

Putting Two and Two Together: Declines in Arithmetic Fluency among Young Canadian Adults, 1993 to 2005

Jo-Anne LeFevre (jo-anne.lefevre@carleton.ca)

Marcie Penner-Wilger (mpennerw@uwo.ca)

Aryn A. Pyke (apyke@andrew.cmu.ed)

Tina Shanahan, and Wendy Ann Deslauriers

Institute of Cognitive Science and Department of Psychology

Carleton University, Ottawa, ON K1S 5B6

Carleton University Cognitive Science Technical Report 2014-01



cogscitechreports@carleton.ca

Institute of Cognitive Science
2201 Dunton Tower
Carleton University
1125 Colonel By Drive
Ottawa, Ontario, K1S 5B6
Canada

Putting Two and Two Together: Declines in Arithmetic Fluency among Young Canadian Adults, 1993 to 2005

Jo-Anne LeFevre (jo-anne.lefevre@carleton.ca)

Marcie Penner-Wilger (mpennerw@uwo.ca)

Aryn A. Pyke (apyke@andrew.cmu.edu)

Tina Shanahan, and Wendy Ann Deslauriers

Institute of Cognitive Science and Department of Psychology
Carleton University, Ottawa, ON K1S 5B6

Abstract

Canadian university students ($N = 461$; aged between 17 and 22) were tested between 1993 and 2005 on a timed multi-digit arithmetic test. This test is closely related to the speed with which people can retrieve answers to arithmetic facts from memory. Scores declined by over 20% in this time period. These findings are discussed in relation to the changes in mathematics curricula that have taken place in North America over the past 20 years.

Introduction

The ability to both quickly and accurately calculate the answers to arithmetic problems is termed *arithmetic fluency* (see also Ramos-Christian, Schleser & Varn, 2008). It has traditionally been assumed that students educated in Canada achieve arithmetic fluency: Just as we expect university students to be able to read words and comprehend text, we expect that they can easily recall simple arithmetic facts (e.g., $3 \times 4 = 12$, $5 + 9 = 14$), and efficiently execute the algorithms for multi-digit addition (e.g., $34 + 98$), subtraction (e.g., $78 - 45$), and multiplication (e.g., 456×4). Arithmetic skills are taught and practiced in elementary school and comprise basic knowledge that is central for further mathematical learning. Evidence suggests, however, that arithmetic fluency may be declining among university undergraduates (Mulhern & Wylie, 2004). The goal of the present research was to evaluate this issue among Canadian undergraduates who were tested between 1993 and 2005 and thus started their arithmetic training (i.e., Grade 1) between 1982 and 1993.

The ability to retrieve basic arithmetic facts from memory and to efficiently execute algorithms for multi-digit arithmetic is a cornerstone of mathematical proficiency (Fuchs et al., 2006; National Mathematics Advisory Panel, 2008; Royer, Tronsky, Chan, Jackson, & Marchant, 1999). Fluency in arithmetic frees up working memory resources, allowing students to focus on higher-level understanding (Walczyk & Griffiths-Ross, 2006). As a result, arithmetic fluency is strongly correlated with performance on mathematical achievement tests for both children (Fuchs et al., 2006; Royer et al., 1999; Zentall, 1990) and adults (Royer et al., 1999).

A decline in arithmetic fluency is of concern for a variety of reasons. First, decline in arithmetic fluency may

adversely affect the preparedness of undergraduates for course work in university and subsequently influence future career opportunities (Parsons & Bynner, 1997). Arithmetic skill is also a strong predictor of employment outcomes (e.g., Finnie & Meng, 2006; Parsons & Bynner, 1997; Rivera-Batiz, 1992). Second, information about trends in arithmetic fluency is essential to educators and social psychologists attempting to assess the effects of curriculum changes and other societal factors on students' performance (Griffin & Callingham, 2006). Third, incorrect assumptions about the arithmetic fluency of adults can be problematic for clinicians assessing cognitive decline and for cognitive researchers generally because arithmetic performance is often used as an index of memory processing. Fourth, arithmetic learning is an important component of the curriculum in elementary school (National Council of Teachers of Mathematics, 2000). Thus, any changes in students' knowledge of arithmetic are of interest to educators at all levels.

