Early Math Assessment@School
Interpretation Guide
Kindergarten to Grade 6

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More Information

For more information on the AIM Centre at Carleton and the EMA@School please see www.carleton.ca/arc or email AIMresearch@carleton.ca. The EMA@School is available to school boards/districts for a modest per-student fee. Support and training can also be provided. Research possibilities are available for interested school partners.

TABLE OF CONTENTS

Executive Summary

The Early Math Assessment@School (**EMA@School**) is a paper-based suite of tasks that measures children's developing mathematical knowledge in kindergarten to grade 6. The assessment was designed and continues to be supported by researchers in the AIM Centre at Carleton University (www.carleton.ca/arc/).

The **EMA@School** is intended to help teachers identify and address gaps in their students' foundational numeracy skills. The EMA@School includes one-on-one assessments, designed to be used with kindergarten and grade 1 students, and group assessments for students in grades 1 to 6. These assessments are based on over 30 years of research in mathematical learning and are rooted in current theory on mathematical cognition and cognitive development. The assessment is classroom based, providing teachers with a relatively quick snapshot of their students' foundational math skills. It typically takes 15-20 minutes to administer to a whole class. The EMA@School can also be used as the pre- to post-test measure for evaluating interventions focused on foundational numeracy skills (AIM Collective, 2024b, 2024c, 2024a).

The **Digital EMA@School (DEMA)** has the same content as the paper-and-pencil version. It has been adapted for digital delivery in collaboration with a Canadian educational technology company, Vretta. In this document, we focus on the paper version; the development of the digital version is ongoing, with the goal of having interchangeability across the two versions. The content is as similar as possible, however, so the theoretical background and general description of the measures is accurate for both versions. More information about the DEMA is available at https://digital.ema-vretta.com.

Why choose the EMA@School or the DEMA as a numeracy screener?

- These screeners are based on research evidence which shows the importance of early skill development.
- Because these measures were developed in Canada, they are sensitive to the demands of Canadian curricula.
- The tests are available in both French and English.
- Students' performance is sensitive to change over time, such that considerable growth can be seen from fall, to winter, to spring administrations. This feature means that the EMA@School can be used to evaluate the success of interventions designed to improve students' numeracy skills.
- The language requirements of the tasks are minimal. This feature means these measures are less biased than those with extensive language demands, because the involvement of language typically disadvantages children from lower-income or other equity-seeking groups (Jordan et al., 1992).
- The instructions can be easily modified to be suitable for languages other than English or French, as required. Tasks have easy-to-follow examples and practice items. Scoring supports are provided for teachers.
- Through Vretta, a school data entry dashboard is available that provides useful reports and feedback for teachers and administrators.

Overview and Scope

Foundational early mathematical skills include knowledge of symbols, relations, and operations (B. L. Devlin et al., 2022; Jordan et al., 2010). Early symbolic knowledge includes numbers (both words and digits), mathematical vocabulary (such as behind, or half), and math-specific symbols such as + and =. Relations includes relative magnitude (e.g., 4 < 5; 6 > 2), and ordinal relations (1 2 3 …). Operations include arithmetic and comprise both conceptual knowledge (e.g., 4 + 2 = 2 + 4; commutativity principle) and procedural skills (e.g., 4 + 2 = 6). The EMA@School includes tasks in each of these subdomains (see Table 1). Although skills in each subdomain are related, each predicts mathematics separately and thus can provide independent insights into children's numeracy development. Note, tasks are classified according to the major knowledge category they represent, but several tap into more than one knowledge category. The EMA@School currently does not address other areas of skill that are adjacent to mathematics (e.g., spatial skills, geometry), on the assumption that number skills are foundational to mathematical learning (Hawes et al., 2021).

Table 1. Assessments Included in the EMA@School.

		Grade						
Name	**Description**	**K**	**1**	**2**	**3**	**4**	**5**	**6**
Symbols								
Dot Counting[1]	Count sets of dots (up to 12) and name the sets, that is, "how many dots?".	√						
Verbal Counting [1]	Recite the count sequence up to 100.	√	√					
Number Naming[1]	Name printed Arabic numbers (e.g., 6, 15, 27).	√	√					
Next Numbers[1]	Continue number sequences orally.		√					
Number Writing[2]	Write down numbers as teachers read them aloud (e.g., 12, 67, 150 etc.).		√	√	√	√	√	√
Symbol Knowledge[2]	Choose the representation the goes with a verbal description (e.g., less than [<]); focus on fractions, decimals, and pre-algebra symbols).					√	√	√
Relations								
Number Lines[2]	Mark the location of a number on a line.	√	√	√	√	√	√	√
Number Comparison[3]	Cross out the larger digit of a pair (e.g., 4 7).	√	√	√	√			
Number Order[3]	Check (√) if a sequence of three single-digit numbers is in increasing order.			√	√			
Rational Numbers[2,5]	Decimals and fractions: mapping symbols to quantities, comparing, and ordering.					√	√	√
Operations								
Addition Fluency[3]	Solve single-digit whole number addition facts		√	√	√			
Subtraction Fluency[3]	Solve simple whole number subtraction		√	√	√			
Combined Addition/Subtraction[3]	Solve single-digit whole number addition and subtraction facts.					√		
Equations[3]	Use conceptual knowledge to solve equations (e.g., 3 + 4 = □ + 3).				√	√	√	√
Combined Multiplication/Division[3]	Solve single-digit whole number multiplication and division facts.					√	√	√
Calculation[3]	Add double-digit numbers.					√	√	√
Total Subtests x grade		5	8	6	7	8	7	7

[1] Tests are administered one-on-one. [2] Item sets differ based on grade. [3] Tests are timed to assess fluency. [5]Task captures both **Symbol** and **Relations** knowledge for rational numbers.

