CEP 11-04

Product Line Rivalry: A Further Analysis

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May 2011

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Ottawa, Ontario, Canada
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May 18, 2011

Abstract

In this paper we conduct a further analysis on the Brander and Eaton (1984) model of product line rivalry by examining two cases that have not been studied previously. The common feature shared by these two cases is asymmetry between firms. Specifically, we examine situations where either a) the firms have different marginal costs, or b) they choose quantity sequentially. Our analysis shows that each of these asymmetries between firms can lead to market interlacing in equilibrium.

Keywords: Product Line Rivalry, Market Interlacing, Market Segmentation

JEL Classification: L10

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I. Introduction

In a seminal paper published in the *American Economic Review*, James A. Brander and Jonathan Eaton (1984) demonstrate, in a multi-stage model of product line rivalry, that market segmentation is “a very reasonable outcome” (Brander and Eaton 1984 p330). At the same time, they also point out that market interlacing is possible if demand conditions are such that firms choose overlapping products or if the firms face the threat of entry.

In this paper we conduct a further analysis on the Brander and Eaton (hereafter B&E) model by examining two cases that have not been studied previously, and show that market interlacing can arise as an equilibrium outcome even in the absence of overlapping products and entry threats. The common feature of the two cases we examine is asymmetry between the firms. In contrast to the cases studied in B&E where the two firms have the same marginal cost and move simultaneously at the quantity-setting stage, we study situations where either a) the firms have different marginal costs, or b) they choose their quantities sequentially. Our analysis shows that each of these asymmetries between the two firms can lead to market interlacing in equilibrium.

Since B&E (1984), a number of papers have addressed issues related to marketing segmentation and/or market interlacing. They include Wernerfelt (1986), Bhatt (1987), Martinez-Giralt and Neven (1988), Klemperer (1992), and Janssen *et al.* (2005). However, none of these papers has explored the role of firm asymmetry as a possible cause for market interlacing. In contrast, the present paper identifies and examines factors that can lead to market interlacing when firms are asymmetric.

The paper is organized as follows. The model is presented in section II. The cases of different marginal costs and sequential quantity decisions are analyzed in sections III and IV respectively. Conclusions are in section V.

II. The Model

For the most part, the model used in our analysis is the same as that of B&E. Specifically, two firms, named A and B, produce a total of four possible products, numbered product 1 through product 4. Product pairs (1, 2) and (3, 4) are close substitutes, while pairs (1, 3), (1, 4), (2, 3), and (2, 4) are more distant substitutes. Let p^i denote the price of good *i* and x^i the quantity of good *i* (*i* = 1, 2, 3, 4). The inverse demand function for good *i* is written as $p^i = p^i(x^1, x^2, x^3, x^4)$. Since goods are substitutes, we have $p^i_j < 0$, where the subscript *j* denote the derivative with respect to x^j. In the case where *i* and *j* are close substitutes and *i* and *k* are more distant substitutes, $p^i_j < p^i_k$. Following B&E, we assume that this inequality holds everywhere in the relevant range of interest. With the exception of these differences in substitutability, the demand structure is perfectly symmetric.

To simplify our analysis, we make an additional assumption that each inverse demand function is separable in its arguments. This assumption implies that $p^i_{jk} = 0$ whenever $j \neq k$. To ensure that the second-order conditions of a firm's profit-maximization problem are satisfied and that a firm's best-response function for quantity is decreasing in its rival's quantity of each product, we assume that $p^i_{jj} \leq 0$ for *i*, *j* = 1, 2, 3, 4. Consistent with the assumption on the degree of substitutability among products, we also assume that $p^i_{jj} \leq p^i_{kk} \leq 0$, where *i* and *j* are close substitutes and *i* and *k* are more distant substitutes.

Each firm makes three decisions in three stages: (i) how many products to produce (the scope decision); (ii) which particular products to produce (the line decision); and (iii) which quantity to produce for each product (the quantity decision). Each firm incurs, at the time of scope decision, a sunk cost K for each product it plans to produce. It incurs a constant marginal cost of production c_i (i = A, B) at the time of quantity decision. To ensure that at least one firm produces positive quantities in equilibrium, we assume that $p^i(0,0,0,0) > \max\{c_A, c_B\}$ for i = 1, 2, 3, 4. In B&E, firms have the same marginal cost, *i.e.*, $c_A = c_B$.

