# Fraction Reduction is Cued by Division but not by Multiplication 

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## Introduction

Reducing fractions involves

- Finding a common factor for a numerator-denominator pair
- Dividing both the numerator and denominator by the common factor
In a previous study to examine adults' knowledge of fraction arithmetic, we collected data online from 677 adults. T Two problems involved reducible fractions
Table 1 presents the percentage of participants who provided correct reduced and correct non-reduced solutions for these problems

Table 1: \% of correct fraction solutions

|  | $2 / 3 \times 3 / 5$ | $4 / 5 \div 3 / 5$ |
| :--- | :---: | :---: |
| Correct-reduced <br> solution | $21 \%$ | $35 \%$ |
| Correct non- <br> reduced solution | $51 \%$ | $18 \%$ |

Why do people use reduction at all, given that it is not required for multiplication or division?
-Does reduction improve performance?
-What factors determine whether solvers reduce fractions? Two factors
(1): required operation (See Table 1)

For multiplication, more correct non-reduced than correct reduced solutions
For division, more correct reduced than correct non-reduced solutions
(2): common denominators

Findings from 6th and 8th grade students (Siegler, Thompson \& Schneider, 2011)) suggested that relationships between denominators also cue strategies. For fraction multiplication and division, solvers' most common errors retain values of denominators.

## Method

59 adults ( 30 women) solved fraction arithmetic problems. If participants asked whether their solutions should be reduced, they were told that "it was up to them".

Solutions were categorized as to whether participants reduced the fractions during the solution process.

Table 2. Example Fraction Problems.

| MULTIPLICATION <br> SAME <br> DENOMINATOR | MULTIPLICATION <br> DIFFERENT <br> DENOMINATOR | DIVISION <br> SAME <br> DENOMINATOR | DIVISION <br> DIFFERENT <br> DENOMINATOR |
| :--- | :--- | :--- | :--- |
|  |  |  |  |
| $3 / 10 \times 2 / 10$ | $2 / 3 \times 5 / 15$ | $3 / 10 \div 2 / 10$ | $2 / 3 \div 5 / 15$ |
| $3 / 10 \times 5 / 10$ | $3 / 4 \times 5 / 10$ | $3 / 15 \div 2 / 15$ | $3 / 4 \div 5 / 10$ |
| $2 / 15 \times 5 / 15$ | $3 / 4 \times 5 / 20$ | $4 / 20 \div 3 / 20$ | $3 / 4 \div 5 / 20$ |

Results
Selection of Reduced vs. Non-reduced Solutions (All Problems)


Summary
Participants were more likely to reduce fractions in division than in multiplication.
Participants were more likely to make errors for same denominator than different denominator problems when multiplying fractions

## Discussion

People were more likely to use reduction on division than on multiplication.

They were equally likely to use reduction on problems with same vs. different denominators. Thus, the relation between denominators did not cue reduction.

They were more like likely to make errors on same denominator than different denominator problems for multiplication. Reduction use appears to be related to participants' ability to generate correct solutions

Division requires an extra solution step (invert second operand, multiply) compared to multiplication. Retaining the inverted operand may require additional mental resources. Hence, reduction may be more likely when problems require more cognitive resources.

## Implications

Encouraging people to use reductioncould increase accuracy on fraction multiplication and division problems.

