The Foundations of Numeracy: Subitizing, Finger Gnosia, and Fine Motor Ability

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Abstract

Butterworth (1999; 2005) proposed that several component abilities support our numerical representations and processes: an innate capacity to represent small numerosities (indexed by subitizing), fine motor ability (indexed here by finger tapping), and the ability to mentally represent one's fingers (indexed by finger gnosia). In the current paper, we evaluated the predictive power of these component abilities in the development of numeration and calculation skills in Grade 1 children (N = 146). Each component ability was found to be a significant unique predictor of number system knowledge, which in turn was related to calculation skill. Finger gnosia was related to calculation skill indirectly through number system knowledge. In contrast, subitizing predicted calculation skill both directly and indirectly through number system knowledge. Our results support Butterworth's view of the foundations of numeracy and have implications for the early identification of children at risk of math difficulties.

Keywords: Numerical representation; Arithmetic Development; Subitizing; Finger Gnosia; Finger Tapping.

Introduction

Mathematical skill is vital in our society. Poor skill in basic mathematics has been shown to have a greater negative effect on employment opportunities and job retention than poor literacy skills (Bynner & Parsons, 1997). In comparison to literacy, however, less is known about the precursor abilities that support math. Butterworth (1999; 2005) proposed that three component abilities support our numerical representations and processes: an innate capacity to represent small numerosities, the ability to functionally use one's fingers, and the ability to mentally represent one's fingers. In the current paper, we evaluate the component abilities underlying the development of numeration and calculation skills.

In Butterworth's view of the development of numerical abilities, the core component is the ability to "categorize the world in terms of numerosities—the number of things in a collection" (1999, p. 6). This ability, he posits, can be indexed by *subitizing*, that is, the ability to quickly enumerate the items in a set without counting. Subitizing generally applies only to sets of up to 3 or 4 items (Mandler & Shebo, 1982). To date, no one has examined the specific relation between subitizing ability and math skill.

To extend our core numerical abilities, Butterworth (1999) asserts, we construct concrete and abstract numerical representations using fingers, number words, and numerals. Across cultures, fingers are used to represent numerosities. The representations of fingers, thus, become tied to the representation of numerosities. Butterworth's position is that children's representation of number beyond the subitizing range hinges on the ability to mentally represent one's fingers, that is, *finger gnosia*. Children use their fingers not only to represent numerosities, but also for counting and arithmetic. Fingers serve as portable manipulatives, providing a bridge from concrete to abstract representations of quantity and of operations. Thus, the fine motor ability necessary to perform such functions may be important for the development of counting and calculation.

Subitizing

Butterworth (1999; 2005) suggested that the core component upon which all other math abilities are built is an innate ability to recognize, represent, and manipulate cardinal values, termed *numerosity*. Deficits are posited to lead to a plethora of math difficulties. In accordance with Butterworth's view, Benoit, Lehalle, and Jouen (2004) concluded that subitizing is a necessary component for the mapping of number words to numerosities, as subitizing "allows the child to grasp the whole and the elements at the same time" (p. 21). The ability to predict math performance based on subitizing ability, however, has not yet been examined.

Koontz and Berch (1996) evaluated children's speed in enumerating sets of items. The authors found that in contrast to normally-achieving children, children with dyscalculia appeared to count the number of dots in a display of 1 – 3 such that their response times increased with each additional dot. These results and similar results from a dyscalculia study by Landerl and colleagues (Landerl, Bevan, & Butterworth, 2004) suggest that a relation will exist between subitizing speed and math skill.

Mental representation of fingers

Consistent with Butterworth's position that children's representation of number hinges on the ability to mentally represent one's fingers, young children of every culture use their fingers to represent quantities and for counting (Butterworth, 1999). As a result of children's use of their fingers to represent numerosity, he posits that fingers come to be associated with numerosities. Fayol and Seron (2005), likewise posit that fingers may be the "missing link" between preverbal representations of number and number words. In support of the importance of fingers in math development, both finger agnosia and fine motor deficits have been linked to math deficits in groups with neuropsychological disorders (Barnes, Smith-Chant, & Landry, 2005). Finger gnosia performance has been found to predict children's math performance both concurrently and longitudinally (Fayol, Barrouillet, & Marinthe, 1998; Noël, 2005).