B&E focus their analysis on situations where each firm produces two products and the firms choose their quantities simultaneously. Market segmentation occurs when firm A produces good 1 and 2 while firm B produces 3 and 4. On the other hand, market interlacing prevails when firm A produces 1 and 3 while firm B produces 2 and 4. They show that both market segmentation and market interlacing are subgame perfect equilibriums if the firms make the line decisions simultaneously (Proposition 2 in B&E), but market segmentation is the only equilibrium if the firms make the line decisions sequentially (Proposition 3 in B&E). B&E conclude that market segmentation is a very reasonable outcome once the multi-stage structure of market rivalry is explicitly recognized.

Following B&E, we also focus on situations where each of the two firms offers two distinct products. Without loss of generality, suppose firm A produces product pair (1, i) and firm B produces (j, 4). Then market segmentation occurs when (i, j) = (2, 3), and market interlacing occurs when (i, j) = (3, 2). To highlight the role played by asymmetry between firms, we restrict our attention to the situations where market segmentation emerges as the only equilibrium in B&E, *i.e.,* where the firms make their line decisions sequentially. To be more specific, suppose that firm A chooses its product line before firm B does.

III. Different Marginal Costs

The purpose of this section is to examine the implications of different marginal costs in the B&E model. To be more specific, we study the situations where $c_A < c_B$. For ease of presentation, we use superscript s to indicate the case of market segmentation (i = 2, j = 3) and t the case of market interlacing (i = 3, j = 2).

Consider firm A's profit-maximization problem at stage 3 (quantity decisions):

$$\max_{x^{1l},x^{il}} \pi_A^l = p_A^{1l}(x^1,x^2,x^3,x^4)x^{1l} + p_A^{il}(x^1,x^2,x^3,x^4)x^{il} - c_A\left(x^{1l}+x^{il}\right) - 2K \qquad (1)$$

where $\left(l,i,j\right)=\left(s,2,3\right)$ or $\left(t,3,2\right)$. The first-order conditions are:

$$\frac{\partial \pi_A^l}{\partial x^{1l}} = p_1^{1l}x^{1l} + p_1^{il}x^{il} + p^{1l} - c_A = 0 \qquad (2)$$

$$\frac{\partial \pi_A^l}{\partial x^{il}} = p_i^{il}x^{il} + p_i^{1l}x^{1l} + p^{il} - c_A = 0 \qquad (3)$$

Similarly, firm B's profit-maximization conditions at stage 3 are:

$$\frac{\partial \pi_B^l}{\partial x^{jl}} = p_j^{jl}x^{jl} + p_j^{4l}x^{4l} + p^{jl} - c_B = 0 \qquad (4)$$

$$\frac{\partial \pi_B^l}{\partial x^{4l}} = p_4^{4l}x^{4l} + p_4^{jl}x^{jl} + p^{4l} - c_B = 0 \qquad (5)$$

Demand symmetry implies that in equilibrium each firm chooses the same quantity for its two products. Hence, we can denote the equilibrium output vector in the case of market segmentation as $X^s = \left(x_A^s, x_A^s, x_B^s, x_B^s\right)$, and the equilibrium output vector in the case of market interlacing as $X^t = \left(x_A^t, x_B^t, x_A^t, x_B^t\right)$. Then from (2) – (5) we obtain:

$$x_A^l = -\frac{p^{1l} - c_A}{p_1^{1l} + p_1^{il}} \qquad (6)$$

$$x_B^l = -\frac{p^{4l} - c_B}{p_4^{4l} + p_4^{jl}} \qquad (7)$$

Given that firm A has a lower marginal cost of production than firm B, intuitively we expect that firm A produces a larger quantity than firm B in equilibrium. The following proposition confirms that this intuition is indeed correct.

Proposition 1: Under both market segmentation and market interlacing, firm A's equilibrium quantity is greater than Firm B's, *i.e.*, $x_A^s > x_B^s$ and $x_A^t > x_B^t$.

To prove Proposition 1, we first establish the following lemma.

Lemma 1: For any $x' > x > 0$, $p^1(x,x,x',x') > p^4(x,x,x',x')$ and $p^1(x,x',x,x') > p^4(x,x',x,x')$.

