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Decomposing the Mean in the Problem-size Effect: An Investigation of Response Time Distributions for a Multiplication Production Task[1]

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Abstract

Response time (RT) distributions for large and small problem sizes were obtained under both speed- and accuracy-stressed instructions in a multiplication production task. Fitting the ex-Gaussian distributional model to the individual RT data allowed for the derivation of quantitative measures of distributional shape. Statistical results indicate that small problem size RT distributions differ from large problem size RT distributions with respect to both the mean of the normal component, μ (larger for large problems) and the size of the tail, τ, (larger for large problems). Accuracy instruction RT distributions also differ from speeded instruction RT distributions with respect to μ (larger under accuracy instructions), and τ (larger under accuracy instructions for large problem sizes only). Results support a strategy-choice explanation of the problem-size effect, and provide suggestions for the comparison of latencies obtained under speed- and accuracy-emphasized instructions.

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Why does it take longer to solve 8 x 9 than 3 x 4? Is speeded arithmetic performance simply faster or fundamentally different? The current analysis examined these questions using the no-report condition of Smith (1996). We undertook a detailed analysis of the problem-size effect and the effect of instructional emphasis on basic arithmetic performance. A response time distributional approach was employed to allow a more thorough evaluation of both these phenomena and existing theories.

Problem-size effect

Larger problems (e.g., 8 x 9) take longer to solve than smaller problems (e.g., 3 x 4). This problem-size effect (Groen & Parkman, 1972) is the most robust phenomenon in mathematical cognition. As such, models must provide an explanation of the problem-size effect. Proponents of direct retrieval have posited that the greater problem frequency of smaller problems gives them an advantage in memory (Ashcraft, 1992), or that differences in the magnitude of large numbers are increasingly difficult to discriminate, complicating retrieval (Campbell, 1995). In contrast, LeFevre, Sadesky & Bisanz (1996) posited that different strategy choices for large and small problems account for the problem-size effect. In LeFevre at al, participants reported using slower procedures involving derived-facts (e.g., 6 x 7 = 6 x 6 + 6) to solve larger problems. The current investigation analyzes response time distributions for small and large problems to provide criteria for evaluating conflicting accounts of the problem-size effect.

Instructional emphasis

Mathematical cognition literature consists of findings from separate labs, using different types of instructions. It is important to determine if separate findings reflect the same cognitive processes and therefore can be directly compared. Reported response times for single-digit arithmetic problems have varied significantly depending on the research laboratory. Campbell (1994) has consistently reported significantly faster response times than LeFevre et al. (1996). Research laboratories differ in the instructions given to participants. Campbell emphasizes speed in order to ensure errors for analysis. LeFevre emphasizes accuracy to maximize valid latencies. The purpose of the current analysis was to determine if speed-emphasized instructions result in a faster, yet similarly shaped distribution of response times, or if the distributional shape was significantly altered under different instructions, suggesting a change in the nature of the task.

Ex-Gaussian distribution

Response time distributions are positively skewed. For the purpose of analyses, some researchers assume response times are normally distributed, and use mean or median response times. Alternatively, researchers trim their extreme scores or perform scale transformations to achieve a normal distribution. Data is trimmed under the assumption that the true underlying distribution is normal and that extreme scores represent nuisance variables. However, it is important to consider that the true distribution may in fact be skewed. If the skew were representative of the underlying process being studied, trimming the data would hamper research. Transforming the data also reduces the meaningfulness of the findings. However, methods to extract quantitative measures of a non-normal distribution are not available in traditional statistical packages.

The ex-Gaussian distribution (Hohle, 1965) provides a good fit to response time data and allows three quantitative measures to be obtained. The ex-Gaussian distribution consists of a normal (Gaussian) component and an exponential tail. Measures derived from this distribution include: mu (μ) the mean of the normal component, sigma (σ) the standard deviation of the normal component, and tau (τ) the mean of the exponential tail. Fitting the ex-Gaussian distribution to response time data provides a more detailed analysis than that provided by examining the means alone. Heathcote (1996) has developed a statistical package, RTSYS, which allows researchers to obtain quantitative measures of response time distributions: mu, sigma and tau. Analyses performed using RTSYS have provided insight into cognitive processing, and allowed for stringent evaluation of existing theories of response time phenomena in a variety of areas (Heathcote et al., 1991; Leth-Steensen et al., 2000).

