Supervising Financial Regulators

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This paper studies optimal supervision of local financial regulators. In the model, local regulators are better informed about local economic and financial conditions but have a bias towards socially excessive leniency. The central insight is that optimal supervision emphasizes rewards over punishments. In particular, a regulator that was relatively more lenient in the past may not be more constrained to exercise leniency in the future. As a result, expected leniency across local regulators may increase in the future when actual leniency differed across local regulators in the past.

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1 Introduction

Supervision of local financial regulators faces the challenge of finding a balance between allocative efficiency of credit and financial stability. Examples of local regulators are state regulators overseeing mortgage originators and state-chartered financial institutions in the United States, and national regulators overseeing national financial institutions in the European Union. In the case of the United States, Agarwal, Lucca, Seru, and Trebbi (2014) find that local regulators enjoy some discretion but are biased toward excess leniency to accommodate local economic conditions, while the cost of local regulatory leniency is partially borne by economy-wide assistance funds. Similar incentive problems led to plans to create a banking union in the European Union (ASC, 2012; Hellwig, 2014). The recent European financial crisis has been providing a testing ground for policies that are potentially useful in a banking union. However, those policies may have been insufficient as European institutions face institutional constraints that could constrain their ability to support Euro-area welfare while also limiting moral hazard. This paper, by addressing the question of how to optimally design central supervision of local financial regulators, can shed light on these concerns.

I build a model where local financial regulators can exercise leniency to ease local credit conditions at the cost of higher expected losses imposed on an economy-wide resolution fund.[1] The social net benefit of leniency is assumed to vary stochastically across localities and over time. As a result, local trade-offs between easing credit conditions and promoting financial stability are not constant but change with local economic and financial conditions. It is assumed that local regulators have superior

[1]Resolution of failed financial institutions, at the very least, requires interim funding which could be provided by economy-wide industry levies (ASC, 2012; Armour, 2014; Hellwig, 2014).

information about local economic and financial conditions such that there is informational asymmetry regarding the net social benefit of leniency across localities.[2] The informational asymmetry matters since, due to reliance on the economy-wide resolution fund, local net benefits of regulatory lenience always exceed economy-wide net benefits. In other words, local regulators have an incentive to overstate local benefits of regulatory lenience.[3] Financial stability may consequently suffer in the sense of excessive leniency and thus excessive reliance on the economy-wide resolution fund.[4] However, while a local regulator does not care about externalities imposed on other localities whenever it exercises leniency, it does care about externalities imposed by other local regulators on its own locality. A supervisor of local regulators can use this fact to better align incentives at each point in time.

I show how optimal supervision of local regulators addresses incentive problems due to informational asymmetries. Each local regulator's discretion to increase leniency when claiming a high need for leniency is increasing in the total number of

[2] Incentive problems that arise from the combination of informational asymmetries and externalities across local regulators are considered important in the policy debate about a European banking union. ASC (2012) summarizes these concerns as follows:

> Cross-border externalities and EU-wide concerns cannot be dealt with effectively by legislation mandating a harmonisation of regulatory and supervisory practices. [...] Exercising judgment requires some flexibility and discretion, for which the legislation must leave room. [...] The distribution of supervisory competencies between national and EU authorities requires a judicious calibration of different concerns, taking account of the information advantages of national supervisors in dealing with their banks.

In Europe, large financial institutions, representing 85 percent of assets held by financial institutions in Europe, will be monitored by a central authority. Banking union members will have an information advantage regarding small local financial institutions and local economic conditions.

[3] For example, local regulators might argue that local borrowers are financially resilient such that a more lenient regulatory stance would have a large positive effect on local economic growth, for given expected losses to the economy-wide resolution fund. Similar, a local regulator might, for example, argue that leniency is needed to support local financial intermediaries which, despite strong balance sheets, suffer from liquidity problems and are not insolvent (see also Foarta, 2014).

[4] In my model, the resolution fund is a "common pool", and financial stability is a public good (Schoenmaker, 2011). In practice, the resolution fund might be administered by a central bank such that financial stability and price stability become interconnected.

local regulators that claim a high need for leniency. By linking leniency across local regulators in this way, supervision can increase the degree to which local regulators internalize the costs of their own local leniency. There are important implications for the dynamics of financial stability. The first implication is that a local regulator that took a more lenient stance in the past may not be required to limit its discretion in any future state. In fact, optimal supervision focuses on rewarding local regulators who exercised less lenience in the past rather than punishing local regulators who exercised more lenience. Rewards take the form of higher discretion as well as being allowed excessive leniency as an immediate within-period response to high leniency by another local regulator. In that sense, local regulators are being rewarded in part by letting them 'supervise' other local regulators by use of state-contingent (ex-post) excessive leniency. The second implication is that when local regulators had different stances with respect to regulatory leniency in the past then expected leniency may increase in the future. In that sense, disagreements about the preferred balance between supporting credit and fostering financial stability can lead to a deterioration of financial stability in the future.

During the recent European financial crisis, a number of European Union member countries faced large costs due to losses generated by their respective financial sectors. Some member countries received assistance from the European Central Bank (ECB) which bought or implicitly guaranteed individual member public debt, enabling member countries to bail out their respective struggling financial sectors (Fratzscher and Rieth, 2015). Since the (expected) cost of central bank assistance is shared across the Euro area – for example via lower ECB profits or lower private savings returns – member countries impose externalities on each other when allowing potentially unsustainably high domestic private or public debt issuance. In 2015, despite these moral hazard concerns, the ECB provided further assistance to Greece, following earlier assistance to that country and other South-European member countries (ECB, 2012, 2015b). The ECB also started in 2015 to buy public debt proportional to ECB equity, i.e. its 'quantitative easing' is directed mainly at public debt issued by large Euro area members such as Germany, France and Italy (ECB, 2015a). While some observers criticized the ECB for helping out Greece again (Feld et al., 2015), others criticized it for not directing public debt purchases more towards Southern European members such as Greece (De Grauwe and Ji, 2015). It seems as if the ECB is doing both too much and too little to help Greece when repeatedly assisting Greece while at the same time also providing assistance to members that need it less, such as Germany. This observation raises concerns that European institutions may have faced political or institutional constraints that limited their ability to support Euro-area welfare during the recent European financial crisis.

The model implications can be used to discuss the policies of European institutions during the European financial crisis. The model implies that, over time, discretion granted to Southern European countries can be maintained when introducing (ex-post) wasteful support for Northern European countries. Having Northern European countries such as France and Germany benefit from quantitative easing is required, in order to maintain incentives, when extending ECB assistance repeatedly to Southern European countries such as Greece. The model also implies that the expected size of the ECB balance sheet increases further following its actions during crisis management. Observed policy actions in Europe seem appropriate, given the analysis in this paper, and they therefore provide useful guidance for the design of a European banking union.

My benchmark model has two periods. An extension illustrates that, under optimal supervision, local regulators incentivize each other with state-contingent (ex-post) excessive leniency even when an infinite horizon is considered.

Related literature:

Amador and Bagwell (2012, 2013) study problems due to informational asymmetries and externalities in the context of a static model of trade tariffs. In my application to supervision of local financial regulators I explicitly allow for policies which exploit the fact that informational asymmetries are not one-sided. A principal can use this fact and effectively let agents supervise each other. For example, a country facing a trading partner that imposes relatively higher import restrictions, after having already done so in the past, should respond immediately by imposing high import restrictions as well even if this leads to lower joint welfare ex-post. However, with a focus limited to a one-sided principal agent problem, as in Amador and Bagwell (2012, 2013), such dynamics remain hidden and repetition of optimal static policies may be optimal when considering longer horizons (Athey et al., 2005; Amador et al., 2006). Foarta (2014) studies supervision of a single local regulator in a delegation problem similar to the one studied in Amador et al. (2006). However, in her model, dynamics – in the form of time-varying, rather than static as in Amador et al. (2006), policy cutoffs – can arise when a local regulator can issue non-contingent debt subject to borrowing constraints. When a local regulator incurs more debt in the model in Foarta (2014) then it trades higher discretion today against reduced discretion in the future. Instead, in my model, dynamics arise because I allow for the actions of multiple local regulators to be coordinated on each other within each period. When local regulators engage in immediate reciprocity in my model, they make incentive-compatible a high degree of discretion for a local regulator that already exercised discretion in the past.

Carletti, Dell'Ariccia, and Marquez (2015) also consider supervision of biased local financial regulators. Their analysis is static and focuses on moral hazard of local regulators with respect to the collection of information while I assume that local regulators possess private information. My focus, however, is on explicitly modeling the bias of local regulators as due to externalities that local regulators impose on each other. There is thus a benefit in my model of linking, across local regulators, any regulatory supervision imposed on local supervisors. This focus generates important novel dynamic implications in my model.[5] Holthausen and Rønde (2004) and Colliard (2014) study static economies with privately informed and biased local regulators (see also Hardy and Nieto, 2011 and Agur, 2013). These papers, in addition, allow for financial institutions to react to changes in regulatory stances explicitly while I do not explicitly model financial institution behavior. Zoican and Gornicka (2015) study the case of a supervisor of local financial regulators who faces a time-inconsistency problem and may be too lenient ex-post in a way that distorts incentives of financial institutions toward more risk-taking. Time-inconsistency in Carletti et al. (2015) implies that a supervisor may be too tough ex-post in a way that distorts incentives of local regulators toward less information collection. I abstract from modelling bank risk-taking or local regulator information collection explicitly, so that these sources of time-inconsistency are not present in my model. Foarta (2015) focuses on political economy frictions such that assistance for a country facing finan-

[5]Linking actions within the period, and not only over time, can strengthen incentives in many different economic environments. Roberts (1985) and Goltsman and Pavlov (2014) show, in a static context, how firms facing oligopolistic competition can partially overcome inefficiencies due to asymmetric information about production costs by linking output within the period. Santoro (2015) studies optimal fiscal policy in a monetary union in a static model with two-sided private information and limited commitment. His model extends the model in Chari and Kehoe (2007) by allowing for private information, and adds to the analysis in Sanguinetti and Tommasi (2004) by allowing for limited commitment.

cial sector difficulties must be designed with problems of local governance in mind. She shows that assistance for a country should be coordinated with both tighter fiscal rules and electoral control.

Supervision partially insures local regulators against the risk of a high need for local regulatory leniency. However, due to the presence of an economy-wide resolution fund the model has implications that go beyond the insights generated by the literature on inter-temporal risk sharing. In many models of risk sharing the aggregate amount of resources available to divide among agents is predetermined at the beginning of each period (Thomas and Worrall, 1990; Atkeson and Lucas, 1992; Taub, 1994), while average leniency can be adjusted instantaneously in my model. Local regulators that exercised more leniency in the past may not experience limited discretion with respect to setting their regulatory stance in the future in my model. In contrast, in the risk-sharing literature debtors often face limited discretion.

The paper is organized as follows. Section 2 presents a model, section 3 characterizes optimal supervision of local regulators, section section 4 extends the model to an infinite time horizon and section 5 concludes.

2 Two-period model

There are two time periods, $t = 1, 2$, and two local financial regulators, $j = 1, 2$. Local regulator $j = 1, 2$ chooses leniency $\ell_{j,t} \in [0, l]$, $l \in (0, \infty)$, and contributes d_t to a resolution fund in period $t = 1, 2$. Contributions to the resolution fund are uniform across localities since local regulators pay for contributions by taxing local agents, for example local financial institutions. Any such taxes must be uniform – otherwise taxed agents would relocate to seek the lowest tax burden – so that contributions cannot be different across localities.

