

# On the Simple Adaptive Control of Flexible-Joint Space Manipulators with Uncertainties

Steve Ulrich and Jurek Z. Sasiadek

**Abstract** This paper addresses the problem of adaptive trajectory tracking control of space manipulators that exhibit elastic vibrations in their joints, and are subject to parametric uncertainties. The proposed composite control methodology combines a decentralized simple adaptive control-based term to stabilize the rigid dynamics with a linear correction term to improve vibration damping at the joints. In particular, this paper illustrates how the injection of knowledge about the reference model via the use of feedforward terms in the adaptive control structure yields improved tracking results. Simulation results demonstrate reduced overshoots when tracking a square trajectory by the end-effector of a two-link manipulator under parametric uncertainties, in comparison with an existing composite adaptive controller without feedforward terms.

**Keywords** Space robotics · Simple adaptive control · Composite control · Trajectory tracking

## 1 Introduction

Although it is well known that space robots equipped with planetary gears exhibit joint vibration effects (Kahraman and Vijayakar 2001), the flexible-joint problem is primarily caused by the use of harmonic drives, i.e. a type of gear mechanism that is increasingly popular for use in space robotic applications, due to its low backlash, low weight, compactness, high torque capability, wide operating temperature range and good repeatability. However, with harmonic drives, the joint vibration problem is significant. Considering the joint flexibility in the analysis and design of control systems is therefore essential, especially when accurate end-effector positioning must be achieved. In some cases, joint flexibility can lead to instability when

---

S. Ulrich (✉) · J.Z. Sasiadek  
Carleton University, 1125 Colonel by Drive, Ottawa, ON K1S 5B6, Canada  
e-mail: Steve.ulrich@carleton.ca

neglected in the control design, as explained by Sweet and Good (1984). Despite the considerable research work that has been done on the dynamics and control of flexible-joint robot manipulators, the topic remains an important area of contemporary research, as noted by Ozgoli and Taghirad (2006), and several constraints and limitations are yet to be overcome. Indeed, most existing flexible-joint control strategies reported in the literature are model-based techniques, and have reasonably good tracking performance only when substantial knowledge of the plant mathematical model and its parameters is available. Consequently, if significant or unpredictable plant parameter variations arise as a result of joint mechanism degradation, or if there are modeling errors due to complex flexible dynamics behaviors, model-based control approaches might perform inadequately.

In this context, this paper considers the adaptive trajectory tracking problem associated with flexible-joint space manipulators subject to uncertainties. Most of the work addressing this problem is based on the singular perturbation theory (Khalil 1992), under which a flexible-joint robot exhibits a two-scale behavior. Using this theory, several authors developed adaptive singular perturbation-based (SPB) controllers (also referred to as adaptive composite controllers) consisting of a slow adaptive control term designed on the basis of the rigid-joint robot model and a fast control designed to dampen the elastic oscillations at the joints. Recent notable studies on adaptive composite control include (Huang et al. 2006; Cao and de Silva 2006).

However, these existing flexible-joint adaptive controllers are based on the indirect adaptive control approach, since they cope with parametric uncertainties by identifying the unknown robot parameters used explicitly in their control algorithm. Adverse consequences of this approach include the increased computational burden associated with real-time computation of unknown parameters.

Unlike these indirect adaptive composite controllers, the proposed design does not require identification of unknown parameters or mathematical models of the system to be controlled. Specifically, the direct adaptive control methodology developed in this paper is based on the model reference adaptive control (MRAC) approach and deals with uncertain parameters by time-varying the controller gains using the recently-developed decentralized simple adaptive control (DSAC)-based adaptation mechanism applicable to nonsquare Euler-Lagrange systems (Ulrich et al. 2012a). This way, the controller gains force the actual robot system response to match the ideal, or reference, model response. In addition, the strategy developed in this paper does not require persistent excitation for the controller gains to converge to their ideal values. The proposed composite control scheme is simple to implement, and renders effective trajectory tracking performance regardless of large parametric uncertainties. Moreover, unlike other flexible-joint control methodologies, the proposed adaptive control methodology is independent of the acceleration and torque signals.

