Macroeconomic Stabilisation:
Fixed Exchange Rates vs Inflation Targeting vs Price Level Targeting*

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Abstract

We argue that the traditional question 'fixed vs. flexible exchange rates?' is not well-defined, because 'flexible exchange rates' does not explicitly specify any particular monetary policy. In traditional analyses, 'flexible exchange rates' was interpreted as implying a fixed money supply. But fixing the money supply (or fixing its growth rate at k%) is rarely advocated nowadays. To reflect today's policy debate, the traditional question should be replaced by the question 'fixed exchange rates vs. inflation targeting vs. price level targeting?'. We then build a simple macroeconomic model of a small open economy. The model incorporates an 'outside lag' in the effect of monetary policy on aggregate demand, so that inflation targeting and price level targeting are always imperfect. We use this model to compare the stabilisation properties of three different monetary rules: a fixed exchange rate, a fixed inflation target, and a fixed price level target. We show that price level targeting is best for stabilising output, the real exchange rate and the real interest rate, relative to their natural rates.

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1. Introduction

A standard exam question is: 'Which is best for macroeconomic stabilisation, fixed or flexible exchange rates?' Though we have used this question ourselves in the past, we have now come to the conclusion that it is not, despite appearances, a well-defined question. This is because 'flexible exchange rates' does not explicitly specify what monetary policy shall be.

Suppose there is a mapping from time paths of the money supply into time paths of the exchange rate. Then, 'fixed exchange rates' refers to that unique time path of the money supply which keeps the exchange rate fixed at its current level, whereas 'flexible exchange rates' can refer to all other possible time paths of the money supply. 'Fixed exchange rates' then tightly specifies what monetary policy shall be (at least in a world of high capital mobility, where sterilisation of reserve flows is ineffective), and is a well-defined policy. But 'flexible exchange rates' allows monetary policy to be almost anything, since any of the alternative time paths for the money supply will result in the exchange rate varying over time. Furthermore, since expected future monetary policy matters at least as much as actual monetary policy, merely saying that the central bank will adopt a policy of flexible exchange rates leaves no anchor for those expectations. Almost anything can happen, and almost anything can be expected to happen.

'Flexible exchange rates' is not a well-defined policy. Therefore, 'fixed vs flexible exchange rates' is not a well-defined pair of alternative policies. To convert it into a well-defined question, we have to specify what particular monetary policy will be followed under 'flexible exchange rates'.

In traditional analyses of fixed vs flexible exchange rates, 'flexible exchange rates' has been used as shorthand for 'fixed (or exogenous) money supply'.[1] Now the question 'fixed exchange rates or fixed money supply?' does have the virtue of creating a well-defined pair of alternative policies. And when Milton Friedman's k% rule for the money supply was a serious candidate for monetary policy, then 'fixed exchange rates vs fixed (k%) money growth' was also an interesting question for policy debates. But the k% rule is no longer widely advocated, and so the traditional interpretation of 'flexible exchange rates' is no longer relevant for macroeconomic policy debates. Instead, the most plausible current alternatives to a policy of fixed exchange rates are inflation targeting and price level targeting. Today then, the best way to interpret the old exam question 'fixed vs. flexible exchange rates' is: 'fixed exchange rates vs inflation targeting vs price level targeting'.

In this paper we compare these three policy rules in the context of a simple macroeconomic model. Our model is the standard Mundell-Fleming model of a small open economy, modified only by including a lag in the effect of the real interest rate and real exchange rate on aggregate demand. We are not aware of any paper which models the question of fixed vs flexible exchange rates, interpreted in this policy-

[1] Cf. the treatment of fixed vs flexible rates in any intermediate textbook treatment of the Mundell-Fleming model.

relevant way.[2] Moreover, the very simplicity and familiarity of our model makes transparent the mechanisms at work. We are asking the right question, and showing how it can be answered. Later researchers may wish to answer the same question using more sophisticated models.

In constructing a simple model, we are forced to ignore many important and interesting issues in the debate over fixed vs flexible exchange rates. Foremost amongst these is the question of the credibility of the policy rules -- whether they could have the institutional support to be followed, and therefore maintain a set of expectations about future prices and exchange rates.[3] In particular, we ignore the very real problem of speculative attacks on a fixed exchange rate.[4] Our simple model assumes without argument that any policy rule is perfectly credible.

'Fixed exchange rates' normally means a policy of keeping the exchange rate constant over time. But in our model, the results would be the same under a crawling peg. Similarly, a price level target could specify a constant price level, or could specify a crawling peg for the price level, so that the central bank commits to trying to attain a CPI of 100 in the year 2000, a CPI of 101 in 2001, 102 in 2002, etc. Similarly, an inflation target could specify zero annual inflation, or (as is more usual) an inflation target of (say) 2% per annum. For simplicity we will talk about a constant price level target, and an inflation target of zero, but our results do not depend on this. And the distinction between a price level target and an inflation target is not that the former targets zero inflation while the latter may target positive inflation. A price level target with a crawling peg of 2% per annum is quite different from a 2% inflation target, as will be explained in the next paragraphs.