Current analysis

The primary focus of the current analysis was to provide a more detailed account of the problem-size effect by fitting the ex-Gaussian distributional model to response time distributions. We expected that this detailed analysis, employing ex-Gaussian parameters, would result in a better understanding of the cognitive processes involved in the problem-size effect. As such, the results would be a useful tool for evaluating current theories of cognitive arithmetic. Specifically, we hypothesized that the size of the exponential tail of the distribution would increase with problem size. This result would be expected if larger problems were solved using less efficient strategies as found by LeFevre et al. (1996).

Response time distributions were also examined under both speed- and accuracy-stressed instructions. The analysis of instructional conditions was undertaken to determine if instructions change the underlying processes and render these different tasks. We expected to find that the mean of the normal component was smaller under speed-emphasized instructions, showing a shift to faster performance. We hypothesized that the size of the exponential tail of the distribution would be longer in the accuracy condition. This result would be expected if participants were checking problems (particularly for large problems) using strategies other than direct retrieval, as suggested by LeFevre et al. (1996).

Method

Data used in the present experiment constitutes the no-report condition from Smith-Chant (1996).

Participants

Thirty-two introductory-psychology students (16 men and 16 women, median age 22 years) participated for course credit or a $12 honorarium. The results presented in the current paper will focus primarily on twenty-one of the 32 original participants. Nine participants were eliminated from the present analysis because the ex-Gaussian distribution failed to fit their response times (most often due to bi-modality of their distributions in at least one of the four experimental conditions). A further participant was eliminated because the chi-square test for the goodness of the ex-Gaussian fit was highly significant for three of the four conditions. A final participant was eliminated due to a highly irregular sigma value in the speed-emphasized, large problem condition (sigma=756.7 with the next largest value in that condition being 159.4). For the remaining twenty-one participants, the ex-Gaussian distribution provided a good fit.

Design and Procedure

The current analysis constitutes a 2 (instructional emphasis: speed, accuracy) x 2 (problem size: large, small) repeated-measures design.

Materials. The set of multiplication problems consisted of all single-digit problems from 2x2 through 9x9, presented twice in each condition. Stimuli were presented in amber on a black computer screen. Two presentation-order lists were created. The order of problems was randomized such that each problem occurred only once in each half of the list. Participants solved the 128-problem set once with accuracy-emphasized instructions and once with speed-emphasized instructions (see appendix A). The order of the instructions (speed/accuracy) was counterbalanced across participants.

Multiplication production task. Participants were seated in front of a computer monitor and wore headphones with an attached microphone. Participants viewed the multiplication problem on the screen and answered vocally. The computer recorded response times, from the presentation of the stimulus until a verbal response was made. In each condition, participants were given ten practice trials before beginning the experimental trials.

Results and Discussion

Dependent measures included: mean response time, standard deviation, and the three ex-Gaussian parameters obtained by fitting the ex-Gaussian distribution to the individual participant data in each of the four experimental conditions. 2 (instructional emphasis: speed, accuracy) x 2 (problem size: large, small) repeated-measures ANOVAs were performed for each dependent measure. Means for each measure are reported in Table 1.

Mean response time

Participants took longer to solve problems under accuracy-emphasized instructions than under speed-emphasized instructions (1121 vs. 938 ms), $F(1, 20)=43.90$, $MSE=15893.8$, $p<.001$. Thus speeded instructions had the desired effect, speeding performance. As expected, the problem-size effect is present in mean response time data. Participants took longer to solve large problems than small problems, (1173 vs. 885 ms), $\underline{F}$ (1, 20)=45.54, MSE=38221.1, $p<.001$. As shown in Figure 1, problem size interacted with instructional-emphasis such that large problems took disproportionately longer to solve under accuracy-emphasized instructions than under speed-emphasized instructions, $\underline{F}$ (1, 20)=14.46, MSE=6110.0, $p<.01$. This interaction is consistent with the hypothesis that answers to large problems are being checked with less efficient procedures (e.g., repeated addition, derived-facts).