Since 1989 when the National Council of Teachers of Mathematics (NCTM) proposed the first version of the *Principles and Standards for School Mathematics*, the trend in mathematics education in North America has been towards an emphasis on conceptual, rather than procedural skills (Star, 2005). As national or provincial curricula were modified to take into account the NCTM guidelines, emphasis shifted away from drill and practice and towards discovery learning, with the goal of motivating children to construct mathematical knowledge. Furthermore, tools such as computers and calculators have become ubiquitous in schools. Thus, as the curriculum expanded to encompass a wider range of mathematical content (National Council of Teachers of Mathematics, 2000), the experiences of North American children in elementary school mathematics have changed dramatically (Schoenfeld, 2004). In the Ontario Mathematics Curriculum (Ontario Ministry of Education, 2003), for example, Number Sense and Numeration is one of five strands that also include Measurement; Geometry and Spatial Sense; Patterning and Algebra; and Data Management and Probability. Although the experiences of any particular child will depend on the proclivities and beliefs of his or her teacher, the curriculum is much more extensive and inclusive than in the past, leaving less time for practice of specific skills within any of the mathematical

sub-domains, including arithmetic.

In support of the view that arithmetic has become much less important in modern elementary schooling, Standing, Sproule, and Leung (2006) found that only 40% of 146 business and economics majors could correctly answer all 10 items on a third-grade arithmetic test from 1932. Rustemeyer and Stoeger (2007) reported similar findings for German students. One interesting potential consequence of these changes in educational policy and practice is that the arithmetic fluency of students who are currently entering universities in Canada may be different from those of earlier cohorts because they have spent less time learning number facts and calculation procedures. The speed with which individuals can solve arithmetic problems, including math facts such as $3 + 4$, and multi-digit problems such as 34×3 , is correlated with performance on mathematics achievement tests (Fuchs et al., 2006; Royer et al., 1999), including the SAT-M among college students (Royer et al., 1999). Thus, changes in computational fluency are relevant for understanding mathematical achievement. We also might expect that students' conceptual understanding and their attitudes towards mathematics may have improved (although the former was not assessed in the current research).

Some research suggests that mathematics skills more generally may also be in decline. Mulhern and Wylie (2004) compared the mathematical skills of two cohorts of undergraduates at Queen's University in Belfast. The first cohort was tested in 1992 and the second in 2002. The students completed a measure that included a variety of components; arithmetic, estimation, algebraic reasoning, graphical interpretation, proportionality and ratio, and probability and sampling. The findings were clear; on all components the 1992 cohort scored higher than the 2002 cohort. Although the cohort differences were significant for all mathematical components, they were smaller for arithmetic than for some of the other measures. Mulhern and Wylie's data suggest that, at least in Ireland, the mathematics skill of university students has declined (see also Mulhern & Wylie, 2006).

For 12 years, we obtained a measure of arithmetic fluency from Canadian undergraduates who participated in experiments on mathematical cognition (e.g., Kalamian & LeFevre, 2007; LeFevre & Morris, 1999; LeFevre et al., 2003; Smith-Chant & LeFevre, 2003). Based on the performance of samples of students across this time period, our intuition was that arithmetic fluency had declined. For the present paper, we analyzed these data to test the hypothesis that the arithmetic fluency of young adults educated in Canada declined over the time period between 1993 and 2005.

Method

Materials

Participants completed the addition and multiplication-subtraction subtests from the Kit of Factor-Referenced

Cognitive Tests (French, Ekstrom, & Price, 1963). Each arithmetic subtest consists of two pages of multi-digit problems. On the addition subtest, the problems have three terms, each either a one- or two-digit number (e.g., $34 + 56 + 27$; $47 + 8 + 92$). They are arranged in columnar form in six rows per page of 10 problems per row (i.e., 60 per page) for a total of 120 problems. Each page of the subtraction-multiplication test has six rows of 10 problems arranged in columnar form. Beginning with the first, every other row consists of two- by two-digit subtraction problems (e.g., $34 - 18$); alternate rows consist of two- by one-digit multiplication problems (e.g., 54×6). Hence, there are a total of 120 problems on the two pages of the subtraction-multiplication subtest. A total correct score for each participant was calculated by summing the number of correct problems across the four pages, with a possible maximum score of 240. The summed score has excellent internal consistency, with Cronbach's alpha across the four pages of .91.

With respect to construct validity, there are substantial and consistent relations between participants' scores on this measure and their speed and accuracy on single-digit arithmetic problems (LeFevre et al., 2003). The correlation between average latency on multiplication problems such as 7×8 and score on this fluency test was $-.51, p < .001$ ($N = 64$; Smith-Chant & LeFevre, 2003). Similarly, the correlation between fluency score and latencies on single-digit addition problems with sums greater than 10 (e.g., $6 + 8$) was $-.67, p < .001$ ($N=34$; Das, LeFevre, & Penner-Wilger, 2010). Thus, performance on the French-Kit test is highly related to how quickly solvers' can respond to simple math facts. The fluency measure is also correlated with performance on mathematical tasks that require reasoning. For example, there was a correlation of $.43, p < .001$ ($N = 80$) between fluency score and performance on the Number Series subtest of the Canadian Cognitive Abilities Test (Multilevel edition, Level H, Form 3; Thorndike, Hagen, & Wright, 1981; LeFevre, 2003).

