# Of computations and dynamic systems - An overview of the dynamicist controversy in cognitive science[1]

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***- Abstract -***

This paper constitutes an overview of two competing conceptual frameworks in the study of cognition, the now standard computational approach and the more recent, and controversial, dynamical hypothesis in cognitive science, championed by T. van Gelder et al. Through such conceptual and methodological disputes about the nature of cognition, a debate about the adequacy of their respective models has been the main ground for disagreements. I propose to explore each framework, or paradigm, in turn, by focusing on their definition and use of a number of critical characteristics of intelligent behavior, namely that of representations, computation, and exactly what is a cognitive feature or process. The conclusions that I have reached are twofold: firstly, the dynamicist view of the computational approach to cognition in no way discredits its relevance to cognitive modeling, since dynamicists are not concerned with the same features of mental processes in their models, and their evaluation of what counts as computational is based on a common misconception, namely a confusion between the abstract and formal concept of computation with that of physical symbol systems. Secondly, the type of explanation used by the dynamicist view is quite different, for it concerns nomological explanations (i.e. explanations through covering laws), whereas the computational view frames its explanations in a mechanistic manner.

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ANN: artificial neural network

CH: computational hypothesis

CHCS: computational hypothesis in cognitive science

CNS: central nervous system

CS: cognitive system

DH: dynamical hypothesis

DHCS: dynamical hypothesis in cognitive science

MCS: mathematical computational system

MDS: mathematical dynamical system

ODE: ordinary differential equations

PDE: partial differential equations

RDS: real dynamical system

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## *- Introduction - Of models and minds*

**Model (abstract)**

**From Wikipedia, the free encyclopedia.**

*An abstract* ***model*** *(or conceptual model) is a theoretical construct that represents physical, biological or social processes, with a set of variables and a set of logical and quantitative relationships between them. Models in this sense are constructed to enable reasoning within an idealized logical framework about these processes and are an important component of scientific theories. Idealized here means that the model may make explicit assumptions that are known to be false in some detail, but by their simplification of the model allow the production of acceptably accurate solutions […]*

What is a conceptual framework? Epistemologists and philosophers of science may not agree on the minutiae of the concepts of concept, knowledge and science, but we can roughly sketch an uncontroversial canvas: it is a set of concepts and methods through which people generate conjectures and theses, and strive to produce descriptions, explanations, and predictions about entities, events, and phenomena.[2] Since we would rather conceptualize and explain phenomena in an interesting manner, that is, with teleological considerations such as accuracy, efficiency, and consistency, many constraints have to be taken into account to establish what constitutes a successful conceptual framework. Science, unarguably the most demanding conceptual and methodological endeavour in the pursuit of knowledge, has numerous constraints through which are filtered what are considered acceptable theses, methods, models, and what counts as evidence. Among these constraints, modern philosophy of science commits us to two universal tenets, (i) an ontological commitment dubbed *materialism*, which states that science's domain is the material world and that it should not bother itself with spiritual or religious phenomena, and (ii) an epistemological claim named *naturalism*, the view that valid explanations or theories ought to make use of, *and only of*, entities accessible to natural science. Much is to be said about and within epistemology and philosophy of science, such as whether or not paradigms are commensurable and continuous, or if there exist radical shifts in conceptual frameworks, on the value of reductionism in view of scientific claims of different levels of description, and on what counts as criteria of justification or proof for such scientific claims.

Allowing myself the luxury of a metaphor, conceptual frameworks can be seen as universes of discourse, following in that the semantic theories in the philosophy of language, spawned from the works of Frege, Peirce, and their successors of the analytic tradition. One semantic view of analytic philosophy, notably, attributes meaning and truth value to propositions validated by models, *viz.* a larger set of propositions mirroring entities and relations between them, that

[2] Here I use the term of conceptual framework in a broader sense than a simple matrix of concepts and relations, so as to include models and methods, akin to Kuhn's concept of paradigm.

represents a 'possible world', or complete state of things. Thus we could view the scientific discourse concerning biology, for example, as a universe or domain of discourse in terms of semantics, and different theories and models of evolutionary biology, such as phyletic gradualism, punctuated equilibria, and creationist models (if there are any that can achieve a reasonable degree of rigor) would validate or invalidate propositions made about the entities, and relations between such entities, of the biotic realm[3]. Philosophy of language or otherwise, the emphasis is that models involve a set of entities and relations by which they purport to accurately describe, explain (and make predictions about) the phenomena under enquiry.

From a somewhat direct lineage of ancestors such as cybernetics, information theory and the study of algorithms in mathematics, computationalism has established itself as a predominant conceptual framework to deal with enquiries concerned with what we understand by intelligence. Computationalism is our more recent conception of intelligence, the view that cognition can be understood as information processing, and has spun models of intelligence inspired by information processing technologies. It reaches as far as the study of biological cognition and even the whole of life sciences altogether, ubiquitous in a way that finds its way into the labelling of our era, the Information Age.

Numerous models of cognition as information processing under the guise of computationalism have been suggested, from the already classic seminal works of Turing (1936) on formal, discrete and machine-like computation, and Rumelhart, McClelland *et al* (1986) on parallel and distributed, brain-like computation, to their philosophical critics and promoters like Fodor and Pylyshyn (1988), and P.M. Churchland (1989, among many other references). Yet, for some skeptics, computationalism is mainly concerned with *simulations* of informational processes, and while it doesn't seem to be controversial for the purpose of developing 'intelligent' devices and technologies, it's being considered as the basis for such

[3] I chose the semantic conception of a model for its simplicity and scope, but the issue is not unproblematic in the details, and much of this paper revolves around the very minutiae of models in cognitive science.

*models* of cognition does not appeal to everyone. Thus, computationalism has been challenged on nearly all possible grounds, with regards to its structure, its constituents and foundations, and its ability to stand for as a qualitative and/or quantitative model of what is meant by cognition (among others, Brooks 1991, Clark 1992, 1998, Dreyfus 1992, Elman 1998, Freeman & Nuñez 1999, Giunti 1995, Piccinini 2003, Stufflebeam 1998, Thelen 1995, van Gelder & Port 1995, Wertsch 1998).

Criticizing is one thing, proposing solutions is another. Has anyone come forth with an alternative framework that might deal with the shortcomings of computationalism and yet bear as much, if more, explanatory and predictive power as required of a rigorous scientific endeavor? Some believe so, and the answer might come from a rather physicalist perspective (it certainly doesn't get any more natural, as in what we mean by naturalism and the naturalization of cognition), that is, the theory of systems dynamics. From the conceptual framework of dynamical systems theory and with the help of its formal and quantitative counterpart, namely dynamical modeling, I will try to assess their position by confronting what as been dubbed the dynamical hypothesis[4] (DH hereafter) about cognition, with the dominant yet quite problematic computational hypothesis (CH hereafter) about cognition.

To this end, the present dissertation is divided in four chapters. The first presents a quick overview of what the computational and dynamical frameworks are, what kind of characteristics and ambiguities define and populate such frameworks, as well as what is entailed by adopting a computational hypothesis in cognitive science (CHCS hereafter), or the dynamical hypothesis in cognitive science (DHCS hereafter). The second chapter is an incursion into the cognitive science of sensorimotor processes, which exposes the application of the previously defined frameworks to empirical research. Through an attempt to link the dynamicists' allegedly revolutionary point of view with neuroscientic findings on the workings of a certain class of low-level cognitive processes, namely sensorimotor control and

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[4] T. van Gelder (1995, and subsequent work).

learning, we will then be able to understand the full extent of the claims of both frameworks on such evidence. This will also make possible a clear, concise ground on which to position ourselves in further characterization of the issues at hand. The third chapter presents formal shortcomings and technical issues of both the CHCS and the DHCS, from areas as different as mathematics and neuroscience. The definitions of computation established in the first chapter, as well as the dynamicist's conceptual repertoire, will be confronted with formal considerations and empirical evidence. To this end, the chapter exposes criticisms, objections, and answers from protagonists of both frameworks, concerning the advantages and shortcomings of their respective stance in cognitive science. The fourth and final chapter presents the controversial class of models that is connectionism. Since proponents of both frameworks insist on claiming connectionism as part of their own view of cognition, the entire chapter is devoted to the clarification of what is at stake in connectionist models, both formally and empirically. In the conclusion, I will attempt to synthesize critical issues of, and possible answers to, the clash of such conceptual frameworks, driven by a scrupulous desire for univocal concepts. I advocate the adoption of a rigorous vocabulary concerning cognitive, computational, and dynamical themes, a point that unfortunately needs to be emphasized notwithstanding its ubiquity in the requirements of a sound academic enterprise. The fact of the matter is that throughout this thesis, I aim to expose a number of incorrectly defended positions criticizing computationalism and promoting dynamics, and such misconceptions undermine the authority of the supported arguments in a way that requires us to redefine the relative advantages, limitations, relevance, and scope of both conceptual schemes.

The debate on whether a dynamical framework is preferable to a computational one can be developed on many avenues, and I have chosen to emphasize epistemological and semantic issues. This decision is based on two observations, namely (i) since, as it is exposed in chapter III, a solution to the disagreements between these two frameworks may be found in the type of explanation held dearest by their respective protagonists, we may assume that a discussion of ontological and formal issues would not focus on the essential divergences between the CHCS and the DHCS that this dissertation aims to disclose, and (ii) such ontological considerations about the nature of cognition, and formal issues in the mathematical treatment of cognitive modeling, would be sufficient grounds to motivate the writing of two other extensive dissertations altogether. Therefore, while this dissertation indeed exploits qualitative and quantitative mathematical issues, as well as essential ontological topics about the mind, all such considerations are secondary to the main line of argumentation. The question of which of the CHCS or the DHCS constitutes the best possible explanatory framework may not be entirely independent of formal and ontological issues, it is nevertheless in a noncommittal stance on such questions that I intend to conduct my examination.

It is worth noting that while I have undertaken an assessment of the conflicting views of the computational and dynamical frameworks, I do not pretend that this schism is 'the' most fundamental issue at hand in cognitive science, with respects to models. For the sake of discussion, I will subsume the biophysical models of neuroscience to the dynamical view (for it is indeed concerned with systems dynamics, and written in the language of calculus), and such models will be a central issue in many parts of the following discussion. The particular status of connectionism will also be addressed along the way, and as it turns out, it will be a critical element in the assessment of the two paradigms' claims, but I won't commit myself to its characterization *yet*. It is less a matter of introducing some element of suspense for the reader, than a preoccupation with mathematical issues that do not lend themselves to a casual overview.

# *- I - Computational and dynamical systems*

## *- I.I - Computation and the computational hypothesis in cognitive science*

**Calculation**

**From Wikipedia, the free encyclopedia.**

*A **calculation** is a deliberate process for transforming one or more inputs into one or more results. The term is used in a variety of senses, from the very definite arithmetical calculation using an algorithm to the vague heuristics of calculating a strategy in a competition or calculating the chance of a successful relationship between two people […]*

**Computation**

**From Wikipedia, the free encyclopedia.**

***Computation** can be defined as finding a solution to a problem from given inputs by means of an algorithm. This is what the theory of computation, a subfield of computer science and mathematics, deals with. For thousands of years, computing was done with pen and paper, or chalk and slate, or mentally, sometimes with the aid of tables […]*

In order to give a fair treatment to the debate between the computational framework in cognitive science and its more recent contender, the dynamical framework, we firstly have to present the two positions in some detail, concerning their technicalities and their history. We thus begin with the dominant view, that of computation. Exactly what is it that the computational hypothesis entails in the realm of cognitive science? In order to answer that question, we have to firstly characterize the theory of computation, secondly, distinguish two complementary yet different conceptions of computation, and thirdly, link this theory to the exploration of cognition. It will also become evident that the term 'computational' refers to a great many things, and perhaps unsurprisingly as such, since its mere formal origins portrayed the concept in a vague, abstract sense.

***- I.I.I - Origins***

Mathematics spawned the concept of computation. The issue at hand, at the dawn of the twentieth century, was the question of which formal problems could be solved, and which couldn't be. Thus a formal method of analysis had to be developed to this end. The issue wasn't trivial at all: defining which set of relations could be solved was quintessential to formal analysis, and to all of the quantitative sciences using such formalisms. Science being dependent on mathematics, defining the class of problems that could in principle be effectively and quantitatively formalized was no mere undertaking. But just what is a formally solvable problem? Fregean logic (and its successors) and most if not all of mathematics model interesting relations as functional relations between arguments and values. As odd as it may appear, the very definition of a function is not that old, it was coined by Leibniz in the late seventeenth century in his development of calculus. Euler later (middle of the eighteenth century) extended the concept to all expressions composed of arguments. When Weierstrass suggested the adoption of arithmetic as a basis for calculus rather than geometry, in the late nineteenth century, Euler's conception of a function took over the entire field of mathematics. Thus, functions are a special subset of relations, linking each element of a set to a unique element of another (or the same) set.[5] Such relations permit effective quantitative analysis and, by extension, effective and workable science.

As we mentioned at the beginning of this chapter, the most abstract definition of computation involves a procedure by which one finds a solution to a problem, given one or more input values (or initial conditions in a broader sense). This procedure is commonly named *algorithm*, and further characterized as a finite and well-defined set of instructions that will produce an equally well-defined result. Mathematicians thus devised models to meet the challenge of computable problems, of which the effective characterization is to be treated as functions and arguments. It was eventually assessed by Alonzo Church (1936ab) and Alan Turing (1936) that the class of computable functions is equivalent to the class of functions defined by the following models:

- recursive functions
- lambda calculus

and that class of computable functions is also definable as algorithms calculable by:

- Markov algorithms
- register machines
- Post systems
- Turing machines[6]

In terms of computation, the preceding formalisms, and algorithms operating over such formal languages, were shown to be equivalent in computational power. In other words, any and all computations that can be 'performed' through one formalism, can in principle also be performed through any other. For the sake of mathematical enquiry, that means that many classes of problems are effectively computable, spanning from partial functions to computable complex numbers, and

[5] Formal definition of a function: a function $f$ from a set $X$ of input values to a set $Y$ of possible output values (written as $f : X \to Y$) is a relation between $X$ and $Y$ which satisfies:

1. $f$ is *total*, or *entire*: for all $x$ in $X$, there exists a $y$ in $Y$ such that $x\,f\,y$ ($x$ is $f$-related to $y$), i.e. for each input value, there is at least one output value in $Y$.
2. $f$ is *many-to-one*, or *functional*: if $x\,f\,y$ and $x\,f\,z$, then $y = z$. i.e., many input values can be related to one output value, but one input value cannot be related to many output values.

[6] Appendix I describes in details the class of computable functions and its equivalents.

find applications even into chaos and quantum related problems. On the other hand, many more magnitudes of formal problems involved in the construction of mathematical proofs and mind-boggling numbers are said to be uncomputable, for a variety of reasons, such as uncountability, computational complexity classes, or the apparent impossibility of subsumption of interesting phenomena under a deterministic formalism (for certain areas of applied mathematics), *et caetera*. This is a very important issue in computability theory with regard to the rest of this essay, for part of the controversy at hand between computational and dynamical enthusiasts is the relevance and scope of the formal tools of computability theory with respect to cognitive science.

***- I.I.II - Queering up the concept of computation***

It is worth mentioning, if not essential to underline the computational equivalence in power of computable functions, and the algorithms defined over them, to the familiar digital computer, with the relevant yet secondary criterion of requiring infinite memory in the definition of an abstract computer, by opposition to the finite constraints of implemented computational devices. The Turing machine is an abstract model of an algorithm which can calculate any and all of the computable functions. But the concept of computation itself is very large, and while a recursive function is calculable by a Turing machine, these two concepts are not identical. Recursive functions are the class of functions, from natural numbers to natural numbers, that are computable, but they are a matter of discrete mathematics almost exclusively, namely number theory and combinatorics (the issue of computable reals and complex numbers will be raised in chapter III). The study of algorithms that are Turing Machines is an extension of applied mathematics and computer science, and as such concern both empirical and formal matters. Thus we can draw a first distinction between the concepts of computable functions (a strictly formal, mathematical concept), their computational equivalent classes (formal systems such as programming languages in computer science, generative grammars in linguistics, *etc*), and the algorithms defined over Turing Machines, which concern implementation issues and are as such beyond the scope of an exclusively formal account. Yet another contrast worth mentioning, related to the aforementioned distinction between abstract and material computers, is between universal Turing machines or UTMs, a definition of an algorithm given infinite resources of calculation, and digital computers, the actual implementation of such abstract devices. For the purpose of clarification, further references to systems based on computation will either call upon a MCS (a mathematical computational system, pertaining to formal models), or a RCS (a real computational system, *viz.* a system which actually performs computations).

We thus draw an elementary distinction between a first, permissive but trivial, formal definition of computability (large computation, *viz.* anything effectively represented as recursive), versus the narrow, and additionally empirical, concept of Turing-computation[7] (symbolic, discrete, and serial computation, both abstract and implemented). Of particular interest to us, then, the Church-Turing thesis as it is commonly named, thus concerns the nature of mechanical devices, beyond mathematical problems. Following Turing, "*Every function which would naturally be regarded as computable can be computed by a Turing machine.*" (Turing, 1936, p. 230) As fundamental as it is, this thesis can *not* be proven or disproven by formal means, since the concept of computable function used in the formulation is too vague. Some view the Church-Turing thesis as a physical law (*viz.* a nomological statement), since it can't be mathematically or logically proven.

Perhaps the problem lies in the fact that the concept of computation, in Turing's sense, relies on another equally vague concept, that of algorithm. A coarse characterization of an algorithm states that: (i) the algorithm consists of a finite set of simple and precise instructions that are described with a finite number of symbols, (ii) the algorithm will always produce the result in a finite number of steps, (iii) the algorithm can in principle be carried out by a human being with only paper and

[7] Hereafter, I shall use the term of (symbolic) Turing-computation to refer to symbolic computation, and computation or computability to refer to the class of computable functions, as is understood by Turing computability.

pencil, and (iv) the execution of the algorithm requires no intelligence of the human being except that which is needed to understand and execute the instructions.[8] Now, as intuitive as it may be, the concept of algorithm is not formally defined, since there is no means to so characterize what is meant by ‘simple and precise instructions’ and ‘required intelligence for the execution of the instructions’.

Those conceptual vagaries, as harmless as they may seem for matters of mathematics and information sciences, are in my opinion not only the source of much of the confusion in the clashes of the many proponents of computationalism in areas such as cybernetics, early cognitive psychology and classic artificial intelligence[9], but also one critical point of dissension between computationalists and dynamicists. Such distinctions in matters of computation will thus be essential in the following discussion.

***- I.I.III - Cognition as computation***

What does all of this have to do with cognition? The computational hypothesis in cognitive science (CHCS), the dominant conceptual framework in cognitive science, is based on the complementary theses of (i) *functionalism*, roughly, the philosophical idea that mental states are functional states (the ontological commitment of cognitive functionalism), and can thus be accounted for without taking into account the underlying physical substrate, but instead by attending to (here, representation-laden)[10] functional states (the epistemological part of functionalism), and (ii) *cognitivism*, the epistemological position in the philosophy of mind which argues that mental functions can be understood by quantitative, positivist and scientific methods (for instance, that such functions can be described through information processing models for the sake of psychological modeling).

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8 A. Markov (1960).

9 Also coined ‘GOFAI’, for good old fashioned artificial intelligence, by John Haugeland (1985, 1997).

10 Most important in the case of the CHCS, a notion that we will explore further in chapter 3.

Intelligent behavior had been, from the early years of the 20th century until the 1950s, studied under the dominating paradigm of behaviorism, a strict empirical approach mainly concerned with the naturalization of human activity through external observation. Heralds and leaders of this paradigm indeed dominated the North-American academic scene during the first half of the 20th century, with figures such as Watson (1913) and Skinner (1938) in psychology, Bloomfield (1933) in linguistics, and the associated endeavors of Carnap (1966) and Hempel (1966), in the form of logical positivism in philosophy. But the shortcomings of its methodology and concepts, along with the evolution of ideas in novel research areas, gave way to the rise of cognitivism. Daring and far reaching projects shifted the obsession with externalism to inward and mechanistic exploration, spawning cybernetics[11], information science[12], cognitive linguistics[13], and the foundations of artificial intelligence[14], to name a few (This list is by no means exhaustive). Further works along interdisciplinary boundaries, concerning conceptual and methodological issues, have since then both enriched and plagued the computational view.

The functionalist thesis stated above is foundational in cognitive science, as mental events are to be distinguished from the physical substrate on some ground (be it properties, if not in terms of substance), albeit to some degree of sophistication that has evolved beyond the traditional philosophical divide imposed by Descartes. Indeed, 20th century sciences of mind and behavior thrived to come to terms with what we call the mind-body problem, but could not escape the boundaries imposed by our intuitions on the matter. That is precisely what led to the adoption of cognitivism: the adoption of a scientifically rooted view of the mind, founded on a functionalist stance, and drawing upon information science to model language,

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11 championed by Rosenblueth, Wiener, and Bigelow (1943, also Wiener, 1948), von Neumann (Aspray & Burks, 1987, for a collection of papers), McCulloch and Pitts (1943)

12 Turing (1950) and Shannon (1948)

13 Chomsky (1957, 1968, among others)

14 pioneered by Newell and Simon (1956, also Newell, 1980), Minsky and Papert (1969, also Minksy, 1968)

memory, perception, sensorimotor processes, *etc.*, or in fewer words (but in a coarse way), everything relating to intelligent behavior.

Most endeavors traded on higher levels of cognitive processes, such as semantics, deliberation, decision-making, *et caetera*, like the works of Chomsky and his generative grammars (1968), Fodor and his language of thought (1975), and some of the abovementioned thinkers in artificial intelligence, too name a few. Such proponents are usually dubbed 'symbolicists', since they championed philosophical and scientific views of cognition in the raw and formal way of Turing, namely the view of cognition as the manipulation of symbols, in the likeness of a universal or implemented Turing machine, exhibiting characteristics such as discreteness, seriality, and intentional (representational) contents individuated through semantic properties.

Thus did computationalism provide a framework for cognitive science that could account for mental phenomena in many advantageous avenues:

- the age-old problem of the separation between mind and body, which had been made quite popular in philosophy through the works of Descartes, was tossed aside through a functionalist conceptualization, abandoning a substantiative conception of everything mental to the benefit of a 'mind is to the brain as what the software is to the computer' stance, in an effort to naturalize cognition,
- a formal account of cognition was able to link such mental-related faculties like language use, logico-mathematical abilities, memory, categorization, *et cetera*, with the machine-like conception of a Turing machine, *viz.* the implementation of an effective, formal and generalizable procedure meant to carry out operations on functions and arguments,
- the formal and technical properties of computational models were meant to reflect cognitive ones, including
    - o the representational nature of mental tokens, which exhibited intentional, content-bearing states, much like language. The symbolic

aspect of Turing-like computation embraced by symbolicists was seen as an essential property of high-level cognition, although the case of lower-level cognition would eventually challenge such a restrictive take on computation,

- the discreteness and seriality of a Turing machine-inspired conception of cognition also seemed to fit well the aforementioned mental faculties of language and logico-mathematical performance. Turing machines (and common digital computers) process information in a serial way (successive operations), over discrete (distinct, non continuous) values, the content of which is individuated by a representational relation, from symbol to object. This relation is thus conventional, arbitrary.

Other thinkers slowly but surely championed alternative views, of which connectionism is the most popular inheritor. On grounds of psychological plausibility, the parallel and distributed nature of information processing in the brain, and the implausibility of content individuation through discrete, symbolic tokens in a significant manner even for simulated cognition, some theorists[15] resurrected the low profile heritage in cognitive neuroscience of individuals such as the abovementioned McCulloch and Pitts (1943), Hebb (1949), and Rosenblatt (1962), to name a few. This would lead to a radical turn in computational modeling, and the pretences of artificial intelligence would thereafter be severely modified. The issue of whether some sophisticated models of connectionism have more to do with computation or dynamical models will be examined throughout this paper, for it has been raised as an argument to support the claims of protagonists of both frameworks. Chapter IV examines the issue of connectionism comprehensively, and with more minutiae.

[15] like Churchland (1986) and Churchland (1989), Rumelhart and McClelland (1986, also Rumelhart, 1989), and Smolensky (1988, 1989)

## *- I.II - Dynamics and the dynamical hypothesis in cognitive science*

**Dynamical system**

**From Wikipedia, the free encyclopedia.**

*In engineering and mathematics, a* ***dynamical system*** *is a deterministic process in which a function's value changes over time according to a rule that is defined in terms of the function's current value […]*

**Dynamics (mechanics)**

**From Wikipedia, the free encyclopedia.**

*In mathematics and physics,* ***dynamics*** *is the branch of mechanics that is concerned with the effects of forces on the motion of objects […]*

The proponents of the dynamical approach to cognitive science are dissatisfied with the dominant view of cognition as computation. Some suggest a radical paradigm shift, pretending that dynamical systems theory and dynamical modeling, inconsistent with the computational view, bear all of the necessary and sufficient concepts and methods for the study of cognition, while others adopt a moderate position, suggesting a number of prescriptions to compensate for the shortcomings of the CHCS, drawing from both the mathematical minutiae and qualitative resources of dynamics. We will firstly characterize what dynamics stand for, secondly, observe two varieties of dynamical systems that account for formal and empirical types of systems, and thirdly, expose the dynamical hypothesis concerning cognitive science.