The upper bound l on local leniency is motivated by a fixed number of local available credit-dependent projects that can be realized as local leniency is increased. These projects cannot be relocated across localities, for example due to decreasing returns, such that local regulators can exercise different degrees of leniency. In other words, I assume that while the tax base for the resolution fund is mobile across localities, credit-dependent projects are not. This paper therefore focuses on common-pool externalities resulting from usage of the economy-wide resolution fund and abstracts from potential race-to-the-bottom externalities due to regulatory arbitrage across localities.[6]

A local regulator trades off the local social benefit from increased leniency against the cost of higher contributions to the resolution fund. It is assumed that this trade-off varies stochastically with local economic and financial conditions. In each period, local regulator $j = 1, 2$ experiences a shock $s_{j,t} \in \{s_L, s_H\}$, with equal probability. Note that shocks are independent across local regulators and over time. $s_{j,t}$ determines the benefit from increasing leniency in locality j relative to the costs imposed on the economy-wide resolution fund. It is assumed that only local regulator j can observe $s_{j,t}$. For example, local financial regulators can better assess the financial health of local borrowers and local financial institutions.

Let $S = \{s_L, s_H\} \times \{s_L, s_H\}$ and let s^t denote the history of realizations of shocks at time $t = 1, 2$ such that $s^t \in S^t$. The function $\theta : \{s_L, s_H\} \to \{\theta_L, \theta_H\}$ maps the shock into a parameter $\theta(s_k) = \theta_k$, $k = L, H$. It is assumed that the expected value of the parameter is $\mu < 1$ and that $\frac{1}{2} < \theta_L < \mu < 1 < \theta_H$. Denote the variance of the

[6] Dell'Ariccia and Marquez (2006) focus on race-to-the-bottom externalities instead. In their model a supervisor can avoid such externalities but is constrained to impose the same policy on each local regulator. As a result, their model delivers a trade-off with respect to the creation of a banking union.

parameters by σ^2 such that $\theta_L = \mu - \sigma$ and $\theta_H = \mu + \sigma$.

Let $\ell_j = \{\ell_{j,t}(s^t)\}_{s^t \in S^t,\, t=1,2}$ denote a path for leniency of local regulator j and let $d = \{d_t(s^t)\}_{s^t \in S^t,\, t=1,2}$ denote a path for contributions to the resolution fund. Local regulator j ranks paths $\{\ell_j, d\}$ according to the local welfare criterion

$$W\left(\{\ell_j, d\}\right) = \sum_{t=1,2} \delta^{t-1} \left(\frac{1}{2}\right)^{2t} \sum_{s^t \in S^t} \left[\theta(s_{j,t})\ell_{j,t}(s^t) - d_t(s^t)\right], \tag{1}$$

where $\delta > 0$ is a discount factor. In each state s^t, contributions to the economy-wide resolution fund are related to leniency exercised by local regulators as follows:

$$d_t(s^t) = \frac{1}{2}\left[\ell_{1,t}(s^t) + \ell_{2,t}(s^t)\right]. \tag{2}$$

The welfare criteria of local regulators one and two can then be written, respectively, as

$$\begin{aligned}
&\sum_{t=1,2} \delta^{t-1} \left(\frac{1}{2}\right)^{2t} \sum_{s^t \in S^t} \left[\left(\theta(s_{1,t}) - \frac{1}{2}\right)\ell_{1,t}(s^t) - \frac{1}{2}\ell_{2,t}(s^t)\right], \\
&\sum_{t=1,2} \delta^{t-1} \left(\frac{1}{2}\right)^{2t} \sum_{s^t \in S^t} \left[\left(\theta(s_{2,t}) - \frac{1}{2}\right)\ell_{2,t}(s^t) - \frac{1}{2}\ell_{1,t}(s^t)\right].
\end{aligned} \tag{3}$$

Joint welfare of the two local regulators, i.e. the sum of the local welfare criteria, is given by

$$\Omega \equiv \sum_{t=1,2} \delta^{t-1} \left(\frac{1}{2}\right)^{2t} \sum_{s^t \in S^t} \left[\left(\theta(s_{1,t}) - 1\right)\ell_{1,t}(s^t) + \left(\theta(s_{2,t}) - 1\right)\ell_{2,t}(s^t)\right]. \tag{4}$$

A local regulator that increases leniency by one (marginal) unit internalizes an increase in its contribution to the economy-wide resolution fund by one-half units.

But it does not internalize the increase of resolution fund contributions by one-half units for the other local regulator. In other words, the local net social benefit of leniency for local regulator j in period t is $\theta(s_{j,t}) - 1/2$, see equation (3), while the economy-wide net social benefit is $\theta(s_{j,t}) - 1$, see equation (4).

2.1 First-best allocation

The first-best is defined as local regulator leniency that maximizes joint welfare Ω and is characterized in lemma 1. Definitions 1 and 2 are helpful in discussing the properties of the first-best.

Definition 1 (Coordination). *Leniency is coordinated across local regulators if leniency of local regulator j in period t depends on the shock experienced by local regulator $i \neq j$ in period t, for some $t = 1, 2$ and some $j = 1, 2$. I.e. leniency is coordinated if $\ell_{j,t}(s^t) \neq \ell_{j,t}(\hat{s}^t)$ for some $\hat{s}^t$ with $s^{t-1} = \hat{s}^{t-1}$ and $s_{j,t} = \hat{s}_{j,t}$, for at least one $j = 1, 2$, $t = 1, 2$, $s^t \in S^t$.*

Definition 2 (Discretion). *Local regulator j is allowed to exercise discretion in period t if its leniency is strictly positive in at least one state in which the economy-wide net social benefit of leniency in locality j is strictly positive. Then local regulator j is allowed to exercise leniency in period t if for some $(s_{1,t}, s_{2,t}) \in S$ with $\theta(s_{j,t}) - 1 > 0$ it is the case that $\ell_{j,t}(s^t) > 0$. Let $\chi_{j,t}(s^{t-1}) = E(\ell_{j,t}(s^t) | s^{t-1}, s_{j,t} = s_H)$ be a measure of discretion exercised by local regulator j in period t, where $E(\cdot|\cdot)$ denotes conditional expectation.*

Definition 3 (Excessive leniency). *Local regulator j is granted excessive leniency in period t if its leniency is strictly positive whenever the economy-wide net social benefit of leniency in locality j is strictly negative. I.e. local regulator j is allowed to exercise leniency in period t if for some $(s_{1,t}, s_{2,t}) \in S$ with $\theta(s_{j,t}) - 1 < 0$ it is the case that $\ell_{j,t}(s^t) > 0$. Let*

$\mathcal{E}_{j,t}(s^{t-1}) = E(\ell_{j,t}(s^t)|s^{t-1}, s_{j,t} = s_L)$ be a measure of excessive leniency granted to local regulator j in period t, where $E(\cdot|\cdot)$ denotes conditional expectation.

Lemma 1. *First-best leniency exercised by local regulators is characterized as follows:*

$$\ell^{FB}_{j,t}(s^t) = \ell^{FB}(s_{j,t}) = \begin{cases} 0, & \text{if } \theta(s_{j,t}) = \theta_L; \\ l, & \text{if } \theta(s_{j,t}) = \theta_H, \end{cases} \qquad s^t \in S^t,\, t = 1,2\,, j = 1,2,$$

and yields joint welfare of $\Omega^{FB} = (1+\delta)(\theta_H - 1)l$.

Proof. From the expression of Ω in equation (4), it can be seen that leniency exercised by local regulator j should be as low as possible whenever $\theta(s_{j,t}) < 1$, and as high as possible whenever $\theta(s_{j,t}) > 1$. Recall that $\theta_L < 1 < \theta_H$. □

First-best leniency is time-independent and leniency of local regulator j depends only on local regulator j's shock. Local regulators are perfectly insured against shocks that affect the local social benefit of leniency. That is, in the absence of informational asymmetry, there is no need to coordinate leniency across local regulators and discretion is maximal at $\chi_{j,t}(s^{t-1}) = l$ for all $j = 1,2$, $t = 1,2$ and $s \in S$.

Corollary 1. *First-best leniency is not incentive compatible.*

Proof. The economy-wide net social benefit of leniency by local regulator j by equation (4) is given by $\theta(s_{j,t}) - 1$, which is positive only for $\theta(s_{j,t}) = \theta_H$. However, local regulator j's local net social benefit of leniency, by equation (3), is given by $\theta(s_{j,t}) - 1/2$, which is always positive. A local regulator thus has an incentive to always claim having received the high shock. □

All else constant, an local regulator prefers its own local leniency to be as high as possible irrespective of its shock. In section 3, coordination of leniency will be essential precisely because it removes the notion of 'all else constant.' This paper shows how coordination of leniency depends on past realizations of shocks across local regulators.

3 Analysis of the two-period model

In this section I formulate a principal-agent problem where joint welfare Ω is maximized subject to local regulators being incentivized to truthfully report their respective shocks (revelation principle). Leniency exercised by each local regulator is then a function of (truthful) reports of shocks by both local regulators, $\ell_{j,t} : S^t \to [0, l]$ for $j = 1, 2$ and $t = 1, 2$. Each period, both local regulators report their respective shock at the same time.

3.1 Static case without coordination

Suppose, for the purpose of this subsection only, that leniency of local regulator j can only depend on local regulator j's current shock, $\ell_{j,t}(s^t) = \ell_{j,t}(s_{j,t})$. Then local regulator j would report the shock that yields it the highest degree of leniency (recall the discussion following corollary 1). Leniency is then constant at some $\bar{\ell} \in [0, l]$.[7] Expected period welfare per local regulator is given by $(\mu - 1)\bar{\ell}$. Since $\mu < 1$, the value $\bar{\ell}$ that yields the highest period welfare is zero. That is, local regulators should receive no discretion at all with respect to exercising leniency. For the remainder of this paper, it is assumed that each local regulator must at least enjoy expected period welfare of zero at the beginning of each period. assumption 1 ensures that a local

[7] $\bar{\ell}$ could also be interpreted as an upper bound on leniency (see Melumad and Shibano, 1991 and Amador and Bagwell, 2013, and also Athey et al., 2005 and Amador et al., 2006).

regulator that chooses, before observing its shock, not to exercise leniency in any state of the world also is not required to contribute to the economy-wide resolution fund.

Assumption 1 (Individual rationality)**.** *At the beginning of each period, before shocks are observed, each local regulator must enjoy expected period welfare of at least zero.*

3.2 Static case with coordination

Suppose, for the purpose of this subsection only, that leniency exercised by local regulator j cannot depend on past realizations of shocks, $\ell_{j,t}(s^t) = \ell_{j,t}(s_{1,t}, s_{2,t})$. Lemma 2 shows how leniency optimally depends on reported shocks in this case.

Lemma 2. *Suppose leniency can only depend on current but not past shocks. Then, joint welfare is maximized by leniency given by*

$$
\begin{aligned}
&\ell_{1,t}(s_H, s_H) = \ell_{2,t}(s_H, s_H) = l, \\
&\ell_{1,t}(s_H, s_L) = \ell_{2,t}(s_L, s_H) = \frac{1-\theta_L}{\theta_L} l, \\
&\ell_{1,t}(s_L, \cdot) = \ell_{2,t}(\cdot, s_L) = 0.
\end{aligned}
$$

Proof. See Appendix A.1. □

Local regulators are fully insured against common shocks but are only partially insured when experiencing different shocks. This improves upon the case in section 3.1, which offered no insurance at all. Note that there is no socially wasteful, or excessive, leniency, $\ell_{1,t}(s_L, \cdot) = \ell_{2,t}(\cdot, s_L) = 0$. It follows directly from Lemma 2 that period welfare for each local regulator is given by

$$v_0 = \frac{1}{4}\sum_{s_1,s_2} (\theta(s_1) - 1)\,\ell_{j,t}(s_1, s_2) = \frac{\theta_H - 1}{4\theta_L} l, \tag{5}$$

which is strictly larger than period welfare of zero obtained in the static case without coordination (section 3.1). The reason for this improvement is that coordinating leniency within the period can relax incentive compatibility constraints.