Participants

Participants were recruited either from the pool of introductory psychology students, such that they received course credit for their participation, or more broadly from the university population and paid typically \$7 to \$10 for 45 min to 1.5 hours. A total of 693 individuals who had been educated in Canada had participated in 25 different experiments on mathematical cognition between 1993 and 2005. These individuals (339 men and 354 women) ranged in age from 17 to 60 years ($Mdn = 20$ years). An estimate of the year that participants started formal schooling was calculated by subtracting their age from the year that they were tested and adding six (the majority of Canadian students start grade 1 in the year that they turn six). Using this criterion, our sample of Canadian-educated students started grade 1 between 1943 and 1994. However, the majority of participants (over 80%) started elementary school between 1980 and 1993.

Procedure

The experimenter instructed the participant to read the first page of instructions and complete several example problems. The participant was then given two minutes to complete each page of problems (timed with a stopwatch). Participants were instructed to solve problems as quickly and accurately as possible. Participants were also instructed to solve the problems in each row, starting at the left and moving to the right, and told not to skip rows. They were permitted to write intermediate results on the paper.

The addition subtest was always completed first. In a few cases ($n = 17$), participants were only given one of the two pages of a subtest. Scores for these individuals were estimated by doubling their score on the single page for that subtest. Total correct trials (summed over the two pages) for each of the two subtests comprised the scores. This value was very similar to the total number of problems attempted, as the participants rarely made errors. Therefore, these measures provide an assessment of combined accuracy and speed—arithmetic fluency.

In most cases, participants completed the arithmetic fluency measure after they had finished some other mathematical activities (these varied considerably across the different experiments). In some cases participants completed the fluency measure at the beginning of the experiment, or between two blocks of simple arithmetic problems.

Results

The mean performance (total number of correct problems) for the 693 Canadian-educated students was 78.1 ($SD = 26.2$; range from 15 to 190), but there was considerable variability across years. To explore patterns by grade 1 year, we limited the sample to the 461 undergraduate students who had started grade 1 between 1982 and 1993 (217 men and 244 women). Using this criterion, each grade 1 year had a sample of at least 20 individuals per year ($M = 38$, range from 22 to 73 per year).

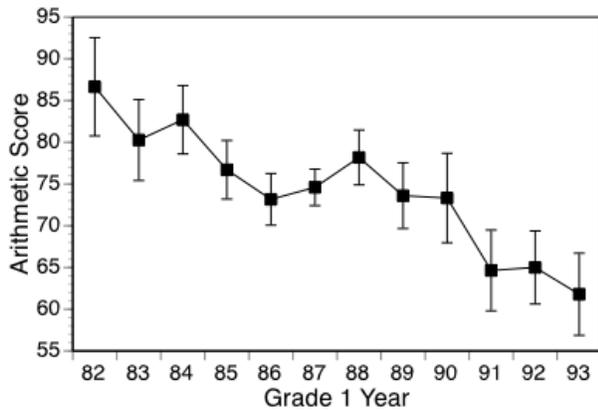


Figure 1. Mean arithmetic fluency scores by year in which participants started Grade 1 ($n = 461$). Whiskers represent standard errors of the mean.

The data in Figure 1 show a clear pattern of decline over time. Fluency scores were analyzed in a 2(gender: men, women) x 12(Grade 1 year: 1982 to 1993) ANOVA. Performance varied with Grade 1 year, $F(11,437) = 2.18$, $MSE = 560$, $p < .05$, $\eta_p^2 = .052$. The significant linear contrast ($p < .001$), indicates that scores declined as Grade 1 year increased. Men scored slightly higher than women on average (76 vs. 71), however this effect was only marginally significant, $F(1, 437) = 3.69$, $p = .055$, and the effect size was small ($\eta_p^2 = .008$). The interaction between gender and grade was not significant, indicating that men and women showed similar patterns of decline with time.

Correlations

As shown below the diagonal in Table 1, grade 1 year was significantly negatively correlated with arithmetic fluency: The more recently students commenced their elementary education, the lower their arithmetic fluency. Age at the time of testing was also correlated with fluency, such that older students tended to have higher scores. These relations support the hypothesis that arithmetic fluency declined over time. In contrast, gender was not significantly related to fluency scores or to the measures of time.

