Nonparametric Identification and Control of Flexible Joint Robot Manipulator

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Abstract— **We discuss nonadaptive and adaptive control strategies for tracking the end effector of a flexible robot with collocated and noncollocated sensors attached at the end effector and 0.5m from the elbow joint. Collocated joint sensors satisfy hyperstability conditions but are unable to capture nonminimum phase (NMP) response which is achieved by noncollocated sensors. Nonadaptive control with collocated and noncollocated sensors results in poor tracking, but adaptive control leads to excellent tracking regardless of sensor location. Nonparametric identification procedure for the flexible robot is also presented and its convergence is discussed.**

I. INTRODUCTION

Flexible robots have become very popular in spacecraft applications lately, however multiple problems emerged. The main challenges in designing controllers of such structures are: link vibrations which make it difficult to achieve precise position control of the end effector, flexural deformation of the robot links and highly nonlinear dynamics of multilink systems. In this paper we consider a nonadaptive and adaptive fuzzy logic system (FLS) control of two-link flexible robot with collocated and noncollocated sensors whose end effector tracks a square trajectory. Link flexibility causes large transient disturbances at 90 degrees changes of directions at square vertices. The flexible robot dynamics is modeled by Euler-Lagrange that include link flexibility based on the dominant assumed mode for an Euler-Bernoulli cantilever beam coupled with rotating rigid-link dynamics [1]. Noncollocated sensors add complexity in end effector position control. We also introduce nonparametric identification procedure for the nonlinear joint stiffness based on Hammerstein model and semi-recursive kernel regression estimation.

II. ROBOT MANIPULATOR AND ITS CONTROL

The main goal of the controller is to assure precise tracking and to reduce vibrations. Residual vibrations are reduced by an input shaping method coupled with an inverse kinematics control strategy for both linear and nonlinear control laws. That approach uses full-order flexible dynamics equations based on a recursive O(n) algorithm for a discretized two-link flexible robot model [2]. A fuzzy logic system (FLS) adapting control for suppressing vibrations is described in [3]. Fuzzy controller presented in described in [4] adapting the control law by varying voltage input to each motor of a two-link flexible robot in response to link acceleration and angular position outputs also suffers from significant transient vibrations.

An improved controller of position and vibration of a single-link flexible manipulator has been implemented in [5]. The controller uses modal expansion of the first three significant vibration modes within a state-space model reference adaptive control (MRAC) strategy and shows reduced positional errors and decreased settling time of transient response to step inputs.

A wave-absorbing controller (WAC) with a noncollocated piezofilm sensor glued to a long flexible beam has been implemented in [6]. It suppresses traveling waves in the beam and provides input to a piezoceramic actuator. It performs better than a collocated WAC and its performance improves with increased length of the noncollocated piezosensor. An extended Popov's hyperstability control concept was proposed in [7] for a single-link flexible robot with reduced-order model for large space structures (LSS) and four vibration modes. In the system, conventional hyperstable control sensors were used in conjunction with performance sensors to enable vision-based stable control of large flexible space robot manipulators. Direct MRAC

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simulations with a single-link flexible robot manipulator with angular displacement sensors collocated at the hub and noncollocated near the midlink and endpoint show excellent tracking, flexibility in choice of adaptive laws and obviate difficulties associated with Lyapunov's stability function [8].

Theoretical and experimental studies were performed on a single-link flexible manipulator using the μ-synthesis control method in [9]. Sensor configurations include collocated sensor feedback control with a hub angle sensor, noncollocated feedback control using both hub angle and endpoint acceleration sensors and noncollocated feedback control using both hub and endpoint sensors for which the hub angle specification is relaxed to reduce hub response overshoot. Experimental results demonstrate that the two noncollocated control designs perform significantly better than collocated hub angle sensor design and robustness is achieved by using endpoint acceleration feedback.

