Intelligent Artificiality
and an Economics of Mental Behavior

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What I Am Not (A Partial List, Part I)

- I am not an economist (though I work alongside a few);

- I am not a neuroscientist, of either wet or dry kinds (though I will work with many of both, soon);

- I am not a psychologist (though I used to work with one, and even tried to impersonate one for 18 months);

- I am not ‘an AI guy (or, gal)’ (though my post docs/RA’s come from a computer science and artificial intelligence lab);
What I Am Not (A Partial List, Part II)

- I am not a neuro-economist (I do not understand what that means);

- I am not a neuropsychologist (they don’t understand what that means);

- I am not an empiricist (but, who is, really?);

- I am not a theorist (see ‘I am not an empiricist’);

- I am not an epistemologist or ‘impartial observer of scientific practice’ (an incoherent concept).
A Gap(ing Hole) in the Core of ‘Rational Choice’ Models

- Choice-theoretic conditions on ‘rational choice’ (antisymmetry, acyclicity, completeness, identity) ‘guarantee’ existence of objective function economic agents are said to maximize *in virtue of* choosing.

- How are we to interpret maximization (optimization)? As a real process whose temporal dynamics *refer* to something?

- If, so, what is it ‘running on’?
  - Brains?
  - Researchers’ desktops? Laptops? iPads?
  - ‘Turing Machines’? (i.e. an imaginary process running on an imaginary device?)
Predict how this Creature will choose from among N options:

**INPUTS:**
- Past choices among similar options
- Revealed preference model
- Rationality conditions

**OUTPUTS:**
- Prediction of choice/behavior

Predict behavior of this device:

**INPUTS:**
- Newton’s Laws
- \( F_g = \frac{G m_1 m_2}{d^2} \)
- Measured values of \( g, l, \Theta, m \)

**OUTPUTS:**
- Prediction of movement trajectory, transient and static

“The Predictive Apparatus Is Faulty”
(Not) A Trick Question
(Illuminative of the question: ‘How does optimization happen?’)

- Suppose Bob must choose between two lotteries:
  - Lottery A pays $1MM with probability 0.1 and $0 with probability 0.9.
  - Lottery B pays $1MM if the 7th digit in the decimal expansion of $\sqrt{2}$ is an 3 and $0$ otherwise.

  - No calculator, SmartPhone or computer;
  - Needs to choose in 2 min.

Bob’s Choice

Lottery A
P($1MM)=0.1
P($0)=0.9;

Lottery B
$1MM if dig7(sqrt(2))=3;
Else, $0.
What If We Know Bob Knows This...?

- Depends on whether or not Bob sees the problem as one solvable by the algorithm;

- Depends on whether or not Bob can correctly perform required operations quickly enough to generate answer in under 2 minutes.

- Depends on whether or not Bob thinks he can correctly perform the operations quickly enough to generate the answer in under 2 minutes.

Problem: Given x such that $x^2 = 2$, find $x$.

[NEWTON'S METHOD]

- Step 1: Form $f(x) = x^2 - 2$

- Step 2: Compute $f'(x) = 2x$

- Step 3: Make first guess at $x$: $x_0 = 1$

- Step 4: (Repeat as necessary) $x_{k+1} = x_k - \frac{f(x_k)}{f'(x_k)}$

  e.g. $x_1 = 1 - \frac{(1 - 2)}{2} = 1.5$

  $x_2 = 1.416667$

  ...

  ...

  Calculator says $x = 1.4142135$. (requires 5 steps)
The Computational Process Model Matters to Whether We Ascribe ‘Rationality’ to Bob

• Each calculation generates new information (2 bits)...
• … that reduces Bob’s uncertainty regarding the true value of the answer...
• …on account of the fact that it actually reduces the instantaneous search space of the problem he is trying to solve.

“A” represents the exact answer, $\varepsilon$ is the diameter of an acceptable “error region” around $A$. \{A_k\} are successive approximations of “A”, outputs of successive iterations of the solution algorithm.
... and ‘the Logical Depth of Calculative Thinking’ Matters to Strategic Payoffs...