Area of Application

As shown in Table 1, for students in kindergarten, the EMA@School assesses counting skills, early number word knowledge, and basic number symbol relations. For older students, the assessments focus on knowledge of large numbers and mathematical symbols (i.e., number writing, symbol knowledge), understanding of the conceptual and procedural inter-relations amongst symbols (i.e., comparison, ordering, and number line), and conceptual and procedural knowledge of operations (i.e., arithmetic, equations, calculation). On all tasks, ‘number’ refers to symbolic number, either spoken (“I counted four dots”), or visually presented (student is shown a number such as 12 or 237 and asked to name it).

The tasks used in the EMA@School are well known in the research literature, and many appear in some form on other assessments. Importantly, to ensure consistency among teachers and other test administrators, instructions are explicit, and marking is fast and easy. The EMA@School was designed to be a brief assessment that provides overall performance rankings, however, there is also additional information from each measure that teachers can use to better understand individual students’ strengths and limitations. Reports can be produced that show performance across areas for individual students, for classrooms, or for school districts.

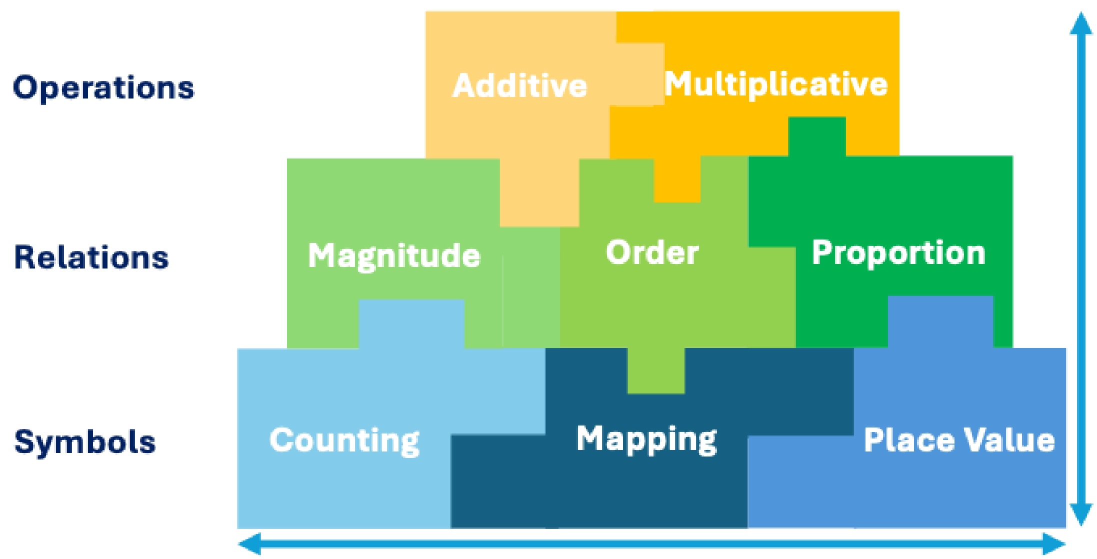
To monitor student development throughout the year and across grades, the EMA@School includes tasks that would be suitable for multiple grades and allow for growth across the year. To ensure that the same tasks are appropriate for multiple grades, time limits and items may vary. Different tasks are used in some grades to align with students’ developing symbol knowledge: Counting in kindergarten; different items sets for Number Writing across grades; fraction knowledge in grade 4; symbol knowledge beyond numbers in grades 4 to 6.

This document should be used along with the **Teacher Guide**, which provides more information about how to support students’ mathematical learning, and the **Activities** Guide, which describes specific interventions, activities, games, and on-line resources. These documents can be provided along with the licensing agreement for the EMA@School.

The Theory and Research Behind the EMA@School

As described above, the EMA@School was designed to capture foundational numeracy skills that support students' acquisition of mathematics from Kindergarten to Grade 6. Accordingly, we assume that there are hierarchical links among the skills. For example, children who do not know the number words will not be able to count correctly. Similarly, students who do not understand how fraction magnitudes reflect the relation between the numerator and denominator will do poorly on tasks that require that understanding, such as placing a fraction on a number line. One way of conceptualizing these hierarchical relations among the foundational skills is shown in Figure 1. Each more advanced skill builds upon earlier ones. These interrelations are particularly obvious as children are learning to count. As illustrated in Figure 1, counting is a cornerstone of further numeracy learning (Aunio & Räsänen, 2016). It requires symbol knowledge (i.e., number words and their order), and both conceptual and procedural skills. It is often assumed to be an informal numeracy skill in that children seem to acquire it without direct instruction, whereas other skills (such as arithmetic) typically involve schooling. However, that assumption is misleading, because counting is complex and requires a long time to master. It is a skill that people continue to use throughout their daily lives but, as adults, we take for granted. Figure 1 shows the skills that students initially learn in relation to whole numbers; notably, the same categories of knowledge apply to rational numbers (i.e., fractions and decimals), which are learned later.