This paper represents an extension to a recent work on the development of a direct adaptive composite controller for flexible-joint manipulators in the presence of parametric uncertainties (Ulrich et al. 2012a). However, in this recent work, the controller gain adaptation mechanism was based on the decentralized modified

simple adaptive control (DMSAC) approach, in which the gains are updated only as a function of the tracking error between the reference model outputs and the system outputs. The objective of this paper is to illustrate how the injection of knowledge about the reference model via the use of feedforward terms in the DSAC structure improves tracking results.

## 2 Dynamics Model

Consider a class of two-link elastic-joint robot manipulator systems, with nonlinear dynamics derived in terms of kinetic and potential energies stored in the system by the Euler-Lagrange formulation. Neglecting gravitational potential energy for space-based applications, and assuming that each joint is modeled as a linear torsional spring of constant stiffness, the resulting dynamics of flexible-joint manipulators with revolute joints which are actuated directly by DC motors are represented by the following second-order differential equations (Spong 1987)

$$M(q)\ddot{q} + C(q, \dot{q})\dot{q} - k(q_m - q) = 0 \quad (2.1)$$

$$J_m\ddot{q}_m + k(q_m - q) = \tau \quad (2.2)$$

where  $M(q)$  denotes the symmetric and positive-definite inertia matrix,  $C(q, \dot{q})$  denotes the centripetal-Coriolis matrix,  $\tau$  denotes the control torque vector,  $q, \dot{q}, \ddot{q}$  represent the link position, velocity and acceleration vector respectively,  $q_m, \dot{q}_m, \ddot{q}_m$  denote the motor position, velocity and acceleration vector respectively,  $J_m$  denotes the positive-definite motor inertia matrix, and  $k$  represents the diagonal positive-definite stiffness matrix of the joints.

In the subsequent development, it is assumed that the Euler-Lagrange system defined in (2.1) and (2.2) is fully-actuated and non-redundant, that the Jacobian matrix denoted by  $J(q)$  has full rank column, and that the link angles and velocities are measurable. Thus, holonomic constraints can be selected in order for the generalized link positions and velocities to satisfy the relation

$$\dot{x}_r = J(q)\dot{q} \quad (2.3)$$

where  $\dot{x}_r$  denote the actual end-effector Cartesian velocity vector defined along the  $x$  and  $y$  axes. Similarly, it is assumed that there exists a mapping allowing the Cartesian position of the end-effector with respect to the robot reference frame along both axes to be obtained as

$$x_r = \Omega(q) \quad (2.4)$$

where  $\Omega(q)$  is the forward kinematic transformation taking the link positions into end-effector Cartesian position. By combining (2.3) and (2.4), joint-space variables

are transformed into the task-space (Cartesian) manipulator system output vector  $y(t)$  through the transformation (Asada and Slotine 1986)

$$y(t) \equiv \begin{bmatrix} x_r \\ \dot{x}_r \end{bmatrix} = \begin{bmatrix} l_1 \cos(q_1) + l_2 \cos(q_1 + q_2) \\ l_1 \sin(q_1) + l_2 \sin(q_1 + q_2) \\ -l_1 \sin(q_1)\dot{q}_1 - l_2 \sin(q_1 + q_2)(\dot{q}_1 + \dot{q}_2) \\ l_1 \cos(q_1)\dot{q}_1 + l_2 \cos(q_1 + q_2)(\dot{q}_1 + \dot{q}_2) \end{bmatrix} \quad (2.5)$$