### *- I.II.I - Origins*

**Calculus**

**From Merriam-Webster Online Dictionary.**

*Function:noun Inflected Form(s): plural* ***cal·cu·li*** *also* ***-lus·es*** *Etymology: Latin, stone (used in reckoning)* ***1 a :*** *a method of computation or calculation in a special notation (as of logic or symbolic logic)* ***b :*** *the mathematical methods comprising differential and integral calculus […]* ***4 :*** *a system or arrangement of intricate or interrelated parts.*

Whereas computability is the domain of applied discrete mathematics and computer science (although it also draws on information science), dynamics are a subset of applied mathematical analysis and the branch of physics concerned with machines or machine-like objects, in the broad sense of the area of study of mechanics, but more specifically within the branch of dynamics, the study of the effects of forces on the motion of objects. Thus on one hand, dynamics are derived from empirical studies in the physics of motion and forces, with their most significant lineage tracing back to Newton, in the late seventeenth century, when he proposed his three laws of motion (the law of inertia, the fundamental law of dynamics, and the law of reciprocal actions). On the other hand, dynamics have much to owe to the mathematical formalism that Newton and Leibniz[16] developed

[16] Some evidence suggests that calculus-related methods and concepts were known by Egyptian and Hellenistic thinkers, notably Eudoxus and Archimedes.

concurrently, but independently: calculus. For Newton, calculus was the necessary means to quantify and express his findings in classical mechanics. Although calculus is thus connected to the advent of Newtonian mechanics, it has thereafter evolved somewhat independently, along the lines of abstract, fundamental mathematics.[17] Calculus is built on studies in algebra and geometry, and relies on the notions of functions and limits. It basically involves the study of two concepts that are indissociable, essentially complementary: that of rates of change and accumulation of quantities. These two concepts are formally expressed by differential and integral calculi, respectively.

The developments of infinitesimal calculus, as it is commonly called, were expanded to all of physics' domains in the following centuries, from particle physics to astrophysics, but even into the life sciences, humanities, and social sciences. On the other hand, mathematicians like Laplace and Lagrange brought the concepts and methods of dynamics to a full bearing into the study of mathematical analysis, the study of real and complex numbers, and of the functions defined over them. Thus the interactions between the empirical applications of dynamics and its formal counterpart, calculus, have been mutually enriching, contributing to great developments in physics and mathematics, while inspiring other disciplines to make use of such conceptual and methodological tools. Following Giunti (1995) and van Gelder and Port (1995), we can conceive of dynamics' contribution to other areas of science as twofold: through the use of (i) dynamical systems theory, we have concepts, qualitative methods and models from which to draw parallels with other phenomena, develop explanations, and make predictions, and (ii) dynamical modeling is the formal means by which we express the relevant features of the phenomena under study, both qualitatively and quantitatively, through infinitesimal calculus, *ergo* differential and integral equations.

### ***- I.II.II - Varieties of dynamical systems***[18]

17 Areas and extensions of calculus include: differential equations, vector calculus, calculus of variations, complex analysis, time scale calculus, infinitesimal calculus, and differential topology.

18 All references for this section, A. Norton (1995).

The following characterizations are meant to form a simple introduction and overview of the relevant features of dynamics. We must first distinguish between a real dynamical system (RDS) and a mathematical dynamical system (MDS), namely the distinction between the phenomenon under observation (a system in which features or elements change over time interdependently, like the weather, an ant colony, the cardiovascular system, *etc*.), and the mathematical model used to represent the system's qualitative changes through variations in the features' or elements' magnitudes. A broad definition of dynamical systems[19] is that they are deterministic processes in which a function's value changes over time, according to a rule that is defined in terms of the function's current value. More precisely, in the words of Norton (1995, p. 45),

> A mathematical dynamical system consists of the space of all possible states of the system together with a rule called the dynamic for determining the state which corresponds at a given future time to a given present state.

The algebraic or geometrical representation of the collection of all possible/relevant values is called the state space of the system.

Of major hindrance to the study of dynamics are the sensitivity to initial conditions and the nonlinearity of most systems. As Poincaré pointed out in the late 19th century, most systems, even composed of a few variables, do not allow for the simple calculation of a solution. Indeed, most nonlinear, and even some piecewise[20] linear systems, exhibit chaotic behavior, *viz.* apparently random, unpredictable behavior from deterministic systems. Poincaré thus suggested that dynamical systems theory could be the basis for a serious qualitative method of analysis, with regards to the intractability of most systems. The concepts thereafter developed involve trajectories, stability, recurrence, attractors and bifurcations, and generic behavior, all of which provide us with useful methodological means of studying the

[19] Source: Wikipedia, under '[dynamical system]'.

[20] A piecewise linear system is a system whose mathematical characterizations allow certain areas, but not its overall state space, to be calculated through simple algebraic functions.

overall behavior of simple and complex systems, and generate explanations and predictions by observing their state space. In Norton's words again (*id.*, p. 47):

> […] the state of mathematical art dictates that any tractable mathematical model should not have too many variables, and that the variables it does have must be very clearly defined. As a result, conceptually understandable models are sure to be greatly simplified in comparison with real systems. The goal is then to look for simplified models that are nevertheless useful.

The calculus-based formalism of dynamics allows for two specific types of systems, if interpreted in terms of continuity and enumerability of time-dependent variable evolution, namely continuous dynamics and discrete dynamics. The formal descriptions of the former are the (algebraic) differential equations and (geometrical) flows, and for the latter, the (algebraic) difference equations and (geometrical) diffeomorphisms. While continuous dynamics are essential to both fundamental mathematics (analysis) and applied mathematics (mechanics), discrete dynamics are a useful tool to predict the qualitative changes of both linear and nonlinear systems. Discrete dynamics also have similarities with the large field of discrete mathematics and computability theory, but are interested in describing and predicting time-dependent changes within the state space of the concerned system. Such similarities and differences will play an important role in the following discussions.

A dynamical system is said to be discrete if its time parameter is measured in discrete steps, *i.e.* that its state space on the time parameter is a metric of evenly spaced discrete jumps. Such systems are modeled through recursive relations[21]. Discrete time dynamics use difference equations, equations defined over integers (for time values) as well as reals (for values of other parameters), by means of recursive functions being iterated on chosen initial values. When a system is modeled has having a continuous time parameter, *i.e.* its metric for time is a continuous progression over the real numbers, it is expressed through ordinary

21 For reference, the logistic map is a simple nonlinear second-degree polynomial, which can be expressed discretely: $x_{t+1}=ax_t(1-x_t)$.

differential equations (ODE) or partial differential equations (PDE)[22]. A differential equation in one variable, or one dimension, is an equation composed of a function x and one or more of its derivatives. Partial differential equations are much more complex, involving partial derivatives of functions of more than one variable. The distinction between linear and nonlinear systems is also very important: linear systems have solutions that form a vector space, and allows the reduction of the problem from calculus to linear algebra. Indeed, one can solve a continuous linear differential equation by reducing it to an algebraic equation, through an algorithm called the Laplace transform method. But as mentioned above, most nonlinear, and some piecewise linear systems challenge our means of calculation: they can't be solved explicitly. The vast majority of natural phenomena being nonlinear under mathematical formalization, we have to rely on sophisticated qualitative and quantitative means of analysis.

Geometrical and topological considerations help greatly in the understanding of such systems. Given a vector field F, one can find the solution trajectories that pass through the field in the proper way (*i.e.* given some initial parameters and the geometrical progression of the relevant differential equations). Each trajectory then corresponds to a set of input parameter values and their solution to the equations (see figures 1, 2 and 3 below for examples of vector fields and solution trajectories). The full solution of an equation is called a flow, using the notation $\phi(t,x)$, and describes the position of a point x on its solution trajectory for a time t. Note that not all solution trajectories are necessary or relevant, and can be defined over a restricted surface, or a manifold for higher dimensional state spaces.

[22] For reference, the logistic map can be defined over the reals through the following ordinary differential equation: $dx/dt=ax(1-x)$.

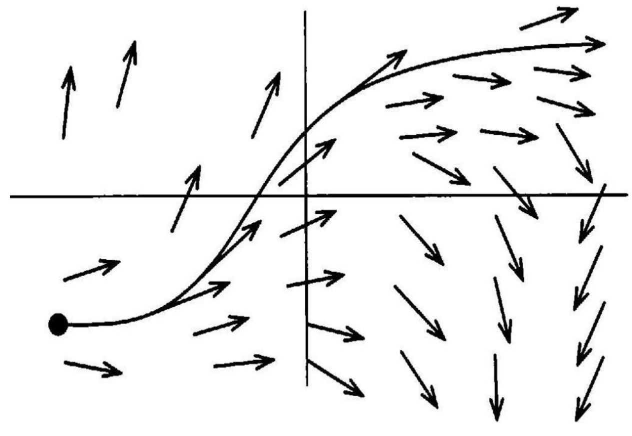


**Figure 1** A vector field on $R^2$ along with a single solution trajectory.

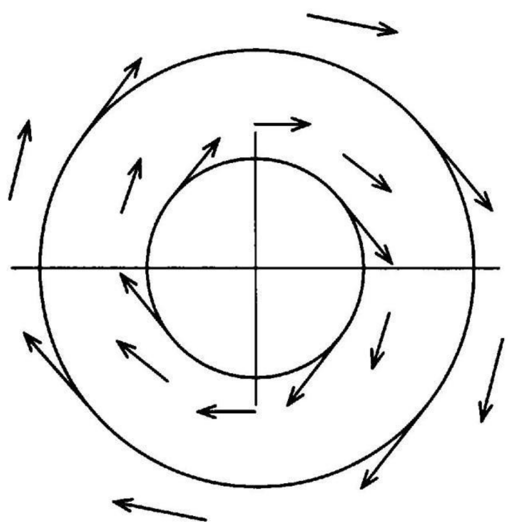


**Figure 2** Two trajectories for the vector field $F(x, y) = (y, -x)$. These trajectories are periodic cycles.

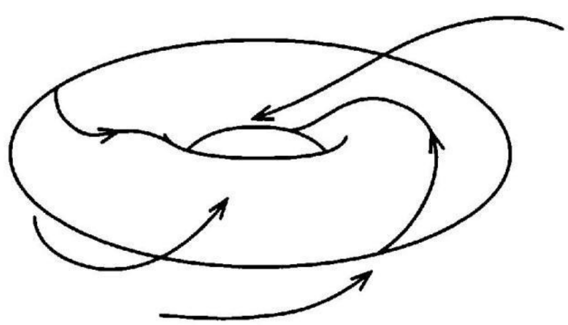
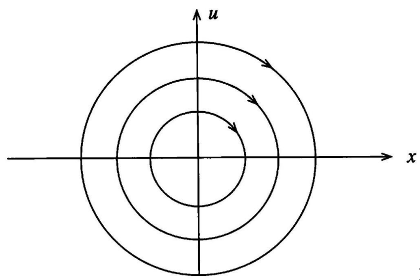


**Figure 3** A toroidal manifold is shown along with the phase portraits of solutions on and approaching the manifold (in this case, a two-dimensional surface configured in three-dimensional space).

Two interesting concepts emerging from geometrical and topological characterizations of dynamical systems are attractors and bifurcations. According to Norton (*id.*, p. 56), "attractors are important because they represent the long-term states of systems." Roughly, attractors can be defined in the following way:

> Let $F$ be a vector field on $R^n$, with flow $\phi$. A closed set $A \subset R^n$ is an attractor for this flow if (i) all initial conditions sufficiently close to $A$ have trajectories that tend to $A$ as time progresses, (ii) all trajectories that start in $A$ remain there, and (iii) $A$ contains no smaller closed subsets with properties (i) and (ii). (*id.*)

An interesting subset of attractors is that of the strange or chaotic attractors, which exhibit diverging nearby trajectories following similar overall directions, and generally possess a fractal structure, where large scale variations are also found on smaller scales. Bifurcations reflect states of transitions in a system, "when a parameter value is reached at which a sudden change in the qualitative type of the attractor occurs." (*id.*, p. 57) Bifurcations can be seen as thresholds where certain parameter values generate different dynamical behaviors. Thus, the system's overall behavior is dependent on the conjunction of the dynamic rule(s) and input parameters. Attractors and bifurcations are of great importance in even the simplest systems. To clarify this point, let us briefly consider Norton's example of a frictionless mass-and-spring system, versus a system which takes friction under consideration. The passage from a MDS of a frictionless mass-and-spring system, whose geometrical representation exhibits a periodic attractor shaped in a circle (for it depends on initial values of position and velocity only), to a more complex system involving the drag force sliding friction, clearly shows that not only is qualitative behavior dependent on the parameters involved, but that the simple addition of a significant real world feature like friction into the dynamics of a system greatly complicates both qualitative and quantitative analysis. (see figures 4 and 5 below)

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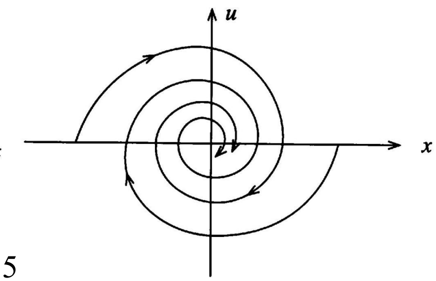


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**Figure 4** (left) is a geometrical representation of the attractor shape of the frictionless mass-and-spring system, given three sets of initial parameters. The abscissa x represents the distance from the rest position, and ordinate u is the velocity. **Figure 5** (right) is the mass-and-spring system influenced by a friction parameter, here a constant in the ordinary differential equations.

### *- I.II.III - Cognition as a real dynamical system*

**Dynamical systems theory**

**From Wikipedia, the free encyclopedia.**

***Dynamical systems theory*** *is an area of mathematics used to describe the behavior of complex systems by employing differential equations.*

*Proponents of the dynamical systems theory approach to cognition [...] believe that differential equations are the most appropriate tool for modeling human behavior. These equations are interpreted to represent an agent's cognitive trajectory through state space. In other words, dynamicists argue that psychology should be (or is) the description (via differential equations) of the cognitions and behaviors of an agent under certain environmental and internal pressures. The language of chaos theory is also frequently adopted.*

What does it mean to have a dynamicist's view of cognition? The DHCS, or dynamical hypothesis in cognitive science[23], is the view that cognitive processes and related states are best described and explained through the conceptual language and models of dynamic systems theory and dynamical modeling. While not incompatible with one of the main tenets of computationalism, *viz.* the thesis of functionalism (by taking into account the essentially embodied nature of cognition), it does clash with the strong claim of cognitivism, in the matter of what type of model best explains cognitive processes. Against the information processing models championed by classic computation and classic artificial intelligence (Haugeland's GOFAI view mentioned above), dynamicists suggest that the mathematical models of dynamics

23 The DH, or dynamic hypothesis, was coined by van Gelder (1998b). The DHCS acronym is used here to contrast with Piccinini's CHCS.

offer a more accurate depiction of cognitive processes, and allows formal and empirical coherence at all levels of cognitive modeling. This section exposes the concepts championed by the core assumptions of the majority of dynamicists, namely the interdependence of context, corporeality, and cognitive processes (embeddedness/situatedness and embodiment), the simultaneity and time-dependent evolution of processes, the emergence of structure and behavior from cognitive processes' interactivity, and the heterogeneity of cognitive time scales.

According to van Gelder (van Gelder and Port, 1995, p. 2)[24], "the heart of the problem is time. Cognitive processes and their context unfold continuously and simultaneously in real time." Now, intuitions and conceptual issues about cognitive processes are one thing, but dynamicists insist that this deeper *a priori* problem is the source of much of the misconceived models of cognition, a legacy of computationally framed cognitivism that favored Turing's metaphor of calculation to generalize it as a theory of mental processes. But computation lacks many features that seem essential to frame cognition properly, according to dynamicists. van Gelder *et al* hold that we already have many reasons to hold on to the DHCS:

> We know, at least, these very basic facts: that cognitive processes always unfold in real time; that their behaviors are pervaded by both continuities and discretenesses; that they are composed of multiple subsystems which are simultaneously active and interacting; that their distinctive kinds of structure and complexity are not present from the very first moment, but emerge over time; that cognitive processes operate over many time scales, and events at different time scales interact; and that they are embedded in a real body and environment. (*id.*, p. 18)

The following issues are meant to illustrate what dynamics have to offer in view of the shortcomings of the computational theory of mind.

*Cognition, time, and the multiplicity of time scales*. Whereas computationalism models cognitive processes in sequences of discrete steps, dynamics help model processes in real time, specifying not only the states of the

24 All references for this section are taken from van Gelder and Port (1995).

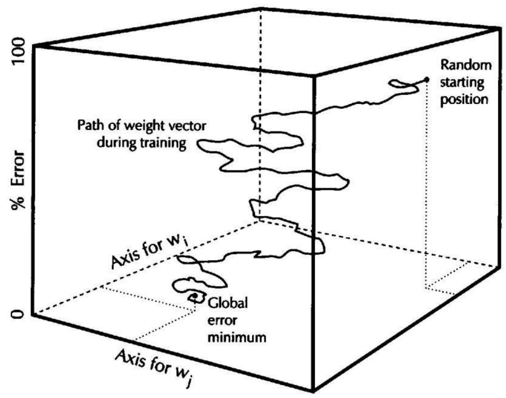
system but also their time evolution. Time is continuous in dynamical models, it is also a quantity, or magnitude, on which other cognitive related magnitudes are dependent, thus providing analyses rich in resolution and detail. Dynamicists stress the issue of cognition occurring *in* time, not simply *over* time like computational models frame cognition. To say that cognition unfolds in time is to hold that cognitive processes are time dependent, and that considerations of simultaneity, embeddedness, and interdependence of a multiplicity of time scales (like neural processes time, perceptual time, decision making time, learning time, and maturation time) are fundamental to cognition itself. Dynamics are precisely the kind of mathematical means to formalize processes that occur over time, with differential calculus pertaining to rates of change, and integral calculus concerning the accumulation of quantities. The interdependence of multiple time scales is formalized by using multiple variables within those equations, which stand for relevant cognitive magnitudes. State variables and parameters can both be seen as changing, thus representing the coevolving nature of processes on different scales.

S*tate continuity, and the multiplicity and simultaneity of interactions.* Dynamics provide formal tools to model both the continuity and discreteness of processes and states, whatever best suits the phenomenon under observation. Dynamicists are concerned with continuity not only in time, but also in state, of the processes underlying cognition. While many cognitive tasks are modeled through discrete dynamics, such as language-related performances and logical and mathematical calculations, a much larger spectrum of processes unfold continuously in time and state, such as sensorimotor processes and related procedural tasks. As pointed out by van Gelder, discreteness of states is also quite often a matter of perceiving seemingly discrete qualitative changes in continuous processes, such as it is conceptualized in dynamics by the use of the term catastrophe, *viz.* sudden and dramatic changes in the behavior of a system, when a parameter’s change in magnitude causes a bifurcation, as seen in the previous section. Similarly, dynamics offers incomparable advantages for the formalization of simultaneity and interactivity, and at all levels of cognitive processes, such as in the modeling of interactive agents in interpersonal tasks, sensorimotor processes, and neurobiological modeling, to name a few. The simultaneity and interactivity of component subsystems, or of cognitive agents, is essential to carrying on individual and collective tasks, all of which fall under the domain of calculus and dynamics in the framing of overall and local behavior, and allows predictions based on both quantitative and qualitative methods.

*Self-organization and emergence.* The organization of cognitive systems and processes exhibits complex and intricate design, or structure. Dynamicists propose to not only describe existing cognitive structures, but also to provide a framework able to explain how such structures came to be in the first place, namely a means to explain the *emergence* of such design. The conceptual and formal tools of dynamics, because they involve the modeling of spatial and temporal structures, offer the possibility of analyzing the time and state evolutions, or morphogenesis, of complex structures in physics, chemistry, and biology. There are therefore good reasons to believe that using dynamical systems theory and dynamical modeling for the purpose of studying the morphogenesis of cognitive processes and structures is a heuristic avenue. Many physical, chemical, and biotic systems are also studied under the stance of self-organization principles, which holds that some structures come into existence with neither a plan, nor an independent, external builder, but through simple formal and empirical principles governing the organization of elements into complex and heterogeneous wholes. Here again, dynamics are of great use to model such phenomena and to elaborate explanations. Dynamicists interested in the study of cognition propose that we have evidence towards such a view of mental processes (chapter II will present an application of such theses in the study of ontogenetic dynamics), and even purport to link cognition and evolution as emergent structures on a shared spectrum,  only pertaining to different time scales dynamics.

*Embeddedness and embodiment.* Dynamicists refuse to hold on a conservative definition of a cognitive system as a strictly internal structure. In order to account for cognitive processes, we must integrate considerations about their neural correlate, their corporeality, and their embeddedness or situatedness in a context, an environment. But neural processes, behaviors, and the whole of environment are *already* quite efficiently and heuristically modeled through dynamics! It thus appears that dynamics, beyond their being desirable in the study of cognition, might even be unavoidable as such. As van Gelder points out, it is also a matter of advantage that is put forward here, since accounts in terms of dynamics for internal matters of cognition find themselves in continuity with the dynamics of behavior and context, thus facilitating the integration of a variety of systems for explanatory and predictive purposes. Such an integration of component systems into what constitutes cognition reflects the interdisciplinary endeavors found in cognitive science, since neither neuroscientific, behavioral or ethological, psychological, or social systems can be exhaustive by themselves of what counts as cognitive. The supporters of the DHCS propose to study precisely the interactions between internal cognitive processes, the body, and different contexts or environments. Dynamicists accuse computationalism of having simply avoided the problems posed by the discontinuity and heterogeneity of systems. By tacitly positing the autonomy of the study of cognition, computationalists have thus avoided, and failed to account for, most, if not all of the abovementioned issues raised by the DHCS. In van Gelder's words,

> […] whenever confronted with the problem of explaining how a natural cognitive system might interact with another that is essentially temporal, one finds that the relevant aspect of the cognitive system itself must be given a dynamical account. It then becomes a problem how this dynamical component of the cognitive system interacts with even more 'central' processes. The situation repeats itself, and dynamics is driven further inward […] (*id.*, p. 30)




Learning by gradient descent in weight/error space. (Axes are shown for only two of the 105 synaptic weights.)

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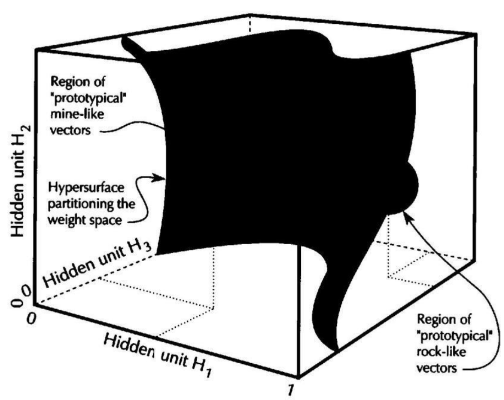


Learned partition on hidden-unit activation-vector space. Axes are shown for only three of the seven hidden-unit activation levels.

Worth mentioning: Churchland (1989) used dynamical representations (**figures 6** and **7**) of the behavior of formal neurons while promoting connectionism against classical (symbolic) Turing-computation models (the symbolic view of cognition). Why bother, a dynamicist might ask, to employ a computational framework, when we have everything we need in biophysics and neuroscience to characterize the (still very mechanistic) functional decomposition of informational processes in nonlinear and differential equations, rightful domain of dynamics? (see Giunti, Piccinini, and van Gelder's arguments in chapter III about that position)

## *- II - Case study: computational and dynamical accounts of sensorimotor cognition*

**Cognitive Science**

**From Wikipedia, the free encyclopedia.**

*The term "cognitive" in "cognitive science" is "used for any kind of mental operation or structure that can be studied in precise terms." (Lakoff and Johnson 1999) This conceptualization is very broad, and should not be confused with how "cognitive" is used in some traditions of analytic philosophy, where "cognitive" has to do only with formal rules and truth conditional semantics. (Nonetheless, that interpretation would bring one close to the historically dominant school of thought within cognitive science on the nature of cognition - that it is essentially symbolic, propositional, and logical.)*

*The earliest entries for the word "cognitive" in the OED take it to mean roughly pertaining to "to the action or process of knowing". The first entry, from 1586, shows the word was at one time used in the context of discussions of Platonic theories of knowledge. Most in Cognitive science, however, presumably do not believe their field is the study of anything as certain as the knowledge sought by Plato.*

The conceptual frameworks of computation and dynamics having been summarily exposed, we have then observed their bearing on the study of cognition. This chapter presents an application of the previously defined frameworks to empirical research on a particular kind of cognitive processes, sensorimotor control and learning. This contextualisation of the aforementioned concepts and models of computational theory and dynamics is essential to the following discussion, presented in chapter III, since it will provide us with a clear picture of the claims and allegations upon which are based most of the misunderstandings and quarrels between supporters of both frameworks. Both the CHCS' and the DHCS' views on such cognitive processes are presented, and the quantitative and qualitative properties of the suggested models will be exposed so as to understand their full extent, and evaluate their respective claims. Since computationalism is rather ubiquitous even in the realm of cognitive neuroscience, I will therefore begin with the exposition of the aforementioned evidence dealing with sensorimotor processes. In a second section, I will then argue about the alleged advantages of switching frameworks for a dynamical account of cognitive processes, by opposition to the traditional computational view. In the last section, I will present a correlated dynamical account of sensorimotor processes which bears explanatory and predictive significance to a higher level of description, that of developmental psychophysiology (more specifically, psychophysics)[25], with the help of Thelen's (Thelen 1995, Thelen, Schöner, Scheier and Smith 2001, and Smith and Thelen 2003) findings and subsequent model. I therefore aim to evaluate the conjectured benefits of the adoption of a dynamical perspective on cognition, relative to the (still controversial) shortcomings of the traditional computational framework, while

[25] Psychophysiology is vaguely defined as the science of understanding the link between psychology and physiology. Psychophysics appears to be more specific, defined as the branch of psychology dealing with the relationship between physical stimuli and their perception. All references online, http://en.wikipedia.org

showing the relevance of said dynamical account in the rather different studies of sensorimotor cognition from neuroscience and psychophysics.