To see this, consider a local regulator with shock s_L who reports s_H instead. Such a local regulator now receives higher leniency, on average, which increases the local regulator's period welfare in expectation by

$$\frac{1}{2}\left[\theta_L - \frac{1}{2}\right] l + \frac{1}{2}\left[\theta_L - \frac{1}{2}\right] \frac{1-\theta_L}{\theta_L} l = \frac{1}{2}\left(\theta_L - \frac{1}{2}\right)\frac{1}{\theta_L} l.$$

But leniency of the other local regulator is now also higher, on average, which decreases the period welfare payoff by $\frac{1}{2}\frac{1}{2}\left(1 - \frac{1-\theta_L}{\theta_L}\right) l = \frac{1}{2}\left(\theta_L - \frac{1}{2}\right)\frac{1}{\theta_L} l$. Thus, the local regulator cannot achieve a net increase in its period welfare by overstating its need for leniency. What keeps a local regulator from overstating its shock is the expected increase in the other local regulator's leniency and the associated increase in contributions to the economy-wide resolution fund. In the model, local regulators cannot free ride on each other's contributions toward the resolution fund if leniency is coordinated in this way.

3.3 Dynamic case with coordination

Optimal supervision of local financial regulators allows each local regulator's leniency to depend on current as well as past shocks experienced by both local regulators. Leniency can thus be coordinated within each period and local regulators can also transfer utility intertemporally. The main result of the paper, which is derived in this section, shows how within-period coordination changes when local regulators transfer utility intertemporally. Policy implications are discussed.

3.3.1 Second period

It is useful to first characterize the set of feasible second-period welfare pairs that can be delivered to local regulators. Let $v \in [0, \bar{v}]$ be second-period welfare to be delivered to local regulator $j = 2$. The lower bound on v is due to assumption 1, and $\bar{v}$ will be defined below. For a given v, let $\ell_j(s_1, s_2)$ be leniency of local regulator $j = 1, 2$ when local regulator one reports shock s_1 and local regulator two reports shock s_2. Local regulator two enjoys second-period welfare of at least v whenever the following promise-keeping constraint holds:

$$\frac{1}{4} \sum_{(s_1,s_2) \in S} \left[\theta(s_2)\ell_2(s_1, s_2) - \frac{1}{2}\ell_1(s_1, s_2) - \frac{1}{2}\ell_2(s_1, s_2) \right] \geq v. \tag{6}$$

Local regulators will report shocks truthfully whenever the following incentive compatibility constraints hold:[8]

$$\frac{1}{2}\sum_{s_2\in\{s_L,s_H\}}\left[\theta(s_L)\ell_1(s_L,s_2)-\frac{1}{2}\ell_1(s_L,s_2)-\frac{1}{2}\ell_2(s_L,s_2)\right]$$
$$\geq\frac{1}{2}\sum_{s_2\in\{s_L,s_H\}}\left[\theta(s_L)\ell_1(s_H,s_2)-\frac{1}{2}\ell_1(s_H,s_2)-\frac{1}{2}\ell_2(s_H,s_2)\right], \tag{8}$$

$$\frac{1}{2}\sum_{s_1\in\{s_L,s_H\}}\left[\theta(s_L)\ell_2(s_1,s_L)-\frac{1}{2}\ell_1(s_1,s_L)-\frac{1}{2}\ell_2(s_1,s_L)\right]$$
$$\geq\frac{1}{2}\sum_{s_1\in\{s_L,s_H\}}\left[\theta(s_L)\ell_2(s_1,s_H)-\frac{1}{2}\ell_1(s_1,s_H)-\frac{1}{2}\ell_2(s_1,s_H)\right]. \tag{9}$$

Let $P(v)$ be the highest second-period welfare that can be delivered to local regulator one given the promise v to local regulator two. That is, $P(v)$ is defined as

$$P(v)=\max_{\{\ell_j\}_{j=1,2}}\frac{1}{4}\sum_{(s_1,s_2)\in S}\left[\left(\theta(s_1)-\frac{1}{2}\right)\ell_1(s_1,s_2)-\frac{1}{2}\ell_2(s_1,s_2)\right], \tag{10}$$

subject to (6), (8), and (9). Then the graph of P, $\{(v_1,v_2)\,:\,v_2\in[0,\bar{v}],\ v_1=P(v_2)\}$, is the Pareto frontier in period two. Note that $P(v)$ is decreasing by the promise-keeping constraint (6) and define $\bar{v}=P(0)$. In the case where both local regulators

[8] When conditions (8) and (9) are satisfied, local regulators will have no incentive to understate shocks as long as the following monotonicity condition holds:

$$\sum_{s_2\in\{s_L,s_H\}}[\ell_1(s_H,s_2)-\ell_1(s_L,s_2)]\geq 0,\ \text{and}\ \sum_{s_1\in\{s_L,s_H\}}[\ell_2(s_1,s_H)-\ell_2(s_1,s_L)]\geq 0. \tag{7}$$

Condition (7) requires that the function ℓ_j increases in s_j in expectation, for $j=1,2$. However, the condition is satisfied at an optimum and can be ignored.

enjoy the same second-period welfare, $P(v_0) = v_0$, the allocation is given by Lemma 2 and v_0 is given by equation (5). Depending on the parameters, there are two cases to consider for how leniency of local regulators is affected when $v \neq v_0$. assumption 2 identifies the relatively more interesting case where leniency or discretion or both have relatively high social benefit.

Assumption 2. *Leniency or discretion matter in the sense that* $\sigma > \sqrt{\mu(1-\mu)}$.

The condition in assumption 2 holds if discretion matters in the sense of σ being large, or if leniency matters in the sense of μ being large. When neither matters then the condition does not hold and the problem studied in this paper – the problem of how to allow local regulators to exercise discretion with respect to leniency – is not a very interesting one. Policy discussions about "leaning against the wind" and "counter-cyclical regulation" suggest that regulatory leniency should depend on economic and financial conditions. In particular the potential for tail events implies that the range of economic and financial conditions considered should not be too narrow, i.e. σ should not be too small, in any discussion of financial regulation (BCBS, 2010). Hence, discretion with respect to regulatory leniency seems to play an important role in practice, see also ASC (2012). assumption 2 is thus maintained throughout the paper.

When $v \neq v_0$, the question arises as to how we can make one local regulator better off than the other along the Pareto frontier. Since local regulators are ex ante identical, it is sufficient to characterize the case $v < v_0$ (or equivalently $P(v) > v_0$) where local regulator one obtains relatively higher second-period welfare along the Pareto frontier. Lemma 3 shows how this is optimally achieved.

Lemma 3. *Suppose $v < v_0$, such that local regulator one is better off along the Pareto frontier. Then, relative to the allocation at v_0,*

1. *$\ell_1(s_H, s_L)$ is strictly higher,*
2. *$\ell_2(s_L, s_H)$ is unchanged for $3\sigma - \mu \leq 0$, and strictly lower otherwise,*
3. *$\ell_1(s_L, s_H)$ is strictly higher, $\ell_2(s_H, s_L)$ remains unchanged at zero,*
4. *leniency of both local regulators is unchanged when both local regulators experience the same preference shock; specifically, $\ell_1(s_L, s_L)$ and $\ell_2(s_L, s_L)$ remain unchanged at zero, and $\ell_1(s_H, s_H)$ and $\ell_1(s_H, s_H)$ remain unchanged at l.*

Proof. See Appendix A.1. $\square$

When $v < v_0$, then local regulator one is better off along the Pareto frontier. Lemma 3 shows how the allocation of leniency changes relative to the symmetric case $v = v_0$ characterized in Lemma 2. Specifically, local regulator one is made better off by allowing it increased discretion, i.e. $\ell_1(s_H, s_L)$ increases while $\ell_1(s_H, s_H)$ remains at l. If $3\sigma - \mu \leq 0$ (region B in Figure 1) then local regulator two is not required to reduce its discretion such that $\ell_2(s_L, s_H)$ is unchanged while $\ell_2(s_H, s_H)$ stays at l. However, both the increased discretion for local regulator one and the fact that local regulator two may not be required to decrease its discretion work toward weakening incentives to report shocks truthfully. Incentives are maintained along the Pareto frontier via state-contingent leniency by local regulator one which is excessive in the sense of having negative economy-wide net social benefit. In particular, local regulator one exercises strictly positive leniency $\ell_1(s_L, s_H)$ in the state where only local regulator two experiences the high shock. Setting $\ell_1(s_L, s_H) > 0$ has the benefit of reducing the stress that discretion by both local regulators puts on incentive compatibility conditions. In contrast, $\ell_1(s_L, s_L)$ remains at zero, since leniency of local regulator one in this state contributes less to maintaining incentives.

The use of state-contingent excessive leniency along the Pareto frontier reduces the need to decrease discretion of the local regulator with lower second-period welfare. Lemma 4 shows that, as a result, expected leniency may increase relative to the symmetric case $v = v_0$. Therefore, if $3\sigma - \mu \leq 0$, a transfer of second-period welfare among local regulators is associated with a deterioration of financial stability in the sense of higher expected leniency.

Lemma 4. *If $3\sigma - \mu \leq 0$, then along the Pareto frontier expected leniency across local regulators is strictly higher when $v \neq v_0$ compared to the case $v = v_0$.*

Proof. See Appendix A.1. □

3.3.2 First period

Local regulators can be assigned different second-period welfare levels, v_1 and v_2, in the first period in a way that encourages them to truthfully reveal the shocks they experience in the first period, as is standard in the risk-sharing literature. That is, second-period welfare can be made contingent on reports of shocks in the first period, $v_j : S \to \mathbb{R}_+$. The Pareto frontier derived in the previous section allows us to express the set of feasible pairs of second-period welfare as

$$\mathcal{P} = \left\{ (v_1, v_2) \in \mathbb{R}^2_+ : v_1 \leq P(v_2) \right\}. \tag{11}$$

Let ℓ_j denote leniency of local regulator j in the first period, $\ell_j : S \to [0, l]$. The

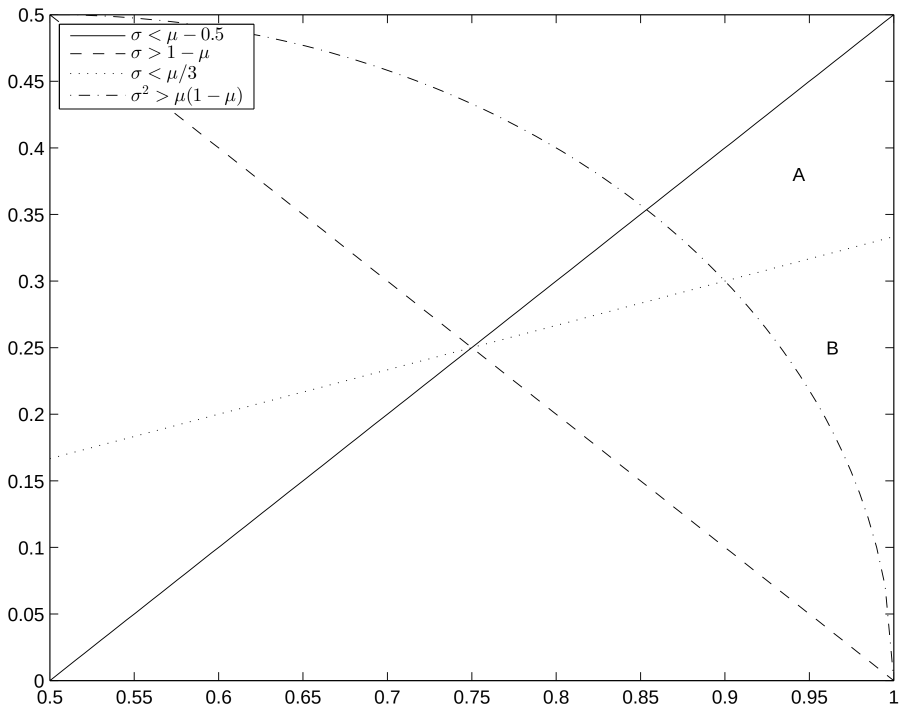


Figure 1: Restrictions on model parameters μ and σ in (μ, σ) space. Parameter pairs in A or B satisfy all restrictions imposed by $\mu < 1$, $\theta_L - \frac{1}{2} > 0$, $\theta_H - 1 > 0$, and assumption 2. The line $\sigma = \frac{\mu}{3}$ divides the region of permissible parameter pairs into two subregions, A and B.