Figure 1. Hierarchical and Integrative Model of Foundational Numeracy Skills



Symbols

Early Symbol Knowledge: Counting

Counting is one of the earliest number skills that children acquire – it involves symbolic knowledge (i.e., the number words), relational knowledge (number order) and operational knowledge (e.g., using counting to add or subtract). Thus, fluent early counting skills provide a framework for further numeracy learning (Aunio & Räsänen, 2016; D. Devlin et al., 2022). Some students arrive in kindergarten with extensive verbal counting skills whereas others are unable to count successfully to 5 (Litkowski et al., 2020). Because these skills can be acquired at home or other preschool experiences, they are often referred to as "informal numeracy" (Purpura et al., 2013; Susperreguy et al., 2020) whereas those skills taught in kindergarten and beyond are referred to as “formal numeracy”. However, both types of skills are foundational and once children enter school, the distinction fades in importance.

Learning to count involves more than just reciting the count sequence. Children need to acquire conceptual knowledge, most importantly, that the purpose of counting is to determine quantity (Gray & Reeve, 2014, 2016). Researchers have identified five counting principles, three of which are necessary to count successfully, and two which are not essential but show that children have a thorough understand how counting works.

To count objects in a meaningful way, children need to know and apply the three *essential* counting principles: a) *stable order* of the count words, that is knowing and using the number words in order, b) *one-to-one correspondence* between the items counted and the count words, that is counting each item once and only once, and c) *cardinality* – the last number word stated in a count indicates quantity, that is, how many items are in the set (Gallistel & Gelman, 1990; Gelman & Gallistel, 1978).

The two non-essential principles are abstraction and order irrelevance. *Abstraction* is recognizing that a group of objects can be counted even if they are not the same (Siegler, 2003). Abstraction allows children to realize that a set of objects (e.g., with three dogs and four cats) is also a set of seven animals. The *order irrelevance* principle is that the order in which items are counted doesn’t matter – the final count will be the same. Order irrelevance may not be acquired by children until mid-elementary school (Kamawar et al., 2010; LeFevre et al., 2006), long after they are successful counters according to the other rules. Adults typically are confident users of all the counting principles but occasionally will insist on counting in a specific order, typically from left to right (Kamawar et al., 2010).

Tips for Teachers – ***Should young children use their fingers to count?***

The answer to this question is an emphatic yes (Frey et al., 2024). *Children who come to grade 1 with better skills typically use their fingers to count. In an intervention study, Frey et al. showed that children with weak numeracy skills in grade 1 who were taught to use finger patterns to represent numbers made more progress than similar children who were not so taught. Of course, finger counting is a temporary support for arithmetic and eventually most students will use it less, or not at all. However, even as adults we use our fingers to keep track of counts, freeing up working memory to name and organize the things-to-be-counted. Fingers are the ideal tool because they are always present, reduce working memory load, and provide a concrete representation of quantity.*

Children learn the counting principles gradually from age 2 to 6 (LeFevre et al., 2006; Purpura et al., 2013). EMA@School kindergarten assessments **Verbal Counting** and **Dot Counting** are useful for ensuring that students have the required counting knowledge and skills before the transition to grade 1. The **Next Number** task, suitable for advanced kindergarten and grade 1 students, requires that students complete a counting sequence. These sequences get progressively more complex, tapping into children’s knowledge of the rules for generating higher numbers (e.g., 12 13 14 15 ___ ___; which numbers come next?), knowledge of other sequences (e.g., 2 4 6 8 __ __), and fluency of access, allowing them to count backwards (e.g., 10 9 8 7 __ __).

Tips for Teachers – ***Counting in Grade 1***

By the end of kindergarten, students should have mastered the essential counting principles, and hence, understand the purpose of counting; they are expected to verbally count to 10 or 20 (or higher, depending on the jurisdiction); and to name number symbols to 10 (or higher)[1]*. The EMA@School assessments can be used to identify which students have not mastered these minimal standards. These students may need intervention to help them acquire foundational skills. For example, students might need pointing and counting practice (for one-to-one correspondence), digit matching games (for*

[1] Specific benchmarks depend on the school district, provincial, state, or country requirements.

mapping and cardinality practice), counting songs, and other applicable activities. Because these skills are foundational for later learning, teachers can use these number knowledge assessments (Verbal Counting, Next Number, and Number Naming) to identify grade 1 students who are unprepared to learn grade 1 material.

Numbers

Starting in Grade 1, students are taught about larger numbers (i.e., 10 to 100 to 1000), although research has shown they also gain knowledge about the structure of large numbers through exposure in everyday life and educational settings (Mix et al., 2023). Students will acquire new rules for mapping number words to number symbols as they are exposed to larger numbers. These rules reflect the structure of the number system but there can be conflict between the verbal labels and the written symbols, especially in some languages. Consider the French number words for 98: quatre-vingt-dix-huit (which translates to four-twenty-eighteen) compared to “ninety-eight” in English or the equivalent in other languages. Language complexity leads to interesting patterns of behaviour -- a familiar number like eighteen is often written as 81 when students are learning. In language such as Dutch or German, where decade numbers such as 42 are named as “four-and-twenty”, such errors are common into grade 2 (Imbo et al., 2014).

