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Are New Keynesian Models Useful When Trend Inflation is Not Very Low? *

Sergio Lago Alves †
Central Bank of Brazil
and Carleton University

Hashmat Khan ‡
Carleton University

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1125 Colonel By Drive
Ottawa, Ontario, Canada
K1S 5B6

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Sergio Lago Alves†
Central Bank of Brazil
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Hashmat Khan‡
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Abstract

The equilibrium in the standard New Keynesian (NK) model with Calvo pricing becomes explosive at low levels of trend inflation (4 to 7 percent). Even halfway before this threshold, optimal prices, price dispersion, and costs rise rapidly to large values, while output plummets. We show that these well-known issues stem not from Calvo pricing itself but from its interaction with the widely used Dixit-Stiglitz demand structure in NK models. Using a framework with general firms' demand functions and Calvo pricing, we demonstrate that for NK models to have a stable equilibrium at any level of trend inflation, the demand function must not increase unboundedly as relative prices decrease — a condition the Dixit-Stiglitz structure fails to meet. We propose a model with price wedges to modify existing demand structures to satisfy this condition. Applying this approach to models with Dixit-Stiglitz and Kimball-demand aggregators, we show that the generalized NK model with price wedges stabilizes price dispersion under rising trend inflation and prevents output from collapsing. Moreover, this model exhibits superior theoretical and empirical properties, aligning better with micro and macro evidence. It also introduces new implications for the slope of the Phillips curve and the effects of monetary shocks.

Keywords: New Keynesian models, Calvo pricing, trend inflation, steady state problem, demand functions.

JEL Codes: E31, E32, E52

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†E-mail: sergio.lago@bcb.gov.br

‡E-mail: hashmat.khan@carleton.ca

1 Introduction

The literature on positive trend inflation has long recognized that the standard New Keynesian (NK) model with Calvo (1983) price setting does not have a stable solution if trend inflation exceeds some low single-digit threshold (in the 4% to 7% range).[1] Even halfway to the threshold where the steady state ceases to exist, optimal prices, price dispersion, and costs all rise rapidly to very high levels, output plummets, the slope of the Phillips curve becomes arbitrarily close to zero, and the role of expectations in driving inflation sharply increases. These problems mean that NK model cannot be used readily when trend inflation is not very low. Most of the existing literature agrees that these properties arise from the assumption of Calvo pricing which implies that a forward-looking firm might not receive the exogenous signal to re-optimize its price for a long period of time, even though with low probability. At the same time, Calvo pricing to characterize nominal rigidity remains popular for monetary policy analysis. So ad hoc remedies like indexation or mechanically increasing price-adjustment frequency are typically adopted to avoid the issues, both of these have little empirical support. Thus, the non-existence of the steady state—the *steady state problem*—remains embedded at the core of NK models.

In this paper, we show that the steady state problem under trend inflation arises not because of the Calvo pricing assumption but due to its interaction with another modelling assumption commonly used in macroeconomic models, namely, the Dixit and Stiglitz (1977) constant-elasticity-of-substitution (CES) consumption aggregator. The CES assumption leads to a tractable constant-elasticity demand function for all goods in the economy and allows for adding monopolistic competition to macroeconomic models. However, under trend inflation and Calvo pricing the implied demand function diverges to infinite relative demand when relative prices approach zero. This property simultaneously creates several problems for the NK model as mentioned above.

We start by considering a model with a general demand structure, in the spirit of Gagliardone, Gertler, Lenzu and Tielens (2023), and prove that the condition for a steady-state equilibrium to always exist *independently* of the level of trend inflation is that demand remains finite when relative prices approach zero. The CES demand structure fails to satisfy this condition, which is the source of the

[1] While the threshold is somewhat higher (7-9%) in models with strategic substitutability in pricing decisions such as King and Wolman (1996) and Ascari (2004), when strategic complementarities are present, as recommended in Woodford (2003), Bakhshi, Burriel-Llombart, Khan and Rudolf (2007) show that the threshold becomes quite low, about 4-5%, especially if output growth is also taken into consideration. See the first part of Section 4 for an analytical discussion on this threshold. Cogley and Sbordone (2008) find that in the US, the time-varying trend in inflation was never above 5% (their Figure 1) between 1960 and 2003, and hence the condition for the existence of the steady state is satisfied for this low trend inflation period.

steady state problem. This is the main theoretical result of our paper.[2]

We then use our result to assess demand structures implied by consumption aggregators. In models with the Kimball (1995) aggregator, for example, Kurozumi and Van Zandweghe (2016, 2024) note that positive trend inflation does not cause the steady state problem. We provide the underlying reason why the Kimball demand function avoids the steady state problem: it is finite only only when the curvature is sufficiently large. But the empirical evidence does not support large curvature, as shown in Klenow and Willis (2016) and Dossche, Heylen and Van Den Poel (2010). Assuming a large curvature leads to an additional issue. It truncates the distribution support of the relative prices and, therefore, cannot match the price distribution observed in microdata. Thus, simply replacing the Dixit-Stiglitz demand structure with the Kimball aggregator does not offer a compelling solution to the steady state problem.

We, therefore, propose a novel approach to augment any existing demand function used in the literature (e.g. Dixit and Stiglitz (1977) or the Kimball (1995)) to make NK models consistent with *any* level of trend inflation. Our premise is that agents never face infinite demand, for consuming requires extra costs that creates a wedge between the sticker price and the effective price. Those costs can arise either from direct monetary causes or from efforts, which then can be translated into a monetary price. And more importantly, they might be resilient even if the sticker price is set at zero. For instance, apple trees can be very tall, requiring an effort to pick apples even when they are free. And an orange tree about the same height requires the same effort, even though being a different good. Since there is only so much fruit individuals can carry down the trees, the extra costs should increase with consumed fruit volumes, as consuming more fruit requires more climbing. This price wedge prevents individuals from consuming infinite amounts of goods, as it keeps the effective price at a strictly positive level even if the sticker price reduces to zero.

Applying price wedges to Kimball aggregation, the elasticities and superelasticities decrease, aligning more closely with micro evidence. In the simpler Dixit-Stiglitz aggregation, price wedges make superelasticities rise to positive levels, inducing the demand function to have a smoothed-out kinked form that does not diverge to infinity. This feature allows Calvo model, augmented with price wedges,

[2] Our solution strongly departs from the two usual remedies to mechanically resolve the steady state problem. First, by assuming full price indexation to trend inflation. Empirical evidence from macro and microdata, however, suggests that there is very small indexation on individual prices (see e.g., Bils and Klenow (2004), Cogley and Sbordone (2005), Klenow and Kryvtsov (2008), Klenow and Malin (2010), and Levin et al. (2005)). So, this is not a satisfactory resolution. Second, by mechanically increasing the Calvo probability of price adjustment with trend inflation, that is, by introducing some state-dependence with respect to trend inflation. But, as Bakhshi, Burriel-Llombart, Khan and Rudolf (2007) show, the elasticity of the Calvo probability with respect to trend inflation needs to be very high. Put differently, one has to assume essentially that prices are near-flexible even at single-digit trend inflation rates, rendering the NK model not useful for any monetary policy analysis since nominal rigidity is essential to account for the effects of monetary policy on an economy.

to be used for all levels of trend inflation. Importantly, this property holds for any level of price wedges, no matter how small. We also find that price wedges strongly attenuate welfare losses and the increase in price dispersion as trend inflation rises, making them more in line with the findings of Nakamura, Steinsson, Sun and Villar (2018) and Sheremirov (2020). We embed the augmented demand function based on Dixit-Stiglitz with price wedges in a textbook general equilibrium NK model as a proof of concept and study the consequences for the output-inflation tradeoff (the slope of the Phillips curve) and the effects of monetary policy shocks at low (3% to 6%), medium (6% to 10%), and high rates (beyond 10%) of trend inflation.

The price-wedge model offsets the decrease in the slope of the Phillips curve that occurs when trend inflation rises. Thus, for any given level of trend inflation, the slope is larger relative to the standard Phillips curve under trend inflation. We also find that, when considering the model as the economy's true data-generating process under different levels of trend inflation, common empirical approaches to estimate the slope of a simple Phillips curve lead to the same pattern found in the literature,[3] i.e. the empirical-based estimated slopes increase with trend inflation. They gradually increase over the low and medium inflation levels. Only in the high inflation range, though, the pattern becomes negatively correlated. These correlations arise independently of the degree of price stickiness, which by construction does not change with the trend inflation in the model.

When the level of trend inflation exceeds about 10%, we find that inflation becomes less responsive to a monetary policy shock, more persistent, and harder to tame, as it lingers much longer and requires a much greater output sacrifice to bring it down. These properties are in line with recent empirical results found by Canova and Forero (2024). The authors estimate a Markov-Switching model for the US with two states (high and low inflation) from 1960 to 2023. They find that, after contractionary monetary policy shocks, inflation rates do not fall as much and become more persistent in high-inflation states when compared to low-inflation states. These results align well with the experience of many countries whose inflation rates that have faced high double-digit inflation rates.

One of the main advantages of our proposed solution is that we can maintain the Calvo pricing assumption while enrich the demand side of the NK model. The Calvo assumption to characterize nominal rigidity remains popular not only in the academic literature on NK models but also in models used at central banks for monetary policy analysis. Besides the well-known theoretical elegance in modelling nominal rigidity, the Calvo model also does a decent job of matching empirical micro evidence. In addition, the time-dependent Calvo model is also shown to be equivalent to a large class of state-dependent pricing models. Klenow and Kryvtsov (2008) show that the Calvo model matches

[3] See e.g. Kurozumi and Van Zandweghe (2024) and Hazell et al. (2022).

six of the eight stylized facts in the microdata underlying the Consumer Price Index, being even better than some state-dependent models. In line with this result, Costain and Nakov (2011, 2023) build and test a model nesting both Calvo (time-dependent) and Golosov and Lucas (2007) fixed menu costs (state-dependent) models. They find that the parameterization that best fits microdata has low state dependence, implying a Phillips curve closer but not the same as the one implied by the Calvo model. Similarly, Gautier and Le Bihan (2022) estimate a industry-specific Calvo Plus model (based on Nakamura, Steinsson, Sun and Villar (2018) hybrid model with time-and state-dependent pricing) with French micro data on prices and find that 60% of price changes are triggered by the Calvo mechanism. Previously, Bakhshi, Khan and Rudolf (2007) showed that the Calvo model approximates the inflation dynamics generated from the Dotsey, King and Wolman (1999) state-dependent model. More recently, Auclert, Rigato, Rognlie and Straub (2024) show that in a broad class of menu cost models, the first-order dynamics of aggregate inflation is first-order equivalent to a mixture of two time-dependent models (e.g. the Calvo model), reflecting the extensive and intensive margins of price adjustment. Our proposed approach, therefore, supports the use of Calvo pricing in a log inflation environment.

The remainder of the paper is organized as follows: Section 2 reviews related literature on micro and macro evidence. In Section 3, we present the model with a general preference structure, assess how marginal costs increase with trend inflation and discuss how Calvo (1983) price setting is affected in this general framework. Section 4 presents the main result of our paper, in the form of a theorem that describes the general conditions that demand functions must satisfy so that there always exists a determinate steady-state equilibrium independently of the level of trend inflation. Section 5 discusses demand functions compatible with the theorem, assessing models such as Kimball aggregation (Section 5.1) and presenting another contribution of our paper, i.e., models with price wedges (Section 5.2). Section 6 presents simulations, and Section 7 concludes.

2 Micro and Macro Empirical Evidence

Before presenting the formal model, in which we consider a general form for demand functions, we present some micro and macro evidence. This evidence motivates the features demand functions should comprise and the macro predictions they should imply when used in NK models.

Recent empirical literature using large scanner data generally finds relatively low, but still positive, values for elasticities (ξ) and superelasticities (η). Micro evidence (e.g. Burya and Mishra (2022), Dossche et al. (2010), Beck and Lein (2015)) suggests that price elasticities likely range in $\xi^{micro} \in [1.0, 5.0]$, while superelasticities lie in the narrower interval of $\eta^{micro} \in [1.5, 2.0]$. It implies that actual demand functions are "kinked", in the sense that superelasticity (curvature) is positive. As for the distribution of relative prices in the US, Kaplan and Menzio (2015) results suggest it is approximately symmetric, leptokurtic (fat-tailed), has large dispersion, even when controlling for exactly the same product (same UPC barcode - Universal Product Code) or allowing for strong substitutability. Most importantly, the authors find that the distribution has a large support, with some actual prices being about twice as large as the average price.

Turning to macro evidence, price dispersion slightly rises at larger levels of trend inflation. For instance, while Nakamura et al. (2018) find that the size of price changes did not increase in response to the Great Inflation of the late 1970s and early 1980s in the United States, Sheremirov (2020) finds that the positive relationship between price dispersion and inflation is only significant for regular prices. Sale prices, which are included in analyses with all prices, actually dampen this effect. And lastly, international evidence from different countries suggests that trend inflation is negatively correlated with per capita consumption levels (e.g., Bleaney (1999)). Even though standard trend-inflation NK models also predict a fall in consumption as trend inflation rises,[4] the predicted fall is implausibly strong.

3 The NK Model with General Demand Functions

Following textbook expositions as in Woodford (2003), Galí (2015) and Walsh (2017), we describe the standard NK model with Calvo (1983) price setting and flexible wages. The economy consists of a representative infinitely-lived household that consumes an aggregate bundle and supplies differentiated labor to a continuum of differentiated firms indexed by $z \in [0,1]$. Firms produce and sell goods in a monopolistic competition environment. We depart from this structure by considering a broader class of demand functions.