problem of a supervisor of local regulators that wishes to maximize joint welfare of local regulators Ω is as follows:

$$\max_{\{\ell_j, v_j\}_{j=1,2}} \frac{1}{4} \sum_{(s_1,s_2)\in S} \left[(\theta(s_1)-1)\ell_1(s_1,s_2) + (\theta(s_2)-1)\ell_2(s_1,s_2) + \delta(v_1(s_1,s_2)+v_2(s_1,s_2)) \right], \tag{12}$$

subject to incentive compatibility

$$\frac{1}{2}\sum_{s_2\in\{s_L,s_H\}} \left[\theta(s_L)\ell_1(s_L,s_2) - \frac{1}{2}\ell_1(s_L,s_2) - \frac{1}{2}\ell_2(s_L,s_2) + \delta v_1(s_L,s_2) \right] \\ \geq \frac{1}{2}\sum_{s_2\in\{s_L,s_H\}} \left[\theta(s_L)\ell_1(s_H,s_2) - \frac{1}{2}\ell_1(s_H,s_2) - \frac{1}{2}\ell_2(s_H,s_2) + \delta v_1(s_H,s_2) \right], \tag{13}$$

$$\frac{1}{2}\sum_{s_1\in\{s_L,s_H\}} \left[\theta(s_L)\ell_2(s_1,s_L) - \frac{1}{2}\ell_1(s_1,s_L) - \frac{1}{2}\ell_2(s_1,s_L) + \delta v_2(s_1,s_L) \right] \\ \geq \frac{1}{2}\sum_{s_1\in\{s_L,s_H\}} \left[\theta(s_L)\ell_2(s_1,s_H) - \frac{1}{2}\ell_1(s_1,s_H) - \frac{1}{2}\ell_2(s_1,s_H) + \delta v_2(s_1,s_H) \right], \tag{14}$$

and feasibility $(v_1(s_1,s_2), v_2(s_1,s_2)) \in \mathcal{P}$ for all $(s_1,s_2) \in S$. Lemma 5 verifies that variation in second-period welfare is in fact used in the first period to make leniency more responsive to shocks in the first period.

Lemma 5. *The solution to the supervisor's problem has the following characteristics:*

1. *When local regulators experience the same shock in the first period, then both local regulators receive second-period welfare of v_0. Leniency is first-best.*

2. *When local regulators experience different shocks in the first period, then second-period*

welfare is varied along the Pareto frontier, i.e. $\theta(s_i) < \theta(s_j)$ implies $v_i > v_j = P(v_i)$. Leniency is not first best but local regulators enjoy more discretion compared to the case in Lemma 2.

Proof. See Appendix A.1. □

When both local regulators experience the same shock, then optimal static leniency given by Lemma 2 already delivers the first-best. Variation in second-period welfare is not beneficial in this case such that both local regulators receive second-period welfare of v_0. In the case where local regulators experience different shocks, it is beneficial to vary second-period welfare in order to improve upon partial insurance provided by the allocation in Lemma 2. Together, Lemmas 3 and 5 yield the main result of the paper. proposition 1 shows how current coordination of leniency across local regulators depends on history.

Proposition 1. *Let $s_1, s_2 \in S$ be shocks in the first period. If $s_1 = s_2$, then optimal leniency in the second period is as given in Lemma 2. If $s_1 \neq s_2$ such that $\theta(s_i) < \theta(s_j)$, then in the second period, compared to the case $s_1 = s_2$,*

1. *local regulator i enjoys strictly higher discretion,*
2. *local regulator j's discretion is unchanged if $3\sigma - \mu \leq 0$ and strictly lower otherwise,*
3. *local regulator i engages in state-contingent excessive leniency; specifically, leniency of local regulator i is strictly positive in the state of the world where only local regulator j experiences a high shock, but zero in the state where both experience the low shock.*

Proof. We know from Lemma 5 that second-period welfare is $v_1 = v_2 = v_0$ whenever $s_1 = s_2$ in the first period. But then second-period leniency is given in Lemma 2.

We know from Lemma 5 that $\theta(s_i) < \theta(s_j)$ implies second-period welfare of $v_i > v_j$ along the Pareto frontier. Then the implications for second-period leniency follow from Lemma 3. $\square$

Inefficiencies due to a past disagreement about the need for leniency, i.e. costly movements along the Pareto frontier away from symmetric second-period welfare, can be mitigated by coordinating leniency more tightly. Coordination is tightened to expand outward the set of feasible second-period welfare pairs and hence to make intertemporal utility transfers cheaper. This is why intertemporal and intratemporal margins for incentive provision should interact at an optimum.

A local regulator is rewarded for exercising less leniency in the first period by granting it more discretion over regulatory leniency in the second period. Period-two incentive compatibility is maintained by allowing that local regulator to discipline the respective other local regulator via state-contingent excessive leniency. A local regulator oversees delivery of its higher second-period welfare by disciplining the respective other local regulator. In that sense, excessive leniency is not a direct punishment of the other local regulator for its past behavior, but rather a means to facilitate intertemporal utility transfers. The model thus gives an example of how short-lived institutions (i.e. one local regulator disciplining the other) can arise endogenously after certain histories within a long-lived relationship (i.e. the ex-ante optimal supervisory arrangement).

Relation to contracting literature on risk sharing:

The tightening of coordination over time, in the sense of state-contingent excessive leniency, following past disagreement is the result of the interaction of two channels for incentive provision that have been studied extensively, albeit separately, in the contracting literature. This paper, on the other hand, focuses on the *interaction* of intratemporal margins (for example Roberts, 1985) and intertemporal margins (for example Taub, 1994) for incentive provision. For instance, many dynamic contracting problems have a solution that features (ex-post) inefficiently high consumption of sufficiently wealthy lenders as a reward for past frugality (e.g. Thomas and Worrall, 1990, Atkeson and Lucas, 1992, Iovino and Golosov, 2013). However, in this paper, such excessive leniency is employed only in certain states, and after certain histories, as a means to provide additional incentives via immediate reciprocity. This feature of my model is novel relative to the existing risk-sharing literature and thus generates novel policy implications.

Discussion of policy implications:

Optimal supervision of local regulators focuses on rewarding lower leniency in the past with higher discretion in the future. On the other hand, higher leniency in the past may not be punished at all by lower discretion the future. The supervisor makes this focus on rewarding good behavior rather than punishing bad behavior incentive-compatible by rewarding not only with higher discretion but also with state-contingent excessive leniency.

There are implications for how the past affects financial stability, as measured by expected leniency across local regulators, in the future. corollary 2 gives sufficient conditions for when past disagreement regarding the need for leniency across local regulators leads to higher expected leniency and thus to a deterioration of financial stability. When shocks are not too far apart, $\theta_H \leq 2\theta_L$, a local regulator will never be required to reduce discretion, even if it exercised relatively more leniency in the past. Figure 2 illustrates that discretion is a convex function of promised second-period welfare, when $\theta_H \leq 2\theta_L$, such that local regulators get rewarded with higher discretion but never punished with lower discretion. As a result, expected leniency is higher whenever local regulators experienced different shocks in the past when $\theta_H \leq 2\theta_L$. In that sense, financial stability deteriorates when local regulators had different regulatory policy stances in the past.

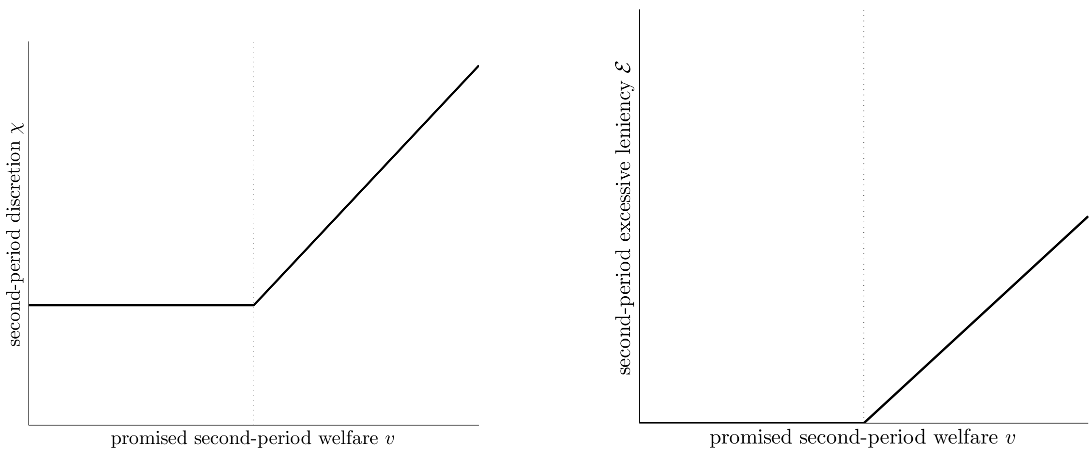
Corollary 2. *Suppose $3\sigma - \mu \leq 0$, or equivalently $\theta_H \leq 2\theta_L$. Let s_1, s_2 be shocks in the first period. If $\theta(s_i) < \theta(s_j)$, i.e. local regulator j was relatively more lenient in the first period, then in period two, compared to the case where $s_1 = s_2$,*

1. *expected leniency across local regulators is strictly higher,*
2. *discretion of local regulator j is not reduced.*

Proof. The result follows from Lemmas 3 and 4 together with Lemma 5. □

4 Extension to infinite horizon

Consider now the case of infinitely many time periods, $t = 0, 1, 2, \ldots$. Suppose each local regulator enjoys instantaneous utility payoffs from local leniency $\ell \in X \subset \mathbb{R}_+$ given by the function $u : \mathbb{R}_+ \times \Theta \to \mathbb{R}$. On the set $\Theta = \{\theta_1, \theta_2, \ldots, \theta_N\}$, an i.i.d., across local regulators and over time, uniformly distributed random variable θ is defined. Its realizations are ordered, $0 < \theta_1 < \cdots < \theta_N < \infty$. Future payoffs are discounted by $\delta \in (0, 1)$. It is assumed that $u(\ell, \theta)$ is twice continuously differentiable and strictly concave in ℓ, strictly increasing in θ, and that $u'(\ell, \theta) = \frac{\partial u(\ell,\theta)}{\partial a}$ is strictly increasing in θ. Payoffs are single-peaked in the sense that, for some $l < \infty$, $\frac{\partial u(\ell,\theta_N)}{\partial \ell} \geq 0$



(a) Discretion $\chi_{j,2}$

(b) Excessive leniency $\mathcal{E}_{j,2}$

Figure 2: Panel 2a illustrates a local regulator's discretion to exercise leniency as a convex function of its second-period welfare in the case where $\theta_H \leq 2\theta_L$ (corollary 2). Discretion is never lower than $\frac{1}{2\theta_L} l$, even for local regulators that exercised relatively more leniency in the first period and therefore were promised second-period welfare lower than v_0. For second-period welfare exceeding v_0, discretion increases up to l. Panel 2b illustrates that only local regulators with relatively higher welfare engage in excessive leniency (proposition 1). Recall that excessive leniency is always state-contingent, i.e. it is only granted if the respective other local regulator experiences a high shock. The dotted line indicates the symmetric value v_0.

for $\ell \leq l$ and $\frac{\partial u(\ell,\theta_N)}{\partial \ell} < 0$ for $\ell > l$. Without loss of generality we can therefore restrict attention to levels of leniency taking values on the interval $X = [0, M]$ for some $M > l$. Specifically, assume that $u(\ell, \theta) = \theta \nu(\ell) - \frac{1}{2}\ell$, where ν is strictly increasing, strictly concave and twice differentiable.[9] Due to joint contributions to the resolution fund the net payoff of local regulator i is given by $u(\ell_i, \theta) - \frac{1}{2}\ell_j$ where j denotes local regulator $j \neq i$ and θ is the parameter of local regulator i. The joint net payoff to local regulators is given by

$$u(\ell_1, \theta^1) + u(\ell_2, \theta^2) - \frac{1}{2}\ell_1 - \frac{1}{2}\ell_2, \tag{15}$$

when ℓ_1, ℓ_2 are the respective actions and θ^1, θ^2 the respective parameters experienced by the two local regulators.

