasticities	
ω	0.75	η	1.2
ω_M	400	θ	6

5. Optimal Simple Rules

The model presented in section 2 can be written compactly as

$$A_1 \begin{bmatrix} X_{1,t+1} \\ E_t X_{2,t+1} \end{bmatrix} = A_2 \begin{bmatrix} X_{1,t} \\ X_{2,t} \end{bmatrix} + BR_{g,t} + \begin{bmatrix} \varepsilon_{t+1} \\ 0_2 \end{bmatrix} \qquad (27)$$

where $X_{1,t}$ is a column vector of predetermined variables, and the initial values of $X_{1,0}$ are all givens. $X_{2,t}$, on the other hand, is a column vector of non-predetermined variables. $R_{g,t}$ is the gap between the short-term interest rate and the Wicksellian natural rate and is the policy instrument. ε_{t+1} is a column vector of innovations to the economic system and 0_2 is a zero column vector. Lastly, A_1 and A_2 are matrices, and B a column vector of the structural parameters, all of which are assumed to be constants.

Assume then that the monetary authority can precommit to a simple policy rule of the form $\hat{R}_t = -FX_t$, where $\hat{R}_t$ is the policy instrument, F is a $1 \times n$ row vector constrained to be sparse in some specified way which depends upon the policy maker's preferences and $X_t = [X_{1t}, X_{2t}]'$. The optimal simple rule will solve for the

values of F that minimize the loss function subject to the economic system denoted by (27).

5.1 Inertial Optimal Simple Rules

When an economy is hit by shocks, the rate of inflation and other economic variables deviate not only from their steady state values but from the values associated with temporary market clearing under perfectly flexible prices. It is in response to this departure from efficiency that the policy maker adjusts its policy instrument, the short-term interest rate, to bring the observed variables back into line with their designated targets. However, given the persistence that exists in the model and the degree of variable interaction that arises through time and finally the lagged information available to the policy maker, optimal policy will often consist of a series of small adjustments in the same direction through time rather than a single immediate jump. This outcome, one that is typically observed in the data, is called monetary policy inertia (Woodford, 1999) and is most often mimicked in the policy rule through the use of a lagged interest rate (interest-rate smoothing). Because we (and many others) find that rules with smoothing perform better than rules without, our comparative analysis of optimal simple rules in this small open economy adopts interest-rate smoothing. At a later stage, the interest-rate smoothing term can be dropped to see how optimal simple rules perform without the effect of smoothing.

In its most general form, the optimal simple interest-rate smoothing rule, written in terms of domestic output prices, can be written as [14]

$$\hat{R}_t = \phi_w \hat{r}^f_{H,t} + \phi_\pi \hat{\Pi}_{H,t} + \phi_x x_t + \phi_q q_{g,t} + \phi_\Psi \hat{\Psi}_t + \phi_r \hat{R}_{t-1}. \qquad (28)$$

We begin by setting ϕ_r equal to 0.8. The case where $0 < \phi_r < 1$ generates interest rate inertia as the policy maker adjusts only partially to the targeted gaps and is a value typically used in empirical studies (see, for example, Clarida et. al., 2000)..

The optimized reaction coefficients and welfare losses corresponding to the five simple monetary policy rules are reported in Tables 2 through 4. We begin by discussing optimal policy for Case 1, the information case where the central bank is assumed to be able

[14] This form differs slightly from that defined in section 2. The reason for this departure is because as Rotemberg and Woodford (1999) argue, that there is no any reason to restrict attention to the case $0 \le \phi_r < 1$, although only in that case can the policy rule can be described as involving partial adjustment toward to a "target" interest rate dependent on current output and inflation. In section 6.3 we find that the optimized value of ϕ_r is greater than one. Finally if ϕ_r is a constant then we can rewrite the policy rule in the general form used previous chapters. In that case, the values of the optimized reaction coefficients, such as ϕ_π, are found using the formula $\phi_\pi = \phi_\pi * /(1-\phi_r)$, where $\phi_\pi *$ is the statistic currently reported in the tables above. For example when $\phi_r = 0.8$, ϕ_π for Rule 1 in Table 6.1. becomes 5.741 instead of 1.1482. The welfare loss remains the same.

to observe or calculate all necessary information. Hence from Table 2, the following general results can be noted.

First, the three optimal simple rules that involve the exchange rate in some form (Rules 1-3) perform best, each working at least as well as the Taylor Rule. Then of the three exchange rate rules, the benchmark rule performs best, outperforming the Taylor rule. If one briefly scans the remaining three tables, it can be seen that the significance of including rather than excluding the exchange rate in the monetary policy rule continues to hold for all four information cases.