3.1 Households

The representative household consumes $c_t(z)$ units of each differentiated good $z \in [0,1]$ at price $p_t(z)$. Consumption over all differentiated goods is aggregated into a bundle C_t. Prices across all firms are aggregated into a consumption price index P_t, which is defined as $P_t C_t \equiv \int_0^1 p_t(z)\, c_t(z)\, dz$.

The household supplies $h_t(z)$ hours of labor to each firm z, at a differentiated nominal wage $W_t(z) = P_t w_t(z)$, where $w_t(z)$ is the real wage. Disutility over hours is $v_t(z) \equiv \chi h_t(z)^{1+\nu} / (1+\nu)$,

[4] See e.g., Ascari (2004), Levin, Lopez-Salido and Yun (2007), Yun (2005), Bakhshi, Burriel-Llombart, Khan and Rudolf (2007), Ascari and Sbordone (2014), Alves (2014, 2018), and Khan, Phaneuf and Victor (2020).

where ν^{-1} is the Frisch elasticity of labor supply. The household's aggregate disutility function is $v_t \equiv \int_0^1 v_t(z)\, dz$. The aggregate consumption bundle C_t provides utility $u_t \equiv \epsilon_t \left(C_t^{1-\sigma} - 1 \right) / (1-\sigma)$, where σ^{-1} is the intertemporal elasticity of substitution and ϵ_t is a preference shock. The household's instantaneous utility is $u_t - v_t$.

The budget constraint is $P_t C_t + E_t q_{t+1} S_{t+1} + B_t \leq S_t + I_{t-1} B_{t-1} + P_t \int_0^1 w_t(z)\, h_t(z)\, dz + d_t$, where E_t is the time-t expectations operator, S_t is the state-contingent value of the portfolio of financial securities held at the beginning of period t, B_t is the stock of government-issued bonds held at the end of period t, d_t denotes nominal dividend income, $I_t = (1 + i_t)$ is the gross nominal interest rate at period t, i_t is the riskless one-period nominal interest rate, and q_{t+1} is the stochastic discount factor from $(t+1)$ to t. Financial markets are complete.

The household chooses the sequence of C_t, $h_t(z)$, B_t, and S_{t+1} to maximize its welfare measure $\mathcal{W}_t \equiv \max E_t \sum_{\tau=t}^{\infty} \beta^{\tau-t} (u_\tau - v_\tau)$, subject to the budget constraint and a standard no-Ponzi condition, where $\beta \in (0,1)$ denotes the subjective discount factor. In equilibrium, the Lagrange multiplier λ_t on the budget constraint and the optimal labor supply function satisfy $\lambda_t = u_t'/P_t$ and $w_t(z) = v_t'(z)/u_t'$, where $u_t' \equiv \partial u_t / \partial C_t$ is the marginal utility of consumption, $v_t'(z) \equiv \partial v_t(z) / \partial h_t(z)$ is the marginal disutility of hours.[5] The optimal consumption plan and the dynamics of the stochastic discount factor, which satisfies $E_t q_{t+1} = 1/I_t$, are described by the Euler equations $1 = \beta E_t \left(\frac{u_{t+1}'}{u_t'} \frac{I_t}{\Pi_{t+1}} \right)$ and $q_t = \beta \frac{u_t'}{u_{t-1}'} \frac{1}{\Pi_t}$, where $\Pi_t \equiv \frac{P_t}{P_{t-1}} = 1 + \pi_t$ is the gross inflation rate at period t.

3.1.1 General Demand Functions

Recall that P_t is the average price of the household's expenditure basket. Let $\wp_t(z) \equiv \frac{p_t(z)}{P_t}$ denote the relative price of firm z's good. For demand considerations, it is also convenient to define an additional price aggregate $P_{s,t}$, describing the state of prices in the economy. It can be implicitly defined as a weighted average of individual prices, with state-dependent weights:

$$P_{s,t} \equiv \int_0^1 g\left(\wp_t(z), \wp_{s,t}\right) p_t(z)\, dz \tag{1}$$

where $\wp_{s,t} \equiv \frac{P_{s,t}}{P_t}$ is the relative price of $P_{s,t}$ and $g(\wp(z), \wp_s)$ are weights, satisfying $g(1,1) = 1$, $g(\wp(z), \wp_s) \in [0,1]$, and $\int_0^1 g(\wp(z), \wp_s)\, dz = 1$. For instance, after considering a particular case of Kimball (1995) consumption aggregation, Dotsey and King (2005), Levin et al. (2007), Harding et al. (2022), and Kurozumi and Van Zandweghe (2024) find a utility-based demand function that depends

[5] As usual, an equilibrium is defined as the equations describing the first-order conditions of the representative household and firms, a transversality condition $\lim_{T\to\infty} E_T q_{t,T} S_T = 0$, where $q_{t,T} \equiv \Pi_{\tau=t+1}^{T} q_\tau$, and the market clearing conditions.

not only on the aggregate price P_t but also the simple arithmetic average of prices $P_{s,t} = \int_0^1 p_t(z)\,dz$. In these cases, $g(\wp_t(z), \wp_{s,t}) = 1$ for all $\wp_t(z)$ and $\wp_{s,t}$.

In the spirit of Gagliardone et al. (2023), we consider a general class of relative demand functions $\frac{c_t(z)}{C_t} = f(\wp_t(z), \wp_{s,t})$, where $f(\wp, \wp_s)$ is continuous and differentiable, satisfying $f(\wp, \wp_s) \geq 0$, $f(1,1) = 1$ and $f_1(\wp, \wp_s) \leq 0$, $\forall(\wp, \wp_s)$ in its domain, where $f_1(\wp, \wp_s) \equiv \frac{\partial f(\wp,\wp_s)}{\partial \wp}$.

Since the aggregate price satisfies $P_t = \int_0^1 p_t(z) \frac{c_t(z)}{C_t} dz$, we obtain a general formulation for P_t:

$$P_t \equiv \int_0^1 p_t(z) f(\wp_t(z), \wp_{s,t})\,dz \tag{2}$$

where we assume that $P_{s,t}$ and P_t grow at the same rate in the steady state. Finally, firm z's price elasticity $\xi_t(z) \equiv -\frac{p_t(z)}{c_t(z)} \frac{\partial c_t(z)}{\partial p_t(z)}$ and the superelasticity of demand $\eta_t(z) \equiv \frac{p_t(z)}{\xi_t(z)} \frac{\partial \xi_t(z)}{\partial p_t(z)}$ are:

$$\xi_t(z) = -\frac{f_1(\wp_t(z),\wp_{s,t})}{f(\wp_t(z),\wp_{s,t})}\wp_t(z) \quad ; \eta_t(z) = 1 + \xi_t(z) + \frac{f_{11}(\wp_t(z),\wp_{s,t})}{f_1(\wp_t(z),\wp_{s,t})}\wp_t(z) \tag{3}$$

3.2 Price Setting

Each firm $z \in [0,1]$ produces a differentiated good using the technology $y_t(z) = \mathcal{A}_t h_t(z)^{\varepsilon}$, where $h_t(z)$ is its demand for labor, $\mathcal{A}_t$ is the aggregate technology shock and $\varepsilon \in (0,1)$. The market clearing condition $c_t(z) = y_t(z)$, $\forall z$, implies that the aggregate output across all firms satisfies $Y_t = C_t$.

Since firm-specific hours $h_t(z)$ are the only production input, the firm's real payroll cost is $co_t(z) = w_t(z) h_t(z)$. Taking wages as given, the firm's real marginal cost $mc_t(z) \equiv \frac{\partial co_t(z)}{\partial y_t(z)}$ is $mc_t(z) = w_t(z) \frac{\partial h_t(z)}{\partial y_t(z)} = \frac{\chi}{\varepsilon} \frac{(Y_t)^{(\sigma+\omega)}}{\epsilon_t(\mathcal{A}_t)^{(1+\omega)}} [f(\wp_t(z), \wp_{s,t})]^{\omega}$, where $\omega \equiv \frac{(1+\nu)}{\varepsilon} - 1$ is a composite parameter. As for the firm's real payroll cost, it can be written as $co_t(z) = \frac{(Y_t)^{(1+\sigma+\omega)}}{\epsilon_t(\mathcal{A}_t)^{(1+\omega)}} [f(\wp_t(z), \wp_{s,t})]^{(1+\omega)}$, where again $\wp_t(z) \equiv \frac{p_t(z)}{P_t}$ and $\wp_{s,t} \equiv \frac{P_{s,t}}{P_t}$.

Under flexible prices, all firms set the same price when maximizing profits $p_t(z) y_t(z) - P_t co_t(z)$. Optimal pricing requires $\left(1 - \frac{1}{\xi_t^n(z)}\right) \wp_t^n(z) = mc_t^n(z)$, where superscript '$n$' denotes natural equilibrium and $\xi_t^n(z)$ is the firm price-demand elasticity. Since all optimal prices are the same, $\xi_t^n(z) = \xi^n$ is constant and we have $\wp_t^n(z) = 1$, $\wp_{s,t}^n = 1$, and $f\left(\wp_t^n(z), \wp_{s,t}^n\right) = 1$. Therefore, the monopolistic static markup under flexible prices is $\mu \equiv \frac{\wp_t^n}{mc_t^n} = \frac{1}{\left(1-\frac{1}{\xi^n}\right)}$. In addition, under flexible prices, all firms produce the same level in equilibrium $y_t^n = Y_t^n = \left(\frac{1}{\mu}\frac{\varepsilon}{\chi}\epsilon_t(\mathcal{A}_t)^{(1+\omega)}\right)^{\frac{1}{(\sigma+\omega)}}$, where Y_t^n is the natural output. Therefore, the marginal cost is $mc_t(z) = \frac{1}{\mu}(X_t)^{(\sigma+\omega)} [f(\wp_t(z), \wp_{s,t})]^{\omega}$, where $X_t \equiv \frac{Y_t}{Y_t^n}$ is the gross output gap.

For the remainder of this paper, we consider the particular time-dependent Calvo price setting before we formally address the steady state problem under general demand functions. With standard Calvo (1983) pricing, with probability $(1-\alpha)$, the firm optimally readjusts its price to $p_t(z) = p_t^*$. With probability α, the firm sets the price with partial indexation according to $p_t(z) = p_{t-1}(z)\,\Pi_t^{ind}$, where $\Pi_t^{ind} \equiv \Pi_{t-1}^{\gamma}$ is the gross indexation rate, $\Pi_t \equiv \frac{P_t}{P_{t-1}}$ is the gross inflation rate, and $\gamma \in (0,1)$.[6] When optimally readjusting at period t, price $p_t(z) = p_t^*$ maximizes the present value of nominal profit flows $E_t \sum_{j=0}^{\infty} \alpha^j q_{t,t+j} \left[\Pi_{t,t+j}^{ind} p_t(z)\, y_{t+j}(z) - P_{t+j} co_{t+j}(z) \right]$, given the demand function and the price setting structure, where $q_{t,t+j}$ is the cumulated nominal stochastic discount factor from period $(t+j)$ to t, recursively defined as $q_{t,t} = 1$, $q_{t,t+1} = q_{t+1}$, and $q_{t,t+j} \equiv q_{t+1} q_{t+1,t+j}$ for $j \geq 1$.

After last optimally readjusting at period t, the marginal cost and demand function at $(t+j)$ are $mc_{t,t+j}(z) = \frac{1}{\mu}\left(X_{t+j}\right)^{(\sigma+\omega)} \left[f\left(\frac{\Pi_{t,t+j}^{ind}}{\Pi_{t,t+j}} \wp_t(z), \wp_{s,t+j} \right) \right]^{\omega}$ and $\frac{y_{t+j}(z)}{Y_{t+j}} = f\left(\frac{\Pi_{t,t+j}^{ind}}{\Pi_{t,t+j}} \wp_t(z), \wp_{s,t+j} \right)$,[7] where $\Pi_{t,t+j}$ and $\Pi_{t,t+j}^{ind}$, for $j \geq 1$, are the cumulated gross inflation and indexation rates from period t to $(t+j)$, recursively defined as $\Pi_{t,t} = \Pi_{t,t}^{ind} = 1$, $\Pi_{t,t+1} = \Pi_{t+1}$, $\Pi_{t,t+1}^{ind} = \Pi_{t+1}^{ind}$, $\Pi_{t,t+j} \equiv \Pi_{t+1}\Pi_{t+1,t+j} = \Pi_{t,t+j-1}\Pi_{t+j}$, and $\Pi_{t,t+j}^{ind} \equiv \Pi_{t+1}^{ind}\Pi_{t+1,t+j}^{ind} = \Pi_{t,t+j-1}^{ind}\Pi_{t+j}^{ind}$. Most importantly, note that $mc_{t,t+j}(z)$ is not the marginal cost $mc_{t+j}(z)$, as the former depends on the state at period t and cumulated rates from t to $(t+1)$.

In this context, all optimally readjusting firms have the same first order condition for $p_t(z) = p_t^*$ in equilibrium. Therefore, it can be conveniently written as in the following system:

$$1 = \frac{\frac{1}{\mu} E_t \sum_{j=0}^{\infty} \alpha^j q_{t,t+j} \mathcal{G}_{t,t+j} \Pi_{t,t+j}^{ind} f_1\left(\frac{\Pi_{t,t+j}^{ind}}{\Pi_{t,t+j}} \wp_t^*, \wp_{s,t+j} \right) \left[f\left(\frac{\Pi_{t,t+j}^{ind}}{\Pi_{t,t+j}} \wp_t^*, \wp_{s,t+j} \right) \right]^{\omega} \left(X_{t+j}\right)^{(\sigma+\omega)}}{E_t \sum_{j=0}^{\infty} \alpha^j q_{t,t+j} \mathcal{G}_{t,t+j} \Pi_{t,t+j}^{ind} \left[f\left(\frac{\Pi_{t,t+j}^{ind}}{\Pi_{t,t+j}} \wp_t^*, \wp_{s,t+j} \right) + \left(\frac{\Pi_{t,t+j}^{ind}}{\Pi_{t,t+j}} \wp_t^* \right) f_1\left(\frac{\Pi_{t,t+j}^{ind}}{\Pi_{t,t+j}} \wp_t^*, \wp_{s,t+j} \right) \right]} \tag{4}$$

where $\wp_t^* \equiv \frac{p_t^*}{P_t}$, $\mathcal{G}_t \equiv \frac{Y_t}{Y_{t-1}}$ denotes the gross output growth rate, and $\mathcal{G}_{t,t+j}$ is the cumulated gross growth rate, defined as $\mathcal{G}_{t,t} = 1$, $\mathcal{G}_{t,t+1} = \mathcal{G}_{t+1}$, and $\mathcal{G}_{t,t+j} \equiv \mathcal{G}_{t+1}\mathcal{G}_{t+1,t+j}$ for $j \geq 1$.

