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A Holographic Model Of Frequency And Interference: Rethinking The Problem Size Effect

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Abstract

In this paper we used a holographic memory system to model Zbrodoff's (1995) findings on the problem size effect, a well-known effect in the area of Math Cognition. The data showed the effects of manipulating both frequency and interference. We successfully modeled this using DHSM (Rutledge-Taylor & West, 2007), which has previously been used to model the fan effect (Anderson, 1974; Rutledge-Taylor & West, 2008). This demonstrates that frequency and interference effects arise naturally as a function of how holographic systems work.

Keywords: fan effect; frequency; holographic memory; interference; arithmetic

The Dynamically Structured Holographic Memory system (DSHM) uses holographic representations as a way of modeling human memory. It is based on Jones and Mewhort's BEAGLE lexicon model (Jones & Mewhort, 2007). The details of DSHM and the similarities to BEAGLE are discussed in Rutledge-Taylor & West (2007). One function that DSHM models well is memory interference. Rutledge-Taylor & West (2008) showed that the *fan effect* (Anderson, 1974) falls naturally out of the DSHM architecture.

The *fan effect* is a term used to describe a memory phenomenon in which the time needed to verify a fact is related to the number of other facts in memory that include concepts in common with the target fact (Anderson, 1974). The fan refers to how many facts share memory elements with the target. For example, if a person's declarative memory contained three propositions: "the hippie is in the park", "the lawyer is in the store", and "the lawyer is in the bank", then the fan of the terms 'hippie', 'park', 'store', and 'bank' are one, while the fan of the term 'lawyer' is two. As first demonstrated by Anderson (1974), larger fans cause slower reaction times in human subjects. This result is consistent with the theory that similar facts cause interference in the retrieval process.

The DSHM model of the fan effect (Rutledge-Taylor & West, 2008) is conceptually similar to the ACT-R model of the fan effect (Anderson & Reder, 1999). Specifically, the emergent behaviors of the DSHM model can be interpreted as being consistent with the spreading activation mechanisms used to produce the fan effect in ACT-R. In brief, DSHM makes use of Holographic Reduced Representations (HRRs) to encode associations between concepts. According to the DSHM model, memory is composed of holographic items. Each item consists, primarily, of two large vectors - an "environmental vector" and a "memory vector." The environmental vector is static after its creation. It is used as the system's representation of the identity of the item in memory. In contrast, the memory vector is dynamic. The memory vector of an item is used to store all the associations between the item and other items in the system.

Associations between items are formed when a set of items is given to the system as input. Many details aside (see Rutledge-Taylor & West, 2008), the memory vector of each item in the input becomes more similar to the environmental vectors of the items it is associated with. Since the vectors can be thought of as coordinates in a high dimensional space, this means that the memory vectors of items move closer to the environmental vectors of items they are associated with. The fan effect results from the encoding process. Items with larger fans get *pulled* in more different directions and this impedes them from moving particularly close to specific vectors. In the example above, the lawyer gets pulled toward both the store and to the bank, whereas the other characters move closer to a single location. In DSHM the inverse of the distance from the question vector to the nearest vector is interpreted as the activation level of that vector and, similar to ACT-R, the activation level determines the speed of retrieval.

The fan effect, proper, addresses only the effect of inter-fact 'interference' on the efficiency of fact retrieval. However, there is another factor that also strongly impacts retrieval speed/efficiency: the person's frequency of exposure to that fact. For example, if a participant reads "the lawyer is in the store" once and "the lawyer is in the bank" four times, the fans of 'store' and 'bank' are each still one. However, one would expect that the association between 'lawyer' and 'bank' to be stronger than the association between 'lawyer' and 'store'. Thus, both fan effects and frequency effects impact the efficiency of fact retrieval.

In ACT-R, frequency of exposure is represented separately by the base level activation function (Anderson & Lebiere, 1998). In DSHM frequency produces an effect by causing a fact to be pulled more in one direction than another. For example, if the lawyer was in the store more often than in the bank, the vector representing the lawyer would be end up closer to the store vector than the bank vector. To test the interaction of frequency and fan in DSHM we modeled the data of Zbrodoff (1995), who manipulated both of these in the context of studying arithmetic cognition (i.e., retrieving simple facts such as "2 + 3 = 5").