### 3 Control Objective

The objective consists in designing a direct adaptive composite controller which ensures that the output vector  $y(t)$  of the flexible-joint manipulator system tracks the time-varying reference model output  $y_m(t)$  regardless of uncertainties in the dynamics model parameters. The reference model, which defines the ideal response to the desired user-defined trajectory, is expressed in terms of the ideal damping ratio  $\zeta$ , and undamped natural frequency  $\omega_n$ , as follows

$$\dot{x}_m(t) = A_m x_m(t) + B_m u_m(t) \quad (3.1)$$

$$y_m(t) = C_m x_m(t) \quad (3.2)$$

where the system matrices are given by

$$A_m = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ -\omega_n^2 & 0 & -2\zeta\omega_n & 0 \\ 0 & -\omega_n^2 & 0 & -2\zeta\omega_n \end{bmatrix} \quad B_m = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -\omega_n^2 & 0 & 0 & 0 \\ 0 & -\omega_n^2 & 0 & 0 \end{bmatrix}$$

$$C_m = I_4$$

and with

$$x_m = \begin{bmatrix} x_{r_d} \\ \dot{x}_{r_d} \end{bmatrix} \quad u_m = \begin{bmatrix} \ddot{x}_{r_d} \\ \dot{x}_{r_d} \end{bmatrix}$$

where  $x_{r_d}, \dot{x}_{r_d}$  denote the desired position and velocity trajectory expressed in the task space, respectively. To quantify the control objective, a tracking error, denoted  $e_y(t)$  is defined as

$$e_y \triangleq y_m - y \quad (3.3)$$

## 4 DSAC-Based Composite Control

In the following, the proposed control strategy uses the singular perturbation-based theory, which transforms a dynamics model into a two-timescale model: specifically, the quasi-steady state subsystem and the boundary-layer subsystem. Here, for brevity, only the final results of the approach are presented. A more comprehensive treatment of the SPB theory is given by Khalil (1992). The SPB theory provides a framework to design composite controllers, in which the control input is given by

$$\tau = \tau_s + \tau_f \quad (4.1)$$

where the subscript  $s$  stands for slow variables defined in the slow time scale  $t$  and the subscript  $f$  stands for variables defined in the fast time scale  $t' = t/\varepsilon$ , with  $\varepsilon$  being a small positive scalar. The slow control torque  $\tau_s$  is designed to stabilize the quasi-steady state subsystem, and the fast-control torque  $\tau_f$  controls the boundary-layer subsystem.

Applied to flexible-joint manipulators, composite controllers typically consist in a fast control term that adds damping to the elastic vibrations at the joints and a slow control term that stabilizes the rigid dynamics (Spong 1989). Indeed, the quasi-steady state subsystem has the form of an equivalent rigid-joint manipulator dynamics model, that is

$$H(q_s)\ddot{q}_s + C(q_s, \dot{q}_s)\dot{q}_s = \tau_s \quad (4.2)$$

where

$$H(q_s) = M(q_s) + J_m \quad (4.3)$$

Based on the above discussion, the proposed control torque input is composed of a rigid-joint direct adaptive term plus a linear damping correction term as

$$\tau = J^T(q) [K_e(t)e_y + K_x(t)x_m + K_u(t)u_m] + K_v(\dot{q} - \dot{q}_m) \quad (4.4)$$

where  $K_e(t)$  represents the stabilizing feedback control gain matrix,  $K_x(t)$ ,  $K_u(t)$  are time-varying feedforward control gain matrices that contribute to reducing the tracking errors, and  $K_v$  is a constant diagonal control gain. Following common practice, each of the adaptive control gains in (4.4) is calculated as the summation of a proportional and an integral term, as follows

$$K_e(t) = K_{Pe}(t) + K_{Ie}(t) \quad (4.5)$$

$$K_x(t) = K_{Px}(t) + K_{Ix}(t) \quad (4.6)$$

$$K_u(t) = K_{Pu}(t) + K_{Iu}(t) \quad (4.7)$$

Adopting herein the DSAC-based adaptation mechanism for nonsquare nonlinear systems recently developed by Ulrich et al. (2012b) yields the following update law for the stabilizing feedback control gains of the slow control term