Table 2

Case 1: the Monetary Authority measures the inflation rate using domestic output prices

$$\hat{R}_t = \phi_w \hat{r}^f_{H,t} + \phi_\pi \hat{\Pi}_{H,t} + \phi_x x_t + \phi_q q_{g,t} + \phi_\Psi \hat{\Psi}_t + \phi_r \hat{R}_{t-1}.$$

Policy Rule	Optimized reaction coefficients					Welfare Loss
	ϕ_w	ϕ_π	ϕ_x	ϕ_q	ϕ_ψ	
1 **(B)**	0.4439	1.1482	-0.0311	0.1023	-0.0542	0.1598
2 **(EX)**	0.4920	1.3348	0.0204	0.0046	0	0.1608
3 **(PT)**	0.4860	1.3553	0.0228	0	-0.0037	0.1608
4 **(TR)**	0.4903	1.3460	0.0229	0	0	0.1608
5 **(IT)**	0.5276	1.4830	0	0	0	0.1610

The second point is that the marginal gain in welfare from incorporating some element of the exchange rate into the policy rule is small for the current case where information is "perfect". For example, the ratio between the smallest welfare loss (Rule 1) and the largest one (Rule 5) in Table 1 is 0.9925. If we compare just those rules that involve the exchange rate and the Taylor rule (Rule 4), the ratio becomes even smaller. For example, the ratio of Rule 1 to Rule 4 is 0.9938. Furthermore, if we compare Rules 2 and 3 to 4, the ratio becomes 1. In other words, the inclusion of some forms of the exchange rate into the policy rule (as in Rules 2 and 3) does not improve the welfare at all relative to the Taylor. This is consistent with the findings of Adolfson (2007) who argues on this basis that the improvement social welfare from incorporating an exchange rate term into an otherwise fully optimized policy rule will be practically zero, irrespective of the degree of the exchange rate pass-through. In response to this challenge, however, several points need to be emphasized. First, the conclusion that adding the exchange rate into the policy rule does not enhance the social welfare holds only under relatively strong conditions. For example, even in the current case some forms of exchange rate use have resulted in welfare improvement. More significantly, however, Adolfson takes neither the lack of information nor its uncertainty into account. As

we will see below, when the central bank is given access to less and less accurate current information for deciding upon policy, the inclusion of the exchange rate in the policy rule is always the welfare enhancing.[15] In some cases it will produce relatively large increases in welfare. Adolfson also finds that an indirect exchange rate response, attained by having policy react to CPI inflation rather than domestic inflation, is welfare enhancing. As we will see below, our results are quite different.

The third general finding that arises first in Table 2 is that the size of the optimized inflation coefficient, ϕ_π, is much larger than the other reaction coefficients. This result is typical of both ours and others findings and not at all unexpected. That is, given the calibrated values of the loss function derived from the model's utility function, the parameter values used in calibrating lead the policy maker to place a much higher weight on inflation in comparison with the other terms in the loss function. For example, given our calibrated values (and the re-scaling of the model so that the weight placed on the inflation rate is 1), the corresponding weights put on the other loss function gaps are: 0.0025 for the output gap; 0.00022 for the real exchange rate gap; and 0.0885 for the interest rate gap. Therefore to minimize the welfare loss the central bank must react strongly to inflation. This asymmetric weighting is also found in Rotemberg and Woodford (1999), for example. They report optimal values for the reaction coefficients corresponding to a Taylor rule with interest-rate smoothing of 1.22, 0.06, and 1.28, respectively, while without the interest-rate smoothing term, the optimal values found are 2.88 and 0.02, respectively. It can also be shown in our model, for example, that by increasing the weights in the utility function given to the output gap, the real exchange rate gap, and the interest rate gap to 0.05, 0.05, and 0.5, respectively, that the optimal parameter values on the output gap and the Wicksellian natural rate corresponding to the Taylor rule (Rule 4) are increased to 0.2943 and 0.7174, respectively, while the reaction coefficient associated with the inflation rate decreases to 1.0291.

A fourth finding is that the signs of the optimized reaction coefficients are not always positive, except for inflation. This is similar to findings noted by Rotemberg and Woodford (1999). While such findings seem counter-intuitive, the likely explanation for the occasional negative coefficient is that when the shocks to the economy result in a positive output gap and higher inflation, the optimal inflation reaction coefficient is so large that it produces an overshooting of the interest rate relative to the output gap, hence requiring a perverse response to the output gap to minimize the loss function. Some support for this interpretation is given when for these perverse cases we change the weighting of the loss function. For

[15] We should add a note of caution in that the arbitrary specificity of the utility function chosen for the analysis means that the welfare implications are better interpreted as ordinal indicators rather than cardinal measures. That is, the analysis is a better indicator of policies that are better or worse rather than as an indicator of exactly how much better or worse each policy is.

example, if we use the revised re-weighting of the loss function reported immediately above, the optimized reaction coefficients corresponding to Rule 1 are found to result in the following rule

$$\hat{R}_t = 0.8\hat{R}_{t-1} + 0.81\hat{r}^f_{H,t} + 1.17\hat{\Pi}_{H,t} + 0.09x_t + 0.37q_{g,t} - 0.19\hat{\Psi}_t,$$

where the sign of optimized coefficient on the output gap now is positive instead of negative. Note that the welfare losses reported in Table 2 are all global minima. Then if we require the ϕ_x in Rule 1 to be positive (as above), the equilibrium is still found to be determinate but will yield a larger welfare loss. In this case, the welfare loss becomes 0.1615.