Learning is about creating connections. Math learning is cumulative and can be viewed, at least in part, as a hierarchy of connections among numerical and other mathematical symbols (Hiebert, 1988; Núñez, 2017; Xu et al., 2023). Connections between symbols and their concrete representations anchor this hierarchy, for example, knowing that the number 4 represents a quantity of ■■■■. Along with successfully connecting the digit 4 with the quantity ■■■■ however, children need to connect the verbal number word “four” with its corresponding number symbol (Hurst et al., 2017; Jiménez Lira et al., 2017). *Mapping* is the term used to describe connecting quantities, digits, and verbal number words. In kindergarten, students complete the **Number Naming** task where they are asked to use verbal number words to name digits. In later grades, **Number Writing** reflects the increasing complexity of number symbols, assessing students’ knowledge of place value (in the base 10 system), and the imperfect connection between number words and digits. For example, students learning math in French will find mastering the count words from 60 to 99 more difficult than will students learning in English, because of the complex decade structure. Variations in the mapping between number words and digit forms can sometimes influence mathematical learning (LeFevre et al., 2002, 2018).

Beyond the verbal labels and the written symbols, children need to understand that the position of a digit indicates its value. Thus, 21 is two tens and one unit, whereas 12 is one ten and two units. Each year, the range of numbers that students learn increases. By grade 2, for example, children in many provinces are expected to know the number system in the hundreds. A student who hears “two-hundred five” and writes “2005” has not yet mastered the rules for transcoding spoken numbers into written digits (Skwarchuk & Anglin, 2002). This common error (called a syntactic error) indicates the student does not understand how the relative positions of written numbers reflect place value, possibly because they have not learned explicit rules for representing larger numbers (Clayton et al., 2020; Simmons et al., 2012; Sowinski et al., 2015). Children with low math performance demonstrate poor understanding of place value rules in the number range they should have learned (Moura et al., 2013). The EMA@School **Number Writing** assessment helps teachers identify students’ transcoding errors (grades 1 to 6) and determine if they have mastered the place value rules to the level indicated in the curriculum.

*Tips for Teachers - **Number Writing***

Students in grades 1 and 2 typically are expected to know numbers to 100; students in grades 3 and 4 are expected to know numbers to 1000 and 10 000 respectively. [Note that these benchmarks depend on the curriculum guidelines of the specific jurisdiction.] Many children exceed these benchmarks, but others do not (Litkowski et al., 2020). Teachers can use the Number Writing task to identify the kinds of mistakes their students make and identify gaps in children's knowledge of larger numbers (e.g., place value issues; writing two hundred five as 2005 or reversing number order; 28 versus 82). In response to these difficulties, teachers can customize number writing supports as needed, using number-matching games, place-value charts, and other activities that highlight the patterns and rules that determine number structure.

Advanced Symbol Knowledge

Mathematics is rife with symbols – numbers are only the beginning. Even in grade 1, students are expected to learn symbols such as '=' and '+'. Symbol knowledge has three facets; the visual, the verbal, and the meaning. For example, '+' is visual, 'plus sign' is verbal, and the meaning is 'add". Research shows that symbol knowledge is a critical aspect of numeracy learning. Every year, students are expected to learn many new symbols, such that by grade 6 students are expected to know, understand, and recognize the symbols for hundreds of mathematical terms (Powell et al., 2021). The Symbol Knowledge task helps teachers determine whether students are acquiring key mathematical knowledge about symbols. In this task, students hear a term, such as 'equal sign' and then choose between four possible foils – only one of which is correct. The selection of incorrect symbol foils is based on common student errors such as confusing similar symbols (e.g., < and >), confusing symbols with multiple meanings (e.g.,– ; subtract, negative integer), confusing different symbols with the same meaning (e.g., multiply; *, ×) and recognizing that symbols can mean different things depending on their position (e.g. 3 in x^3, $3x$) (Rubenstein & Thompson, 2001). Connecting the visual, verbal, and meaning of mathematical terms involves language skill and so students whose first language is not the language of instruction may need extra support to learn the correct terms for symbols.

Tips for Teachers – Use Precise Mathematical Language

Powell and her colleagues (Driver & Powell, 2015; Hughes et al., 2016; Powell et al., 2019, 2021; Powell & Fluhler, 2018) have done a great deal of research on the importance of mathematical language, which is an important facet of symbol knowledge. In two publications (Hughes et al., 2016; Powell et al., 2019) they provide useful information about the ways in which math language can be made as precise as possible. For example, they recommend not referring to 'the numbers' in a fraction such as ½; instead, say "this fraction is a number" and connect it to the quantity it represents (e.g., on a number line). Fractions represent magnitudes so a fraction is not two separate numbers. Similarly, rather than saying 2 over 3 for the fraction 2/3, they suggest referring to the numerator and the denominator. These researchers stress the importance of teachers using correct, precise mathematical language to refer to mathematical symbols for students with learning difficulties, but this knowledge is important for all students.

Relations

Numbers can be related to each other in many ways (Merkley & Ansari, 2017). Consider the numerals 1, 2, and 3. These numerals have cardinal (quantity/magnitude) relations (e.g., 3 > 1; 2 < 3), ordinal relations (e.g., 2 comes after 1 and before 3), and arithmetic relations (e.g., 1 + 2 = 3; 3 – 1 = 2). As children acquire various associations among symbolic numbers, including cardinal, ordinal, and arithmetical connections, these associations form an increasingly interconnected mental number network (Hiebert, 1988; Siegler & Lortie-Forgues, 2014; Xu et al., 2019, 2025; Xu & LeFevre, 2021a).

Quick and accurate access to quantities from symbolic numbers supports the development of other number skills like ordinal knowledge, arithmetic, and fractions. For kindergarten to grade 3, relation tasks use whole numbers; from grade 4 to 6, the tasks shift to using rational numbers (fractions and decimals).