Definition 4. *First Best leniency is defined by the function $\ell^{FB} : \Theta \to X$ such that $u'(\ell^{FB}(\theta), \theta) = \frac{1}{2}$, for all $\theta \in \Theta$. Privately optimal leniency is defined by the function $\ell^* : \Theta \to X$ such that $u'(\ell^*(\theta), \theta) = 0$, for all $\theta \in \Theta$. Note that $\ell^{FB} < \ell^*$ uniformly on Θ.*

Note that a local regulator's net payoff is highest when its leniency is ℓ^*, for given leniency of the respective other local regulator. Also note that the sum of net payoffs across regulators is highest when leniency is given by ℓ^{FB} for each local regulator. Define $\underline{v}$ as the discounted expected value of net payoffs to either local regulator when leniency is privately optimal, $\ell_i = \ell_i^*$ for $i = 1, 2$, at all times.

4.1 Recursive formulation

Optimal supervision of local regulators is characterized by solving for the Pareto set of discounted expected values of net payoffs to each local regulator. As is standard, values in the Pareto set will be expressed as a combination of current leniency and

[9] In section 2, $\nu(\ell) = \min(\ell, l)$ and the effective upper bound on leniency, $l \in \mathbb{R}_{++}$, was arbitrary.

future (continuation) values. For a given current value of local regulator two, $v \in V$, define the functions for leniency $\ell_i : \Theta^2 \to X$ and continuation values $v_i : \Theta^2 \to V$, $i = 1, 2$. Let $V = [\underline{v}, \bar{v}]$, where the lower bound on V is motivated by assumption 3, which is an analogue to assumption 1, and the upper bound is defined below.

Assumption 3. *At the beginning of each period, before observing its parameter, either local regulator has the option to demand that the supervisor prescribes privately optimal policies for both local regulators from the current period onward. That is, any policy that a supervisor prescribes must yield values of at least $\underline{v}$ for either local regulator.*

Define an operator T on $C(V)$, the space of continuous, decreasing, and concave functions on V, by

$$(Tf)(v) = \max_{\ell_1, \ell_2, v_1, v_2} \frac{1}{N^2} \sum_{(\theta_i, \theta_j) \in \Theta^2} \left[u(\ell_1(\theta_i, \theta_j), \theta_i) - \frac{1}{2} \ell_2(\theta_i, \theta_j) + \delta v_1(\theta_i, \theta_j) \right] \tag{16}$$

subject to

$$\frac{1}{N^2} \sum_{(\theta_i, \theta_j) \in \Theta^2} \left[u(\ell_2(\theta_i, \theta_j), \theta_j) - \frac{1}{2} \ell_1(\theta_i, \theta_j) + \delta v_2(\theta_i, \theta_j) \right] \geq v \tag{17}$$

$$\frac{1}{N} \sum_{\theta_j \in \Theta} \left[u(\ell_1(\theta_i, \theta_j), \theta_i) - \frac{1}{2} \ell_2(\theta_i, \theta_j) + \delta v_1(\theta_i, \theta_j) \right] \tag{18}$$

$$\geq \frac{1}{N} \sum_{\theta_j \in \Theta} \left[u(\ell_1(\theta_{i+1}, \theta_j), \theta_i) - \frac{1}{2} \ell_2(\theta_{i+1}, \theta_j) + \delta v_1(\theta_{i+1}, \theta_j) \right], \quad i = 1, 2, \ldots, N-1$$

$$\frac{1}{N} \sum_{\theta_i \in \Theta} \left[u(\ell_2(\theta_i, \theta_j), \theta_j) - \frac{1}{2} \ell_1(\theta_i, \theta_j) + \delta v_2(\theta_i, \theta_j) \right] \tag{19}$$

$$\geq \frac{1}{N} \sum_{\theta_i \in \Theta} \left[u(\ell_2(\theta_i, \theta_{j+1}), \theta_j) - \frac{1}{2} \ell_1(\theta_i, \theta_{j+1}) + \delta v_2(\theta_i, \theta_{j+1}) \right], \quad j = 1, 2, \ldots, N-1$$

$$v_1(\theta_i, \theta_j) \leq f(v_2(\theta_i, \theta_j)), \quad v_2(\theta_i, \theta_j) \in V, \text{ for all } i, j = 1, 2, \ldots, N. \tag{20}$$

The maximization problem yields the highest possible value for local regulator one given that local regulator two receives value of at least v where 'feasibility' of the (continuation) values v_1, v_2 is determined by the function f.[10] The upper bound on feasible values is then given by $\bar{v} = P(\underline{v})$.

Lemma 6. *T has a unique fixed point P that is strictly decreasing, strictly concave, and continuously differentiable on V. The Pareto set $\{(v_1, v_2) : v_1 = P(v_2),\ v_2 \in V\}$ is self-generating (see Abreu et al., 1990) in the sense that constraint* (20) *is always binding for $f = P$.*

Proof. See appendix A.1. $\square$

Definition 5. *The set of feasible (continuation) values is defined by $\{(v_1, v_2) : v_1 \leq P(v_2),\ v_2 \in V\}$.*

Definition 6. *Denote the symmetric (promised) value by $\hat{v} = P(\hat{v})$.*

The properties of P that lemma 6 establishes are helpful since optimal leniency and continuation values can be characterized by examining first-order conditions from problem (16) where $f = P$. The optimality conditions for leniency ℓ_1 and ℓ_2 respectively are

$$u'(\ell_1(\theta_i, \theta_j), \theta_i)\left[1 + \psi_1(\theta_i)\right] - u'(\ell_1(\theta_i, \theta_j), \theta_{i-1})\psi_1(\theta_{i-1}) = \frac{1}{2}\left[\tau + \psi_2(\theta_j) - \psi_2(\theta_{j-1})\right], \tag{21}$$

$$u'(\ell_2(\theta_i, \theta_j), \theta_j)\left[\tau + \psi_2(\theta_j)\right] - u'(\ell_2(\theta_i, \theta_j), \theta_{j-1})\psi_2(\theta_{j-1}) = \frac{1}{2}\left[1 + \psi_1(\theta_i) - \psi_1(\theta_{i-1})\right], \tag{22}$$

where τ, ψ_1, and ψ_2 are the Lagrange multipliers on constraints (17), (18), and (19) respectively. The optimality conditions for continuation payoffs v_2 and envelope

[10] Lemma 8 in appendix A.1 shows that, as long as leniency of each local regulator is increasing in its shock, it is sufficient to focus on adjacent upward incentive constraints.

conditions are given by

$$-P'\left(v_2(\theta_i,\theta_j)\right) = \frac{\tau + \psi_2(\theta_j) - \psi_2(\theta_{j-1})}{1+\psi_1(\theta_i) - \psi_1(\theta_{i-1})}, \tag{23}$$

$$-P'(v) = \tau. \tag{24}$$

The following lemma 7 summarizes some straightforward implications of these conditions.

Lemma 7. *Conditions* (21)-(24) *have the following implications.*

1. *The geometric mean of realized marginal net payoffs is always bounded above by $\frac{1}{2}$, the First-Best marginal utility,*

   $$\sqrt{u'(\ell_1(\theta_i,\theta_j),\theta_i)u'(\ell_2(\theta_i,\theta_j),\theta_j)} \leq \frac{1}{2},$$

   for $i, j = 1, 2, \ldots, N$ with equality only if $i = j = 1$.

2. *If $v = \hat{v}$ then $v_2(\theta_i,\theta_i) = v = \hat{v}$ for $i = 1, 2, \ldots, N$.*

3. *$\ell_1(\theta_i,\theta_j) \leq \ell_1(\theta_i,\theta_{j+1})$ if and only if $v_2(\theta_i,\theta_j) \geq v_2(\theta_i,\theta_{j+1})$, for $i = 1, 2, \ldots, N$ and $j = 1, 2, \ldots, N-1$.*

4. *$\ell_2(\theta_i,\theta_j) \leq \ell_2(\theta_{i+1},\theta_j)$ if and only if $v_1(\theta_i,\theta_j) \geq v_1(\theta_{i+1},\theta_j)$, for $j = 1, 2, \ldots, N$ and $i = 1, 2, \ldots, N-1$.*

5. *$v_1(\theta_i,\theta_j) > \hat{v}$ implies $u'(\ell_1(\theta_i,\theta_j),\theta_i) \leq \frac{1}{2}$, and $v_2(\theta_i,\theta_j) > \hat{v}$ implies $u'(\ell_2(\theta_i,\theta_i),\theta_j) \leq \frac{1}{2}$, for $i, j = 1, 2, \ldots, N$.*

Proof. See appendix A.1. $\square$

Lemma 8 in appendix A.1 shows that, as long as leniency of each local regulator is increasing in its shock – as in the numerical example considered below – it is sufficient to focus on adjacent upward incentive constraints in order to ensure incentive compatibility of the allocation. In that case, lemma 7 characterizes features of optimal supervision of local regulators.

When leniency of each local regulator is increasing in its shock then lemma 7 gives a sense in which, on average, local regulator leniency is higher than first best. It also shows that if both local regulators enjoy the same value along the Pareto frontier then they will continue to do so as long as they receive the same shock, independent of the level of that shock. It is further shown that if optimal supervision uses continuation values to incentivize a local regulator then it also uses leniency of the respective other local regulator to incentivize the former, and vice versa. Finally, lemma 7 shows that a regulator that is awarded a higher continuation value in a particular state is also allowed to engage in excessive leniency in that state. Such a local regulator has a relatively higher current value, i.e. has received relatively lower shocks in the past, or is experiencing a relatively lower shock in the current period, or both. When both is the case then lemma 7 suggests an infinite-horizon analogue to proposition 1. That is, the local regulator that exercised relatively less leniency in the past is allowed to engage in state-contingent excessive leniency in the current period.

4.1.1 Numerical exercise

In the two-period version of the model one can solve for optimal supervision by hand and characterize analytically an emphasis on rewards over punishment. Recall that corollary 2 gives conditions under which discretion is a convex function of second-period welfare in the two-period version of the model as shown in figure 2.

This section presents a numerical example with $v(\ell) = 2\sqrt{\ell}$, $\delta = 0.8$, $N = 5$, and $\Theta = \{0.5, 0.625, 0.75, 0.875, 1\}$. Figure 4 shows that discretion χ is convex in promised value around the symmetric value $\hat{v}$, and that excessive leniency $\mathcal{E}$ tends to be increasing in promised value. The insights from section 3 are confirmed: local regulators get rewarded with increased discretion rather than punished with reduced discretion, which is made incentive-compatible by allowing the local regulator that was relatively less lenient in the past to engage in state-contingent excessive leniency.[11] Figure 4c shows, in an analogue to corollary 2, that expected leniency across local regulators is higher the more regulatory stances differed in the past, i.e. the further current promised values are apart.