### *- II.I - The evidence*

**Kinematics**

**From Wikipedia, the free encyclopedia.**

*In physics, **kinematics** is the branch of mechanics concerned with the motions of objects without being concerned with the forces that cause the motion. In this latter respect it differs from dynamics, which is concerned with the forces that affect motion […]*

**Inverse kinematics**

**From Wikipedia, the free encyclopedia.**

***Inverse kinematics** is the process of determining the movement of interconnected segments of a body or model. For example, with a 3D model of a human body, if the hand is moved from a resting position to a waving position, how do the connected fingers, forearm, upper arm and main body move in response? It is a subject of programming and animating. It is approached often in game programming and 3D modeling […]*

A standard way to describe neuroscientific evidence is through neurobiological modeling (Montague & Dayan, 1998), which in turn relies heavily on formal characterizations of more or less sophisticated computational design, when imported into the arena of cognitive science. By contrasting the conceptual language of neuroscientific studies of sensorimotor processes involved in control and learning with that of the CHCS, it becomes apparent that neuroscience draws upon a mathematical language that extends beyond that of computation, towards dynamics. The case study concerns the problem of inverse kinematics, addressed from both computational and dynamical perspectives.

Whether we are concerned with what is traditionally considered a high level (language and decision-making, to name just a few) or low level (emotional responses, sensorimotor activity) cognitive process, neurobiological modeling is computation-laden. As we have seen in chapter I, both the original forms and contemporary offshoots of cognitive science - including some cognitive neuroscience - (i) view mental states as functional states, and (ii) conceive these functional states as computational, *viz.* to be modeled and explained through information processing concepts and schemes.[26] Computational functionalism is the conjunction of the two, but they are logically independent theses, according to Piccinini (2003, 2004b, 2004d). Part of the ambition of this chapter is indeed concerned with showing that while a functionalist account of cognition can be appreciated from within a computational framework, it does not entail that computationalism is the only functionalist model of mental processes, and there are different aspects of cognition that are worth scrutinizing.

According to Albright (1993, p. 178), "motion processing serves a number of behavioral goals, from which it is possible to infer a *hierarchy of computational steps*." (My emphasis) The initial step for motion perception is said to be motion detection, more precisely, the perception of motion direction. What purposes might motion perception serve? Albright lists a few, such as establishing the volumetric structure of a scene, posture and balance control, the appraisal of one's own trajectories and possible collisions, segregating visual inputs into objects and background, and identifying and predicting the motion of objects to respond accordingly, among others. Each of these sensorimotor functions can be in turn described computationally and neurophysiologically in detailed steps. Research on motion bestows significance to sensorimotor behavior in a causal and mechanistic way. For example, parietal cortical stream (areas MT and MST) activity and motor control of ocular globes activation (by means of dorsolateral pons) suggests a causal relationship from the former to the latter. Functional decomposition is an essential part of a mechanistic explanation, shared by a plethora of sciences, both computational and noncomputational. Thus, following Albright, we can say that the main function of motion perception for our purpose is the affordance of motor control. Yet, Albright also attempts to characterize sensorimotor processes in the

[26] A more precise, and therefore different, commitment of cognitivism, which conflates the CHCS and the possibility evoked by cognitivism of a sound empirical, scientific account of mental processes. This is not unproblematic, since dynamicists claim to the naturalization of cognition on different grounds. It is therefore important to distinguish computationalism as part, but also a stricter form, of cognitivism.

larger perspective of (implicitly) an agent or organism and (explicitly) an environment (this will turn out to be less trivial than it might sound at first):

> Detection and interpretation of these motions are not only crucial for predicting the future state of one's dynamic world […] but also provide a wealth of information about the 3-D structure of the environment. (*id.*, p. 179)

Can a computational account of cognition, based on functional decomposition, be exhaustive of the actual inner workings of sensorimotor processes? What counts as a good computational explanation, if not the effectiveness of a simulation inspired by neurobiological modeling, with the aim of matching inputs and outputs to and from the cognitive unit under scrutiny? Another question that immediately follows, then, is: is there any more, or any less computation actually going on in this collection of cognitive processes? Perhaps some reversal of perspective is needed to accurately characterize these processes.

According to Bizzi, Mussa-Ivaldi and Giszter (1991)[27], some neurons must calculate the relative positions of body/limbs and objects in order to achieve an adequate sensorimotor activity (based on egocentric sensorimotor perception and producing a behavioral output). The CNS also has neurons involved in the calculation of body/limbs- independent perception, or allocentric perception, in the representation or signal emission of the concerned objects. In their words:

> Recent psychophysical evidence supports the hypothesis that the planning of limbs' movements constitutes an early and separate stage of information processing. [...] during planning the brain is mainly concerned with establishing movement kinematics, *a sequence of positions* that the hand is expected to occupy at different times within the extrapersonal space. […] The analysis of arm movements has revealed *kinematic invariances*. […] The data derived from straight and curved movements indicate that the *kinematic*

[27] See also Bizzi and Mussa-Ivaldi 1998, Bizzi, Tresch, Saltiel, and d'Avella 2000, Mussa-Ivaldi and Bizzi 2000, and Gandolfo, Li, Benda, Padoa Schioppa, and Bizzi 2000 for further references on Bizzi and colleagues' research on sensorimotor processes. It should be noted that at no point does Bizzi advocate a dynamical stance over a computational one. These orthogonal considerations are the author's designs.

> *invariances could be derived from a single organizing principle based on optimizing endpoint smoothness.* (*id.*, p. 287. My emphases)

Notwithstanding its computation-laden imagery, this evidence is hardly consequent of functional decomposition through a strict computational commitment. It translates to computations and interactions between the CNS, afferent visual and kinaesthetic inputs, and musculoskeletal outputs – have them calculate on analog or discrete quantities, whichever is more appealing – the overall picture suddenly appeals to a different mathematical characterization, that of the changes within and outside a system according to thresholds, invariants, nonlinearity and obviously, motion. Such is the mathematical language of dynamics. We witness the emergence of concepts of the likes of optimization gradients, thresholds and invariances (known as attractors in systems dynamics), and complex behavioral plasticity resulting from simple nonlinear 'organizing principles' (which are nevertheless computationally mind-boggling for our commonly linear reductivism).

One central theme cherished by proponents of the application of dynamics to cognition is the interactivity between cognitive agents and their environment. Remarkably, following again Bizzi and his colleagues, the CNS is *not* the source of coordinates in space; it relies on extrinsic information, as in:

> […] actions are planned in *spatial or extrinsic coordinates,* [then] for the execution of movements, the CNS must convert the desired direction and velocity of the limbs into signals that control muscles. (*id.*, my emphasis)

This has rather interesting consequences. Given extrinsic coordinates- and kinematic invariances- reliance for the CNS to actually 'do' something sensorimotorwise, (i) sensorimotor cognition is better studied in specific contexts, supporting the claims of proponents of embodied/situated cognition, and (ii) the world provides enough 'affordances' in the language of J. J. Gibson (1966, 1979), and 'structure' for a cognitive agent to navigate without having to build a new world from scratch. In the equations of dynamicists, the world operates as a whole system itself, albeit not a cognitive one, and influences continuously and inexorably the cognitive agent within it, and it works the other way around too, in feedback loops. Granted, discharging some of the weight of information and information processing (because the world has structural invariants at all levels) does not make cognition any easier to model, nor to understand. There is evidence that the CNS calculates inverse kinematic and inverse dynamic problems in the generation of motion, so much for the dignity of empirical sensorimotor enquiry and even more of a burden for cognitive modeling and simulations. Is the account of inverse dynamics computation by the CNS satisfactory or, to put the issue at hand in other words: do our brains and computational models of cognitive processes deal the same way with such informational complexities?

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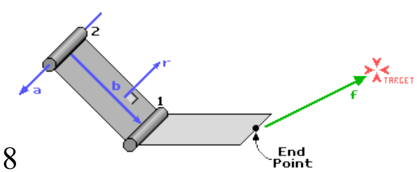


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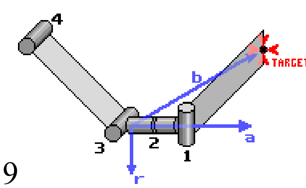


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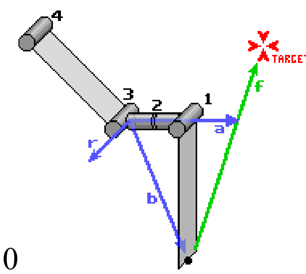


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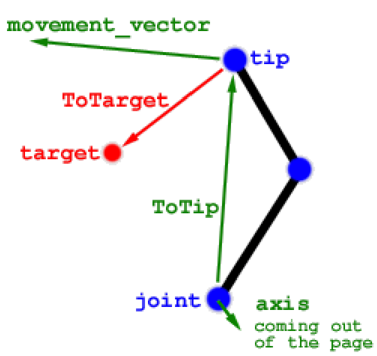


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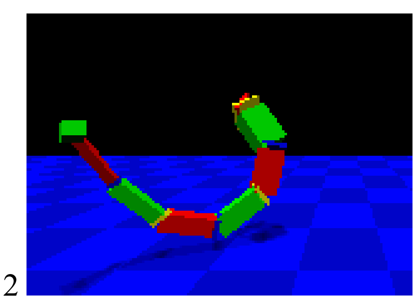

**Figures 8, 9, 10** and **11** are geometrical representations of some parameters of inverse kinematics problems such as they are studied by computer scientists, engineers and roboticists. **Figure 12** is a still frame of a 3D rendered simulation of inverse kinematics where a "tail' tries to reach and touch a small green cube, illustrating the complexity of devising such an algorithm.

No. Just as you thought it was over with conceptual and methodological pitfalls, it's not that simple. In Bizzi's words:

> One way to compute inverse dynamics is based on carrying out *explicitly* the algebraic operations after representing variables such as position, velocity acceleration, torque, and inertia. This hypothesis, however, is unsatisfactory because there is no allowance for the *inevitable mechanical vagaries associated with any interaction with the environment*. Alternative proposals have been made that *do not depend on the solution* of the complicated inverse-dynamic problem. Specifically, it has been proposed that the CNS may transform the desired hand motion into a series of *equilibrium positions*. […] According to the equilibrium-point hypothesis […] (Feldman, 1974) limb movements result from a shift in the *neurally specified equilibrium point.* (*id.*, p. 289. My emphases)

Computation is again constrained by structural invariants of the body, as well as invariances in the world.

> […] With respect to control, the *elastic properties* of the muscles provide *instantaneous correcting forces* when a limb is moved away from the intended trajectory by some external perturbation. With respect to computation, the [same] elastic properties offer the brain an opportunity to deal with the inverse-dynamics problem. (*id.* My emphases)

Well! There is more to meat than first transpires! Conclusions? (i) Our cognitive processes are constrained and mediated by some useful designs and useful physical properties that *afford* them – much of sensorimotor cognition does not require symbolic processing, and (ii) there is less to be paranoid about the amount of computation and information processing that the CNS must carry out for the information- and computation- obsessed theorist. Things you can suddenly do without if you are a central nervous system, *vis-à-vis* sensorimotor limb control: parameters and variables of inertial forces, gravity, viscosity, required effort expenditures, *et cetera*… How does that fit the simulation model? Worth noting is the stubbornness and resilience of computational schemes to be dealt away with, even for such cognitive neuroscientists, in saying: "[…] in this context, a representation in the CNS [of the previous variables] contained in the equations of motion is no longer necessary." (*id.*) A radical dynamicist's reply could be: "In this context, a representation in the CNS of the previously discussed parameters and variables in the equations of motion is *nowhere to be found*, there is no such thing, bottom line, the brain does *not* need it, get over it and change your model…"

It is fascinating that psychophysical research models its observations from an ambiguous middle ground in the midst of computational and dynamical stances, such as in the case of du Lac and colleagues (1995, p. 411): "The iterative process of improving motor performance by executing movements, *identifying* errors, and *correcting* those errors in subsequent movements is called motor learning." (Note the vernacular of intentionally and informationally phrased definitions here, my emphasis) Yet, elsewhere and about the same topic:

> The simplest form of motor learning is adaptation, in which muscular force generation changes to compensate for altered mechanical loads or sensory inputs. Adaptation can involve movements across either a single joint or multiple joints, and can occur in both reflexive and voluntary movements. (*id.*, p. 418)

It can clearly be seen that whereas the first definition involves a functional decomposition that is essentially computational in flavour, the second one is quite recoverable by an effective and quantitative dynamical characterization. As we have seen with Bizzi and colleagues, the danger of following a model to its deeper logical conclusions is falling prey to over-characterization and bearing little into matters of empirical correspondence. Consider the following:

> For complex movements, motor learning is required to select and coordinate the appropriate muscular contractions, to link together motor subroutines, and to create new motor synergies by combining forces generated across multiple joints in novel spatial and temporal patterns. (*id.*, p. 416)

Well, the illusion here is that there is an ongoing calculation at every single step of the described process, and it contradicts the previous precisions on motor control intricacies; what matters is that the functional decomposition of sensorimotor processes, as mechanistic as it gets, does not entail what I will dub an *omniparametrism*, or more specifically, an *omnicomputationalism* of processes under functional scrutiny.

***- II.II - The argument***

In this second section, we are confronted with the arguments of a proponent of the dynamical view of sensorimotor control and learning, Esther Thelen[28]. The broad range of arguments from the DHCS will be exposed against the treatment of sensorimotor learning from a computational perspective[29], as Thelen tries to answer a most important question: how do we relate sensorimotor ontogenetic dynamics with cognition? Dynamicists aim to provide a more biologically plausible framework for cognitive science while also aiming for a gain in explanatory and predictive strength through their models. Thelen's work is primarily concerned with setting up empirical psychophysical experimentations to provide enough support for the following claim: that ontogenetic dynamics are *the very source of cognition*. Her premises: (i) *embodiment is a necessary condition for cognition*, as we have seen time and again, and (ii) as is conceived through the study of infant psychophysical development, the *major developmental task that tops them all is to gain control of the body*.

Many attempts at modeling sensorimotor processes from a dynamical framework have been made, such as those of Bingham on visual event recognition, Grossberg on the neurodynamics of motion perception, recognition learning, and spatial attention, Saltzman on sensorimotor coordination, Thelen on the development of sensorimotor (embodied) cognition, and Turvey and Carello on haptic perception and coordinated movement[30]. For the sake of a workable basis of reference, we will only explore Thelen's work on developmental dynamics, an arbitrary choice, albeit a perfect exemplar of the relevance of dynamics to cognition.

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28 E. Thelen (1995), Thelen, Schöner, Scheier and Smith (2001), and Smith and Thelen (2003).

29 Computational solutions of the inverse kinematics problem, and the dynamical equations of Thelen *et al* concerning sensorimotor dynamics, can be found in appendix II.

30 All of the preceding authors, 1995, found in Port, R. F., and van Gelder, T., Eds., *Mind as Motion*. Cambridge, MA: MIT Press.

Port and van Gelder (1995, p. 69), say that “[…] Thelen argues that taking up the dynamic perspective leads to dramatic reconceptualization of the general nature of cognitive development, and indeed of the product of development, mind itself.” Thelen’s original contribution to developmental psychology has been of integrating the theoretical tools of dynamics (that is, dynamical systems theory coupled with quantitative dynamical modeling) within a research program and setting up experiments to gather empirical support in order to confirm or refute the relevance of what van Gelder properly named the dynamical hypothesis in cognitive science, as exposed in chapter I. As we have seen before, the dynamical hypothesis is the conjecture that cognition and its related processes might better be described in the conceptual framework of dynamics, rather than, say, a computational one. Obviously, discontent with computational endeavors motivated this departure, and as mentioned in the introduction, was spawned by the many shortcomings of such models. On Thelen’s take on developmental dynamics again:

> Changes in behavior come to be understood in terms of attractors, stability, potential wells, parameter adjustment and so forth. Taking over this vocabulary facilitates a whole new way of seeing how sophisticated capacities emerge. New abilities take shape in a process of gradual adjustment of the dynamics governing the range of movements currently available; this adjustment is effected by exploratory activity itself. (*id.*)

Thus, the vocabulary of dynamic systems theory might be understood as a bridge between outward, externalist descriptions of behavioral changes on one hand, and inward, internalist descriptions of informational processes on the other hand. Dynamics is a language of systematicity, above all, it groups, joins, couples homogeneous (similar entities or processes understood as) systems, or heterogeneous (different hierarchical levels of interactions between entities or processes understood as) systems.

Let’s develop further on the arguments supporting a dynamical account of cognition where developmental psychophysics and psychology are concerned.

Thelen argues that her ambition is to take on not only a noncomputational stance on the ontogenesis of cognition, but to go as far as deny the traditional view of genetic determination of such development; a view supported by Piagetian objectivism and the maturationist account of development. Again, in van Gelder's words:

> Since infants can begin this process of adjustment from very different starting points, it is highly unlikely that there is any predetermined, genetically coded program for development. It is rather a self-organizing process in which solutions emerge to problems defined by the particular constraints of the infants' immediate situation. (*id.*)

The question that begs to be answered is: how do we relate sensorimotor ontogenetic dynamics with cognition, though? Thelen does recuperate a thesis of Piaget's according to which "thought grows from action, and that activity is the engine of change." (*id.*, p. 73) As such, Piaget's fundamental thesis is what we nowadays call that of *embodiment*. Piaget's mistake, though, was to admit a fundamentally Cartesian separation to the end-state of development in the characterization of a mature objective mind, thus recreating a discontinuity essential of what is meant by the Cartesian dualism of mind and body. Cognitive dynamics avoid exactly such a problem, or it could be said that it is an answer to the very re-enactment of mind-body dualism that permeates the standard model of cognitive science. So, an embodied cognition appears to be the only way to solve the mind-body discontinuities, found even in the weakest form that is positing 'mental properties'.

Thelen's is a radical position: beyond a noncomputational account of cognition, she also promotes an antirepresentational stance of cognition. Since this does not turn out to be a common ground to all or even most dynamicists' views on cognition, we will proceed without making this assumption. It is also not a necessary criterion to promote the dynamical hypothesis as such. A better depiction would be to say that dynamicists, up to and including Thelen, are against symbolic representations when it comes to characterizing biological cognition. Another consequence of Thelen's arguments is that it trivializes such concepts like the modularity of knowledge in cognitive processes, the consequences of which are up to interpretation and will not be discussed here. On the other hand, it certainly does well in dealing away with the annoyance of the archaic introduction of a distinction between semantic and episodic knowledge (know-that knowledge) versus procedural knowledge (know-how knowledge). Seemingly pointless (excluding matters of neurological localization of relevant faculties and processes) or irksome distinctions of such nature are always a good test to the relevance of a paradigmatic shift from one conceptual framework to another. Other problems that might be better solved through a dynamical framework are ones concerning individual differences in cognitive processes, context sensitivity of cognition, cognitive tasks such as categorization, and the integration and seamlessness of cognition and behavior (internal and external states), or further, that of behavioral changes, ontogenetic 'learning', and ontogenetic physiological changes. The methodology? Correlating continuities in time between 'physical' and 'mental' events or processes. Thus, it can be said that dynamicists aim to provide a more biologically plausible framework for cognitive sciences while also aiming for a gain in explanatory and predictive strength through their models.

But the meat of Thelen's work in setting up empirical psychophysical experimentations is that it provides enough support for the following claim: that ontogenetic dynamics are *the very source of cognition*! The premises are indeed compelling, if not intuitive: (i) *embodiment is a necessary condition for cognition*, as we have seen time and again, and (ii) as is conceived through the study of infant psychophysical development, the *major developmental task that tops them all is to gain control of the body*.[31] To prove such claims, it is in turn necessary to demonstrate the origins of certain mental processes, and Thelen argues that an analysis of the various time-scales' dynamics of psychophysical processes shows them to be interdependent and profoundly embedded structures. There is no place for discontinuities, in her own words. It should be noted that this is a departure from

[31] Worth noting is that both points (i) and (ii) are not uncontroversial, and only lightly elaborated on by dynamicists, including Thelen.

earlier positions concerning development in psychophysically related research: motor development was mistakenly considered as a strictly biological phenomenon and secondary to the brain or CNS development. This thesis, conveniently dubbed the *maturationist* account of development, thus viewed motor control as a by-product of autonomous brain development, in a rather Piagetian stage-like process of emergence. It is pretty obvious that the spectre of dualism was ubiquitous even in the mostly empirical of ontogenetic accounts of cognitive developments. Piaget's alternative introduced the idea that mental life is built upon those sensorimotor processes, but maintained the undesirable dualism by also claiming that mental processes are distinct, separate phenomena characterized as the end-state development of abstract and objective mental structures. The whole of cognitive science, it seems, has a bad tendency of getting drawn back to (or should I say drowned into) Descartes' legacy.

A dynamical account of cognition necessarily calls onto a deep commitment to the thesis of embodied cognition. The legacy of Cartesian dualism is found in some assumptions of classic and contemporary forms of cognitive science, like

> The denial of the relevance of the physical body in all its instantiations through movement, feeling, and emotion […] [and] the separation of intelligent behavior from the subjective self, from consciousness, imagination, and from commonsense understanding. (*id.*, p. 74)

This legacy has long denied embodiment as a necessary condition for cognition, and is a consequence of its sharing continuity with the methodology of a strictly formal and functional characterization that is the (classic) computationalist view.

## - II.III - The model

*"1. We cast the mental events involved in perception, planning, deciding, and remembering in the analogic language of dynamics. This situates cognition within the same continuous, time-based, and nonlinear processes as those involved in bodily movement, and in the large-scale processes in the nervous system […] Finding a common language for behavior, body, and brain is a first step for banishing the specter of dualism once and for all.*

*2. Because perception, action, decision, execution, and memory are cast in compatible task dynamics, the processes can be continuously meshed together. This changes the information-processing flow from the traditional input-transduction-output stream to one of time-based and often shifting patterns of cooperative and competitive interactions. The advantage is the ability to capture the subtle contextual and temporal influences that are the hallmarks of real life behavior in the world.*

*3. We address specifically the developmental origins of cognition. Since Piaget […], it has been widely acknowledged that all forms of human thought must somehow arise from the purely sensorimotor activities of infants. But it is also generally assumed that the goal of development is to rise above the "mere sensorimotor" into symbolic and conceptual modes of functioning. The task of the developmental researcher, in this view, has been to unearth the "real" cognitive competence of the child unfettered by performance deficits from immature perception, attention, or motor skills. This division between what children really "know" and what they can demonstrate they know has been a persistent theme in developmental psychology […] We argue here that these discontinuities are untenable. Our message is: if we can understand this particular infant task and its myriad contextual variations in terms of coupled dynamic processes, then the same kind of analysis can be applied to any task at any age. If we can show that "knowing" cannot be separated from perceiving, acting, and remembering, then these processes are always linked. There is no time and no task when such dynamics cease and some other mode of processing kicks in. Body and world remain ceaselessly melded together."*

- Thelen, E., Schöner, G., Scheier, C. & Smith, L. B. (2001) *The Dynamics of Embodiment: A Field Theory of Infant Perseverative Reaching*. *Behavioral and Brain Sciences* 24 (1)

This section develops a correlated dynamical account of sensorimotor processes which bears explanatory and predictive significance to a higher level of description, that of developmental psychophysics, with the help of Thelen's findings on infants sensorimotor cognition and the subsequent model developed to deal with the empirical findings. Exposed here are the benefits of adopting of a dynamical perspective on cognition, relative to some shortcomings of a computational framework, while showing the relevance of said dynamical account in the rather different studies of sensorimotor cognition from neuroscience and psychophysiology. The formal and qualitative characterizations of the dynamical hypothesis in cognitive science are thus applied directly to the studies of developmental psychophysics.