Fayol, Barrouillet, and Marinthe (1998) examined the predictive power of finger-related measures on math performance in a longitudinal study of 5- and 6-year-old children (N = 177). Neuropsychological tests of perceptuo-tactile performance, including tests for simultagnosia, digital gnosia, digital discrimination, and graphisthesia, were given in June preceding entry into Grade 1. Developmental tests, including the Lozenge Drawing Test and Goodenough's Person Drawing Test, were also given during this session. These tests were combined to derive a neuropsychological score and a developmental score, which were used to predict math outcome both concurrently and longitudinally eight months later in Grade 1. Math outcome was measured by children's performance on tests including: write down all numbers you know, completing sequences of numbers, identifying cardinal values, and verbal word problems. The neuropsychological score was the best predictor of mathematical scores in preschool and Grade 1 (r = .49 and .46 respectively). This evidence supports a role for fingers in math development.

Noël (2005) examined the predictive power of finger gnosia on numerical tasks in a longitudinal study of children from the beginning of Grade 1 to the end of Grade 2 (6 – 7 year olds, N = 41). Noël refined the finger gnosia task, eliminating the confound of number labels for identifying the fingers. Children were given the finger gnosia test in Grade 1 and again in Grade 2. Numerical tasks were given in Grade 2 and included: magnitude comparison (range 1- 9, with dots and with Arabic digits), number transcoding (write dictated number in Arabic digits, range 2- to 3-digit numbers), subitizing (range 1-8), finger counting (identifying conventional and unconventional patterns of raised fingers), and single-digit addition. A numerical error factor was created from accuracy measures (comparison, subitizing, number transcoding, finger counting, and addition), and a numerical speed factor was created from response time measures (comparison, finger counting). Finger gnosia scores in Grade 1 were correlated with both the numerical error factor (-.48) and the numerical speed factor (-.30). Finger gnosia scores in Grade 2 were correlated with the numerical error factor (-.36). The predictive power of finger gnosia was specific to numerical skills and was not correlated with reading skill. Thus, even once the confound of number labels is removed, finger gnosia remains a good predictor of children's numerical abilities in the early grades. In summary, existing research supports the view that children's mental representation of fingers is related to the development of mathematical skill.

Functional role of fingers

Consistent with Butterworth's position that fingers play a functional role in the development of numeracy, fine motor deficits are associated with math disabilities. Barnes et al. (2005) investigated the link between fine motor ability and math in children with Spina Bifida. Spina Bifida is a neurodevelopmental disorder that produces, among others, deficits in fine motor ability. Moreover, children with Spina Bifida show math difficulties early in development and these difficulties persist into adulthood.

In a group of 120 children aged 8 – 16 years (60 with Spina Bifida and 60 age- and grade-matched controls) Barnes et al. found that fine motor ability, measured with the Purdue Pegboard task, was correlated with skill in multi-digit calculation, accounting for 28 % of the variance. In a group of younger children (aged 36 months old) fine motor ability measured with the Visual Motor Integration test (Beery & Beery, 2004) predicted significant unique variance in children's nascent quantitative skills including: object counting, finger counting, quantitative vocabulary, and counting concepts. Thus, there is a link between fine motor skill and a variety of numerical tasks that is robust across measures and development.

Present Research

What component abilities underlie the development of numeration and calculation skills? Butterworth posits a role for three component skills: subitizing, finger gnosia, and fine motor ability. In the current paper we investigated the joint and independent contributions of each component ability to the numeration and calculation skills of children in Grade 1. Numeration skills include counting, ordering, recognizing numerals, sequencing, and place value. These tasks make use of the representations of number, which are posited to be formed based on subitizing ability and extended via finger gnosia, and of the procedures facilitated by fine motor ability. We hypothesized that these component abilities make independent contributions to the development of numerical skill. We further hypothesized that the component abilities relate to calculation skill indirectly via numeration. In contrast, on the view that subitizing forms the core of all numerical skills, we hypothesized that subitizing may also relate to calculation skills directly.

Method

Participants

Grade 1 children (*N* = 146) were selected from an ongoing longitudinal study. The testing sessions took place in the late Spring, by which time the children had participated in nine months of mathematical instruction. The children (71 girls and 75 boys) ranged in age from 5 to 7 years old (*M* = 82 months).