$$K_{Pe}(t) = W^T \text{diag}\{e_y, e_y^T\} \Gamma_{Pe} \quad (4.8)$$

$$\dot{K}_{Ie}(t) = W^T \left[ \text{diag}\{e_y, e_y^T\} \Gamma_{Ie} - \text{diag}\{\sigma_e WK_{Ie}\} \right] \quad (4.9)$$

where  $W$  is a scaling matrix defined by

$$W \triangleq [I_2 \quad I_2]^T \quad (4.10)$$

With the DSAC scheme, the components of the feedforward control gain matrices are updated as follows

$$K_{Px}(t) = W^T \text{diag}\{e_y, x_m^T\} \Gamma_{Px} \quad (4.11)$$

$$\dot{K}_{Ix}(t) = W^T \left[ \text{diag}\{e_y, x_m^T\} \Gamma_{Ix} - \text{diag}\{\sigma_x WK_{Ix}\} \right] \quad (4.12)$$

$$K_{Pu}(t) = W^T \text{diag}(e_y, u_m^T) \Gamma_{Pu} \quad (4.13)$$

$$\dot{K}_{Iu}(t) = W^T \left[ \text{diag}\{e_y, u_m^T\} \Gamma_{Iu} - \text{diag}\{\sigma_u WK_{Iu}\} \right] \quad (4.14)$$

In (4.8), (4.9), and (4.11–4.14),  $\Gamma_{Pe}$ ,  $\Gamma_{Ie}$ ,  $\Gamma_{Px}$ ,  $\Gamma_{Ix}$ ,  $\Gamma_{Pu}$ , and  $\Gamma_{Iu}$  are positive-definite diagonal parameter matrices that control the rate of adaptation. Also, diagonal damping coefficient matrices  $\sigma_e$ ,  $\sigma_x$ , and  $\sigma_u$  have been introduced in the adaptation law algorithm. With this adjustment, the integral gains increase as required (due to large tracking errors, for example) and decrease when large gains are no longer necessary. This way, divergence of the integral control gains is avoided. Indeed, without these coefficients, the integral adaptive gains would increase for as long as there is a tracking error. When the integral gains reach some values, they have a stabilizing effect on the system and the tracking error starts decreasing. However, if, for some reasons, the tracking error does not reach zero, the integral gains would continue to increase. Furthermore, every sudden change in the desired trajectory would result in greater values of these adaptive gains, which could eventually lead to instability. With this adjustment, the integral control gains are obtained as a first-order filtering of the tracking error, and cannot diverge, unless the tracking errors diverge.

## 5 Simulation Results

This section presents the simulation results obtained by implementing the adaptive composite controller developed in this paper. To validate the trajectory tracking performance, a 12.6 m  $\times$  12.6 m square trajectory was required to be tracked in 60 s in a counterclockwise direction starting from rest at the lower-right-hand corner. Compared with other type of Cartesian trajectories, such as straight lines, circles, or sine-type waveforms, square trajectories represent greater control challenges as each corner of the square represents an abrupt change in directions. Hence, it is required that the end-effector reaches each corner and then redirects itself along an orthogonal direction with minimum overshoot and settling time. For this reason, square trajectories are ideal for studying the control of transient vibrations in flexible systems and are commonly used in the literature to assess the trajectory tracking performance of flexible space manipulators. The speed of the commanded square trajectory is fast enough to render the nonlinearities and flexibility effects significant (15 s for each side of the square).

The physical properties of the robot (the length and the mass of the links) are adopted from a previous study on the control of space robots (Ulrich et al. 2012a) and the flexible parameters are based on Cao and de Silva (2006), which are representative of manipulators with highly flexible joints.