Thelen gives three examples of embodied cognition and the relevance of dynamics to characterize cognition: the *containment* example, one on *symmetry*, and another one on *forces*. Following Johnson (1987), she points towards psychological studies from an embodiment perspective, to show how obviously pervading are the recurrent features and constraints of our physical world not only in our actions, but even in our language and thoughts as well. Indeed, Johnson puts forward the idea of what I call 'embodiment semantics', such as when we use prepositions like 'in', 'out', 'over', 'near', 'under', *etc*. To see the extent of such implications, consider the following: *"I don't want to leave any relevant data* out *of my argument."* Metaphor? Yes, but there's no chicken-and-egg problem here, the world came first, and then this embodiment-laden cognition… that's containment for you, right there. Similarly with physical and bodily symmetry and polarization, we can see the extent of such categories as far as in our cultural artifacts, beyond our actual cognitive processes. Just take a hint and think about literature and poetry, cinema and music, and if you don't see schematic and spatial cognition in that, you can keep hoping that disembodied AI will come up with emergent poetry-writing software... Last but not least, consider the concept of force embodiment. Forces reach into cognition as essentially as they involve our every physiological interaction: what about the semantics of verbs? Language and thought are dynamically-laden, think again of the following sentence: *"I'm* attracted *to the ideas of Tim van Gelder."* Johnson calls this prelinguistic meaning, semantics drawn from experience.

It should be noted that in Thelen's view, a dynamical account of cognition is still compatible with a functionalist account from a mechanistic perspective.[32] Indeed, attributing characteristics to an entity as being a real dynamical system is an ontological commitment as much as a model of such entity; it makes claims about the nonlinear, emergent, and embedded properties of such an entity, its *intrinsic* dynamics. Dynamics are the mathematical study of patterns of flow, expressed in nonlinear calculus equations. It is concerned with motions and forces, quite a physicalist level of description and explanation. Yet, from the design of dynamical systems, complex structures and emergent properties arise, and a nontrivial qualitative characterization is possible on top of the quantitative bearing of dynamical modeling. Thelen notes that in order to bear any scientific adequacy and explanatory power, the dynamical equations must *fit* the observed behavioral data. The dynamical hypothesis in cognitive science not only posits that cognition is essentially a dynamical phenomenon, but that dynamics is the best explanatory framework so far for the scientific study of cognition. Thus, (real) complex nonlinear systems can be studied through mathematical dynamical systems (MDSs), and such MDSs can explain changes as the result of coupled magnitudes fluctuating interdependently. One fundamental assumption from the adoption of the dynamical hypothesis is that *pattern emerges only in process*: it thus rejects symbols, computational structures[33], and developmental stages in the programmatic view of computationalism as ontologically unacceptable in the study of the brain and of cognitive processes.

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32 But the problem lies in that dynamicists such as Thelen would have their formal account *replace* any other formal model of cognition, *i.e.* they actually believe their dynamical account to be a good candidate for a functionalist account. Such a claim is hardly supported, and perhaps ill-fated, as we will see in chapter III with the help of Bechtel's arguments.

33 Or does it? We will explore the claim of compatibility under arguments about the types of explanation involved in both frameworks in chapter III, section II.II.

> There can be no description of a purely 'inner life': every mental and behavioral act is always emergent in context […] Perception, action, and cognition form a single process, with no distinction between what people really 'know' and what they perform. (van Gelder and Port, 1995, p. 72)

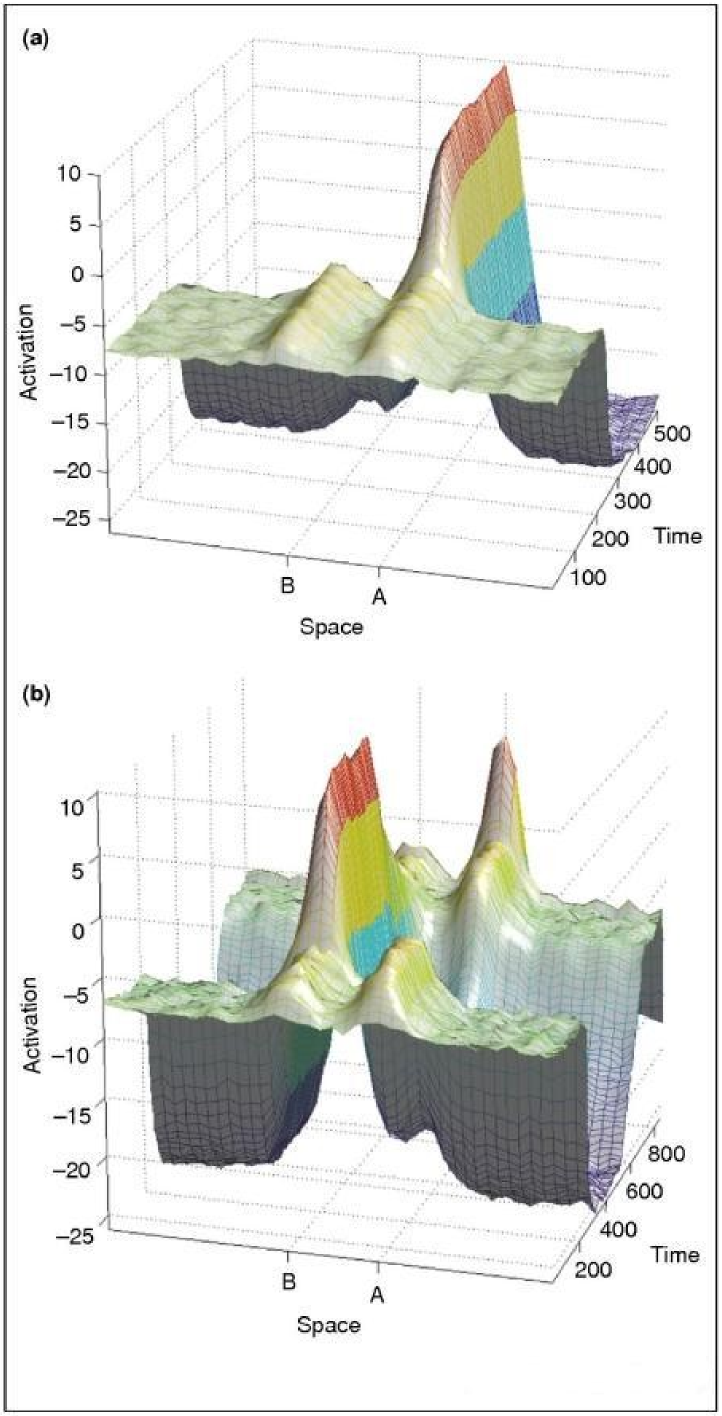
How exactly do cognitive systems translate into dynamical ones?

> A fundamental assumption in a dynamical approach to development is that behavior and cognition, and their changes during ontogeny, are not represented anywhere in the system beforehand either as dedicated structures or symbols in the brain or as codes in the genes. (*id.,* p. 76)

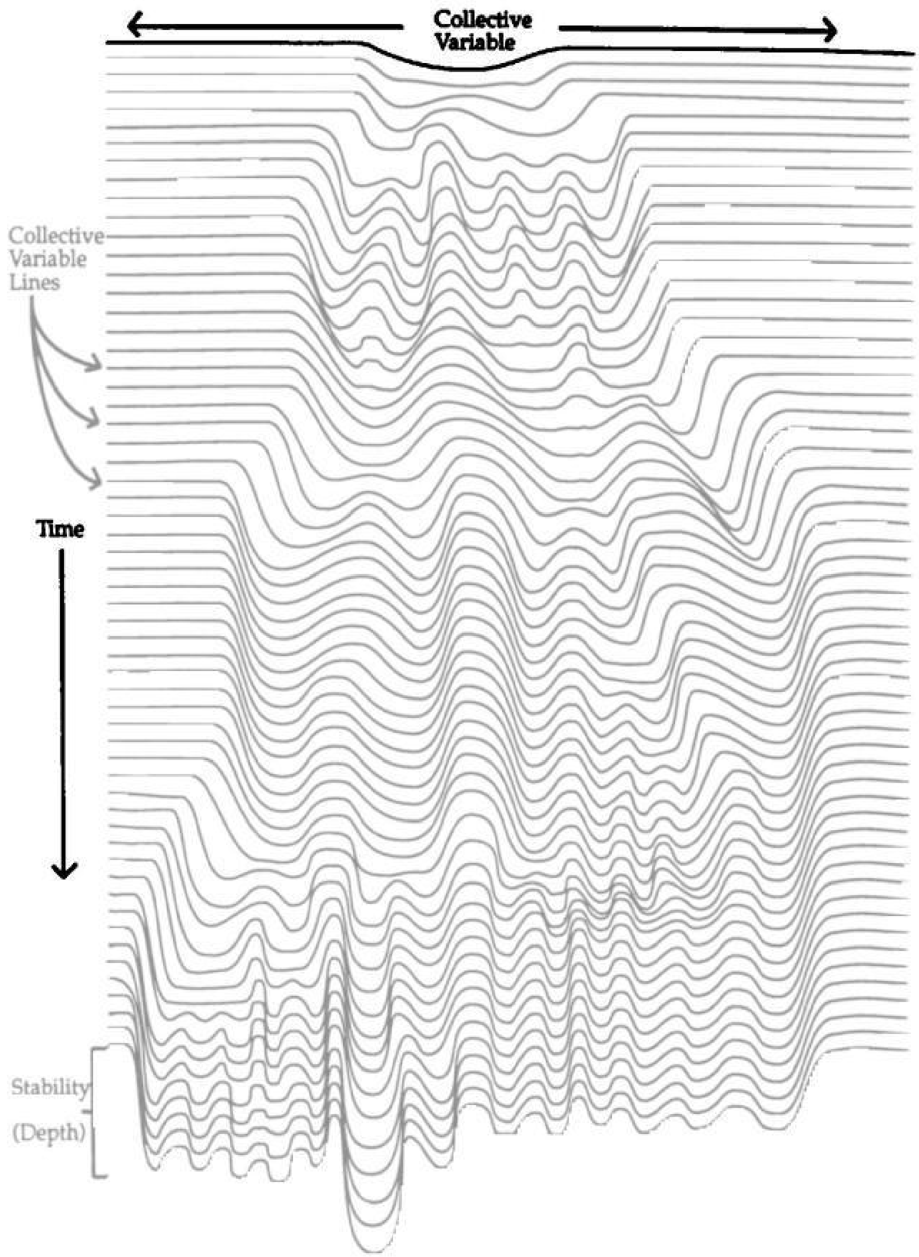
Cognitive processes and behavioral outputs are better thought of as dynamical patterns of activity, function of the context at hand coupled with the intrinsic dynamics of an agent. These intrinsic dynamics are in turn the product of the current architecture of the system and its history of prior activity. Thus, the "behavior represents a reduction of the degrees of freedom of the contributing subsystems into a pattern that has form over time." (*id.*) Every fascinating aspect of cognitive processes finds its place in a compelling dynamical characterization: the stability of an action or thought is considered as the intrinsically preferred states, or attractors, in the behavioral state space of a system. Strong attractors represent patterns of cognitive activity that are more likely to be manifested, the more consistent behaviors. Weaker attractors express instability and perturbation, as well as the variability and unreliability of such patterns. Development itself is the changing landscape of preferred behavioral states. Thelen argues that some of these preferred behavioral states are so ubiquitous within and across individuals of our species that they are interpreted as discrete developmental stages, such as the ones traditionally described by orthodox developmental psychophysics and psychology. These stages are merely high probability states in the behavioral space of cognitive processes. But attractors in a state space cannot be too rigid and stable, otherwise change wouldn't be possible: the combination of moderate attractors and pattern instability could be a good dynamical translation of the concept of behavioral and neural plasticity, in my opinion. Behavior is

> The product of the confluence of components within a specific problem context […] [and] development is likewise a series of both gains and losses as old ways of solving problems are replaced by more functional forms. (*id.,* p. 78)

**Figures 13, 14,** and **15** (following pages) are geometrical MDS representations of different sensorimotor cognitive tasks. **Figure 13** (page 49) represents the performance of an infant in an object-hiding task named the 'A-not-B error', with regards to the development of object permanence (Smith and Thelen 2003). **Figure 14** (page 50) is a general ontogenetic landscape, where development is seen as a series of evolving and dissolving attractors over time (Thelen 1995). **Figure 15** (page 51) is a more sophisticated depiction of the 'A-not-B error' task. It shows some properties of a movement field without specific input (following that there were no cues or training over the decision to make) (Thelen, Schöner, Scheier and Smith 2001).

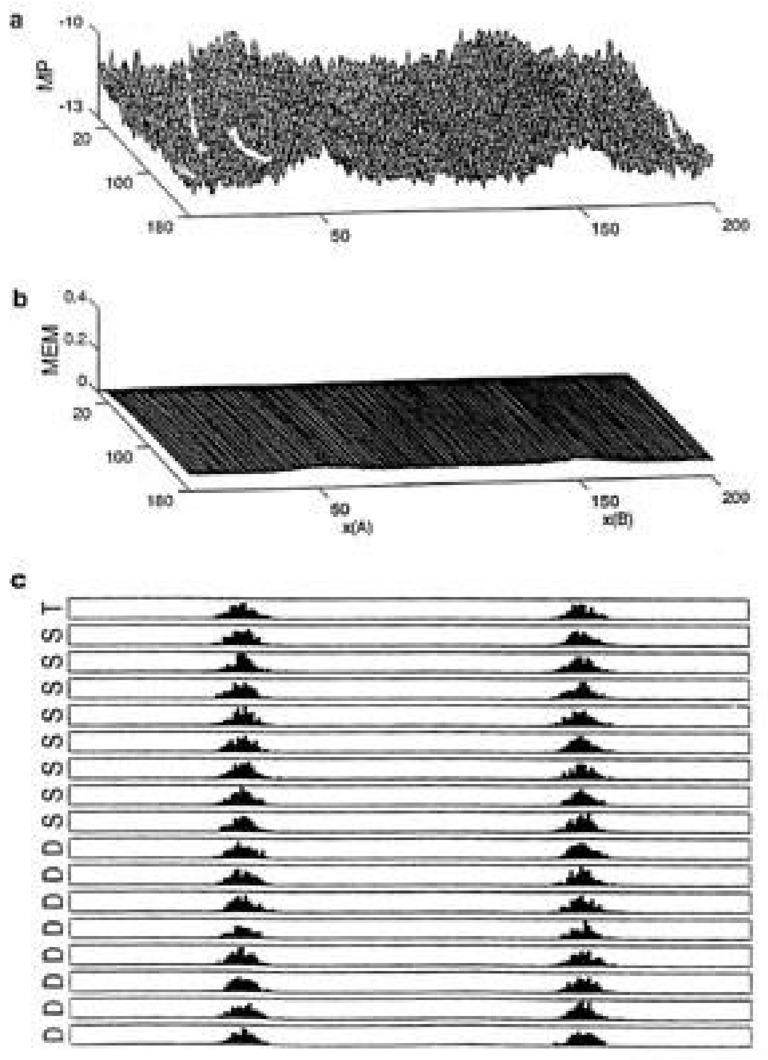



(a) The time evolution of activation in the planning field on the first A trial. The activation rises as the object is hidden and, owing to self-organizing properties in the field, is sustained during the delay. (b) The time evolution of activation in the planning field on the first B trial. There is heightened activation at A before the hiding event, owing to memory for prior reaches. As the object is hidden at B, activation rises at B, but as this transient event ends, owing to the memory properties of the field, activation at A declines and that at B rises.




An ontogenetic landscape; development is depicted as a series of evolving and dissolving attractors. Time moves from back to front. Each horizontal line portrays the probability at any point in time that the system (as indexed by a collective variable) will be in various attractor states. Deep and steep attractors are very stable. Note that the attractor states must flatten out—the system must lose stability—before a new landscape furrow develops. As time progresses the landscape develops multiple stable behavioral attractors. (From Muchisky, M., Gershkoff-Stowe, L., Cole, E., et al., in press.)

(a) Motor field dynamics in non-cooperative regime with task input only (no specific input) at the first reach to A (A1). Parameters: $S_{spec} = 0$, $S_{task} = 1$, $S_{mem} = 3$ (a). Motor planning field evolution (b) Corresponding evolution of memory field. In this figure and subsequent figures, x axis denotes field location, y is time, z is activation. On the y axis a letter code indicates the input present at different moments in time: *T,* task input, *S*, specific input (none added here), and *D*, the delay where no specific input is added. (c). Histograms of decisions to A or B from an ensemble of 500 simulations per trial showing the read-out of the field as a function of time. The decision to reach to A or B is probabilistic; in this case, A or B is equally likely at any point in time.



Thelen also aims to show the essential chronometric properties of cognitive processes, that is, that cognition and behavior are best explained through a set of interdependent variables changing at a number of quantitatively different, embedded time-scales. She distinguishes (but to further reintegrate) what I dub ontogenetic time (learning and development time scale) and cognitive time (task resolution time scale, *i.e.* real time), time scales that are continuous in her model, for they share the same dynamics. The embeddedness of time scales is fundamental to cognition and overall development, and is made available to our quantitative proclivity through and only through dynamical modeling. Thus is expressed not only the confluence of behavior and cognition in a given context, conceptualized as 'local dynamics', but this confluence also shapes, affects the overall internal-external pattern configuration, which we have already named 'intrinsic dynamics'. Shorter version: the cognitive time scale (local dynamics) shapes ontogenetic time (intrinsic dynamics), which feeds back on every subsequent local, real time cognitive processes. Thelen draws an example from the modeling of a simple damped mass-spring[34] to model early spontaneous limb movements in infants, an activity that leads to coordinated sensorimotor control through exploration and selection of values matching the affordances (allowing ourselves a Gibsonian analogy) of the environment coupled with the goal of the task at hand. See figure 14 for an idea of an ontogenetic landscape generated through exploratory and selective experience, where coordination is learnt by exploring "the many different values of the spring parameters generated by [the infant's] spontaneous movements and movements produced in the presence of a goal." (*id.*, p. 80)

The values 'selected' from exploration become attractors in a given class of actions, the clearest depiction of the causal relationships between local and ontogenetic dynamics. The mathematical dynamical system (MDS) fits of course the real dynamical system (RDS) that is the infant's psychophysiological 'substrate':

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[34] The mathematical expression of which is $mx + kx + sx = f(t)$, $x$ being the displacement of the spring and its derivatives, $m$ is the mass, $k$ is the damping coefficient (friction), $s$ is the stiffness of the spring, and $f(t)$ is the time-dependent quantity of energy produced by the contraction of the muscle.

> Activity changes the biochemistry and the anatomy of muscles and bones […] These changes occur over a more prolonged time scale than do changes in behavior, but they are part and parcel of the same dynamic. (*id.*, p. 81)

This model can even explain phase shifts and discontinuities of the dynamic specifications of the sensorimotor system of the infant through the simple damped mass-spring analogy: Thelen elaborates on the example where newborns are held upright and make step-like movements. These motions then disappear over the next few months, which can be explained by an increase in leg mass at a faster rate than muscle strength. In terms of the damped mass-spring, the parameter $m$ (mass) is increasing faster than parameter $f$ (energy burst from muscle contraction). Parameters $m$ and $k$ are constant in local time, but they change over ontogenetic time, whereas $s$ and $f$ change over both time scales. The consequent behavioral shift (disappearance of step-like movements when held upright) in ontogenetic dynamics caused by the faster increase in leg mass than muscle strength is again subject to a phase shift at a later age (latter part of the first year), when the gain in muscle strength relative to leg mass is reversed, enabling the child not only to lift their legs when held upright, but eventually to support their own weight. To summarize, cognitive time scales are continuous in such a way that doesn't allow for a clear distinction between them, hence the thesis of embeddedness.

Sensorimotor activity urges us to conceive behavior, real time cognition, and ontogenetic dynamics as seamless and indissociable even in their conceptualization. Isolating one for the sake of analysis is risking a considerable loss of explanatory resolution. But Thelen's plan is to assimilate higher cognitive processes in the same way, to show that beyond biomechanics, developmental processes of sensorimotor coordination are (i) the same for *all* psychophysical and cognitive levels of processing, and (ii) that sensorimotor coordination itself is the *foundation* of all mental activities. These serious conjectures indeed provide a ground for further empirical enquiry, and for the analysis of the relevance and scope of the dynamical hypothesis concerning cognitive science. But what is under examination here is the commensurability of two conceptual frameworks concerning the study of cognition, a type of 'comparative theoretical cognitive science', if you would allow such an exotic epithet, and I will now address the very controversies sustained by proponents of both stances towards cognition. While this chapter was meant to expose the adoption of a contending conceptual framework in the exploitation of cognitive phenomena, we have but barely undertaken a rigorous comparison between the foundational arguments supporting both frameworks. The arguments of Thelen concerning embodiment, embeddedness and the precedence and determination of sensorimotor control over higher-level cognitive processes are most certainly compelling, but we will see that they are only secondary to the epistemological and semantic issues opposing computationalism and dynamicism in cognitive science.

## *- III - Issues, controversies, and answers concerning the framing of cognition in a computational or a dynamical model*

The CHCS and DHCS still raise many debates in cognitive science, and this chapter presents some of the formal and empirical issues raised for and against them from areas such as mathematics, neuroscience, and philosophy. The definitions of computation established in the first chapter, as well as the dynamicist's conceptual repertoire, will be brought into play to assess the significance of such arguments.

In chapter II, I have exposed a certain number of controversial and not-so controversial ideas concerning cognitive science that are related to the issue of conceptual clashes between computationalism and dynamicism, namely: (i) the extrinsic nature of many variables related to cognitive processes of sensorimotor design, versus the interior processing and mapping of everything, leading to (ii) the idea of environmentally- and physiologically- constrained cognitive processes, versus an omniparametrism, or rather an omnicomputationalism of such processes, (iii) some empirical evidence in neuroscience and psychophysics points toward the adoption of dynamical concepts and models to further our understanding of cognition, and also (iv) toward the integration, in the study of cognition, of context, corporeality, and systematicity.

But dynamicism is also silent on many things, namely, (i) it does not provide a theoretical framework for implementation: the dynamical hypothesis is not an implementation theory, much like the debate from Fodor and Pylyshyn (1988) on matters of computationalism *versus* connectionism. Indeed, connectionism can be conceived as an attempt to model the implementation of informational processes into a biologically inspired design. It does not follow that all connectionist models are or should be computational, even if we invoke the polysemous concept of computation as it is exposed in the first section of chapter I. Some supporters of the DHCS claim that some nontrivial types of artificial neural networks have more to do with nonlinear differential equations and dynamics than with digital computation (Grossberg 1995, van Gelder 1990, 1998abc, 1999abc), and that will play an important part in both the following discussion and chapter IV.

Also, (ii) dynamicism, despite having put forward strong arguments for the relevance of dynamics to cognition, does not constitute in itself a definitive rebuttal of computationalism or a vindication of the dynamical hypothesis above all other frameworks. Much more work is needed to this end, through an assessment of the accuracy and explanatory power of the framework, *and* the rebuttal of a possible compatibility, or coexistence, of both stances towards cognition. This will also turn out to be a fundamental issue in the resolution of this comparative analysis.

Finally, (iii) although advocates of dynamicism claim to avoid restricting their framework to mechanistic explanations of cognitive processes by observing mathematical correlations between systems, internal and external, there are accusations from computationalists of dynamics being a sophisticated new avatar of behaviorism. This issue will also be addressed in this section, drawing on arguments about the types of explanation involved in both frameworks.

***- III.I - On the nature of cognition* vis-à-vis *computation and dynamical systems***

This first section deals with the arguments concerning the adequacy of mathematical formalisms with regards to cognition and cognitive processes. The main concepts and subject matters under scrutiny are those of cognitive processes and continuous or discrete time dynamics, symbolic representations and neural computation, and the role of representations in cognitive dynamics. Here will be argued (i) that the dynamicists' conceptions of computation and representation are inadequate, on conceptual and methodological grounds, (ii) that computational cognitive science needs not be rejected on grounds that its symbolic avatar, spawned from the early days of artificial intelligence, cannot account adequately for biological cognition, and (iii) that while representations in models of cognition may be different in format, they are still required to account for cognitive processes, even in a dynamical view.

***- III.I.I - Giunti and van Gelder on the mathematical properties required to properly model cognitive systems and processes***

Giunti has presented many arguments to promote the study of cognitive systems as dynamical systems. While his position has become more moderated with time with regards to the relevance of computational models of cognition, we propose to expose his earlier arguments on cognition as best studied through dynamics. This account will help understand the mathematical issues at stake in modeling cognitive processes, as well as constituting a preliminary acknowledgment of flaws and confusions concerning the concepts of computation and cognition. Giunti exposes two sufficient conditions for a system not to be computational, and suggests that both the time and state space values of computational models lack analytical resolution (continuity, density, *viz.* properties exclusive to real numbers). The remainder of the section follows van Gelder's discussion with his antagonists on objections to the definitions of dynamical systems and digital computers that he champions. Topics of interest concern the scope of such definitions (too narrow or too broad definitions), and the minute distinctions between computational and dynamical models concerning the temporality of cognition, their state space, and considerations on quantification.