III. Flexible Robot

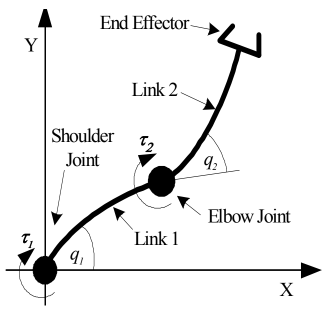


Figure 1. Flexible robot. [1]

The flexible robot shown in Fig. 1 can perform planar motion and has vibration modes. Gravity is neglected. The robot has the following parameters: length of each link $L_1 = L_2 = 4.5$m, flexural rigidity EI = 1676 N/m^2 and link mass density ρ = 0.335 kg/m [1].

A. Flexible Dynamics

We now derive a closed form of nonlinear dynamics of a multilink robot with flexible links using the Euler-Lagrange equations expressed in terms of kinetic and potential energies stored in the system [1]. Given an independent set of generalized coordinates, $q_i = q_1,......,q_n$, the total kinetic energy T and potential energy U stored in the system the Lagrangian [9], [10], [11] is defined by

$$L\left(q_i, \dot{q}_i\right) = T - U, \quad i = 1,......,n. \quad (1)$$

From (1) we derive robot dynamic equations of motion when it is subjected to generalized force F_i acting on a generalized coordinate q_i

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{q}_i} - \frac{\partial L}{\partial q_i} = F_i, \quad i = 1,......,n. \quad (2)$$

Accurate tracking control of a two-link robot is compounded by distributed flexibility of its links, flexural vibrations and nonminimum phase response (NMP). For precision control the residual vibrations must be suppressed while compensating for NMP response [13], [14]. We apply modal expansions which are widely used in the derivation of Euler-Lagrange dynamics equations for rotating flexible robot links and are used in this paper for an Euler-Bernoulli cantilever beam modeling of each link [10], [11], [12]. Assumed modes follow changes in configuration during operation, whereas natural modes must be continually recomputed.

Elastic deformations of the robot links are modeled by a finite series of space-dependent, *admissible functions,* multiplied by a specific set of time-dependent amplitude functions, resulting in a deformation function. A selected set of admissible functions should satisfy, at least, the flexible robot geometric boundary conditions and form basis functions throughout its operational workspace, provided the boundary conditions are consistent. An approximate deformation of the robot links subjected to transverse vibrations is given by [11], [12], [13], [14].

$$u(x,t) = \sum_{i=1}^{n} \phi_i(x) q_i(t) \quad (3)$$

where $\phi_i(x)$ are assumed mode shapes.

Transverse beam vibration theory implies cantilever mode shapes [10], [11], [12]

$$\phi_{ci}(x) = A\left[\cosh\lambda_{ci}x - \cos\lambda_{ci}x - k_{ci}(\sinh\lambda_{ci}x - \sin\lambda_{ci}x)\right]$$

where A = 0.1 is an arbitrary constant, $\lambda_{ci}L = (i - 0.5)\pi$, are numerically approximated roots of the characteristic equation $\cos(\lambda_{ci}L)\cosh(\lambda_{ci}L) + 1 = 0$ and $k_{ci} = \cos\lambda_{ci}L + \cosh\lambda_{ci}L / \sin\lambda_{ci}L + \sinh\lambda_{ci}L$, i = 1,……,n. Modal frequencies are given by

$$\omega_{ci} = (\lambda_{ci}L)^2 \sqrt{EI/\rho L^4} \quad . \quad (4)$$

The deformation of an Euler-Bernoulli cantilever beam $u(x,t)$ in (3) is used to derive the Euler-Lagrange flexible robot dynamics matrix equations [9]

$$\tau = M(q)\ddot{q} + C(q,\dot{q}) + Kq \quad (5)$$

where M is a matrix of rigid and flexible-link inertia elements, C is a matrix of rigid Coriolis and centrifugal forces where second-order terms of interacting elastic modes are neglected, K is a stiffness matrix and q is a generalized coordinate vector of joint angles and elastic deformations. Complete finite element derivation of the dynamics equations for a multi-degree-of-freedom (multi-dof) manipulator was presented in [15]. Alternatively, the dynamics equations in (5) may be expressed as:

$$\begin{bmatrix}\tau\\0\end{bmatrix}=M(\theta,\delta)\begin{bmatrix}\ddot{\theta}\\\ddot{\delta}\end{bmatrix}+\begin{bmatrix}C_1(\theta,\dot{\theta})\\C_2(\theta,\dot{\theta})\end{bmatrix}+\begin{bmatrix}0&0\\0&K\end{bmatrix}\begin{bmatrix}\theta\\\delta\end{bmatrix} \quad (6)$$

where C_1 and C_2 are partitions of C and the zero-dynamics, where torque is zero in the control law vector, constitute the internal dynamics when the input and output of the system is identically zero. The internal dynamics of the flexible robot include elastic deformations of the links $_i$ [13].