The correlations shown below the diagonal in Table 1 may overestimate the declines in performance across this time period because participants who are older may have higher fluency scores than younger participants for a variety of reasons. First, older participants in the current sample were more likely to be graduate students and thus potentially more mathematically skilled than undergraduates. Second, older students had more opportunities to practice arithmetic. Thus, a more stringent test of whether arithmetic fluency declined during this time period is to restrict the analysis to those individuals who were undergraduates (i.e., aged between 17 and 22) at the time of testing ($n = 491$). Correlations for this subset of individuals are shown above the diagonal in Table 1. The patterns are very similar to those in the larger group.

Table 1: Correlations among Arithmetic Fluency, Grade 1 Year, Age, Calculator Use (Calc), Nervousness about Math (Nerv), and Gender

	Fluency	Grade 1	Age	Gender	Calc ¹	Nerv ²
Fluency		-.22**	.09*	-.10*	.19**	.31**
Grade 1	-.33**		-.50**	.19**	-.42**	-.08
Age	.26**	-.90**		-.08	.11*	.04
Gender	-.02	-.05	.01		-.09	-.18**
Calc	.19**	-.39**	.19**	-.08		.09
Nerv	.31**	.00	-.01	-.18**	.01	

Notes. Correlations below the diagonal are for all of the 693 students educated in Canada. Correlations above the diagonal are for the subgroup of 491 students who were aged 17 to 22 at the time of testing. Age was measured in years. Gender was coded as 1 for men and 2 for women. Calculator use was coded as 1=started in elementary school; 2 = started in middle school; 3 = started in

high school or later. Nervousness was rated on a 7-point scale, where 1 = very nervous and 7 = not at all nervous.

¹ $n = 350$; ² $n = 462$.

* $p < .05$; ** $p < .01$

As in the full sample, the correlation between arithmetic fluency and grade 1 year is negative. Thus, even when other factors (such as amount of practice) are eliminated, the data support the hypothesis that arithmetic scores of undergraduates have declined over the last decade. The correlations shown above the diagonal in Table 1 suggest that gender is weakly related to fluency in the age-restricted sample (consistent with the ANOVA); however, gender is also positively correlated with year of testing because more females participated in later years than males. In order to assess the relation between arithmetic scores and grade 1 year as stringently as possible, arithmetic scores were analyzed in a multiple regression analysis with grade 1 year while controlling for age and gender. In the multiple regression, only grade 1 year was a significant unique predictor of fluency. The regression indicated that, for each increase in grade 1 year, fluency scores declined by 1.5 problems. Although this change may seem small, it represents an average decline of 23.5% over the period of time represented in this sample, from an average total fluency score of approximately 85 to 65. Thus, even with a very stringent criterion for inclusion of participants, the prediction that arithmetic fluency had declined over time was supported. The change was not due to an overall decrease in the age of participants over time, to a change in the proportion of males versus females, or to inclusion of graduate students.

However, what evidence is there that the mathematical experiences of these students have changed over time? One potential source of change is the extent to which calculators were used throughout schooling. Many of the individuals ($n=319$) in the age-restricted sample had answered the question: "When did you start using a calculator? (e.g., Grade 6, first year university)". These responses were categorized as follows: elementary school (grades 1 to 6), middle school (grades 7 and 8), or high school and beyond (grades 9 to 13 or university). To evaluate the hypothesis that use of calculators had changed over time, the reports of calculator use were examined in relation to grade 1 year. As shown in Figure 2, students' reports of when they started using a calculator varied with grade 1 year, $\chi^2(6) = 57.44$, $p < .01$. Students whose elementary education started in the early 1980s reported starting to use calculators in high school. This pattern shifted substantially over time, such that the majority of students whose education started in the early 1990s reported starting to use calculators in elementary school. In a multiple regression, the variance in fluency accounted for by onset of calculator use was shared with Grade 1 year. Thus, one contributing factor in the changes in students' arithmetic fluency over time may be the extent to which arithmetic was practiced with the assistance of a calculator. This suggestion is consistent with experimental evidence that practice at generating answers

without a calculator is more effective than practice with a calculator for committing answers to memory (e.g., Crutcher & Healy, 1989; Pyke, LeFevre & Isaacs, 2008; see also Pyke & LeFevre, 2011).