In Case 2 we remove information from the monetary authority by assuming that the ability to observe contemporaneous market values is insufficient to determine the flexible price values required to determine the targeted gaps in the money rules. Hence Table 3 reports the results for the five monetary rules when the policy maker must replace the Wicksellian natural interest rate and the other unobserved flexible price variables in the money rules with their steady state values.

As one would expect, a comparison of the welfare losses in Table 3 with those reported for the same measure of inflation in Table 2 shows that the welfare losses in Table 3 are always bigger than those in Table 2. That is, the less information that is available to the authority making the policy decision, the larger will be the resulting welfare loss (the smaller the welfare gain that can be made from pursuing monetary policy optimally). But while the overall welfare losses are all larger, the same pattern of welfare losses continues to arise across the five different monetary rules. In particular, the three exchange rate rules now always dominate the Taylor and Inflation Targeting money rules.

Table 3

Case 2: Steady state values while targeting inflation measured in domestic output prices

$$\hat{R}_t = \phi_\pi \hat{\Pi}_{H,t} + \phi_x \hat{Y}_t + \phi_q \hat{q}_t + \phi_\psi \hat{\Psi}_t + \phi_r \hat{R}_{t-1}$$

Policy Rule	Optimized reaction coefficients				Welfare Loss
	ϕ_π	ϕ_x	ϕ_q	ϕ_ψ	
1 **(B)**	7.6458	0.1583	-0.0125	-0.0576	0.1888
2 **(EX)**	7.8389	0.2052	-0.0811	0	0.1889
3 **(PT)**	7.6229	0.1517	0	-0.0624	0.1888
4 **(TR)**	7.7623	0.1733	0	0	0.1890
5 **(IT)**	10.6935	0	0	0	0.1944

Finally, we note that the results for the optimal monetary rules in Table 3 also generate reaction coefficient values for the exchange rate and the law of one price that are often

negative. In part, it may be that because these negative coefficients arise in those cases where the inflation reaction coefficient is quite large that the interest rate is overshooting with respect to the exchange rate and so requires perverse exchange rate adjustments to preserve determinacy. It also suggest, however, that sign (as well as the size) of the reaction coefficients may be affected not only by the loss function chosen but also by the information set assumed available to the policy maker.

In Tables 4 and 5 we repeat the procedure followed in Tables 2 and 3, except for the information that is assumed to be available to the policy maker. In Tables 2 and 3, we assumed that the policy maker could access current period information on market variables, while in Tables 4 and 5, we assume that the monetary authority can observe or access market values only with a one period lag. Case 3 follows Case 1 in assuming that the monetary authority can use its market information to calculate the flexible price values (but with a one period lag), while Case 4 follows Case 2 in assuming that the monetary authority cannot calculate the flexible price values and must use steady states values.

Broadly speaking, the removal of contemporaneous information does not change the basic ranking of monetary policy rules observed in Tables 2 and 3. That is, in all these Tables at least one of the three exchange rate rules dominates the Taylor and Inflation Targeting money rules. The results then reinforce the earlier suggestion that for a small open economy some incorporation of external constraints on the economy in the exchange rate rule will dominate money rules that focus only on the traditional "closed" economy features of the money rule.

However, while the general pattern of results may remain the same, three additional observations are worth noting. First, as expected, the withdrawal of information from the monetary authority increases the overall welfare loss. Comparing directly comparable tables, such as Tables 4 and 2, Tables 5 and 3, we see that the minimum welfare losses in Tables 4 and 5 are all larger than found in Tables 2 and 3.

Second, it is interesting to note that the welfare losses in Case 3 are all significantly smaller than those found for Case 2. Note, for example, that the welfare loss arising from the use of Rule 1 in Case 3 (see Table 4) is 0.1617, while being 0.1888 in Case 2 (see Table 3). The former is only 85.65% of the latter while the same ratio for the Taylor rule is 85.82%. This general finding is of particular interest because it suggests that the ability to use flexible price values in the money rule (even if that information is old) can dominate the ability to access more current information but use steady state values in the policy rule. Implicitly, even though Case 3 assumes that the central bank uses one period lagged information, the

amount of information it has access to in using flexible price values incorporates more relevant information than does the use of current information in Case 2.[16]

Finally, as would be expected, the welfare loss in Case 4 is the largest of all four cases considered here. This is the case where the monetary authority unambiguously has the least amount of information.