B. Linear Joint Model

In the linear-joint dynamics model (Spong [17]) each joint is modeled as a linear torsional spring of constant stiffness and the resulting dynamics of the flexible joint manipulators consists of two second-order differential equations. Let q_m denote the vector of angular displacements of the motor shaft angles, where the elastic joint vibrations vector is defined as $q - q_m$. The joint flexibility is taken into account by augmenting the kinetic energy of the rigid-joint space robot presented earlier with the kinetic energy of the rotors due to their own rotation and by considering the elastic potential energy of the flexible joints. The kinetic energy of the rotors, T_e, and the elastic potential energy, U_e, are given by

$$T_e=\frac{1}{2}\dot{q}_m^T J_m(q)\dot{q}_m$$
$$U_e=\frac{1}{2}(q-q_m)^T k(q-q_m)$$

where J_m denotes the positive definite motor inertia matrix and stiffness of the flexible joints is modeled by the diagonal positive definite stiffness matrix k. Combining elastic terms with rigid-dynamics equations, the following dynamics equations of a flexible joint space robot with revolute joints actuated directly by DC motors are obtained by

$$M(q)\ddot{q}+C(q,\dot{q})\dot{q}=k(q_m-q)$$
$$J_m\ddot{q}_m+k(q_m-q)=\tau$$

C. Nonlinear Joint Model

Since the development of the linear stiffness model by Spong [17] a more precise dynamics models of flexible effects in the joints of robotic manipulators have been studied. As nonlinear effects become prevalent in the system dynamics, their accurate modeling is key to the successful design of advanced flexible joint control laws. The most relevant nonlinear effects are related to nonlinear stiffness. To replicate experimental joint stiffness curves, several studies recommend approximating the stiffness torque by a nonlinear cubic function. Another important characteristic related to nonlinear stiffness was investigated by Kircanski and Goldenberg [16] who observed that the torque C torsion characteristic is deformed toward the torque axis in the region from 0 to 1 Nm. In this region, the stiffness is lower due to the soft-windup effect. Besides nonlinear-joint stiffness effects, friction torques have an important impact on the behavior of the robot manipulator system and need to be considered for accurate modeling and control. One of the most complete friction models recently proposed by Makkar et al. [18] includes all dominant friction components such as static friction, Coulomb friction, viscous friction, and the Stribeck effect. By combining the cubic nonlinear stiffness torque term, soft-windup effects, inertial coupling, and frictional torques, the following nonlinear-joint dynamics representation was obtained by Ulrich et al. [19]:

$$M(q)\ddot{q}+S\ddot{q}_m+C(q,\dot{q})\dot{q}+f(\dot{q})-k(q,q_m)(q_m-q)=0 \quad (7)$$
$$S^T\ddot{q}+J_m\ddot{q}_m+k(q,q_m)(q_m-q)=\tau \quad (8)$$

where q denotes the link angle vector, q_m represents the motor angle vector, M(q) denotes the symmetric positive definite link inertia matrix, $C(q,\dot{q})$ represents the centrifugal-Coriolis matrix, J_m denotes the positive-definite motor inertia matrix, and τ represents the control torque vector. The inertial coupling matrix is given by

$$S=\begin{bmatrix}0&J_{m2}\\0&0\end{bmatrix}$$

In Eqs. (7) and (8), the nonlinear stiffness torque term is

$$k(q,q_m)(q_m-q)=a_1\begin{bmatrix}(q_{m1}-q_1)^3\\(q_{21}-q_2)^3\end{bmatrix}$$
$$+a_2(q_m-q)+K_{sw}(q,q_m)(q-q_m)$$

where a_1 and a_2 are positive definite diagonal matrices of stiffness coefficients and $K_{sw}(q,q_m)$ is the soft-windup correction factor that is modeled as a saddle-shaped function

$$K_{sw}(q,q_m)=-k_{sw}\begin{bmatrix}e^{-a_{sw}(q_{m1}-q_1)^2}&0\\0&e^{-a_{sw}(q_{m2}-q_2)^2}\end{bmatrix}$$

