Asset Encumbrance, Bank Funding, and Covered Bonds*

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First version: November 2012

This version: October 2015

Abstract

How does secured funding affect the fragility of intermediaries subject to rollover risk? We offer a model of covered bonds that features the replenishment of assets and dual recourse for secured creditors. Encumbering assets allows a bank to raise cheap secured funding and expand profitable investment but it concentrates risk on unsecured debt holders, exacerbating fragility. Deposit insurance and (implicit) guarantees induce the bank to excessively encumber, shifting risks to the guarantor. Policies to limit asset encumbrance correct this externality. Testable implications relate asset encumbrance to conditions in unsecured funding markets and investment characteristics.

Keywords: wholesale funding, rollover risk, covered bond, asset encumbrance.

JEL classifications: D82, G01, G21.

*We thank Douglas Gale, Agnese Leonello, Frank Milne, Jean-Charles Rochet, and seminar participants at Alberta School of Business, Amsterdam (UvA), Bank of Canada, Berlin (Humboldt), Danmarks Nationalbank, FDIC Annual Bank Research Conference 2015, Frankfurt School, IBEFA 2015, IRMC 2015, Monetary Authority Singapore, HEC Montreal, McGill, NFA 2015, Reserve Bank of New Zealand and Wellington (Victoria) for comments. We also thank Fabio Piamonte for data assistance. Anand and Gai gratefully acknowledge financial support from the University of Auckland Faculty Research Development Fund (FRDF #3000875). The views expressed in this paper are those of the authors and do not necessarily reflect the views of the Bank of Canada or the Deutsche Bundesbank.

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1 Introduction

The unhappy experience with securitization markets in the wake of the global financial crisis has led to renewed focus on the collateralization of bank balance sheets. In many jurisdictions, banks have turned to secured funding markets to finance their activities (CGFS, 2013). In the United States, covered bonds have been advocated as a means of revitalizing mortgage finance (Paulson, 2009; Soros, 2010). Indeed, Campbell (2013) forcefully argues that "the US has much to learn from mortgage finance in other countries, and specifically from the Danish implementation of the European covered bond system." (p.1)

Covered bonds are "secured senior debt" issued by banks. These are claims on originating banks, collateralized by a pool of mortgages that remain on balance sheet. This cover pool is ring-fenced, or encumbered, and treated as bankruptcy-remote. Critically, the cover pool is *dynamic* – banks must replenish non-performing assets with performing ones of equivalent value and quality to maintain the requisite collateralization. Covered bond holders are also protected by *dual recourse*. That is, if the value of the cover pool is insufficient to meet obligations, covered bond holders have a claim of the shortfall on unencumbered assets, with the claim equal in seniority to other creditors. These institutional features provide strong incentives for banks to underwrite mortgages carefully, avoiding some of the pitfalls with the originate-to-distribute model (Bernanke, 2009).[1]

Covered bonds have formed a cornerstone of bank funding in Europe since the late 18th century. They are especially dominant in Germany, where the *Pfandbrief* system was established by Frederick the Great following the Seven Years War (1756–63) to supply credit to Prussian landowners, and in Denmark where they emerged to finance reconstruction following the Great Fire of Copenhagen in 1795. Over the past two centuries, covered bonds have experienced no defaults and delayed payments to investors have been rare.[2]

[1] Keys et al. (2010) provide evidence to suggest that the originate-to-distribute model underpinning the securitized mortgage system in the United States lead to lax screening by lenders.

[2] See Wandschneider (2014) for a historical discussion of the *Pfandbriefe* system in 18th century Prussia. Mastroeni (2001) details the origins of covered bonds in other European jurisdictions.

Despite their longevity as a financial instrument, there has been no theoretical analysis of covered bonds.[3] This paper fills that gap by developing the first model of bank funding and asset encumbrance in which covered bonds assume center stage. We model the institutional features of dynamic replenishment and dual recourse and show how they make for a safe asset for investors and a cheap funding source for banks. We demonstrate a two-way interaction between secured and unsecured funding markets. Covered bond issuance influences the incidence of runs by unsecured creditors and, in turn, conditions in the unsecured funding market influence the choice of asset encumbrance and covered bonds by banks.

Our approach steps outside the Modigliani and Miller (1958) framework by assuming market segmentation, whereby unsecured and secured funding markets each have distinct investor clienteles. In the spirit of Vayanos and Vila (2009), we suppose that investors in the unsecured funding market, whom we liken to money market funds run by fund managers, are risk-neutral. Covered bond investors, by contrast, are infinitely risk-averse, reflecting the highly restrictive mandates of pension funds and other large institutional investors. Our model also uses global game methods (Carlsson and van Damme, 1993; Morris and Shin, 2003; Rochet and Vives, 2004; Goldstein and Pauzner, 2005). The ex-post rollover decisions of unsecured creditors constitutes a coordination problem that is characterized by a unique equilibrium at which a debt run occurs. We link the incidence of runs to the bank's ex-ante choice of covered bond issuance, and solve for the unique cost of unsecured debt. Our characterization of unsecured demandable debt follows Rochet and Vives (2004).

Our analysis suggests that covered bonds may not be the panacea that the proponents of such instruments might hope for. We highlight two effects of asset encumbrance and covered bond issuance. The first is a *risk concentration* effect: as more covered bonds are issued, credit risk is asymmetrically concentrated onto unsecured creditors, which increases the incidence of an unsecured debt run. The second is a *bank funding* effect: greater issuance

[3] The existing literature on covered bonds is empirical. Carbo-Valverde et al. (2011) examine the extent to which covered bonds substitute for mortgage-backed securities. And Prokopczuk and Vonhoff (2012) study how market liquidity and asset quality affect covered bond pricing.

of covered bonds finances more profitable investment, which increases the expected equity value and reduces the potential for a run. The optimal choice of encumbrance balances these opposing effects. Moreover, the dual recourse provision is never called upon in equilibrium since infinitely risk-averse investors only evaluate the expected utility of holding a covered bond at the largest possible credit shock. This result is consistent with the finding of Wandschneider (2014), who notes that dual recourse has never called upon in practice.

A number of countries, notably Australia, Canada and New Zealand, have introduced formal limits on asset encumbrance. These prudential policies safeguard against the possibility that banks, with large pools of guaranteed deposits, issue secured debt in order to shift risks to the guarantor. Our framework clarifies the logic of this position, identifying precisely how the expected costs to the guarantor depends on the level of asset encumbrance and the coverage of the guarantee scheme. We show how the bank optimally responds to the guarantee scheme, choosing a level of asset encumbrance that is excessive. The analysis suggests that, without prudential safeguards, such as a cap on asset encumbrance, covered bonds may not have the desired effects of reviving mortgage financing.

The model generates testable implications about secured funding. Asset encumbrance and covered bond issuance are increasing in the returns and liquidation value of assets. However, as the conservatism of unsecured debt holders and credit shocks increase, the bank encumbers fewer assets and issues fewer covered bonds. These findings are consistent with the broad trends for covered bond issuance in the euro area in the period 2003–12. We illustrate the general mechanism for the case of more conservative unsecured debt holders. As their incentive to roll over debt is reduced, the fragility of the bank increases. In response, the bank encumbers fewer assets and issues fewer covered bonds, foregoing profitable investment opportunities to reduce the endogenous probability of a debt run.

The issue of bank funding remains relatively unexplored in the recent academic debates. A seminal contribution is Greenbaum and Thakor (1987) who present a signaling model in which the choice of deposit funding (on-balance sheet) versus securitized funding (off-balance sheet) is based on the quality of the project. Borrowers choose between funding modes. Under deposit-based funding, the borrower's risk-adjusted loan rate reflects the value of collateralization and screening costs incurred by banks and depositors. If the borrower instead chooses securitized funding, the degree to which the bank provides recourse in the event of default is chosen, which signals quality to non-bank investors.

More recent work has begun to examine the interplay between secured and unsecured funding. Gai et al. (2013) and Eisenbach et al. (2014) explore the relationship between bank fragility and funding structures using partial equilibrium frameworks. Gai et al. (2013) use global game techniques to examine how the liquidity and solvency risk of a bank changes with the composition of funding. They distinguish between short- and long-term funding in the repo market and show how "dashes for collateral" by short-term secured creditors can occur. Eisenbach et al. (2014) develop a balance sheet approach to explore alternative funding structures that is amenable to graphical analysis. They treat creditor behavior as exogenous and study how bank stability depends on various balance sheet characteristics including leverage, debt maturity structure, and the liquidity of the bank's asset portfolio.

Matta and Perotti (2015) have recently developed a model on the implications of secured repo funding for bank fragility that is also related to the present work. Like us, they draw on global game techniques and model the ex ante funding mix of the bank. While the insights of their paper has more immediate resonance for the analysis of financial stability policy in market-based financial systems, our explicit focus on the institutional features of covered bonds allows us to focus on the financial stability concerns of bank-based systems.