5 Conclusion

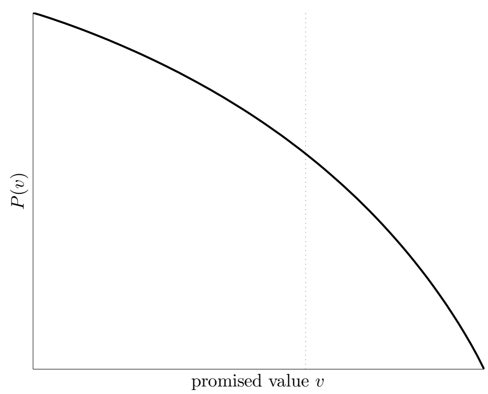
A supervisor of local financial regulators can, and should (ASC, 2012), do better than imposing harmonized rules. Under optimal supervision, local regulators enjoy some discretion with respect to their policy stance. In order to strengthen incentives for local regulators to share relevant information, optimal supervision coordinates

[11] The definitions for discretion and excessive leniency from section 2 need to be adapted to the infinite horizon model. The following definitions of discretion and excessive leniency for local regulator two given its promised value, up to a constant, have definitions 2 and 3 as a special case when $N = 2$ and $v(\ell) = \min(\ell, l)$.

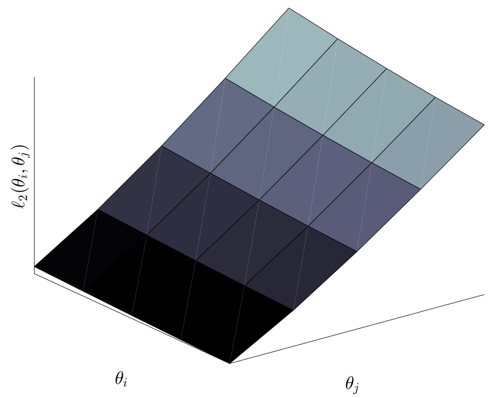
$$\chi = \frac{1}{N}\sum_{i=1}^{N}\left[\ell_2(\theta_i, \theta_N) - \min\left(\ell_2(\theta_i, \theta_1), \ell^{FB}(\theta_1)\right)\right],$$

$$\mathcal{E} = \frac{1}{N}\sum_{j=1}^{N}\left[\ell_2(\theta_N, \theta_j) - \ell_2(\theta_1, \theta_j)\right] \cdot \mathbb{1}_{\ell_2(\theta_N, \theta_j) \geq \ell^{FB}(\theta_N)},$$

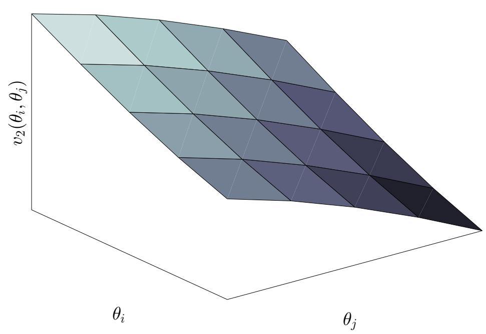
where $\mathbb{1}$ is the indicator function. Note that $\ell^{FB}(\theta) = \theta^2$ when $v(\ell) = 2\sqrt{\ell}$. The role of the min-operator in the definition of χ is to avoid that a higher measure of excessive leniency reduces measured discretion. The role of the indicator function in the definition of $\mathcal{E}$ is to focus on immediate reciprocity that involves excessive leniency.



(a) Pareto frontier P

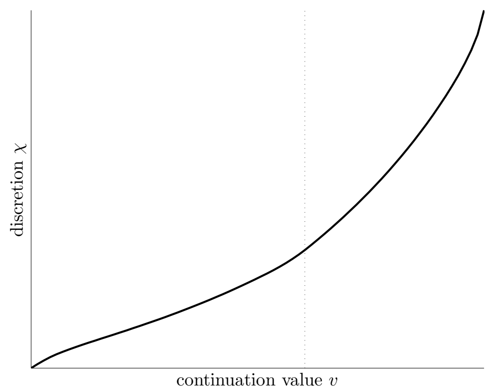


(b) Leniency of local regulator two ℓ_2

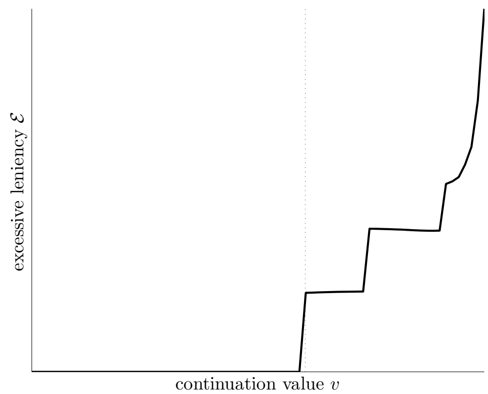


(c) Continuation value of local regulator two v_2

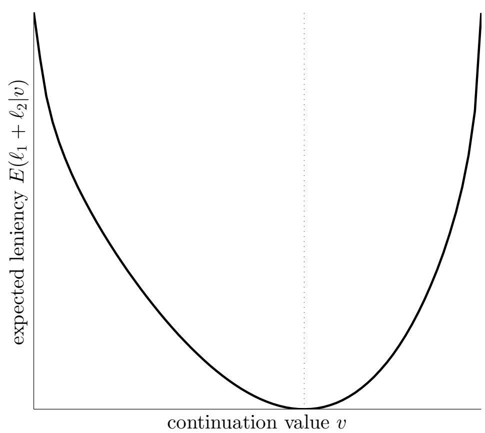
Figure 3: The dotted line in panel 3a indicated the symmetric value $\hat{v}$. Panels 3b and 3c show ℓ_2 and v_2, respectively, when the value promised to local regulator two is $\hat{v}$.



(a) Discretion χ



(b) Excessive leniency $\mathcal{E}$



(c) Expected leniency $E(\ell_1 + \ell_2|v)$

Figure 4: Panel 4a and 4b confirm the intuition gained from figure 2. Local regulators get rewarded with increased discretion relatively more than punished with reduced discretion. Incentive-compatibility is maintained by allowing the local regulator that was relatively less lenient in the past to engage in state-contingent excessive leniency. Figure 4c shows that, as a result, expected leniency across local regulators is higher the further apart promised value are. The dotted line indicates the symmetric value $\hat{v}$.

regulatory leniency across local regulators within each period in addition to varying discretion awarded to each local regulator over time.

The first major insight is that optimal supervision focuses more on rewarding past restraint than on punishing past leniency. A local regulator that exercised relatively more leniency in the past may therefore not be required to reduce its discretion to exercise leniency in the future. The second major insight is that past disagreement about regulatory policy stances across local regulators affects financial stability in the future. When local regulators exercised different degrees of leniency in the past then expected leniency in the future may increase.

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A Appendix

A.1 Proofs

Proof of Lemma 1. This is immediate from the assumption that $\theta_k < 1 < \theta_H$. $\square$

Proof of Lemma 2. Since local regulators have symmetric preferences, and leniency is static, it follows that $\ell_1(s_1, s_2) = \ell_2(s_2, s_1)$. The problem can then be written as

$$\max_{\ell_1(s_1,s_2)\in[0,l]} 2(1+\delta)\frac{1}{4}\left[(\theta_k - 1)(\ell_1(s_L, s_L) + \ell_1(s_L, s_H)) + (\theta_H - 1)(\ell_1(s_H, s_L) + \ell_1(s_H, s_H))\right],$$

subject to incentive compatibility

$$\begin{aligned}&\left(\theta_k - \frac{1}{2}\right)(\ell_1(s_L, s_L) + \ell_1(s_L, s_H)) - \frac{1}{2}(\ell_1(s_L, s_L) + \ell_1(s_H, s_L)) \\ &\quad \geq \left(\theta_k - \frac{1}{2}\right)(\ell_1(s_H, s_L) + \ell_1(s_H, s_H)) - \frac{1}{2}(\ell_1(s_L, s_H) + \ell_1(s_H, s_H)).\end{aligned}$$

Since this is a linear program, it is sufficient to verify that its first-order conditions are satisfied by the allocation proposed in the lemma. Letting $\psi > 0$ denote the multiplier on the incentive compatibility constraint, these conditions are

$$
\begin{aligned}
\ell_1(s_L, s_L): &\quad \theta_k - 1 + \left(\theta_k - \frac{1}{2}\right)\psi - \frac{1}{2}\psi = -(1-\theta_k)(1+\psi) < 0, \\
\ell_1(s_L, s_H): &\quad \theta_k - 1 + \left(\theta_k - \frac{1}{2}\right)\psi + \frac{1}{2}\psi < 0, \\
\ell_1(s_H, s_L): &\quad \theta_H - 1 - \left(\theta_k - \frac{1}{2}\right)\psi - \frac{1}{2}\psi = 0, \\
\ell_1(s_H, s_H): &\quad \theta_H - 1 - \left(\theta_k - \frac{1}{2}\right)\psi + \frac{1}{2}\psi = \theta_H - 1 + (1-\theta_k)\psi > 0, \\
&\quad \psi \cdot \left[-\frac{1}{2}\frac{1-\theta_k}{\theta_k}l - \left(\left(\theta_k - \frac{1}{2}\right)\left(\frac{1-\theta_k}{\theta_k}l + l\right) - \frac{1}{2}l\right)\right] = 0.
\end{aligned}
$$

The first, fourth and fifth conditions clearly hold. To see that the second holds given the third, note that the third condition can be solved for $\psi = \frac{\theta_H - 1}{\theta_k}$ such that

$$
\theta_k - 1 + \left(\theta_k - \frac{1}{2}\right)\psi + \frac{1}{2}\psi = \theta_k - 1 + \theta_k\psi = \theta_k - 1 + \theta_H - 1 = 2(\mu - 1) < 0,
$$

since $\mu < 1$. $\square$

Proof of Lemma 3. To make notation simpler, denote $\ell_i^{jk} = \ell_i(s_j, s_k)$ for $j, k \in \{L, H\}$. For use throughout this appendix, let us write out first-order conditions for leniency (for given v).

$$\ell_1^{LL}: \qquad \theta_k - \frac{1}{2} - \frac{1}{2}\tau + \left(\theta_k - \frac{1}{2}\right)\psi_1 - \frac{1}{2}\psi_2 \lesseqqgtr 0, \tag{25}$$

$$\ell_1^{LH}: \qquad \theta_k - \frac{1}{2} - \frac{1}{2}\tau + \left(\theta_k - \frac{1}{2}\right)\psi_1 + \frac{1}{2}\psi_2 \lesseqqgtr 0, \tag{26}$$

$$\ell_1^{HL}: \qquad \theta_H - \frac{1}{2} - \frac{1}{2}\tau - \left(\theta_k - \frac{1}{2}\right)\psi_1 - \frac{1}{2}\psi_2 \lesseqqgtr 0, \tag{27}$$

$$\ell_1^{HH}: \qquad \theta_H - \frac{1}{2} - \frac{1}{2}\tau - \left(\theta_k - \frac{1}{2}\right)\psi_1 + \frac{1}{2}\psi_2 \lesseqqgtr 0, \tag{28}$$

$$\ell_2^{LL}: \qquad -\frac{1}{2} + \left(\theta_k - \frac{1}{2}\right)\tau - \frac{1}{2}\psi_1 + \left(\theta_k - \frac{1}{2}\right)\psi_2 \lesseqqgtr 0, \tag{29}$$

$$\ell_2^{LH}: \qquad -\frac{1}{2} + \left(\theta_H - \frac{1}{2}\right)\tau - \frac{1}{2}\psi_1 - \left(\theta_k - \frac{1}{2}\right)\psi_2 \lesseqqgtr 0, \tag{30}$$

$$\ell_2^{HL}: \qquad -\frac{1}{2} + \left(\theta_k - \frac{1}{2}\right)\tau + \frac{1}{2}\psi_1 + \left(\theta_k - \frac{1}{2}\right)\psi_2 \lesseqqgtr 0, \tag{31}$$

$$\ell_2^{HH}: \qquad -\frac{1}{2} + \left(\theta_H - \frac{1}{2}\right)\tau + \frac{1}{2}\psi_1 - \left(\theta_k - \frac{1}{2}\right)\psi_2 \lesseqqgtr 0, \tag{32}$$

where τ is the Lagrange multiplier on the promise-keeping constraint for local regulator two and ψ_i is the multiplier on the incentive compatibility constraint for local regulator $i = 1, 2$. We will compute the optimal allocation for $v \in [\underline{v}, v_0]$, where the case $v \in [v_0, \bar{v}]$ follows from symmetry. P will be piece-wise linear with kinks $\underline{v}_j$, $j = 0, 1, 2$, and $\underline{v}_2 < \underline{v}_1 < v_0$. Below we will guess and verify optimal leniency that attains $P(v)$ for each $v \in [\underline{v}, v_0]$.