In Eq. (7), $f(\dot{q})$ denotes the friction vector which is modeled by (see Makkar [29])

$$f(\dot{q})=\gamma_1\tanh(\gamma_2\dot{q})-\tanh(\gamma_3\dot{q})+\gamma_4\tanh(\gamma_5\dot{q})+\gamma_6\dot{q}$$

where γ_i, i = 1, . . . ,6 denote positive parameters defining different friction components.

IV. NON-MINIMUM PHASE

A common problem with flexible robot control is non-minimum phase response. It happens when torque actuating a robot joint induces flexure and momentary acceleration of the end effector in a direction opposite to that expected and causes a time delay in subsequent control action. The delay is caused by the time it takes for a mechanical wave to travel through the link from joint to end effector. This behavior occurs when transfer function has zeros in the right half s-plane and is termed a phase shift or, transport lag, between the actuator and end effector. When the zero-dynamics are asymptotically stable the system is minimum phase with transfer function zeros occurring in the left half plane. Otherwise, it is non-minimum phase with transfer function zeros occurring in the right half s-plane and may become unstable [13], [14]. In this case, some form of internal dynamics control is needed to stabilize the system. One successful technique is input shaping which provides residual vibration damping in the forward dynamics [1]. Other control strategies include integral manifolds, input-output decoupling, observer-based decoupling, inverse dynamics sliding resulted in robust closed-loop performance and reduced end effector position tracking errors, but they lead to increased complexity of control system and to increased computational load on the system unsuitable for real-time applications [13], [14].

NMP response depends on the distance between position sensor and actuator. Noncollocated sensors are located somewhere on the robot links to measure position. Collocated sensors are mounted on joints. They measure rotation angle displacements and are more suited for fixed-base rigid-link robots operating at speeds where flexibility is not an issue. Flexible robots exhibit significant deformation at the end effector necessitating accurate control by adjusting for phase shift between actuator and end effector [20]. When a sensor is collocated with a joint actuator on a flexible robot such a robot forms a *hyperstable* system. Hyperstable systems require sensors and actuators in equal numbers, matching types of sensors and actuators and sensor/actuator collocation [7], [8] but that makes proper control of flexible robots more challenging. The flexible robot described by (5) and (6) for the dominant mode of vibration and with a noncollocated sensor is considered stable by the Routh-Hurwitz criterion [7].

V. CONTROL STRATEGIES

A. Nonadaptive control

Figure 2 presents a block diagram of the nonadaptive control strategy [1]. Torque feeds into the flexible dynamics equations and actuates each robot joint for acceleration output. Slew angles θ_1 and θ_2 together with flexural deformations δ_1 and δ_2 are fed back into the flexible dynamics equations and also transform into x and y end effector positions through the direct kinematics (12) and (13). Slew angles θ_1 and θ_2 and slew rates $\dot{\theta}_1$, $\dot{\theta}_2$ are fed back to form position and velocity errors; e_x, e_y and $\dot{e}_x, \dot{e}_y$.

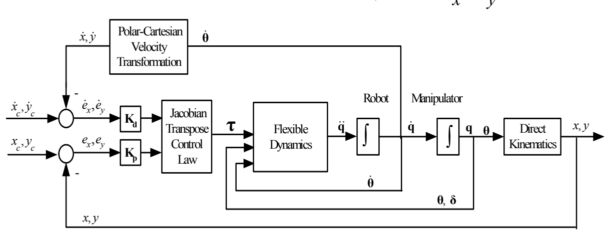


Figure 2. Nonadaptive control strategy [1].