*Two sufficient conditions for a system not to be computational*. Giunti's (1995) earlier endeavors in the promotion of the dynamical approach of cognition was very clever: in order to support dynamics, he proposed a formal, comparative analysis, both qualitative and quantitative, of the two frameworks with respect to cognition. His principal thesis, that all cognitive systems are dynamical systems, is uncontentious. It is his secondary thesis that poses a problem: that computational systems are a subclass of restricted dynamical systems that would only gain in explanatory power, if they were 'released' from the shortcomings of the computational framework. While there is no problem *in principle* with the analysis of mathematical models from other areas within mathematics, it does not follow that cognitive science would benefit from the analysis of computational systems in terms of dynamics. Nevertheless, the exercise is original and enlightening, and here is a summary of the argument.

As distinguished in the first chapter, there are RCSs and RDSs in the world, and the mathematical models are said to 'realize' the regularities observed in real dynamical systems. Giunti elaborates on the mathematical characterization of dynamical systems: they have three elements, namely a time set $T$, a state space $M$, and a set of functions $\{g^t\}$. Now, both a MDS and a MCS[35] *instantiate* one or more aspects of a real system, for purposes of simplification and tractability. Thus, it can be said that different MDSs and MCSs can describe the same real world system independently, depending on the parameters and variables of interest. A discrete MDS, also called a *cascade*, is thus a MDS $<T\ M\ \{g^t\}>$ where functions are expressed in the form $g^{t+1}(x)=g(g^t(x))$, and the time set is a set defined on the (non-negative or complete) integers. From a dynamical perspective, thus, a Turing machine (the foremost exemplar in the computational theory of mind, according to Giunti and van Gelder) 'is' a cascade, *viz.* a discrete, mathematical, dynamical system. But dynamicists hold that in order to better understand cognitive processes, we have to have access to the time evolution of the state space of such processes.

Now, according to the tools made available by dynamical systems theory, discrete dynamical systems, or cascades, only appeal to a limited part of dynamics, such as the qualitative concepts of state space, time evolution in terms of periodic, eventually periodic, or aperiodic 'orbits', and attractors. But that's it, since Turing machines and symbolic processors lack any interesting topological and metric properties, according to Giunti. But computational systems have an additional essential characteristic of being effectively describable. In Giunti's words: "Intuitively, this means that the constitution and operations of the system are purely mechanical or that the system can always be identified with an idealized machine." (Giunti, *id.*, p. 559) Thus, a computational system can be more specifically defined

35 A mathematical computational system. The acronym will be used hereafter. Note that the 'mathematical' part of the expression MCS is debatable, since dynamicists specifically appeal to (symbolic) Turing-computation, which conflates symbolic logic, mathematical computation, and essential properties of algorithms.

as an effective cascade, or effective discrete dynamical system. This requires, with regards to fundamental issues in the mathematics of computation, that the state space *M* must be a decidable set, and that each state transition function $g^t$ is effective/computable. The two sufficient conditions for a (dynamical) system not to be computational, then, concern whether its time set or state space is continuous or not: a system is not computational if (i) its time set is defined over real numbers, and/or (ii) its state space is not effectively denumerable. Apparently, we should be satisfied with such scarce formalities.

*On the definitions of dynamical and computational systems.* Before we move to some criticism, I want to complement Giunti's arguments with van Gelder's[36] on similar grounds. van Gelder is struggling with his critics on the topic of the proper treatment of dynamical models and computational ones. He argues that dynamical systems are significantly different from digital computers, the implementation exemplar championed by proponents of the CHCS, in that the state space of a computational system is quite different from a metric space, such as the integers. For a Turing machine, the relevant set of variables is

> […] head state, head position, and locations on the tape. These are the things which change over time in the operation of the machine. The state space of the Turing machine is the set of all possible combinations of values of this set of variables. Ontologically, this is wholly different than the integers. (van Gelder 1998a, p. 2)

So to speak, van Gelder claims that the state space of a computational model cannot be equivalent to a metric space except in a trivial sense, and consequently does not constitute a quantitative system. Further, a system's metric should be independent of its behavior, otherwise "we can't know what the distances are in the state space until we know how the system behaves." (van Gelder, *id.*, p. 3) van Gelder thus accuses Turing machines of having entirely *post hoc* and uninteresting metric properties. Therefore, the criterion of a dynamical system pertaining to its being quantitative in

[36] van Gelder, 1998abc, 1999abc. For an overall perspective, 1998a.

state should require the additional condition of having a behavior- independent metric.

Another issue of concern is that of the confusion over dynamical systems being continuous or discrete. van Gelder claims that it is not the issue at hand while one attempts to discriminate between computational and dynamical systems, but rather a matter of having *quantitative* systems, which is a property of dynamical systems 'not' shared with computational ones. He is perfectly aware that continuity or discreteness of states are significant in dynamics, but also claims, much like Giunti, that dynamics allows the study of both types of systems, whereas computability theory is only interested in discrete systems, and moreover, *"interpreted formal systems"* (van Gelder, *id.*, p. 4) on top of that. On the other hand, there are discrete dynamical systems that have been proposed as models in cognitive science (van Geert, 1995, for an example), and discreteness alone does not make a Turing machine, or a digital computer. The essential temporality of cognition is directly dependent on such matters of quantitative modeling, in van Gelder's words again:

> The fundamental point is that in systems exhibiting quantitative state-time interdependence, the time set is not merely an ordered set used to specify the order of change in which system states are occupied. Rather, it is a metric space, such that amounts of change in state are systematically related to amounts of change in time as measured by that metric. (van Gelder, *id.*, p. 14)

*Objections*

While Giunti and van Gelder's efforts in promoting the dynamical hypothesis in cognitive science are bold and appealing, on grounds of what dynamics have to offer to cognitive science, there are quite a few foibles in the arguments above, which I have split in three categories: (i) arguments in the observation of computational systems as dynamical systems, or in the comparative advantages of dynamics and computability for the study of cognition, (ii) matters of state space, time sets, metrics, and on continuity and discreteness, and (iii) methodological problems related to the interpretation and use of concepts such as cognition and computation. The following considerations address such issues.

*On the relative advantage of dynamics in comparison with computability theory, on empirical and pragmatical grounds*. There are *prima facie* two problems with Giunti's argumentation on the promotion of quantitative dynamical systems 'over' simulation[37] models of cognition: (i) observing correlations between magnitudes, and the interdependent time evolution of features of such processes *does not make it any more fundamental to cognition in any way*, and (ii) dynamics still *does not answer what counts as cognition in the first place*, begging the question of which cognitive magnitudes we are supposed to care about. He admits to point (ii) in his conclusions, while not providing a rigorous argument to waive the issue raised by the first point. Also, Giunti and van Gelder seem to suggest that the relative advantage of dynamics over computation is up for grabs on empirical and pragmatical grounds, beyond formal and conceptual matters. Indeed, they concede that it may turn out that cognition, or a subset of cognitive processes, might best be accounted for in terms of computation, and then argue that it's not a big deal, since computability can also be explained through features and models of dynamics, a more powerful and resourceful mathematical language. Roughly, we could make the following syllogistic inference to sketch this vague and unconvincing point of view:

- All MCSs are MDSs (not really an issue),
- Some MDSs are MCSs (again, not controversial),
- All cognitive systems (CSs hereafter) are RDSs (we have but to agree with that too),
- BUT, I ask, what if most, or all CSs turned out to be possibly modeled as MCSs, in a way that is both necessary and sufficient for our concerns?

Remember, the DHCS is appealing for its formal and conceptual resources, but it does not entail that cognitive phenomena might be relevantly modeled through dynamics, as empirical and pragmatical concerns may waive dynamics in favor of

[37] Giunti calls quantitative, continuous dynamical systems 'Galilean models', by opposition with the limited qualitative and discrete character of symbolic models, which he calls 'simulation models'.

computability. Thus, one very critical epistemological argument supporting the supremacy of dynamics over computability, as far as cognition is concerned, is the following:

- CSs are best described through models that are MDSs, *but* are not *also* MCSs.

Now, postponing criticism on conceptual and formal matters concerning the proper treatment of such mathematical models, here are unanswered questions about the aforementioned empirical and pragmatical issues: if both computational models and dynamical models can, in principle, account for a given cognitive feature, or set of features, which one is preferable, and on what ground? Aren't mathematical models defined with an arbitrary degree of resolution, or precision, *and* chosen on grounds of the type of features, and results, that we are interested in?

*On conceptual and formal issues concerning the divergences in explanatory power of both mathematical models*. Piccinini disagrees with the temporal constraints of computational models, by comparison with the alleged advantage of mathematical dynamical systems. Piccinini claims that

> This objection trades on an ambiguity between the mathematical representation of time and real time. Computations are temporally unconstrained in the sense that they can be defined and individuated in terms of computational steps, independently of how much time it takes to complete a step. But this is not due to the fact that the process being defined is computational. The same is true of any mathematically described process, whether computational or not. Differential equations contain time variables, but *per se* these do not correspond to real time any more than the time steps of a Turing machine correspond to any particular real time interval. In order for the time variables of differential equations to correspond to any particular real time, a temporal scale must be specified (*e.g.*, whether time is being measured in seconds, nanoseconds, light years, or what-have-you). By the same token, the time steps of a digital computing mechanism can be made to correspond to real time by specifying an appropriate time scale. (Piccinini 2004e, p. 10)

Thus, the temporality of cognition need not be exclusive to dynamics, since the allegedly uninteresting and *post hoc* properties of the metric of Turing machines can be made far more interesting by incorporating a relevant time scale into the computational model. This time metric need not be specified in terms of the actual computational steps of the process, and can be made to match the content of what is being computed.

Furthermore, computational models can be made to perform over continuous values, either by design (analog computation), or in the specification of algorithms to this end (computation over continuous values by a digital mechanism, such as neural networks). Real computation, that is, a hypothetical mechanism computing over real numbers, is simply not possible outside of its abstract formulation, since the implementation of such a mechanism defies many levels of physical phenomena, from macrophysical noise to quantum uncertainty effects. Giunti claims that

> An immediate consequence [of the state space not being denumerable] is that *any finite neural network whose units have continuous activation levels is not a computational system* […] A computational system can, of course, be used to *approximate* the transitions of a network [with continuous activation levels]. Nevertheless, if the real numbers involved are not computable, we cannot conclude that this approximation can be carried out to *an arbitrary degree of precision.* (Giunti, 1995, p. 561)

But the interdependent evolution of variables defined over reals is itself computable for a large enough class of numbers and functions. Indeed, Glymour summarizes:

> Suppose we consider a dynamical system as a function *f(w,t)*, where *t* is the real variable representing time, *w* is some *n*-tuple of numerical quantities, including possibly integers, real or complex numbers, taking values in a space of *k*-tuples, u, of similar objects […] Computable complex numbers are defined in terms of computable reals; a computable real number *r* can be defined in various ways—as a computable sequence of rationals converging to *r* with a computable bound on the error at any stage in the sequence; as a number whose digits in some base – say 2 – can be computed by an infinitary generalization of a Turing machine (essentially a multi-tape Turing machine

> that need never stop reading input or printing output), and in other ways. [38] (Glymour 1997, p. 6)

Glymour stresses that not all definitions of computability are equivalent, for they depend on the representations involved, “and for computation on the reals, to the measure of approximation.” (*id.*, pp. 6-7) For example, the simple operation of multiplication by 3 is not computable in the decimal notation of reals, but actually is in binary notation!

There *are* uncomputable systems, including dynamical ones. But such characterizations depend on formal factors that need not concern us here, and in fact *may not concern cognition at all.* For one thing, even many chaotic systems are computable, and so are some quantum phenomena. What about cognitive systems? Glymour holds that the functions proposed to model cognitive processes are expected to be computable, if only for the fact that we may tend to postulate computable systems, “or because natural dynamical systems, including people, are mostly computable.” (*id.*, p. 7) Thus, although we can obviously postulate uncomputable systems in the world, one has yet to come forward with an empirically grounded observation of a cognitive process which can be modeled only through an uncomputable dynamical system. The burden of proof should be on the dynamicists, as this seems like a logically dubious relation, a variation on a faulty generalization, or inductive fallacy: the possibility of uncomputable dynamical systems suspiciously supporting the claim that cognitive systems are uncomputable. Note that van Gelder seems to be rather unfair to computability when it comes to observing similar claims on the relationship between mathematical model and real world phenomena:

> The fact that sequences of discretized states of continuous dynamical systems can be given (digital) computational descriptions is certainly interesting, but all it really shows is that we can set up complicated mappings between the

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[38] A real number is said to be computable if it can be approximated by some algorithm in the following sense: given any integer $n \geq 1$, the algorithm produces an integer $k$ such that: $(k\text{-}1)/n \leq a \leq (k+1)/n$. Another way is for an algorithm to produce a rational number $r$, given any real error bound $\varepsilon > 0$, such that $| r\text{-}a \leq \varepsilon |$.

> realms of dynamics and digital computation. It doesn't show that the dynamical system *is* a digital computer, any more than the fact that we can simulate the solar system on a digital computer shows that the solar system is a digital computer. (van Gelder, *id.*, p. 5)

So, according to van Gelder, it is fair to say that a RDS 'realizes' a MDS that stands as a model of the RDS, but the solar system could not be said to 'realize' a MCS? It seems that MDSs can realize MCSs, but not the opposite. It's too bad, I guess, that I can't appreciate why real world phenomena, dynamical models, and computational ones, coexist in such an irreconcilable asymmetry…

*On the proper treatment of cognition and computation*. One thing really suspicious about the discussion on the comparative advantages of computational and dynamical models so far is an apparent lack of consistency in the use of the concepts of cognition and computation. Indeed, authors on both sides of the divide move back and forth along different intensions and extensions of such concepts, perhaps unknowingly, out of carelessness, or by an outright commitment to the reduction of the many senses of the concepts to some core definition shared by all of its subspecimens. Nevertheless, would the latter case be the actual motivation to do so, their lack of explicitness should be proof enough to the contrary. Through all the literature on computation and dynamics, for example, connectionist models are usually claimed by both sides on grounds of characteristics that they share in common, somewhat exclusively. But as it is becoming obvious through the arguments of Giunti, Thelen, van Gelder, and other dynamicists, their arguments against computability rest on a somewhat archaic intension of the concept of computation, that is, *Turing computation*, or a symbolic view of both functionalist and cognitivist commitments to the CHCS. But as we have seen in chapter I, computation need not be Turing's thesis on the implementation of decidable functions through an abstract mechanism, operating over symbols! That is just one of many interesting properties of the theory interested in computability, and does not constitute a strictly formal account, but also empirical criteria on realization and instantiation considerations. Such considerations will be discussed in section I.III of this chapter, and the controversial issue of connectionism will be covered in the last chapter.

On the issue of cognition, it seems that different authors switch back and forth between what counts as cognitive, be it internal processes, from neurological processes to higher level cognition such as decision making or language use, or behavioral and social processes, involving other agents and an environment which must be inescapably included to study the relevant cognitive features. It can be said, thus, that arguments about, and drawn from, the study of cognition, are very sensitive to the level of description with which they are concerned. For one thing, arguments about the proper treatment of cognition in matters of modeling may not turn out to cover all levels of what counts as cognitive: developing a symbolic information processing model of sensorimotor processes, all things considered, does sound superfluous, and so does observing the continuous correlations of external cognitive features, whatever they might be, when studying the processes by which one performs long division in mathematical problems. Sadly, it seems that many computationalists and dynamicists think that explanations framed in their respective concepts and models can deal with any sort of evidence or phenomena. So far, the strategy of both sides has been to find some cognitive features that can heuristically be explained through their respective framework, and poorly dealt with from the 'adverse' perspective. Thelen, in championing a radical antirepresentational account of cognition, would have all of cognition reduced to basic organizational principles of systems dynamics in a largely behavioral viewpoint, focusing on the coevolving correlations of internal and external magnitudes. Can all of cognition be reduced to dynamical principles? We will see in section II of this chapter that epistemological and methodological issues determine, far beyond this type of semantic warfare, an accurate account of the relevance of both frameworks. Piccinini is no exception, on the matter of computational explanations of neurological processes, and although he is not a supporter of the DHCS explicitly, he does favor a departure from computation-laden models to a more favorable mathematical model. We will see, in the next section, how such an account constitutes a clever empirical support to the aforementioned formal considerations of Giunti and van Gelder, but that it ultimately avoids the problem altogether by refusing to integrate the full extent of computation as a mathematical tool that reaches far beyond symbolicism (in section I.III), and constitutes a different type of explanation altogether (in section II.II).

***- III.I.II - Piccinini on symbols, strings, and neural spikes***

This section deals with Piccinini's (Piccinini 2004e) account of neuroscientic models, which contrasts the biophysical models of Rashevsky *et al* with McCulloch and Pitts' computational endeavors. Piccinini considers that the computational models in neuroscience are inadequate, based on the definitions of the concepts drawn from computation theory. His reasoning, leading to the conclusion that neurons do not compute, is roughly as follows: (i) computation is the manipulation of strings or symbols (ii) neural spikes aren't symbols, spike trains (or sets) aren't strings (iii) the manipulation of spike trains is therefore not computational. He also suggests that we have no reason to believe that other aspects of neural activity are computational, and that we therefore have no reason to believe that neural activity is computation. Some objections will be raised against his conception of what counts as computation and cognition, much as in the section about Giunti's take on the same concepts above.

According to van Gelder *et al*, the dynamical hypothesis in cognitive science is not contrived to a mere externalist and observational characterization of cognition, it is in fact a powerful qualitative and quantitative framework that allows the coupling of various systems, and such systems can be internal informational processes much in the same way that functional decomposition from a computational perspective would have it.[39] Our own foray into sensorimotor cognition and behavior, in chapter II, clearly hints towards the possibility of a dynamical outlook. Piccinini elaborates on how computationalism was absent of

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[39] This issue is actually quite controversial, and is the subject matter of section II.II below.

pioneering work in biophysics (Rashevsky 1938, Householder and Landahl 1945) that provided a framework for neuroscientific modeling. Mathematical biophysics is the formal means to model the change in behavior of biological phenomena, inspired by the concepts and methods applied in physical sciences. Rashevsky and his colleagues used such means to complement a full account of neural mechanisms in the neuroscience of the 1940s, and of the psychological phenomena that supervene on them. Neither the concept of computation, nor any considerations derived from computability theory, were involved in such an undertaking. Rather, ordinary differential and partial differential equations, along with integral calculus, were the formal tools constitutive of biophysical accounts of neuromechanics.

The adoption of computation-laden models was the original contribution of Pitts and McCulloch in neuroscience, drawing from their research and interests in cybernetics. Since computability theory already meant to a considerable extent the modeling of informational processes through Turing's view of computation, *i.e.* through operations on symbols, Pitts and McCulloch purported to explain neural and mental processes in a coherent framework, and thus viewed neural activity as informational processes in much the same way. Neural spikes, namely the activation peaks of electrochemical processes in the nervous system, were thus considered as mathematical symbols, and spikes sets (commonly dubbed spikes trains) were equivalent to strings of symbols. Piccinini grants that similarities between mathematical symbols and neural spikes were easily found, such as their discreteness, and unambiguous individuation relative to the processes in which they take part. But this account is not satisfactory according to him: we need to look at their inherent differences too, which are significant enough to undermine a computational view of neural processing. This observation needs not be surprising at all, since contemporary neuroscientific modeling is much more similar to Rashevsky's mathematical biophysics than to McCulloch and Pitts' computational neuroscience. The problem lies in the fact that while today's neuroscientists seldom treat neural spikes as symbols, it has 'contaminated' the rest of mainstream cognitive science into adopting such a view according to Piccinini, as the CHCS is exactly the view that mental processes are computational processes realized by the brain. Piccinini claims that neuroscience is noncomputational, and gives a detailed account of the shortcomings of identifying neural spikes as symbols, and spikes sets as strings of symbols.

He starts by elaborating an account of computationalism as the manipulation of symbols, and strings of symbols, and emphasizes two properties of (symbolic) Turing-computation relevant to his endeavors: (i) a symbol's content or role[40] is unambiguous, relative to the behavior of the system, and (ii) an output of a computational process depends solely on the following combination: the internal state of the system, the input symbols, and the way in which those symbols are concatenated in a string, for a specific step of the process or a particular time interval. Piccinini then draws on two features of neural mechanisms that will pave the way to support his argument on the 'noncomputationality' of neural pathways: neural spikes are all-or-none events (that is, neither 'simply' symbolic and discrete, nor analog, *i.e.* time-dependent, continuous variables), and neural processes include a large amount of spontaneous activity, which doesn't allow for the simple individuation of input-output matching processes, or the identification of functionally relevant media to carry them out. What follows is that points (i) and (ii) above are simply not found in neural processes, since it is nigh impossible to individuate either functional units in neural signals, or a concatenation relation that would establish sets (strings) of such units. Time-dependence (the absence of clear boundaries for the beginning and end of a signal, of consistency of intervals, of synchronicity), allegedly nondeterministic processes, and the unwarranted significance of the presence or absence of a token or a string, in view of spontaneous activity[41], are all disincentives for a computational account of neural processes.

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[40] Piccinini distinguishes between a semantic view and a functional view of computationalism, an issue which is briefly discussed in section II.I.

[41] Not to be confused with 'noise', as spontaneous activity might in fact be functionally relevant, by opposition to the principled irrelevance of noise for functional purposes, in signal processing.

The worse part of the story, according to Piccinini, is not that neuroscientific phenomena has been wrongly given a computational account, but that this account has led, through the formulation of the CHCS, to a number of conclusions about cognition that are ill-founded. Among others, (i) that we posses an explanatory framework that can accurately account for mental processes through the Turing conception of cognition, (ii) that the Church-Turing thesis puts neural processes on the same ground as digital computation for the sake of an explanation of cognition, and (iii) consequently, it is in principle possible that digital computation might eventually realize identical cognitive prowesses. I could not agree more with such arguments. The problem is, as it has been hinted in the previous section on mathematics and cognition, every single (contemporary) computationalist knows so. Such is the subject matter of the following section.

***- III.I.III - Bechtel and Eliasmith on the issue of representations in dynamical systems***

What transpires so far about the dynamicists' rebuttal of computation as an adequate framework for cognition is its apparent lack of distinction between (symbolic) Turing-computation, championed by the symbolicists quite a while ago (the era of GOFAI, so to speak), and subsymbolic or nonsymbolic models of cognition that are nevertheless computational, in the much larger (to the extent of being somewhat trivial) sense debated on in the first chapter. Piccinini, much like Giunti and van Gelder, makes an unarguably good case against symbolic, or Turing computation, at the expense of being a bit behind schedule. More a case of a straw man argument, then, or as I shall call it specifically, the Don Quixote case against computationalism.

The flaw in this line of argumentation is thus a matter of conflating digital computers and the mathematical conception of computability. Piccinini makes a good case about neural spikes and spikes sets being non- [Turing] computational, but does he make a case against computability in its largest sense? Again, like in the case of Giunti and van Gelder, we should differentiate between what constitutes a computational account of something, from the level of explanation at which we study something. Symbolic computation may account for logico-mathematical skills, the use of language, and almost all of the inner workings and behaviors of a digital computer, it does not appear to fit most of the rest of biological cognition. Nevertheless, it can be said that we do have computational models of cognition in the large, albeit more trivial, sense of the word. Indeed, we have *subsymbolic* models of cognition, realized on digital computers, for one thing. The digital computer, in such cases, is *not* the model, just a platform from which we design the relevant models, at the relevant level of enquiry, thus symbolic computation is nothing but a canvas on which are painted appropriate textures and colors mirroring our conception of cognition, if you allow me the use of such a metaphor.

Bechtel (Bechtel 1998) and Eliasmith (Eliasmith 1997) accuse some proponents of the DHCS of being antirepresentationalists, much to the demise of cognitive science, and based on a misconception of the very concept of representation. While it is incorrect that van Gelder *et al* are against representations in the modeling of cognitive processes, much has to be said concerning the role and format of representation to clear up this fundamental issue. van Gelder does confuse computation and symbol manipulation, and as he unsuccessfully tries to deal away with the wrong concept of representation (a strict, symbolic conception of representations not necessary at all for computational cognitive science), he (as are most dynamicists) is left with a vague, non-operational definition (one where anything can count as a representation, such as attractors and trajectories in the state space of the dynamics of a system). While the DHCS integrates representations in its mathematical characterizations (by means of interpretation of concepts such as attractors and trajectories), it further requires an implementation theory (in Fodor's and Pylyshyn's 1988 sense of a cognitive architecture), like connectionism or other possible implementation models in cognitive science.