[Cournot-Nash Duopoly Without Logical Omniscience]

\[
\begin{align*}
q_N &= \frac{a-c}{2} - q_N \frac{1}{2} ; \\
q_o &= 0
\end{align*}
\]

Which results from joint maximization of profits

\[\Pi_i = (a-c-q_i-q_j)q_i\]

Generate using series

So, if firm 1 says, “I will sell \(\frac{a-c}{2}\), firm 2 will credibly retort, “I will sell \(\frac{a-c}{4}\); which would lead to losses relative to the \(\left[\frac{a-c}{3}, \frac{a-c}{3}\right]\) solution

NASH EQUILIBRIUM

Moldoveanu, 2009, Thinking Strategically About Thinking Strategically
Computational Landscapes for Interactive Problem Solving (Duopoly)

- Computational Landscape of Cournot Nash Equilibrium, 2 firms, $a=3$, $c=1$.

- Horizontal axes represent number of iterations for each firm. Vertical axis is the profit level of firm 1. Profit levels of firm 2 are symmetrical.

- Landscape converges to Nash Equilibrium output of $(a-c)/3$. 

Moldoveanu, 2009, *Thinking Strategically About Thinking Strategically*
If All Problem Solving Processes Had These Dynamics, We Would Be Programming on Brains Right Now.

A Model of Calculation as an Information-Producing Process: Each Individual Operation Reduces the Uncertainty (Conditional Entropy of pdf(loc(answer(SearchSpace)))) – Associated with Creature’s Guess at an Answer, whose exact value is denoted by A.
What if Bob Had to Make a Different Choice with Procedural Implications?

- $1MM for finding the shortest Path connecting Canada’s 4663 cities in 1 day of less, OR
- One day’s consulting fees guaranteed

- Total number of operations required
  \[ K \sim 2^{4663} \sim 5 \times 10^{1403} \]

His computational prowess \( R \sim 10^{12} \text{ ops/second} \)

His computational budget
\( (10^{12} \text{ ops / second}) (3600 \text{ sec / H}) (24h / \text{ day}) \times (365 \text{ days / yr}) \sim 3 \times 10^{20} \text{ ops} \)

He can solve this problem in \( 1.6 \times 10^{1383} \text{ years} \)
\( \Rightarrow \text{not worth it!} \)
UNLESS, Bob Had Some Kind of a Short Cut

- Non-exhaustive
- Non-deterministic
- Non-universal (will not be optimal for other NP-hard optimization problems)
- Locally exportable (to other TSP’s)
- Hardware-adaptable (more/less RAM, and operations per I/O cycle);

4663 city TSP, solved using Lin-Kernighan (meta) algorithm
The NP Class Reads like a Who’s Who of Everyday Problems (Solved by Creatures with Brains)

- **kSAT** (Cook, 1976)
  - Reduces to **3 SAT**
  - Models: Optimal scheduling with lumpy activities and constraints, Optimal task design, via **Knapsack**

**3 SAT**
- Reduces to **Partition**
- Reduces to **Vertex Cover**
- Reduces to **Clique**

**Partition**
- Models: Optimal task design, via **Knapsack**

**Vertex Cover**
- Models: Hamiltonian circuit

**Clique**
- Models: Nash Equilibrium calculation for minimum payoff, Circle of trust discovery, clique discovery (social reasoning)

**Traveling Salesman** problem
‘Generalized Problem Solver, Version 2.X’
Modeling Toolkit for Problem Solving Processes: An Associative Map

Payoffs [Problems x Procedures]
What Could Computational Payoffs Look Like? Two Separate Payoff Structures…

Algorithmic advantage: $f_A(a,s) \geq f_B(a,s) \forall a, S$

Algorithmic advantage: $f_A(a,p) \geq f_B(a,p) \forall a, p$
...Combine into One 3D Measure

$a =$ Accuracy of solution

$S =$ Speed of convergence

$P =$ Probability of Convergence

Algorithmic Advantage: $f_A(a,s,p) \geq f_B(a,s,p), \forall a,s,p$
Getting Closer: How Would a Chip Designer Think About Embodied Problem Solving?