Table 4

Case 3: lagged information while targeting inflation measured in domestic output prices

$$\hat{R}_t = \phi_w \hat{r}^f_{H,t-1} + \phi_\pi \hat{\Pi}_{H,t-1} + \phi_x x_{t-1} + \phi_q q_{g,t-1} + \phi_\Psi \hat{\Psi}_{t-1} + \phi_r \hat{R}_{t-1}$$

Policy Rule	Optimized reaction coefficients					Welfare Loss
	ϕ_w	ϕ_π	ϕ_x	ϕ_q	ϕ_ψ	
1 **(B)**	0.5307	1.4317	-0.0411	0.0925	-0.0475	0.1617
2 **(EX)**	0.5813	1.5925	0.0015	0.0079	0	0.1622
3 **(PT)**	0.5727	1.6103	0.0054	0	-0.0020	0.1622
4 **(TR)**	0.5763	1.6076	0.0055	0	0	0.1622
5 **(IT)**	0.5896	1.6525	0	0	0	0.1623

Table 5

Case 4: lagged information, steady state targets and inflation in domestic output prices

$$\hat{R}_t = \phi_\pi \hat{\Pi}_{H,t-1} + \phi_x \hat{Y}_{t-1} + \phi_q \hat{q}_{t-1} + \phi_\Psi \hat{\Psi}_{t-1} + \phi_r \hat{R}_{t-1}$$

Policy Rule	Optimized reaction coefficients				Welfare Loss
	ϕ_π	ϕ_x	ϕ_q	ϕ_ψ	
1 **(B)**	3.5134	0.0635	-0.0167	-0.0834	0.1955
2 **(EX)**	3.5598	0.1331	-0.1268	0	0.1966
3 **(PT)**	3.5061	0.0552	0	-0.0899	0.1955
4 **(TR)**	3.5071	0.0879	0	0	0.1981
5 **(IT)**	4.2006	0	0	0	0.2030

A summary of our findings thus far leads to the following tentative conclusions. First, in a small open economy where asset markets are not perfect and where there the degree of exchange rate pass through is incomplete, the exchange rate is an important endogenous variable that the policy maker should incorporate into its policy rule. Doing so lowers welfare

[16] We will show below that when ϕ_r is optimized instead of simply setting it to be 0.8 here, the finding will be modified.

losses. Second, while it is not surprising that the better is the information used by the policy maker, the larger is the welfare gain associated with using monetary policy optimally, what is interesting is that the better the information accessible to the policy maker, the less will be the marginal welfare gain from incorporating the exchange rate into the policy rule. Our analysis then suggests that the use of the exchange rate in the policy rule will be more valuable to those small open economy monetary authorities with less good information on the current state of their economy. Finally, when considering the choice of the price index to be used as the measure of inflation in the monetary rule, our analysis suggests that the central bank should measure inflation in terms of domestic output prices.

5.2 Optimal Smoothing Rules

Strictly speaking the "optimal simple rules" presented in Tables 2 through 5 are optimal only in a restricted sense since the reaction coefficient on the interest rate smoothing term was assigned. However, even though most estimates of ϕ_r have fallen in a small range about 0.8,[17] it is of interest to ask how the optimal simple policy rules would respond to a lifting of that restriction. The results of this re-optimization are presented in Tables 6 through 9. Because the results are broadly similar across the different cases, we restrict specific comment to the observations that can be seen from Table 6 below.

Table 6

Case 1: the Monetary Authority measures the inflation rate using domestic output prices

$$\hat{R}_t = \phi_w \hat{r}^f_{H,t} + \phi_\pi \hat{\Pi}_{H,t} + \phi_x x_t + \phi_q q_{g,t} + \phi_\Psi \hat{\Psi}_t + \phi_r \hat{R}_{t-1}.$$

Policy Rule	Optimized reaction coefficients						Welfare Loss
	ϕ_w	ϕ_π	ϕ_x	ϕ_q	ϕ_ψ	ϕ_r	
1 **(B)**	-0.0570 (0)	2.1011 (2.0456)	0.0107 (0.008)	0.0036 (0.0089)	0.0302 (0.0245)	1.9290 (1.8246)	0.1586 (0.1586)
2 **(EX)**	0.0725 (0)	1.9345 (2.0073)	-0.0077 (-0.0092)	0.0392 (0.0412)	0	1.6606 (1.7857)	0.1587 (0.1587)
3 **(PT)**	-0.0657 (0)	2.1143 (2.0625)	0.0125 (0.0126)	0	0.0324 (0.0289)	1.9464 (1.8304)	0.1586 (0.1586)

[17] For example, Clarida et al (2000) find that the estimate of ϕ_r is 0.68 for pre-Volcker period, and 0.79 for Volcker-Greenspan era. Across six different quarterly U.S. data samples (differing in their definition of output gap), Kozicki (1999) reports a range for ϕ_r from 0.75 to 0.82, while across 16 different quarterly samples of U.S. data (differing in output gap, inflation, and sample period definition), Amato and Laubach (1999) report a range of ϕ_r from 0.78 to 0.92.