Socio-economic status (as measured by parent education levels) was relatively high across all of the schools in the sample. Seventy-three percent of the participants' parents provided education level information, of these, only 19% did not hold university or college degrees, while 25% held post-graduate degrees.

Procedure

Most children completed the computer measures in one half-hour session and the rest of the measures in a separate half-hour session on a different day. Some children completed all of the measures in a single session of approximately an hour.

All of the computer tasks were presented using software developed specifically for this project. For the computer tasks, children initiated the trials themselves by pressing the spacebar. Response times were measured from the point at which the stimuli appeared, until the experimenter pressed the stop-timer key (using a separate keyboard) when the child spoke their response. The experimenter then typed in the child's response.

Materials

Subitizing On each trial the computer displayed a set of 1 to 6 circular red target objects. The children were instructed to respond with the number of objects, out loud, as quickly as possible. The child initiated each trial by pressing the space bar. To promote accuracy, the targets remained on the display until the child's response was entered by the experimenter. There were 18 trials, preceded by two practice trials of two and seven objects. Half of the trials were within the subitizing range (1 to 3), and half in the counting range (4 to 6). Although the subitizing range is often defined as from 1 to 4 objects (Trick & Pylyshyn, 1994), previous research with children in the Grade 1 age range shows accuracy and speed declined for four items (Trick, Enns, & Brodeur, 1996).

The measure of interest for the present research is the response time (RT) slope as a function of set size. To compensate for the variability in the response times of this small number of trials, median response times were calculated for each set size (1 – 3 items) for each child. The best-fitting regression line through these medians was calculated for each child. The slope values were used as the dependent measure.

Finger Gnosia The Finger Gnosia measure is based on one designed by Noël (2005). Ten trials were conducted on each hand, beginning with the dominant hand. In each trial, two fingers were lightly touched below the first knuckle. The child's view of the touches was obstructed with a cloth cover raised from the child's wrist. After the cloth cover was lowered, the child pointed to the two fingers that had been touched. A point was awarded for each correct identification of a touched finger in a trial, with a maximum of 20 points per hand. The score for the non-dominant hand was used as the dependent measure.

Finger Tapping To isolate fine motor ability from visual-spatial performance, we developed a computer-game version of the Finger Tapping Test (Baron, 2004). The game was presented as a canal-digging exercise, where the longer the canal, the bigger the fish that will swim in it. Each tap on the space bar was a 'dig of the shovel', and increased the length of the canal, with an indicator of the maximum length from any previous trials. Children were instructed and encouraged to tap as fast as they could. Timing began upon the first press of the space bar and continued for 10 seconds (Baron, 2004), until an animated fish appeared. The tapping score determined the type of fish, with a prized killer whale animation appearing for scores over 50 taps. Tapping scores were collected for three trials on each hand, beginning with the dominant hand. The maximum number of taps achieved across the three non-dominant hand trials was used as the dependent measure.

Digit Recognition/Next Number The Digit Recognition and Next Number tasks were performed on the computer. Each of these tasks had 18 trials of increasing difficulty. The task ended if the child made three sequential errors. In each trial, a number was displayed on the screen. The first half of the trials were under 100, then increased in difficulty through the hundreds, thousands and for Next Number, into the ten and hundred thousands, ending at 407,276. For Digit Recognition, the experimenter asked the child, "What number is that?" Later during the testing session, for the Next Number task, the child was asked to respond with the number "that comes next when counting." Responses that were spoken as digits–for example, "one oh oh" instead of "one hundred"–were marked as errors. The total number of correct responses was used as the dependent measure.

Place Value Knowledge of place value was measured through a test designed for this study. Each page of the testing easel displayed an image of a set of proportionately sized unit, ten, hundred or thousand blocks. Above the blocks was a two- to four-digit number with one digit underlined. The child was asked to identify whether the number of blocks was consistent with the underlined number. A consistent trial would show the number 352 with an image of 3 hundred-unit blocks. An inconsistent trial would show the number 352 with an image of 3 ten-unit blocks. Two example trials and two practice trials precede the twelve test trials. The task taps children's understanding of the values associated with the location of digits in numbers. The total number of correct trials was used as the dependent measure.