PROOF of Lemma 1: Given symmetric demand functions, $p^4(x,x,x',x') = p^1(x',x',x,x)$ Using demand symmetry and the mean value theorem we obtain: $p^1(x,x,x',x') - p^4(x,x,x',x') =$ $p^1(x,x,x',x') - p^1(x',x',x,x) =$ $p_1^1 \cdot (x - x') + p_2^1 \cdot (x - x') + p_3^1 \cdot (x' - x) + p_4^1 \cdot (x' - x) =$ $\left[p_1^1 + p_2^1 - p_3^1 - p_4^1\right] \cdot (x - x')$, which is positive because $0 > p_4^1 = p_3^1 > p_2^1 > p_1^1$ and $x < x'$. Hence, $p^1(x,x,x',x') > p^4(x,x,x',x')$. Similarly, we can show that $p^1(x,x',x,x') > p^4(x,x',x,x')$.

QED

PROOF of Proposition 1: It is clear that $x_A^l = x_B^l$ ($l = s, t$) cannot satisfy (6) – (7) because $c_A \neq c_B$. Here we only need to rule out that $x_A^l < x_B^l$ ($l = s, t$). Observe that $x_A^l < x_B^l$ would imply the following for the numerators and denominators of (6) and (7). In terms of the numerators, $p^{1l} - c_A > p^{4l} - c_B$ by Lemma 1 and $c_A < c_B$. In terms of the denominators, $x_A^l < x_B^l$ would imply

$p_4^{4l} + p_4^{jl} < p_1^{1l} + p_1^{il} < 0$ because p_j^i and p_{jj}^i depend on the value of x^j only , and $p_j^i < 0, p_{jj}^i \leq 0$.

Then (6) and (7) would imply $x_A^l > x_B^l$, which leads to a contradiction. Hence, a necessary condition to satisfy both (6) and (7) is $x_A^l > x_B^l$. **QED**

Since firm A chooses its product line before firm B does, it is able to preempt firm B's decision on product line by its choice between product pairs (1, 2) and (1, 3). Accordingly, our analysis will focus on the profitability of firm A under market interlacing relative to market segmentation. The following proposition suggests that the difference in output levels between firm A and firm B creates an incentive for firm A to choose market interlacing over market segmentation.

Proposition 2: For any $x > x' > 0$, $p^1(x, x', x, x') > p^1(x, x, x', x')$ and $p^3(x, x', x, x') > p^2(x, x, x', x')$.

PROOF: By the mean value theorem, we have,

$$p^1(x, x', x, x') = p^1(x, x, x', x') + p_2^1 \cdot (x' - x) + p_3^1 \cdot (x - x') \qquad (8)$$

Since $p_3^1 > p_2^1$ and $x > x'$, this implies that

$$p^1(x, x', x, x') - p^1(x, x, x', x') = (p_3^1 - p_2^1)(x - x') > 0 \qquad (9)$$

Since demand functions are symmetric and separable, $p^3(x, x', x, x') = p^1(x, x', x, x')$ and $p^2(x, x, x', x') = p^1(x, x, x', x')$. Hence, $p^3(x, x', x, x') > p^2(x, x, x', x')$. **QED**

Propositions 1 and 2 imply that, if both firms were to maintain the same output level under market interlacing as under market segmentation, firm A would receive higher prices under the former than under the latter. This effect, which we will call the "price effect", gives firm A a reason to favor market interlacing.

This price effect is distinct from the effect examined in B&E (1984) and arises only when firms produce different quantities. B&E's analysis suggests that competition is more intense under market interlacing than under market segmentation; hence firm A's profits are unambiguously lower under the former than under the latter. This effect, which we will call the "competition effect," is still present here. But our analysis shows that when firms are asymmetric, an additional factor comes into play. That is, the firm that produces a larger quantity gains an advantage under market interlacing through the price effect.

In what follows we will first examine the implications of the price effect for firm A's output level. Then we will explore the conditions under which market interlacing emerges as the equilibrium outcome.

Proposition 3: Firm A produces a larger quantity under market interlacing than under market segmentation, *i.e.*, $x_A^t > x_A^s$.