Many of the students in the age-restricted sample ($n = 461$) had also been asked to indicate whether math made them nervous, on a seven-point scale where 1 = very nervous and 7 = not at all nervous. Students who rated themselves as more nervous also had lower fluency scores, as shown in Table 1. In a multiple regression, both nervousness about math and Grade 1 year accounted for unique variance in fluency. The least nervous students had mean fluency scores that were approximately 20 points higher than the most nervous students ($p < .001$). However, even after accounting for the rise in nervousness about mathematics, each increase in the year students started Grade 1 was still associated with a 1.6-point decrease in fluency scores ($p < .001$).

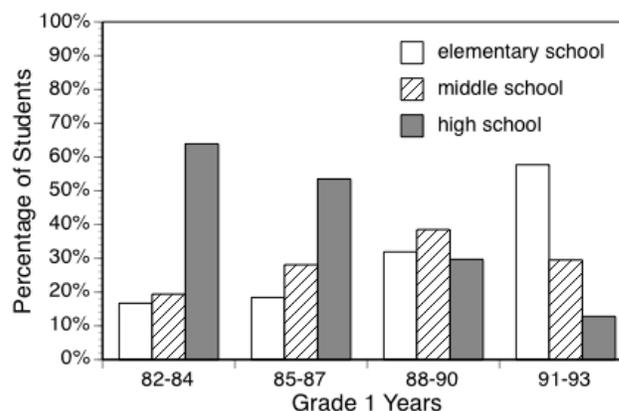


Figure 2. Percentage of students who reported that they started using a calculator in elementary, middle, or high school by year they started Grade 1.

Discussion

The math curricula experienced by North American students has changed dramatically in the last 20 years. One aspect of this change is that emphasis on learning calculation procedures and on drill and practice of arithmetic facts has declined (Star, 2005). The data from the present study are consistent with the view that this change in curricula, coupled with increases in students' calculator use, is reflected in the arithmetic calculation scores of Canadian undergraduates. Students whose elementary training in mathematics occurred largely before major curricular shifts (i.e., from 1982 until approximately 1989) had higher scores on a multi-digit calculation test than students whose elementary education was more strongly influenced by curricular reform (i.e., after 1989). As evidence that the curriculum had changed, students reported using calculators earlier in their schooling as years passed.

We could not test whether this decline in arithmetic fluency was accompanied by any changes in other forms of mathematical knowledge, although for Irish undergraduates, Mulhern and Wylie (2004) found a concomitant decline in

all aspects of mathematical knowledge that they assessed. Notably, arithmetic showed smaller declines than some of the other skills. The change in arithmetic fluency that we observed among undergraduates cannot be attributed to a decline in the overall quality of the academic skills of entering cohorts. The average high school grades of students accepted to the university in this study have actually increased substantially over the same time period, because entrance requirements were deliberately raised. Thus, minimally, we expect a similar level of arithmetic skill across time.

It seems likely that changes in curricula, driven by changes in educators' views of what is most important in mathematical learning, are responsible for these changes in arithmetic fluency. Schoenfeld (2004) described the tumultuous history of mathematics education in the United States: Curricular reform in the 1980s was characterized by over-reaction to a failed 'back to basics' movement in the 1970s that followed the failure of the 'new math' of the 1960s. Schoenfeld notes that in the 1990s and beyond, the U.S. has been engaged in 'Math Wars' wherein traditionalists and reformers are in competition for control of the educational curriculum. All of these upheavals are presumably related to dissatisfaction among educators and the public with mathematics education (see also Latterall, 2005). Most recently, the National Mathematics Advisory Panel in the United States released a report in which concrete suggestions for streamlining and improving mathematics education were outlined (National Mathematics Advisory Panel, 2008). Notably, some of their recommendations relate to establishing fluent arithmetic skills as a foundation for more advanced topics in mathematics.

Are similar factors at work in Canada? According to Robitaille and Sherrill (1980), the new math and back-to-basics movements also influenced Canadian education. Similarly, curricular reforms in the 1990s were implemented at least in part in response to the Standards published by the National Council of Teachers of Mathematics (McDougall, 2000; O'Shea, 2003). Further, there does seem to be ongoing concern about mathematics curricula. In Ontario, for example, where most of the students in the current study were educated, a new mathematics curriculum was developed in 1997 in conjunction with the implementation of province-wide testing of children in mathematics and literacy. The Ontario government commissioned a report in 2003 on early mathematics ("Early Mathematics Strategy", Ontario Ministry of Education, 2003) and in 2005, revised the 1997 math curriculum for grades 1 through 8 (Ontario Ministry of Education, 2005). One interpretation of the current results is that changes in the math curricula in Canada over the past 20 years have shifted emphasis from arithmetic to other aspects of mathematics knowledge. However, it is difficult to link the present results with specific experiences for students because of the shifting views about mathematics education and the consequently variable curricula over the past 20 years (Griffin &

Callingham, 2006). The impact of major curricular shifts on students' performance accumulates slowly. Students who have experienced the revised mathematics curriculum (implemented in 1998) throughout their education in Ontario will only arrive in universities in 2010.