Standard Deviation

Participant's standard deviations were larger under accuracy-emphasized instructions than under speed-emphasized instructions (380 vs. 290), $\underline{F}$ (1, 20)=13.06, MSE=12927.2, $p<.01$. Standard deviations were larger for large problems than small problems, (439 vs. 232), $\underline{F}$ (1, 20)=30.91, MSE=28999.5, $p<.001$. As shown in Figure 1, problem size interacted with instructional-emphasis such that large problems had disproportionately larger standard deviations under accuracy-emphasized instructions than under speed-emphasized instructions, $\underline{F}$ (1, 20)=4.46, MSE=21530.2, $p<.05$.

Accuracy

Participants had higher accuracy rates under accuracy-emphasized instructions than under speed-emphasized instructions (93 vs. 89%), $\underline{F}$ (1, 20)=10.09, MSE=32.9, $p<.001$. Accuracy rates were lower for large problems than small problems, (86 vs. 97%), $\underline{F}$ (1, 20)=39.21, MSE=68.2, $p<.01$, as shown in Figure 1.

Mu

The mean of the normal component of the fitted ex-Gaussian distributions was larger under accuracy-emphasized instructions than under speed-emphasized instructions (779 vs. 682 ms), F (1, 20)=31.36, MSE=6251.0, p<.001. The constant 100 ms decrease in mu, shown in Figures 4 and 5 for Vincentized data, under speed-emphasized instructions for small and large problems suggests a constant factor such as motor response (speech) is being affected, but that cognitive processing remains the same. Mu was larger for large problems than small problems, (775 vs. 687 ms), F (1, 20)=25.61, MSE=6333.4, p<.001. Thus the problem-size effect is observed in mu. In contrast to the analysis of mean response times, problem size did not interact with instructional emphasis for mu, as shown in Figure 1. The lack of an interaction in mu suggests that the interaction present in mean response times is a result of greater skew of the response time distribution rather than a shift in the mode.

Sigma

The standard deviation of the normal component of the fitted ex-Gaussian distributions was larger for large problems than small problems, (90 vs. 56), F (1, 20)=9.60, MSE=2496.3, p<.01. In contrast to the analysis of standard deviations, no main effects or interactions were associated with instructional emphasis, as shown in Figure 1.

Tau

As shown in Figure 2, the size of the tail of the fitted ex-Gaussian distributions was larger under accuracy-emphasized instructions than under speed-emphasized instructions (344 vs. 257 ms), F (1, 20)=13.21, MSE=12294.1, p<.01. As hypothesized, this result suggests participants were checking problems in the accuracy condition. Tau was larger for large problems than small problems, (401 vs. 199 ms), F (1, 20)=32.37, MSE=26583.4, p<.001. This result is consistent with the theory that larger problems were solved using less efficient strategies. As shown in Figure 1, problem size interacted with instructional-emphasis such that the tail of the distribution was disproportionately larger under accuracy-emphasized instructions than under speed-emphasized instructions, $\underline{F}$ (1, 20)=9.84, MSE=10048.0, p<.01. This finding is consistent with checking of answers, under accuracy-emphasized instructions, with procedures such as repeated addition or derived facts, which would be more time consuming for larger problems.

General Discussion

The fitting of an ex-Gaussian distributional model to response time data was successful in adding to our knowledge of the problem size effect and effect of instructional emphasis. The present analysis lends support to a strategy-choice definition of the problem-size effect. Tau in particular seems to be a reflection of strategy choice. This information may be useful in developing an alternative indicator of strategy use instead of the controversial self-report method currently used.

The examination of the effects of instructional emphasis show that findings under different types of instructions are comparable, taking into account the 100 ms shift, and processes occurring after an answer is obtained under accuracy-emphasized instructions (i.e., answer checking). This finding is encouraging, as the mathematical cognition literature is comprised of results obtained under various types of instructions.

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Table 1

Dependent Measures by instruction type for small and large problems.