The paper proceeds as follows. Section 2 sets out the model. Section 3 studies the ex-post rollover decision of unsecured creditors, and solves for the equilibrium in the secured and unsecured funding markets. Section 4 considers the normative issue of prudential safeguards to internalize externalities posed by the incentives of the bank to shift risk on to a guarantor, such as the deposit insurance fund. Section 5 reports on recent trends in covered bond markets and presents several testable implications of the model. A final section concludes.

2 Model

Time is discrete and extends over four dates, $t = 0, 1, 2, 3$. There are three agents – a penniless and risk-neutral banker, and two groups of wholesale investors, each of unit mass. To fix ideas, these clienteles may be thought of as money market mutual funds (MMFs) and pension funds, respectively. MMFs receive a unit endowment at $t = 0$ and are risk-neutral. Pension funds, by contrast, receive a unit endowment at $t = 1$ and are infinitely risk-averse, reflecting their mandate for high-quality and safe assets. The banker consumes at $t = 3$, while investors are indifferent between consuming at any date and do not discount time.

The banker seeks funding from wholesale investors to finance profitable and high-quality investment opportunities that arise at $t = 0$ and $t = 1$. All investments mature at $t = 3$, where the return is $R > 1$. Premature liquidation of investments at $t = 2$ is costly and yields a fraction ψ of the return at maturity, where $1/2 < \psi R < 1$. The cost of liquidation may, for example, reflect efficiency losses as asset ownership is transferred from skilled bankers to relatively unskilled institutional investors (Diamond and Rajan, 2001).

MMFs deposit their endowment with the banker at $t = 0$ to receive an unsecured demandable debt claim as in Rochet and Vives (2004). Unsecured debt can be withdraw at $t = 2$ or rolled over until $t = 3$. The decision to roll over debt is taken by a group of professional fund managers, indexed by $i \in [0, 1]$. These managers face strategic complementarity in their decisions, with an individual manager's incentive to roll over increasing in the proportion of managers who roll over. The relative cost to managers of rolling over debt, $0 < \gamma < 1$, plays an important role in this decision. The higher is γ, the more jittery (or conservative) are fund managers, and the less likely that unsecured debt is rolled over. The face value of unsecured debt, $D_u \geq 1$, is independent of the withdrawal date.

The banker can make additional investment at $t = 1$ by attracting finance from pension funds. Specifically, the banker issues covered bonds by encumbering (or ring-fencing) a fraction $0 \leq \alpha \leq 1$ of assets and placing them in the cover pool – a bankruptcy-remote vehicle on the bank's balance sheet. The level of asset encumbrance is publicly observed at $t = 2$. We denote by $B \geq 0$ the total amount of covered bond funding raised, and by D_b the face value of a covered bond at $t = 3$. Table 1 illustrates the bank's balance sheet at $t = 1$ after all wholesale funding is raised.

(cover pool)	α	B
(unencumbered assets)	$1 - \alpha + B$	1

Table 1: Balance sheet at $t = 1$

A defining feature of covered bonds is the *dynamic replenishment* of the cover pool after an adverse shock. Replenishment requires the banker to maintain the value of the cover pool at all dates, replacing non-performing assets in the cover pool with performing unencumbered assets. Covered bond holders are, thus, protected and effectively become senior debt holders. But replenishment hurts unsecured debt holders, since the entire shock is concentrated on them.

We suppose that the balance sheet of the bank is subject to a shock $S \leq R$ at $t = 3$. The shock has a continuous probability density function $f(S) > 0$ and a cumulative distribution function $F(S)$. Moreover, $f'(S) \leq 0$, so that small shocks are more likely than larges ones. The banker observes the shock at $t = 2$ and replenishes the cover pool to ensure that its value is maintained. Table 2 shows the bank's balance sheet at $t = 3$ for a small shock, $S > 0$, in which all unsecured debt is rolled over at $t = 2$. The equity of the bank is denoted by $E(S)$.

(cover pool)	$R\alpha$	$D_b B$
(unencumbered assets)	$R\big(1 - \alpha + B\big) - S$	D_u
		$E(S)$

Table 2: Balance sheet at $t = 3$ after a shock

Unlike the banker, wholesale investors cannot observe the shock before it materializes. However, fund managers receive a noisy private signal, x_i, about the shock at $t = 2$ upon which they base their rollover decisions. Specifically, they receive the signal $x_i \equiv S + \epsilon_i$, where ϵ_i is idiosyncratic noise drawn from a continuous distribution G with support $[-\epsilon, \epsilon]$, for $\epsilon > 0$. The idiosyncratic noise is independent of the shock and independently and identically distributed across fund managers. Such imperfect information facilitates a unique solution to the coordination game between fund managers (Morris and Shin, 2003).

A second defining feature of covered bonds is *dual recourse*. Under bankruptcy, the bank is closed and covered bond holders receive the market value of the cover pool, $\alpha\psi R$, at $t = 3$. If, however, this is insufficient to meet their claims worth $D_b B$, then covered bond holders have a claim on the bank's unencumbered assets for the residual amount, $D_b B - \alpha\psi R$, at $t = 3$, with equal seniority to unsecured debt holders.

If a proportion $\ell \in [0, 1]$ of unsecured debt is not rolled over at $t = 2$, the banker must liquidate the amount $\ell D_u/\psi > \ell D_u$ to serve withdrawals. Thus, the condition for bankruptcy is

$$R\big(1 - \alpha + B\big) - S - \frac{\ell D_u}{\psi} < \big(1 - \ell\big) D_u + \big(D_b B - \alpha R \psi\big). \tag{1}$$

The value of the unencumbered assets at $t = 3$ is $R\big(1-\alpha+B\big)-S$. At $t = 3$, the banker must serve the remaining $(1-\ell)$ unsecured debt, with face value D_u, along with the residual claims of covered bond holders as required by dual recourse. Table 3 summarizes the timeline.

$t = 0$	$t = 1$	$t = 2$	$t = 3$
1. Unsecured debt issuance	1. Asset encumbrance	1. Banker observes credit shock	1. Investment matures
2. Investment	2. Secured debt issuance	2. Dynamic replenishment	2. Credit shock materializes
	3. Additional investment	3. Private signals about shock	3. Banker honors debts
		4. Unsecured debt withdrawals	4. Banker consumes equity

Table 3: Timeline of events.

3 Equilibrium

We proceed to solve the model backwards. First, we analyze the rollover decisions by fund managers at $t = 2$, taking as given the amount and cost of secured funding, the level of asset encumbrance, and the cost of unsecured funding. Second, we analyze the choice of secured funding by the banker at $t = 1$. The banker chooses the amount of covered bond funding, B, the level of asset encumbrance, α, and the face value of secured debt, D_b, to maximize the expected value of bank equity, for a given cost of unsecured funding and subject to the participation constraint of pension fund investors. Third, we solve for the cost of unsecured funding, D_u, at $t = 0$, which maximizes the expected value of equity, subject to the participation constraint of MMFs.

3.1 Rollover risk of unsecured debt

If the shock was common knowledge, then the rollover behavior of fund managers would be characterized by multiple equilibria. Figure 1 illustrates tripartite classification of the shock under this circumstance, with the bounds above and below which the dominant strategy for fund managers is to withdraw or rollover, respectively.

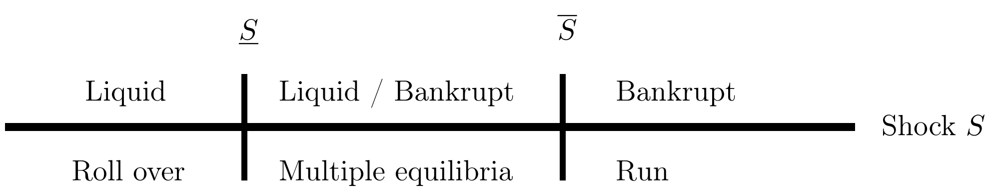

Figure 1: Tripartite classification of the shock

If all unsecured debt is rolled over, $\ell = 0$, bankruptcy occurs when the shock is larger the *upper bound*, $\overline{S} \equiv R\big(1 - \alpha + B\big) - D_u - \big(D_b\, B - \alpha R\psi\big)$. When $S > \overline{S}$ it is a dominant strategy for fund managers not to roll over unsecured debt. Likewise, when $\ell = 1$, all deposit are withdrawn and bankruptcy is avoided whenever the shock is smaller than the *lower bound,* $\underline{S} \equiv R\big(1-\alpha+B\big) - \frac{D_u}{\psi} - \big(D_b\, B - \alpha R\psi\big) < \overline{S}$. When $S < \underline{S}$ it is a dominant strategy for fund managers to roll over unsecured debt. Both dominance regions are well defined for all funding choices at $t=0$ and $t=1$ since $\underline{S} > -\infty$ and $\overline{S} < R$.

Under imperfect information about the shock, there is a unique Bayesian equilibrium for each unsecured debt rollover subgame in $t=2$, as summarized by Proposition 1.