Note that infinite sums involving $f\left(\frac{\Pi_{t,t+j}^{ind}}{\Pi_{t,t+j}} \wp_t^*, \wp_{s,t+j} \right)$, $f_1\left(\frac{\Pi_{t,t+j}^{ind}}{\Pi_{t,t+j}} \wp_t^*, \wp_{s,t+j} \right)$ and $mc_{t,t+j}^*$ do not generally allow for recursive representations, and so steady state computations must be done numerically after considering a finite sum $j = \{0, ..., J\}$, for a large J. This is true even in commonly used models based on Kimball aggregation. Lastly, price aggregations (1) and (2) imply

$$\frac{\wp_{s,t}}{(1-\alpha)} = \sum_{j=0}^{\infty} \alpha^j g\left(\frac{\Pi_{t-j,t}^{ind}}{\Pi_{t-j,t}} \wp_{t-j}^*, \wp_{s,t} \right) \frac{\Pi_{t-j,t}^{ind}}{\Pi_{t-j,t}} \wp_{t-j}^* \quad ; \; \frac{1}{(1-\alpha)} = \sum_{j=0}^{\infty} \alpha^j f\left(\frac{\Pi_{t-j,t}^{ind}}{\Pi_{t-j,t}} \wp_{t-j}^*, \wp_{s,t} \right) \frac{\Pi_{t-j,t}^{ind}}{\Pi_{t-j,t}} \wp_{t-j}^* \tag{5}$$

[6] We allow for price indexation even though empirical evidence from macro and micro data suggest that there is very small indexation on individual prices. Full indexation is the particular case in which $\gamma = 1$.

[7] That is, considering cumulative indexation, the relative price is $\frac{\Pi_{t,t+j}^{ind} p_t(z)}{P_{t+j}} = \frac{\Pi_{t,t+j}^{ind}}{\Pi_{t,t+j}} \frac{p_t(z)}{P_t}$.

The Online Appendix A assesses how general demand functions affect real rigidity.

3.3 Quarterly Benchmark Calibration

We calibrate the model parameters at the quarterly frequency. As in Cooley and Prescott (1995), we set the subject discount factor at $\beta = 0.99$ and the elasticity to hours at the production function at $\varepsilon = (1 - 0.36)$. We set $\alpha = 0.70$ as the degree of price stickiness, which is consistent with micro and macro evidence.[8] Since empirical evidence from macro and micro data suggest that there is non-existent or very small indexation on individual prices, we set $\gamma = 0$.[9] Using central estimates (the modes of the posterior distributions) obtained by Smets and Wouters (2007), we set the reciprocal of the elasticity of intertemporal substitution at $\sigma = 1.39$. As for the reciprocal of the Frisch elasticity, we set it at $\nu = 1$ for a compromise between micro estimates and macro evidence on total hours fluctuation over the business cycle.[10] Finally, based on median estimates from Cogley and Sbordone (2008) and Ascari and Sbordone (2014), we set the monopolistic static markup of $\mu = 1.12$.[11]

4 Steady State Convergence

We search for conditions the demand function $f(\wp_t(z), \wp_{s,t})$ must meet in order to always ensure the existence of a determinate steady-state equilibrium, with no output growth ($\bar{\mathcal{G}} = 1$), regardless the value of the parameter set $[\beta, \sigma, \nu, \chi, \varepsilon, \alpha, \gamma, \mu]$ and the level of trend inflation $\bar{\Pi} = (1 + \bar{\pi})$. We define determinate steady-state equilibria as those in which all infinite summations in the steady-state equation implied by (4) and (5) converge. For notation purposes, barred variables stand for steady state level.

Except for the general demand function, the model previously described is otherwise a typical example of the standard NK model with Calvo staggered price setting: it has monopolistic compe-

8 Using US microdata from 1980 to 2023 Blanco et al. (2024) find the average quarterly frequency of price changes to be 29.4%, which implies $\alpha = 0.706$. Nakamura and Steinsson (2008), using microdata from 1988 to 2005, estimate the median duration between price changes at roughly 4.5 months (including sales) and 10 months (excluding sales). Their findings are similar to those obtained in Bils and Klenow (2004). The median durations τ_m are consistent with $\alpha = 0.63$ and $\alpha = 0.81$ in quarterly fequency, using $\tau_m = -\log(2) / \log(\alpha)$. As for the macro evidence, Cogley and Sbordone (2008), for instance, report $\alpha = 0.588$ as their median estimate, while Smets and Wouters (2007) report $\alpha = 0.65$ as the mode estimate, using the full sample period from 1966:1 to 2004:4.

9 For instance, this evidence is found in Bils and Klenow (2004), Cogley and Sbordone (2005), Cogley and Sbordone (2008), Klenow and Kryvtsov (2008), Klenow and Malin (2010), Levin et al. (2005) and Smets and Wouters (2007).

10 In this regard, even though Chetty et al. (2011) finds a smaller value for ν^{-1} (i.e. a larger value for ν) on the micro side, recent evidence suggests that earlier estimates of micro elasticities for ν^{-1} might be downwardly biased, as their inference approaches did not account for important features in households composition between: (i) male and female workers; (ii) age; and (iii) primary and secondary earners. See, for example, Keane and Rogerson (2012), Peterman (2016), and Bredemeier et al. (2023).

11 In a Dixit and Stiglitz (1977) aggregation model, with the elasticity of substitution set at $\theta = 9.5$, the markup is $\mu = \frac{\theta}{(\theta-1)} = 1.12$.

tition, standard functional forms, only one source of nominal rigidity and shocks to preferences and technology. Under those circumstances, the generally accepted paradigm in the literature on trend inflation is that there is a low upper limit for trend inflation consistent with a determinate steady-state equilibrium (see e.g. Ascari and Sbordone (2014)). In the standard NK model with Dixit and Stiglitz (1977) aggregator, the demand function is $f\left(\wp_t\left(z\right), \wp_{s,t}\right) = \left(\wp_t\left(z\right)\right)^{-\theta}$, where $\theta = \frac{\mu}{(\mu-1)}$. Given a trend inflation level $\bar{\Pi}$, the steady state equilibrium with no output growth only exists if $\bar{\Pi} < \min\left[\left(\frac{1}{\alpha}\right)^{\frac{1}{(\theta-1)(1-\gamma)}}, \left(\frac{1}{\alpha\beta}\right)^{\frac{1}{\theta(1+\omega)(1-\gamma)}}\right]$. Using the benchmark calibration, the annualized upper limit for trend inflation is 5.16%. If we had assumed a calibration more compatible with micro evidence in the labor market,[12] with $\nu = \frac{1}{0.59} = 1.69$, the annualized upper limit would be much smaller, at 3.80%.

As we formally show below, this inflation upper bound in standard NK models arises because the usual Dixit and Stiglitz (1977) demand function have a singularity point at $\wp_t\left(z\right) \to 0$. In Theorem 1, we formalize the idea that convergence at any level of trend inflation requires the general demand function to be always finite, even in the limits $\wp_t\left(z\right) \to 0$ and $\wp_t\left(z\right) \to \infty$.

Assumption 1: Under the Calvo staggered price setting $(\alpha > 0)$ with partial indexation $(\gamma < 1)$, as previously described, consider the generic relative demand function $\frac{y}{Y} = f\left(\wp, \wp_s\right)$ described in Section 3.1.1, where $\wp_s \equiv \frac{P_s}{P}$, where P_s and P grow at the same rate in any steady state, such that $f\left(\wp, \wp_s\right)$ is a non-negative, continuous and differentiable function in $(\wp, \wp_s) \in \left(\mathbb{R}_+^* \times \mathbb{R}_+^*\right)$ and non-increasing in $\wp \in R_+^*$. Let $f_1\left(\wp, \wp_s\right) \equiv \frac{\partial f(\wp, \wp_s)}{\partial \wp}$ denote the partial derivative of f with respect to $\wp$. In addition, as we assume in Section 3.1.1, the weight function is bounded, i.e. $g\left(\wp, \wp_s\right) \in [0, 1]$. And so, $g\left(0, \wp_s\right)$ and $\lim_{\wp \to \infty} g\left(\wp, \wp_s\right)$ exist and are both finite.

Theorem 1 *If $f\left(\wp, \wp_s\right)$ and $\wp \cdot f_1\left(\wp, \wp_s\right)$ are finite and defined at all their domain, including at $\wp \to 0$ and $\wp \to \infty$, according to Assumption 1, there always exists a steady state equilibrium for any value of the parameter set and any level of trend inflation ($\bar{\Pi} = 1 + \bar{\pi}$), provided that it is not extremely negative, i.e. $\bar{\Pi} > (\alpha)^{\frac{1}{(1-\gamma)}}$. For any other level of trend inflation, including all positive values, both the optimal relative price and the output-gap converge to finite steady state levels.*

Proof. Consider that all shocks are kept at their means, i.e. $\epsilon_t = \bar{\epsilon}$ and $\mathcal{A}_t = \overline{\mathcal{A}}$, at all periods, so that there are no stochastic uncertainties. Also consider that gross trend inflation is kept constant at $\bar{\Pi} = 1 + \bar{\pi}$. Since we assume that P_s and P grow at the same rate in any steady state, it must be the case that $\bar{\wp}_s$ is independent of j.

For simplicity sake, let us define the function $\tilde{f}\left(\wp, \wp_s\right) \equiv \wp \cdot f_1\left(\wp, \wp_s\right)$. Given the theorem assumptions, for a fixed value $\bar{\wp}_s$, let $f_0 \equiv \lim_{\wp \to 0} f\left(\wp, \bar{\wp}_s\right)$, $f_\infty \equiv \lim_{\wp \to \infty} f\left(\wp, \bar{\wp}_s\right)$, $\tilde{f}_0 \equiv \lim_{\wp \to 0} \tilde{f}\left(\wp, \bar{\wp}_s\right)$,

[12] See Table 1 in Chetty et al. (2011).

$\tilde{f}_\infty \equiv \lim_{\wp\to\infty} \tilde{f}(\wp,\bar{\wp}_s)$, $g_0 \equiv \lim_{\wp\to 0} g(\wp,\bar{\wp}_s)$, and $g_\infty \equiv \lim_{\wp\to\infty} g(\wp,\bar{\wp}_s)$ denote the implied finite limits.

In this case, the cumulated rates satisfy $\bar{q}_{t,t+j} = \left(\frac{\beta}{\bar{\Pi}}\right)^j$, $\bar{\Pi}^{ind}_{t,t+j} = (\bar{\Pi}^\gamma)^j$, and $\bar{\mathcal{G}}_{t,t+j} = 1$. Therefore, if existent, the pricing steady state relations implied by the system in (4) and (5) are:

$$1 = \frac{-\frac{(\bar{X})^{(\sigma+\omega)}}{\mu\bar{\wp}^*}\sum_{j=0}^{\infty}(\alpha\beta)^j\left[-\tilde{f}\left(\frac{\bar{\wp}^*}{\bar{\Pi}^{(1-\gamma)j}},\bar{\wp}_s\right)\right]\left[f\left(\frac{\bar{\wp}^*}{\bar{\Pi}^{(1-\gamma)j}},\bar{\wp}_s\right)\right]^\omega}{\sum_{j=0}^{\infty}\left(\frac{\alpha\beta}{\bar{\Pi}^{(1-\gamma)}}\right)^j f\left(\frac{\bar{\wp}^*}{\bar{\Pi}^{(1-\gamma)j}},\bar{\wp}_s\right) - \sum_{j=0}^{\infty}\left(\frac{\alpha\beta}{\bar{\Pi}^{(1-\gamma)}}\right)^j\left[-\tilde{f}\left(\frac{\bar{\wp}^*}{\bar{\Pi}^{(1-\gamma)j}},\bar{\wp}_s\right)\right]}$$

$$\frac{\bar{\wp}_s}{(1-\alpha)} = (\bar{\wp}^*)\sum_{j=0}^{\infty}\left(\frac{\alpha}{\bar{\Pi}^{(1-\gamma)}}\right)^j g\left(\frac{\bar{\wp}^*}{\bar{\Pi}^{(1-\gamma)j}},\bar{\wp}_s\right) \quad ; \; \frac{1}{(1-\alpha)} = (\bar{\wp}^*)\sum_{j=0}^{\infty}\left(\frac{\alpha}{\bar{\Pi}^{(1-\gamma)}}\right)^j f\left(\frac{\bar{\wp}^*}{\bar{\Pi}^{(1-\gamma)j}},\bar{\wp}_s\right)$$

where $\tilde{f}\left(\frac{\bar{\wp}^*}{\bar{\Pi}^{(1-\gamma)j}},\bar{\wp}_s\right) = \left(\frac{\bar{\wp}^*}{\bar{\Pi}^{(1-\gamma)j}}\right) f_1\left(\frac{\bar{\wp}^*}{\bar{\Pi}^{(1-\gamma)j}},\bar{\wp}_s\right)$.