Zbrodoff's Experiments

In arithmetic cognition research, it is often found that small sums, like 2 + 3 = 5, are more quickly retrieved than large sums, like 5 + 7 = 12. This is the so-called *problem-size effect* (reviewed by Zbrodoff & Logan, 2005). It is known that small problems are presented more frequently in math texts than large problems (Hamann & Ashcraft, 1986), so this effect could be due to frequency of exposure, however, interference between memory elements has also been proposed as an explanation (Seigler, 1987; Vergats & Fias, 2005). Zbrodoff's (1995) goal was to investigate the extent to which different retrieval times for different math facts should be attributed to fan effects (which she termed 'interference') or frequency effects (which she termed 'strength') or an interaction of these two effects

Zbrodoff (1995) conducted four experiments that manipulated the effects of strength and interference to assess their relative contribution to the problem-size effect. In each experiment participants were shown mathematical problems with a potential answer on a computer screen. The participant's task was to press one key if the problem was correct, and press a different key if the problem was false. To manipulate frequency and interference for facts in memory, and to eliminate pre-experimental practice effects, instead of using regular arithmetic, Zbrodoff's stimuli were alphabet arithmetic facts (e.g., A + 3 = D, which indicates that the number three letters past A is D). The first addend was always a letter of the alphabet; the operator was always addition; the second addend was 2, 3 or 4; and, the sum was a letter of the alphabet. The problem was considered true if translating the letters to numbers according to their index in the alphabet resulted in a true math fact. For example, "A + 2 = C" is true because translating 'A' to 1 and 'C' to 3, results in the true math fact "1 + 2 = 3". Participants were told that they could determine whether a problem was true or false by starting with the first addend (e.g., A) and then counting through the alphabet the number of letters specified by the second addend (e.g., 3).

In each experiment, participants were exposed to large blocks of problems, each of which consisted in repeated instances of a group of 12 unique problems. Each group consisted of six true and six false problems. A False problem was a problem for which the answer was incorrect. In Experiments 1 and 2, each problem consisted of the combination of one of six letter addends with one of the digit addends (2, 3 or 4). Each letter was paired with only one digit, and each digit was paired with two letters. False problems were generated by setting the incorrect answer to one letter past the correct answer (e.g., B + 3 = F). The groups of problems were counterbalanced so that each letter addend was paired with each numerical addend equally frequently Table 1 provides an example of a group of problems for Experiments 1 and 2.

Table 1: Experiment 1 & 2 group of problems

Letter addend		Number addend		True answer	False answer
A	+	2	=	C	D
B	+	3	=	E	F
C	+	4	=	G	H
D	+	2	=	F	G
E	+	3	=	H	I
F	+	4	=	J	K

Groups of problems for Experiments 3 and 4 were similar to those for Experiments 1 and 2 with two changes. In each stimulus set only two unique letter addends were used (e.g., A and B). Each letter was paired with each of the three digit addends. The false answers were either one letter past the correct answer or one letter before it. Table 2 provides an example of a group of problems for Experiments 3 and 4.

Table 2: Experiment 3 & 4 group of problems

Letter addend		Number addend		True answer	False answer
A	+	2	=	C	D
A	+	3	=	D	E
A	+	4	=	E	F
B	+	2	=	D	C
B	+	3	=	E	D
B	+	4	=	F	E

Re-Analysis of Zbrodoff's Experiments

Zbrodoff (1995) used a method of analyzing the data that was not useful for our purposes. Specifically, she plotted a straight line through the reaction times associated with the different addends and used the slope of this as an index of the magnitude of the Problem Size effect. This is a common