4 **(TR)**	0.0691 (0)	2.0736 (2.1472)	0.0115 (0.011)	0	0	1.6583 (1.7767)	0.1589 (0.1589)
5 **(IT)**	0.0609 (0)	2.1932 (2.2534)	0	0	0	1.7155 (1.818)	0.1589 (0.1589)

Note that the values in parenthesis are the optimized reaction coefficients and welfare loss yielded by the same rule when ϕ_w is set equal to zero.

The first and the most interesting observation is that the optimized value of ϕ_r is quite different from the one reported in the literature and the value used in section 5.1. Here the optimized coefficient is greater than, rather than less than, one. This implies that the monetary policy process is not stable for all given initial values of the interest rate. As Rotemberg and Woodford (1999) point out, the interesting feature is that the "explosive" monetary rule does not produce explosive 'equilibria'. Rather, the explosive nature of the rule implies that the interest rate set must jump to a value that preserves the expected equilibrium value of the interest rate. That is, intuitively, the prospect of an exponential increase in real rate leads to a substantial reduction in expected future aggregate demand through the aggregate demand function, which in turn induces firm to cut prices today. That decline in prices ensures that the interest rate need not rise but instead converge on its equilibrium value.

The second change is that the absolute values of the optimized coefficient on the Wicksellian natural rate ϕ_w become much smaller when the coefficients on the lagged interest rate ϕ_r are optimized. They were much larger, around 0.5, when ϕ_r was set equal to 0.8. This suggests that that there might be some degree of substitutability in how the two forms of interest rate appear in the money rule—the optimized values of the coefficients on the Wicksellian natural rate versus the optimized value the lagged interest rate coefficient. To test whether there is a trade-off arising between the Wicksellian rate and the lagged interest rate in the money rule, we experiment with different values in terms of Rule 2. Table 6 currently shows that the optimized values of ϕ_w and ϕ_r are 0.0725, and 1.6606, respectively. Suppose now that that we simply decrease ϕ_r and set it at 1.2. Re-optimization over the other values in the rule results the optimized value of ϕ_w increasing to 0.284. Next, Table 2 has shown that when ϕ_r was set at 0.8, ϕ_w becomes 0.4920, and when we decrease ϕ_r even further to 0.2, ϕ_w now jumps to 1.572. Finally, Table 10 shows that ϕ_w further jumps to 3.128 when ϕ_r is set to be zero. This exercise is consistent with the hypothesis that the lagged interest rate term performs some of the same functions as the Wicksellian natural rate in the money rule.

A third observation arising when Table 6 is examined in conjunction with Table 7 is that when the lagged interest rate coefficient is optimized, difference in policy performance for each of the different monetary policy rules in Cases 1 and 2 is negligible despite the information loss arising in the second case. There appear to be two important reasons for this particular finding. First to the extent that the optimized lagged interest rate substitutes for the Wicksellian rate, the elimination of the Wicksellian rate in Case 2 should have relatively little effect on the overall outcome. This prediction, arising from the observation in point two above, can be tested by imposing a zero coefficient on the Wicksellian rate in Table 6 and then re-optimizing. The results of doing this are reported as the second line (in parentheses) of each row in Table 6. As the table shows, the removal of the Wicksellian natural rate term does not change the size of the welfare loss for each money rule. The second difference that arises between Cases 1 and 2 is in the use of flexible price output and law of one price gap in Case 1 versus the use of the deviation from the steady state in Case 2. Then because the weights in the loss function ascribed to these two dimensions are quite small, the overall change in the welfare loss from using steady state values versus flexible price values is negligible. This latter point can be confirmed by increasing the weights placed on the output gap and the law of one price gap in the loss function. Doing so does now produce a larger deadweight loss for each money rule.

Table 7

Case 2: contemporaneous prices and steady state values

$$\hat{R}_t = \phi_\pi \hat{\Pi}_{H,t} + \phi_x \hat{Y}_t + \phi_q \hat{q}_t + \phi_\Psi \hat{\Psi}_t + \phi_r \hat{R}_{t-1}.$$

Policy Rule	Optimized reaction coefficients					Welfare Loss
	ϕ_π	ϕ_x	ϕ_q	ϕ_ψ	ϕ_r	
1 **(B)**	2.0541	-0.0015	0.0184	0.0257	1.8238	0.1586
2 **(EX)**	2.0807	-0.0143	0.0385	0	1.7888	0.1587
3 **(PT)**	2.1000	0.0077	0	0.0327	1.8374	0.1586
4 **(TR)**	2.2281	0.0024	0	0	1.8026	0.1589
5 **(IT)**	2.2534	0	0	0	1.8180	0.1589

Fourth, the incorporation of optimal interest rate smoothing reduces even further the marginal welfare differences arising across the five different monetary policy rules. What this means for our earlier policy conclusion is that while optimal policy should still include some form of exchange rate consideration, the marginal gain from doing so is now considerably smaller. For example, the ratio of the welfare losses yielded by Rule 1, the smallest welfare loss, compared to the Taylor rule (Rule 4) in Table 6 is 0.9981; compared to Rule 2 and Rule 4 is 0.9987, and the ratio of Rule 3 to Rule 4 is exactly the same as the ratio

of Rule 1 to Rule 4. This result may not be that surprising. Below we will show that when we compare non-inertia policy rules to rules with interest rate smoothing, the policy rules with inertia always perform better. However, the marginal welfare gain is largest when any form of interest rate smoothing is adopted. The marginal gain then becomes smaller as the policy maker moves from an arbitrary reaction coefficient to one that is optimize. This mirrors our finding that the gain in the performance of the money rule increases at a decreasing rate as the policy maker access more and more information relevant to their policy making decision.