This pricing system involves five types of different non-negative power series. Each has a general format $\mathcal{F}\left(\rho;\bar{\wp}^*,\bar{\Pi}^{(1-\gamma)},\bar{\wp}_s\right) \equiv \sum_{j=0}^{\infty}(\rho)^j b\left(j;\bar{\wp}^*,\bar{\Pi}^{(1-\gamma)},\bar{\wp}_s\right)$, where $\rho \in \left\{\alpha\beta, \frac{\alpha\beta}{\bar{\Pi}^{(1-\gamma)}}, \frac{\alpha}{\bar{\Pi}^{(1-\gamma)}}\right\}$, and $b\left(j;\bar{\wp}^*,\bar{\Pi}^{(1-\gamma)},\bar{\wp}_s\right) \geq 0$ is a well-defined, finite and non-negative sequence in j, that can be one of the following functions: $f\left(\frac{\bar{\wp}^*}{\bar{\Pi}^{(1-\gamma)j}},\bar{\wp}_s\right)$, $\left[-\tilde{f}\left(\frac{\bar{\wp}^*}{\bar{\Pi}^{(1-\gamma)j}},\bar{\wp}_s\right)\right]$, $\left[-\tilde{f}\left(\frac{\bar{\wp}^*}{\bar{\Pi}^{(1-\gamma)j}},\bar{\wp}_s\right)\right]\left[f\left(\frac{\bar{\wp}^*}{\bar{\Pi}^{(1-\gamma)j}},\bar{\wp}_s\right)\right]^\omega$, or $g\left(\frac{\bar{\wp}^*}{\bar{\Pi}^{(1-\gamma)j}},\bar{\wp}_s\right)$.

In order to show that all five power series converge, we use the Ratio test. But before, some considerations are necessary. Since the relative demand and weight functions are general, there is nothing precluding $b\left(j;\bar{\wp}^*,\bar{\Pi}^{(1-\gamma)},\bar{\wp}_s\right)$ to be zero at some points. Therefore, we resort to an auxiliary power series $\mathcal{F}_\zeta(\rho) \equiv \sum_{j=0}^{\infty}(\rho)^j\zeta$, defined for an arbitrary fixed and strictly positive value $\zeta > 0$. Obviously, $\mathcal{F}_\zeta(\rho)$ converges as long as $|\rho| < 1$. We need it to be the case for all $\rho \in \left\{\alpha\beta, \frac{\alpha\beta}{\bar{\Pi}^{(1-\gamma)}}, \frac{\alpha}{\bar{\Pi}^{(1-\gamma)}}\right\}$. And so, since $\beta < 1$, the condition is $\alpha < \bar{\Pi}^{(1-\gamma)}$.

Let us consider the augmented power series $\mathbb{F}\left(\rho;\bar{\wp}^*,\bar{\Pi}^{(1-\gamma)},\bar{\wp}_s\right) \equiv \mathcal{F}_\zeta(\rho) + \mathcal{F}\left(\rho;\bar{\wp}^*,\bar{\Pi}^{(1-\gamma)},\bar{\wp}_s\right)$. By construction, all terms in this power series are strictly positive. It implies that convergence can be verified using the Ratio test. That is, $\mathbb{F}\left(\rho;\bar{\wp}^*,\bar{\Pi}^{(1-\gamma)},\bar{\wp}_s\right) = \sum_{j=0}^{\infty}(\rho)^j\left[\zeta + b\left(j;\bar{\wp}^*,\bar{\Pi}^{(1-\gamma)},\bar{\wp}_s\right)\right]$ converges if $T_{ratio} \equiv \lim_{j\to\infty}\left|\frac{(\rho)^{(j+1)}\left[\zeta + b\left((j+1);\bar{\wp}^*,\bar{\Pi}^{(1-\gamma)},\bar{\wp}_s\right)\right]}{(\rho)^j\left[\zeta + b\left(j;\bar{\wp}^*,\bar{\Pi}^{(1-\gamma)},\bar{\wp}_s\right)\right]}\right| < 1$, and diverges if $T_{ratio} > 1$.

Note that $\lim_{j\to\infty}\frac{\bar{\wp}^*}{\bar{\Pi}^{(1-\gamma)j}} = 0$ if $\bar{\Pi}^{(1-\gamma)} > 1$, whereas $\lim_{j\to\infty}\frac{\bar{\wp}^*}{\bar{\Pi}^{(1-\gamma)j}} = \infty$ if $\bar{\Pi}^{(1-\gamma)} < 1$, and $\lim_{j\to\infty}\frac{\bar{\wp}^*}{\bar{\Pi}^{(1-\gamma)j}} = 1$ if $\bar{\Pi}^{(1-\gamma)} = 1$. Therefore, since the limits f_0, f_∞, $\tilde{f}_0$, $\tilde{f}_\infty$, g_0, and g_∞ exist and are finite, it is easy to verify that the limiting ratio is $T_{ratio} = |\rho|$ for all five augmented power series, regardless the level of gross trend inflation $\bar{\Pi}$. And so, the ratio test predicts that each augmented power series converge if $|\rho| < 1$ and diverge if $|\rho| > 1$. Since the auxiliary power series $\mathcal{F}_\zeta(\rho)$ converges, it implies that all five original power series also converge under the same conditions.

Therefore, we conclude that the system implies a convergent relation and provides an implicit solution for the steady state levels $\bar{\wp}^*$, $\bar{\wp}_s$ and $\bar{X}$, as long as $\alpha < \bar{\Pi}^{(1-\gamma)}$. Since $\bar{\Pi} = 1 + \bar{\pi}$, convergence is achieved if trend inflation $\bar{\pi}$ is not extremely negative, i.e. when $\bar{\pi} > (\alpha)^{\frac{1}{(1-\gamma)}} - 1$. ■

The requirement that trend inflation is extremely negative is easily satisfied, and so the feasibility inequality in Theorem 1 does not pose a practical restriction. For instance, if $\alpha = 0.60$ and $\gamma = 0$ (no indexation), the steady state levels cease to exist if $\bar{\pi} > \alpha - 1 = -40\%$ in quarterly frequency (-97% annually).

Hahn (2022) adopts an alternative approach to cope with the fact that the Dixit and Stiglitz (1977) demand function diverges to infinity when relative prices approaches zero. His approach, however, is to keep the standard demand function while allowing firms not to satisfy demand all the times. The author introduces an optimal rationing mechanism, by curtailing supply to its optimal level. By contrast, our approach is to investigate the root of the steady state problem, and propose conditions for demand functions in the standard approach used in the literature of supply meeting demand at any price level in equilibrium.

5 Demand Functions Consistent With Theorem 1

Here, we focus on demand functions that simultaneously satisfy: (i) Theorem 1 conditions, especially those related to finite demand and slope at zero relative price; and (ii) micro and macro empirical support, as described in Section 2.

Under monopolistic competition models with a continuum of firms, as we consider in this paper, the class of Kimball (1995) demand functions, especially the one proposed by Dotsey and King (2005), has been often used in recent literature.[13] Using Theorem 1, we show that having a sufficiently large curvature parameter is the necessary condition for Kimball demand functions to be consistent with all levels of trend inflation. However, under large curvature, Kimball demand functions have three features that are at odds with what micro and macro evidence suggests (see Section 2): (i) superelasticities become much larger than the micro evidence range; (ii) they fail to accommodate a sizable mass of relative prices found in the US empirical distribution; and (iii) if used in NK macroeconomic models with Calvo pricing in lieu of Dixit and Stiglitz (1977) demand functions, Kimball-based NK models predict that the distorted output (due to nominal rigidities) becomes much larger than the flexible-prices output as trend inflation rises. These facts are in line with the recent critiques and find-

[13] If we were to extend our modelling approach to also consider oligopoly models with a finite number of firms, instead of only monopolistic competition models with an infinite number of firms in the continuum $z \in (0, 1)$, we have a broad set of demand functions satisfying Theorem 1 conditions, as oligopoly demand functions are typically bounded. In this regard, the Atkeson and Burstein (2008) and Wang and Werning (2022) oligopoly models with N firms are strong candidates to be applied to NK models with trend inflation in future extensions.

ings on Kimball-based NK models found in the literature (see e.g., Dossche et al. (2010), Beck and Lein (2015), Klenow and Willis (2016), and Kurozumi and Van Zandweghe (2016, 2024)).

5.1 Kimball Aggregator

Within the broad class of Kimball (1995) consumption aggregation, Dotsey and King (2005) propose a particular functional form that has been frequently used in the literature (e.g. Levin et al. (2007), Harding et al. (2022) and Kurozumi and Van Zandweghe (2016, 2024)). As we present in detail in the Online Appendix B, the implied demand function is $\frac{c_t(z)}{C_t} = \frac{1}{(1+\varphi)}\left(\frac{\wp_t(z)}{\wp_{k,t}}\right)^{\varpi} + \frac{\varphi}{(1+\varphi)}$ if $\left(\frac{\wp_t(z)}{\wp_{k,t}}\right) \leq (-\varphi)^{\frac{1}{\varpi}}$, or $\frac{c_t(z)}{C_t} = 0$ otherwise. Here, $\wp_t(z) \equiv \frac{p_t(z)}{P_t}$ is the relative price of firm z, $\wp_{k,t} = \frac{P_{k,t}}{P_t}$ is the auxiliary composite relative price of $P_{k,t} \equiv (1+\varphi) P_t - \varphi P_{s,t}$, $P_{s,t} \equiv \int_0^1 p_t(z)\, dz$ is the average price, and P_t is the aggregate price, implicitly defined by $(P_{k,t})^{(1+\varpi)} = \int_0^1 (p_t(z))^{(1+\varpi)}\, dz$. The composite parameters are $\varpi \equiv \frac{\mu_k(1+\varphi)}{(1-\mu_k)}$ and $\mathfrak{m} \equiv \frac{\mu_k(1+\varphi)}{(1+\mu_k\varphi)}$, where $\mu_k \geq 1$ is the elasticity parameter, which matches the implicit markup rate μ under flexible prices, and $\varphi \leq 0$ sets the aggregation curvature. If $\varphi = 0$, the demand curve simplifies into the standard Dixit and Stiglitz (1977) form. When $\varphi < 0$, Kimball demand function has positive superelasticities (see the Online Appendix B), which makes it qualitatively in line with micro evidence, as described in Section 2.

Note that $f(\wp, \wp_s) = \frac{1}{(1+\varphi)}\left(\frac{\wp}{(1+\varphi)-\varphi\wp_s}\right)^{\varpi} + \frac{\varphi}{(1+\varphi)}$ if $\left(\frac{\wp}{(1+\varphi)-\varphi\wp_s}\right) \leq (-\varphi)^{\frac{1}{\varpi}}$, or $f(\wp, \wp_s) = 0$ otherwise. Therefore, Theorem 1 conditions are satisfied only when its curvature is sufficiently large, i.e. when $\varphi < -1$. This condition is generally met in the macroeconomic literature for the US, as φ is usually estimated/calibrated at large values, typically set in the range $\varphi \in [-16, -2]$.[14] Large curvature levels, however, lead to elasticities and superelasticities that are much larger than their empirical microdata counterparts, i.e. $\xi^{micro} \in [1.0\,,\, 5.0]$ and $\eta^{micro} \in [1.5\,,\, 2.0]$. To illustrate, consider that the static markup is set at the usual low levels of $\mu = \mu_k = 1.12$. Under flexible prices, the model's elasticity and superelasticity are $\xi^n = \frac{\mu_k}{(\mu_k-1)}$ and $\eta^n = (-\varphi)\frac{\mu_k}{(\mu_k-1)}$. If $\varphi < -1$, we find that $\xi^n = 9.3$ and $\eta^n > 9.3$. And is $\varphi = -2$, the smallest curvature in the macroeconomic range, the implied superelasticity is $\eta^n = 18.6$.

In addition, a large macro curvature induces the theoretical distribution of relative prices to be strongly asymmetric to the left and imposes zero demand for prices that are set slightly above the average price (see e.g. Klenow and Willis (2016)). This prediction is at odds with empirical micro evidence found by Kaplan and Menzio (2015). See a detailed discussion in Online Appendix B.3, in

[14] Some typical values for the US are the following: (i) $\varphi = -12.2$ in Harding et al. (2022); (ii) $\varphi = -2.6$ in Kurozumi and Van Zandweghe (2024); (iii) $\varphi = -8$ in Levin et al. (2007); and (iv) $\varphi = -3.79$ in Smets and Wouters (2007). In addition, obtaining a better marginal likelihood statistics for model comparison, Harding et al. (2022) re-estimate Smets and Wouters (2007) model with a different prior distribution and obtain $\varphi = -16.37$.

which we propose an alternative approach to test the plausibility of Kimball's upper limit on relative prices.

In light of those results, we propose in the next section a remedy to attenuate the issues induced by Kimball demand functions.

5.2 Sticker and Effective Prices

Between purchasing a good and consuming it within a specific period, it is not uncommon for individuals to face extra costs that create a wedge between the sticker price and the effective price. Those costs can rise either from direct monetary causes or from efforts, which then can be translated into a monetary price. And more importantly, they might be resilient even if the sticker price is set at zero. For instance, apple trees can be very tall, requiring an effort to pick apples even when they are free. And an orange tree about the same height requires the same effort, even though being a different good. Since there is only so much fruit individuals can carry down the trees, the extra costs should increase with consumed fruit volumes, as consuming more fruit requires more climbing.

We can indirectly compute the extra price added to the sticker price by quantifying the effort (energy, abilities, etc.) needed to climb the tree in every period we want to consume a fruit. And we highlight that acquiring them has a complementary nature with consuming the fruit, as individuals would not "buy" more effort goods and less fruit unities if effort becomes relatively cheaper than fruits.