\[
x[k] = \frac{1}{M} \sum_{n=1}^{M} x[n] e^{i2\pi kn/M}; \quad M=2048
\]

\[
A(x) = "Radix - 2 FFT"
\]
Using Application-Specific Chip Design as a Paradigm for Mind-Brain Investigation

- No operation without implementation;

- No algorithmic change without architectural consequence;

- Capacity limit (Ops/sec, $M$) part of every hardware decision;

- Hardware Adaptable to Algorithm/I-O requirements (more/less RAM, operations per I/O cycle, precision of internal representation of coefficients);

- Average-case performance far more important than worst case performance (e.g. dynamic range extremes of the input $x[n]$).

\[
x[k] = \frac{1}{M} \sum_{n=1}^{M} x[n] e^{j2\pi kn/M}, \ M = 2048 \\
A(x) = "Radix – 2 FFT"
\]
“Simulation” Is Not Just “Modeling”:
It Has Bite, Which Is Why We Call It EMULATION

\[
x[k] = \frac{1}{M} \sum_{n=1}^{M} x[n] e^{j2\pi kn/M}, M=2048
\]

\[A(x) = "Radix - 2 FFT"
\]

\[
x[k] = \frac{1}{M} \sum_{n=1}^{M} x[n] e^{j2\pi kn/M}, M=2048
\]

\[Algo(x) = "Radix 2 FFT"
\]
Of Course, Humans Can Choose Whether or Not to Proceed with an Algorithmic Computation at Many Points…

\[
H_k(a_j) = \sum_j p(a_j) \log_2 \frac{1}{p(a_j)}
\]

‘uncertainty’

\[
ALG_j \bigcap k^{th} \text{ operation}
\]

Information gain

\[
\beta_k = H_{k-1} - \beta_k
\]

‘uncertainty threshold’

\[
H_k \leq H_T? \quad \text{yes} \rightarrow \text{stop}
\]

\[
EV(k+1) > 0 \quad \text{no}
\]

Threshold test for engaging in next operation

\[
EV(k+1) = C_0 (H_{k-1} - H_k) - C_1 (H_k - H_k - H_{k-1}) - C_m
\]

Along the way

anxiety cost

\[
C_{ATWA}(k+1)
\]

Estimate

Decision Point 2

no

Yes

Decision Point 3

no
A Goal for Intelligent Artificiality: A Brain Emulator/Co-Processor

- No ‘model’ of mental behavior without architectural and behavioral consequences;

- Brain states on which mental states supervene can be tracked, not only ‘modelled’: prediction/control supersedes ‘explanation as regulative goal.

- ‘Hardware changes’ (TMS, ECT, stimulus protocols, psycho-pharm) can be emulated, enabling point predictions about mental behavior.
We Need an Anatomically Informed Model of ‘Brainware’…

- Layered connectivity for the associative cortices;

- Cross-layer forward and backward connections (sparser), intra-layer connections (denser);

- Some (parametrizable) asymmetry between forward and backward connections;

- Architectural levers include strength of synaptic connections, ‘plastic’ formation of new circuits.
... That Is ‘Emulable’ via a Well Understood Structure (Recurrent Neural Network)
... Which Extremizes an Objective Function Familiar to Self-Organizing (Entropy-Increase-Defying) Systems...

\[
\min_{\{\alpha, s\}} F(x,s/\alpha) = -\langle \ln p(x,\theta/\alpha) \rangle_q + \langle \ln q(\theta,s) \rangle_q
\]

Kullback-Leibler divergence of \( p,q \)

“Spread of \( q \)” (ENTROPY)

Predictions

Dynamically Adaptive Nodes

Sensory Nodes

\{x\}
... to Provide an Extremis and That ‘Works’ at Different Space-Time Scales and in Different Domains of Being.

$$\min F(x, s/\alpha) = \min -\langle \ln p(x, \theta/\alpha) \rangle \downarrow q + \langle \ln q(\theta, s) \rangle \downarrow q$$

$$q = q(\theta \downarrow p, S \downarrow p) \ast q(\theta \downarrow m, S \downarrow m) \ast q(\theta \downarrow M, S \downarrow M)$$

- **“pico”:** synaptic weight changes
- **“micro”:** attentional shifts
- **“macro”:**
Now, If We Could Only Explain Away ‘Complexity Mismatches’ – Which We Can!