In all simulations, the integral structure of the integral control gains in (4.9), (4.12) and (4.14) is computed online via a standard Tustin algorithm. In addition, all integral control gains were initialized to zero. The reference model parameters were selected as  $\omega_n = 10$  rad/s and  $\zeta = 0.9$ . The control parameters and coefficients were selected as follows

$$\begin{aligned} \Gamma_{Pe} &= \text{diag}[80 \quad 80 \quad 50 \quad 50] \\ \Gamma_{Ie} &= \text{diag}[120 \quad 120 \quad 150 \quad 150] \\ \Gamma_{Px} &= \Gamma_{Pu} = \text{diag}[0.1 \quad 0.1 \quad 2 \quad 2] \\ \Gamma_{Ix} &= \Gamma_{Iu} = \text{diag}[0.2 \quad 0.2 \quad 2 \quad 2] \\ \sigma_e &= \text{diag}[0.75 \quad 0.75 \quad 0.15 \quad 0.15] \\ \sigma_x &= \sigma_u = \text{diag}[0.9 \quad 0.9 \quad 0.4 \quad 0.4] \\ K_v &= 120I_2 \end{aligned}$$

These controller gains, parameters, and coefficients were selected in numerical simulations to provide satisfactory tracking performance when applied to the two-link flexible-joint robot modeled with the nominal linear joint stiffness dynamics representation described by (2.1) and (2.2). In addition, the  $\sigma$  coefficients were selected to be very small, since they are only to prevent the integral adaptive gains from reaching

excessively high values or diverging in time. Note that the controller may be sensitive to the selected control parameters. As a result, different trajectory tracking performance may be obtained by choosing different parameters than those listed above.

The trajectory tracking results obtained with the proposed controller when applied to the linear joint stiffness dynamics model are depicted in Fig. 1. In this figure, the dashed line corresponds to the desired end-effector position  $x_{r_d}$  (i.e. the input of the reference model), and the solid line corresponds to the actual end-effector position  $x_r$ . As shown in Fig. 1, the trajectory exhibits minimal overshoots of 0.031, 0.030 and 0.030 m for the first, second, and third direction changes, respectively. Moreover, tracking is close to a straight line along each side of the trajectory.

For completeness and comparison purposes, the controller in (4.4) was implemented without including the feedforward adaptive control gains. The adaptation law defined in (4.8) and (4.9) was used to update the stabilizing control gain terms. Thus, by neglecting feedforward gains, the control torque input takes the following DMSAC form (Ulrich et al. 2012b)

$$\tau = J^T(q)K_e(t)e_y + K_v(\dot{q} - \dot{q}_m) \quad (5.1)$$

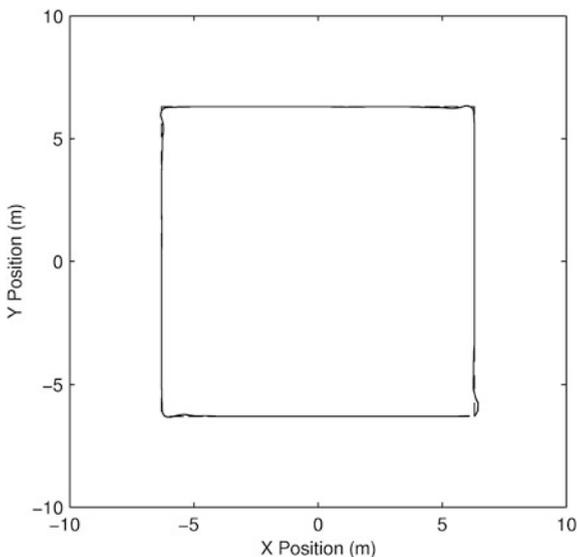
The gains for this controller were determined as follows