Sometimes, the costs can be directly measured in monetary units, for consuming the good might require post-purchase accompanying extra cost from handling, shipping and storing the goods within the period. Again, even if the good's sticker price is set at zero, those extra costly activities still remain. And their cost in many cases depend on good volumes and weights, rather than good types. Those properties characterize complementarity rather than substitutability between consumed goods and the extra cost sources.

Of course, features such as rarity, fragility and perishability also matters. The extra costs might also vary across different individuals and across time.[15] In this paper, for simplicity, we abstract from

[15]The extra costs can be also be generated if, for consuming goods, individuals are required to buy extra services or goods that do not reflect extra utility-bearing consumption, in the spirit of Michaillat and Saez (2015) when they model a case in which consuming one service unit requires buying a total of $(1+\tau)$ service units. For instance, household storage rooms and refrigerators can generate this effect, as their associated costs are related to volumes and not to the specific goods they store. As our alternative case, we could assume that $\delta c_t(z)$ represents the storage volume required to keep $c_t(z)$ units of utility-bearing goods, and there is no price wedge. And so, paralleling Michaillat and Saez (2015) results, consuming $c_t(z)$ units would require buying a total of $(1+\delta)\,c_t(z)$ units. Even though this alternative approach also embeds the complementary nature between utility-bearing and non-utility bearing consumption, it requires changing the market clearing condition to account for both types of produced goods.

those possibilities and do not specify any particular source of the realistic nature of extra costs. In all cases, extra costs prevent individuals from consuming infinite amounts of goods, even though that is what they would like to if sticker prices were to approach zero in the absence of extra costs.[16] And lastly, the extra costs might be simply wasted (deadweight loss) or might be recovered somehow into the economy. Notice that the first type of extra costs generates more distortions than the second type, as there are no firms or individuals able to accrue the losses individuals bear.

For short, we use the term ”price wedges” to characterize this class of extra-cost models. Here, we consider a simple structure and let the extra costs to be recovered by firms in order to minimize implied distortions. It allows usto make the case that this class of models meets the requirements of Theorem 1, and so can be used to assess the economy at all levels of trend inflation. As will be clear from the steps shown in the next section, applying price wedges for any general demand framework is straightforward.

5.2.1 Price Wedges Model

Here, we assess the properties of the simplest structure in the class of price wedge models. We assume that consuming $c_t(z)$ units of good z at sticker price $p_t(z)$ requires paying a extra price wedge δP_t to firm z for processing, handling and storing, where $\delta \geq 0$ is the wedge rate. As the surcharge only depends on volumes, independently of the good type, each unit has the same price wedge δP_t. Therefore, the household's total expenditure is $\int_0^1 (p_t(z) + \delta P_t)\, c_t(z)\, dz$. Since the market clearing condition is $y_t(z) = c_t(z)$, for each firm z, its revenue is now $(p_t(z) + \delta P_t)\, y_t(z)$. As shown in Section 5.2.2, even a small value for δ produces important changes in the resulting demand function.

We need small changes to adapt the general results shown in Section 3. In this context, let us initially define the aggregate price P_t as in $(1+\delta) P_t C_t \equiv \int_0^1 (p_t(z) + \delta P_t)\, c_t(z)\, dz$. This definition is necessary so that P_t can be interpreted as an average of sticker prices $p_t(z)$. On the household side, after substituting the total expenditure $(1+\delta) P_t C_t$ for $P_t C_t$ in the budget constraint, the optimal labor supply curve becomes $w_t(z) = (1+\delta)\, v_t'(z) / u_t'$. Firms results also change a little bit, as we show further on in Section 5.2.3. We first assess the consequences of having price wedges under Kimball and Dixit-Stiglitz aggregation. In this context, Section 5.2.2 below derives the resulting demand curves and studies their properties.

[16] Since firms are assumed to satisfy any demand level, and all individuals face the same price, there is no incentive for over purchasing goods intended for reselling.

5.2.2 Demand function under price wedges

Let us start with the emplest case in which the representative household is subject to price wedges and has Dixit and Stiglitz (1977) consumption aggregation $C_t = \left(\int_0^1 c_t(z)^{\frac{\theta-1}{\theta}} dz\right)^{\frac{\theta}{\theta-1}}$, where $\theta = \frac{\mu_k}{(\mu_k-1)} > 1$ is the elasticity of substitution between goods. Therefore, minimizing total expenditure $(1+\delta) P_t C_t = \int_0^1 (p_t(z) + \delta P_t) c_t(z) dz$, subject to $C_t = \left(\int_0^1 c_t(z)^{\frac{\theta-1}{\theta}} dz\right)^{\frac{\theta}{\theta-1}}$, leads to the demand function $\frac{c_t(z)}{C_t} = \left(\frac{p_t(z)+\delta P_t}{(1+\delta)P_t}\right)^{-\theta}$, where P_t now satisfies $[(1+\delta) P_t]^{(1-\theta)} = \int_0^1 (p_t(z) + \delta P_t)^{(1-\theta)} dz$.[17]

In the context of Theorem 1, considering the relative price notation $\wp_t(z) \equiv \frac{p_t(z)}{P_t}$, this demand function can be conveniently written as $\frac{c_t(z)}{C_t} = \left(\frac{\wp_t(z)+\delta}{1+\delta}\right)^{-\theta}$. Since there is no extra relative price $\wp_s$ in this demand function, $f(\wp) = \left(\frac{\wp+\delta}{1+\delta}\right)^{-\theta}$ and $\wp f_1(\wp) = -\frac{\theta\wp}{(1+\delta)}\left(\frac{\wp+\delta}{1+\delta}\right)^{-(1+\theta)}$. Note that, as long as $\delta > 0$, $f(\wp)$ and $\wp f_1(\wp)$ are finite and defined for all $\wp \geq 0$. That is, $\lim_{\wp\to 0} f(\wp) = \left(\frac{\delta}{1+\delta}\right)^{-\theta}$, $\lim_{\wp\to\infty} f(\wp) = 0$, $\lim_{\wp\to 0} \wp f_1(\wp) = 0$, and $\lim_{\wp\to\infty} \wp f_1(\wp) = 0$. Therefore, as long as $\delta > 0$, $f(\wp)$ and $\wp f_1(\wp)$ always satisfy the Theorem 1 conditions, no matter how small δ is. It implies that the NK model with Dixit and Stiglitz (1977) aggregation and non-zero price wedges $(\delta > 0)$ can be used at all levels of trend inflation. In addition, as we show further on, price wedges allows the demand function with Dixit and Stiglitz (1977) aggregation to be quasi-kinked and more in line with empirical micro-evidence, as presented in Section 2.

If the representative household is subject to price wedges and has Kimball-type preferences (see Section 5.1), total expenditure minimization gives us the demand function $\frac{c_t(z)}{C_t} = \frac{1}{(1+\varphi)}\left(\frac{\wp_t(z)+\delta}{\wp_{k,t}+\delta}\right)^{\varpi} + \frac{\varphi}{(1+\varphi)}$ if $\left(\frac{\wp_t(z)+\delta}{\wp_{k,t}+\delta}\right) \leq (-\varphi)^{\frac{1}{\varpi}}$, or $\frac{c_t(z)}{C_t} = 0$ otherwise, where P_t is now satisfies $(P_{k,t} + \delta P_t)^{(1+\varpi)} = \int_0^1 (p_t(z) + \delta P_t)^{(1+\varpi)} dz$. Therefrore, as long as $\delta > 0$, it is easy to verify that the Theorem 1 conditions always hold, not matter the curvature level φ. For each relative price $\wp_t(z)$, consider the auxiliary variable $\mathfrak{p}_{\delta,t}(z) \equiv \frac{\wp_t(z)+\delta}{\wp_{k,t}+\delta}$. The firm z's price elasticity and superelasticity are: (i) $\xi_t(z) = -\frac{\varpi\left(\frac{\wp_t(z)}{\wp_t(z)+\delta}\right)}{1+\varphi(\mathfrak{p}_{\delta,t}(z))^{-\varpi}}$ and $\eta_t(z) = \left[-\frac{\varphi\xi_t(z)}{(\mathfrak{p}_{\delta,t}(z))^{\varpi}} + \frac{\delta}{(\wp_{k,t}+\delta)}\frac{1}{\mathfrak{p}_{\delta,t}(z)}\right]$, if $\mathfrak{p}_{\delta,t}(z) \leq (-\varphi)^{\frac{1}{\varpi}}$; or (ii) $\xi_t(z) = 0$ and $\eta_t(z) = 0$, if $\mathfrak{p}_{\delta,t}(z) > (-\varphi)^{\frac{1}{\varpi}}$.

Figure 1 depicts the demand function (log-log), price elasticities and price superelasticities for different levels of price wedge rates $\delta \in [0, 0.50]$ and curvature parameters $\varphi \in \{0, -2.0\}$, keeping the static markup $\mu = \frac{\theta}{(\theta-1)(1+\delta)}$ fixed at 1.12 (see Section 5.2.3).

[17] Solving $\min_{\{c_t(z)\}} \int_0^1 (p_t(z) + \delta P_t) c_t(z) dz + \lambda_t \left[C_t - \left(\int_0^1 c_t(z)^{\frac{\theta-1}{\theta}} dz\right)^{\frac{\theta}{\theta-1}}\right]$, the first order condition is $(p_t(z) + \delta P_t) = \lambda_t \left(\frac{c_t(z)}{C_t}\right)^{-\frac{1}{\theta}}$. Since $(1+\delta) P_t C_t = \int_0^1 (p_t(z) + \delta P_t) c_t(z) dz$, we obtain $\lambda_t = (1+\delta) P_t$ and $\frac{c_t(z)}{C_t} = \left(\frac{p_t(z)+\delta P_t}{(1+\delta)P_t}\right)^{-\theta}$. Plugging it into $(C_t)^{\frac{\theta-1}{\theta}} = \int_0^1 c_t(z)^{\frac{\theta-1}{\theta}} dz$ leads to $[(1+\delta) P_t]^{(1-\theta)} = \int_0^1 (p_t(z) + \delta P_t)^{(1-\theta)} dz$.

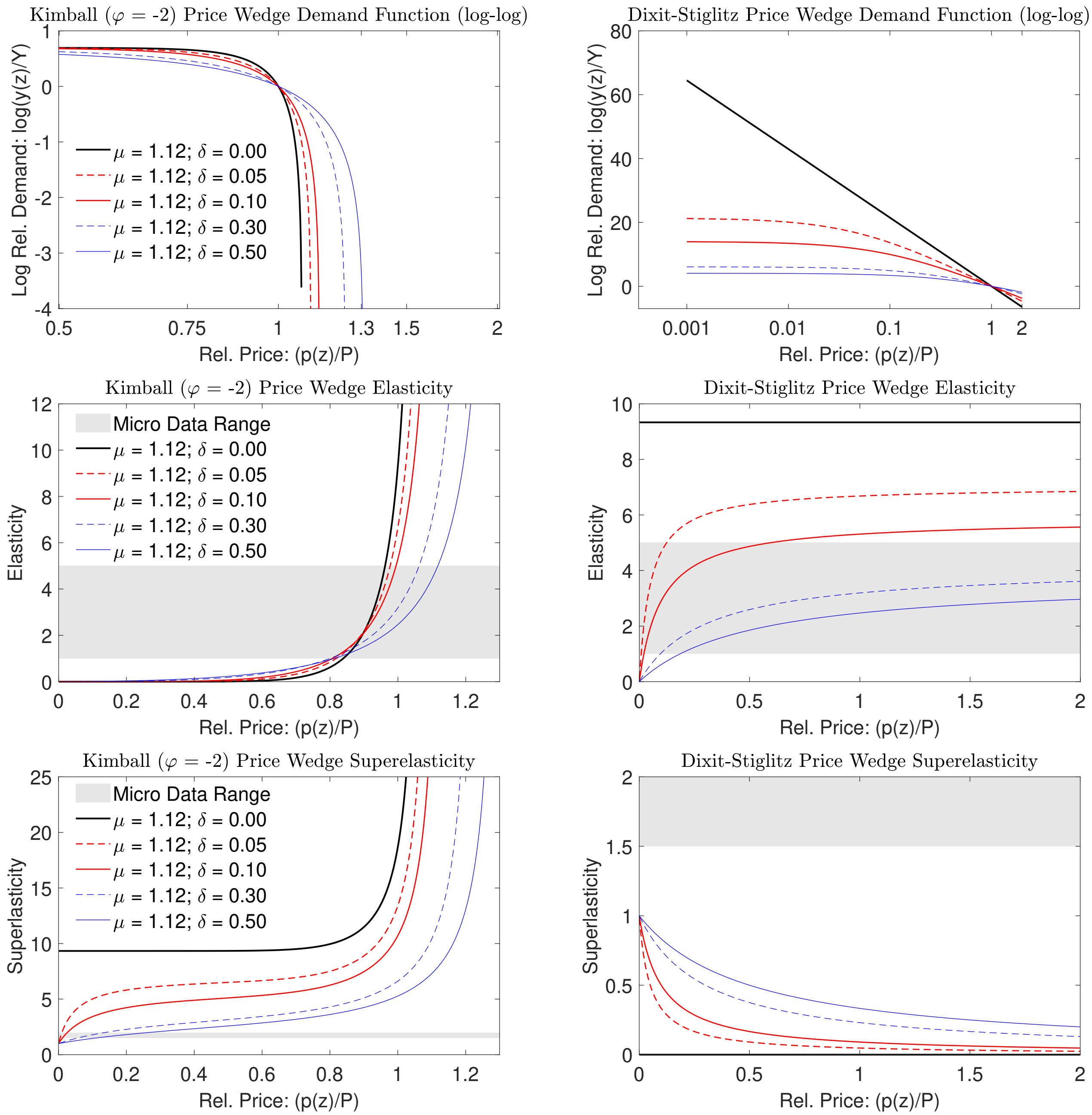

Figure 1: Demand, Elasticities and Superelasticities - Price Wedges

Notes: In top panels, the demand function is plotted using $\log(y(z)/Y)$.and $\log(p(z)/P)$. For generating the figures, we fix $\wp_k = 1$ and use $\mu = \frac{\theta}{(\theta-1)(1+\delta)}$ (see Section 5.2.3) to recompute μ_k for each value of δ in order to keep the static markup at $\mu = 1.12$.