Bechtel insists on two essential characteristics of representations in information processing systems, independent of the nature of the concerned system:

(i) the aspect of representations that we usually express as 'standing-for' something else, and (ii) the format of such representations. While we can adopt different views towards what counts as representations, he argues that the former point is necessary as such, and that dynamics do not deal away with representations at all. But supporting the claim of informational systems requiring representations as Bechtel does is unnecessary, since van Gelder does not, in fact repudiate their relevance. Only a number of radical dynamicists, such as Thelen, repudiate the use of representation-laden cognitive systems, and such an ambition trades on a misreading of computational models being conflated with symbolicism, as stated above, more than a sound account against the very concept of representation. Even van Gelder's landmark example of a dynamical system, Watt's centrifugal governor, which is used to counter the computational explanation of an allegedly inherent dynamical nature of cognition, can thus be said to be representational. Indeed, the various components, and interactions, of this type of mechanism nevertheless indicate (stand for) magnitudes of physical phenomena, and determine its operation (are meant to operate on, or produce, a spectrum of outcomes depending on the relevant magnitudes determined by the system).

On the latter point, concerning the format of representations, Bechtel agrees that dynamicists are innovative in promoting non-symbolic, quantitative values to stand for informationally driven systems. Granted, this non-symbolic acceptation of the concept of representation makes it otherwise ubiquitous, and can be said to be some sort of 'minimal (as in low-level) representation'. It is nevertheless operational for the purpose of the framing explanations, relating to "any organized system which has evolved or been designed to coordinate its behavior with features of its environment." (Bechtel, *id.*, p. 16) So representations need not be strictly a matter of propositional format, and moreover, static ones. The proposition that trajectories and attractors might stand for representations, to the benefit of the overall behavior of a system, thus constituting dynamical representations (*i.e.* representations that change over time, influenced by other features of the processes involved) marks this original contribution from dynamics to the study of cognition. Bechtel additionally points out that:

> van Gelder and Port also stress that in DST systems the processes within the system are *not defined over representations*. […] DST, like connectionist modeling as well as much work in neuroscience is concerned with representations that *figure in processes.* (Bechtel, *id.*, p. 9. My emphases)

Eliasmith's criticism is even less reverent towards van Gelder, whom he accuses of being completely beside the point, on his characterization of connectionism, for one thing. As van Gelder first confuses computation with symbolic and digital processing, he then wrongly claims that connectionism has more to do with dynamics than computation. Apparently, van Gelder overlooked Newell's (Newell, 1980, 1990) distinction between the *type* of computer postulated to realize cognitive processes, from the *family* of universal computers. Newell, as did all symbolicists, postulated just that kind of representational systems, *i.e.* symbol systems, *not computational systems*. On the other hand, to say that connectionist models are noncomputational on that (misconceived) ground is preposterous: every and all connectionists have always considered their models to be computational, for it is indeed the very point of connectionism to model information processing in a biologically plausible way, in order to better understand cognition. Connectionists *are* committed to complex dynamical analysis, as a means to account for essential features of information processing in neural networks, but the ultimate goal is to address computational problems (see Churchland and Sejnowski, 1992).

Bottom line is, connectionists, and even symbolicists, have generally had a much broader conception of computation than dynamicists, such as van Gelder. As such, there is no computational *versus* noncomputational division between symbolicism and connectionism, it is a mistaken characterization on the part of certain dynamicists. On the issue of representations, Eliasmith also sharply disagrees with van Gelder's view of dynamical systems having representations in the loose sense of trajectories and attractors, for if any kind of pattern or element of a system might be said to be representational, then the very meaning and use of representations become patently trivial. Bechtel's account of representations, above, should be the preferred view, since it constitutes a minimalist and deflationist account that is nevertheless operational, and allows for representations in both frameworks, their only dissimilarities pertaining to format.

### *- III.II - On the type of explanation involved in computational and dynamical models*

The second section deals with methodological arguments concerning the motivation and scope of both frameworks, as many writers have tried to dissociate, negate, or complement their models in the study of cognitive science. The topics of discussion concern distinctions between functionalism and computationalism, mechanistic and covering laws explanations, and the complementary value of computational and dynamical models. We will firstly consider Piccinini's account of functionalism in cognitive science, in light of his conflation of computation and symbolic processing. In a second section, I will draw on Bechtel's clever characterization of the types of explanation involved in the two conceptual frameworks, a crucial step in the development of this thesis, if not its main grounds for argumentation.

#### *- III.II.I - Piccinini on functionalism and computationalism as independent characterizations in cognitive science*

Piccinini is concerned with what he calls the semantic view of computation (wrongly carried into philosophy, the view that computational states should be individuated by their semantic properties) leads him to propose a strictly functionalist account of computation. However, functionalism and computationalism have traditionally been conflated as one concept, and computation turns out to be only one type of functional explanation, according to the author. The important distinction between computational explanation and computational modeling is also introduced, but I will argue that here again, and similarly to van Gelder and most dynamicists, Piccinini in fact still conflates the empirical thesis of implementation of computer science's computation (*i.e.* it's much stricter symbolic, serial, and discrete account of what constitutes computation), with the mathematical class of computable functions, along with the related algebraic and geometrical properties of its analysis (not to be confused with mathematical analysis' sense of the study of real and complex numbers, and related functions). This will turn out to be of direct consequence with respect to Piccinini's thesis, concerning the CHCS, that (i) (if) any nontrivial computational theory of mind is committed to the existence of appropriate mechanisms that realize the computations, and (ii) (if) the manipulation of spike trains, according to neuroscience, is not computational (section I.II), (iii) (then) there is no nontrivial computational theory that survives the empirical test (according to *his* definition of the functional account of computationalism).

Having criticized above, in section I.II, Piccinini's account of neural processes with respect to the concepts of computability theory, the section is then concerned with Piccinini's (Piccinini 2003, 2004bcd) position on the role of computational explanations in cognitive science. As seen before, Piccinini disagrees with the type of consequences that can be drawn from the foundational theses of the CHCS, for he refuses a computational account of neurological mechanisms that would support cognition. We have already commented on his twofold shortcomings, one being his unwarranted conflation of (symbolic) Turing-computation with the class of formal definitions of computable functions and numbers, the other pertaining to his reluctance to address anything above neural mechanisms as possibly being both cognitive and computational. Indeed, he sticks to neural mechanisms, while debating over the formulation of the CHCS:

> According to the [CHCS][42], neural mechanisms perform computations, and neural computations explain mental capacities more or less in the way that the computations performed by calculators and computers explain the capacities that are peculiar to them." (Piccinini 2004d, p. 2)

[42] Piccinini actually uses the acronym CTMB, for a 'computational theory of mind and brain', which carries ontological commitments that are not under evaluation here. I therefore consider, as mentioned in the introduction, only the epistemological and semantic issues raised by what is meant by the CHCS, to be on par with the DHCS, in this dissertation.

Since I have already commented on the matter of whether computation has any bearing on neurological processes, my endeavors here are to address what Piccinini's insights, concerning the type of explanation involved by the CHCS, entail for the debate over the comparative advantages and flaws of the computational and dynamical frameworks.

Computational explanations are usually defined as postulating mechanisms operating over representations. A first formulation of computationalism, which Piccinini names the *semantic* view of computational explanation, holds that "computations are individuated at least in part by their semantic properties." (*id.*, p. 3) But such an account in untenable, as symbolic ascriptions to representational processes is arbitrary and observer-relative. This first definition of computationalism needs not concern us, since (i) Piccinini conflates formal computation and (symbolic) Turing-computation, and (ii) to argue that neural mechanisms aren't computational on grounds of not being interpretative mechanisms operating on arbitrary symbols is patently evident to us anyways. How about computational explanations being warranted in virtue of a system possibly being modeled as computational? The problem is that too many things would turn out to be computational! That would thus trivialize computational explanations, claims Piccinini. Also, it would establish the phenomena of concern, here being cognitive processes, as computational *a priori*, in a dogmatic way. Again, I feel obligated to reply that some criteria on pragmatical grounds can be invoked, such as Stufflebeam's (Stufflebeam 1998) take on what constitutes intrinsic, *versus* extrinsic computation: Stufflebeam argues that since anything can be modeled as computa*ble*, that is, given an interpretation as a computational system, we should be concerned only with what makes a system computation*al*, *viz.* intrinsically to such a system. It turns out that whatever can be said to *perform* computations, either by design or by an apparently inexorable tendency to be considered as such (thus, on pragmatical grounds), *should* be considered as such. Information processing models should be involved in, and only in, explanations pertaining to entities that process information in a relevant sense. In short, yes, it's up to us to decide what *is* computing, but not everything need, or should, be considered as a computational process.

So, which processes *do* deserve to be called computations in a relevant sense? Piccinini feels compelled to peek into physiology and engineering, to parallel his study of neural mechanisms and computers, and claims that we must observe the type of explanation involved in such scientific endeavors to better understand computation. Now, a general explanatory strategy in applied sciences is to appeal to functional explanations:

> A functional analysis involves the partition of a mechanism into components, the assignment of functions to those components, and the identification of organizational relations between the functioning components. For any capacity of a mechanism, a functional explanation invokes appropriate functions of appropriate components of the mechanism, which, when appropriately organized under normal conditions, generate the capacity to be explained. The components' capacities to fulfill their functions may be explained by the same strategy, namely in terms of the *components'* components, functions, and organization. The process of functional analysis bottoms out in components whose capacities are no longer functionally analyzable; they are to be explained by other explanatory strategies (e.g., subsumption under physical laws). (*id.*, p. 8)

How then, are computational explanations related to functional explanations?

Mechanistic and nomological explanations co-occur quite abundantly in science: "More generally, mathematical descriptions can be employed in conjunction with functional analyses to yield theories and models of functionally analyzed systems." (*id.*, p. 9) The relevance of mathematical descriptions, according to Piccinini, is threefold: (i) to specify the time evolution of mechanisms, or their features, (ii) to observe relations of dependence or interdependence between variables (expressing features of the mechanism), such as input-output matching, and (iii) to observe how the state space of a mechanism changes, or develops, *viz.* to observe trajectories, attractors, and bifurcations. So Piccinini agrees that *mathematical descriptions are complementary to functional analysis* (*id.*), by providing a means to observe the behavior of a mechanism, or the relations between its components, both qualitatively and quantitatively. He then proceeds to trying to convince us that (i) computational and functional explanations are traditionally conflated (in the literature of cognitive science, he claims), and that (ii) computational explanations are but a particular type of functional explanations. While I find the latter to be intuitively sound and uncontroversial, I disagree with the former point, part of which obviously pertains to Piccinini's own flawed, conflated account of the concept of computation. So, functional explanations appeal to the internal states, processes, and inputs of a system. The problem is thus a matter of choosing the appropriate type of functional analysis, for refrigeration, digestion, or photosynthesis have little to do with computation, to name a few. But, avoiding again the debate on matters of formal computation and symbolic processing, is it not just stating the obvious, much to the advantage of computational explanations, their being concerned only with information processing mechanisms in general? By avoiding Piccinini's flawed concept of computation, we thus have little left to argue about, since such derived conclusions were spun by untenable premises.

The fact that Piccinini constantly appeals to (symbolic) Turing-computation to prove his point about our functional explanations having to shift towards another explanatory framework, when neural mechanisms are concerned, makes it hard to disagree with, for symbolicism (the GOFAI era) has faded in popularity quite a while ago. What is *not* fair, however, is Piccinini's treatment of connectionism as being doomed on the same grounds: connectionism is different enough from (symbolic) Turing-computation, or symbolicism, to deserve an analysis of its own, as we shall see in chapter IV. On the bright side, Piccinini has opened up a very important issue about the complementary value of mathematical descriptions and functional analysis, which is the subject of the following section.

### *- III.II.II - Bechtel, on mechanistic explanations versus nomological explanations in cognitive science*

The clash between computationalism and dynamicism may not be about the alleged 'nature' of cognition after all. As we have seen in the previous section, methodological concerns turn out to be a preeminent issue when comparing the two frameworks with respect to the study of cognition. For one thing, is the choice between computability theory and dynamics necessarily one of mutual exclusivity, or aren't such concepts and models more a matter of phenomena of concern? Let's rewind a bit and think about the data on which Thelen builds her arguments supporting dynamics, in chapter II: the evidence of concern is psychophysical phenomena. Now, if we pay attention to the very definition of psychophysics,

**Psychophysics**

**From Wikipedia, the free encyclopedia.**

*Psychophysics is the branch of psychology dealing with the relationship between physical stimuli and their perception. […]*

*Psychophysics studies* psychological scales *for physical stimuli. Hot and cold, for example, are psychological scalings of temperature stimuli for which such* physical measures *as degrees Celsius provide only physical units.*

*Areas of investigation include* sensory thresholds, methods of measurement of sensitivity, and signal detection theory. (My emphases)

Is it not now clearer that such a *modus operandi*, and the concepts relevant to the dynamical framework in cognitive science, have more to do with behavioral observations, and the mathematical description of patterns and regularities of the parameters of a system? Psychophysics, for one thing, differs greatly from cognitive neuropsychology, or task-specific enquiries into psychological faculties, for example, which are areas of research purporting to identify mechanisms relevant to the observed cognitive processes. Psychophysics also incorporate cognitive features, of course, but such cognitive features are *already given*, the concern is to observe correlations of physiological and cognitive magnitudes *already arbitrarily defined and chosen*. One might jut say that in observing sensory thresholds, sensitivity measurement, and signal detection, we are observing the dynamics of a cognitive and sensorimotor system, but in no way is such an endeavor able to produce a constitutive account of the design of a system. As Glymour states about van Gelder's (representative of the DHCS in general) view:

> One way of abiding by some of van Gelder's prohibitions is to adopt a kind of neo-behaviorism […]. Skinner's version of behaviorism tried to confine scientific inquiry and conjecture to functional--indeed dynamical--descriptions of how human and animal action depends on the environmental history to which the creature has been exposed. Doubtless van Gelder's behaviorism would differ considerably from Skinner's in what it lets in, but van Gelder appears to agree with Skinner in wishing to prohibit any inquiry into the internal mechanisms by which the creature does what it does or thinks what it thinks. (Glymour 1997, p. 10)

Dynamicism, thus, in its most radical form (such as Thelen's), would be some sort of sophisticated behaviorism, albeit integrative of some internal considerations (which are *also* to be modeled through correlational observations, not through design and components).

Bechtel (Bechtel 1998)[43] also presents strong arguments about the conception of explanation championed by the dynamicists. He holds that while it is indeed compelling and useful to adopt the concepts and methodologies of dynamics in cognitive science, it is in no way a refutation of the CHCS and the computational approach in general, since their respective type of explanation are orthogonal ways of conducting research. Indeed, while computational models of cognition are interested in mechanistic characterizations of the processes involved, through localization and functional decomposition, dynamical models are of another type of explanation, namely the explanation of cognition through what Bechtel calls covering laws (following the type of explanation championed by the neopositivists). Indeed, a more traditional view of science (Hempel 1966) has been to conceptualize scientific laws as universally true statements, also called nomological statements. But covering law explanations pose a problem when we depart from physics, such as

[43] See also Craver (forthcoming) for a similar argumentation in favor of mechanistic explanations in neuroscience.

in the domains of life and cognitive sciences, where subsuming phenomena under universal laws is not as much the goal as is the discovery of particular processes at work in a given system. The main difficulty, while observing the behavior of complex phenomena, lies in that we have no way to distinguish statements that might be universally true from accidentally true statements, and that low-level physical laws are too simple to be constitutive of nontrivial accounts of such complex phenomena. So, by appealing to mechanistic explanations, we can analyze the processes of a system through *component* processes, "described either physically or functionally." (Bechtel, *id.*, p. 10)

Two underlying assumptions of mechanistic explanations are that of *decomposition* and *localization*, that is, (i) "the assumption that the overall activity results from the execution of component tasks", and (ii) "the assumption that there are components in the system that perform these tasks." (*id.*, pp. 10-11) Complex phenomena such as biological and cognitive systems have been studied for quite a while through mechanistic explanations, for it narrows down many conjectures in testing them through empirical enquiry, and helps formulate sound conclusions concerning the role of component processes into the overall behavior of a system. In Bechtel's words:

> This explanatory strategy is common not just in information processing psychology but in much of contemporary neuroscience; researchers try to decompose the tasks performed by the brain into component tasks and then seek evidence that these tasks are actually performed by neural components. […] These studies accordingly are seeking to identify hypothesized component psychological processes with specific brain regions." (*id.*, p. 11)

Dynamicists hold a somewhat holistic view of cognitive systems and processes, which they claim is incompatible with mechanistic decomposition and localization on conceptual grounds. But mechanistic models need not be simple linear and serial processes, they too can (and ultimately do, in both life and cognitive sciences) be sophisticated accounts of integrated and nonlinear systems! The information processing metaphor is important to mechanistic explanations in that it models “particular components in the system as carrying information about processes elsewhere in the system.” (*id.*, p. 13)

Perhaps one of the most enlightening developments in Bechtel’s comment is about a contrasting feature of dynamical models, in comparison with computational ones, as found in his discussion between connectionist and dynamical models:

> The difference and differential equations in [Townsend and Busemeyer’s[44]] models are intended to describe patterns of linked change in the values of specified parameters in the course of the system’s evolution over time. The parameters do not correspond to components of the system which interact causally. They are, rather, features in the phenomenon itself. (*id.*, p. 14)

In other words, the parameters of dynamics refer to magnitudes, themselves drawn from features of behavioral concern in a system’s process, but do not pertain to the mechanisms’ componential characterization! One links an infant’s capacity to grasp objects with regards to coordination factors such as perception, motor control, and physiological features (mass, strength, *etc.*), as Thelen does, or the arbitrary valuation of motivational features relative to particular consequences or expected outcomes, such as in Townsend and Busemeyer’s model. But none of these parameters appeal to the nature or role of the underlying component processes! In short, dynamics does well at describing correlations and overall tendencies of arbitrarily chosen magnitudes relevant to a system’s behavior, but do not answer ‘why’ questions about such processes (since dynamics are not interested in what does what, to what else, and for what reason), only *partly* ‘how’ questions (since causal and organizational issues are not addressed either).

But Bechtel favors, even supports as essential, the complementarity of mechanistic and nomological endeavors:

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[44] In reference to Townsend and Busemeyer’s (1995) study of decision-making using dynamical representations.

> Assume that we have a correct [dynamical] account of motor behavior […], of motor development […], of perception […], or of decision-making […]. Each of these invites a further question: how is the underlying system able to instantiate the laws identified in these [dynamical] accounts? One way to answer this question is to pursue a mechanistic explanation by trying to decompose the overall behavior and localize subtasks. (*id.*, p. 15)

Accordingly, such methodologies are essentially complementary, and one does not have any kind of 'priority' over the other:

> If a [dynamical] account provides an account at this level [of description of processes], its legitimacy is not undercut by learning how the various components in the system operate and perform their individual roles. (*id.*)

Bechtel even adds a further value, or advantage, of co-opting dynamical (nomological) explanations in the study of cognition: some research conjectures may be doomed to modeling cognitive processes in a way that is inaccurate, and having a fair account of the behavior of a system may warrant a good mechanistic explanation of it *a fortiori*. This argument is equivalent to that of proponents of the need for the ecological validity of models in cognitive science (a good example of which is the characterization of the inverse kinematics problem as seen in chapter II).

Are the CHCS and DHCS incompatible, mutually translatable, or simply orthogonal characterizations of the same phenomena? Bechtel provides us with good reasons to believe not only in their compatibility, but even to an essential complementarity of both frameworks.

> Most cognitive science research has been devoted to determining the nature of the mechanisms underlying cognitive performance, whereas some DST (dynamical systems theory) accounts are rather directed toward identifying laws that relate different parameters in a system. But while there is a difference here between DST accounts and other cognitive accounts, this does not render the two approaches incompatible. Indeed, they are complementary. We want to know both what the regularities are in the phenomena, and what mechanisms underlie them. (*id.*, p. 16)

## *- IV - Close encounters of the third kind: connectionism*

This fourth and final chapter presents the controversial class of connectionist models. Since proponents from both frameworks insist on claiming connectionism as part of their own view of cognition, the entire chapter is devoted to the clarification of what is at stake in connectionist models, both formally and empirically, and in what way it has anything to do with the comparative analysis of computational and dynamical models.

### *- IV.I - Misunderstandings so far: on representation and computation, types of explanation, and the special case of connectionism*

Now that we have spent a lot of ink on conceptual issues concerning the nature, format, role, and explanations involving computation, representation, and dynamics, we have to assess how all of this might claim lineage with yet another type of applied mathematical models, that of connectionism. Connectionism, as we have seen throughout this paper, has pretenders on both sides of the computational-dynamical divide. This chapter's aim is to show that connectionism can be seen as an exemplar model of a particular type of cognitive processes, namely neural mechanisms, and that such a model combines elements of both conceptual frameworks in an essentially complementary way, in its aim to provide us with an accurate account of the biological substrate of psychological processes. But before we even start characterizing connectionist models, let us summarize what we have gathered from our earlier reflections on computation and cognition. Firstly, we have established that computational models need not be (but *can* indeed be) symbolic models, as the latter are but a special type of the former, more generic family of formal models. Secondly, representations are not only necessary features of any mechanistic explanation involving information processing models, but we can conceive them in an operational and minimalist, albeit ubiquitous, way that avoids the aforementioned symbolic characterization and is quite compatible with a dynamical explanation. Thirdly, cognition is meant to refer to many things, since cognitive processes aren't circumscribed to brain processes, or task performances, and can pertain to environmental and social features too. While connectionism constitutes a computational view in a nonsymbolic sense, and involves representations much like in Bechtel's discussion above, it concerns *only* internal, low-level cognitive processes, namely that of neural mechanisms, and how they exhibit features that can be informative about higher level psychological features.

Piccinini's (Piccinini 2004e) comments on the adequacy of computational models to represent neural processes have left us in doubt, since his account of what counts as computational is severely biased. But, like everyone else, he still holds that a functional account of neural mechanisms should be, in principle, possible, albeit not a (symbolicist) Turing-computational one. So, if it is in principle possible to model neural mechanisms through an adequate enough mathematical model, the question that remains is: which one? Connectionist models, much like the ones from computational and dynamical theories of cognition, also have a dual commitment to a formal thesis (which involves both computable functions and mathematical analysis) and an empirical thesis (that the realization of computation in biologically inspired information processing involves parallelism and large-scale distribution, among other things). Glymour sketches the outlines of the motivations and pretensions pertaining to connectionism, with regard to cognition:

> The most obvious and most important fact about cognitive psychology is that on almost every dimension this aim [to figure out cognition], and even more specialized pieces of it, are radically underdetermined by this sort of evidence [traditional sorts of evidence available to psychologists] […] Psychologists have given three sorts of responses to [various sources of] evidences of underdetemination. One is to ignore alternatives and treat speculation as nearly established fact; another is to try to establish only more modest, but still relevant, claims about mental processes and their development; and a third is to try to connect models of mind, in so far as possible, with biology, in the hope that biology will eventually so constrain such mechanisms that together with psychological experiments many of the big questions about how cognition works can be answered. Connectionism is the oldest and most influential instance of the third strategy. (Glymour 1997, p. 2)

Connectionism is built on the assumption that the brain, in fact the whole nervous system, is the substrate of our cognitive, here psychological, processes, and that such processes are carried out in a way that is best captured through a sophisticated computational model, thus appealing to information processing models. Connectionism is defined as

**Connectionism**

**From Wikipedia, the free encyclopedia.**

*Connectionism […] refers to an approach in the fields of cognitive psychology, cognitive science and philosophy of mind which models mental or behavioral phenomena with neural networks […]*

The main assumptions are that (i) a mental state can be represented by a n-dimensional vector of numeric activation values, over neural units interconnected in a network, and (ii) psychological processes commonly referred to as learning and memory are represented by the modification of the strengths (or weights), or the architecture[45], of the connections between such units. Connection weights are themselves described as *NxN*-dimensional matrices. The state of a neural unit

> is a function of the weighted sums of states of its parents, the function roughly approximating how changes in cell potentials depend on inputs. Learning takes place by any of several essentially local algorithms that adjust weights. Memory resides in the weight values. (*id.*)

The motivation for connectionist modeling came from the application of information processing concepts and methods to the physiology of the nervous system, as we have seen in chapter III, section I.II. But we must also be aware of the limitations of connectionist models. For one thing, not all of the local learning algorithms are representative of real neural processes, and artificial neural networks (ANNs hereafter) are simplified, coarse grained versions of such processes. Nevertheless, as stated above, the aim is to constrain the explanations about psychological phenomena through our knowledge of neurobiological processes, an endeavor that is still in progress, and has already shown considerable success over

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[45] By the creation of new connections (representing synaptogenesis) or new neurons (neurogenesis), paralleling actual neurobiological processes. For references on the biological bases of connectionism, see Cline (Cline 2001) on neurogenesis and synaptogenesis, Kandel (Kandel, Jessell, and Sanes 2000) on the neural mechanisms underlying behavior and cognition, Shultz's models of cognitive development (Shultz 2003), and Stein, Wallace, and Stanford (1998) on single neuron electrophysiology.

the last decades. For example, some neurobiological evidence suggests that local neurosynaptic learning might be one of two mechanisms, the other one being global processes, which should be featured in our explanations of neural mechanisms. But as global processes such as hormonal transmission (Glymour 1997) and interactions with glial cells (Fields 2004) are modeled into neurobiological and neurophysiological explanations, they can also find corresponding features in connectionist models. Such sciences, and their models, inform each other and evolve slowly but surely. Global processes, Glymour argues, might just turn out to make connectionist modeling easier, rather than more difficult.