The central bank has almost instantaneous information on the exchange rate, and the exchange rate reacts almost instantaneously to the central bank's actions. This means that (assuming away speculative attacks, and assuming that fiscal policy is consistent with monetary policy) the central bank can control the exchange rate almost perfectly. 'Fixed exchange rates' is a monetary policy which (given our assumptions) can be implemented on a day-to-day basis. But central banks cannot perfectly control the inflation rate or price level. Because of lagged information on prices (the 'inside lag'), and lags in the effect of monetary policy on prices (the 'outside lag'), central banks can only implement inflation targeting or price level targeting over a longer time period, of perhaps two or more years. It is this fact which creates the distinction between inflation targeting and price level targeting.

To understand the difference between price level targeting and inflation targeting, suppose that there is an unforeseen shock which causes the price level (and hence the inflation rate) to rise above the bank's target. Under a policy of price level targeting, the bank will try to reverse that shock in the following period to push the price level back to its target path, and so the price level will follow a stationary

[2] While the recent literature on fixed vs flexible exchange rates, e.g., Devereux and Engel (1999) and references cited therein, employs sophisticated modeling techniques, it is not directly relevant to the current policy debate in small open economies such as Canada, where fixed money growth is not on the policy agenda. The nearest papers that we can find to our question are those which, like Boyer (1978) and Roper and Turnovsky (1980), ask, what is the optimal *degree* of foreign exchange market intervention? They do not, of course, explicitly consider modern policies such as inflation or price level targeting.

[3] Cf. Laidler (1999).

[4] Cf. Osakwe and Schembri (1998).

process (superimposed on a time trend under a crawling peg). But under a policy of inflation targeting, the bank will accommodate that previous undesired rise in the price level, and so the price level will follow a random walk (or a random walk with drift under a positive inflation target).

We wish to see which of the three policy rules best stabilises real output. We *could* calculate the variance of real output under each of the three policy rules. However, not all fluctuations in output are undesirable; fluctuations which would occur in a flexible price, neoclassical, competitive equilibrium are presumably efficient, and should be accommodated. So instead we calculate the variance of output *relative to the natural rate.* That policy rule is best which minimises the variance of deviations of output from the natural rate.

But perhaps stability of *aggregate* output is not the only measure of successful stabilisation. Some advocates of fixed exchange rates argue that a policy of flexible exchange rates puts too great a burden of macroeconomic adjustment on the traded goods sector, and too little burden of adjustment on the non-traded goods sector.[5] It is even conceivable that a particular monetary policy might perfectly stabilise aggregate output, but drastically destabilise each individual component of aggregate output. For example, if a particular monetary policy caused output in both the traded and non-traded goods sectors to have a very high variance, but be perfectly negatively correlated, aggregate output could be perfectly stabilised, but this would not necessarily be desirable.

To examine the relevance of this argument, we also compare the variance of real exchange rates, and the variance of real interest rates, under each of the three policy rules. The variance of real exchange rates should give us an indicator of stabilisation in the traded goods sector, and the variance of real interest rates should give us an indicator of stabilisation in the interest-sensitive sector. But, as is true for output, not all fluctuations in real exchange rates and real interest rates are bad. Relative prices ought to fluctuate in the face of real shocks. Fluctuations in real exchange rates and real interest rates which would happen in a flexible price, neoclassical, competitive equilibrium (i.e., fluctuations in their 'natural rates') are presumably efficient. To allow for this, we look for the lowest variance of real exchange rates and real interest rates relative to their respective natural rates. If the real exchange rate and real interest rate are always equal to their (possibly fluctuating) natural rates, we deem them to be perfectly stabilised.

In traditional analysis, the desirability of fixed vs flexible exchanges rates (i.e., fixed exchange rates or fixed money supply) depends on the contemporaneous responsiveness of the economy to various different shocks (e.g., the slopes of *IS*, *LM*, and *BP* curves), and on the relative variances and covariance of such shocks. Whether fixed or flexible exchange rates best stabilise the economy is parameter-dependent. In our analysis the main results are not parameter-dependent. Price-level targeting best stabilises output, the real exchange rate, and the real interest rate, relative to their natural rates. Moreover, the traditional literature's emphasis on the importance of the sources of shocks disappears when 'fixed vs flexible exchange rates' is interpreted in this new way. What matters instead is whether a given shock is observed or unobserved.

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[5] Cf. Courchene and Harris (1999).