Conclusions

In summary, the arithmetic fluency of Canadian undergraduates has declined significantly over the past twenty years -- by 23%¹. One contributing factor to this decline may have been the earlier introduction of calculators over this same time period. Moreover, comfort with basic mathematics has also declined significantly. These findings have implications for educators and education policy makers at all levels, as well as for student outcomes, in relation to both academics and employment (e.g., Finnie & Meng, 2006; Parsons & Bynner, 1997; Walczyk, 2000; Walczyk & Griffith-Ross, 2006).

One implication of the decline in fluency is that students will find tasks that involve arithmetic relatively more demanding of their cognitive resources than was the case for previous cohorts. Much of the reduced efficiency on multi-digit calculation occurs because the single-digit facts are not memorized. Instead, students use relatively slower and more error-prone procedures such as counting or computing from a known fact (e.g., $9 + 3 = 10, 11, 12$; $6 \times 7 = 6 \times 6 + 7$; Smith-Chant & LeFevre, 2003). In several studies, speed of solving basic arithmetic facts has been correlated with children and adults' performance on mathematics achievement tests (Fuchs et al., 2005; Royer et al., 1999; Zentall, 1990). Accessing arithmetic facts from memory should free working memory resources for other aspects of problem solving (DeStefano & LeFevre, 2004). Accordingly, Imbo and LeFevre (2009) showed that Chinese-educated students, whose fluency scores are, on average, about twice as good as Canadian-educated students, solve computational addition problems (e.g., $34 + 29$) with only minimal requirement for working memory resources. Canadian-educated students, in contrast, required substantial working memory resources to solve these problems. Lack of fluency with basic arithmetic will slow students' comprehension of examples because it forces them to rely on compensatory actions such as pausing, looking back, finding and using a calculator, or needing to search memory to re-activate related information (Walczyk & Griffith-Ross, 2006). Thus, educators at all levels need to be aware of the decline in young adults' arithmetic fluency.

The decline in arithmetic fluency also has implications for students' academic outcomes. Arithmetic fluency is an important foundation for the acquisition of conceptual skills in mathematics and related fields (e.g., science, geography, and technological design). Walczyk and Griffith-Ross (2006) showed that weak computational skills were related

¹ Analysis of additional data collected between 2005 and 2010 showed a further decline in scores, such that the overall decline from 1993 to 2010 was approximately 37%.

to poorer performance (speed and accuracy) for university students solving algebraic inequalities under time pressure (see also Royer et al., 1999, who claim that males' superiority on high stakes mathematics tests can be traced back to their higher levels of arithmetic fluency relative to females). In Walczyk and Griffith-Ross' work, although the students with weaker computational skills could compensate when they had unrestricted time to solve the problems, such compensatory strategies divert cognitive resources that would otherwise be available for learning, especially in situations involving more difficult problems, distracting environments, limited time, or when individuals are not strongly motivated (Walczyk, 2000). On this view, inefficient arithmetic skills may negatively influence students' learning of a variety of subjects in high school and university.

Finally, the decline in arithmetic fluency has implications for students' employment outcomes. Being able to use basic numerical skills in every day life is central to a variety of real world activities required in a variety of employment settings (McCloskey, 2007). Duncan and colleagues (2007) found that early numeracy skills, measured in kindergarten, were better predictors of both literacy and numeracy outcomes than reading scores. Accordingly, numeracy skills, which include arithmetic as well as applied problem solving, are strong predictors of economic success (Parsons & Bynner, 1997; Statistics Canada, 2005; Steen, 1997). Bynner and Parsons (1997) reported that the impact of poor numeracy skills might be greater than that of poor literacy skills. They found that adults in the U.K. with poor numeracy skills (and competent literacy skills) had more difficulty finding and retaining full-time positions, earned lower salaries, and were more likely to be employed in manual trades than adults with competent numeracy skills and poor literacy skills. The growth in mathematics-intensive jobs in the US is outpacing overall job growth by 3:1 (National Mathematics Advisory Panel, 2008). In summary, given the importance of mathematical proficiency and interest in mathematics to the economic outcomes of individuals and the nation, our findings suggest that people with poor numeracy skills may face increasingly greater employment difficulties. Thus, a decline in both arithmetic fluency and comfort with mathematics of young adults has broad implications for the Canadian economy.