Finally, as we would expect, the welfare losses arising when the optimized coefficient is used (in Tables 6 through 9) are all smaller than those found when the 'traditional sized' smoothing rule is adopted (in Tables 2 through 5). For example, when the welfare losses summarized in Table 6 are compared directly to those listed for the same information case in Table 2, we find welfare gains across the five rules ranging from 0.9% to 1.4%.

The results for the remaining information cases are presented below for comparison. In general, they simply reinforce the points made above. However, there is one new finding of interest--the welfare losses in Case 2 (see Table 7) are now smaller (rather than larger, see Table 3) than those arising in Case 3 (see Table 8). This implies that even though the monetary authority in Case 2 is constrained to use steady state values in its targeted gaps, the use of an optimized (rather than arbitrary) coefficient for the lagged interest rate more than offsets the ability to use lagged flexible price values in the money rule. This finding, however, is again somewhat conditional on the loss function used. It can be shown, for example, that by increasing the utility weights given to the output gap, the real exchange rate gap, and the interest rate gap to 0.05, 0.05, and 0.5, respectively, as we did earlier, yield welfare losses for Case 2 that are larger than those in Case 3 and also larger than those generated in Case 1. The welfare losses in Rule 2 from using the new weights in the loss function in Cases 1 to 3 are 1.0388, 1.0714, and 1.0633, respectively.

Table 8

Case 3: lagged money prices and flexible price gaps

$$\hat{R}_t = \phi_w \hat{r}^f_{H,t-1} + \phi_\pi \hat{\Pi}_{H,t-1} + \phi_x x_{t-1} + \phi_q q_{g,t-1} + \phi_\Psi \hat{\Psi}_{t-1} + \phi_r \hat{R}_{t-1}.$$

Policy Rule	Optimized reaction coefficients						Welfare Loss
	ϕ_w	ϕ_π	ϕ_x	ϕ_q	ϕ_ψ	ϕ_r	
1 **(B)**	0.4367	1.5505	-0.0296	0.0673	-0.0297	1.0040	0.1616
2 **(EX)**	0.3450	1.6785	-0.0078	0.0240	0	1.2157	0.1617
3 **(PT)**	0.3295	1.7321	0.0043	0	0.0095	1.2500	0.1618
4 **(TR)**	0.3687	1.7285	0.0037	0	0	1.1589	0.1618

5 **(IT)**	0.3746	1.7613	0	0	0	1.8887	0.1618

Table 9

Case 4: lagged money prices and steady state values

$$\hat{R}_t = \phi_\pi \hat{\Pi}_{H,t-1} + \phi_x \hat{Y}_{t-1} + \phi_q \hat{q}_{t-1} + \phi_\Psi \hat{\Psi}_{t-1} + \phi_r \hat{R}_{t-1}.$$

Policy Rule	Optimized reaction coefficients					Welfare Loss
	ϕ_π	ϕ_x	ϕ_q	ϕ_ψ	ϕ_r	
1 **(B)**	1.8047	-0.0024	0.0158	0.0197	1.7923	0.1620
2 **(EX)**	1.8221	-0.0122	0.0311	0	1.7573	0.1621
3 **(PT)**	1.8344	0.0054	0	0.0255	1.8006	0.1621
4 **(TR)**	1.9116	0.0014	0	0	1.7523	0.1623
5 **(IT)**	1.9247	0	0	0	1.7613	0.1623

5.3 Robustness Analysis

In Section 5.1 we highlighted two key findings of our analysis: first, the more relevant was the information incorporated in the policy decision, the larger was the welfare gain from using monetary policy optimally; and second, that in our small open economy, the exchange rate was an important endogenous variable whose incorporation into the policy rule improved the economy's performance. These outcomes are also present in Section 5.2. In this section we examine the robustness of these findings as a basis for conclusions respecting the policy performance in small open economies such as ours.

One feature of our findings that could be seen as undermining confidence in the robustness of our exchange rate inclusion conclusion is the tendency, first observable in Table 2, for the welfare loss to decrease as additional variables are incorporated into the policy rule. This feature is observed in the other tables as well. This suggests the following counter hypothesis. Perhaps the reason that the benchmark rule (Rule 1) results in a smallest welfare loss is simply because adding more variables will always improve the performance of a rule rather than because the exchange rate is itself an important targeting variable for the policy rule.[18] Such a possibility suggests the following test: if the 'more variables better outcome' hypothesis is true, then the addition of one or more 'significant' variables into the policy rule should yield an even smaller welfare loss.