2. The Model

We want our model to be as simple as possible. But we want it to have the following two essential features.

First, the central bank must have an *effective* informational advantage over the private sector (due to, e.g., nominal wage contracts). Without this feature, all (deterministic) monetary policy rules would have the same variance of real output (the policy ineffectiveness proposition), and the results would be uninteresting.

Second, whereas the central bank can influence the exchange rate instantaneously, it can only influence output and the price level with a lag. Without this lag, the central bank would have instantaneous control over aggregate demand, prices, and output, and so macroeconomic stabilisation could be perfect. Whenever output started to rise above trend, the central bank could tighten monetary policy, and instantly bring output back to a smooth trend. Without this lag, stabilisation policy would be not only perfect, but trivially easy. Clearly stabilisation policy is neither perfect nor trivially easy. To ignore lags would then obviously bias the results against fixed exchange rates.

Lags are generally divided into 'inside lags' and 'outside lags'. The inside lag is the time it takes information on prices and output to reach the central bank, for a decision to be made on how to respond to new information, and for that decision to be implemented. The outside lag is the time it takes for the change in monetary instruments to affect output and prices. Our prior is that the outside lag is much longer, and hence presumably much more important, than the inside lag. Wishing to keep the model as simple as possible, we therefore incorporate an outside lag only. We assume that the central bank has full contemporaneous information, and can react instantly to new information, but that monetary policy takes time to affect output and prices, and so it cannot perfectly stabilise the economy.

We use a very simple model of the economy, comprising two equations only: an aggregate demand equation, and an aggregate supply equation. With one exception, the aggregate demand side of our model is a standard, textbook, Mundell-Fleming model of a small open economy. That one exception is the assumption of an 'outside lag' in the effect of monetary policy on aggregate demand. The aggregate supply equation is also a standard, textbook, New Keynesian supply equation in which output responds to the difference between the actual price level and the (two-period) prior expectation of the price level. We assume rational expectations. For simplicity, we assume the model is linear.

We assume that aggregate demand depends linearly on the (log) real exchange rate, on the real interest rate, and on a serially correlated aggregate demand shock. We need to create a lag in the effect of monetary policy on output and the price level. We therefore assume that there is a one-period lag in implementing spending plans, so that firms and households decide this period how much goods and services to buy next period. This is akin to the 'time to build' metaphor in real business cycle theory. More precisely, our story is of a 'time to start building'. Agents decide this period what to 'build', but do not actually start to build until next period. Since expenditures on investment goods do not start until next period, investment decisions are based not on the current real interest rate, but on the expectation of next period's real interest rate. Similarly, since production of net exports does not start until next period, production decisions are based not on the current real exchange rate, but on the expectation of next period's real exchange rate.[6]

The (log) real exchange rate can be written as the (log) nominal exchange rate S, plus the (log) foreign price level P^*, minus the (log) domestic price level P. Assuming perfect capital mobility and hence uncovered interest rate parity, the real interest rate can be written as the foreign interest rate i^*, plus the expected change in the exchange rate, minus the expected inflation rate. Thus the aggregate demand equation becomes:

$$(1)\ Y_{t+1} = a + bE(S_{t+1} + P^*_{t+1} - P_{t+1})/ \mathrm{I}_t - cE(i^*_{t+1} + S_{t+2} - S_{t+1} - P_{t+2} + P_{t+1})/ \mathrm{I}_t + u_{t+1}$$

where $a>0$, $b>0$, $c>0$, and u is an AR(1) process with serial correlation parameter p, so that $u_{t+1} = pu_t + w_{t+1}$, where w_{t+1} is a white noise disturbance with zero mean.

The second equation is an aggregate supply equation. It says that output responds positively to the difference between the actual and expected price level, and negatively to a serially correlated aggregate supply shock. There is a two-period lag in expectations adjustment. We need this two period lag to give the central bank an effective informational advantage; a one-period lag is not enough, since it takes one period for monetary policy to affect aggregate demand. This two-period lag could be the result of Fischer-style two-period nominal wage contracts.[7] Note that we have defined a *positive* aggregate supply shock as one which raises prices but lowers output. This is simply to make the equations look neater.

$$(2)\ Y_{t+1} = (1/g)(P_{t+1} - \mathrm{E}[P_{t+1} / \mathrm{I}_{t-1}]) - v_{t+1}$$

where $g>0$, and v is an AR(1) process with serial correlation parameter r, so that $v_{t+1} = rv_t + z_{t+1}$, where z_{t+1} is a white noise disturbance with zero mean.