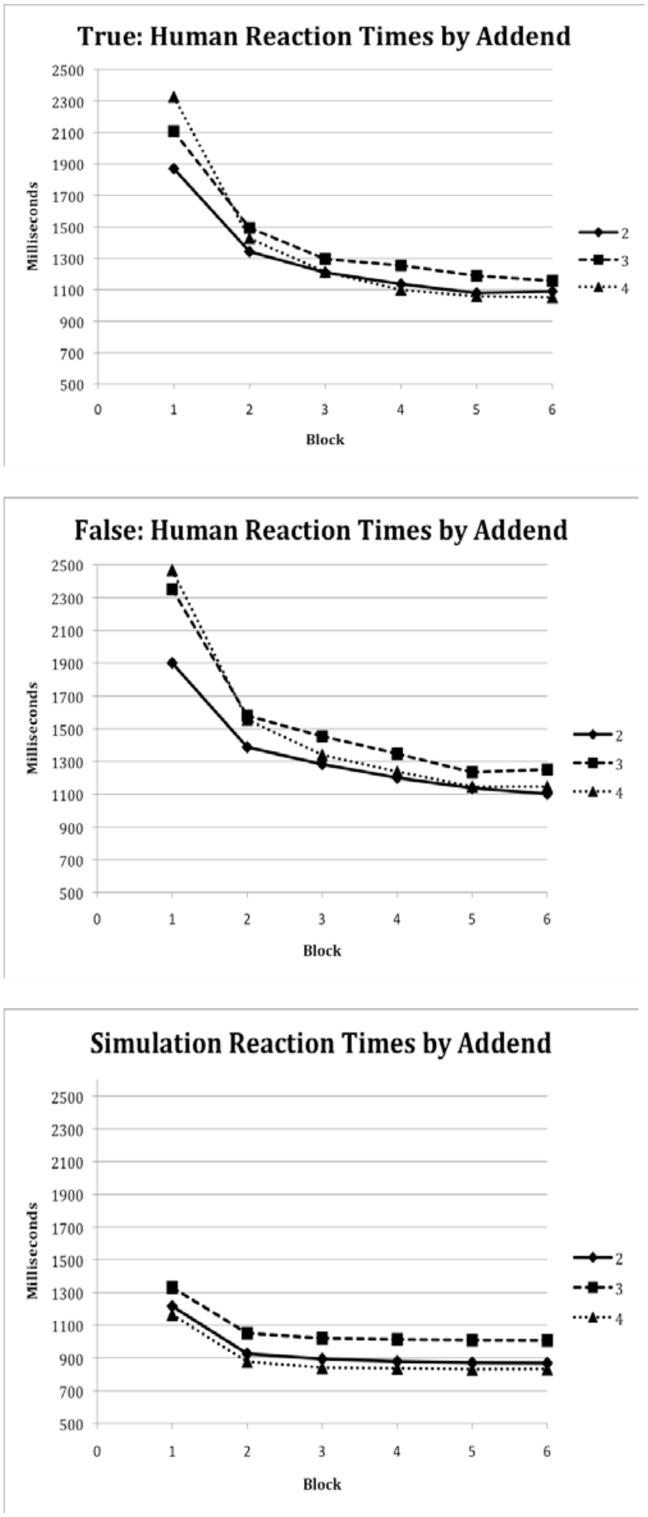

Figure 1: Data and Simulation for Experiment 4

way of analyzing the Problem Size effect within the area of Math Cognition. However, since we were interested in testing our model, not the Problem Size effect, we re-analyzed the reaction time data (which is provided in the paper) and came up with a somewhat different interpretation of the results.

Experiment 1

Experiment 1 was focused on learning in the short term. In it participants were presented with three blocks of problems. Each consisted of 96 true and 96 false problems. This was problematic for us because the learning process involved initially doing the calculations to get the answers. Therefore, these blocks represent a mixture of calculating and memorizing. By the third block we assume subjects were using memory, but may have still been relying on calculation as well. Since the DSHM models memorizing only, we could not represent the effect of calculating.

Also, we do not as yet have a theory about the relationship between presenting a stimulus to a human and adding a vector representing that stimulus to DSHM. In our simulations we assumed a one to one mapping for convenience but there is no theoretical or empirical reason to support this. Therefore, our goal was not to model the learning curve. Instead we were focused on the long term learning trends. Because of this, we did not include Experiment 1 in our analysis. However, the results of Experiment 1 showed that frequency of exposure to a problem eventually resulted in faster reaction times, regardless of whether the addend was large or small. This is consistent with the DSHM model since, if all other things are held equal (as they were in Experiment 1), more exposures results in faster recall.

Experiment 2

The purpose of Experiment 2 was to determine whether the frequency effect demonstrated in Experiment 1 held up once performance had reached asymptote. That is, with enough practice does response time performance converge despite differences in frequency of exposure. To test this, participants were presented with 15 blocks of problems (3 blocks per day) identical to those from the *Standard* condition of Experiment 1, which was designed to mimic real world math learning conditions where smaller numbers are encountered more frequently than larger numbers. Specifically, the addend 2 was presented 24 times per block, the addend 3 was presented 16 times per block, and the addend 4 was presented 8 times per block. The results showed that performance did converge as it reached an asymptote. However, we also noticed that reaction times were much lower at asymptote in Experiment 2 than in Experiments 3 and 4 (approximately 600 msec in Experiment 2; 1000 msec in Experiments 3 and 4). To explain this discrepancy we examined the stimuli and found that in Experiment 2 each letter-answer was uniquely associated with a different letter-addend in the question (see Table 1). Therefore subjects could memorize a pairing between the letter-addend in the question and the letter-answer. Given that this was not the case in Experiments 3 and 4 we feel the learning and use of this strategy is a likely explanation for the faster reaction times in Experiment 2.