Encoding \((p,q)\) via Kolm \((p,g)\):

\[
\text{Kolm} \downarrow M (x) = \text{length} (A \downarrow M, \text{min} (x))
\]

"Efficient coding":

\[
p(x) = 2^{\uparrow - \text{Kolm} \downarrow M (x)} \quad \quad \quad [\text{Kraft–McMillan coding}]
\]

Information content of \(p(x)\):

\[
- \sum K \uparrow \downarrow p(x) \log_2 p(x) = \sum \uparrow \downarrow 2^{\uparrow - \text{Kolm}(x)} \text{Kolm} \downarrow M (x) = \langle \text{Kolm} \downarrow M (x) \rangle \downarrow p
\]

Let \(M = \text{CORT} \rightarrow K \downarrow M (\cdot) = K \downarrow \text{CORT} (\cdot)\)
We Can Rebuild a ‘Theory of Computation’ Using ‘Brainware’ as the Computational Substrate
... and Fill in the Gaps of Both Symbolic Representation and ‘Rational Choice’ Approaches

\[ \text{not } \max_{x,y,z,...} U(x,y,z,...) s.t \ B(x,y,z,...;t) \leq B^{\uparrow \star} : \]

PROCEDURALLY OPAQUE;
ARCHITECTURALLY INDETERMINATE;
PHYSICALLY UNREALIZABLE IN MANY CASES OF INTEREST

\[ \text{not } \max_{\{P\},\{A\}} V(P\downarrow 1, P\downarrow 2, ... P\downarrow m / A\downarrow 11, A\downarrow 12 ... A\downarrow mn) s.t \ Comp (A\downarrow jk / P\downarrow k) \leq Con \]

PROCEDURALLY UNREALISTIC;
ARCHITECTURALLY INAPPLICABLE;
WORST CASE EMPHASIS UNREASONABLE IN MOST CASES OF INTEREST
Circumventing Logically Deep Equilibrium Calculations: Beauty Contest Example

- \( N \) players, 1 period game;
- Each player submits number from 0 to \( N \) to an honest clearing house.
- Winner (gets \( $N \times $1000 \)) of the game is the player that submits the number that is closest to \( \frac{2}{3} \) of the average of all the other numbers.
- Iterated dominance reasoning:
  
  \[
  \text{if I submit } x \text{ and others submit } (y, z, w, \ldots) \text{ then winner would have had to have submitted } z, \text{ so I should have submitted } y. 
  \]

- Equilibrium submission (‘strategy’) is \( 0: (2/3)(0) = 0 \)

\[ 
\begin{align*}
\text{e.g. } N=100, x\downarrow n \sim [0,100] \\
x\downarrow \text{ave} = 50, x\downarrow, \text{ave} = 33.33\ldots \\
x\downarrow \downarrow +1, \text{ave} = 2/3 \times x\downarrow, \text{ave} \rightarrow x^* = 0 
\end{align*}
\]
Circumventing Logically Deep Equilibrium Calculations: Beauty Contest Example

- Encode *others* via Types (Ho, 2004)
- **Type 0** players do not think of what others think;
- **Type 1** players think only of what others think;
- **Type 2** players think of what **Type 0** and **Type 1** players think only;
- **Type k** players think of what **Type (k-1, k-2,...)** players think only.
- Define \( Q(\text{this group type set}) \) as estimate of density of **Type k** players in **this group**.
- Refine \( Q(\text{types}) \) (*mode, spread*) according to cues.
Intelligent Artificiality

A Foundation for Mind-Brain Design, Diagnostics and Development