Rigid-link robot kinematic equations relating end effector positions x, y to joint angles θ_1, θ_2 shown in Fig. 1 are given by

$$x = L_1\cos(\theta_1) + L_2\cos(\theta_1 + \theta_2) \quad (9)$$

$$y = L_1\sin(\theta_1) + L_2\sin(\theta_1 + \theta_2) \quad (10)$$

Position and velocity errors are multiplied by proportional and derivative (PD) gains, $\mathbf{K_p}$ and $\mathbf{K_d}$ yielding a Jacobian transpose control law

$$\tau = J^T(\theta)\left(K_p\begin{pmatrix} e_x \\ e_y \end{pmatrix} + K_d\begin{pmatrix} \dot{e}_x \\ \dot{e}_y \end{pmatrix}\right) \quad (11)$$

Proportional and derivative gains for the dominant cantilever assumed mode frequency ω_{c1} are given by

$$K_p = \text{diag}\left[\omega_{c1}^2 \quad \omega_{c1}^2\right] = \text{diag}\left[150.79, 150.79\right] \quad (12)$$

$$K_d = \text{diag}\left[2\zeta\omega_{c1} \quad 2\zeta\omega_{c1}\right] = \text{diag}\left[17.364, 17.364\right] \quad (13)$$

where ω_{c1} = 12.28 Hz and damping ratio ζ = 0.707.
Jacobian J can be derived from (9) and (10) the is given by (see [10])

$$J = \begin{bmatrix} \frac{\partial x}{\partial \theta_1} & \frac{\partial x}{\partial \theta_2} \\ \frac{\partial y}{\partial \theta_1} & \frac{\partial y}{\partial \theta_2} \end{bmatrix} = \begin{bmatrix} -L_1\sin\theta_1 - L_2\sin(\theta_1+\theta_2) & -L_2\sin(\theta_1+\theta_2) \\ L_1\sin\theta_1 + L_2\cos(\theta_1+\theta_2) & L_2\cos(\theta_1+\theta_2) \end{bmatrix}.$$

D. Fuzzy adaptive control

Figure 3 shows the Mamdani Fuzzy Logic System (FLS) controller with three membership functions (MF) shown in Figure 4 and containing 9 rules for input and output variables [1]. Universes of discourse range from -5m to 5m for δ_1 and δ_2 and from 0 to 1 for λ. Verbal descriptors for positive maximum (PMAX), positive very very high (PVVH), positive very high PVH), positive high (PH), positive medium (PM), positive low (PL), positive very low (PVL), positive very very low (PVVL), positive (P), zero (ZERO), negative very high (NVH), negative high (NH), negative medium (NM), negative low (NL), negative very low (NVL) and negative (N) are used to generate fuzzy rules typically of the form. 'IF δ_1 is NL AND δ_2 is PL THEN λ is PM'. The FLS uses antecedent composition (MIN), implication (MIN), aggregation (MAX) and defuzzification (CENTROID). As elastic link deformations δ_1 and δ_2 vary positively or negatively they complement or counter deformation of the other link to produce a resultant deformation within a range from ZERO for zero deformation to PMAX for the largest deformation thereby forming a symmetric fuzzy rule matrix shown in Table I. The value of λ is determined by the fuzzy

rules according to the values of δ_1 and δ_2 whose MF base widths are adjusted by K_s to adapt the Jacobian transpose control law.
The adapted form of control law τ_r in (11) is given by [3]

$$\tau_f = K_s \lambda \left(J^T(\theta) \left(K_p \begin{pmatrix} e_x \\ e_y \end{pmatrix} + K_d \begin{pmatrix} \dot{e}_x \\ \dot{e}_y \end{pmatrix} \right) \right)$$