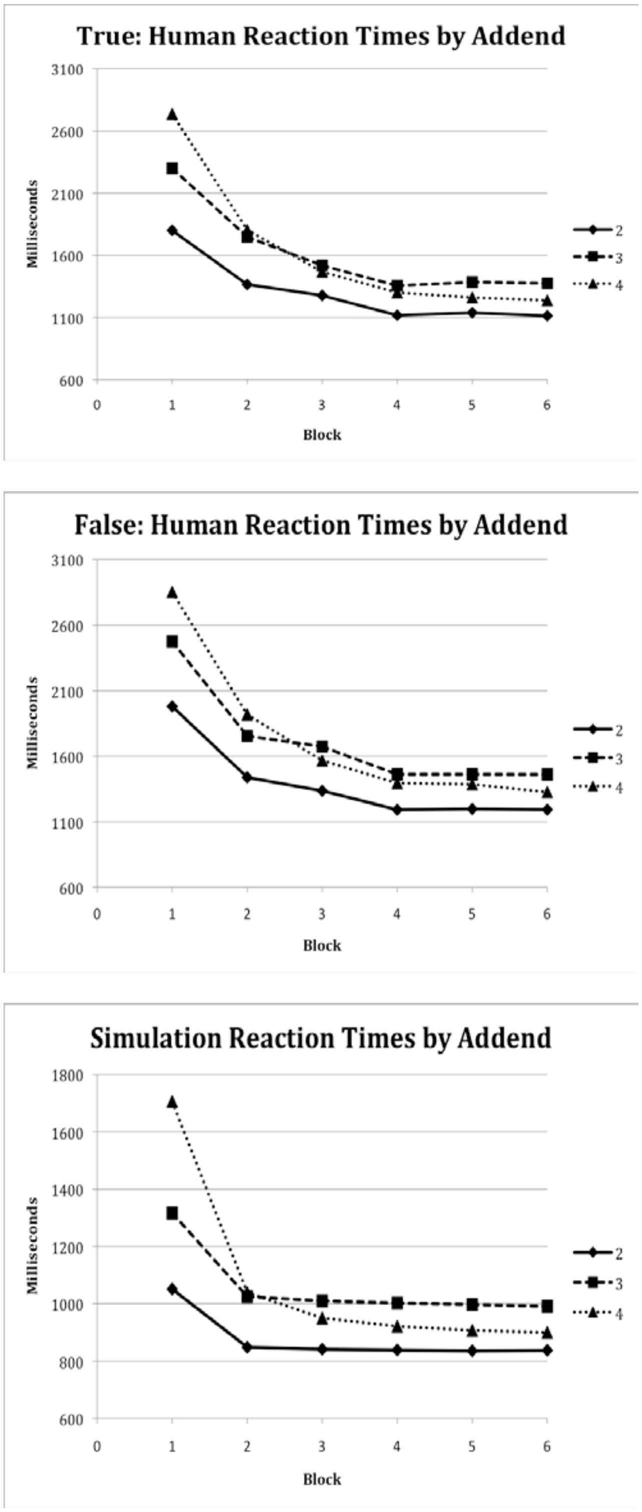

Figure 2: Data and Simulation for Experiment 3

The DSHM could model this but doing a simulation was unnecessary. Because this strategy ignores the addend manipulation the model would predict no differences at the asymptote because there would be nothing to produce a difference.

Experiment 4

We begin our modeling results with Experiment 4, because it is less complex than Experiment 3. In Experiment 4 all of the problems in Table 2 were presented with equal frequency. To model this, each problem, including the answer and whether the answer was true or false, was represented as a random vector and entered into the DSHM, so that one entry equaled one presentation to a subject. As noted above, this was done for convenience. The actual correspondence between presenting to subjects and placing vectors in DSHM may be greater or smaller than this.

There were two ways the model could decide if a question was true. One was to submit a question vector with the problem plus the answer and a *blank* for whether it was true or false. The model would then return whether or not it believed the question was true or false. The second way was to submit the question with the answer as a *blank* and whether or not it was true filled in with *true*. In this case the model would return what it believed to be the correct answer (note, the model can make errors but this data is not presented here). The second method fit the data better than the first, suggesting that people were recalling the answers to see if the questions were true or false. In this case the model makes the same predictions for true and false questions. Consistent with this, the human data was very similar for the true and false questions. To get accurate reaction times from the model the inverse of the activation levels were scaled up by a factor of 400. Note that this represents a claim that the activation levels of the model translate directly into reaction times.