Numeration Concepts such as quantity, order, and place value were measured with the Numeration subtest of a multi-domain diagnostic test, the KeyMath Test-Revised, Form B (Connolly, 2000).

Calculation Skill Mathematical skill was assessed with the Calculation subtest from the Woodcock-Johnson Psycho-Educational Battery—Revised (Woodcock & Johnson, 1989). This subtest begins with small mathematical problems in both horizontal and vertical formats. The problems progress in difficulty and include addition, subtraction, and multiplication.

Processing Speed To assess processing speed, we implemented a computer-based simple choice reaction time task (Petrill, Luo, Thompson, & Detterman, 2001). Two types of stimuli (an X or an O) were displayed for 1 second, preceded by a half second fixation point. The display then cleared and the next trial began automatically 1 second later. There were 24 trials. The child positioned the index finger of their dominant hand on the keyboard key with an 'X' sticker (the 'X' key) and their middle finger on the key above it labeled 'O' (the 'D' key). Left-handed children used similar stickers on the right side of the keyboard. The median response time for pressing the correct key in response to the stimuli was used as the dependent measure.

Vocabulary Receptive language was measured using the Peabody Picture Vocabulary Test–Third Edition (Dunn & Dunn, 1997). It was included primarily as a measure of verbal, non-mathematical knowledge.

Results

Descriptive statistics for each measure are shown in Table 1. Correlations among the measures are shown in Table 2. Notably, there were no significant correlations among the three precursor abilities, subitizing, finger tapping, and finger gnosia, supporting Butterworth's view of separate component abilities (Butterworth, 1999). Subitizing and finger gnosia were correlated with both number system knowledge and calculation skill. Finger tapping was correlated with number system knowledge.

Table 1. Descriptive information (*N* = 146)

Measure	Max	M	SD
Subitizing Slope[4]		100.8	129.4
Finger Gnosia[2]	20	15.0	2.4
Finger Tapping		38.8	5.5
Calculation Skill[1]		99.8	15.1
Processing Speed[4]		781.2	209.4
Vocabulary[1]		109.5	11.5
Number System Knowledge [Component Measures]			
Digit Recognition[2]	18	14.5	2.6
Next Number[2]	18	10.2	3.8
Place Value[2]	12	7.2	2.1
Numeration[3]	17	12.6	3.7

[1] Standardized score; [2] Number correct; [3] Scaled Score; [4] Milliseconds

Table 2. Correlations among measures (*N* = 146)

Task	1	2	3	4	5	6	7
1. Gender[1]	—						
2. Vocabulary	.13	—					
3. Processing Speed	-.08	-.06	—				
4. Finger Gnosia	-.15	.10	-.16	—			
5. Finger Tapping	-.01	.06	-.113	.06	—		
6. Subitizing	-.03	-.05	-.10	-.10	.04	—	
7. Number System Knowledge	.23**	.43**	-.23**	.27**	.18*	-.30**	—
8. Calculation Skill	.16**	.23**	-.10	.19**	.12	-.31**	.61**

*[1] Gender coding: Female = 1 and Male = 2. Significance levels: *p<.05, **p<.01.*

Data Reduction

Tasks that index number system knowledge (i.e. digit recognition, next number, place value, and KeyMath numeration) were entered into a principal components factor analysis with varimax rotation. A one-factor solution emerged that accounted for 66% of the variance among the measures with the following loadings: digit recognition (.89), next number (.86), place value (.59), and KeyMath numeration (.87). Factor scores were saved and used as the Number System Knowledge measure in the subsequent regressions.

Regressions

To test the hypothesis that the finger tasks and subitizing would each predict number system knowledge, a regression was conducted with the number system knowledge factor as the dependent variable and subitizing, finger gnosia, and finger tapping as predictors. In both this and the following regression, gender, receptive vocabulary, and processing speed were included as control variables. The pattern of significant predictors is shown on the left of Figure 1. Consistent with Butterworth's view, each precursor ability was a significant unique predictor of number system knowledge. Overall, the model accounted for 36.4 % of variance in number system knowledge.