With the standard Dixit and Stiglitz (1977) aggregation, i.e. when $\varphi = 0$, we obtain $\xi_t(z) = \frac{\theta\wp_t(z)}{(\wp_t(z)+\delta)} \geq 0$ and $\eta_t(z) = \frac{\delta}{(\wp_t(z)+\delta)} \geq 0$. Therefore, when $\delta \neq 0$, the demand elasticity $\xi_t(z)$ and superelasticity $\eta_t(z)$ are price-dependent even if $\varphi = 0$. As Figure 1 shows, accounting for price wedges allows demand functions derived from Dixit and Stiglitz (1977) aggregation to be quasi-kinked.

In the remainder of the paper, we assess the dynamic properties of price wedge models when firms have sticky prices. For that, we consider the Dixit-Stiglitz aggregation with price wedges as a proof of concept to study the implied NK model for small and large levels of trend inflation.

5.2.3 Firms

Consider the case with Dixit-Stiglitz aggregation and price wedges. Using the market clearing condition $y_t(z) = c_t(z), \forall z$, the aggregate and average levels of output satisfy $Y_t = C_t$. Therefore, firm z's demand function is $\frac{y_t(z)}{Y_t} = f(\wp_t(z), \wp_{s,t}) = \left(\frac{\wp_t(z)+\delta}{1+\delta}\right)^{-\theta}$.

The firm's revenue is now $[p_t(z) + \delta P_t]\, y_t(z)$. As we mentioned before, optimal labor supply curve under price wedges is $w_t(z) = (1+\delta)\frac{\chi}{\epsilon_t} h_t(z)^{\nu} (Y_t)^{\sigma}$. Adapting the results shown in Section 3.2, optimal pricing under flexible prices now requires $\left[\frac{(1+\delta)}{\mu_k} + \delta\left(\frac{1}{\wp_t^n} - 1\right)\right]\wp_t^n = mc_t^n$, where $mc_t^n = \frac{(1+\delta)\chi}{\varepsilon}\frac{1}{\epsilon_t(\mathcal{A}_t)^{(1+\omega)}}\left(\frac{\wp_t^n+\delta}{1+\delta}\right)^{-\theta\omega}(Y_t^n)^{(\sigma+\omega)}$ is the marginal cost under flexible prices. Since $\wp_t^n = 1$ under flexible prices, the natural output evolves according to $(Y_t^n)^{(\sigma+\omega)} = \frac{1}{(1+\delta)\mu}\frac{\varepsilon}{\chi}\epsilon_t(\mathcal{A}_t)^{(1+\omega)}$, where $\mu \equiv \frac{\wp_t^n}{mc_t^n} = \frac{\mu_k}{(1+\delta)}$ is the static markup under flexible prices and price wedges, and $\mu_k \equiv \frac{\theta}{(\theta-1)}$ is the basic markup that would prevail in the absence of price wedges.

This last result allows us to design a strategy to calibrate θ as a function of markup μ and price wedge rate δ: $\theta = \frac{\mu(1+\delta)}{[\mu(1+\delta)-1]}$. Note that, for a given a steady state markup μ, the elasticity of substitution θ monotonically decreases with δ. In particular, at the benchmark low markup level $\mu = 1.12$, the elasticity of substitution can be as low as $\theta = 5$ even for a small price wedge rate of $\delta = 0.10$. If $\delta = 0.33$, the elasticity of substitution θ falls to about $\theta = 3$, which is consistent with microdata estimates in Broda and Weinstein (2006). The authors find that consumers have low elasticities of substitution across similar goods in most categories, with the median elasticity being estimated at about $\theta = 3$.

With Calvo price setting under price wedges, the firm z's optimal pricing decision is:

$$1 = \frac{E_t \sum_{j=0}^{\infty} \alpha^j q_{t,t+j} \Pi_{t,t+j}^{ind} \mathcal{G}_{t,t+j} \left(\frac{z_{t,t+j}^*}{1+\delta}\right)^{-(1+\bar{\theta}_1)} (X_{t+j})^{(\sigma+\omega)}}{E_t \sum_{j=0}^{\infty} \alpha^j q_{t,t+j} \Pi_{t,t+j}^{ind} \mathcal{G}_{t,t+j} \left(\frac{z_{t,t+j}^*}{1+\delta}\right)^{-\theta}} \quad ; \; z_{t,t+j}^* \equiv \frac{\Pi_{t,t+j}^{ind}}{\Pi_{t,t+j}} \wp_t^* + \delta \tag{6}$$

where $\theta_1 \equiv \theta(1+\omega)$ is a composite parameter, and again $\omega = \frac{(1+\nu)}{\varepsilon} - 1$, $\wp_t^* = \frac{p_t^*}{P_t}$ and $X_t = \frac{Y_t}{Y_t^n}$. In this framework, price aggregation $(1+\delta)^{1-\theta} = \int_0^1 \left(\frac{p_t(z)}{P_t} + \delta\frac{P_{s,t}}{P_t}\right)^{1-\theta} dz$ evolves according to:

$$(1+\delta)^{-(\theta-1)} = (1-\alpha)\sum_{j=0}^{\infty}\alpha^j\left(z_{t-j,t}^*\right)^{-(\theta-1)} \quad ;\ z_{t-j,t}^* \equiv \frac{\Pi_{t-j,t}^{ind}}{\Pi_{t-j,t}}\wp_{t-j}^* + \delta \tag{7}$$

Note that the auxiliary variables $z_{t,t+j}^*$ and $z_{t-j,t}^*$ enter systems (6) and (7) raised to non-positive integer powers. This fact prevents the equations to have recursive forms. In order to cope with that, we present a precise approximation in next section, allowing those terms to have recursive forms in log-linearizations.

5.2.4 Aggregates and Welfare

Let $h_t \equiv \int_0^1 h_t(z)\,dz$ denote the aggregate working hours. Given the production function $y_t(z) = \mathcal{A}_t h_t(z)^{\varepsilon}$ and demand function $\frac{y_t(z)}{Y_t} = \left(\frac{\wp_t(z)+\delta}{1+\delta}\right)^{-\theta}$, we conclude that aggregate hours evolve according to $h_t = (1+\delta)^{\frac{\theta}{\varepsilon}}\left(\frac{Y_t}{\mathcal{A}_t}\right)^{\frac{1}{\varepsilon}}\Lambda_{y,t}$, where $\Lambda_{y,t} \equiv \int_0^1 (\wp_t(z)+\delta)^{-\frac{\theta}{\varepsilon}}\,dz$. Therefore, following the vast literature of price dispersion, we can write the aggregate output as $Y_t = \frac{1}{\mathfrak{d}_{y,t}}\mathcal{A}_t(h_t)^{\varepsilon}$, where $\mathfrak{d}_{y,t} \equiv (1+\delta)^{\theta}(\Lambda_{y,t})^{\varepsilon}$ is the production-relevant metric of price dispersion. Using Calvo price setting, note that $\Lambda_{y,t} = (1-\alpha)\sum_{j=0}^{\infty}\alpha^j\left(z_{t-j,t}^*\right)^{-\frac{\theta}{\varepsilon}}$.

As for welfare considerations, recall that $\mathcal{W}_t \equiv (u_t - v_t)$ is the relevant instantaneous welfare metric, where $u_t \equiv \epsilon_t\frac{(Y_t)^{(1-\sigma)}-1}{(1-\sigma)}$ is the consumption utility and $v_t \equiv \int_0^1 v_t(z)\,dz$ is the aggregate disutility of working hours, in which $v_t(z) \equiv \frac{\chi}{(1+\nu)}h_t(z)^{(1+\nu)}$. Given the production and demand functions, we can write the aggregate disutility as $v_t = \mathfrak{d}_{v,t}\frac{\chi}{(1+\nu)}(h_t)^{(1+\nu)}$, where $\mathfrak{d}_{v,t} = \frac{\Lambda_t}{(\Lambda_{y,t})^{(1+\nu)}}$ is the welfare-relevant metric of price dispersion, $\Lambda_t \equiv \int_0^1 (\wp_t(z)+\delta)^{-\theta_1}\,dz$. Under Calvo price setting, note that $\Lambda_t = (1-\alpha)\sum_{j=0}^{\infty}\alpha^j\left(z_{t-j,t}^*\right)^{-\theta_1}$.

In the equilibrium with flexible prices ($\alpha = 0$), we obtain $\Lambda_{y,t}^n = (1+\delta)^{-\frac{\theta}{\varepsilon}}$, $\Lambda_t^n = (1+\delta)^{-\theta_1}$, and $\mathfrak{d}_{y,t}^n = \mathfrak{d}_{v,t}^n = 1$. In this equilibrium, the instantaneous welfare evolves according to $\mathcal{W}_t^n \equiv (u_t^n - v_t^n) = \epsilon_t\frac{(Y_t^n)^{(1-\sigma)}-1}{(1-\sigma)} - \frac{\chi}{(1+\nu)}\left(\frac{Y_t^n}{\mathcal{A}_t}\right)^{(1+\omega)}$, where $u_t^n = \epsilon_t\frac{(Y_t^n)^{(1-\sigma)}-1}{(1-\sigma)}$, $v_t^n = \frac{\chi}{(1+\nu)}(h_t^n)^{(1+\nu)}$, and $h_t^n = \left(\frac{Y_t^n}{\mathcal{A}_t}\right)^{\frac{1}{\varepsilon}}$.

Therefore, following Schmitt-Grohe and Uribe (2007), we can compute the consumption-equivalent welfare metric as a distorted output level Y_t^{eq} that would prevail in a equilibrium with flexible prices in order to keep the welfare level as the one obtained with sticky prices ($\mathcal{W}_t$). That is, Y_t^{eq} satisfies:

$$\epsilon_t\frac{\left(Y_t^{eq}\right)^{(1-\sigma)}-1}{(1-\sigma)} - \frac{\chi}{(1+\nu)}\left(\frac{Y_t^{eq}}{\mathcal{A}_t}\right)^{(1+\omega)} = \mathcal{W}_t = \epsilon_t\frac{(Y_t)^{(1-\sigma)}-1}{(1-\sigma)} - \mathfrak{d}_{v,t}\frac{\chi}{(1+\nu)}\left(\mathfrak{d}_{y,t}\frac{Y_t}{\mathcal{A}_t}\right)^{(1+\omega)}$$

In this regard, we define $X_t^{eq} \equiv \frac{Y_t^{eq}}{Y_t^n}$ as the consumption-equivalent output gap.

5.2.5 Steady State Properties

Using the steady state relations shown in Appendix C.2, Figure 2 shows how steady state output gap $\bar{X}$ and production-relevant price dispersion $\bar{\mathfrak{d}}_y$ vary with different levels of trend inflation $\bar{\pi}$ and different price wedge rates δ. For this, we consider the benchmark calibration defined in Section 3.3 and use $\mu = \frac{\theta}{(\theta-1)(1+\delta)}$ (see Section 5.2.3) to recompute θ for each value of δ, keeping the static markup at $\mu = 1.12$.

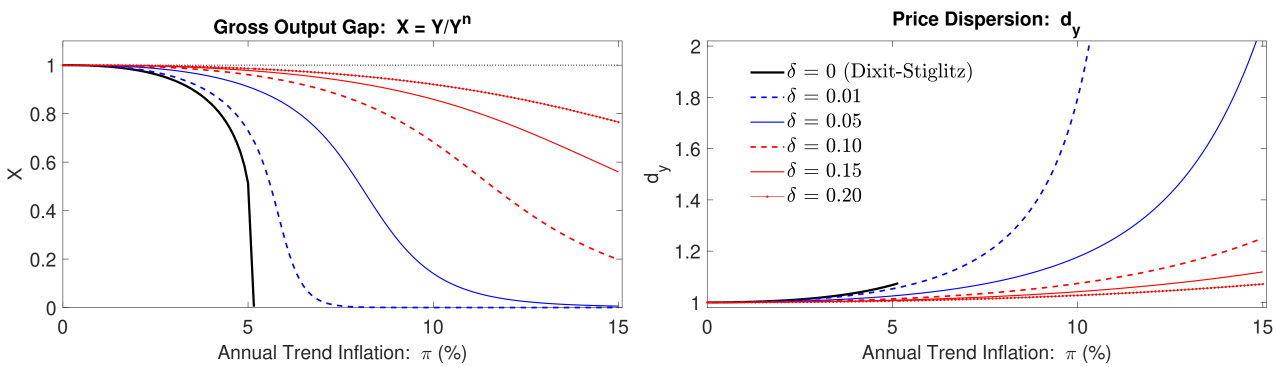

Figure 2: Steady State Levels - Price Wedges

As predicted, steady state levels now exist for all levels of trend inflation as long as $\delta > 0$. In the left panel, note that the gross output gap falls as trend inflation rises. It is interesting to note that using larger values for δ makes output smoothly decline with respect to the natural output, avoiding the sharp fall observed under $\delta = 0$ (Dixit-Stiglitz). If δ is very small, the model is able to present a seamless continuation of what standard NK models (Dixit-Stiglitz) predict for the steady state, but now without the upper limit on trend inflation, which is 5.16% using the benchmark calibration. In the right panel, note that the presence of price wedges strongly attenuates the price dispersion caused by trend inflation. We highlight these results as recent micro evidence on price dispersion suggests that it only weakly increases as inflation rises (e.g., Nakamura et al. (2018) and Sheremirov (2020)).