For $v \in [\underline{v}_1, v_0]$ we have that $\ell_1^{LH}, \ell_1^{HL}, \ell_2^{LH}$ take interior values such that (26), (27) and (30) hold with equality. The remaining first-order conditions yield corner solutions $\ell_1^{LL} = \ell_2^{LL} = \ell_2^{HL} = 0$ and $\ell_1^{HH} = \ell_2^{HH} = l$. Both incentive compatibility constraints bind such that multipliers are given by

$$\tau = 2\mu - 1, \quad \psi_1 = \frac{\sigma^2 - \mu(1-\mu)}{(\mu - \sigma)(1 - \mu + \sigma)}, \quad \psi_2 = \frac{(2\mu - 1)(\sigma^2 + \mu(1-\mu))}{(\mu - \sigma)(1 - \mu + \sigma)}.$$

Note that ψ_1 is strictly positive, since discretion matters (by assumption 2). To verify that $\ell_2^{HL} = 0$, note that the left-hand side of (31), whenever the left-hand side of (26) is zero, can be written as $\theta_k(\tau - 1) - (1-\theta_k)(\psi_2 - \psi_1) < 0$.

The interior policies for a given $v \in [\underline{v}_1, v_0]$ are

$$\ell_1^{LH} = 4(v_0 - v), \quad \ell_1^{HL} = \frac{1-\mu+\sigma}{\mu-\sigma} l + 4(v_0 - v), \quad \ell_2^{LH} = \frac{1-\mu+\sigma}{\mu-\sigma} l.$$

We have $\ell_1^{HL} = w$ at $v = \underline{v}_1$ where

$$\underline{v}_1 = \frac{3\sigma - \mu}{4(\mu - \sigma)} l.$$

Note that $\underline{v}_1$ can be either positive or negative. In the former case, we are interested in $v \in [0, \underline{v}_1)$ as well. Then ℓ_1^{LH}, ℓ_2^{LH} take interior values such that (26) and (30) hold with equality. The remaining first-order conditions yield corner solutions $\ell_1^{LL} = \ell_2^{LL} = \ell_2^{HL} = 0$ and $\ell_1^{HL} = \ell_1^{HH} = \ell_2^{HH} = l$. Only the second local regulator's incentive compatibility constraint binds such that multipliers are given by

$$\tau = 2\mu - 1 - \frac{\sigma^2 - \mu(1-\mu)}{\sigma}, \quad \psi_1 = 0, \quad \psi_2 = \frac{\sigma^2 + \mu(1-\mu)}{\sigma},$$

where it is easy to verify that $\tau \in (0,1)$. We have $\ell_2^{HL} = 0$ for the same reason as above, since again $\psi_2 - \psi_1 > 0$. To see that $\ell_1^{HL} = l$, note that the left-hand side of (27), whenever the left-hand side of (26) is zero, can be written as $\theta_H - \theta_k - \psi_2 = (\sigma^2 - \mu(1-\mu))/\sigma > 0$.

The interior policies for a given $v \in [\underline{v}_2, \underline{v}_1]$ are

$$\ell_1^{LH} = (2(\mu - \sigma) - 1)\,\frac{4v + l}{2\sigma}, \quad \ell_2^{LH} = \frac{4v + (1-2\sigma)l}{2\sigma}.$$

Note that both are decreasing as v decreases – however, $\ell_1^{LH} > 0$ throughout, such that local regulator one will still engage in state-contingent socially wasteful leniency. We have $\ell_2^{LH} = 0$ at $v = \underline{v}_2$, but this value does not satisfy assumption 1, since

$$\underline{v}_2 = -\frac{1-2\sigma}{4} l < 0.$$

Thus, as v decreases in $[0, v_0]$, we have ℓ_1^{HL} increasing, $\ell_1^{LH} > 0$, and ℓ_2^{LH} non-increasing, whenever discretion matters (in the sense of assumption 2). $\square$

Proof of Lemma 4. When $3\sigma - \mu \leq 0$, then $\underline{v}_1$ in the proof of Lemma 3 is negative. By assumption 1 it follows that second-period welfare of any local regulator must be larger than $\underline{v}_1$. But then average leniency across local regulators is uniformly higher when $v \neq v_0$ compared to the case where $v = v_0$, and strictly higher in states where local regulators receive different preference shocks. It follows that expected leniency is strictly higher when $v \neq v_0$ compared to the case where $v = v_0$. $\square$

Proof of Lemma 5. To see that the constrained optimization problem is convex, note that the objective is linear and that the non-linear constraints can be written as

$$v_1(s_1, s_2) - P(v_2(s_1, s_2)) \leq 0, \quad (s_1, s_2) \in S^2,$$

where the left-hand side is convex whenever P is concave in v_2. To see that P is concave, note that we can use the results from the proof of Lemma 3 to define P as

$$P(v) = \begin{cases} v_0 - \tau_0(v - v_0) & \text{if } v \in [\underline{v}_1, v_0] \\ P(\underline{v}_1) - \tau_1(v - \underline{v}_1) & \text{if } v \in [\underline{v}_2, \underline{v}_1), \end{cases}$$

where

$$\tau_0 = 2\mu - 1,\ \tau_1 = 2\mu - 1 - \frac{\sigma^2 - \mu(1-\mu)}{\sigma},\ \underline{v}_1 = \frac{3\sigma - \mu}{4(\mu - \sigma)}l, \text{ and, } \underline{v}_2 = -\frac{1 - 2\sigma}{4}l.$$

Due to symmetry in the first period, the first-order conditions for leniency look almost the same as in the proof of Lemma 2:

$$\begin{aligned}
\ell_1(s_L, s_L): &\quad -(1-\theta_k)(1+\psi) < 0, \\
\ell_1(s_L, s_H): &\quad \theta_k - 1 + \theta_k\psi < 0, \\
\ell_1(s_H, s_L): &\quad \theta_H - 1 - \theta_k\psi \geq 0, \\
\ell_1(s_H, s_H): &\quad \theta_H - 1 + (1-\theta_k)\psi > 0,
\end{aligned}$$

where $\psi = 0$ if (13) is slack (or equivalently if (14) is slack). Note that ψ is the Lagrange multiplier on either incentive compatibility constraint. The second line is implied by the third, since $\mu < 1$. If the third line would not hold, then $\ell_1(s_H, s_L) = 0$ such that $\psi = 0$ (but then $\ell_1(s_H, s_L)$ should be increased, a contradiction). Hence we know that $\psi \leq \frac{\theta_H - 1}{\theta_k}$.

Ignoring feasibility, the first-order effect of $v_j(s_L, s_L)$ is $\frac{\delta}{4}(1+\psi)$, which is positive. But then $v_1(s_L, s_L) = P(v_2(s_L, s_L)) = v_0$. Similarly, ignoring feasibility, the first-order effect of $v_j(s_H, s_H)$ is $\frac{\delta}{4}(1-\psi)$, which is positive, since $\psi < 1$. But then $v_1(s_H, s_H) = P(v_1(s_H, s_H)) = v_0$ as well. Simplifying notation slightly, we can write (13) as

$$\ell_1^{HL} = \min\left\{l, \frac{1-\theta_k}{\theta_k}l + \frac{\delta}{\theta_k}\left(v_1^{LH} - v_1^{HL}\right)\right\}.$$

Ignoring feasibility, the first-order effects of v_1^{LH} and v_1^{HL} are $\frac{\delta}{4}(1+\psi)$ and $\frac{\delta}{4}(1-\psi)$, respectively. Both are positive such that, using symmetry, $v_1^{LH} = P\left(v_2^{LH}\right) = P\left(v_1^{HL}\right)$ and $v_1^{HL} = P\left(v_2^{HL}\right) = P\left(v_1^{LH}\right)$. Hence we know that second-period welfare pairs are always on the Pareto frontier.

Finally, we need to show that $\Delta_v \equiv v_1^{LH} - v_1^{HL} > 0$ at the optimum. At $v_1^{HL} = v_1^{LH} = v_0$, suppose we reduce v_1^{HL} by a small $dv > 0$. Then $\Delta_v = \left(-P'\left(v_1^{HL}\right) + 1\right) dv$. The effect on first-period welfare per local regulator is

$$\Delta_W = \frac{1}{4}\left[(\theta_H - 1)\frac{\delta}{\theta_k}\Delta_v + \delta\left(-P'\left(v_1^{HL}\right) - 1\right) dv\right] = \frac{\delta}{4\theta_k}\left[-(2\mu - 1)P'\left(v_1^{HL}\right) - (1-2\sigma)\right] dv.$$

For $dv < v_1^{HL} - \underline{v}_1$ we have $-P'\left(v_1^{HL}\right) = \tau_0 = 2\mu - 1$ such that $\Delta_W = \frac{\delta}{2\theta_k}\left(\sigma - 2\mu(1-\mu)\right) dv > \frac{\delta}{2\theta_k}\sigma(1-2\sigma)$, which is positive. Then $\Delta_v > 0$ at an optimum such that local regulators' second-period welfare will be varied to increase discretion in period one (ℓ_1^{HL} is higher compared to the allocation in Lemma 2). $\square$

Proof of lemma 6. We first show that T maps $C(V)$ into itself. That Tf is decreasing follows from the promise-keeping constraint (17). Tf is continuous by corollary 5 in Milgrom and Segal (2002). To see that Tf is concave take any $v_0', v_0'' \in V$, $\alpha \in (0,1)$. Denote the corresponding policy rules and continuation payoffs be $\ell_1', \ell_2', v_1', v_2'$ and $\ell_1'', \ell_2'', v_1'', v_2''$ respectively. While the set of incentive compatible allocations is generally not convex, we can construct alternative policies and continuation payoffs $\ell_1^a, \ell_2^a, v_1^a, v_2^a$, that satisfy (17)-(20) for $v_0 = \alpha v_0' + (1-\alpha)v_0''$, as follows

$$u(\ell_1^a(\theta_i,\theta_j),\theta_i) - \frac{1}{2}\ell_2^a(\theta_i,\theta_j) = \alpha\left[u(\ell_1'(\theta_i,\theta_j),\theta_i) - \frac{1}{2}\ell_2'(\theta_i,\theta_j)\right] + (1-\alpha)\left[u(\ell_1''(\theta_i,\theta_j),\theta_i) - \frac{1}{2}\ell_2''(\theta_i,\theta_j)\right],$$

$$u(\ell_2^a(\theta_i,\theta_j),\theta_j) - \frac{1}{2}\ell_1^a(\theta_i,\theta_j) = \alpha\left[u(\ell_2'(\theta_i,\theta_j),\theta_j) - \frac{1}{2}\ell_1'(\theta_i,\theta_j)\right] + (1-\alpha)\left[u(\ell_2''(\theta_i,\theta_j),\theta_j) - \frac{1}{2}\ell_1''(\theta_i,\theta_j)\right],$$

$$\ell_k^a(\theta_i,\theta_j) > \alpha\ell_k'(\theta_i,\theta_j) + (1-\alpha)\ell_k''(\theta_i,\theta_j), \quad k = 1,2,$$

$$v_k^a(\theta_i,\theta_j) = \alpha v_k'(\theta_i,\theta_j) + (1-\alpha)v_k''(\theta_i,\theta_j), \quad k = 1,2.$$

The alternative allocation satisfies (17) for $v_0 = \alpha v_0' + (1-\alpha)v_0''$ and, since f is concave, it also satisfies (20). To see that (18), evaluated at the alternative allocation, holds note that

$$\frac{1}{N}\sum_{\theta_j\in\Theta}\left[u(\ell_1^a(\theta_i,\theta_j),\theta_i) - u(\ell_1^a(\theta_{i+1},\theta_j),\theta_i) - \frac{1}{2}(\ell_2^a(\theta_i,\theta_j) - \ell_2^a(\theta_{i+1},\theta_j)) + \delta(v_1^a(\theta_i,\theta_j) - v_1^a\left(\theta_{i+1},\theta_j\right))\right]$$
$$\geq \frac{1}{N}\sum_{\theta_j\in\Theta}(\theta_{i+1}-\theta_i)\left[\nu(l_1^a(\theta_{i+1},\theta_j)) - \alpha\nu(l_1'(\theta_{i+1},\theta_j)) - (1-\alpha)\nu(l_1''(\theta_{i+1},\theta_j))\right] > 0$$

where the first inequality follows from the fact that (18) holds for allocations $\ell_1', \ell_2', v_1', v_2'$ and $\ell_1'', \ell_2'', v_1'', v_2''$ respectively and by construction of the alternative allocation, and the second inequality follows by construction of the alternative allocation and the fact that ν is increasing and convex. A similar argument shows that (19) is strictly slack evaluated at the alternative allocation. But then $(Tf)(\alpha v_0' + (1-\alpha)v_0'') \geq \alpha(Tf)(v_0') + (1-\alpha)(Tf)(v_0'')$ and this inequality can be made strict since (18) and (19) are strictly slack when evaluated at the alternative allocation (the alternative allocation can be strictly improved upon). Hence Tf is (strictly) concave and $T : C(V) \to C(V)$.