PROOF: We prove this proposition by demonstrating that $x_A^t \le x_A^s$ cannot be satisfied in equilibrium. We need to rule out the following two cases: $\left\{x_A^t \le x_A^s, x_B^t \le x_B^s\right\}$ and $\left\{x_A^t \le x_A^s, x_B^t > x_B^s\right\}$. First, consider the case $\left\{x_A^t \le x_A^s, x_B^t \le x_B^s\right\}$. Using the mean value theorem we obtain:

$$\begin{aligned} &p^{1t}\left(X^t\right) - p^{1s}\left(X^s\right) = p_1^1 \cdot \left(x_A^t - x_A^s\right) + p_2^1 \cdot \left(x_B^t - x_A^s\right) + p_3^1 \cdot \left(x_A^t - x_B^s\right) + p_4^1 \cdot \left(x_B^t - x_B^s\right) \\ &> p_1^1 \cdot \left(x_A^t - x_A^s\right) + p_4^1 \cdot \left(x_B^t - x_B^s\right) + p_3^1 \cdot \left[\left(x_A^t - x_A^s\right) + \left(x_B^t - x_B^s\right)\right] \ge 0 \end{aligned} \tag{10}$$

In other words, $p^{1t} > p^{1s}$. Using this and $p_{jj}^i \le 0$ to compare (6) in the case $l = s$ with that in the case $l = t$, we find that $x_A^t > x_A^s$, which leads to a contradiction.

Second, consider the case $\left\{x_A^t \le x_A^s, x_B^t > x_B^s\right\}$. This, together with Proposition 1, implies that $x_A^s \ge x_A^t > x_B^t > x_B^s$. Using the mean value theorem, we obtain

$$\Delta p^1 = p^{1t} - p^{1s} = p_1^1 \cdot \left(x_A^t - x_A^s\right) + p_2^1 \cdot \left(x_B^t - x_A^s\right) + p_3^1 \cdot \left(x_A^t - x_B^s\right) + p_4^1 \cdot \left(x_B^t - x_B^s\right) \quad (11)$$

$$\Delta p^4 = p^{4t} - p^{4s} = p_1^4 \cdot \left(x_A^t - x_A^s\right) + p_2^4 \cdot \left(x_B^t - x_A^s\right) + p_3^4 \cdot \left(x_A^t - x_B^s\right) + p_4^4 \cdot \left(x_B^t - x_B^s\right) \quad (12)$$

Then,

$$\Delta p^4 - \Delta p^1 = \left(p_1^4 - p_1^1\right)\left(x_A^t - x_A^s\right) + \left(p_2^4 - p_2^1\right)\left(x_B^t - x_A^s\right) + \left(p_3^4 - p_3^1\right)\left(x_A^t - x_B^s\right) + \left(p_4^4 - p_4^1\right)\left(x_B^t - x_B^s\right) \quad (13)$$

which has a negative sign given $x_A^s \geq x_A^t > x_B^t > x_B^s$ and the assumptions on the signs and relative magnitudes of p_j^i. In other words, $\Delta p^4 < \Delta p^1$.

On the other hand, from the first-order conditions (2) – (5), we obtain:

$$\Delta p^1 = p^{1t} - p^{1s} = (p_1^{1s} + p_1^{2s})x_A^s - (p_1^{1t} + p_1^{3t})x_A^t \quad (14)$$

$$\Delta p^4 = p^{4t} - p^{4s} = (p_4^{4s} + p_4^{3s})x_B^s - (p_4^{4t} + p_4^{2t})x_B^t \quad (15)$$

These imply

$$\Delta p^4 - \Delta p^1 = \left\{\left[p_4^{4s} + p_4^{3s}\right]x_B^s - \left[p_1^{1s} + p_1^{2s}\right]x_A^s\right\} - \left\{\left[p_4^{4t} + p_4^{2t}\right]x_B^t - \left[p_1^{1t} + p_1^{3t}\right]x_A^t\right\} \quad (16)$$

Define $F(x) = -[p_1^1(x) + p_1^2(x)]x$ and $G(x) = -[p_1^1(x) + p_1^3(x)]x$, where p_j^i only depends on x_j due to the separability of the demand functions. We have $F(x) > G(x) > 0$ and $F'(x) > G'(x) > 0$ given the assumptions on the relative magnitudes of p_j^i and p_{jj}^i. Then

$$\Delta p^4 - \Delta p^1 = \int_{x_B^s}^{x_A^s} dF(x) - \int_{x_B^t}^{x_A^t} dG(x) > \int_{x_B^t}^{x_A^t} F'(x)dx - \int_{x_B^t}^{x_A^t} G'(x)dx = \int_{x_B^t}^{x_A^t}\left[F'(x) - G'(x)\right]dx > 0, \quad (17)$$

which is in direct contradiction to the conclusion reached in the previous paragraph. **QED**