Substituting the *AD* equation (1) into the *AS* equation (2) and rearranging gives us an expression for the price level:

$$(3) P_{t+1} = E(P_{t+1} \mid I_{t-1}) + g[a + bE\{(S_{t+1} + P^*_{t+1} - P_{t+1}) \mid I_t\} - cE\{(i^*_{t+1} + S_{t+2} - S_{t+1} - P_{t+2} + P_{t+1}) \mid I_t\} + u_{t+1} + v_{t+1}]$$

Our *AD* equation (1) can be understood as the result of substituting the *BP* equation into the *IS* equation of the Mundell-Fleming model, and introducing an outside lag. We do not need to introduce the LM

[6] Svensson (1998) has a similar model of a small open economy. In his model, as in ours, aggregate demand is a function of expected interest rates and exchange rates, and monetary policy has an outside lag. His model is more complicated than ours, and he uses it to examine different methods of inflation targeting, rather than to compare fixed exchange rates and price level targeting to inflation targeting.

[7] Strictly speaking, overlapping two period nominal wage contracts should generate an aggregate supply equation containing both one- and two-period lagged expectations, but we ignore the former here for simplicity.

curve into the analysis, since the money supply is endogenous under each of our three policy regimes (fixed exchange rates, inflation targeting, and price level targeting). Under fixed exchange rates, the central bank directly targets the exchange rate. Under inflation or price level targeting, the central bank will use the exchange rate as an intermediate target to try to hit its ultimate inflation or price level target.

Since the money supply does not appear directly in the model, it is best to think of the central bank using the spot exchange rate as its policy instrument. But notice that the *actual* exchange rate has no effect on aggregate demand. Instead, it is the *expected future* exchange rates (one and two periods in the future) which affect aggregate demand. In this model, therefore, we should think of monetary policy as a feedback rule for the actual exchange rate, which generates a time-path for the expected exchange rate. The rule for fixed exchange rates is simple; the bank simply commits to holding the exchange rate constant. More complex feedback rules for the exchange rate will generate the policies of inflation targeting or price level targeting. We will solve for those feedback rules below.

Because of the lag we have built into our *AD* equation, output in any period is predetermined by aggregate demand in the previous period. The short-run aggregate demand curve (taking the previous period's expectations as given) is therefore vertical. This is consistent with policymakers' views of how monetary policy affects a small open economy with perfect capital mobility. Monetary policy immediately affects the exchange rate, and expected future exchange rates (and possibly also the real interest rate via its effect on expected future exchange rates and prices), but these have a *lagged* effect on aggregate demand, and on real output and the price level. And because monetary policy has a lagged effect, macroeconomic stabilisation is necessarily imperfect.

We assume that the information set at time t, I_t , both for the central bank and for the public, contains all variables dated t and earlier. In particular, this means that the shocks to *AD* and *AS*, u_{t+1} and v_{t+1}, are unobserved by the bank and the public at date t (although their serial correlation allows for imperfect forecastability). Initially, we will assume that the parameters a, b, c, and the foreign variables P^* and i^*, stay constant. We shall return to this assumption in section 8.

3. Fixed Exchange Rate

Under a fixed exchange rate policy, we assume that the central bank adjusts monetary policy to maintain the value of the nominal exchange rate at a constant, given level, and that it can do so instantaneously, i.e., $S_t = \bar{S} \forall t$. This implies that the expected change in the exchange rate is zero. We impose this on equation (3) above. We find the following solution for the nominal price level:

$$(4) P_{t+1} = \bar{S} + P^* + (a/b) - (ci^*/b) + g(w_{t+1} + z_{t+1}) + \frac{gp(b+c)}{(b+c(1-p))(1+g(b+c))} w_t + \frac{gr(b+c)}{((b+c(1-r))(1+g(b+c))} z_t + \frac{p^2}{b+c(1-p)} u_{t-1} + \frac{r^2}{b+c(1-r)} v_{t-1} .$$

Notice that the coefficients on the demand shock and supply shock enter additively for the contemporaneous shock and with symmetric coefficients for the lagged shocks.

Next, we can determine the equilibrium level of aggregate output:

$$(5) Y_{t+1} = (1/g)\{P_{t+1} - E(P_{t+1}) \mid I_{t-1}\} - v_{t+1}$$
$$= w_{t+1} + z_{t+1} + \frac{p(b+c)}{(b+c(1-p))(1+g(b+c))} w_t + \frac{r(b+c)}{(b+c(1-r))(1+g(b+c))} z_t - v_{t+1}$$

To interpret this equation, notice that output plus the contemporaneous supply shock ($Y_{t+1} + v_{t+1}$), depends on the white noise components of the contemporaneous and lagged demand and supply shocks.