The essential assumption of computation throughout such models is supported not only through formal and conceptual considerations, as we have stated on so many occasions, but also on the sheer success of such a perspective, judging by the ubiquity of computational explanations in cognitive science, and beyond, even in life sciences. In Glymour's words again:

> I think no one with scientific experience can read the papers reviewed by Churchland and Sejnowski, or many other sources, and doubt that it is real science, or that computational and representational ideas are essential to it […] In almost all of [the research in cognitive neuroscience], an essential assumption is that cognition depends on computable biological processes. (Glymour 1997, p. 4)

But connectionism need not be pulled into the allegedly opposite directions of computationalism and dynamicism, it does actually constitute a field rich in mathematical enquiries that is determined by its one motivation: to frame a biologically adequate explanation of cognition. The end justifies the means, and such means are drawn from many branches of mathematics, even statistical analysis![46] Bechtel illustrates this through the example of Elman's (Elman 1995)

46 A remark from Glymour concerning the mathematical inspirations and requirements of connectionism: "[…] the two directions scarcely exhaust the methods by which people try to understand why connectionist systems behave the way they do – at least as important, perhaps more so, is the application of rather conventional statistical techniques to try to gain a qualitative understanding of the causal relations among features of a complex connectionist system." (Glymour 1997, pp. 4-5, note 2)

connectionist model of a language related tasks, the prediction of successive words in a sentence.

Elman uses a recurrent[47] neural network which has both computational and dynamical features. There is no doubt that such a model is mechanistic, for one thing, since it appeals to neural mechanisms and the functional role of their components. The model is also obviously computational for the same reasons stated throughout this paper. But dynamics are here used to analyze the behavior of such mechanisms:

> The question motivating this research is whether recurrent connections provide sufficient information for the network to predict words of grammatically appropriate categories. Elman demonstrated that when an appropriate training regime was used the network's predictions would respect even fairly long range grammatical dependency relations. (Bechtel 1998, p. 15)

Elman is curious about the way this is achieved. How does the network manage to do so? Since his network is significantly complex (involving many neural units, and many more connections between them), the 'information' stored by the network is bound to be massively distributed, and single unit investigation is therefore pointless. Elman consequently uses formal strategies issued from dynamics and statistics, such as cluster analysis and principal components analysis, to observe a reduced state space of its behavior (by observing the qualitative features of the dynamics of a certain number of variables). By comparing the behavior of the network on nearly identical tasks, it is then possible to pinpoint relevant differences in processing, and thus give a satisfactory account of the performance of such a complex mechanism. Elman's combination of mechanistic assumptions, *viz.* that of decomposition and localization, with dynamics' heuristics of cluster and components analyses, provides him with compelling information from which he can then give a detailed account of the phenomena under observation. Note that such dynamical features need not be strictly methodological, and external, characteristics. For one

[47] The next section develops on such specifications.

thing, the recurrence of the network itself is a dynamical feature, and so are many of the features of component learning algorithms, as we will see in the next section.

To summarize in a more concise, disambiguated vocabulary, connectionist models (i) involve subsymbolic (a contrasting feature with GOFAI computation) computational models, (ii) are simplified models of real neural networks, but nevertheless exhibit many of their interesting features (otherwise, there wouldn't be any point to pursue such venture), (iii) are traditionally simulated (here, implemented) on digital computers to make use of their computational power, (iv) realize a mathematical model that involves complex, nonlinear algebraic calculations, and exhibit parallelism and massively distributed representations, and (v) may, or may not (at the risk of excessive simplicity, or triviality) involve essential dynamical features, and/or appeal to dynamics for the purpose of framing adequate explanations. We need to put enough emphasis on that last point, as many researchers, such as Smolensky (one of the founders of parallel and distributed processing), have argued that the direction connectionist models will take is towards fully continuous, high-dimensional, nonlinear dynamic systems approaches.[48]

## *- IV.II - Types of connectionist models, and what makes them more or less dynamical*

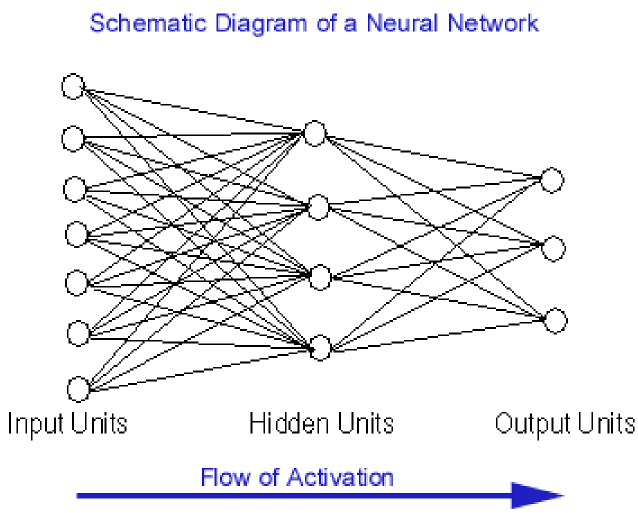
**Neural network**

[48] On that topic, see Elman, Bates, Johnson, Karmiloff-Smith, Parisi, and Plunkett, 1997, and Elman 1998.

**From Wikipedia, the free encyclopedia.**

*A neural network is an interconnected group of neurons. The prime examples are biological neural networks, especially the human brain. In modern usage the term most often refers to artificial neural networks (ANN) […]*

*An artificial neural network is a mathematical or computational model for information processing based on a connectionist approach to computation.[…] It involves a network of relatively simple processing elements, where the global behavior is determined by the connections between the processing elements and element parameters. The original inspiration for the technique was from examination of bioelectrical networks in the brain formed by neurons and their synapses. In a neural network model, simple nodes (or "neurons", or "units") are connected together to form a network of nodes — hence the term "neural network".*


Schematic Diagram of a Neural Network
Input Units
Hidden Units
Output Units
Flow of Activation

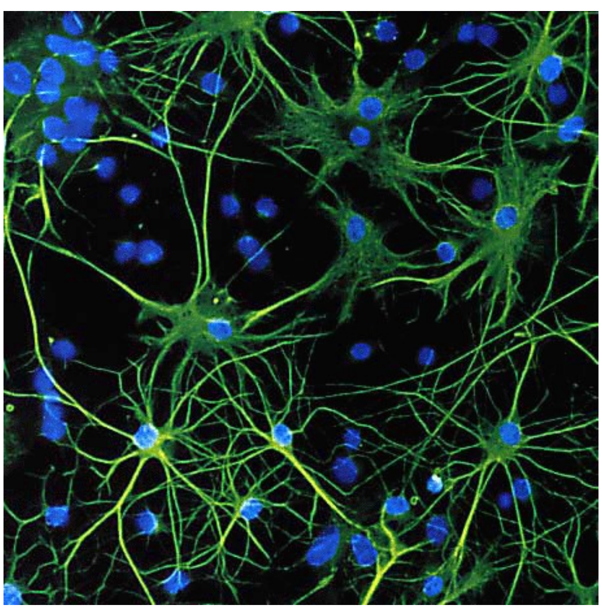


**Figure 16** (left) A neural network, an interconnected group of nodes, akin to the vast network of neurons in the human brain. **Figure 17** (right) Neurosynaptic pathways, illustrated through immunofluorescence.[49]

Connectionists are interested in modeling cognitive processes through neural networks, which are designed to incorporate a variety of parameters and constraints, but their sophistication also depends on the mathematical and informational interests of their engineers. Thus not all artificial neural networks are meant to model cognitive processes, since ANNs are now general computing tools to facilitate the solution of problems in any and all areas one might think of, from engineering design to management, and automated navigation. This section presents an overview of a number of connectionist models, with the aforementioned considerations in mind. The purpose is to assess the potential of connectionism as a model of choice for cognitive science, and its 'situation' in view of computational and dynamical models.

*Prima facie*, what are connectionist models, namely ANNs, used for generally, in cognitive science? Glymour (Glymour 1997) sketches four mainstream types of connectionist research to link neural network models and the study of cognition: (i) *systems simulation*, which endeavors to "describe as completely as possible the nerve connections of very simple animals and simulates them on a computer" (*id.*, p. 3), (ii) *functional analysis*, which focuses on the physiological properties of single neurons, by framing them into information processing explanations, constituting a significant part of neurobiological research (also referred to as single neuron electrophysiology, *e.g.* Stein, Wallace, and Stanford, 1998), (iii) *serial implementation*, which purports to establish serial, and sometimes discrete processes onto parallel and distributed processes, and (iv) *abnormal cognition simulation*, which attempts to simulate the evidence gathered from abnormal neuropsychology in neural networks through the characterization of similar features,

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49 Sources: http://www.psych.utoronto.ca/~reingold/courses/ai/nn.html and http://www.discip.crdp.ac-caen.fr/svt/cgaulsvt/travaux/travmich/synweb/reseau.html.

such as the graceful degradation of artificial networks being quite similar to brain lesions.

Now, artificial neural networks are much simpler and smaller than actual neural subsystems (as can be seen in figures 16 and 17 above, for example), but connectionists are interested in bridging functional features of such models with actual cognitive features and performances, most prominent of which probably is learning, on smaller scales. Formal neural units operate similarly to biological neurons, typically in layers (at least three layers are necessary to exhibit any interesting kind of calculation, namely input, hidden, and output layers[50]), by the summation of weighted synaptic inputs, which may or may not be sufficient to activate a given unit, depending on a given threshold value (usually determined by a sigmoid function). The process is repeated on a massive scale, for all interconnected units. Thus, artificial networks are mathematically designed over 'transfer' functions, representing the activation relation between biological neurons. Such calculations span from very simple (algebraically outputting 0s and 1s) to rather complex (if they are to be representative of biological processes, or computationally useful in any way). *Sigmoid* and *tanh* (hyperbolic tangent) functions are usually employed as transfer functions since they introduce nonlinearity in the calculations of a network, while restricting the domain and codomain's values to a range of [0,1] for the sigmoid function, or [-1,1] for the hyperbolic tangent. A derived advantage of such functions is that their derivatives are simple, and as such allow easier error-correction calculations for neural networks. Such calculations are usually set on random initial values (the state of a system which has no information), then 'trained' by feeding input values that are to be matched to output values through the gradual modification of synaptic weights, in order to obtain a network that can be said to have 'learnt' useful associative patterns, thus constituting some kind of representations, or memory, of relevant data.

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50 The information processing usually occurs in both hidden and output layers, as the input layer typically only serves the purpose of feeding the information to the rest of the network.

Neural networks exhibit many heuristic features of biological neural mechanisms, as well as their 'higher-level' cognitive counterparts: (i) *learning*, through exposure to an environment by means of sensory inputs, (ii) *auto-organized representations*, which result from the learning process on repeated exposition to diverse sources of inputs, (iii) *fault-tolerance*, as representations are redundantly formed as prototypical informations, which are massively distributed, local damage to the network does not impair the network's overall performance, (iv) *flexibility* and *scalability*, as noisy and partial inputs are handled efficiently through such models, and they can handle different problems related to a similar inputs-outputs environment, and (v) *real-time processing*, as the implementation of an ANN can be made to operate on bounded real-valued (continuous) data. As mentioned above, different types of networks can achieve different types of tasks, with a computational might proportional to the degree of sophistication involved in its mathematical and architectural design. We can sketch the following taxonomy as a coarse characterization of the various types of neural networks[51] (such a classification is by no means exhaustive of the ever expanding field of parallel and distributed architectures in computer science):

***Feed-forward networks***

Feed-forward models are ANNs with inputs to outputs activation, the information flowing only in one way. Thus, the outputs from all neurons go to following but not preceding layers, so there are no feedback loops. While such networks can be useful for simple calculation tasks, they do not qualify as dynamical in the preferred way described throughout this paper.

*Single-layer perceptron*

Frank Rosenblatt's (1958) first attempt at modeling parallel and distributed, neural-like information processing, the very simple single-layer perceptron is built as a unique layer of output neurons, to which input values are

[51] All references for such models: Elman 1998, Elman, Bates, Johnson, Karmiloff-Smith, Parisi, and Plunkett 1997, Gurney 1997, Haykin 1998, Rumelhart 1989, Rumelhart and McClelland 1986, Smolensky 1989, Stein, Wallace, and Stanford 1998, Sun 1998.

directly fed, through a set of weights. The output values are simply either ‘activated’, or ‘deactivated’, which are given through a rudimentary learning algorithm named the delta rule. Such networks can only solve linearly separable problems.

*Multi-layer perceptron*

A network usually possessing at least three layers, input, hidden, and output, where all the units from one layer are interconnected with each unit in the subsequent layer. Learning is formalized through the back-propagation algorithm, which compares output values with the expected values to calculate an error-function. This error calculation is in turn used to adjust the weights of the connections in order to minimize the error of further network computations. This weight adjustment algorithm is known as gradient descent calculation.

***Feedback (recurrent) networks***

Recurrent networks are designed to include bi-directional data flow, whereby a function of the output signal of a system is passed (fed back) to the input. This is done in order to control the dynamic behavior of the network. Such networks obviously fit well into our discussion on the complementarity of computation and dynamics.

*Simple recurrent network (SRN)*

Designed like feed-forward multi-layer networks, such models also include ‘context’ units in the input layer. Such units are used to maintain a copy of the previous values of the hidden units, allowing complex computations involving sequence prediction, for instance.

*Fully recurrent network*

A non-layered network where every unit is connected to everyone else. Some subset of the network’s units also receives external inputs, whereas another subset performs the opposite task of outputting values outside of the interconnections.

*Hopfield network/Boltzmann machine*

Such recurrent networks have symmetrical connections, and exhibit dynamical properties quite useful for complex calculations. A Boltzmann machine has the additional feature of involving noisy variables, making it a stochastic network.

*Cascade-correlation*

Another example of recurrent networks, cascade-correlation is a constructive learning algorithm. It starts as a minimal network, consisting only of an input and an output layer. Minimizing the overall error of the network through backpropagation, it adds ('recruits') at each computational step new hidden units to the hidden layer, until the network has assimilated its training input vectors. This allows cascade-correlation networks to learn much faster.

***Integrated networks***

*Committee of machines (CoM)*

Tricks of design can be greatly beneficial to connectionism's computing endeavors, and integrating many networks together is one such clever idea. The idea is to have multiple networks sharing the same architecture, but different initial random weights and input training values, 'vote' together on a given problem. While it doesn't translate into faster processing, it has the advantage of greater output stability over its many calculations.

***Time-based networks***

Whereas recurrent networks offered a first dynamic outlook to computational processes by including feedback and continuous, simultaneous interactivity, networks that integrate the timing and latency of processes have an even greater edge in the race to account for cognitive processes. Such networks can thus be said to have essential dynamic features, on par with the type of continuous dynamical models championed by the proponents of the DHCS.

*Spiking neural networks (SNN)*

Spiking networks propose to model the intrinsic timing of neural spikes, and spike trains, properties essential to the dynamics of biological neural networks. Thus are considered the latency of inputs, the all-or-nothing type of event that is neural activation, and the processes are achieved continuously. Such a model would probably do well in confronting Piccinini's arguments in chapter III, section I.II.

*Adaptive time-delay neural network (ATDNN)*

A type of multi-layer, feed-forward or recurrent network, the ATNN's architecture has a set of neurons which can 'store' their energy level, and are connected to other neurons. In a standard network, each neuron can be connected to any number of neurons in the next layer, but they can only have a single connection to any given neuron. The difference with an ATNN is the use of delayed weights. That is, a given neuron in a preceding layer can be connected to a neuron in further layer many times with different weights. With each weight is associated a time delay, which acts as a memory. Thus the ATNN offers a significant gain in memory, but also the possibility of changing the delay values during training, another significant gain in flexibility.

***Hybrid networks***

*Autonomous robotics control, using Continuous-time recurrent neural networks (CTRNNs) and genetic algorithms (GAs)*

Want to make your artificial neural network even more 'biologically inspired'? No problem, combine it with yet another type of biocomputing model, a genetic algorithm. Genetic algorithms are a subset of evolutionary algorithms, which find solutions to optimization problems through heuristics inspired by biological phenomena such as inheritance, mutation, natural selection, and recombination. By combining a CTRNN, which is similar to the time-based networks mentioned above (with the additional advantage of being capable, in principle, of approximating any dynamical system), with a genetic algorithm, one can design very sophisticated computational models to achieve an artificial sensorimotor control system. The genetic algorithm's input values are the neural networks' parameters, and fitness is measured through the comparative adequacy of outputted motor behaviors.

While the aforementioned list of connectionist models is by no means as nearly sophisticated as are biological neural mechanisms, they do show how much progress can be, and has been, achieved in modeling cognitive processes, or for other purposes, such as developing computing applications and tools. Are connectionist models computational? They are, in fact, essentially so, and were always meant to model cognition as informational processes. Some may be dynamical, in van Gelder *et al*'s view of dynamics, appealing to the continuous, time-based coevolution of variables. Ultimately, connectionist models, as far as they can be used in explanatory endeavors in the study of cognition, are but one type of models of intelligent processes and behaviors. While we were concerned with the comparative advantages and limitations of the computational and dynamical frameworks at large, *viz.* on all levels of description relevant to cognitive science, connectionism constitutes but one such level of cognitively related phenomena. Nevertheless, this examination has only served the purpose of summarily showing how cognition can be appropriately studied through both computational and dynamical concepts and methods. The following sections offer a more comprehensive summary of just how exactly connectionism *is* both computational and dynamical, and moreover, constitutes a good candidate as a model supporting a theory of cognition.

*Connectionism as a computational framework.* Connectionism is a mathematical model that departs from symbolic computation, but is nevertheless built on the latter. The symbolic approach of the GOFAI, mostly concerned with a literal understanding of Turing's thesis on a formal manipulation of symbols being functionally implemented by mechanical means, was a preliminary approach to a theory of cognition, emphasizing a strong resemblance between symbolic processors and the linguistic and logico-mathematical competences of natural cognitive agents[52]. But the machine metaphor was too narrow and restrictive, and a more biologically inspired model of cognition was needed. Connectionism, with its massively parallel and distributed architecture of informational processes, emphasizes on an integrated account of cognition spanning from neurological, psychological, and linguistic[53] considerations on how mental processes should be modeled, while keeping essential features of its computational origin. Most importantly, connectionism is still

- a functionalist account of cognition,
- an information processing model,
- based on representations, albeit not strictly symbolic ones,
- a model that *realizes* a formal representation of neural processes, *implemented* on a computational architecture,
- a good candidate to accommodate higher level symbolic processing as originally conceived by symbolicists.

Thus, connectionism *is* a computational theory, because (i) its concepts are still integral to a functional and informational stance on cognition, while enjoying a considerable gain in explanatory power in the field of cognitive science (we could say that connectionism is a conservative extension of a symbolic theory of cognition, with a much larger scope), and (ii) its models, *viz.* artificial neural networks, are computational ones, *i.e.* they perform calculations by means of an idealized formal model implemented on a computational machine (a digital computer usually). Those calculations are vectorial transformations, which are made in the language of linear algebra. Indeed, connectionist computations usually involve algorithms defining functions of linear and nonlinear algebra, and dynamical and statistical analyses are generally used to observe relevant qualitative and quantitative features of such algorithms, or enhance their computational performance.

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52 Noteworthy is the absence of sensorimotor processes, agency and perception from such an approach to cognition, which were relegated to cybernetics and early robotics. Likewise, a theory of animal cognition was but awkwardly possible under such a framework, since formal and semantic properties of cognition could hardly be attributed to nonhuman animals.

53 I include logical and semantic properties as linguistic ones, for the sake of simplicity.

*Connectionism as a subset of dynamical systems.* Any connectionist theory of cognition that pretends to a nontrivial degree of explanatory power must draw upon dynamical systems theory and dynamical modeling to better describe informational processes, and the intricate relationships between cognition, body, context, and environment. While there can be dynamical models of cognition that deal with nonconnectionist issues, such as behavioral and psychophysiological phenomena, the opposite is hardly relevant anymore to model cognition, as seen in the abovementioned feed-forward models.[54] This is due to the intrinsically dynamic nature of cognitive processes, including the relevant ones for connectionism, namely the neurobiological processes involved in natural cognition. Neural information processing occurs in feedback loops, through the reinforcement of synaptic connections (by means of electrochemical catalysts), or the impoverishment of such connections (by means of inhibitory electrochemical reactions), and such connections may in turn be globally cooperative or antagonistic in the activation of yet other neural processes. Since the architecture of any nontrivial artificial neural network requires the use of formal concepts and tools drawn from dynamics and statistical analysis, we can support without further ambiguities the claim that connectionism is indeed dynamical in nature.

That connectionist models are dynamical is hardly surprising then, and computationalists have as such never denied it, as seen in chapter III. Thus was the issue only a matter of emphasis, as dynamicists would have some connectionist models (the nontrivial, relevant ones in cognitive science, incidentally) belong exclusively to their view. We have seen that such a claim is untenable, if we correctly distinguish between the narrow definition of symbolic computation, and the larger definition of computability. The fact is that connectionism *is* a computational theory of cognitive processes, albeit one that is not concerned only with symbols, and incorporates formal elements drawn from many areas of pure and

[54] Feed-forward models were a first good approximation of just how neural processes might be designed, but are nowadays relegated to purely instrumental purposes, such as the design of inductive algorithms for task-specific computational problems.

applies mathematics, both qualitative and quantitative. It so happens that as real cognitive processes are dynamical, *i.e.* they happen in time, are complex systems whose variables fluctuate interdependently, and can not be reduced to simple linear equations. Any pretender to an accurate account of cognition must thus explain (and consequently be able to model) the dynamical nature of mental phenomena.

*Connectionism as a theory of cognition, and neural networks as models of cognitive processes*. I had intended to present connectionism as but one example of a theory in cognitive science that is both computational and dynamical for very specific reasons, namely the popularity and dominance of connectionist accounts of specific cognitive processes on one hand, but mostly because connectionism was claimed on both sides of the controversy concerning which account of cognition is more accurate, between the CHCS and the DHCS. A fair and impartial account of the benefits of adopting a connectionist theory of cognition must delimitate its scope: connectionism is concerned with, *and only with*, cognitive phenomena occurring at the level of neurological events, and even then, at the one postulated to be relevant to just the kind of phenomena of interest to cognitive science, namely the informational level involved in perception, memory, language use, deliberation, *et caetera*. So on the one hand, connectionism may take into account neurobiological processes of a lower level, such as electrochemical dynamics, and indirect neurobiological events such as synaptic reinforcement through interactions with glial cells for example, but such lower or indirect levels of biological activity are not the focus of such a theory. On the other hand, connectionism is scarcely exhaustive in the modeling of all psychological phenomena, and does not pretend to be able to explain linguistic, sensorimotor, or social cognition solely through the workings of neural processes.

That connectionism does not offer a unified and complete account of cognition is again hardly surprising, as such an account would be quite suspicious from a mechanistic outlook anyhow, and more so, what counts as cognitive is rather inclusive, or permissive. Cognitive science is for this very reason an interdisciplinary endeavor, and connectionism is but one theory among many, sometimes competitive, oftentimes complementary accounts of cognitive phenomena. Being focused on specific aspects of such complex phenomena is but a necessity while threading mechanistic explanations.

## *- Conclusion - Strange bedfellows? Computational and dynamical models in cognitive science*

*"11h15, restate my assumptions: 1. Mathematics is the language of nature. 2. Everything around us can be represented and understood through numbers. 3. If you graph these numbers, patterns emerge. Therefore: There are patterns everywhere in nature."* - Maximillian Cohen, in the movie *Pi* (Darren Aronofsky, 1998)

*"Hold on. You have to slow down. You're losing it. You have to take a breath. Listen to yourself. You're connecting a computer bug I had with a computer bug you might have had and some religious hogwash. You want to find the number 216 in the world, you will be able to find it everywhere. 216 steps from a mere street corner to your front door. 216 seconds you spend riding on the elevator. When your mind becomes obsessed with anything, you will filter everything else out and find that thing everywhere […] But, Max, as soon as you discard scientific rigor you are no longer a*

*mathematician. You become a numerologist."* - Sol Robeson, in the movie *Pi* (Darren Aronofsky, 1998)

***Cleaning up: dealing with some conceptual vagaries in cognitive science***

Early on in the course of my argumentation, I have exposed fine distinctions pertaining to the definitions of concepts like cognition, computation, and that of representation, concepts that find themselves equivocal in the literature of cognitive science. I have also argued that part of the conflict between the proponents of both computational and dynamical models of cognition originated from such conceptual vagaries.