Acknowledgments

This research was supported by the Natural Sciences and Engineering Research Council of Canada through grants to Jo-Anne LeFevre and a postgraduate scholarship to Marcie Penner-Wilger. We thank Velian Pandeliev and Alexis Given for their help collating the data.

References

Bynner, J., & Parsons, S. (1997). *Does numeracy matter?* London: The Basic Skills Agency.

- Crutcher, R. J. & Healy, A. F. (1989). Cognitive operations and the generation effect. *Journal of Experimental Psychology: Learning, Memory, and Cognition*, 15, 669-675.
- Das, R., LeFevre, J.-A., & Penner-Wilger, M. (2010). Negative numbers in simple arithmetic. *Quarterly Journal of Experimental Psychology (2006)*, 63(10), 1943-52. doi:10.1080/17470210903564359
- Duncan, G. J., Dowsett, C. J., Claessens, A., Magnuson, K., Huston, A. C., Klebanov, P., et al. (2007). School readiness and later achievement. *Developmental Psychology*, 43(6), 1428-46. doi:10.1037/0012-1649.43.6.1428
- Finnie, R. & Meng, R. (2006). *The Importance of Functional Literacy: Reading and Math Skills and Labour Market Outcomes of High School Drop-outs* (Analytical Studies Branch Research Paper Series 2006275e). Ottawa, ON: Statistics Canada, Analytical Studies Branch.
- French, J. W., Ekstrom, R. B., & Price, I. A. (1963). *Kit of reference tests for cognitive factors*. Princeton, NJ: Educational Testing Service.
- Fuchs, L. S., Fuchs, D., Compton, D. L., Schatschneider, D., Powell, S. R., Seethaler, P. M., Capizzi, A. M., & Fletcher, J. M. (2006). The cognitive correlates of third-grade skill in arithmetic, algorithmic computation, and arithmetic word problems. *Journal of Educational Psychology*, 98, 29-43.
- Griffin, P., & Callingham, R. (2006). A 20-year study of math achievement. *Journal for Research in Mathematics Education*, 37(3), 167-186.
- Imbo, I., & LeFevre, J.-A. (2009). Cultural differences in complex addition: efficient Chinese versus adaptive Belgians and Canadians. *Journal of Experimental Psychology: Learning, Memory, and Cognition*, 35, 1465-1476. doi:10.1037/a0017022
- Kalaman, D. A., & LeFevre, J. (2007). Working memory demands of exact and approximate addition. *European Journal of Cognitive Psychology*, 19, 187-212.
- Latterell, C. M. (2005). *Math wars: A guide for parents and teachers*. Westport, CT: Greenwood/Praeger.
- LeFevre, J. (2003). *Individual differences in working memory and arithmetic*. Unpublished manuscript, Carleton University.
- LeFevre, J., & Morris, J. (1999). More on the relation between division and multiplication in simple arithmetic: Evidence for mediation of division solutions via multiplication. *Memory & Cognition*, 27, 803-812.
- LeFevre, J., Smith-Chant, B. L., Hiscock, K., Daley, K. E., & Morris, J. (2003). Young adults' strategic choices in simple arithmetic: Implications for the development of mathematical representations. In A. J. Baroody, & A. Dowker (Eds.), *The development of arithmetic concepts and skills: Constructing adaptive expertise* (pp. 203-228). Mahwah, NJ: Lawrence Erlbaum Associates.
- McCloskey, M. (2007). Quantitative literacy and developmental dyscalculias. In D. B. Berch & M. M. Mazzocco, (Eds.), *Why is math so hard for some*