Intuitively, the impact of the price effect and, respectively, the competition effect on firm A's quantity can be seen from the numerator and, respectively, the denominator of (6). Suppose the firms produce the same quantities under market interlacing as under market segmentation. Then Propositions 1 and 2 suggest that $p^{1t} - c_A > p^{1s} - c_A$ (the price effect), while the assumptions on the degree of substitutability among products imply that $-(p_1^{1t} + p_1^{3t}) <$ $-(p_1^{1s} + p_1^{2s})$ (the competition effect). From (6) we can see that both the price effect and the competition effect cause firm A to produce a larger quantity under market interlacing than under market segmentation.

On the other hand, the price effect and competition effect have opposing impacts on firm A's profits. While higher prices improve firm A's profits, intensified competition reduces them. As a result, market interlacing will emerge as the equilibrium outcome only if the price effect dominates the competition effect.

Next we proceed to derive a sufficient condition under which market interlacing is the equilibrium outcome. Given that the price effect here is caused by the difference in the marginal costs of the two firms, intuitively we expect that the price effect should be larger when the gap between the firms' marginal costs is wider. However, if this gap becomes too wide, firm B produces no output, in which case firm A becomes a monopolist and the notion of market interlacing and market segmentation becomes vacuous. The following lemma establishes such critical conditions in terms of c_B.

Lemma 2: For a given c_A, there exist $c_B^s(c_A)$ and $c_B^t(c_A)$ such that $x_B^l = 0$ for $c_B \geq c_B^l(c_A)$ (l = s, t). Moreover, $c_B^s > c_B^t > c_A$.

PROOF: Let x_A^{lm} denote firm A's monopoly output under market interlacing ($l = t$) and market segmentation ($l = s$); that is, x_A^{lm} is a solution to (6) with the value of x_B^l set to zero. Then from (7) we know that $c_B^s = p^{4s}(x_A^{sm}, x_A^{sm}, 0, 0)$ and $c_B^t = p^{4t}(x_A^{tm}, 0, x_A^{tm}, 0)$. Obviously, $c_B^l > c_A$ for both $l = t$ and $l = s$.

Using the mean value theorem, we can easily show that $p^{4s}(x_A^{sm}, x_A^{sm}, 0, 0) >$ $p^4(x_A^{sm}, 0, x_A^{sm}, 0)$. Since $x_A^{sm} < x_A^{tm}$ by Proposition 3 and $p_i^j < 0$, we obtain $p^4(x_A^{sm}, 0, x_A^{sm}, 0) >$ $p^{4t}(x_A^{tm}, 0, x_A^{tm}, 0)$. Hence, $c_B^s = p^{4s}(x_A^{sm}, x_A^{sm}, 0, 0) >$ $p^{4t}(x_A^{tm}, 0, x_A^{tm}, 0) = c_B^t$. **QED**

Lemma 2 implies that firm B produces positive quantities under both market interlacing and market segmentation as long as $c_B < c_B^t(c_A)$. Given our interest in product line rivalry, we will only consider situations where both firms produce positive quantities; in other words, we will focus on the case where $c_B < c_B^t(c_A)$.

Proposition 4: There exists $c_B^* < c_B^t(c_A)$ such that market interlacing is the equilibrium outcome for any $c_B \in (c_B^*, c_B^t(c_A))$.