Number Comparison

Number Comparison is a timed task where students quickly and accurately cross off the numerically larger digit in a pair. Number Comparison is used from kindergarten to grade 3, with appropriate modifications in timing. Number Comparison combines knowledge of how symbols are connected to quantities with the ability to compare those quantities mentally. Ansari and colleagues have shown that number comparison is a foundational mathematical skill (Hawes et al., 2019; Nosworthy et al., 2013). Moreover, number comparison can be used to identify children with persistent developmental dyscalculia (Bugden et al., 2021).

Tips for Teachers - ***Number Comparison***

Students in kindergarten and grade 1 with the weakest number comparison skills lack automatic/direct associations between the digit symbols and quantities. For these students, teachers can provide activities that support linking digits and quantities such as card games like War, board games like Sorry, dice games – or other activities where students practice linking visual symbols with quantities (Gasteiger & Moeller, 2021a).

Number Order

Number Order is a timed task where students quickly and accurately judge if three digits are in increasing order (e.g., 2 3 5 vs. 3 1 2). Knowledge of the sequential relations amongst numbers is distinct from knowledge about quantity (Lyons & Ansari, 2015; Lyons & Beilock, 2013). As children's number relations become more sophisticated and integrated, they can use ordinal skills (e.g., what number comes after 4?) on increasingly complex ordering tasks (Lyons et al., 2014; Xu & LeFevre, 2021b). **Number Order** is included in the EMA@School for students in grades 2 and 3. Some students in grade 1 can do the Number Order task, however, like children in kindergarten, many find it difficult to understand that non-adjacent numbers such as 2 4 6 are nevertheless 'in order' (Hutchison et al., 2022; Xu & LeFevre, 2021b).

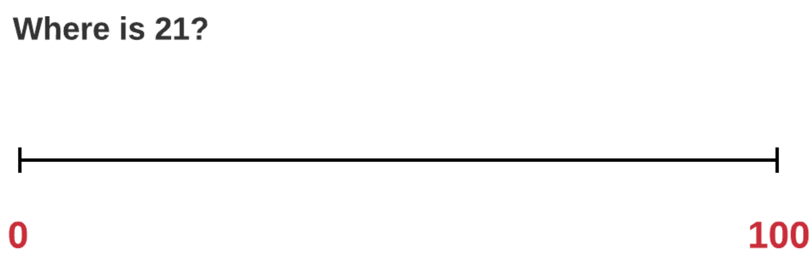
Tips for Teachers - ***Number Order***

Teachers can help students develop and use number ordering relations in various contexts such as games involving sequences -- ordering numbers, what comes before, what comes next, and so on. Flexibility in using both order and magnitude knowledge to determine if sequences are ordered develops further beyond grade 3 (Lyons et al., 2014). Notably however, practicing arithmetic strategies (e.g., solving 3 + 4 by counting on from 3; 5, 6 7) also strengthens order knowledge.

Number Line

Estimating the position of a number on a **Number Line** captures two aspects of a student's mathematical knowledge. First, it measures their understanding of the ordinal relations among numbers (i.e., 3 comes after 2 and before 4; 50 comes after 40 and before 60). Second, number line estimation requires proportional reasoning skills. Proportional reasoning is used to place the number accurately on the line (e.g., 49 is close to 50 and 50 is one-half of 100 so an appropriate strategy is to divide the line in half). An example of a number line question is shown in Figure 2.

Figure 2. *The Number Line Task*



In the EMA@School, the **Number Line** task is used to assess students' developmentally appropriate understanding of ordinal and proportional relations amongst numbers. For kindergarten students, the number range is 0-10, for students in grade 1 the range is 0-100, for students in grade 2 the range number lines include 0-100 and 0-1000 number lines and for students in grade 3, the number range is 0-1000. For students in kindergarten to grade 3, estimation errors greater than 20% (i.e., placing 7 either below 5 or above 9 on a 0-10 number line; placing all numbers above 100 near to 1000) indicates they have a weak grasp of the tested number range (Xu, 2019).

In grades 4, 5, and 6, students see both 0-10,000 whole number lines and 0-1 and 0-2 fraction number lines. These students should show more precise placement than younger students, and so estimation errors greater than 15% index poor understanding. At these later grades, the number lines include fractions. In general, number line performance is a sensitive index of developing mathematical knowledge (Booth & Siegler, 2006), in part because the task is novel to many students, requiring them to invent a strategy. Proportional reasoning is a superior strategy that is used by more-skilled students whereas less-skilled students will attempt to use counting from either the lower or upper anchor (e.g., 0 or 100 on a 0-100 number line), resulting in variable performance across the number line range (Xu, Di Lonardo Burr, et al., 2023).

Tips for Teachers - ***Number Line***

Teachers can support students' understanding of how number magnitudes and number line locations are connected with activities that promote understanding of ordinal relations such linear board games like Snakes and Ladders (Siegler & Ramani, 2009) *for younger students, often call "number path games" (see https://www.naeyc.org/resources/pubs/tyc/apr2019/number-path-games). In general, number lines are useful teaching tools and provide a concrete representation for learning about fractions and decimals for older students (Jordan et al., 2017a; Rodrigues et al., 2024).*

Operations

Procedural fluency and conceptual knowledge of arithmetic principles are mutually reinforcing (Rittle-Johnson & Alibali, 1999). Both types of knowledge are central to the development of strong mathematical understanding (Crooks & Alibali, 2014; Fyfe et al., 2012). Thus, children's acquisition of the arithmetic operations are foundational predictors of later fraction and algebra skills (Schneider et al., 2017). To assess these skills in grade 1 and beyond, the EMA@School includes **Number Facts.** In later grades, students complete the **Calculation Addition task.** These tasks are measures of fluency. **Equations**, which is used in grades 3 and up, is a measure of conceptual knowledge of arithmetic.