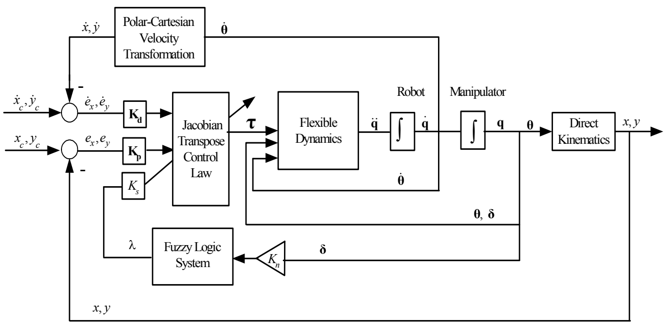
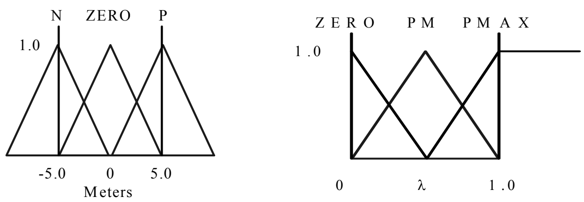


Figure 3. Fuzzy adaptive control strategy [1].



(a) (b)

Figure 4. Fuzzy membership functions: (a) δ_1 and δ_2, (b) Output variableλ [1].

TABLE I
FLS RULE MATRIX

		δ_2		
	λ	N	ZERO	P
δ_1	N	PMAX	PM	PMAX
	ZERO	PM	ZERO	PM
	P	PMAX	PM	PMAX

An operational space control strategy, shown in Figures 2 and 3, is used to demonstrate control with a noncollocated sensor as joint angles and rates are transformed through the direct kinematics equations into end effector positions and velocities control law computation. Time delays are implemented in the feedback loop to model the effect of NMP because there is no provision for traveling wave velocity and associated time delays in the robot dynamics equations and the system output variables are joint rotation angles and link deformations. Therefore, it is necessary to model sensors located at points on the links by including direct kinematics equations with link lengths, L_1 and L_2, as the distance between joint and sensor to feedback x and y coordinate measurements of the sensor. Direct kinematics equations for the end effector are included to track the end effector trajectory. Transport delay blocks (not shown) with a second-order Padé approximation are implemented in Matlab/Simulink™ models [20]. NMP response is corrected by time delays for command signals input to the control law. Time delays are calculated using the transverse beam vibration traveling wave velocity c given by

$$c = \sqrt{\frac{E}{\rho}} = \sqrt{\frac{1745833}{21}} = 288.33 \text{ m/s.}$$

VI. HAMMERSTEIN MODEL

Modeling of nonlinear dynamic system is a classical system theory problem. Two general approaches: Volterra series approach and Wiener approach [22] impose strong constraints on the identified systems such as smoothness limiting their applicability to continuous and smooth systems and thus eliminating important nonlinearities such as hard and soft limiters and quantizers. Both approaches incur heavy computational load on the identification system. In order to avoid these problems a lot of attention in recent years have been devoted to simpler models capable of representing large class of nonlinear dynamics. The most popular models are Wiener system made of a linear dynamic system followed by a memoryless nonlinearity [22] and Hammerstein system consisting of a memoryless nonlinearity followed by a linear dynamic system [23,24], see Fig. 5.

Identification of nonlinear systems has been investigated over several decades, see Haber and Unbehauen [25] for a comprehensive survey. Identification techniques maybe divided into parametric and nonparametric. Even though parametric identification approaches are most popular they suffer from serious limitations. One typically considers polynomial or trygonometric nonlinearities which are excluding important nonlinearities such as dead-zone limiters, quantizers and hard-limiters. Nonparametric identification techniques relax smoothness constraints. They do not require parametric form of the nonlinearity and we are able to identify all measurable L_2 functions, where from now onwards L_2 denotes the square integrable functions. Nonparametric identification algorithms are typically derived from nonparametric regression estimators which have been studied in statistics since over the last five decades [26,27]. The most common nonparametric regression estimation estimates include kernel estimate, k nearest neighbor, Fourier and Hermite series estimates and neural networks [27]. Nonparametric methods have been applied to identification of cascades of memoryless systems [28], Hammerstein systems [29] and Wiener systems [30]. A good survey of nonparametric identification methodology is provided in Greblicki and Pawlak [24].

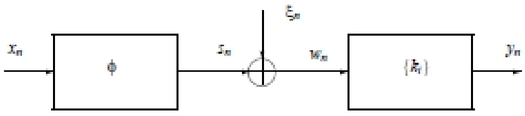


Figure 5. Hammerstein system.