$$\Gamma_{P_e} = \Gamma_{I_e} = \text{diag}[150 \quad 150 \quad 25 \quad 25]$$

$$\sigma_e = \text{diag}[0.008 \quad 0.008 \quad 0.023 \quad 0.023]$$

$$K_v = 35I_2$$

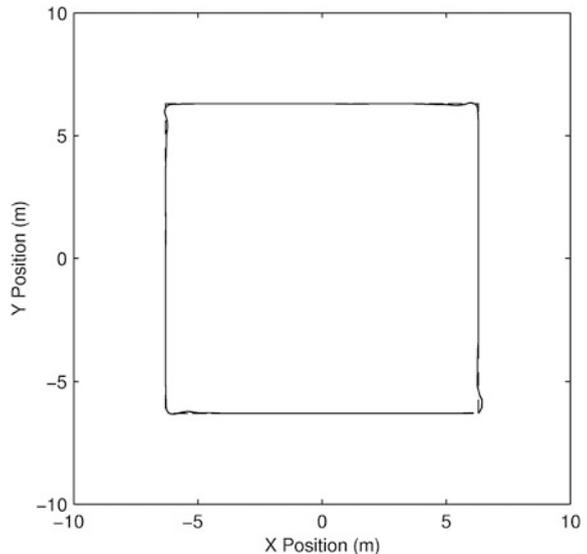
**Fig. 1** DSAC trajectory tracking results, nominal joint stiffness matrix



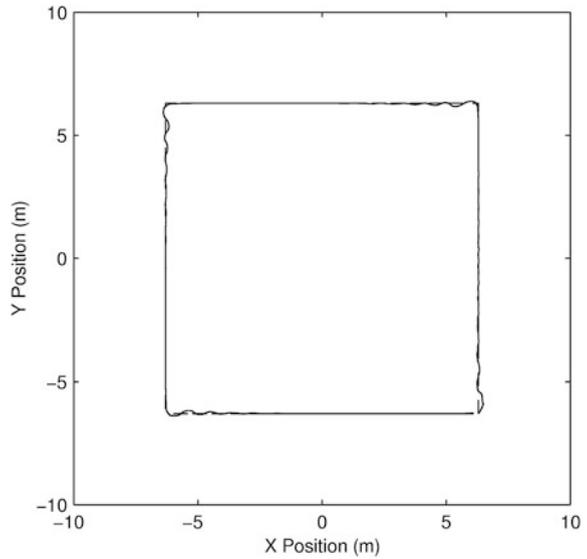
The results obtained with the DMSAC approach can be found in Ulrich et al. (2012b) and are reported in Fig. 2 for the nominal robot model, i.e., the linear joint stiffness dynamics model. When compared to the results shown in Fig. 1, superior tracking performance with smaller maximum end-effector deflection when tracking a square trajectory is obtained with the DSAC methodology. Hence, the response obtained with the DSAC composite control approach is much closer to a square than that of the DMSAC response. As anticipated, these results suggest that injecting information about the reference model into the controller structure improves the tracking performance.

Numerical simulations using a manipulator with joints of significantly lower stiffness were performed in order to assess the performance of the proposed controller when parametric uncertainties in the plant are included. The same composite controller tuned previously with the nominal robot manipulator was applied to a robot with a joint stiffness matrix of  $k = 200I_2$  Nm/rad, representing an uncertainty of 60 %. The results are shown in Figs. 3 and 4 for the DSAC and DMSAC methodologies, respectively. As demonstrated in these two figures, the trajectory obtained with the DSAC-based composite controller yields smaller overshoots than those obtained with the DMSAC strategy, i.e. 0.080, 0.083 and 0.083 m for the first, second, and third direction changes with the DSAC approach, as opposed to 0.095, 0.112 and 0.094 m for the first, second, and third direction changes with the DMSAC approach. While the DSAC-based composite controller achieved superior tracking performance, its residual oscillations are larger, in comparison with the DMSAC-based strategy. This may be explained by the larger control inputs associated with the DSAC strategy due to the introduction of additional feedforward terms in the control law. Indeed, these terms increase the overall control effort, as