- children? *The nature and origins of mathematical learning difficulties and disabilities* (pp. 415-429). Baltimore, MD, US: Paul H. Brookes Publishing.
- McDougall, D. (2000). Impact math: A model for implementing new curriculum. *Orbit*, 31, 51.
- Mulhern, G., & Wylie, J. (2004). Changing levels of numeracy and other core mathematical skills among psychology undergraduates between 1992 and 2002. *British Journal of Psychology*, 95, 355-370.
- Mulhern, G., & Wylie, J. (2006). Mathematical prerequisites for learning statistics in psychology: Assessing core skills of numeracy and mathematical reasoning among undergraduates. *Psychology Learning & Teaching*, 5, 119-132
- National Council of Teachers of Mathematics. (2000). *Principles and standards for school mathematics*. Retrieved May 10, 2006 from <http://standards.nctm.org/document/index.htm>
- National Mathematics Advisory Panel (2008). *Foundations for Success: Report of the National Mathematics Advisory Panel*. Retrieved April 22, 2008 from <http://www.ed.gov/about/bdscomm/list/mathpanel/index.html>
- Ontario Ministry of Education. (2005). *The Ontario curriculum: Grades 1 to 8 mathematics*. Retrieved April 25, 2006 from www.edu.gov.on.ca/eng/curriculum/elementary/math18cur.pdf
- Ontario Ministry of Education. (2003). *Early math strategy: The report of the expert panel on early math in Ontario*. Retrieved April 25, 2006 from www.edu.gov.on.ca/eng/document/reports/math/math.pdf
- O'Shea, T. (2003). The Canadian mathematics curriculum from the new math to the NCTM standards. In G. M. A. Stanic & J. Kilpatrick (Eds.), *A history of school mathematics* (pp. 843-890). New York: National Council of Teachers of Mathematics.
- Parsons, S., & Bynner, J. (1997). Numeracy and employment. *Education and Training*, 39, 43-51.
- Pyke, A. A., & LeFevre, J.-A. (2011). Calculator use need not undermine direct-access ability: The roles of retrieval, calculation, and calculator use in the acquisition of arithmetic facts. *Journal of Educational Psychology*, 103(3), 607-616. doi:10.1037/a0023291
- Pyke, A. A., LeFevre, J., & Isaacs, R. (2008). Why do the math? The impact of calculator use on participants' actual and perceived retention of arithmetic facts. In B. C. Love, K. McRae, & V. M. Sloutsky (Eds.), *Proceedings of the 30th Annual Conference of the Cognitive Science Society* (pp. 811-816). Austin, TX: Cognitive Science Society.
- Ramos-Christian, V., Schlessner, R. & Varn, M. (2008). Math fluency: Accuracy versus speed in preoperational and concrete operational first and second grade children. *Early Childhood Education Journal*, 35, 543-549.
- Rivera-Batiz, F. (1992). Quantitative literacy and the likelihood of employment among young adults in the United States. *Journal of Human Resources*, 27, 313-328.
- Robitaille, D. F., & Sherrill, J. M. (1980). The teaching of mathematics in British Columbia. *Canadian Journal of Education*, 5, 14-26.
- Royer, J. M., Tronsky, L. N., Chan, Y., Jackson, S. J., & Marchant, H., III. (1999). Math-fact retrieval as the cognitive mechanism underlying gender differences in math test performance. *Contemporary Educational Psychology*, 24, 181-266.
- Rustemeyer, R., & Stoeger, H. (2007). Does hand calculator use explain why university students cannot perform elementary arithmetic? *Psychological Reports*, 100, 1270-1272.
- Schoenfeld, A. H. (2004). The math wars. *Educational Policy*, 18, 253-286.
- Siegler, R. S., & Booth, J. L. (2005). Development of numerical estimation: A review. In Campbell, J. I. D. (Ed.), *Handbook of mathematical cognition* (pp. 197-212). New York, NY: Psychology Press.
- Smith-Chant, B. L., & LeFevre, J. (2003). Telling it like it is and doing as they are told: Self-reports in mental arithmetic. *Memory & Cognition*, 31, 516-528.
- Standing, L.G, Sproule, R.A., & Leung, A. (2006). Can Business and Economics students perform elementary arithmetic? *Psychological Reports*, 98, 549-555.
- Star, J. R. (2005). Reconceptualizing procedural knowledge. *Journal for Research in Mathematics Education*, 36, 404-411.
- Statistics Canada. (2005). *Building on our competencies: Canadian results of the international literacy and skills survey*. Retrieved April 26, 2006 from www.statcan.ca/english/freepub/89-617-XIE/89-617-XIE2005001.pdf
- Thorndike, R. L., Hagen, E., & Wright, E. N. (1981). *Canadian Cognitive Abilities Test*. Toronto: Nelson Canada Limited.
- Steen, L. A. (1997). *Why Numbers Count: Quantitative Literacy for Tomorrow's America*. New York: The College Board.
- Walczyk, J. J. (2000). The interplay between automatic and control processes in reading. *Reading Research Quarterly*, 35, 554-566.
- Walczyk, J. J., & Griffith-Ross, D. A. (2006). Time restriction and the linkage between subcomponent efficiency and algebraic inequality success. *Journal of Educational Psychology*, 98, 617-627.
- Zentall, S. (1990). Fact-retrieval automatization and math problem solving by learning disabled, attention-disordered, and normal adolescents. *Journal of Educational Psychology*, 82, 856-865.