PROOF: Consider the case $c_B = c_B^t(c_A)$. The equilibrium output levels are $\left(x_A^t, x_B^t, x_A^t, x_B^t\right) =$ $= \left(x_A^{tm}, 0, x_A^{tm}, 0\right)$ under market interlacing and $\left(x_A^s, x_A^s, x_B^s, x_B^s\right)$ with $x_B^s > 0$ under market segmentation. Note $\pi_A^t\left(x_A^{tm}, 0, x_A^{tm}, 0\right) > \pi_A^t\left(x_A^{sm}, 0, x_A^{sm}, 0\right)$ by the definition of x_A^{tm} and $\pi_A^t\left(x_A^{sm}, 0, x_A^{sm}, 0\right) > \pi_A^s\left(x_A^{sm}, x_A^{sm}, 0, 0\right)$ by the implication of Proposition 2. Furthermore, $\pi_A^s\left(x_A^{sm}, x_A^{sm}, 0, 0\right) > \pi_A^s\left(x_A^s, x_A^s, x_B^s, x_B^s\right)$ by the definition of x_A^{sm}. Hence, we have $\pi_A^t\left(x_A^{tm}, 0, x_A^{tm}, 0\right) > \pi_A^s\left(x_A^s, x_A^s, x_B^s, x_B^s\right)$. By continuity, there exists $c_B^* < c_B^t(c_A)$ such that $\pi_A^t > \pi_A^s$ for $c_B \in (c_B^*, c_B^t(c_A))$. In other words, firm A chooses products 1 and 3 at stage 2 for any $c_B \in (c_B^*, c_B^t(c_A))$. In this case, the best response of firm B is to choose products 2 and 4 at stage 2 because producing the same product(s) as firm A would further intensify the competition and lead to even smaller profits for firm B. Hence, market interlacing is the equilibrium outcome if $c_B \in (c_B^*, c_B^t(c_A))$. **QED**

The preceding analysis shows that the quantity difference caused by the difference in marginal costs can lead to market interlacing under some circumstances. Of course, a difference in marginal costs is not the only factor that can lead firms to produce different quantities. Sequential quantity choices by the two firms can also lead to different levels of output. This we examine in the next section.

IV. Sequential Quantity Decisions

In this section we consider the case where firms make their decisions sequentially both at the stage of product line choice and at the stage of quantity choice. To be more specific, suppose that at each stage firm A moves first, and firm B moves the second. To highlight the role of sequential output decisions, we revert back to B&E's original assumption that firms have the same marginal cost c, *i.e.*, $c_A = c_B = c$. We will first identify the incentives for firm A to choose market interlacing using the general model, and then explore the conditions under which market interlacing emerges as the equilibrium outcome using a more specific model where the demand functions are linear.

The profit-maximization problem of firm B (the Stackelberg follower) at stage 3 is the same as that in section III. Accordingly, the profit-maximization conditions (4) and (5) determine the best-response functions of firm B: $\tilde{x}^{jl}(x^1, x^i)$ and $\tilde{x}^{4l}(x^1, x^i)$, where (l, i, j) = (s, 2, 3) and (t, 3, 2). Using comparative statics, we can show that $\partial\tilde{x}^{kl} / \partial x^1 < 0$ and $\partial\tilde{x}^{kl} / \partial x^i < 0$ for $k = j$ and 4.

When choosing its output levels, firm A takes into consideration firm B's best response functions. Hence, firm A's profit-maximization problem is

$$\max_{x^{1l}, x^{il}} \pi_A^l = [p^{1l}(x^1, x^2, x^3, x^4) - c]x^{1l} + [p^{il}(x^1, x^2, x^3, x^4) - c]x^{il} - 2K \quad (18)$$

subject to $x^j = \tilde{x}^{jl}(x^1, x^i)$ and $x^4 = \tilde{x}^{4l}(x^1, x^i)$. (19)

The first-order conditions are:

$$\frac{\partial \pi_A^l}{\partial x^{1l}} = p_1^{1l} x^{1l} + p^{1l} + p_1^{il} x^{il} - c + x^{1l}(p_j^{1l} \frac{\partial \tilde{x}^{jl}}{\partial x^1} + p_4^{1l} \frac{\partial \tilde{x}^{4l}}{\partial x^1}) + x^{il}(p_j^{il} \frac{\partial \tilde{x}^{jl}}{\partial x^1} + p_4^{il} \frac{\partial \tilde{x}^{4l}}{\partial x^1}) = 0 \quad (20)$$

$$\frac{\partial \pi_A^l}{\partial x^{il}} = p_i^{il} x^{il} + p^{il} + p_i^{1l} x^{1l} - c + x^{1l}(p_j^{1l} \frac{\partial \tilde{x}^{jl}}{\partial x^i} + p_4^{1l} \frac{\partial \tilde{x}^{4l}}{\partial x^i}) + x^{il}(p_j^{il} \frac{\partial \tilde{x}^{jl}}{\partial x^i} + p_4^{il} \frac{\partial \tilde{x}^{4l}}{\partial x^i}) = 0 \quad (21)$$