Proposition 1. Unsecured debt rollover subgame. *If private noise vanishes,* $\epsilon \to 0$*, there exists a unique Bayesian equilibrium in each subgame. It is characterized by a bankruptcy threshold,* S^**, and a signal threshold,* x^**. Fund manager* i *rolls over unsecured debt if and only if* $x_i < x^*$ *and bankruptcy occurs if and only if* $S > S^* \equiv R\big(1-\alpha+B\big) - \kappa D_u - \big(D_b B - \alpha R\psi\big) \in \big(\underline{S}\,,\,\overline{S}\big)$*, where* $\kappa \equiv 1+\gamma\left(\frac{1}{\psi}-1\right) \in \left(1, \frac{1}{\psi}\right)$ *and* $x^* \to S^*$*.*

Proof. See Appendix A. ■

The following corollary is immediate.

Corollary 1. *The bankruptcy threshold* S^* *decreases in the level of asset encumbrance and the cost of secured and unsecured funding; but it increases in the amount of secured funding if* $D_b < R$*:*

$$\frac{\partial S^*}{\partial \alpha} = -R(1-\psi) < 0, \quad \frac{\partial S^*}{\partial D_b} = -B < 0, \quad \frac{\partial S^*}{\partial D_u} = -\kappa < 0, \quad \frac{\partial S^*}{\partial B} = R - D_b. \tag{2}$$

Proof. See Appendix A. ■

The signs of the partial derivatives are intuitive. First, greater asset encumbrance reduces both the amount of unencumbered assets available to meet withdrawals by fund managers and the net claim of covered bond holders under dual recourse. Because of over-collateralization, the overall effect of greater encumbrance is that fund managers withdraw deposits for a larger range of shocks. Second, more costly secured funding raises the residual value of covered bond holders claims at $t=3$ due to dual recourse. This induces withdrawals of unsecured debt at $t = 2$ to prevent a dilution of their claims. Third, more costly unsecured funding implies a greater degree of strategic complementarity among fund managers, which induces them to withdraw unsecured debt more often. And finally, more secured funding increases both the amount of unencumbered assets and the claim to covered bond holders under dual recourse. The former effect dominates if $D_b < R$, resulting in fewer withdrawals.

3.2 Secured funding and asset encumbrance

We derive the banker's objective function. For values of the shock below the bankruptcy threshold, $S \leq S^*$, the equity value is positive and equal to the value of investments net of the shock and total debt repayments to investors, $E(S) = R(1 + B) - S - D_b B - D_u$. For shocks above the bankruptcy threshold, the value of equity is zero due to limited liability.

Participation by pension fund investors requires that the expected utility from holding a covered bond must be, at least, equal to the outside option of consuming the endowment. Each cover bond promises a face value D_b and is backed by an equal share of the liquidated cover pool, $\alpha\psi R/B$, along with dual recourse on the bank's unencumbered assets in the event of bankruptcy. The banker's problem at $t = 1$ is thus

$$\begin{aligned} \max_{\{\alpha,B,D_b\}} \quad & \pi \equiv \int_S E(S)dF(S) = F(S^*)\left[R\big(1+B\big) - D_u - D_b B\right] - \int_{-\infty}^{S^*} S dF(S), \\ & \text{s.t.} \\ 1 \leq \min_S & \left\{ D_b, \frac{\alpha R \psi}{B} + \max\left\{0, \frac{D_b}{B D_b + (1-\ell^*(S))D_u}\psi\left(R(1-\alpha+B) - S - \frac{\ell^*(S)D_u}{\psi}\right)\right\}\right\}, \end{aligned} \tag{3}$$

where $\ell^*(S) = 0$ for $S \leq S^*$ and $\ell^*(S) = 1$ otherwise.

Critically, the dual recourse provision is never called upon in equilibrium.[4] Since pension fund investors are infinitely risk-averse, they evaluate the expected utility from holding the covered bond at the largest shock. For $S = R$, it is strictly dominant for all

[4] This result is consistent with the finding of Wandschneider (2014) that in over two-hundred years of covered bonds, the dual recourse clause has never been invoked.

fund managers to withdraw unsecured MMF deposits at $t = 2$, so that $\ell^*(R) = 1$. In this case, the value of unencumbered assets is zero by limited liability ($B^* < \alpha^* + D_u/\psi R$). As a result, dual recourse has zero value to pension fund investors, and their participation constraint simplifies to

$$1 \leq \min\left\{D_b, \frac{\alpha R\psi}{B}\right\}. \tag{4}$$

The constraint always binds in equilibrium, as the proof of Lemma 1 shows.

Lemma 1. Covered bond issuance. *In any equilibrium, the face value of covered bonds is $D_b^* = 1$ and its issuance volume is $B^* = \alpha^* R\psi$.*

Proof. See Appendix B. ■

Lemma 1 highlights a *bank funding* effect. Encumbering more assets allows the banker to issue more covered bonds. As more secured funding is attracted, the banker expands the balance sheet via additional investment, which improves its expected equity value. Moreover, the participation constraint of pension fund investors binds in equilibrium, $D_b^* = 1$, and the banker's problem can be reduced to an unconstrained optimization problem.

Lemma 2. Risk concentration. *Encumbering more assets reduces the bankruptcy threshold:*

$$\frac{dS^*}{d\alpha} = \frac{\partial S^*}{\partial \alpha} + \frac{\partial S^*}{\partial B^*}\frac{dB^*}{d\alpha} = -R(1-\psi R) < 0. \tag{5}$$

Proof. See Appendix B. ■

In contrast, Lemma 2 highlights a *risk concentration* effect of secured funding. Issuing covered bonds asymmetrically concentrates the shock on unsecured debt holders. Dynamic replenishment of the cover pool makes covered bonds more senior than unsecured debt. Therefore, greater asset encumbrance leads to higher incidence of runs on the bank. This, in turn, lowers the bankruptcy threshold. The banker takes both these effects into account when choosing the optimal level of asset encumbrance. Figure 2 illustrates with an example.

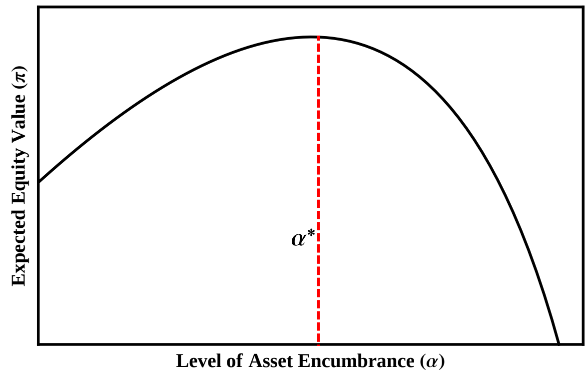

Figure 2: Expected value of equity as a function of the level of asset encumbrance. Additional parameters were $R = 3$, $\psi = 0.2$, $\gamma = 0.01$, and $D_u = 1.1$, where the shock follows a truncated exponential distribution with rate $\lambda = 1.1$.

Proposition 2. Optimal asset encumbrance. *There exists a unique level of asset encumbrance. If $\underline{R} < R < \overline{R}$, the solution is interior, $\alpha^* \in (0, 1)$, and implicitly given by:*

$$\frac{F\left(S^*\left(\alpha^*\right)\right)}{f\left(S^*\left(\alpha^*\right)\right)} = \frac{(1-\psi R)}{\psi(R-1)}\left[(\kappa-1)D_u + \alpha^*(1-\psi)R\right]. \tag{6}$$

Proof. See Appendix C, where the bounds on investment profitability are derived. ■

3.3 Endogenous cost of unsecured funding

Having established the equilibrium in the market for secured funding, we turn to the equilibrium cost of unsecured funding attracted by the banker at $t = 0$.

Figure 3 shows how the repayment and partial default on unsecured debt depends on the shock size. Unsecured debt holders receive the promised payment D_u if the bank is solvent, or an equal share of the proceeds from liquidating unencumbered assets. By limited liability, investors receive zero for a large credit shock, $S > S^*_D(\alpha^*) \equiv R[1 - \alpha^*(1 - R\psi)]$, where we used the equilibrium in the secured funding market. Taken together, the unsecured debt claim is

$$\min\left\{D_u, \max\left[0, \psi\big(S^*_D(\alpha^*) - S\big)\right]\right\}. \tag{7}$$

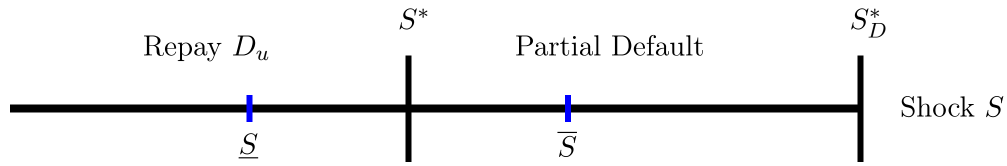

Figure 3: The shock and unsecured debt claim.