We also compute the equilibrium values for the expected real exchange rate and the expected real interest rate under this regime:

$$(6) E_t RER_{t+1} = -(a/b) + (ci^*/b) - \frac{gp(b+c)}{(b+c(1-p))(1+g(b+c))} w_t$$
$$- \frac{gr(b+c)}{((b+c(1-r))(1+g(b+c))} z_t - \frac{p^2}{b+c(1-p)} u_{t-1} - \frac{r^2}{b+c(1-r)} v_{t-1}$$
$$(7) E_t RIR_{t+1} = i^* + \frac{gp(b+c)(1-p) - p^2}{b+c(1-p)(1+g(b+c))} w_t + \frac{gp(b+c)(1-r) - r^2}{b+c(1-r)(1+g(b+c))} z_t + \frac{(1-p)p^2}{b+c(1-p)} u_{t-1} + \frac{(1-r)r^2}{b+c(1-r)} v_{t-1}$$

where we substitute from equation (4) and the expectation over (4). Equations (5) – (7) above give us expressions for output, the real exchange rate, and the real interest rate, that we shall return to later when we compare the variability of output, real exchange rate, and real interest rate under the three regimes.

4. Inflation Target

Under inflation targeting, we assume that the price level follows a random walk (implicitly assuming therefore that the inflation target is zero), and so expected future price levels are equal to the current price level. The central bank will be unable to keep inflation exactly constant, because the unforecastable components of the demand and supply shocks will affect the price level. We impose the restriction $E(P_{t+1} \mid I_t) = P_t \forall t$ on equation (3) and, again, we proceed by conjecturing the form of the equilibrium solutions for the nominal price level (consistent with the random walk) and also for the expected exchange rate. We find the following solutions:

$$(8) P_{t+1} = P_t + g(w_{t+1} + z_{t+1}),$$
$$(9) E(S_{t+1} \mid I_t) = -(a/b) - P^* + (ci^*/b) + (1 + 1/(g(b+c)))P_t - (1/g(b+c))P_{t-1}.$$

In turn, we find the equilibrium levels of output, (expected) real exchange rate, and (expected) real interest rate:

$$(10) Y_{t+1} = w_{t+1} + z_{t+1} + w_t + z_t - v_{t+1},$$
$$(11) E_t RER_{t+1} = ci^* - (a/b) + (1/(g(b+c))(P_t - P_{t-1}),$$
$$(12) E_t RIR_{t+1} = i^* - (1/(g(b+c)))(P_t - P_{t-1}).$$

Just as in the case of fixed exchange rates, output, plus the supply shock, is an AR(2) process. The only difference between fixed exchange rates and inflation targeting is the coefficient on the lagged white noise components of the shocks.

If we think of the exchange rate as the central bank's policy instrument, equation (9) tells us the reaction function that the bank must follow if it wants to achieve inflation targeting. The normal way to solve a macroeconomic model is first to assume a policy reaction function for the bank's instrument (the exchange rate in this case), and then solve for the implied dynamic equation for the price level. We have instead started by assuming what the bank wants to achieve (a random walk in the price level), and then solved for the policy reaction function that will implement that objective. But note that any reaction function equal to the right hand side of equation (9) *plus any white noise random component*, would achieve exactly the same inflation targeting objective. This is because only the expected, not the actual, exchange rate influences aggregate demand in our model.

5. Price Level Target

The third and final case that we consider is a price level target. Under this regime, we assume that the price level is uncorrelated white noise, because the central bank reverses the effects of random shocks, and so the expected future price level is a constant. We impose this assumption about expected future prices on the aggregate demand and supply equations, and then solve the model for the one-period effect of shocks on the price level, and for the dynamic equation for the exchange rate.

We impose the restriction $E(P_{t+1} \mid I_t) = \overline{P} \forall t$ on equation (3), and find the following solutions:

$$(13) P_{t+1} = \overline{P} + g(w_{t+1} + z_{t+1}),$$
$$(14) E(S_{t+1} \mid I_t) = \overline{P} - P^* + (ci^* - a)/b - \frac{p}{b - cp + c} u_t - \frac{r}{b - cr + c} v_t.$$

The equilibrium levels of output, the (expected) real exchange rate, and (expected) real interest rate are given by:

$$(15) Y_{t+1} = w_{t+1} + z_{t+1} - v_{t+1},$$
$$(16) E_t RER_{t+1} = (1/b)(ci^* - a) - (p/(b - cp + c))u_t - (r/(b - cr + c))v_t,$$

$$(17)\, E_t RIR_{t+1} = i^* + ((p(1-p))/(b-cp+c))u_t + ((r(1-r))/(b-cr+c))v_t\,.$$

We now make a similar observation as we did in the case of inflation targeting. If we think of the exchange rate as the central bank's policy instrument, equation (14) tells us the reaction function that the bank must follow it wants to achieve price level targeting. We have assumed what the bank wants to achieve (a price level target), and then solved for the policy reaction function that will implement that objective. Again note that any reaction function equal to the right hand side of equation (14) *plus any white noise random component*, would achieve exactly the same price level targeting objective. This is because only the expected, not the actual, exchange rate influences aggregate demand in our model.