Given the symmetric demand functions, the equilibrium quantities of the two products produced by each firm are equal. With a slight abuse of notations, we will continue to use x_A^l and x_B^l ($l = s, t$) to denote the equilibrium quantity of each product produced by firm A and firm B, respectively. Using the symmetry and separability of the demand functions we rewrite firm A's first-order conditions as:

$$(p_1^{1s} + p_1^{2s})x_A^s + p^{1s} - c + x_A^s(p_3^{1s} + p_4^{1s})\frac{\partial(\tilde{x}^{3s} + \tilde{x}^{4s})}{\partial x^1} = 0 \quad (22)$$

in the case of market segmentation, and

$$(p_1^{1t} + p_1^{3t})x_A^t + p^{1t} - c + x_A^t(p_2^{1t} + p_4^{1t})\frac{\partial(\tilde{x}^{2t} + \tilde{x}^{4t})}{\partial x^1} = 0 \quad (23)$$

in the case of market interlacing.

Using the above conditions, one can easily verify that under both market segmentation and market interlacing, the Stackelberg leader (firm A) produces a larger quantity than the follower (firm B); that is, $x_A^s > x_B^s$ and $x_A^t > x_B^t$. Then Proposition 2 applies and firm A has a motive for choosing market interlacing over market segmentation because of the price effect.

The purpose of examining the case of sequential quantity choices, however, is to show that firm A has a second motive for favoring market interlacing, as implied by the following proposition.

Proposition 5: At $x^1 = x^i = x_A$, and $x^j = x^4 = x_B$ for $(i, j) = (2, 3)$ and $(3, 2)$,

$$\frac{\partial(\tilde{x}^{2t} + \tilde{x}^{4t})}{\partial x^1} < \frac{\partial(\tilde{x}^{3s} + \tilde{x}^{4s})}{\partial x^1} < 0 \text{ and } \frac{\partial(\tilde{x}^{2t} + \tilde{x}^{4t})}{\partial x^3} < \frac{\partial(\tilde{x}^{3s} + \tilde{x}^{4s})}{\partial x^2} < 0$$

PROOF: Conducting comparative static analysis on equation system (4) – (5) for each of $(l, i, j) = (s, 2, 3)$ and $(t, 3, 2)$, we find, after simplifications using the separable and symmetric properties of the demand functions, that at $x^1 = x^i = x_A$, and $x^j = x^4 = x_B$

$$\frac{\partial(\tilde{x}^{2t} + \tilde{x}^{4t})}{\partial x^1} = -\frac{p_1^{2t} + p_1^{4t}}{x_B(p_{44}^{4t} + p_{44}^{2t}) + 2p_4^{4t} + p_2^{4t} + p_4^{2t}} < 0 \qquad (24)$$

$$\frac{\partial(\tilde{x}^{3s} + \tilde{x}^{4s})}{\partial x^1} = -\frac{p_1^{3s} + p_1^{4s}}{x_B(p_{44}^{4s} + p_{44}^{3s}) + 2p_4^{4s} + p_3^{4s} + p_4^{3s}} < 0 \qquad (25)$$

The signs of (24) and (25) are negative because $p_j^i < 0$ and $p_{jj}^i \leq 0$ for $i, j = 1, 2, 3, 4$. Separability and symmetry of the demand functions imply that at $x^1 = x^i = x_A$, and $x^j = x^4 = x_B$, $p_1^{4t} = p_1^{4s}$, $p_4^{4t} = p_4^{4s}$, $p_{44}^{4t} = p_{44}^{4s}$, $p_2^{4t} = p_4^{2t}$, and $p_3^{4s} = p_4^{3s}$. The assumptions $p_1^2 < p_1^3 < 0$, $p_3^4 < p_2^4 < 0$, and $p_{44}^3 \leq p_{44}^2 \leq 0$ imply that $\frac{\partial(\tilde{x}^{2t} + \tilde{x}^{4t})}{\partial x^1} < \frac{\partial(\tilde{x}^{3s} + \tilde{x}^{4s})}{\partial x^1}$. Using the same reasoning we can show that $\frac{\partial(\tilde{x}^{2t} + \tilde{x}^{4t})}{\partial x^3} < \frac{\partial(\tilde{x}^{3s} + \tilde{x}^{4s})}{\partial x^2}$. **QED**

Proposition 5 suggests that a unit increase in firm A’s output elicits a larger reduction in firm B’s total output in the case of market interlacing than in the case of market segmentation.