The banker sets the face value of unsecured debt D_u at $t = 0$ to maximize its expected value of equity, subject to the participation constrained of MMF investors. The expected equity value decreases in the face value of unsecured debt, $d\pi / dD_u = -F(S^*(\alpha^*)) - \kappa f(S^*(\alpha^*))[(\kappa - 1)D_u + R(1 - \psi)\alpha^*] < 0$, so the banker chooses the smallest value, D_u^*, consistent with participation of risk-neutral investors:

$$1 = F(S^*(\alpha^*, D_u^*))D_u^* + \psi \int_{S^*(\alpha^*, D_u^*)}^{S_D^*(\alpha^*)} [S_D^*(\alpha^*) - S] \, dF(S). \tag{8}$$

Proposition 3. Equilibrium cost of unsecured funding. *There exists a unique face value of unsecured debt, $D_u^* > 1$, if the investment return satisfies $R \leq \tilde{R}$ and MMF investors accept unsecured debt claims when promised all unencumbered assets.*

Proof. See Appendix D, where we derive and state the sufficient conditions. ■

The first sufficient condition, $R \leq \tilde{R}$, suffices for the value of the debt claim, denoted by $V(D_u)$ and given by the right-hand side of equation (8), to increase in the face value of unsecured debt. This condition ensures that at most one solution exists. Since $V(D_u = 1) < 1$, existence follows if $V(D_u = S_D) \geq 1$ for all encumbrance levels, yielding the second sufficient condition. Because of (partial) default for some credit shocks, the face value promised to investors must exceed unity.

4 Public guarantees and limits to asset encumbrance

In many jurisdictions, unsecured debt holders enjoy the benefits of explicit (or perhaps implicit) public guarantee schemes. Such schemes, which usually apply to retail deposits, often extend to unsecured wholesale depositors during times of crisis.[5] But deposit insurance schemes do not typically incorporate the effects of collateralized bank balance sheets. A bank with a large deposit base may, therefore, find it optimal to issue secured funding in order to shift risks to the deposit guarantee scheme. Since deposits are guaranteed, unsecured debt holders do not factor in the consequences of increased asset encumbrance and the benefits of public guarantees are externalized. Prudential safeguards include caps on asset encumbrance (Australia and New Zealand), ceilings on the amount of secured funding (Canada and the United States), and inclusion of encumbrance levels in deposit insurance premia (Canada).

Our model provides a natural framework with which to examine these normative issues. We focus on the secured funding market at $t = 1$ and show how prudential safeguards, in the specific form of a limit to asset encumbrance, serves to establish constrained efficiency. That is, given the information structure and rollover behavior of fund managers, and given the cost of unsecured and guaranteed debt, both the banker and a social planner choose the same level of asset encumbrance.

Suppose that a fraction $0 < m < 1$ of unsecured debt is guaranteed, so that these debt holders have no need to withdraw at $t = 2$. The guarantor (the government) is deep-pocketed and always pays. Denoting the face value of the guaranteed debt by D_g, the bankruptcy condition now becomes

$$R\,[1 - \alpha + B] - S - \frac{\ell(1-m)D_u}{\psi} < (1-\ell)(1-m)D_u + mD_g + (BD_b - \alpha R\psi). \qquad (9)$$

[5]For example, during the 2007/8 crisis, Australia, Canada and New Zealand were prominent among countries that enacted special arrangements for banks to have new and existing wholesale bank funding guaranteed by the government until market conditions normalized. Recent analysis of the interplay between government guarantees and financial stability include König et al. (2014) and Allen et al. (2015).

The value of unencumbered assets is unchanged at $t = 3$ and equals $R\,[1-\alpha+B]-S$. At $t = 2$, a fraction ℓ of $(1-m)$ non-guaranteed unsecured debt is withdrawn, while the remainder is rolled over. At $t = 3$, the banker must service the non-guaranteed unsecured debt claims that were rolled over, $(1-\ell)(1-m)D_u$, the guaranteed unsecured claims, mD_g, and claims to covered bond holders that stem from dual recourse.

In equilibrium, the bankruptcy threshold at $t = 2$ is:

$$S^*_m = R\,[1-\alpha+B] - mD_g - (1-m)\kappa D_u\,. \tag{10}$$

If $\kappa D_u > D_g$, then the bankruptcy threshold increases in the proportion of deposits that are guaranteed, $dS^*_m/dm > 0$, leading to a reduced incidence of runs. This is a consequence of the lower cost and greater stability of guaranteed funding. First, the banker has to serve fewer non-guaranteed debt claims at $t = 2$, which are associated with rollover risk ($\kappa > 1$). And second, guaranteed debt is cheaper, as the risk is not priced ($D_u > D_g$).[6]

The equilibrium in the secured funding market at $t = 1$ yields $D^*_b = 1$ and $B^* = \alpha R\psi$ as before. The risk concentration effect specified in Section 2 thus remains unchanged, $dS^*_m/d\alpha = -R(1-R\psi) < 0$.

Let $C(\alpha, m)$ be the expected cost of guaranteeing a fraction m of unsecured debt. Suppose that guaranteed debt has seniority over non-guaranteed claims. The banker will have sufficient unencumbered assets to pay the guaranteed debt holders if $S \leq \hat{S}_m \equiv R\,[1-\alpha(1-R\psi)] - \frac{m}{\psi}$. To ensure $\hat{S}_m > S^*_m$, we impose an upper bound on guaranteed deposits, $m < \overline{m} \equiv \frac{1+\gamma\left(\frac{1}{\psi}-1\right)}{1+(1+\gamma)\left(\frac{1}{\psi}-1\right)} \in (0,1)$. Partial default, and thus costs to the guarantor, occur for $\hat{S}_m < S \leq \hat{S}_m + m$, while full default occurs for larger shocks. Thus, the expected cost to the guarantor is:

$$C(\alpha; m) \equiv \int_{\hat{S}_m}^{\hat{S}_m+m} (S-\hat{S}_m)\; dF(S) + mD_g \int_{\hat{S}_m+m}^{R} dF(S). \tag{11}$$

[6] It is easy to see that the endogenous face value of guaranteed unsecured debt at $t = 0$ is $D^*_g = 1$.

Lemma 3. Guarantee cost. *The expected cost to the guarantor increases in both the level of asset encumbrance and the fraction of guaranteed unsecured debt:*

$$\frac{\partial C}{\partial \alpha} > 0\,, \quad \text{and} \quad \frac{\partial C}{\partial m} > 0\,. \tag{12}$$

The expected cost of the guarantee is weakly convex in the level of encumbrance, $\frac{\partial^2 C}{\partial \alpha^2} \geq 0$*, and has a positive cross-derivative,* $\frac{\partial^2 C}{\partial \alpha \partial m} > 0$.

Proof. See Appendix E. ■

Lemma 3 summarizes the key features of the cost of a guarantee. First, as more assets are encumbered, the bound $\hat{S}_m$ decreases, so the guarantor must pay out for a larger range of shocks. Second, an increase in the fraction of guaranteed debt has two effects: (i) a decrease in $\hat{S}_m$ and thereby increase the range of shocks over which the guarantee is paid; and (ii) an increase in the coverage of the guarantee. Third, the impact of asset encumbrance on expected costs is greater, the more unsecured debt is included in the guarantee.

In establishing the optimal choice of asset encumbrance, the banker ignores the guarantee cost, whilst taking into account the stabilizing influence of guaranteed unsecured debt on rollover behavior. The banker's problem can be written as:

$$\alpha_m^* = \underset{\alpha \in [0,1]}{\arg\max}\ \pi(\alpha; m) \equiv F(S_m^*)\big[R(1 + \alpha(R-1)\psi) - m - (1-m)D_u\big] - \int_{-\infty}^{S_m^*} S\, dF(S).$$

Proposition 4. Privately optimal asset encumbrance and guarantees. *There is a unique privately optimal level of encumbrance,* $\alpha_m^* \in (0,1)$*, that is implicitly defined by:*

$$\frac{F(S_m^*(\alpha_m^*))}{f(S_m^*(\alpha_m^*))} = \frac{1 - \psi R}{\psi(R-1)}[(\kappa - 1)(1-m)D_u + \alpha_m^*(1-\psi)R]. \tag{13}$$

More guaranteed unsecured debt induces the banker to encumber more assets, $\frac{d\alpha_m^*}{dm} > 0$.

Proof. See Appendix F. ■

Proposition 4 clarifies why policymakers (e.g., CGFS, 2013) have emphasized the importance of prudential safeguards to mitigate the risks of heavy asset encumbrance. The intuition for the proposition relates to the cost and stability of funding. As the proportion of guaranteed unsecured debt increases, the bankruptcy threshold, S_m^*, increases. As a smaller proportion of unsecured debt is uninsured, the rollover risk is reduced. The bank is less fragile and bankruptcy occurs for a smaller range of shocks. Consequently, the banker encumbers more assets, expanding its balance sheet to increase its expected value of equity.