To see that T is a contraction note that $Tf \leq Tg$ whenever $f \leq g$ and that, since (20) always binds, $T(f+c) = Tf + \delta c$ for $c \in \mathbb{R}_+$, where $\delta < 1$. Suppose (20) does not bind for some (θ_i, θ_j) such that $v_1(\theta_i, \theta_j) < f(v_2(\theta_i, \theta_j))$ then the first-order optimality conditions for $v_1(\theta_i, \theta_j)$, $v_2(\theta_i, \theta_j)$ and $c_1(\theta_i, \theta_j)$ yield $\left(u'(\ell_1(\theta_i, \theta_j), \theta_i) - u'(\ell_1(\theta_i, \theta_j), \theta_{i-1})\right)\psi_1(\theta_{i-1}) = 0$, where ψ_1 is the Lagrange multiplier on (18). Since there is a unique finite bliss point $\ell_1^*(\theta_{i'})$ for each $i' = 1, 2, \ldots, N$ and $\theta_i > \theta_{i-1}$ it must be the case that $\psi_1(\theta_{i-1}) = 0$, and a similar argument shows that $\psi_2(\theta_{j-1}) = 0$. But then $v_1(\theta_i, \theta_j)$ and $v_2(\theta_i, \theta_j)$ should be as high as possible from the first-order conditions for v_1 and v_2. Hence $v_1(\theta_i, \theta_j) = f(v_2(\theta_i, \theta_j))$ for all (θ_i, θ_j).

Note that $(C(V), \|\cdot\|_\infty)$, with $\|f\|_\infty = \sup_{v \in V} |f(v)|$, is a complete metric space such that T has a unique fixed point P. Note that P is strictly concave since Tf is strictly concave for any $f \in C(V)$ (see corollary 1 on page 52 in Stokey and Lucas, 1989). To establish differentiability note that the set of maximizers (and multipliers) generated by $(TP)(v_0)$ is a singleton for each v_0 due to strict concavity of u and P. Then the directional derivatives given by corollary 5 in Milgrom and Segal (2002) coincide.[12] Hence P is continuously differentiable on int$V$.

That the Pareto set is self-generating since (20) binds strictly for any $f \in C(V)$). $\square$

Lemma 8. *Local regulators report shocks truthfully as long as the inequalities* (18) *and* (19) *are satisfied if local regulator leniency is increasing in the local shock.*

[12] Since there is no enforcement problem in the setup considered the regularity condition of the corollary (non-empty interior of the constraint set at every $v_0 \in \text{int}V$) is always satisfied. See Kocherlakota (1996) for a setup where this is not the case.

Proof of lemma 8. Local regulator one with parameter θ_i cannot benefit from reporting θ_{i-1} instead if the following expression is positive.

$$\frac{1}{N}\sum_{\theta_j\in\Theta}\left[u(\ell_1(\theta_i,\theta_j),\theta_i)-\frac{1}{2}\ell_2(\theta_i,\theta_j)+\delta v_1(\theta_i,\theta_j)\right]-\frac{1}{N}\sum_{\theta_j\in\Theta}\left[u(\ell_1(\theta_{i-1},\theta_j),\theta_i)-\frac{1}{2}\ell_2(\theta_{i-1},\theta_j)+\delta v_1(\theta_{i-1},\theta_j)\right]$$

Since (18) is satisfied with equality, the expression can be written as

$$\frac{1}{N}\sum_{\theta_j\in\Theta}\left(\theta_i-\theta_{i-1}\right)\left(\ell_1(\theta_i,\theta_j)-\ell_1(\theta_{i-1},\theta_j)\right),$$

which is positive for ℓ_1 increasing in the shock experienced by regulator one. A similar argument applies to the case of local regulator two.

Iterate on (18) to see that local regulator one with parameter θ_i cannot benefit from reporting θ_{i+k} instead, with $i+k\leq N$, whenever ℓ_1 is increasing in the shock experienced by regulator one.

□

Proof of lemma 7. From (21) we have that

$$\frac{1}{2}\left[\tau+\psi_2(\theta_j)-\psi_2(\theta_{j-1})\right]>u'(\ell_1(\theta_i,\theta_j),\theta_i)\left[1+\psi_1(\theta_i)-\psi_1(\theta_{i-1})\right]$$

since marginal utilities are strictly increasing in the parameter. Similarly, from (22) we obtain

$$\frac{1}{2}\left[1+\psi_1(\theta_i)-\psi_1(\theta_{i-1})\right]>u'(\ell_2(\theta_i,\theta_j),\theta_j)\left[\tau+\psi_2(\theta_j)-\psi_2(\theta_{j-1})\right].$$

Substituting the second into the first inequality and rearranging yields the first result. The second result follows from conditions (23) and (24) and the fact that local regulators are ex-ante identical.

Assume that $\ell_1(\theta_i,\theta_j)\leq\ell_1(\theta_i,\theta_{j+1})$. Suppose $v_2(\theta_i,\theta_j)<v_2(\theta_i,\theta_{j+1})$, then $-P'(v_2(\theta_i,\theta_j))<$

$-P'(v_2(\theta_i, \theta_{j+1}))$ and therefore $\psi_2(\theta_j) - \psi_2(\theta_{j-1}) < \psi_2(\theta_{j+1}) - \psi_2(\theta_j)$. It follows that

$$
\begin{aligned}
& u'(\ell_1(\theta_i, \theta_j), \theta_i) \left[1 + \psi_1(\theta_i)\right] - u'(\ell_1(\theta_i, \theta_j), \theta_{i-1}) \psi_1(\theta_{i-1}) \\
& \quad < u'(\ell_1(\theta_i, \theta_{j+1}), \theta_i) \left[1 + \psi_1(\theta_i)\right] - u'(\ell_1(\theta_i, \theta_{j+1}), \theta_{i-1}) \psi_1(\theta_{i-1}) \\
\Leftrightarrow & \left[u'(\ell_1(\theta_i, \theta_j), \theta_i) - u'(\ell_1(\theta_i, \theta_{j+1}), \theta_i)\right] \left[1 + \psi_1(\theta_i)\right] \\
& \quad \left[u'(\ell_1(\theta_i, \theta_j), \theta_{i-1}) - u'(\ell_1(\theta_i, \theta_{j+1}), \theta_{i-1})\right] \psi_1(\theta_{i-1}).
\end{aligned}
$$

This expression can be simplified as follows.

$$
\left[\nu'(\ell_1(\theta_i, \theta_j)) - \nu'(\ell_1(\theta_i, \theta_{j+1}))\right] \theta_i \left[1 + \psi_1(\theta_i)\right] < \left[\nu'(\ell_1(\theta_i, \theta_j)) - \nu'(\ell_1(\theta_i, \theta_{j+1}))\right] \theta_{i-1} \psi_1(\theta_{i-1})
$$

Since $1 + \psi_1(\theta_i) - \psi_1(\theta_{i-1}) > 0$ the expression yields a contradiction for $\ell_1(\theta_i, \theta_j) \leq \ell_1(\theta_i, \theta_{j+1})$. Hence $v_2(\theta_i, \theta_j) \geq v_2(\theta_i, \theta_{j+1})$. Assume now conversely that $v_2(\theta_i, \theta_j) \geq v_2(\theta_i, \theta_{j+1})$. Then

$$
\begin{aligned}
& u'(\ell_1(\theta_i, \theta_j), \theta_i) \left[1 + \psi_1(\theta_i)\right] - u'(\ell_1(\theta_i, \theta_j), \theta_{i-1}) \psi_1(\theta_{i-1}) \\
& \quad \geq u'(\ell_1(\theta_i, \theta_{j+1}), \theta_i) \left[1 + \psi_1(\theta_i)\right] - u'(\ell_1(\theta_i, \theta_{j+1}), \theta_{i-1}) \psi_1(\theta_{i-1})
\end{aligned}
$$

and

$$
\left[\nu'(\ell_1(\theta_i, \theta_j)) - \nu'(\ell_1(\theta_i, \theta_{j+1}))\right] \theta_i \left[1 + \psi_1(\theta_i)\right] \geq \left[\nu'(\ell_1(\theta_i, \theta_j)) - \nu'(\ell_1(\theta_i, \theta_{j+1}))\right] \theta_{i-1} \psi_1(\theta_{i-1}),
$$

such that $\ell_1(\theta_i, \theta_j) > \ell_1(\theta_i, \theta_{j+1})$ cannot be the case and therefore $\ell_1(\theta_i, \theta_j) \leq \ell_1(\theta_i, \theta_{j+1})$. This proves the third result. The fourth result is proved similarly.

If $v_2(\theta_i, \theta_j) > \hat{v}$ then $-P'(v_2(\theta_i, \theta_j)) > 1$ and

$$
\begin{aligned}
& u'(\ell_1(\theta_i, \theta_j), \theta_i) \left[1 + \psi_1(\theta_i)\right] - u'(\ell_1(\theta_i, \theta_j), \theta_{i-1}) \psi_1(\theta_{i-1}) \\
& \quad > u'(\ell_2(\theta_i, \theta_j), \theta_j) \left[\tau + \psi_2(\theta_j)\right] - u'(\ell_2(\theta_i, \theta_j), \theta_{j-1}) \psi_2(\theta_{j-1}) \geq u'(\ell_2(\theta_i, \theta_j), \theta_j) \left[\tau + \psi_2(\theta_j) - \psi_2(\theta_{j-1})\right].
\end{aligned}
$$

Suppose $u'(\ell_2(\theta_i,\theta_j),\theta_j) > \frac{1}{2}$ then

$$\begin{aligned}
& u'(\ell_1(\theta_i,\theta_j),\theta_i)\left[1+\psi_1(\theta_i)\right] - u'(\ell_1(\theta_i,\theta_j),\theta_{i-1})\psi_1(\theta_{i-1}) \\
& \quad > \frac{1}{2}\left[\tau + \psi_2(\theta_j) - \psi_2(\theta_{j-1})\right] \\
& \quad = u'(\ell_1(\theta_i,\theta_j),\theta_i)\left[1+\psi_1(\theta_i)\right] - u'(\ell_1(\theta_i,\theta_j),\theta_{i-1})\psi_1(\theta_{i-1}).
\end{aligned}$$

This expression yields a contraction such that $u'(\ell_2(\theta_i,\theta_j),\theta_j) \leq \frac{1}{2}$. The case for local regulator one is proved similarly. This proves the fifth result. $\square$

′