[18] Adolfson (2007) raises a similar point. He points out that if the policy rule is excessively simple (i.e., is suboptimal) then the inclusion of any additional state variable is likely to yield an improvement in the rule. However, optimization of the reaction coefficients reduces the sub-optimality of the simple rule, which partly mitigates such a problem.

To test this hypothesis, we first add "consumption" to Rule 1 in Case 1 when the monetary authority uses domestic output prices to target the inflation rate with the partial interest-rate smoothing (Table 2, row 1). In that case the welfare loss associated with the revised optimal policy rule was 0.1598. Second if we instead add "labor" to Rule 1, the welfare loss is also found to be 0.1598. In both cases the welfare result is exactly the same as the welfare loss arising in their absence. Hence the simple inclusion of additional variables in the policy rule does not bring down the welfare loss further as would be expected. This set of findings gives us greater confidence that the reason that the benchmark rule yields better policy performance is that the incorporation of the exchange rate and the law of one price gap into the policy rule incorporates more relevant information.

To further test this hypothesis, we re-optimize after taking the real exchange rate and the law of one price gap out of the policy rule and substituting "consumption" or "labor supply" in their place. Should these substitutions improve the performance of policy, our information hypothesis would be contradicted. We begin by replacing the real exchange rate in Case 1 Rule 1 with "consumption" in the policy rule and then doing the same for the law of one price gap. Our simulation results show that using "consumption" to replace either the real exchange rate or the law of price gap yields a larger welfare loss. Their respective values found were to be 0.1603 and 0.1603, larger than the 0.1598 the welfare loss for the benchmark Rule 1 reported in Table 2. Next, we use "labor" to replace the real exchange rate and the law of one price gap in Rule 1 and obtain the similar results. Here the welfare losses associated with each test are 0.1608 and 0.1608, respectively; both larger than the original 0.1598 loss arising under Rule 1 in Case 1. Finally, using both "consumption" and "labor" to replace the real exchange rate and the law of one price gap in Rule 1 improves performance (on their separate use) but also yields a larger over welfare loss, 0.1601, compared to the benchmark rule. Together these findings are strongly consistent with our interpretation of the information content of the openness variables—the real exchange and the law of one price gap—for the effectiveness of the money rule. As such, these findings increase our confidence in this interpretation of our results.

In our small economy model external considerations influence domestic outcomes in two ways: first through the assumption of incomplete capital markets and second through the law of one price gap. While the former forms part of the structure of the model, it has no one simple parameter by which the intensity of incompleteness can be adjusted. In this sense, the latter external complication is more susceptible to manipulation. In particular, the law of one price gap affects welfare via incomplete exchange rate pass-through, which in turn arises from the assumption that the importers face quadratic adjustment costs in setting prices in the domestic currency. In terms of the specification adopted, the parameter ω_M scales the

magnitude of import price adjustment costs such that the degree of exchange rate pass-through increases as ω_M decreases. When ω_M is set equal to zero, the degree of exchange rate pass-through is 100 percent, which implies that the law of one price gap will be zero. In this case, adding to the policy rule the law of one price gap would not be expected to enhance the social welfare. Similarly since the size of the distortion in the real exchange rate is positively related to the law of one price gap, the marginal welfare gain from incorporating the real exchange rate into the policy rule should also fall as ω_M decreases, but because the real exchange rate is just partially related to the law of one price gap, therefore the marginal welfare gain from incorporating the real exchange rate will be expected to be less significant than those from Rules 1 and 3 as ω_M varies .

To test this implication of the analysis, we vary the parameter ω_M between 400, which represents a low degree of exchange rate pass-through, and 1, representing a high degree of the exchange rate pass-through.[19] The welfare losses WL associated with the corresponding optimized simple rules and the relative welfare losses are presented in Tables 10. Table 10 shows that in relative terms the marginal social welfare gain from incorporating the exchange rate and/or the law of one price gap into the Taylor Rule is large when the degree of the exchange rate pass-through is low, while the gain is small under a high degree of the exchange rate pass-through.