Unlike the banker, the planner must account for the expected costs of the guarantee. The planner chooses the level of asset encumbrance that maximizes the expected equity of the bank net of the expected costs of the guarantor. Formally,

$$\alpha_P^* = \arg\max_{\alpha\in[0,1]} W(\alpha; m) \equiv \pi(\alpha; m) - C(\alpha; m). \tag{14}$$

A limit on the level of asset that the bank can encumber allows the externality to be internalized. Proposition 5 summarizes.

Proposition 5. Excessive encumbrance and regulation. *The privately optimal level of asset encumbrance is excessive, $\alpha_m^* > \alpha_P^*$. Imposing a limit on the level of assets the banker encumbers,*

$$\alpha \leq \alpha_P^* \,, \tag{15}$$

establishes constrained efficiency. Moreover, the gap between the constrained efficient and privately optimal level of asset encumbrance increases in the coverage of the guarantee,

$$\frac{d(\alpha_m^* - \alpha_P^*)}{dm} > 0 \,. \tag{16}$$

Proof. See Appendix G. ■

5 Testable implications

The comparative static results on the optimal level of asset encumbrance and covered bond issuance of our model are consistent with recent trends in the European covered bonds market. Figure 4 shows the evolution of covered bonds issuance since the start of 2003. We identify four distinct episodes and describe these together with the following propositions.

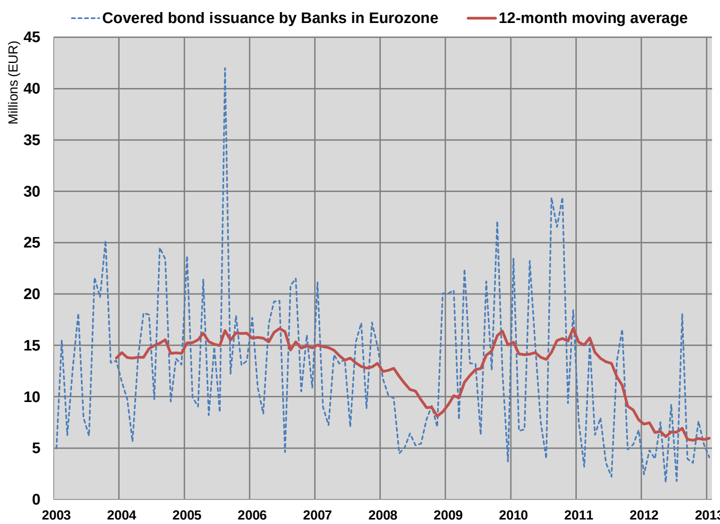

Figure 4: Time-series of covered bond issuance in Europe (in million EUR). Source: ESRB.

Proposition 6. Investment returns. *An increase in the investment return, R, leads to an increase in the level of asset encumbrance.*

Proof. See Appendix H. ■

A higher investment return increases the value of unencumbered assets on the bank's balance sheet, for any given level of asset encumbrance. Since more resources are available to meet withdrawals by fund managers, unsecured debt is rolled over for a larger range of shocks, which reduces the fragility of the bank. As a result, the banker encumbers more assets to issue more covered bonds and benefit from profitable investment that increases its expected value of equity. Overall, there are fewer but more profitable unencumbered assets.

The period of 2003–07, which preceded the financial crisis, was marked by strong growth in mortgage lending to European households. This development is consistent with robust investment returns in the language of our model. The total amount outstanding of covered bonds increased by 35% to a total of some EUR 2 trillion over that period, with issuance especially strong in Spain and Ireland (Martin et al., 2014). Figure 4 also confirms the robust issuance of covered bonds during this period.

Proposition 7. Conditions in unsecured debt markets. *An increase in the degree of conservatism of fund managers, γ, leads to a decrease in the level of asset encumbrance.*

Proof. See Appendix H. ■

As the degree of conservatism increases, fund managers are more jittery and less inclined to roll over unsecured MMF deposits, for any given level of encumbrance. Therefore, the probability of bankruptcy is higher. The banker responds to this heightened fragility in a precautionary manner by reducing the level of encumbrance. It forgoes profitable investment opportunities through the issuance of covered bonds in return for more stable unsecured funding, which reduce its fragility and the risk of bankruptcy.

The height of the crisis in 2007–2008 was arguably a time of turmoil in unsecured debt markets and heightened fund manager conservatism. As Figure 4 shows, covered bond issuance decreased markedly during this episode.

Proposition 8. Liquidation value. *An increase in the liquidation value, ψ, leads to an increase in the level of asset encumbrance.*

Proof. See Appendix H. ■

A higher liquidation value lowers the degree of strategic complementarity among fund managers, for any given level of encumbrance. Withdrawals by some managers, and the resulting liquidation of assets, cause less damage to other unsecured debt holders. As a result, the bank's fragility is reduced and bankruptcy occurs for a smaller range of shocks. Therefore, the banker encumber more assets to increase the expected value of equity. Overall, there are fewer but more liquid unencumbered assets on the bank's balance sheet.

Covered bond issuance picked up after 2008, which is partly due to the ECB's Covered Bond Purchase Program announced in May 2009. This program was designed to encourage banks to expand lending and improve market liquidity in important segments of the private debt securities market (Beirne et al., 2011). To this end, European banks encumbered more assets, issued more covered bonds, and invested in more high-quality assets. By stimulating the supply of credit by banks to the real economy, the program, in effect, helped raise the liquidation value of assets, which further contributed to an increase in covered bond issuance. During the first half of 2010, the fraction of covered bonds issued relative to uncollateralized debt in the euro area rose from 47% to 83% when compared with the same period of 2009.

Proposition 9. Credit risk. *Suppose that the shock follows a truncated exponential distribution with rate $\lambda > 0$. A lower value of λ decreases the level of asset encumbrance.*

Proof. See Appendix H. ■

A lower rate λ increases the average shock and thus the probability of bankruptcy, for any given level of encumbrance. Responding to the heightened fragility, the banker encumbers fewer assets and issues fewer covered bonds, which reduces the rollover risk associated with unsecured funding.

The period between 2011-2013 was characterized by the sovereign debt crisis in Europe. The associated deteriorating in the macroeconomic environment and investor confidence increased their credit risk of banks, which results in a sharp decline in covered bond issuance.

Proposition 10. Tail risk and unsecured funding costs. *Consider two distributions, F and $\tilde{F}$. If $\tilde{F}$ first-order stochastically dominates F, whereby $\tilde{F}(S) = F(S)$ for all $S \leq R - \kappa$*

and $\tilde{F}(S) > F(S)$ *for all* $R - \kappa < S < R$, *then* $\tilde{D}_u^* < D_u^*$.

Proof. See Appendix I. ■

Proposition 10 describes how the distribution of the shock affects the cost of unsecured funding. Both distributions, F and $\tilde{F}$, assign the same probability to small shocks, $S \leq R-\kappa$. However, the two distributions differ for large shocks, with large shocks being less likely under $\tilde{F}$ than under F (lower tail risk). The banker's optimal choice of asset encumbrance is unaffected, and unencumbered assets have a higher expected value in bankruptcy under $\tilde{F}$. Thus, risk-neutral MMF investors accept a lower face value of unsecured debt.

6 Conclusion

We present the first model of bank-based financing using covered bonds and explore their implication for financial stability. We decompose the influence of covered bonds into two distinct balance sheet effects. First, covered bond issuance asymmetrically shifts credit shocks to unsecured debt holders (risk concentration effect), resulting in greater fragility. Second, greater asset encumbrance allows a bank to issue more covered bonds and make more profitable investment, increasing its expected value of equity (bank funding effect). The unique equilibrium choice of asset encumbrance balances these two effects.

We derive normative implications of asset encumbrance in the context of guaranteed unsecured deposits. The privately optimal level of asset encumbrance exceeds the social optimum, as the banker does not internalize the effect of encumbrance on the cost of the guarantee. To establish constrained efficiency, a limit on the level of asset encumbrance may be imposed, which is consistent with some jurisdictions like Australia, Canada and New Zealand. Absent prudential safeguards, banks have strong incentives to issue covered bonds in order to shift risk onto the guarantor. Accordingly, proposals that emphasize covered bond systems as a means of reviving mortgage finance need to be carefully designed.

Our model points to some testable implications about asset encumbrance. Higher profitability of investment, lower liquidation costs, lower credit risk, and greater confidence in unsecured debt markets each reduce the fragility of the bank. As a result, the bank is induced to encumber more assets and issue more covered bonds to fund profitable investment. These implications of our model are consistent with recent trends in the European covered bonds market. Formal empirical tests of these implications is a fruitful area for future work.

Our main insights on the trade-off between bank fragility and expanding profitable investment generalize to other settings. First, in our model the banker has all bargaining power in funding markets. In other market structures, the amount of secured funding raised by encumbering assets may be lower, reducing investment and equity value. The bank funding effect would be attenuated and the banker would encumber fewer assets. Second, the mix of assets that back covered bonds are often heterogeneous, including mortgages and public debt. Then, following a shock to the balance sheet, the replenishment of the cover pool not only affects the amount of unencumbered assets, but also its risk profile. Since lower-risk assets would be swapped into the cover pool first, the risk concentration effect would be exacerbated, raising fragility and inducing the banker to encumber fewer assets.