In the present paper we identify Hammerstein systems by a recursive kernel approach [31]. We develop identification algorithm in two stages. In the first stage we develop identification algorithm for a static nonlinearity and establish its convergence. In the second stage we develop the identification algorithm for the dynamic Hammerstein systems driven by a stationary, white noise by estimating the linear dynamic subsystem and nonlinear memoryless nonlinearity from the input and output observations of the complete system. The parameters of the linear subsystem are identified by correlation approach and memoryless nonlinearity is identified by the algorithm based on the semi-recursive kernel regression estimate. The Hammerstein system identification algorithm is used to identify static nonlinearities and parameters of linear dynamical subsystem in two-link and two-joint flexible space robot manipulator.

VII. NONPARAMETRIC IDENTIFICATION OF STATIC NONLINEARITIES

Assume that memoryless system S shown in Fig. 6 is driven by a multivariate random input X and single output Y. Next we develop nonparametric identification procedure for memoryless nonlinearity S.

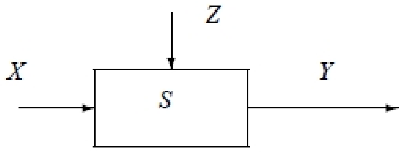


Figure 6. Memoryless system.

Let X have a density f and assume E|Y| < ∞. Let m(x)=E(Y|X=x) be a regression function and f, g ∈ L_2, where g(x) = m(x)f(x). Given sequence (X_1, Y_1),…,(X_n, Y_n) of independent identically distributed samples of an R^dxR-valued random vector (X,Y) we identify regression function m(x) = E(Y|X = x) by the semi-recursive kernel regression estimate

$$m_n(x) = \frac{\sum_{i=1}^{n} Y_i K_i((x - X_i)/h_n)}{\sum_{i=1}^{n} K_i((x - X_i)/h_n)} \qquad (14)$$

where K is an absolutely integrable kernel and h_n is the bandwidth. The estimate was introduced by Wolverton and Wagner [32] and was studied among others by Krzyżak and Pawlak [31] and 2_{22} et a_l. [27]. The estimate can be computed recursively as follows

$$m_0(x) = g_0(x) = 0$$

$$g_n(x) = g_{n-1}(x) + K_{h_n}(x - X_n)$$

$$m_n(x) = m_{n-1}(x) + g_n^{-1}(x)(Y_n - m_{n-1}(x))K_{h_n}(x - X_n)$$

The next theorem states conditions for the pointwise convergence of estimate (14) , see Krzyżak and Partyka [33].

Theorem 1

Let K be a symmetric and integrable kernel satisfying

$$\alpha H(\|x\|) \le K(x) \le \beta K(x)$$

or let K satisfy

$$\alpha I_{S_{0,R}} \le K \le \beta I_{S_{0,R}}$$

for some 0<α<β<∞ and positive R. Assume further that

$$h_n \to 0, \ \sum_{n=1}^{\infty} h_n = \infty. \qquad (15)$$

Then

$$\lim_{n\to\infty} E\int (m_n(x) - m(x))^2 dx = 0$$

for all distributions of (X,Y).

If, in addition, m satisfies Lipschitz condition of order p, 0<p≤1 then

$$|m_n(x) - m(x)| = O\left(n^{-p/(2p+1)}\right) \textit{in probability}.$$

VIII. NONPARAMETRIC IDENTIFICATION OF HAMMERSTEIN SYSTEM

In this section we develop identification procedure for a Hammerstein system shown in Fig. 5. The output of nonlinear memoryless subsystem (I) is given by:

$$W_n = \phi(X_n) + \xi_n, n = 0, \pm 1,$$

where X_n is R-valued stationary white noise and ξ_n is a stationary white noise with zero mean and finite variance. No correlation is assumed between ξ_n and X_n and assume that ϕ is a scalar function. The linear subsystem dynamic (II) is described by the autoregressive moving average (ARMA) model:

$$Y_n + a_1 Y_{n-1} + \ldots + a_l Y_{n-l} = b_0 W_n + b_1 W_{n-1} + \ldots + b_l W_{n-l}$$