*Quining (some) concepts.* In an entertaining take on the problem of *qualia* in philosophy and cognitive science, Daniel Dennett (Dennett 1988) advocated a radical position by pretending to simply eliminate such a problem, or in his own sarcastic vernacular, by 'quining' it, that is, by refusing to deal with a seemingly important issue on grounds of its not being a real problem in the first place: *"quine, v. To deny resolutely the existence or importance of something real or significant"*. While I do not pretend to do the same about concepts relevant to cognitive science, I do want to adopt a rigorous and strict, positivist-like view on the aforementioned conceptual vagaries: we simply can *not* allow ourselves the extravagance of polysemic references in science, however useful that might turn out to be for analogical reasoning and scientific revolutions. We thus need necessary and sufficient criteria for concepts like cognition, computation, and representation, an issue which has only been covered superficially in the present paper. Many of the arguments proposed by supporters of the CHCS and the DHCS are based on conveniently vague and questionable definitions of what such concepts stand for. For mechanists, there simply isn't a question of whether cognition extends outside the brain, or the body, since most of them are strictly interested in design and functional issues of the internal structures underlying intelligent behavior. Not that it need be so, but the point is that you can't accuse someone of neglecting embeddedness or social cognition on the basis of their choice of a level of enquiry that focuses on internal processes. Granted, it may turn out (and is a liable hypothesis) that social and environmental factors have direct consequences in the shaping of cognitive processes, but the study of neural pathways or syntactical performance does not entail that one has to include such top-down considerations in every aspect of their studies. Better start rigorously at the bottom and work your way up, while nevertheless be wary of external factors that might turn out to be essential in the understanding of your domain of enquiry. Dynamicists, on the other hand, seem to often toss away considerations about the underlying mechanisms of cognition in favor of behavioral and systematic descriptions of what counts as cognitive to them, but neither can they be guilty of being concerned with such factors. What both sides are guilty of, if it is the case, is being parsimonious in their conception of what counts as cognitive, and to what extent one should be concerned with it. This intransigence towards different intensions of cognition doesn't indeed facilitate the debate. Cognitive science doesn't restrict itself to a theory of behavior, a theory of brain, or a theory of mental faculties, it aims to integrate such endeavors.

*Don Quixote's take on computation*. Likewise, computation has been mistreated from the very beginning of the discussion, since the dynamicists' take on the CHCS was obstinately directed towards one very narrow characterization of computability, namely symbolic computation, or Turing's formal account of computable systems. The formal and empirical components of both frameworks aren't equivalent, and some theoretical exemplars of such frameworks (such as digital computers and Watt's centrifugal governor) translate poorly as exemplars of biological cognition. By focusing strictly on symbolicism, and conflating computation and symbolic processes, supporters of the DHCS did little in discrediting computational endeavors in cognitive science, struggling with straw men. Not only did most of the dynamicists' arsenal of arguments not hold the road in convincing us of potential shortcomings in computability theory with regards to the study of cognition, it also marks an embarrassing anachronism in that such an issue was already being debated decades before, in the discussion on the relative advantages of connectionism *versus* symbolicism. More so, connectionists always explicitly endorsed a commitment to the use of dynamics in modeling the cognitive processes of concern. So much for a revolutionary stance. On the issue of representations, we have seen that while such a problem did indeed pervade the arguments supporting both stances towards cognition, it was necessary to posit them in mechanistic explanations. By simply adopting a minimalist concept of representation, we can then only concern ourselves on matters of representational format, which seems to indeed play a significant role into the framing of accurate explanations of cognitive phenomena.

*Everything you've always wanted to know about maths but never dared to ask*. Another important issue concerns the mathematics involved in both conceptual repertoires. Time and again, we have seen that cognitivists clash on formal issues supporting their respective position. Some of those arguments are essential to the debate on the relative advantages of such and such model of cognition, but find themselves squarely regressing back to foundational issues in theoretical mathematics. Continuity and discreteness, integers and reals, algorithms and computable functions, isomorphism and effective representations, and the formalisation of time are many such grounds of dispute. It comes down to a *petitio principii* on the possibility of adequately representing cognitive phenomena through a given mathematical model, and connectionism has been shown to develop in heuristic avenues, to mention only one such model.

*What about the symbolic view?* So much has been said against an essentially symbolic approach to cognition, but is there still a place for it in cognitive science? Indeed, since no one would reasonably doubt the fundamentally symbolic nature of *some* aspects of cognition, namely the acquisition and use of language, high-level processes such as deliberation and logical reasoning, parts of mnemonic and perceptual processes, social cognition, *et caetera*. Regardless of one's semiotic take on the role of icons, indices, and symbols, deeply representation-laden topics in cognitive science are as important as concerns about psychophysics and neurological studies. Since today's cognitive science has a broader conception of what constitutes cognitive phenomena, it should be considered as a conservative extension of older definitions of what is involved in the realm of the mental.

***Of evidence and use***

*If the recipe works, why change it? Dynamicism, flawed logical reasoning, and the burden of proof*. van Gelder *et al* profess the openness of the dynamical hypothesis, *viz.* that only future research and evidence in cognitive will prove the righteousness of adopting a dynamic stance towards cognition. On the other hand, dynamicicts accuse computational models of failing to meet the standards of cognitive science, two rather dubiously prejudiced takes on the same ground of argumentation. As Glymour most acerbically states it:

> In almost all of [cognitive science's] work, an essential assumption is that cognition depends on computable biological processes. And here is where the radical character of van Gelder's thesis begins to come home: van Gelder's thesis is that the thousands of papers on the computational biology of nervous systems relevant to cognition are scientific junk, pursued under some fundamental metaphysical error. […] make no mistake about the boldness (or, to be less generous, the crankiness) of his claim. What should replace a century's scientific investigation of cognitive physiology and its computational aspects is 'dynamical systems'. (Glymour 1997, pp. 4-5)

Likewise, on the issue of cognitive systems possibly belonging to the class of uncomputable dynamical systems, the burden of proof should be on the dynamicists, for the successes so far gathered in computation-based cognitive science don't seem to stress any cognitivist's possible angst towards such an issue. In fact, since most radical dynamicists rely on the promotion of dynamics as a 'larger' framework, encompassing even computable systems, and then taking credit in that cognitive processes *might* be uncomputable but nevertheless still always dynamical processes. Such a line of thinking has more to do with rhetoric than a sound, reasonable position. Promoting the DH as an open empirical investigation project *is* reasonable, as is bearing in mind some pragmatical concerns about the evolution of empirical enquiry in cognitive science anyway. But doing so, far from invalidating the computational stance towards cognition, actually bestows additional merit to computability theory, having progressed that far, and through controversies spanning more than half a century already. A more lenient pragmatical position would be to say, as Bechtel does, that it has proven to be useful to adopt both views towards cognitive processes, and that their complementarity promises even more sophisticated means of explanation in the study of cognition.

***(Not so) strange bedfellows?***

*The best of both worlds*. After behaviorism's downfall around the middle of the twentieth century, mentalistic explanations, of which the CHCS is an offspring, gained popularity because they offered more and more accurate and useful depictions of cognitive phenomena. Obviously, the paradigmatic shift was to eventually reintegrate what it had tossed away, by means of a renewed interest in matters of cognitive performances in given contexts and environments. Dynamics offer one such opportunity of explaining cognition in a systematic, embodied and embedded way. The mechanists that are computationalists posit actual functional features and components in cognitive systems, bearing a strong realism in their type of explanation, while the more nomologically inclined dynamicists have a skeptical outlook to their humean empiricist take on causal explanations. It would seem that this whole debate might revolve around the philosophical problem of causal relations, with optimistic realists clashing with noncommittal empiricists. Indeed, the whole debate might just be formulated as following: is it preferable to be noncommittal towards causal explanations such as the ones involved in mechanistic explanations, by simply sticking to a correlational stance, for a wide array of phenomena?

Or maybe we should shed some dennettian light on the debate, by drawing on an analogy derived from Dennett's (Dennett 1987) take on explanations in the philosophy of mind and cognitive science. Dennett's classic argument consisted of a harmonious division in three 'stances' towards cognitive phenomena, an intentional stance, a design stance, and a physical stance. Now, the intentional stance appeals to mentalistic explanations, ones involving the intentional vernacular of propositional attitudes, such as the attribution of beliefs, desires, and intentions. Unlikely to be dismissed on pragmatical grounds, such a level of explanation of cognition is nevertheless not the preferred means of scientific enquiry in cognitive science. The design stance, on the other hand, posits causal relations and functional roles much in the same way that the aforementioned mechanistic type of explanation involved in the CHCS does, and it was indeed the motivation of Dennett to segregate it in view of the role of computational explanations. The third stance, that of physicalism, is interested in the properties of the substrate of cognition, and appeals to the covering laws of physics and the regularities observed in the life sciences. Obviously, the design and physical stances translate well into our 'competing' schemes that are computational and dynamical explanations. But just as Dennett, some twenty years ago, advocated the complementarity of all such stances both on conceptual and pragmatical grounds, so do I for computability and dynamics, in the wake of Bechtel. All such distinctions, from Dennett to Bechtel, were meant to emphasize the various contributions championed in the name of cognition, and their complementarity is also consequent of just how far reaching the concept of cognition can be, as discussed above. Cognitive science is said to be interdisciplinary precisely for such reasons, bearing on many levels of description, and drawing on concepts and methodologies from all concerned areas of research.

***- Finale -***

To summarize our concluding position regarding the comparative advantages and limitations of the computational and dynamical stances in the study of cognition, we can say firstly that much of the dispute between proponents of computationalism and dynamicism is based on conceptual confusions of the nature of what counts as cognitive, on the nature and role of computation, and also on the nature, role, and format of representations. Secondly, that the complementarity of computational and dynamical models of cognition has been established by virtue of the type of explanation involved through such characterization, and as such, no exclusive claim on cognition can be made by their respective proponents. Such explanations have been segregated as mechanistic and nomological types of analysis, and find their place in the study of cognition by means of integration. Thirdly, the aforementioned integration has a theoretical exemplar in the form of the applied mathematical and informational theory that is connectionism. While we do not claim that such a framework is an exclusive means to model cognition, our endeavors were to show the possibility of drawing upon all available formal and empirical tools and evidence to frame cognitive phenomena in a heuristic, informative manner. Fourthly, the functional account of connectionism can be reciprocally constrained and informed by and with psychophysical evidence, as behavior and internal processing must be integrated into a coherent framework, should there be any claim of an interdisciplinary account of cognition. Similarly, mechanistic and nomological concepts and methods should be employed in concert even where behavior and psychophysical enquiries are concerned, as computational and dynamical tools of analysis are not restricted to the study of internal processes.

***- Appendices -***

## - *Appendix I - Definitions of computation*[55]

The class of computable functions is equivalent to the class of functions defined by the following models:

- *recursive functions*

Class of functions from natural numbers to natural numbers.

Axioms and operators:

(i) The constant function 0 is primitive recursive;

(ii) The successor function $S$, which takes one argument and returns the succeeding number as given by the Peano postulates, is primitive recursive;

(iii) The projection functions $P_i^n$, which take $n$ arguments and return their $i$th argument, are primitive recursive;

(iv) Composition: Given $f$, a $k$-ary primitive recursive function, and $k$ $l$-ary primitive recursive functions $g_0,...,g_{k-1}$, the composition of $f$ with $g_0,...,g_{k-1}$, i.e. the function $h(x_0,...,x_{l-1}) = f(g_0(x_0,...,x_{l-1}),...,g_{k-1}(x_0,...,x_{l-1}))$, is primitive recursive;

(v) Primitive recursion: Given $f$ a $k$-ary primitive recursive function and $g$ a $(k+2)$-ary primitive recursive function, the $(k+1)$-ary function defined as the primitive recursion of $f$ and $g$, i.e. the function $h$ where $h(0,x_0,...,x_{k-1}) = f(x_0,...,x_{k-1})$ and $h(S(n),x_0,...,x_{k-1}) = g(h(n,x_0,...,x_{k-1}),n,x_0,...,x_{k-1})$, is primitive recursive;

(vi) Extension to partial functions: $f$ is *many-to-one*, or *functional*: if $x\,f\,y$ and $x\,f\,z$, then $y = z$. i.e., many input values can be related to one output value, but one input value cannot be related to many output values. The function $f$ need not be *total*, or *entire* (for all $x$ in $X$, there exists a $y$ in $Y$ such that $x\,f\,y$ ($x$ is $f$-related to $y$), i.e. for each input value, there is at least one output value in $Y$);

[55] References for computable functions equivalents and algorithms: J. L. Hein (1996), and R. G. Taylor (1998).

(vii) Unbounded search operator: If $f(x,z_1,z_2,...,z_n)$ is a partial function on the natural numbers with $n$+1 arguments $x$, $z_1,...,z_n$, then the function $\mu x\, f$ is the partial function with arguments $z_1,...,z_n$ that returns the least $x$ such that $f(0,z_1,z_2,...,z_n)$, $f(1,z_1,z_2,...,z_n)$, ..., $f(x,z_1,z_2,...,z_n)$ are all defined and $f(x,z_1,z_2,...,z_n) = 0$, if such an $x$ exists; if no such $x$ exists, then $\mu x\, f$ is not defined for the particular arguments $z_1,...,z_n$.

- *lambda calculus*

A formal system designed to investigate the definition and applications of functions, as well as the concept of recursion. The lambda calculus consists of a single transformation rule (variable substitution) and a single function definition scheme.

Axioms and operators:

(i) Composed of a countably infinite set of identifiers, for example, {a, b, c, ..., x, y, z, $x_1$, $x_2$, ...};

(ii) The set of all lambda expressions can be described by the following context-free grammar:

1) <expr> ::= <identifier>
2) <expr> ::= (λ <identifier> . <expr>)
3) <expr> ::= (<expr> <expr>)

The first two rules generate functions, while the third describes the application of a function to an argument. Usually the brackets for lambda abstraction (rule 2) and function application (rule 3) are omitted if there is no ambiguity under the assumptions that (1) function application is left-associative, and (2) a lambda binds to the entire expression following it. For example, the expression ((λ $x$. ($x$ $x$)) (λ $y$. $y$)) can be simply written as (λ $x$. $x$ $x$) λ $y$.$y$;

(iii) Lambda expressions such as λ $x$. ($x$ $y$) do not define a function because the occurrence of the variable $y$ is *free*, i.e., it is not *bound* by any λ in the expression. The binding of occurrences of variables is (with induction upon the structure of the lambda expression) defined by the following rules:

1) In an expression of the form $V$ where $V$ is a variable this $V$ is the single free occurrence.
2) In an expression of the form $\lambda\ V.\ E$ the free occurrences are the free occurrences in $E$ except those of $V$. In this case the occurrences of $V$ in $E$ are said to be bound by the λ before $V$.
3) In an expression of the form $(E\ E')$ the free occurrences are the free occurrences in $E$ and $E'$;

(iv) Over the set of lambda expressions an equivalence relation (here denoted as ==) is defined that captures the intuition that two expressions denote the same function. This equivalence relation is defined by the so-called alpha-conversion rule (v) and the beta-reduction rule (vi);

(v) alpha-conversion rule (expresses the idea that the names of the bound variables are unimportant): if $V$ and $W$ are variables, $E$ is a lambda expression, and $E[V/W]$ means the expression $E$ with every free occurrence of $V$ in $E$ replaced with $W$, then $\lambda\ V.\ E == \lambda\ W.\ E[V/W]$;

(vi) beta-reduction rule (expresses the idea of function application): $((\lambda\ V.\ E)\ E') == E\ [V/E']$ if all free occurrences in $E'$ remain free in $E\ [V/E']$. The relation == is then defined as the smallest equivalence relation that satisfies these two rules;

(vii) Eta-conversion rule (expresses the idea of extensionality): two functions are the same iff they give the same result for all arguments. Eta-conversion converts between $\lambda\, x\ .\, f\, x$ and $f$, whenever $x$ does not appear free in $f$.

The class of computable functions is also definable as algorithms calculable by:

- *Markov algorithms*

A string rewriting system that uses grammar-like rules to operate on strings of symbols.

Vocabulary and operations:

(elements of the Markov algorithm)

(i) A vocabulary composed of symbols/strings of symbols, and

(ii) grammatical rules;
(operations)
(iii) Check the rules in order from top to bottom to see whether any of the strings to the left of the arrow can be found in the symbol string;
(iv) If none are found, stop executing the algorithm;
(v) If one or more is found, replace the leftmost matching text in the Symbol string with the text to the right of the arrow in the first corresponding Rule;
(vi) Return to step (iii) of operations and iterate.
- *register machines*

An abstract machine used to study decision problems. Also called counter machines, Minsky machines, or program machines.

Vocabulary and operations:
(i) A register machine consists of a finite set of registers $r_1$ ... $r_n$, each of which can hold a non-negative integer, and
(ii) a finite list of instructions $I_1$ ... $I_m$. Each instruction can only be either:

a) INC ($j$, $k$) — increment the value of $r_j$ by 1, then jump to instruction $I_k$;
b) DEC ($j$, $k$, $z$) — check if the value of $r_j$ is zero. If so, jump to instruction $I_z$; otherwise, decrement $r_j$ by 1 and jump to $I_k$;
c) HALT — halts the computation.

- *Post systems*

A deterministic finite automaton with a queue. There is no separate input tape.

(mechanical description)
(i) At the start of the computation, the input string x is loaded on the queue. The input string is followed by a special symbol Zo. At the start of the computation, the contents of the queue are xZo. The first symbol of x is at the front of the queue and Zo is at the end of the queue;
(ii) A transition of a Post machine depends on the symbol at the front of the queue and on the state. Each transition will delete the symbol at the front of the queue. A

transition has two components: the next state and the string to be added at the end of the queue;

(iii) This string can be the empty string.

- *Turing machines*

An abstract machine introduced by Turing to give a mathematically precise definition of an algorithm.

(mechanical description)

(i) A Turing machine consists of:

1) A *tape* which is divided into cells, one next to the other. Each cell contains a symbol from some finite alphabet. The alphabet contains a special *blank* symbol (here written as '0') and one or more other symbols. The tape is assumed to be arbitrarily extendible to the left and to the right, i.e., the Turing machine is always supplied with as much tape as it needs for its computation. Cells that have not been written to before are assumed to be filled with the blank symbol;
2) A *head* that can read and write symbols on the tape and move left and right;
3) A *state register* that stores the state of the Turing machine. The number of different states is always finite and there is one special *start state* with which the state register is initialized;
4) An *action table* (or *transition function*) that tells the machine what symbol to write, how to move the head ('L' for one step left, and 'R' for one step right) and what its new state will be, given the symbol it has just read on the tape and the state it is currently in. If there is no entry in the table for the current combination of symbol and state then the machine will halt.

(ii) Note that every part of the machine is finite, but it is the potentially unlimited amount of tape that gives it an unbounded amount of storage space.

(formal definition)

(iii) A (one-tape) Turing machine is a 7-tuple $M = (Q,\Gamma,\Sigma,s,b,F,\delta)$, where

a) $Q$ is a finite set of states ;

b) $\Gamma$ is a finite set of the tape alphabet ;

c) $\Sigma$ is a finite set of the input alphabet ($\Sigma \subseteq \Gamma$) ;

d) $s \in Q$ is the initial state ;

e) $b$ is the blank symbol ($b \in \Gamma \setminus \Sigma$) ;

f) $F \subseteq Q$ is the set of final or accepting states ;

g) (for a one-tape Turing machine) $\delta : Q \text{ X } \Gamma \rightarrow Q \text{ X } \Gamma \text{ X } \{L, R\}$ is a partial function called the transition function, where L is left shift, R is right shift, or

h) (for a k-tape Turing machine) $\delta : Q \text{ X } \Gamma^k \rightarrow Q \text{ X } (\Gamma \text{ X } \{L, R, S\})^k$ is a partial function called the transition function, where L is left shift, R is right shift, S is no shift.

***- Appendix II - Computational and dynamical models of low-level cognitive processes***

What follows are formal representations of, respectively, an inverse kinematics problem from a computational perspective (related to figures 8, 9, 10, and 11 in chapter II), and a MDS (mathematical dynamical system) of the 'A-not-B error' task (figures 13 and 15 in chapter II).[56]

*Partial code for the inverse kinematics problem: (a) gradient by measurement, (b) gradient by calculation, (c) alternative (faster) gradient following, (d) defining a target through a vector field. One can think of a simple neural network that would implement such algorithms rather easily.*

(a)

```
function Calc_Distance(angle_A, angle_B)
    work out the tip position for joint A = angle_A  and  joint B = angle_B
    return distance from calculated tip position to target
  end function
  dist = Calc_Distance(a, b)
  while (dist > 0.1)
  {
    gradient_a = Calc_Distance(a+1, b) - Calc_Distance(a-1, b)
    gradient_b = Calc_Distance(a, b+1) - Calc_Distance(a, b-1)
    a -= gradient_a
    b -= gradient_b
    dist = Calc_Distance(a, b)
  }
```

(b)

```
for each joint
    if 3D:  axis = axis of rotation for this joint
    if 2D:              axis = (0, 0, 1)
    ToTip = tip - joint_centre
    ToTarget = target - tip
    movement_vector = crossproduct(ToTip, axis)
    gradient = dotproduct(movement_vector, ToTarget)
  end loop
```

(c)

```
dist = Calc_Distance(a, b)
```

56 The references for these codes and formulas are http://freespace.virgin.net/hugo.elias/models/m_ik.htm, http://freespace.virgin.net/hugo.elias/models/m_ik2.htm, and Thelen, Schöner, Scheier and Smith 2001.

```
old_gradient_a = 0
old_gradient_b = 0
while (dist > 0.1)
{
   gradient_a = Calc_Distance(a+1, b) - Calc_Distance(a-1, b)
   gradient_b = Calc_Distance(a, b+1) - Calc_Distance(a, b-1)
   have we gone past it?
   if sign(old_gradient_a) != sign(gradient_a) then
      a -= speeda * old_gradient_a / (gradient_a-old_gradient_a)
      speeda = 0
   else
      speeda += ga
   if sign(old_gradient_b) != sign(gradient_b) then
      b -= speeda * old_gradient_b / (gradient_b-old_gradient_b)
      speedb = 0
   else
      speedb += gb
   move
   a -= speed_a
   b -= speed_b
   dist = Calc_Distance(a, b)
}
```

(d)

```
constant  POINT = 1
   constant  PLANE = 2
   constant  RING  = 3
   structure TARGET
      integer   Target_Type
      vector    centre
      vector    axis
      number    size
   end structure
   function to_target(TARGET T, vector Tip_Position)
      if T.Target_Type = POINT
         v = T.centre - Tip_Position
         return v
      end if
      if T.Target_Type = PLANE
         p = T.centre - Tip_Position
         v = T.axis * dotproduct(p, T.axis)
         return v
      end if
      if T.Target_Type = VECTOR_FIELD
         return vector at this Tip_Position
      end if
   end function
```

*Algebraic characterizations of the MDS for the 'A-not-B error' task (see figures 13 and 15): equation (i) is the dynamic field of the 'A-not-B error' task when inputs are added together, (ii) time scale parameter, (iii) interactions within the dynamic field (cooperation), (iv) interaction kernel of cooperation function, (v) threshold function of cooperation function, (vi) isolated cooperation function, (vii) motor field evolution function, including time scale, cooperativity, inertia, and sensory inputs, (viii) overall field dynamics (precedent function coupled with Gaussian noise), (ix) input sources function for motor planning field dynamics, (x) task input specification function for 'A-not-B error' task, (xi) specific input function during time interval T, (xii) memory field dynamics (another bias of the dynamic field for the 'A-not-B error' task).*

(i)

$$\tau \dot{u}(x,t) = -u(x,t) + S(x,t)$$

$$u(x,t) = \Delta u(x) \exp\left(-\frac{t}{\tau}\right) + S_0(x)$$ (ii)

(iii)

$$\tau \dot{u}(x,t) = -u(x,t) + S(x,t) + g_{intrafield}[u(x');x']$$

(iv) $$w(x-x') = -w_i + w_e \exp\left[-\frac{(x-x')^2}{2\sigma_w^2}\right]$$

(v) $$f(u) = \frac{1}{1+\exp[-\beta u]}$$

(vi) $$g_{intrafield}[u] = \int w(x-x') f(u(x'))\,dx'$$

(vii) $$\tau \dot{u}(x,t) = -u + S(x,t) + \int w(x-x') f(u(x'))\,dx' - u(x,t)$$

(viii)

$$\tau \dot{u}(x,t) = -u + S(x,t) + \int w(x-x')f(u(x'))dx' + h + q\xi(x,t) - u(x,t)$$

(ix)

$$S(x,t) = S_{task}(x,t) + S_{specific}(x,t) + S_{memory}(x,t)$$

$$S_{task}(x,t) = S_{task,0}(\exp[-\frac{(x-x_A)^2}{2\sigma_{task}^2}] + c_{A/B}\exp[-\frac{(x-x_B)^2}{2\sigma_{task}^2}])$$

(x)

(xi)

$$S_{spec}(x,t) = S_{spec,0}\exp[-\frac{(x-x_{spec})^2}{2\sigma_{spec}^2}]$$

(xii)

$$\tau_{mem}\dot{u}_{mem}(x,t) = -u_{mem}(x,t) + \Theta(u(x,t) - u_0)$$

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