6. Macroeconomic stabilisation

Macroeconomic stabilisation normally focusses exclusively on stabilising aggregate output. However, Courchene and Harris (1999) have drawn attention to the question of stabilising the real exchange rate. They argue that a policy of flexible exchange rates induces excessive volatility of the real exchange rate and therefore destabilises the tradeables sector. A similar argument could be made for the investment (interest-sensitive) sector. Accordingly, we will examine stabilisation of aggregate output, the expected real exchange rate, and the expected real interest rate.[8]

Not all fluctuations in output are undesirable. Variations in the natural rate of output, caused for instance by supply shocks, should be accommodated, because, even in a fully classical world (where allocative efficiency holds), these would cause output to vary. The bank should try to minimise the variance of deviations of output from the natural rate. This means that the bank should try to minimise the variance of price level surprises, $(P_{t+1} - E_{t-1}(P_{t+1}))$, on the grounds that these distort agents' decisions.

A similar argument can be made about the real exchange rate and the real interest rate.[9] Variations in these that reflect variations in their underlying 'fundamental', natural rates should be accomodated. We define these natural rates as those that would obtain in a classical world in which price level surprises, $(P_{t+1} - E_{t-1}(P_{t+1}))$, are zero. Henceforth, by 'stabilisation' we shall denote minimising the variance of a given macroeconomic variable relative to its natural rate.

Setting $P_{t+1} - E_{t-1}(P_{t+1}) = 0$, from equation (2), we solve for the natural rate of output, and from equation (3), and taking expectations as of date t, we can solve for the natural real exchange rate and real interest rate. The natural values are given by:

$$(18)\ YN_{t+1} = -v_{t+1}$$
$$(19)\, RERN_{t+1} = (1/b)(ci^* - a) - (p/(b-cp+c))u_t - (r/(b-cr+c))v_t$$
$$(20)\, RIRN_{t+1} = i^* + ((p(1-p))/(b-cp+c))u_t + ((r(1-r))/(b-cr+c))v_t$$

[8] Since in our model spending decisions are based on the expected real exchange rate and real interest rate, we will focus on stability of the expected, rather than the actual, real exchange rate and real interest rate.

[9] We thank David Laidler for bringing this point to our attention.

We now compute the deviations of the actual from the natural values, for our three macroeconomic variables, in the three policy regimes.

Under fixed exchange rates:

$$(21)\ Y_{t+1} - YN_{t+1} = w_{t+1} + z_{t+1} + \frac{p(b+c)}{(b+c(1-p))(1+g(b+c))} w_t + \frac{r(b+c)}{(b+c(1-r))(1+g(b+c))} z_t$$

$$(22)\ E_t RER_{t+1} - RERN_{t+1} = \frac{p}{(b+c(1-p))(1+g(b+c))} w_t + \frac{r}{(b+c(1-r))(1+g(b+c))} z_t$$

$$(23)\ E_r RIR_{t+1} - RIRN_{t+1} = -\frac{p}{(b+c(1-p))(1+g(b+c))} w_t - \frac{r}{(b+c(1-r))(1+g(b+c))} z_t$$

Under inflation targeting:

$$(24)\ Y_{t+1} - YN_{t+1} = w_{t+1} + z_{t+1} + w_t + z_t$$

$$(25)\ \begin{aligned} E_t RER_{t+1} - RERN_{t+1} &= [\frac{1}{b+c} + \frac{p}{b+c(1-p)}]w_t + [\frac{1}{b+c} + \frac{r}{b+c(1-r)}]z_t \\ &+ \frac{p^2}{b+c(1-p)} u_{t-1} + \frac{r^2}{b+c(1-r)} v_{t-1} \end{aligned}$$

$$(26)\ \begin{aligned} E_t RIR_{t+1} - RIRN_{t+1} &= -[\frac{1}{b+c} + \frac{p(1-p)}{b+c(1-p)}]w_t - [\frac{1}{b+c} + \frac{r(1-r)}{b+c(1-r)}]z_t \\ &- \frac{p^2(1-p)}{b+c(1-p)} u_{t-1} - \frac{r^2(1-r)}{b+c(1-r)} v_{t-1} \end{aligned}$$

Under price level targeting:

$$(27)\ Y_{t+1} - YN_{t+1} = w_{t+1} + z_{t+1}$$

$$(28)\ E_t RER_{t+1} - RERN_{t+1} = 0$$

$$(29)\ E_t RIR_{t+1} - RIRN_{t+1} = 0$$

These immediately give us the corresponding variances:

Under fixed exchange rate:

(30)

$$\operatorname{var}(Y_{t+1} - YN_{t+1}) = [1 + (\frac{p(b+c)}{(b+c(1-p))(1+g(b+c))})^2]\operatorname{var}(w) + [1 + (\frac{r(b+c)}{(b+c(1-r))(1+g(b+c))})^2]\operatorname{var}(z)$$

(31)

$$\operatorname{var}(E_t RER_{t+1} - RERN_{t+1}) = \frac{p^2}{[(b+c(1-p))(1+g(b+c))]^2}\operatorname{var}(w) + \frac{r^2}{[(b+c(1-r))(1+g(b+c))]^2}\operatorname{var}(z)$$

(32)

$$\operatorname{var}(E_t RIR_{t+1} - RIRN_{t+1}) = \frac{p^2}{[(b+c(1-p))(1+g(b+c))]^2}\operatorname{var}(w) + \frac{r^2}{[(b+c(1-p))(1+g(b+c))]^2}\operatorname{var}(z)$$

Under inflation targeting:

(33) $\operatorname{var}(Y_{t+1} - YN_{t+1}) = 2\operatorname{var}(w) + 2\operatorname{var}(z)$

(34)
$$\begin{aligned}\operatorname{var}(E_t RER_{t+1} - RERN_{t+1}) &= [(\frac{1}{b+c} + \frac{p}{b+c(1-p)})^2 + (\frac{p^2}{(b+c(1-p))^2(1-p)^2})]\operatorname{var}(w)\\ &+ [(\frac{1}{b+c} + \frac{r}{b+c(1-r)})^2 + (\frac{r^2}{(b+c(1-r))^2(1-r)^2})]\operatorname{var}(z)\end{aligned}$$

(35)
$$\begin{aligned}\operatorname{var}(E_t RIR_{t+1} - RIRN_{t+1}) &= [(\frac{1}{b+c} + \frac{p(1-p)}{b+c(1-p)})^2 + \frac{p^2}{(b+c(1-p))^2}]\operatorname{var}(w)\\ &+ [(\frac{1}{b+c} + \frac{r(1-r)}{b+c(1-r)})^2 + \frac{r^2}{(b+c(1-r))^2}]\operatorname{var}(z)\end{aligned}$$

Under price level targeting:

(36) $\operatorname{var}(Y_{t+1} - YN_{t+1}) = \operatorname{var}(w) + \operatorname{var}(z)$
(37) $\operatorname{var}(E_t RER_{t+1} - RERN_{t+1}) = 0$
(38) $\operatorname{var}(E_t RIR_{t+1} - RIRN_{t+1}) = 0$

Notice that, under price level targeting, the expected real exchange rate and the expected real interest rate are identical to their natural rates. The reason for this is that, under price level targeting, deviations of P_{t+1} from $E(P_{t+1}|I_{t-1})$ are white noise (they persist for only one period), so that, *pari passu*, deviations of output from its natural rate are white noise. Therefore, deviations of *actual* real exchange rate and real interest rate from their natural rates may be white noise, but then expectations of these deviations, one period prior, must be zero. It follows that deviations of the expected real exchange rate and real interest rate from their respective natural rates are zero, and *a fortiori* the variances of these deviations are zero.

Comparing the variances of output deviations, it is immediate that price level targeting dominates the other two policy rules, because it delivers the smallest variance. It is not obvious which of the other two is second best. It depends upon the parameters. For instance, if g approaches infinity, so that the *AS* curve becomes classical, or if both shocks approach white noise, then a policy of fixed exchange rates becomes as good as price level targeting. At the other extreme, if g becomes zero, so that the *AS* curve becomes extreme Keynesian, and if both shocks approach random walks, then a policy of fixed exchange rates is worse than inflation targeting.

Comparing the variances of the expected real exchange rate deviations and the expected real interest rate deviations, it is clear that price level targeting dominates the other two policy rules, because it delivers the smallest variance, indeed, the smallest possible variance (zero). The second best contest between the other two policy rules is again parameter-dependent. For instance, if g approaches infinity, or if both shocks approach white noise, then a policy of fixed exchange rates becomes as good as price level targeting, both for the real interest rate and the real exchange rate. If g becomes zero, or if both shocks approach random walks, then a policy of fixed exchanges rates is worse than inflation targeting, both for the real interest rate and the real exchange rate.

7. Shocks to P and i**

So far we have assumed that P^* and i^* are constant. How do our results change if we allow them to vary?