Regarding our normative implications, an interesting area for further work is uncertainty about the deep pocket of the guarantor. If unsecured (but guaranteed) bank creditors are concerned about the guarantor's ability to pay, they may also run. This additional fragility would reduce the incentives of the bank to encumber assets and issue covered bonds.

Finally, our model assumes that the banker always invests in high-quality activities. However, moral hazard considerations played a critical role in the development of the Prussian *Pfandbfrief* market in the 18th century (Wandschneider, 2014). Extending our analysis to allow for moral hazard by the banker, and to explore how asset encumbrance is related to the trade-off between ex-ante moral hazard and the disciplining effect of ex-post runs by unsecured creditors, would be an interesting avenue for future work.

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A Proof of Proposition 1

In each rollover subgame, it is sufficient to establish the existence of a unique Bayesian equilibrium in threshold strategies for sufficiently precise private information. Morris and Shin (2003) show that only threshold strategies survive the iterated deletion of strictly dominated strategies; see also Frankel et al. (2003). Specifically, we consider the limiting case of vanishing private noise, $\epsilon \to 0$. Each fund manager i uses a threshold strategy, whereby unsecured debt is rolled over if and only if the private signal suggests that the credit shock is small, $x_i < x^*$. Hence, for a given realization $S \in [\underline{S}, \overline{S}]$, the proportion of fund managers who do not roll over debt is:

$$\ell\Big(S, x^*\Big) = \text{Prob}\left(x_i > x^* \Big| S\right) = \text{Prob}\left(\epsilon_i > x^* - S\right) = 1 - G\Big(x^* - S\Big).$$

A critical mass condition states that bankruptcy occurs when the credit shock reaches a threshold S^*, where the proportion of unsecured debt not rolled over is evaluated at S^*:

$$R\Big[1 - \alpha + B\Big] - S^* - \ell\Big(S^*, x^*\Big)\frac{D_u}{\psi} = \Big(1 - \ell\Big(S^*, x^*\Big)\Big)D_u + \Big(D_b B - \alpha R \psi\Big) \tag{17}$$

The posterior distribution of the credit shock conditional on the private signal is derived using Bayes' rule. The indifference condition states that the fund manager who receives the critical signal $x_i = x^*$ is indifferent between rolling over and not rolling over unsecured debt:

$$\gamma = \Pr\left(S < S^* | x_i = x^*\right). \tag{18}$$

Using the definition of the private signal $x_j = S + \epsilon_j$ of the indifferent fund manager, we can state the conditional probability as follows:

$$
\begin{aligned}
1-\gamma &= \Pr\left(S \geq S^{*} | x_i = x^{*}\right) = \Pr\left(S \geq S^{*} | x_i = x^{*} = S + \epsilon_j\right) && (19)\\
&= \Pr\left(x^{*} - \epsilon_j \geq S^{*}\right) = \Pr\left(\epsilon_j \leq x^{*} - S^{*}\right) && (20)\\
&= G\Big(x^{*} - S^{*}\Big) && (21)
\end{aligned}
$$

The indifference condition implies that $x^{*} - S^{*} = G^{-1}\Big(1-\gamma\Big)$. Inserting the indifference condition into $\ell\big(S^{*}, x^{*}\big)$, the proportion of fund managers who do not roll over when the credit shock is at the critical level S^{*} is perceived by the indifferent fund manager to be:

$$
\ell\Big(S^{*}, x_i = x^{*}\Big) = 1 - G\Big(x^{*} - S^{*}\Big) = 1 - G\Big(G^{-1}\Big(1-\gamma\Big)\Big) = \gamma. \tag{22}
$$

Therefore, the bankruptcy threshold is $S^{*} = R\Big[1 - \alpha + B\Big] - \kappa D_u - \big(D_b B - \alpha R \psi\big)$. If private noise vanishes, the signal threshold also converges to this value. The partial derivatives of the bankruptcy threshold S^{*} are immediate.

B Proof of Lemma 1 and Lemma 2

We will prove that $B^{*} = \frac{\alpha^{*} R \psi}{D_b^{*}}$ and $D_b^{*} = 1$ in any equilibrium. We also derive the total effect of asset encumbrance on the bankruptcy threshold. We guess and verify that $D_b^{*} < R$. Consider the partial derivatives of the objective function with respect to B and D_b:

$$
\begin{aligned}
\frac{\partial \pi}{\partial B} &= (R - D_b)\left[F(S^{*}) + f(S^{*})E(S^{*})\right] > 0 && (23)\\
\frac{\partial \pi}{\partial D_b} &= -B\left[F(S^{*}) + f(S^{*})E(S^{*})\right] < 0, && (24)
\end{aligned}
$$

where the value of bank equity at the bankruptcy threshold is $E(S^{*}) = (\kappa - 1)D_u + \alpha R(1 - \psi) > 0$ and the inequalities arise for a positive amount of covered bond funding, $B > 0$.

We prove $B^* = \frac{\alpha^* R\psi}{D_b^*}$ by contradiction. First, suppose that $D_b^* > \frac{\alpha^* R\psi}{B^*}$. Then, infinitely risk-averse investors value the covered bond claim at $\frac{\alpha^* R\psi}{B^*}$ since bankruptcy occurs with positive probability ($S^* < R$). This violates the supposed optimality of D_b^*, since lowering the face value would raise the objective function ($\frac{\partial \pi}{\partial D_b} < 0$) without affecting the constraint. Contradiction. Thus, $D_b^* \leq \frac{\alpha^* R\psi}{B^*}$. Second, suppose $D_b^* < \frac{\alpha^* R\psi}{B^*}$. Then, infinitely risk-averse investors value the covered bond claim at D_b since the bank is solvent with positive probability. This violates the supposed optimality of B^*, since raising the issuance volume of covered bonds would raise the objective function ($\frac{\partial \pi}{\partial B} > 0$) without affecting the constraint. Contradiction. Thus, $D_b^* \geq \frac{\alpha^* R\psi}{B^*}$. Taking together, we have $B^* = \frac{\alpha^* R\psi}{D_b^*}$.

As a result, the problem of the banker reduces to:

$$\max_{\{\alpha, B\}} \quad \pi(\alpha, B) \equiv F(S^*(\alpha, B)) \left[R\big(1 + B - \alpha\psi\big) - D_u\right] - \int_{-\infty}^{S^*(\alpha,B)} SdF(S) \quad (25)$$

$$\text{s.t.}$$

$$S^*(\alpha, B) = R[1 - \alpha + B] - \kappa D_u$$

$$1 \leq \frac{\alpha R\psi}{B}.$$

Since $\frac{\partial \pi(\alpha,B)}{\partial \alpha} < 0$, the participation constraint of risk-averse investors binds in equilibrium, so $B^* = \alpha^* R\psi$ and $D_b^* = 1 < R$, which verifies the supposition.

Using these results, the total effect of asset encumbrance on the bankruptcy threshold $S^*(\alpha) = R[1 - \alpha(1 - \psi R)] - \kappa D_u$ is $\frac{dS^*(\alpha)}{d\alpha} = -R(1 - \psi R) < 0$, which we label the risk concentration effect.

Observe that, since the maximum supply of covered bonds is $\psi R < 1$, a minimum mass of risk-averse investors of this amount suffices to absorb any secured funding issued by the bank. Also observe that $S \leq R$ ensures that there is never default on covered bonds.

C Proof of Proposition 2

This proof continues from the preceding proof of Lemma 1 and Lemma 2. Using $D_b^* = 1$ and $B^* = \alpha^* \psi R$, the banker's problem reduces to a simple unconstrained optimization problem. Both the risk concentration and the bank funding effects are taken into account:

$$\max_{\alpha \in [0,1]} \quad \pi(\alpha) \equiv F(S^*(\alpha)) \left[R\big(1 + \alpha(R-1)\psi\big) - D_u\right] - \int_{-\infty}^{S^*(\alpha)} S dF(S) \tag{26}$$

s.t.

$$S^*(\alpha) = R[1 - \alpha(1 - \psi R)] - \kappa D_u.$$

The first and second derivative of the objective function value with respect to the level of asset encumbrance are:

$$\frac{1}{R}\frac{d\pi}{d\alpha} \equiv F(S^*(\alpha))\psi(R-1) - (1-\psi R) f(S^*(\alpha))[(\kappa - 1)D_u + \alpha(1-\psi)R] \tag{27}$$

$$\frac{1}{R^2(1-\psi R)}\frac{d^2\pi}{d\alpha^2} \equiv (1-\psi R) f'(S^*(\alpha))\big[(\kappa-1)D_u + R(1-\psi)\alpha\big] - (R-\psi) f(S^*(\alpha)) < 0$$

The sign of the second-order derivative is ensured by two of our assumptions, $f' \leq 0$ and $\psi R < 1$. Therefore, the objective function is globally concave and there exists at most one solution. If a solution exists, it is a (global) maximum. By continuity of the objective function π, and the closed set $[0, 1]$ over which the banker maximizes, a solution α^* exists. This establishes the existence and uniqueness of a global maximum.