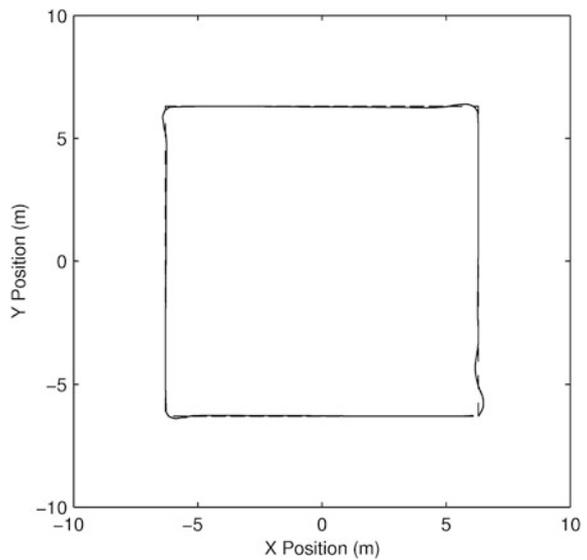
**Fig. 2** DMSAC trajectory tracking results, nominal joint stiffness matrix (Ulrich et al. 2012b)



**Fig. 3** DSAC trajectory tracking results, uncertain joint stiffness matrix



**Fig. 4** DMSAC trajectory tracking results, uncertain joint stiffness matrix (Ulrich et al. 2012b)



required to improve the tracking performance. However, this makes the controller more sensitive to small tracking errors and to sudden changes in the desired trajectory, which in turn results in greater oscillations when applied to a manipulator with excessively low joint stiffness coefficients, as demonstrated by the obtained results.

## 6 Conclusions

A decentralized simple adaptive control-based composite control algorithm is developed for a flexible-joint space manipulator. The primary objective of the developed control methodology is to incorporate additional feedforward terms in the adaptive control structure, to force the errors between the plant outputs and the reference model outputs to converge to zero. The proposed controller is validated in numerical simulations while considering large parametric uncertainties in the joint stiffness coefficients. Obtained results demonstrate superior tracking performance with smaller maximum end-effector deflection when tracking a square trajectory, in comparison with an existing composite control scheme based on the decentralized modified simple adaptive control methodology. However, the latter approach might represent a better strategy for cases where the objective is to reduce residual oscillations. Future efforts will focus on extending the control method to flexible-joint robots modeled as nonlinear square systems, thereby guaranteeing closed-loop stability.

## References

- Asada H, Slotine JJE (1986) Robot analysis and control. Wiley, New York
- Cao Y, de Silva CW (2006) Dynamic modeling and neural-network adaptive control of a deployable manipulator system. *J Guidance Control Dyn* 29:192–194
- Huang L, Ge SS, Lee TH (2006) Position/force control of uncertain constrained flexible joint robots. *Mechatronics* 16:111–120
- Kahraman A, Vijayakar S (2001) Effect of internal gear flexibility on the quasi-static behavior of a planetary gear set. *J Mech Design* 123:408–415
- Khalil HK (1992) Nonlinear systems. Macmillan, New York
- Ozgoli S, Taghirad HD (2006) A survey on the control of flexible joint robots. *Asian J Control* 8:332–344
- Spong MW (1987) Modeling and control of elastic joint robots. *J Dyn Syst Meas Control* 109:310–319
- Spong MW (1989) Adaptive control of flexible joint manipulators. *Syst Control Lett* 13:15–21
- Sweet LM, Good MC (1984) Re-definition of the robot motion control problem: effects of plant dynamics, drive system constraints, and user requirements. In: Proceedings of 23rd IEEE conference on decision and control, pp 724–732
- Ulrich S, Sasiadek JZ, Barkana I (2012a) Decentralized simple adaptive control for nonsquare Euler-Lagrange systems. In: Proceedings of American control conference, pp 232–235
- Ulrich S, Sasiadek JZ, Barkana I (2012b) Modeling and direct adaptive control of a flexible-joint manipulator. *J Guidance Control Dyn* 35:25–39



<http://www.springer.com/978-3-319-13852-7>

Aerospace Robotics II

Sasiadek, J. (Ed.)

2015, XIV, 198 p. 110 illus., 92 illus. in color., Hardcover

ISBN: 978-3-319-13852-7