Arithmetic fluency reflects how quickly and efficiently students can retrieve number facts or apply efficient strategies (McNeil et al., 2025). Practicing arithmetic strategies and increasing fluency strengthens all aspects of early numeracy and supports stronger relational skills. Importantly, arithmetic fluency is a core ability and is closely related to many other mathematical skills (Price et al., 2013), including problem solving (Lin, 2020). Students who know their number facts have more working memory available to focus on the problem they are solving compared to students who need to laboriously calculate on simple problems. Importantly, fast and accurate fact retrieval indicates mastery of foundational arithmetic skills and mastery is a more sensitive metric than accuracy when it comes to monitoring growth in students' skill (VanDerHeyden & Peltier, 2023; Vanderheyden & Solomon, 2023).

Number Facts assessments include single digit addition and subtraction (grades 1 to 3), multiplication (grade 3) and combined addition and multiplication and/or multiplication and division (grade 4 and higher).

Tips for Teachers - ***Number Facts***

The Number Facts assessments identify students with weak arithmetic fluency (addition, subtraction, multiplication, division). Performance on ***Number Facts*** *can be used by teachers to gauge the current arithmetic skills of their group of students and plan lessons accordingly. Teachers can support fact fluency with card or board games (e.g., cribbage), classroom practice, and other applicable activities (Gasteiger & Moeller, 2021b). Access to online, gamified practice supports students' fluency, but there are many other ways to encourage mastery of number facts including practicing timed retrieval (Codding et al., 2011; Riccomini et al., 2017). Building fluency early can also reduce or prevent math anxiety because students realize that they can be successful (Codding et al., 2023).*

Calculation Addition includes multi-digit addition and is a measure of students' computation skills and their understanding of place value. To solve these problems, students need to manipulate the numbers. They can use standard algorithms or other strategies such as breaking the number down (e.g., 83 + 27 = [7 + 3] + [80 + 20] = 110). As students' number skills develop, their solution strategies become more efficient (Hickendorff et al., 2019). To quickly and efficiently solve calculation problems that involve regrouping, students need to process place-value information. Students who experience math difficulties take more time and make more errors when solving multi-digit arithmetic problems than their peers (Lambert & Moeller, 2019).

Equations

Equations assess students' knowledge of five arithmetic principles. These include the basic principles of *identity*, that is, adding or subtracting zero, or multiplying or dividing by 1 does not change the number and *negation*, that is, subtracting a number from itself equals 0 or dividing it by itself equal 1 (Robinson, 2017). The principle of *inversion* arises from an understanding of identity and negation, such that students can easily solve problems like 3 + 54 – 54 and 21 x 5 ÷ 5 without calculating (Crooks & Alibali, 2014; Robinson & LeFevre, 2011).

Two other principles, *commutativity* and *equivalence*, allow students to flexibly evaluate equations presented in a variety of formats. *Commutativity* is the principle that the order of operands in addition doesn't matter, so that 3 + 4 will have the same answer as 4 + 3. Students implicitly understand this principle when they know they can solve both 2 + 5 and 5 + 2 by counting on from 5 (Siegler & Braithwaite, 2017). Multiplication is also commutative, and that understanding can be used to simplify learning the answers to multiplication facts. There are half as many facts to learn when students are taught the "small x large" facts and the commutativity rule (Campbell & Xue, 2001; LeFevre & Liu, 1997).

Equivalence is the rule that the information on both sides of an equation must be equal. Equivalence is signalled by the equal sign and it is fundamental for flexible thinking about numbers (Kirkland et al., 2024) and for learning algebra (Crooks & Alibali, 2014; Star & Rittle-Johnson, 2008). Knowledge of equivalence starts out very simply, for example, knowing that 4 = 4 is a valid equation. At first, students may think that the equal sign in a problem like 3 + 2 = __ means something like “add these two numbers and put the answer in the blank space.” That operational notion must be replaced by a relational understanding -- the equal sign really means “the quantity on the left side and the quantity on the right side should be the same”. This relational understanding is critical but does not fully develop until grade 6 or later in many countries (Simsek et al., 2021). Students who acquire relational understanding earlier and therefore develop flexible problem-solving skills will find learning about fractions, decimals, and algebra easier (Crooks & Alibali, 2014; McNeil et al., 2006; Prather & Alibali, 2009).

*Tips for Teachers - **Equations***

The Equations task provides information about which principles are familiar to students. Helping students to learn these principles and become flexible problem solvers will prepare them for learning about fractions and algebra in later grades (Robinson & Dubé, 2012). Providing frequent opportunities for students to see equations in many different forms and fostering a relational interpretation of the equal sign will support students' progress (Star & Rittle-Johnson, 2008). Even older students may need to be reminded about the meaning of the equal sign when they encounter it in new contexts.