Figure 1 presents the results. Note that the reaction time for the addend 3 questions is slowest in both the human data and the simulation. Zbrodoff (1994) concluded that there were no differences because the method of taking the slopes of the reaction times across the addends only works if reaction time goes up or down linearly with the addends. Because the reaction time went up then down with the addends the method was inappropriate and occluded the results. The reason for the result is that the fans of the questions are not equal if you consider the answers as contributing toward the fan. In particular, the fans for the addends 2 and 4 are lower than 3. We can also see that model learns faster than the human subjects and very quickly asymptotes. As noted above, this is because the model does not calculate the answers. To avoid making the graphs too small only the first six blocks are presented. However, after 6 blocks the human data was at asymptote.

Experiment 3

Experiment 3 was the same as Experiment 4 except that frequency was manipulated in the same way as in Experiment 2. That is, the questions with the smaller numerical addends were presented more frequently. The model used here was exactly the same as the one used to model Experiment 4, where only the fan was manipulated. No parameters were altered!

Figure 2 shows the human data and the simulation results. Overall, the model does a good job of accounting for the results. The only exception occurs in the later blocks (not shown on the graphs) where the model continues to have the addends 2 and 4 close together with the addend 3 higher. In contrast, in the human data, the addend 4 moves back up closer to the addend 3. This result is difficult to interpret. It could be that the model does not predict well for long term learning, although it did accurately predict long-term learning for Experiment 4. Another possibility is that subjects were using a rehearsal strategy between sessions. If subjects were recalling the questions and checking them by calculation, or rehearsing them, it could produce this effect since the addend-4 questions would be harder to recall due to the low frequency of presentation (for random recall without a cue, interference should not play a role). Therefore, the addend-4 questions would not be practiced as much. Given that the model fits well in every other respect, further tests using other sources of human data need to be done.

Conclusions

Frequency and interference are key predictors of human memory performance (e.g., retrieval time) in many cognitive tasks. The individual and joint effects of these factors are clearly evidenced in arithmetic fact learning. The present simulations demonstrated that the DSHM modeling system is not only well suited to capture interference (fan) effects (see also Rutledge-Taylor & West, 2008) but also frequency effects in conjunction with fan effects. Both of these effects arise naturally out of the holographic system, suggesting that this way of representing memory is in some way analogous to the way memories are represented in the human brain. Our results also suggest that cognitive modeling may be the way to move ahead in the Math Cognition area.

References

Anderson, J. R. (1974). Retrieval of propositional information from long-term memory. *Cognitive Psychology, 6,* 451-474.

Anderson, J. R. & Lebiere, C. (1998). *The atomic components of thought.* Mahwah, NJ: Erlbaum. [Chapter abstracts, model source code, and web-based simulations]

Anderson, J. R. & Reder, L. R. (1999). The fan effect: New results and new theories. *Journal of Experimental Psychology: General, 128(2),* 186-197.

Hamann, M. & Ashcraft, M. (1986). Textbook presentations of the basic addition facts. *Cognition and Instruction*, 3, 173-192.

Siegler, R. (1987). The perils of averaging data over strategies: An example from children's addition. *Journal of Experimental Psychology: Genera*l, 116, 250-264.

Verguts, T. & Fias, W. (2005). Interacting neighbours: A connectionist model of retrieval in single-digit multiplication. *Memory & Cognition*, 22, 1-16.

Jones, M. N., & Mewhort, D. J. K. (2007). Representing word meaning and Order information in a composite holographic lexicon. *Psychological Review, 114*, 1-37.

Rutledge-Taylor M., & West, R. L., (2007). DSHM: Dynamically Structured Holographic Memory. *The proceedings of the International Conference on Cognitive Modeling*.

Rutledge-Taylor, & West, R. L. (2008). Modeling The Fan Effect Using Dynamically Structured Holographic Memory. *The Proceedings of Cognitive Science.*

Zbrodoff, N. J. (1995). Why is 9 + 7 harder than 2 + 3? Strength and interference as explanations of the problem-size effect. *Memory and Cognition*, 23(6), 689–700.

Zbrodoff, N. J. & Logan, G. D. (2005). What everyone finds: The problem-size effect. In J. I. D. Campbell (Ed.), *Handbook of Mathematical Cognition* (pp. 331–346). New York: Psychology Press.