6 Simulations

In this section, we assess the price-wedge model dynamics using the log-linearized model presented in Online Appendix C.3. Using the benchmark calibration, recall that the upper limit for annualized trend inflation is $\bar{\pi} = 5.16\%$ under standard Dixit and Stiglitz (1977) preferences and zero price wedges ($\delta = 0$).[18] However, as we show in Section 5.2.2, the demand function function under price wedges satisfies Theorem 1, and so setting $\delta > 0$ is a sufficient condition for the existence of steady state equilibrium at any level of trend inflation.

Hatted variables represent log-deviations from steady state levels. In this context, we assume that the central bank has a mandated inflation target $\bar{\pi} \geq 0$ and follows a (log-linearized) Mixed Taylor rule $\hat{\imath}_t = \phi_{\mathrm{i}}\hat{\imath}_{t-1} + (1-\phi_{\mathrm{i}})\left[\phi_{f\pi}E_t\hat{\pi}_{t+1} + \phi_{gx}\left(\hat{x}_t - \hat{x}_{t-1}\right)\right] + \hat{\epsilon}_{\mathrm{i},t}$, roughly based on Coibion and Gorodnichenko (2011) baseline specification,[19] where $\hat{\epsilon}_{\mathrm{i},t}$ is the monetary policy shock, $\phi_{\mathrm{i}} \in (0,1)$ is the policy smoothing parameter, and the response parameters $\phi_{f\pi}$ and ϕ_{gy} are consistent with stability and determinacy in equilibria with rational expectations under positive trend inflation. Based on Coibion and Gorodnichenko (2011) estimates for the post-1982 period, we set $\phi_{\mathrm{i}} = 0.86$, $\phi_{f\pi} = 2.20$ and $\phi_{gx} = 1.56$. We highlight that reacting to output gap growth $(\hat{x}_t - \hat{x}_{t-1})$ is in line with the findings of Coibion and Gorodnichenko (2011) and **?** as it generates more stabilizing properties when the trend inflation is not zero. In addition, reacting to growth is in line with Walsh (2003), and Orphanides and Williams (2007).

The log-linearized price-wedge New-Keynesian Phillips Curve under trend inflation has an infinite number of ancillary recursive equations. However, as we show in Appendix C.3, we can approximate it into a more didactic and compact form. Under a vast range of parametrizations, we verify that this approximation is very accurate if trend inflation is no larger than 11%. For larger levels, the full log-linearized model presented in Appendix C.3 must be used.

The approximated price-wedge NKPC under trend inflation is:

$$\begin{aligned}\left(\hat{\pi}_t - \hat{\pi}_t^{ind}\right) \approx\ & \beta E_t\left(\hat{\pi}_{t+1} - \hat{\pi}_{t+1}^{ind}\right) + \bar{\kappa}\hat{x}_t \\ & + \left(\frac{\bar{\Phi}_{N1}}{\bar{\Phi}_{D1}} - 1\right)\bar{\kappa}\bar{\alpha}_1\bar{\Phi}_{D1}\beta E_t\hat{\omega}_{1,t+1} + \left(\frac{\bar{\Phi}_{N2}}{\bar{\Phi}_{D2}} - 1\right)\bar{\kappa}\bar{\alpha}_2\bar{\Phi}_{D2}\beta E_t\hat{\omega}_{2,t+1} \\ & + \left(\frac{\bar{\alpha}_1\bar{\Phi}_{D1}}{\bar{\alpha}_2\bar{\Phi}_{D2}} - 1\right)\bar{\kappa}\bar{\alpha}_2\bar{\Phi}_{D2}\beta E_t\left(\hat{\omega}_{3,t+1} - \hat{\omega}_{2,t+1}\right) + \left(\frac{\bar{\Phi}_{D1}}{\bar{\Phi}_c} - 1\right)\beta E_t\left(\hat{\pi}_{t+1} - \hat{\pi}_{t+1}^{ind}\right)\end{aligned} \tag{8}$$

where the composite parameters are all functions of steady state levels, shown in Appendix C.3. As for hatted variables, they are log-deviations from their steady state levels. That is, $\hat{\pi}_t$ is the inflation rate, $\hat{\pi}_t^{ind} = \gamma\hat{\pi}_{t-1}$ is the partial indexed inflation rate, $\hat{x}_t$ is the output gap, $\hat{\mathfrak{g}}_t = \left(\hat{Y}_t - \hat{Y}_{t-1}\right)$ is the output growth, and $\hat{\epsilon}_t$ is the demand shock. In addition, $\hat{\omega}_{1,t}$, $\hat{\omega}_{2,t}$ and $\hat{\omega}_{3,t}$ are three ancillary variables,

[18] See Section 4.

[19] In Coibion and Gorodnichenko (2011), monetary policy responds to output growth instead.

which evolve according to:

$$\hat{\omega}_{1,t} = \bar{\alpha}_1 \bar{\Phi}_{N1} \beta E_t \hat{\omega}_{1,t+1} + \hat{x}_t - \bar{\lambda}_1 \left[\left(\hat{\pi}_t - \hat{\pi}_t^{ind} \right) + (\sigma - 1) \hat{\mathfrak{g}}_t - (\hat{\epsilon}_t - \hat{\epsilon}_{t-1}) \right]$$
$$\hat{\omega}_{2,t} = \bar{\alpha}_2 \bar{\Phi}_{N2} \beta E_t \hat{\omega}_{2,t+1} + \bar{\lambda}_2 \left(\hat{\pi}_t - \hat{\pi}_t^{ind} \right)$$
$$\hat{\omega}_{3,t} = \bar{\alpha}_2 \bar{\Phi}_{D2} \beta E_t \hat{\omega}_{3,t+1} + \bar{\lambda}_3 \left(\hat{\pi}_t - \hat{\pi}_t^{ind} \right)$$

If trend inflation is zero, the composite parameter ratios have the same value $\frac{\bar{\Phi}_{N1}}{\bar{\Phi}_{D1}} = \frac{\bar{\Phi}_{N2}}{\bar{\Phi}_{D2}} = \frac{\bar{\alpha}_1 \bar{\Phi}_{D1}}{\bar{\alpha}_2 \bar{\Phi}_{D2}} = \frac{\bar{\Phi}_{D1}}{\bar{\Phi}_c} = 1$. In this case, the three ancillary variables play no role in influencing inflation dynamics. If $\bar{\pi} \gtrless 0$, we have $\frac{\bar{\Phi}_{N1}}{\bar{\Phi}_{D1}} \gtrless 1$ and $\frac{\bar{\Phi}_{N2}}{\bar{\Phi}_{D2}} \gtrless 1$. As for $\frac{\bar{\alpha}_1 \bar{\Phi}_{D1}}{\bar{\alpha}_2 \bar{\Phi}_{D2}}$ and $\frac{\bar{\Phi}_{D1}}{\bar{\Phi}_c}$, they are different from 1 only if $\delta > 0$ and $\bar{\pi} \neq 0$. If either trend inflation is zero ($\bar{\pi} = 0$) or there are no price wedges ($\delta = 0$), the approximation (8) matches the full log-linearized equation.

6.1 Slope of the Phillips Curve

How does trend inflation $\bar{\pi}$ affect the slope of the Phillips Curve in the price wedge model? To answer this question, we investigate the role of δ on the slope.

The first and simplest answer is obtained by assessing $\bar{\kappa}$, the composite coefficient on contemporaneous output gap in the Phillips Curve. As known in the literature of trend inflation (e.g. Ascari and Sbordone (2014)), $\bar{\kappa}$ decreases with trend inflation $\bar{\pi}$. Here, we find that the net effect of δ is to slightly increase this composite parameter. However, empirical evidence has suggested that trend inflation actually increase the slope of the Phillips Curve (e.g. Romer et al. (1988), Ball and Mazumder (2011) and Kurozumi and Van Zandweghe (2024)). And so, the fact that the contemporaneous slope $\bar{\kappa}$ decreases with trend inflation is at odds with empirical evidence. The literature explains this apparent puzzle by the fact that the frequency of price readjustments should increase with trend inflation, reducing the slope. Since this channel is absent in Calvo pricing, $\bar{\kappa}$ actually decreases with trend inflation.

Nonetheless, as a contribution of this paper, we show that this puzzle is only apparent. In the empirical literature, the simplest empirical approach used in the literature is carried out by using observed measures of inflation rate and output gap to estimate the slope κ_0 with a specification based on the zero-trend inflation NK Phillips Curve (NKPC) $\hat{\pi}_t = \beta \hat{\pi}_t^e + \kappa_0 \hat{x}_t + \varepsilon_{0,t}$, where the parameter β is calibrated, $\hat{\pi}_t^e$ is an empirical metrics for expected inflation, and $\varepsilon_{0,t}$ is an error term.[20]

[20]For instance, Ball and Mazumder (2011) and Kurozumi and Van Zandweghe (2024) use $\beta = 1$ and consider the 4-quarter moving-average of realized inflation $\frac{1}{4} \sum_{j=1}^{4} \hat{\pi}_{t-j}$ as a proxy for $\hat{\pi}_t^e$. For inference, they use US Congressional Budget Office's output-gap measure for $\hat{x}_t$ in a Kalman Filtering approach to estimate time-varying coefficients $\kappa_{0,t}$. Hazell et al. (2022), on the other hand, consider the employment gap for $\hat{x}_t$, set $\beta = 0.99$ and use the rational expectations approach $\hat{\pi}_t^e = E_t \hat{\pi}_{t+1}$

Alternatively, one can also estimate the reduced-form Phillips Curve $\hat{\pi}_t = \kappa_x \hat{x}_t + \varepsilon_{x,t}$. As Hazell et al. (2022) explain, this specification arises is assuming that $\hat{x}_t$ follows a reduced-form AR(1) dynamics $\hat{x}_t = \rho_x \hat{x}_{t-1} + \nu_{x,t}$ in equilibrium. If the term $\nu_{x,t}$ is white noise, then the rational expectations solution $\hat{\pi}_t = \kappa_0 \sum_{j=0}^{\infty} \beta^j \hat{x}_{t+j} + \varepsilon_{0,t}$ is equivalent to $\hat{\pi}_t = \frac{\kappa_0}{(1-\beta\rho_x)} \hat{x}_t + \varepsilon_{0,t}$. It implies that the reduced-form slope satisfies $\kappa_x = \frac{\kappa_0}{(1-\beta\rho_x)} \geq \kappa_0$. Hazell et al. (2022) argue that this is the rationale to explain why estimated reduced-form slope coefficients κ_x can be much larger than κ_0.

Therefore, the econometrician exercise is to estimate time-varying slope coefficients $\kappa_{0,t}$ and $\kappa_{x,t}$ and plot it against a time-varying measure of trend inflation (e.g. Cogley and Sbordone (2008), Chan et al. (2018)). As mentioned before, this leads to the empirical evidence that $\kappa_{0,t}$ and $\kappa_{x,t}$ are positively correlated with the level of trend inflation. In this context, we pose the question whether the theoretical dynamics implied by the NK model with price wedges are also consistent with a positive correlation between empirics-based slope metrics κ_0 (or κ_x) with trend inflation $\bar{\pi}$, when replicating the empirical approach. If so, is the theoretical model also consistent with estimates for κ_x being larger than κ_0, i.e. in line with Hazell et al. (2022) rationale?

As we show below, the answer is yes to both questions when trend inflation lies in the range observed in most economies, i.e. between 0% and 10%. Using the model to generate endogenous variables as observables, the empirics-based slope metrics not only take in account the effect of contemporaneous $\bar{\kappa}$, but also the effect of monetary policy and the remaining variables and shocks (current and lagged) in general equilibrium in the way they influence fluctuations of $\hat{\pi}_t$, $E_t \hat{\pi}_{t+1}$ and $\hat{x}_t$.

Therefore, we assess what the price-wedge model predicts for κ_0 and κ_x, had the empirical data used by econometricians been generated by the model. Since we also test the predictions with $\delta = 0$, we also contribute to the literature on trend inflation by reconciling the standard trend inflation NK model with the evidence that empirical estimates of the slope is positively correlated with the level of trend inflation.

In what follows, we assess asymptotic estimates for κ_0 and κ_x, obtained using the functional forms $(\hat{\pi}_t - \beta E_t \hat{\pi}_{t+1}) = \kappa_0 \hat{x}_t + \varepsilon_{0,t}$ and $\hat{\pi}_t = \kappa_x \hat{x}_t + \varepsilon_{x,t}$. The first step is to recognize that current $\hat{x}_t$ is endogenous to $\varepsilon_{0,t}$ and $\varepsilon_{x,t}$. And so instruments are in need for estimating κ_0 and κ_x. For that, we use lagged endogenous variables produced by the model as instruments and consider the 2SLS estimator. As it turns out, we always obtain the same estimates for κ_0 and κ_x once we consider instrumentaliza-

to solve the zero-trend inflation NKPC forward , obtaining $\hat{\pi}_t = \kappa_0 \sum_{j=0}^{\infty} \beta^j \hat{x}_{t+j} + \varepsilon_{0,t}$ when assuming that the error term $\varepsilon_{0,t}$ is white noise. For inference, they replace expected future employment gaps with their realized values and an expectation error, and use lagged employment gaps as instrumental variables in a GMM approach. As for the infinite sum, they truncate it at $T = 20$, estimating the functional form $\hat{\pi}_t = \kappa_0 \sum_{j=0}^{20} \beta^j \hat{x}_{t+j} + \varepsilon_{0,t}$.

tion, no matter which endogenous variable we use or which lag we choose for the instrument. That is, using lags of $\hat{x}_{t-j}$, $\hat{\pi}_{t-j}$, or $\hat{\imath}_{t-j}$ as instruments, we always obtain the same estimates. In addition, this result is also robust to using different shocks to generate fluctuations: monetary policy shock $\hat{\epsilon}_{i,t}$, utility (demand) shock $\hat{\epsilon}_t$ and technology (supply) shock $\widehat{\mathcal{A}}_t$.