with an unknown order l. Coefficients a_1,…,a_l are chosen to guarantee the asymptotic stability of the linear subsystem. Consequently Y_n is weakly stationary provided that W_n is weakly stationary process. We can also represent the Hammerstein system by the state equations

$$\hat{X}_{n+1} = A\hat{X}_n + bW_n$$

$$Y_n = c^T \hat{X}_n + d_1 W_n$$

where $\hat{X}_n$ is l-dimensional state vector and A is asymptotically stable matrix, thus state vector and output Y_n are weakly stationary provided that W_n is weakly stationary. That happens when the dynamic subsystem is asymptotically stable and $EW_n^p < \infty$, which in turn is implied by the condition $E\,|\,\phi(X)\,|^p < \infty.$

We identify both static nonlinearity ϕ and parameters of linear subsystem by observing random inputs and outputs of the whole system, i. e., the sequence (X_i, Y_i), i=0, 1,…, n-1 of n correlated observations of the input and output. The linear subsystem identification procedure has been described in [34] and is omitted.

Next we introduce identification algorithm for the static nonlinearity in the Hammerstein system. Note that the state equations implies

$$E\{Y_n \mid X_n\} = d_1\phi(X_n) + \alpha = m(X_n) \qquad (16)$$

where $\alpha = E\phi(X)c^T (I - A)^{-1} b.$

In order to identify ϕ we first estimate the regression function m given in (16) by the kernel semi-recursive regression estimate and then we obtain the estimate of ϕ. Undetermined coefficients d_1 and α in (16) are the result of inability to measure signal W_n. They could be identified if ϕ were known at two points x_1 and x_2, such that $\phi(x_1) \neq \phi(x_2)$ and such that $m_n(x)$ converges. Thus we estimate d_1 and α as follows

$$d_{1,n} = \frac{m_n(x_1) - m_n(x_2)}{\phi(x_1) - \phi(x_2)},$$

$$\alpha_n = m_n(x_1) - \frac{\phi(x_1)}{\phi(x_1) - \phi(x_2)}(m_n(x_1) - m_n(x_2))$$

Hence we can identify ϕ by

$$\phi_n(x) = (m_n(x) - \alpha_n) / d_{n,1},$$

where regression function m is estimated by the semi-recursive kernel estimate (14). Similar identification problem with in which regression function was estimated by the classical kernel regression estimate was considered in [34]. The next theorem provides convergence conditions for algorithm (14).

Theorem 2

Let $EY^2 < \infty$, A be asymptotically stable and f, g ∈ L_2. If kernel K satisfies the conditions of Theorem 1 and smoothing sequence satisfies (15) then

$$\lim_{n\to\infty} E\int (\phi_n(x) - \phi(x))^2\, dx = 0$$

for all distributions of (X,Y).

If, in addition, m satisfies Lipschitz condition of order p, 0<p≤1 then

$$|\,\phi_n(x) - \phi(x)\,| = O\left(n^{-p/(2p+1)}\right) \textit{in probability.}$$

for almost all x.

IX. NONPARAMETRIC IDENTIFICATION OF ROBOT MANIPULATOR

In practice in real flexible link manipulator the torque is applied to the joints using geared transmission. The friction in these joints is a nonlinear function of position and can be modeled by a static nonlinearity. The whole link can be modeled by the Hammerstein system in which static part represents nonlinear flexibility and friction at the joints and dynamic part represents flexibility of the structure. We can thus apply the identification algorithm developed in this paper for the Hammerstein system to identify static nonlinearities from input-output excitations. The results of experiments with real flexible link manipulators will be reported in the future.

X. CONCLUSIONS

Nonadaptive control has no effect on suppressing the transient flexural deformation of the robot links. Tracking results are greatly improved with flexural vibrations suppressed and non-minimum phase instability effects negated by an FLS adaptive control strategy with either a collocated joint rotation sensor or a noncollocated position sensor at the end effector. FLS control is unaffected when the position sensor is moved to point 0.5m from the elbow joint. Additional improvements in control of flexible robots can be obtained by identification of nonlinearities in the joints by nonparametric techniques proposed in the present paper.

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