Rational Numbers: Going Beyond Whole Numbers

Whole number knowledge forms the foundation for building strong rational number skills (i.e., fractions, decimals, and percentages) and prepares students for later mathematical success (Malone et al., 2019). Fraction skills predict students' later understanding of algebra, and their overall math achievement in high school (Booth & Newton, 2012; Jordan et al., 2017b; Siegler et al., 2013). The **Rational Number** task measures students' understanding of how fraction symbols connect to fraction magnitudes, that is, how fraction symbols are related to the quantities they represent.

The magnitude of a fraction is based on the relation between the numerator and denominator (i.e., $3/8 < 1/2$ even though 3 > 1 and 8 > 2). Thus, even though each fraction includes two or more numerals, the fraction symbol represents a number (i.e., a single magnitude). Whole number knowledge can make it difficult for students to grasp how fraction symbols represent magnitude. Those students who are less skilled will show evidence of this 'whole number bias' (Braithwaite & Siegler, 2023; Siegler & Braithwaite, 2017).

Fractions are also complex in that they are used to represent multiple different quantities. For example, $3/8$ can represent a discrete quantity (3 out of 8 students), part of a whole (3 slices of an 8-slice pizza) or a continuous quantity ($3/8$ of a litre).

Finally, unlike whole numbers, fractions with different integers can describe the same magnitude (e.g., $3/8 = 6/16$). Students' ability to inhibit their knowledge of whole number concepts may predict their fraction understanding (Di Lonardo Burr et al., 2022b; Xu, LeFevre, et al., 2023). In the Rational Numbers task, students match quantities to fractions or decimals and compare magnitudes and orders and a single score is produced. However, teachers can explore performance across the individual items, allowing them to see exactly which aspects of rational number knowledge are weak among their students.

Tips for Teachers – ***Common Fraction and Decimal Errors***

Students' mistakes can provide information about gaps in their understanding. Teachers can address these misconceptions directly and help students to develop strong conceptual knowledge about fractions (Deringöl, 2019). Certain errors are particularly common.

1. **Fractions: Confusing the denominator and the numerator**. When representing fractions, students may misunderstand what the numerator and denominator represent. For example, a student may incorrectly identify 3 out of 8 slices of pizza as $3/5$, or even $5/8$.
2. **Fractions: Using whole number knowledge to interpret fraction magnitudes**. Students may not understand that the magnitude of the fraction is based on the relation between the numerator and denominator. Instead, they may rely on their understanding of whole numbers to compare fractions. For example, students may assume that a larger denominator indicates a larger fraction (e.g., thinking $1/3$ is greater than $1/2$), or a larger numerator indicates a larger fraction (e.g., thinking $5/8$ is greater than $2/3$), or they might look at the gap between the numerator and denominator and assume a smaller gap indicates a larger fraction (e.g., thinking $2/3$ is greater than $7/9$) (Di Lonardo Burr et al., 2022a; Rinne et al., 2017). Showing fractions on a number line and having students compare the quantities shown can be helpful in overcoming the whole number bias.
3. **Decimals**: **Place value confusion**. Students may confuse the number of digits with the size of the number in decimal numbers. For example, they may think that 0.3 is smaller than 0.27 because 3 is smaller than 27.

Technical Characteristics of the EMA@School

Reliability

In general, reliability is an indication of how consistently students respond across all of the items on a test. Item-level data was collected and analyzed for 608 students in 13 schools in 2021-2022. As shown in Table 2, item-level reliability was generally excellent (> .80) for all but the number line task where it was acceptable in grade 2 and good in grade 3. There were only five items for the number line measure.

Table 2: Reliability Coefficients (Cronbach's Alpha)

	All students	**Grade 2 (n = 313)**	**Grade 3 (n = 295)**
Number comparison	.858	.852	.825
Number order	.903	.823	.913
Number writing	.833	.836	.817
Number line	.752	.649	.775
Addition fluency	.947	.834	.900
Subtraction fluency	.947	.805	.955

Note. For number comparison and number writing, reliability is based on the totals across pages. For number line, reliability was based on scores for each of five items, for number writing on scores for all 13 items, and for the fluency measures on scores for all 39 items. Similar results were found if fluency was based on fewer items.

Test-retest reliability is reflected in correlations for the same test at different time points. As described below in the Validation Study section, performance from fall to winter and from winter to spring testing shows high correlations. This pattern suggests that the measure is stable across the school year. Nevertheless, it is also responsive to interventions.

Validity

Validity is an indication of how well a test captures what it is supposed to measure. For an early numeracy screener, the assessment should capture students' early numeracy skills that are related to their mathematical development. Evidence for screener validity can include content, convergent, and consequential validity.

- The EMA@School has strong *content* validity because it was designed based on 30+ years of research, as outlined above. The skills assessed are based on a theory of early numeracy that includes three subdomains: symbols, relations, and operations.
- *Convergent validity* is established when an assessment is strongly correlated with other similar assessments. The EMA@School is related to other math assessments and to provincial test scores (see below).
- *Consequential* validity means that test scores can be used to make meaningful decisions. Evidence of consequential validity comes from intervention research using the EMA to identify students for intervention and to establish the effectiveness of the interventions (AIM Collective, 2024d).

Validation Study

The EMA@School was used in a validation study with the Fort Vermilion School Division (FVSD) in Alberta (AIM Collective, 2024d). Most of the students in kindergarten to grade 4 participated, from eleven schools in the division. Kindergarten students (*n* = 220) completed the EMA@School (i.e., the PNSA[2]) and another early numeracy assessment, the Preschool Early Numeracy Screener (PENS-

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[2] The EMA@School content is licensed to Alberta Education and made available province wide as the Provincial Numeracy Screening Assessment (PNSA). The content of the EMA@School and the PNSA is the same although the form and presentation of the two measures differ slightly.