We now study whether this solution α^* is interior. If the solution is interior, it is characterized by the first-order condition $\frac{d\pi}{d\alpha}\big|_{\alpha^*} = 0$:

$$F(S^*(\alpha^*))\psi(R-1) = (1-\psi R) f(S^*(\alpha^*))[(\kappa-1)D_u + \alpha^*(1-\psi)R]. \tag{28}$$

For an interior solution, we first require $\alpha^* > 0$. Rewriting $\frac{d\pi}{d\alpha}\big|_{\alpha=0} > 0$ and using $S^*(\alpha = 0) = R - \kappa D_u$ yields:

$$\frac{\psi(R-1)}{(1-\psi R)}\frac{F[R-\kappa D_u]}{f[R-\kappa D_u]} > (\kappa-1)D_u. \tag{29}$$

Focusing on the left-hand side of condition (29) – the numerator increases in R, while the denominator decreases in R. Thus, the left-hand side increases in R, while the right-hand side is independent of R. Consequently, there exists a $\underline{R}$ such that $\alpha^* > 0$ for all $R > \underline{R}$, where $\underline{R}$ is the unique solution to:

$$\frac{\psi(\underline{R}-1)}{(1-\psi \underline{R})}\frac{F[\underline{R}-\kappa D_u]}{f[\underline{R}-\kappa D_u]} \equiv (\kappa-1)D_u. \tag{30}$$

Note that $\underline{R} \in (1, \frac{1}{\psi})$, since the left-hand side of condition (29) is zero at $R = 1$ and positive infinity for $R \to \frac{1}{\psi}$, while the right-hand side is positive but finite.

Second, we require $\alpha^* < 1$ for an interior solution. Rewriting $\frac{d\pi}{d\alpha}\big|_{\alpha=1} < 0$ and using $S^*(\alpha = 1) = \psi R^2 - \kappa D_u$:

$$\frac{\psi(R-1)}{(1-\psi R)}\frac{F[\psi R^2-\kappa D]}{f[\psi R^2-\kappa D]} < (\kappa-1)D_u + R(1-\psi). \tag{31}$$

As before, the left-hand side of condition (31) increases in R. However, the right-hand side now also increases in R. Since S^* decreases in α (Lemma 2), the left-hand side in condition (31) is smaller than the left-hand side in condition (29). Moreover, the right-hand side in condition (31) is larger than the right-hand side in condition (29) because of the additional term. As a result, we have $\overline{R} > \underline{R}$, where $\overline{R}$ solves:

$$\frac{\psi(\overline{R}-1)}{(1-\psi \overline{R})}\frac{F[\psi \overline{R}^2-\kappa D_u]}{f[\psi \overline{R}^2-\kappa D_u]} \equiv (\kappa-1)D_u + \overline{R}(1-\psi). \tag{32}$$

Since the left-hand side of this expression again goes to positive infinity for $R \to \frac{1}{\psi}$, and the right-hand side remains positive but finite, an upper bound $\overline{R} < \frac{1}{\psi}$ exists. In sum, $\alpha^* \in (0, 1)$ for any $R \in \left(\underline{R}, \overline{R}\right)$.

D Proof of Proposition 3

The equilibrium face value of unsecured debt D_u^* is implicitly defined by the binding participation constraint of risk-neutral investors, $V(D_u^*) = 1$. The proof of existence and uniqueness of D_u^* is in four steps.

First, for any given D_u, the value of the unsecured debt claim decreases in the level of asset encumbrance:

$$\frac{\partial V}{\partial \alpha^*} = D_u[1-\kappa\psi]f(S^{**})\frac{dS^{**}}{d\alpha^*} - \psi R(1-\psi R)\int_{S^{**}}^{S_D^*} dF(S) < 0. \tag{33}$$

Intuitively, more asset encumbrance reduces both the pool of unencumbered assets and the range of credit shocks for which unsecured debt holders are repaid in full, so the overall effect on the value of the unsecured debt claim is negative.

Second, risk-neutral investors never accept a debt claim with face value $D_u = 1$:

$$\begin{aligned} V(D_u = 1) &= F(R[1-\alpha^*(1-\psi R)] - \kappa) + \psi \int_{S_D^*-\kappa}^{S_D^*} (S_D^* - S)dF(S) && (34)\\ &< F(R[1-\alpha^*(1-\psi R)]) + \psi\kappa[F(S_D^*) - F(S_D^* - \kappa)] < 1. && (35)\end{aligned}$$

Third, the value of the unsecured debt claim changes with its face value according to $\frac{dV}{dD_u} = \frac{\partial V}{\partial \alpha^*}\frac{\partial \alpha^*}{\partial D_u} + \frac{\partial V}{\partial D_u}$, where

$$\begin{aligned} \frac{\partial V}{\partial D_u} &= F(S^{**}) - \kappa(1-\psi\kappa)D_u f(S^{**}) && (36)\\ &= f(S^{**})\frac{1-\psi R}{\psi(R-1)}[(\kappa-1)D_u + \alpha^*(1-\psi)R] - \kappa(1-\psi\kappa)D_u f(S^{**}), && (37)\end{aligned}$$

where we used the first-order condition for α^*. Since the indirect effect via α^* is positive, $\frac{\partial V}{\partial \alpha^*}\frac{\partial \alpha^*}{\partial D_u} > 0$, and since $\alpha^*(1-\psi)Rf(S^{**}) \geq 0$, a sufficient condition for $\frac{dV}{dD_u} > 0$ is that the term multiplying $f(S^{**})D_u$ is non-negative, $-\kappa(1-\psi\kappa) + \frac{1-\psi R}{\psi(R-1)}(\kappa - 1) \geq 0$. Rewriting yields the stated upper bound on investment profitability:

$$R \leq \tilde{R} \equiv \frac{\kappa - 1 + \kappa\psi(1-\kappa\psi)}{\psi(\kappa-1) + \kappa\psi(1-\kappa\psi)} \in \left(1, \frac{1}{\psi}\right). \tag{38}$$

This condition ensures the monotonicity of the unsecured debt claim in its face value and suffices for uniqueness of D_u^*.

Fourth, existence requires that there is a feasible face value such that risk-neutral investors accept the debt claim. At most the total value of unencumbered assets, S_D^* can be credibly promised. Thus, we require $V(D_u = S_D^*) > 1$ because of monotonicity. Note that $S^{**} = -(\kappa-1)S_D^*$, so:

$$1 < S_D^* F\left[-(\kappa-1)S_D^*\right] + \psi \int_{-(\kappa-1)S_D^*}^{S_D^*} (S_D^* - S)dF(S). \tag{39}$$

Since greater asset encumbrance dilutes the unsecured debt claim, a sufficient condition in terms of exogenous parameters of the model can be obtained by evaluating this inequality at $\alpha^* = 1$, which yields

$$\psi R^2 F\left[-R(\kappa - \psi R)\right] + \psi \int_{-R(\kappa-\psi R)}^{\psi R^2} (\psi R^2 - S)dF(S) \geq 1. \tag{40}$$

This condition suffices for the existence of D_u^*. Figure 5 illustrates.

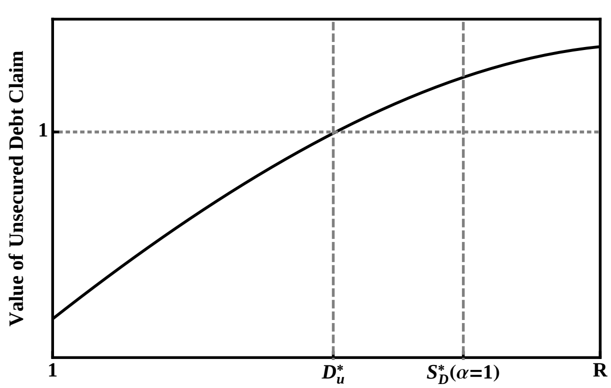

Figure 5: Shock and unsecured debt claim.

E Proof of Lemma 3

Greater asset encumbrance and a larger proportion of guaranteed debt both increase the expected cost to the guarantor:

$$\frac{\partial C}{\partial \alpha} \equiv C_\alpha = R(1-R\psi)\big[F(\hat{S}_m+m)-F(\hat{S}_m)\big] > 0 \tag{41}$$

$$\frac{\partial C}{\partial m} \equiv C_m = 1-F(\hat{S}_m+m)+\frac{F(\hat{S}_m+m)-F(\hat{S}_m)}{\psi} > 0 \tag{42}$$

Moreover, we have the following results for second-order derivatives:

$$\frac{\partial^2 C}{\partial \alpha^2} \equiv C_{\alpha\alpha} = -R^2(1-R\psi)\big[f(\hat{S}_m+m)-f(\hat{S}_m)\big] \geq 0 \tag{43}$$

$$\frac{\partial^2 C}{\partial \alpha \partial m} \equiv C_{\alpha m} = -R(1-R\psi)\left[\left(\frac{1}{\psi}-1\right)f(\hat{S}_m+m)+\frac{1}{\psi}f(\hat{S}_m)\right] < 0, \tag{44}$$

since $f(\hat{S}_m+m) \leq f(\hat{S}_m)$ because of $f' \leq 0$.