Next, a second regression was conducted with calculation skill as the dependent variable, and with number system knowledge and the three precursor abilities as the predictors. As shown in Figure 1, number system knowledge was related to calculation skill. Finger gnosia was related to calculation skill indirectly through number system knowledge. This finding is consistent with Butterworth's view that the ability to mentally represent fingers is an integral building block for the representation of numbers beyond the subitizing range. In contrast, subitizing predicted calculation skill both directly and indirectly through number system knowledge. This finding is consistent with Butterworth's view that the capacity to represent small numerosities is the core ability upon which all other mathematical skills are built. Overall, the model accounted for 36.0 % of variance in calculation skill.

In summary, as predicted, all component skills (subitizing, finger gnosia, and finger tapping) contributed uniquely to number system knowledge. As predicted, finger gnosia was indirectly related to calculation skill. Finger tapping, however, was not related to calculation skill. This finding may reflect that our operational definition of fine-motor skill as indexed by finger tapping was not ideal. In contrast, subitizing exerted influence on calculation skill beyond that accounted for by number system knowledge.

Discussion

In the current paper we investigated the joint and independent contributions of each component ability (subitizing, finger gnosia, and fine motor ability) to the numeration and calculation skills of children in Grade 1. No correlations were found among the component abilities, consistent with the view that each reflects a separate ability. Further, each component ability was found to predict significant unique variance in children's number system knowledge.

Subitizing was correlated with both number system knowledge and calculation skill. Thus, children able to enumerate 1 - 3 items without counting performed better in mathematics. Butterworth (1999) proposes that this is the core ability upon which math is built. Indeed, both young infants and many non-human animals exhibit the ability to distinguish small sets of items based on magnitude (for a review see Butterworth, 1999; Dehaene, 1997). Despite the evidence supporting subitizing as a developmental and evolutionary precursor to mathematics, our findings are the first that we are aware of to show a correlation between subitizing and mathematical ability in an unselected population.

Finger-based representations of magnitude are posited to extend the representation of quantity beyond the subitizing range. Finger gnosia was correlated with both number system knowledge and calculation skill. Thus, children able to use their fingers as representational tools performed better in mathematics. Butterworth (1999) proposes a functional relation between finger gnosia and math; the two are related because children use their fingers to represent quantities. Alternative neural views of the relation between finger gnosia and math have also been proposed (see Anderson & Penner-Wilger, 2007; Dehaene, et al. 2003). Anderson and Penner-Wilger suggest that the two tasks share a common underlying neural circuit, which though originally purposed for finger representation has been redeployed for the representation of magnitude.

Children with greater finger agility, as measured by finger tapping, would be in a better position to use their fingers to perform counting and arithmetic procedures. We found that finger tapping was correlated with number system knowledge. The numeration component of number system knowledge involved counting and simple arithmetic problems.

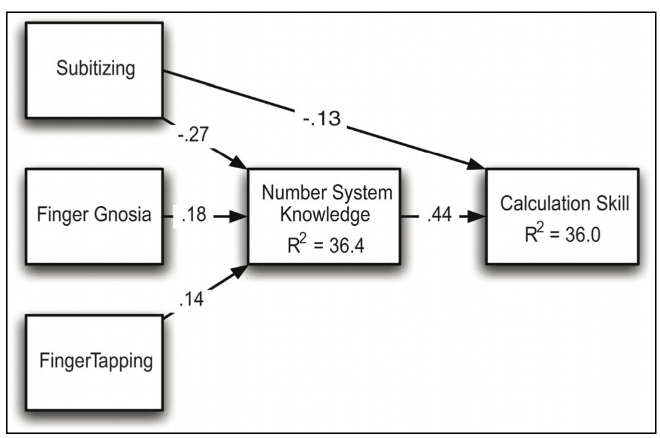

Figure 1: Regression model with semi-partial regression coefficients.

Conclusion

What component abilities underlie the development of mathematical skills? Overall, the results of the present study suggest independent contributions for the ability to enumerate small sets of items, fine motor ability, and the ability to mentally represent one's fingers. These predictors hold promise for the early identification of children with math difficulties. Mathematical skills form a complex mosaic, however, and the component abilities investigated constitute only a portion of the answer to why some children are better at math than others. In the reported work we examined the components concurrent with mathematical skill in Grade 1; we are currently investigating whether these predictors hold longitudinally.

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