Table 10

Case 1 when ω_M varies

	Rule 1 **(B)**		Rule 2 **(EX)**		Rule 3 **(PT)**		Rule 4 **(TR)**
	WL_1	WL_1/WL_4	WL_2	WL_2/WL_4	WL_3	WL_3/WL_4	WL_4
$\omega_M = 400$	0.1975	0.9841	0.1995	0.9940	0.1982	0.9875	0.2007
$\omega_M = 100$	0.2038	0.9869	0.2062	0.9985	0.2044	0.9898	0.2065
$\omega_M = 50$	0.2059	0.9932	0.2072	0.9995	02061	0.9942	0.2073
$\omega_M = 25$	0.2066	0.9995	0.2067	1	0.2066	0.9990	0.2067
$\omega_M = 10$	0.2037	0.9985	0.2040	1	0.2038	0.9990	0.2040
$\omega_M = 1$	0.1968	1	0.1968	1	0.1968	1	0.1968

As mentioned above, there is no one parameter by which we can vary the incompleteness of capital markets. However, the risk premium that the representative

[19] In terms of Adolfson (2007) estimation, the degree of the exchange rate pass-through is about 10% when $\omega_M = 400$, and 70% when $\omega_M = 1$.

household must pay to borrow on world markets due to incomplete capital markets is variable and can be affected by the size of the shock to the risk premium, its net foreign asset position at the time of the shock, and the constant, b, which governs the size of the risk premium.[20] Suppose then that while other conditions remain fixed, we let the size of the shock to the risk premium increase. Then since the larger the shock to the risk premium, the greater will be the effect of incomplete capital markets on the model leading to the larger the size of the welfare loss associated with that distortion. It then follows that the marginal gain from incorporating one of the external gaps into the money rule should increase as the size of shock to the risk premium increases.[21]

To test this implication, we redo Table 11 for different sized shocks to the risk premium. The welfare losses WL associated with the corresponding optimized simple rules and the relative welfare losses are presented in Tables 11. Scanning Table 11 we see that the marginal welfare gains from adding one of the external gaps to the policy rules become more significant as the size of shock to the risk premium increases. For example, in Case 1 when $\varepsilon_{2t} = 0.05^2$, the relative gain associated with using the exchange rate in Rule 2 compared to the Taylor rule is 1/0.9983 = 1.002, while it becomes 1/0.9191 = 1.088 when the size of the shock rises to $\varepsilon_{2t} = 1^2$.

Table 11

Case 1 when ε_{2t} changes

Risk premium Shock	Rule 1 **(B)**		Rule 2 **(EX)**		Rule 3 **(PT)**		Rule 4 **(TR)**
	WL_1	WL_1 / WL_4	WL_2	WL_2 / WL_4	WL_3	WL_3 / WL_4	WL_4
$\varepsilon_{2t} = 0.05^2$	0.1719	0.9954	0.1724	0.9983	0.1722	0.9971	0.1727
$\varepsilon_{2t} = 0.1^2$	0.1833	0.9897	0.1840	0.9935	0.1836	0.9914	0.1852
$\varepsilon_{2t} = 0.5^2$	0.5381	0.9236	0.5656	0.9708	0.5447	0.9349	0.5826
$\varepsilon_{2t} = 1^2$	1.6441	0.9047	1.6705	0.9192	1.6702	0.9191	1.8173

6. Concluding Remarks

At present the rule most often advocated in the literature for monetary policy is the Taylor Rule, even for the case of small open economies subject to external shocks. However, the conjecture that the Taylor rule will continue to perform best when the openness of the

[20] The risk premium is defined as $\exp(-bNFA_t + \ln \chi_{2t})$. In this sense, capital markets are complete when the risk premium is one.

[21] Cecchetti et al. (2000) show that financial disturbances may cause the exchange rate to have destabilizing effects that should be mitigated by monetary policy, while Adolfson (2007) does not support it.

economy encounters imperfections and the reaction coefficients are found optimally has proved misleading if not entirely wrong.[22] In our model, it is always optimal to augment the Taylor rule with some factor that allows for the incorporation of external complications — in our case either the exchange rate gap and/or the law of one price gap. This finding is robust across our four cases of different levels of information availability.

Second, the addition of the lagged interest rate to an optimal simple policy rules typically enhances the level of social welfare and hence policy performance. The reason why this is the case was explored at length and appears to be for a combination of reasons – its implicit ability to capture information missed in other policy instruments, the fact that welfare depends directly in interest rate variability, and as a way of establishing and maintaining commitment. Perhaps most controversially, we find in section 5.2 that the optimized value of the coefficient on the lagged interest rate is greater than one. This is quite different from the empirical estimate usually found in the literature where the coefficient is most often found in the range between 0.7 and 0.9. To the extent that this model captures some of the more important features of small open economies, it would imply that the central banks of these economies have been too timid in practice.

Finally, our analysis suggests that the marginal welfare gain from incorporating foreign exchange considerations and/or lagged interest rates falls as the relevant information available to the policy maker improves. This is perhaps our most practical policy finding. As a practical proposition, it suggests that the less well developed is the statistical information available to a small open economy, the larger will be the gain from including some measure of the size of the external shock facing the economy and/or lagged interest rates in the monetary policy rule.

[22] It may be of interest to note that if "typical" reaction coefficients are assigned to the money rule in this small open economy model, then the loss functions measures of the five monetary policy rules will suggest that the Taylor Rule does work best. As we have seen above, this result disappears when the reaction coefficients are determined optimally.

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