B; Purpura, 2020). Students in grades 1 to 4 (*n*s ~ 240 per grade) completed the EMA@School and the Wide-Range Achievement Test, Version 5 (WRAT-5; Wilkinson & Robertson, 2017).

The results indicated that:

1. The EMA@School has strong stability (Fall to Winter and Winter to Spring) in all grades, with correlations of .77 and .87 for kindergarten and ranging from .80 to .94 for grades 1 to 4.
2. The EMA@School has strong convergent validity with the PENS-B in kindergarten. Scores on the EMA@School were significantly correlated with student performance on the PENS-B in Fall and Spring respectively (*r*s = .78 and .70).
3. The EMA@School has good convergent validity with the WRAT-5 in grades 1 to 4. Scores on the EMA@School were significantly correlated with student performance on the WRAT-5 (correlations ranging from .64 to .75 in Winter and .66 to .80 in Spring). However, the WRAT-5 classified more students in grades 3 and 4 as 'at risk' than the EMA@School (i.e., based on provincial norms). The EMA@School seems to be more appropriate for students in the FVSD than the WRAT-5, because the latter is mainly a measure of computation whereas the EMA@School taps a broader range of numeracy skills and has more room for variability.
4. The EMA@School has good predictive validity as indicated by significant correlations from Fall to Spring with the PENS-B (*r* = .60), and from Winter to Spring with the WRAT-5 (*r*s = .50 to .79). The EMA@School provides teachers with more detailed information about the specific subdomains in which their students may have difficulty, whereas the other measures provide a single score.

In summary, the FVSD study indicates that the EMA is both a reliable and valid assessment for students in kindergarten through grade 4. It has better alignment with kindergarten students' skills than the PENS-B, because that measure was developed with preschoolers and shows ceiling effects by the end of kindergarten. It also has better alignment with students' scores in grades 3 and 4, possibly because the WRAT-5 gets very difficult quickly and moves beyond the curriculum that these students experience. Thus, the WRAT-5 may underestimate students' skills.

Intervention Studies

The Carleton team has supported intervention studies with several school boards in Ontario and Alberta. For example, in 2023-24, we supported an intervention with grade 3 students in the Upper Grand District School Board (UGDSB). The intervention was designed by the board's math and STEM program lead and administered to hundreds of children. Details are provided in a report on the AIM website (AIM Collective, 2024a). The students showed more improvement in their EMA@School scores during the intervention, beyond the usual increase over time. Among the students who were selected to participate in the small group intervention, 72% were below the 25th percentile before the intervention, compared to 37% after the intervention.Thus, the intervention helped many students improve their numeracy scores.

As evidence of external validity, the EMA@School scores were significantly correlated with Ontario Report Card Grades and with the Ontario Grade 3 EQAO Assessment (AIM Collective, 2024a). In summary, the EMA@School is useful for understanding the overall performance levels of students and for providing feedback for teachers. It can be used as a sensitive index of improvements in foundational skills that were targeted by the intervention. Similar results were found in other intervention studies where the EMA@School was used as the pre- and post-test assessment (AIM Collective, 2024b, 2024c).

Further Reading and Resources

These resources and websites provide additional information about students' early learning and are readily accessible online.

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Frykholm, J. (2010) Learning to Think Mathematically with the Number Line. A resource for teachers, a tool for young children. Math Learning Centre, Salem, Oregon. https://www.mathlearningcenter.org/sites/default/files/pdfs/LTM_Numberline.pdf

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Meadows Centre for Preventing Educational Risk (2017) *10 Key Mathematics Practices for All Elementary Schools.* Austin, TX: The Meadows Centre for Preventing Educational Risk https://repositories.lib.utexas.edu/bitstream/handle/2152/74228/10Keys_ElemMath_Web.pdf?sequence=2&isAllowed=y

Additional Websites of interest:

* AIM Research Centre (ARC). https://carleton.ca/arc/
* AIM Collective. https://aimcollective.ca
DREME Network [include lessons and other helpful information with a focus on kindergarten to grade 2]. https://dreme.stanford.edu
Erikson Early Math. https://earlymath.erikson.edu
*Fraction Learning Pathways. https://www.fractionslearningpathways.ca
*JUMP Math. https://jumpmath.org/ca/
Learning to Think Mathematically. https://www.mathlearningcenter.org/educators/free-resources/lessons-publications/learning-think-mathematically
Learning Trajectories. https://www.learningtrajectories.org
Math Learning Centre. https://www.mathlearningcenter.org
National Centre for Intensive Intervention. https://intensiveintervention.org/implementation-intervention/math-lessons#numbers
* Robertson Program [includes many lessons for early numeracy]. https://wordpress.oise.utoronto.ca/robertson/early-years/
Science of Math. https://www.thescienceofmath.com

* Canadian resources.

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AIM Collective. (2024b). *Assessment of the MATmatics intervention in the Grande Prairie Public School Division*. https://www.aimcollective.ca/reportgppsd

AIM Collective. (2024c). *Evaluating the MATmatics Intervention in the Kenora Catholic District School Board*. https://www.aimcollective.ca/reportkcdsb

AIM Collective. (2024d). *Validation of the Provincial Numeracy Screening Assessment in the Fort Vermilion School Division*. https://www.aimcollective.ca/research/reports/reportfvsd-pnsa

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