In other words, the Stackelberg follower is more accommodating of the leader's output expansion under market interlacing than under market segmentation. This property, which we will call "quantity effect," confers firm A (the Stackelberg leader) an advantage that it can exploit under market interlacing.

To gain more insight into the conditions under which firm A will indeed choose product pair (1, 3) over (1, 2), we consider a system of linear demand functions: $P^T = A^T - BX^T$, where $P = (p^1, p^2, p^3, p^4)$, $A = (a, a, a, a)$, and

$$B = \begin{pmatrix} b & d & e & e \\ d & b & e & e \\ e & e & b & d \\ e & e & d & b \end{pmatrix} \qquad (26)$$

The assumptions on the degree of substitutability between pairs of products imply that $b > d > e > 0$.

Note that parameter d measures the degree of substitutability within a pair of products, and e measures the degree of substitutability between pairs of products. The closer d is to b, the higher is the substitutability between products 1 and 2 (or between 3 and 4). The closer e is to d, the higher is the substitutability between pair (1, 2) and pair (3, 4).

It is straightforward to derive the equilibrium prices, quantities, and profits in cases of market segmentation and market interlacing. Comparison of firm A's profits in these two cases reveals:

Proposition 6: Suppose that demand functions are linear. For d in the interval $[\sqrt{3}b/3,\ b)$, there exists $e^* \in [0,\ d)$ such that for $e \in (e^*,\ d)$, market interlacing is the equilibrium outcome. Furthermore, $e^* = 0$ for d sufficiently close to b.

PROOF: Firm A's equilibrium level of profits is

$$\pi_A^s = \frac{(a-c)^2(b+d-e)^2}{2(b+d)[(b+d)^2-2e^2]} - 2K \qquad (27)$$

under market segmentation, and

$$\pi_A^t = \frac{(a-c)^2(2b-d+e)^2}{4(b+e)[2(b+e)^2-(d+e)^2]} - 2K \qquad (28)$$

under market interlacing. Define $H(d,e) \equiv \pi_A^s - \pi_A^t$ and examine the sign of $H(d, e)$ for e in the interval $[0, d]$. Firm A would use its first-mover advantage to choose product pair (1, 3) if $H(d, e) < 0$. It is easy to verify that $H(d, e) = 0$ at $e = d$. Furthermore,

$$\left.\frac{\partial H}{\partial e}\right|_{e=d} = \frac{(a-c)^2 db}{2(b+d)^2(b^2+2bd-d^2)^2}(3d^2-b^2), \qquad (29)$$

which has a positive sign if $d > \sqrt{3}b/3$. Given the latter condition, by continuity there exists $e^* \in [0, d)$ such that $G(d, e) < 0$ for $e \in (e^*, d)$. To prove the second part of this proposition, note that when evaluated at $d = b$, $H(d, e) < 0$ for all $e > 0$. By continuity, for d sufficiently close to b, $H(d, e) < 0$ for all $e \in (0, d)$, *i.e.*, $e^* = 0$. **QED**

Proposition 6 highlights the role played by the degrees of demand substitutability in affecting firm A's decision to choose market interlacing over market segmentation. To the firm, the advantage of market segmentation is that competition intensity is lower than under market interlacing. But this advantage is smaller if there is a higher degree of substitutability either between the two pairs of products (*i.e.*, a larger e) or within each pair of products (*i.e.*, a larger d), in which case the price effect and quantity effect associated with the Stackelberg leadership are more likely to dominate. That is why market interlacing is the equilibrium outcome when the values of d and e are sufficiently large.

V. Conclusions

This paper extends B&E's analysis by examining two cases where the firms are asymmetric in some aspects. We show that market interlacing can emerge as the equilibrium outcome even in situations where market segmentation would have been the only possible outcome had the firms been symmetric. Our analysis identifies two factors that come into play when firms are asymmetric. First, the firm with a larger output receives, *ceteris paribus*, higher prices for its products under market interlacing than under market segmentation. Second, when firms choose quantity sequentially, the follower becomes more accommodating of the quantity expansion by the Stackelberg leader, a property that the leader can exploit under market interlacing. Our analysis demonstrates the circumstances under which these two factors lead to market interlacing in equilibrium.

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