Notice that it is the expected rather than the actual levels of the foreign price level and rate of interest which affect aggregate demand in our model. This has important implications for stabilisation policy. We assume that the bank has the same information as the public and, accordingly, will hold the same expectations as the public. Therefore, if there is a rise in $E(P^*_{t+1} \mid I_t)$, the bank will know it. Under inflation or price level targeting, the bank will respond to this change in public expectations by tightening monetary policy to prevent any increase in aggregate demand, and will thereby prevent any increase in the price level. The bank can, and will, perfectly offset changes in $E(P^*_{t+1} \mid I_t)$. But, under a fixed exchange rate, the bank can, and will, do nothing, so aggregate demand will increase, and the price level will rise and real output will rise above the natural rate. This holds, *mutatis mutandis*, for a change in $E(i^*_{t+1} \mid I_t)$.

The implication is that, by assuming the constancy of the foreign price level and rate of interest, we have made the assumptions most favourable to a policy of fixed exchange rates. Relaxing this assumption would further strengthen the case for price level targeting and inflation targeting over fixed exchange rates.

It is instructive to compare the shocks in our formulation to the shocks in traditional analyses. Traditional analyses of aggregate demand stabilisation under fixed vs flexible exchange rates place great emphasis on the distinctions between various shocks. In traditional analyses, there are foreign shocks, domestic shocks, *IS* shocks, *LM* shocks, *BP* shocks, terms of trade shocks, etc., and all these separate categories are deemed to be important in their own peculiar ways. Our analysis is much simpler; the nice distinctions of traditional analysis are irrelevant. There is only one aggregate demand shock. This is not to deny that there may be many different underlying *sources* of the aggregate demand shock, but only to deny that the source of the shock is relevant.

For example, *LM* shocks appear in traditional analyses of flexible exchange rates because the traditional interpretation translates "flexible exchange rates" as "fixed money supply", and so shocks to money demand will affect interest rates, exchange rates, and aggregate demand. But *LM* shocks do not appear in our analysis because the money supply is endogenous under each of the three policy regimes. Any shock to money demand would be immediately offset by a matching change in money supply.

In our model, the demand and supply shocks always enter the solutions symmetrically. Indeed, if the demand and supply shocks had the same serial correlation, we could simply add them together and, effectively, there would be only one shock. The only distinction we need to make is between observed shocks and unobserved shocks. In the case of observed shocks, the bank can respond under inflation and price level targeting but not under fixed exchange rates. In the case of unobserved shocks, the bank cannot respond at all.

We have assumed that the parameters a, b, c, g, p, r, are known by the bank, and by private agents, with certainty. In our model, the inability of the bank to control the price level or inflation perfectly is caused solely by the lag in *AD* and its inability to forecast the white noise components of the shocks. In the real world, the bank and private agents also face uncertainty about the parameters (to say nothing about the structure of the model). Parameter uncertainty will not affect the bank's ability to fix the exchange rate, but it will affect its ability to achieve a price level or inflation target. Ignoring this source of uncertainty could be argued to bias our results against fixed exchange rates. It is hard to model parameter uncertainty,[10] and would be especially hard to do so in a model such as ours in which private agents' rational expectations would also be affected by both their and the bank's parameter uncertainty.

9. Conclusion

Central banks cannot perfectly control output and the price level. Any model which tries to compare macroeconomic stabilisation under fixed exchange rates, inflation targeting, and price level targeting, must somehow incorporate lags. We incorporate an outside lag in our model. It takes time before changes in monetary policy affect aggregate demand. This is the most crucial feature of our model, and the way it differs from those found in any textbook.

Given our one-period lag in aggregate demand, and our two-period lag in the expectations-augmented aggregate supply, we find that targeting the price level clearly dominates both fixed exchange rates and inflation targeting for macroeconomic stabilisation, either of aggregate output or of the real exchange rate or real interest rate. The reason is that under price level targeting, the central bank is always trying to restore the price level to validate the constant expected price level that is built into the expectations-augmented aggregate supply curve. Indeed, we would have sympathy with a critic who claimed we have stacked the deck in favour of price level targeting by assuming an aggregate supply equation in which

[10] Cf. Srour (1999).

deviations from the natural rate occur when there is an unexpected change in *the price level*. If we had instead assumed an aggregate supply equation in which deviations from the natural rate occur when there is an unexpected change in *the exchange rate*, then a policy of fixed exchange rates would have best stabilised output. But such an aggregate supply equation would be neither standard nor plausible.

Price level targeting is best for stabilising output: it is also best for stabilising the expected real exchange rate. The view that a policy of fixed exchange rates will best stabilise real exchange rates is shown to be unwarranted. Having a constant *nominal* exchange rate does not mean having a constant *real* exchange rate, either logically or in terms of the model. We have to look at what happens to the price level, and this means looking at the whole system. If there is a shock to aggregate demand, then something has to adjust, either the real exchange rate, or the real interest rate, or else output. It turns out that the policy which best stabilises one, stabilises all three.

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