F Proof of Proposition 4

The first-order condition is a direct extension of the main model. The comparative static follows from the implicit function theorem, where

$$\begin{aligned}\pi_{\alpha\alpha} &\equiv \frac{dS_m^*}{d\alpha}[\psi R(R-1)f(S_m^*)-R(1-R\psi)[(\kappa-1)(1-m)D_u+\alpha_m^*(1-\psi)R]f'(S_m^*)]+ \\ &\quad \cdots - f(S_m^*)(1-R\psi)R^2(1-\psi) < 0 \end{aligned} \tag{45}$$

$$\pi_{\alpha m} \equiv R\frac{dS_m^*}{dm}\big(\psi(R-1)f(S_m^*)-(1-R\psi)f'(S_m^*)\big)-f(S_m^*)(1-R\psi)R^2(1-\psi) > 0. \tag{46}$$

Thus, by the implicit function theorem, $\frac{d\alpha_m^*}{dm} = -\frac{\pi_{\alpha m}}{\pi_{\alpha\alpha}} > 0$.

G Proof of Proposition 5

The constraint efficient level of asset encumbrance, α_P^*, solves the first-order condition $\pi_\alpha(\alpha_P^*) = C_\alpha(\alpha_P^*)$, which can be expressed as

$$\begin{aligned}\frac{\psi(R-1)}{1-\psi R}F(S_m^*(\alpha_P^*)) &= f(S_m^*(\alpha_P^*))[(\kappa-1)(1-m)D_u + \alpha_P^*(1-\psi)R] \\ &\quad + \big[F(\hat{S}_m(\alpha_P^*)+m) - F(\hat{S}_m(\alpha_P^*))\big].\end{aligned} \tag{47}$$

The constrained efficient level of encumbrance exists and is the unique global maximum, since $W_{\alpha\alpha} \equiv \pi_{\alpha\alpha} - C_{\alpha\alpha} < 0$.

To establish constrained inefficiency of the private level of encumbrance, note that $W(\alpha_m^*; m) = -C(\alpha_m^*; m) < 0$. Since α_P^* solves $W(\alpha_P^*; m) = 0$, and the objective function of the planner is globally concave, $W_{\alpha\alpha} < 0$, it follows that $\alpha_P^* < \alpha_m^*$.

For the second part of the proposition, the claim can be established if we show that $\frac{d\alpha_P^*}{dm} < \frac{d\alpha_m^*}{dm}$. Let $W_{\alpha m} \equiv \pi_{\alpha m} - C_{\alpha m}$. Using the implicit function theorem for both α_P^* and α_m^*, this inequality requires

$$-\frac{W_{\alpha m}}{W_{\alpha\alpha}} < -\frac{\pi_{\alpha m}}{\pi_{\alpha\alpha}} \tag{48}$$

$$-\frac{\pi_{\alpha m} - C_{\alpha m}}{\pi_{\alpha\alpha} - C_{\alpha\alpha}} < -\frac{\pi_{\alpha m}}{\pi_{\alpha\alpha}} \tag{49}$$

$$C_{\alpha m}\pi_{\alpha\alpha} < \pi_{\alpha m}C_{\alpha\alpha}, \tag{50}$$

which always holds.

H Proof of Propositions 6–9

We compute comparative statics of α^* with respect to four variables (γ, D_u, ψ, R), for the case $R \in \big(\underline{R}, \overline{R}\big)$, which guarantees an interior solution $\alpha^* \in (0,1)$. The implicit function defining α^* is:

$$IA \equiv \frac{F(S^{**})}{f(S^{**})} - \frac{(1-\psi R)}{\psi(R-1)}[(\kappa-1)D_u + \alpha^* R(1-\psi)] = 0\,, \tag{51}$$

where $S^{**} = S^*(\alpha^*)$, which satisfies

$$\begin{aligned}
\frac{\partial S^{**}}{\partial D_u} &= -\kappa < 0 && (52)\\
\frac{\partial S^{**}}{\partial R} &= 1-\alpha^* > 0 && (53)\\
\frac{\partial S^{**}}{\partial \gamma} &= -\left(\frac{1}{\psi}-1\right) D_u < 0 && (54)\\
\frac{\partial S^{**}}{\partial \psi} &= R^2\alpha^* + \frac{\gamma D_u}{\psi^2} > 0, && (55)
\end{aligned}$$

Using the shorthand $\Xi = (\kappa-1)D_u + \alpha^*(1-\psi)R$, the partial derivatives of IA with respect to α^*, γ, D_u, ψ and R are

$$\begin{aligned}
\frac{\partial IA}{\partial \alpha^*} &= \psi(R-1)f(S^{**})\frac{\partial S^{**}}{\partial \alpha} - (1-\psi R)f'(S^{**})\frac{\partial S^{**}}{\partial \alpha}\Xi \\
&- (1-\psi R)f(S^{**})(1-\psi)R < 0\,, && (56)\\
\frac{\partial IA}{\partial \gamma} &= \psi(R-1)f(S^{**})\frac{\partial S^{**}}{\partial \gamma} - (1-\psi R)f'(S^{**})\frac{\partial S^{**}}{\partial \gamma}\Xi < 0\,, && (57)\\
\frac{\partial IA}{\partial D_u} &= \psi(R-1)f(S^{**})\frac{\partial S^{**}}{\partial D_u} - (1-\psi R)f'(S^{**})\frac{\partial S^{**}}{\partial D_u}\Xi \\
&- (1-\psi R)f(S^{**})(\kappa-1) < 0\,, && (58)\\
\frac{\partial IA}{\partial \psi} &= (R-1)F(S^{**}) + \psi(R-1)f(S^{**})\frac{\partial S^{**}}{\partial \psi} + Rf(S^{**})\Xi \\
&- (1-\psi R)f'(S^{**})\frac{\partial S^{**}}{\partial \psi}\Xi - (1-\psi R)f(S^{**})D_u\frac{d\kappa}{d\psi} > 0\,, && (59)\\
\frac{\partial IA}{\partial R} &= \psi F(S^{**}) + \psi(R-1)f(S^{**})\frac{\partial S^{**}}{\partial R} + \psi f(S^{**})\Xi \\
&- (1-\psi R)f'(S^{**})\frac{\partial S^{**}}{\partial R}\Xi - (1-\psi R)f(S^{**})\alpha^*(1-\psi) > 0\,. && (60)
\end{aligned}$$

Using the implicit function theorem, we obtain that

$$\frac{d\alpha^*}{d\gamma} < 0, \quad \frac{d\alpha^*}{dD_u} < 0, \quad \frac{d\alpha^*}{d\psi} > 0, \quad \text{and} \quad \frac{d\alpha^*}{dR} > 0. \tag{61}$$

Consider an exponential distribution of the shock with rate parameter $\lambda > 0$, where the distribution is truncated at $S = R$. Thus, $f(S) = \frac{\lambda e^{-\lambda S}}{1-e^{-\lambda R}}$ and $F(S) = \frac{1-e^{-\lambda S}}{1-e^{-\lambda R}}$. Then,

$$\frac{\partial IA}{\partial \lambda} = \frac{1 + e^{\lambda S^{**}}(-1 + \lambda S^{**})}{\lambda^2} > 0. \tag{62}$$

It thus follows that

$$\frac{d\alpha^*}{d\lambda} > 0. \tag{63}$$

I Proof of Propositions 10

The bound $\breve{S} = R - \kappa$ is constructed to ensure $\breve{S} > S^*$. Since $\alpha^* \geq 0$ and $D_u^* > 1$, we always have $\breve{S} > S^* = R[1 - \alpha(1 - \psi R)] - \kappa D_u$. Since both F and $\tilde{F}$ are identical for any $S < \breve{S}$, it follows from equation (28), which defines the optimal level of asset encumbrance, that $\alpha^* = \tilde{\alpha}^*$. Thus, the effect of the change in distribution affects the value of the unsecured debt claim only via the liquidation value in bankruptcy, but not via changes in asset encumbrance.

Next, observe that $V(D_u|\tilde{F}) > V(D_u|F)$, for any given D_u, because of the lower tail risk under $\tilde{F}$. Since risk-neutral MMF investors always break even in expectation, we have $V(\tilde{D}_u^*|\tilde{F}) = 1 = V(D_u^*|F)$. Finally, since $\frac{dV}{dD_u} > 0$ as showed before, it follows that $\tilde{D}_u^* < D_u^*$.