	Accuracy Instructions		Speed Instructions	
Measure	Small	Large	Small	Large
Mean	944.6	1297.4	827.2	1050.3
SD	243.4	517.7	221.4	360.3
Mu	735.5	823.4	638.8	726.7
Sigma	63.6	90.4	50.0	90.7
Tau	209.2	480.2	189.8	323.7

Appendix A

Instructions for speed and accuracy conditions, Smith-Chant (1996)

Speed. You are being tested on how quickly you can solve simple multiplication problems. First, you will see an asterisk in the centre of the computer screen. The asterisk will begin to flash. This signals that the asterisk will be replaced by a single digit multiplication problem like '3 x 4'.

I would like you to say the answer as quickly as you possibly can, without making any mistakes. Occasional mistakes are normal when people go fast, so do not be too concerned if you make a mistake. It is important that you respond as quickly as possible.

Accuracy. You are being tested on how accurately you can solve simple multiplication problems. First, you will see an asterisk in the centre of the computer screen. The asterisk will begin to flash. This signals that the asterisk will be replaced by a single digit multiplication problem like '3 x 4'.

I would like you to say the answer as quickly as you possibly can, without making any mistakes. Occasional errors are normal, but please try to avoid making mistakes. It is important that you respond as accurately as possible.

Figure 1. Traditional and ex-Gaussian measures as a function of problem size for accuracy- and speed-emphasized instructions

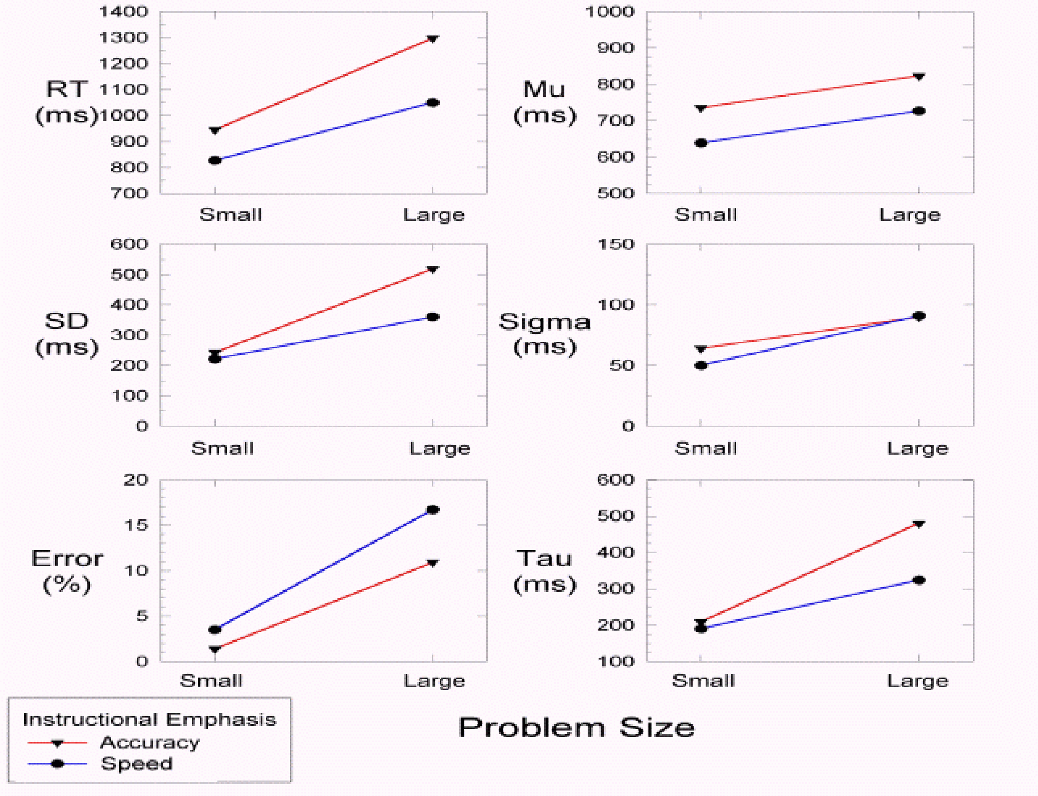

Figure 2. Vincentized response time histograms for small and large problems under speed- and accuracy-emphasized instructions with ex-Gaussian model fitted (curve shown)