Using an instrument $\mathfrak{z}_t \in \left\{\hat{x}_{t-j}, \hat{\pi}_{t-j}, \hat{\imath}_{t-j}\right\}$, the asymptotic estimates for κ_0 and κ_x are:

$$\kappa_0 = plim\left(\hat{\kappa}_0\right) = \frac{Cov[\mathfrak{z}_t\ ,\ (\hat{\pi}_t - \beta E_t \hat{\pi}_{t+1})]}{Cov[\mathfrak{z}_t\ ,\ \hat{x}_t]} \quad ; \kappa_x = plim\left(\hat{\kappa}_x\right) = \frac{Cov[\mathfrak{z}_t\ ,\ \hat{\pi}_t]}{Cov[\mathfrak{z}_t\ ,\ \hat{x}_t]} \tag{9}$$

For computing those metrics, we use the theoretical unconditional covariances implied by the model. Figure 3 shows how trend inflation $\bar{\pi}$ and price wedge δ affects three model-implied slope metrics: $\bar{\kappa}$, κ_0 and κ_x.

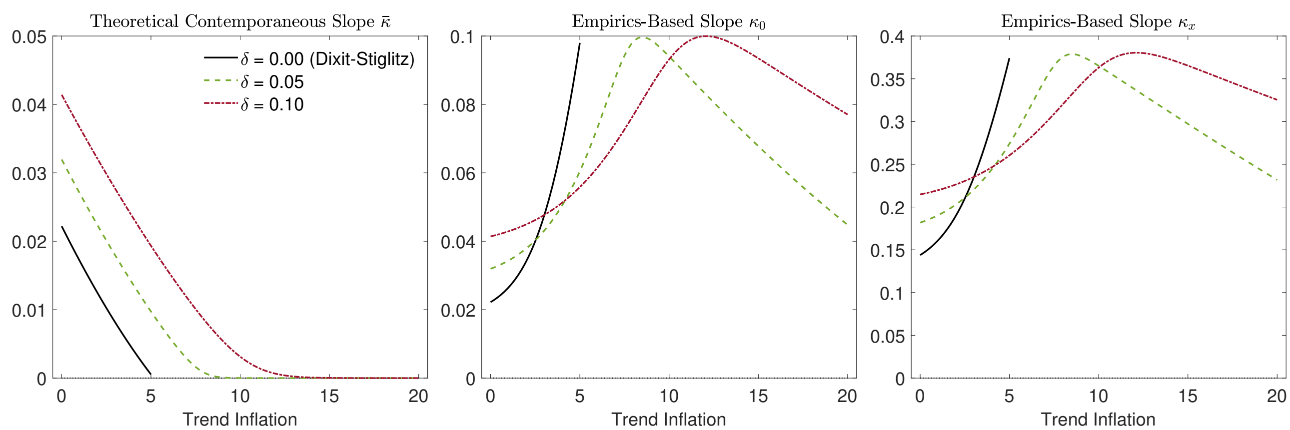

Figure 3: Slope of The Phillips Curve

Notes: Slope metrics of the Phillips Curve. Slope $\bar{\kappa}$ is the contemporaneous metric directly obtained from the composite parameter of $\hat{x}_t$ in equation (8). Slope κ_0 is the asymptotic value obtained when estimating the zero-trend inflation Phillips Curve $\hat{\pi}_t = \beta E_t \hat{\pi}_{t+1} + \kappa_0 \hat{x}_t + \varepsilon_{0,t}$. Slope κ_x is the asymptotic value obtained when estimating the reduced form Phillips Curve $\hat{\pi}_t = \kappa_0 \hat{x}_t + \varepsilon_{x,t}$. The asymptotic values κ_0 and κ_x are obtained with observations for $\hat{\pi}_t$, $E_t \hat{\pi}_{t+1}$, and $\hat{x}_t$ endogenously generated by the price wedge model. Since $\hat{x}_t$ is endogenous to the error terms, we use a lagged variable $z_t \in \{\hat{x}_{t-j}, \hat{\pi}_{t-j}, \hat{\imath}_{t-j}\}$ as an instrument. The estimates are consistent regardless of the lagged variable used as an instrument or the shock generating the fluctuations.

Note that, even though $\bar{\kappa}$ reduces with trend inflation, the empirics-based slope metrics κ_0 and κ_x increase with trend inflation when it is not large. However, the slope starts to fall once trend inflation is sufficiently high. Depending on the value of δ, this turning point happens at about 8% to 12%. We also highlight that, for the standard NK model with trend inflation ($\delta = 0$), the slope is always increasing up to the model's trend inflation threshold. Since inflation in most economies evolve in the range from 0% to 10%, the model predicts that empirical assessments of κ_0 and κ_x will generally increase with trend inflation. In particular, the κ_0 metrics obtained with $\delta = 0.05$ closely matches the empirical values for the US, as computed by Kurozumi and Van Zandweghe (2024). The pattern and values also match those obtained with with state dependent models (e.g. Blanco et al. (2024), Karadi et al. (2024)).

6.2 Impulse responses

Here, we exploit the dynamic consequences of the slope findings shown in Section 6.1. Since slope considerations are more important for assessing the impact of monetary policy, we retrieve impulse responses to monetary policy shock $\hat{\epsilon}_{i,t}$, considering a range of different levels of annual trend inflation. In order not to strongly depart from results consolidated in the literature of standard NK models with trend inflation, we set the price wedge rate at a very low value, i.e. we use $\delta = 0.05$.[21] Setting a low value for δ, as we do, implies that the dynamics are very similar to those obtained under standard NK models when trend inflation are smaller than the upper limit. However, as $\delta > 0$, it allows us to explore the dynamics at larger long-run inflation rates, past the usual upper limit, which in this case is $\bar{\pi} = 5.16\%$.

As Alvarez, Beraja, Gonzalez-Rozada and Neumeyer (2019) show, the frequency of price changes is invariant to trend inflation when the latter is low. Therefore, we consider it reasonable to assume that α remains roughly constant when trend inflation varies from 0% to 10%. And for making the point that even a low price wedge rate is enough to ensure the existence of steady state equilibria for all levels of trend inflation, we extend the trend inflation range to not low levels from 10% to 20%. For the latter range, we recognize that the frequency of price changes must respond endogenously to the increased level of trend inflation, but we keep α at the same calibrated value for illustrative purposes.

Therefore, for retrieving the impulse responses, we use the full log-linearized model presented in Appendix C.3.We highlight that using the approximated Phillips curve under price wedges delivers about the same results when trend inflation varies from 0% to 10%.

Figure 4 shows the responses to unitary monetary policy shock $\hat{\epsilon}_{i,t}$ under different levels of trend inflation. The top two rows show the responses of aggregate output $\hat{Y}_t$, annualized inflation $4\hat{\pi}_t$ and annualized nominal interest rate $4\hat{\imath}_t$ under low annual trend inflation, from 0 to 10%. And the bottom two rows assesses responses under high trend inflation, from 10 to 20%. We highlight that responses

[21] Recall that the markup result $\mu = \frac{\theta}{(\theta-1)(1+\delta)}$ (see Section 5.2.3) allows us to calibrate θ as a function of markup μ and price wedge rate δ.

for trend inflation larger than 5.16% are not possible under the standard NK model ($\delta = 0$). Here, using a non-zero value for δ allows us to explore the economy dynamics past the usual threshold.

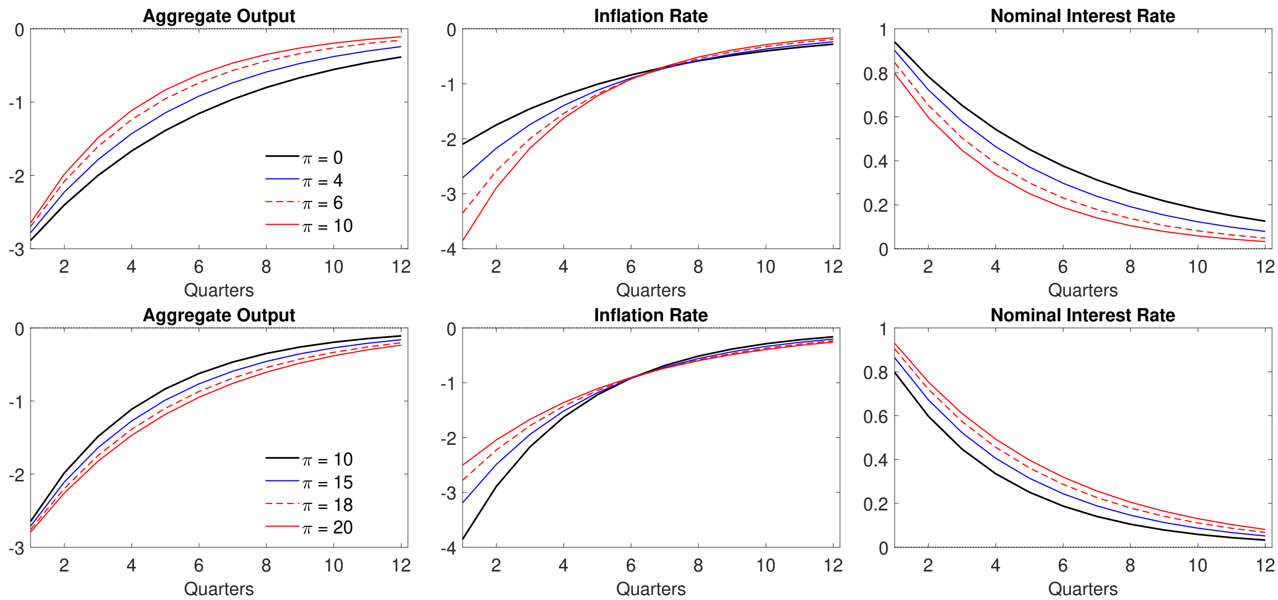

Figure 4: Impulse responses to monetary policy shocks

Notes: Impulse responses to Monetary Policy shock with $\delta = 0.05$, in the Dixit-Stiglitz model with Price Wedges. In the top row, we consider low levels of trend inflation ($0 \leq \bar{\pi} \leq 10$). In the bottom row, we consider "not low" levels of trend inflation ($10 \leq \bar{\pi} \leq 20$). One-off shocks are unitary, i.e. $\hat{\epsilon}_{i,1}= 1$ at period 1, for all levels of trend inflation.

The impulse responses are consistent with the results we obtained when assessing the empirics-based slope metrics. When trend inflation is smaller than 10%, responses to monetary policy shocks behave similarly to what we observe in standard NK models, in the sense that the amplitude of inflation rate responses increase with trend inflation, whereas output has its response amplitudes decreased. However, we see that there is a reversal in this pattern at high levels of trend inflation. From this point on, amplitudes of inflation responses decrease, while that of output increase, as trend inflation gets higher. It means that it becomes harder for central banks to curb inflation hikes and bring it down, when the average inflation sits above the 10% level. We highlight that these properties are in line with recent empirical results found by Canova and Forero (2024). The authors estimate a Markov-Switching model for the US with two states (high and low inflation) from 1960 to 2023. They find that, after contractionary monetary policy shocks, inflation rates do not fall as much and become more persistent in high-inflation states when compared to low-inflation states.

7 Conclusion

We provide a resolution to a well-known problem: the steady state of the widely-studied New Keynesian models based on Calvo-pricing does not exist beyond a low single-digit trend inflation threshold, rendering them not useful for monetary policy analysis when trend inflation is not very low. The main contribution of the paper is to establish that the root of the steady state problem originates from the interaction of Calvo pricing with the popular Dixit-Stiglitz demand structure in NK models. We present a general demand structure with the feature that demand remains finite when relative prices increase and show that the steady state always exists with Calvo pricing for any trend inflation level. Using this framework, we assess the properties of the Kimball-demand aggregator, which avoids the steady state problem but creates new ones. We then present a model with price wedges to augment the Dixit-Stiglitz and Kimball-demand aggregators and show that it resolves the steady state problem. Our findings show that modification of the demand structure can ensure that NK models are useful in evaluating alternative monetary policies for reducing inflation when trend inflation is not very low.

References

Alvarez, F., Beraja, M., Gonzalez-Rozada, M. and Neumeyer, P. A.: 2019, From Hyperinflation to Stable Prices: Argentina's Evidence on Menu Cost Models, *The Quarterly Journal of Economics* **134**(1), 451–505.

Alves, S. A. L.: 2014, Lack of divine coincidence in New Keynesian models, *Journal of Monetary Economics* **67**, 33–46.

Alves, S. A. L.: 2018, Monetary policy, trend inflation, and unemployment volatility, *Journal of Money, Credit and Banking* **50**(4), 637–673.

Ascari, G.: 2004, Staggered prices and trend inflation: Some nuisances, *Review of Economic Dynamics* **7**(3), 642–667.

Ascari, G. and Sbordone, A. M.: 2014, The macroeconomics of trend inflation, *Journal of Economic Literature* **52**(3), 679–739.

Atkeson, A. and Burstein, A.: 2008, Pricing-to-Market, Trade Costs, and International Relative Prices, *The American Economic Review* **98**(5), 1998–2031.

Auclert, A., Rigato, R., Rognlie, M. and Straub, L.: 2024, New pricing models, same old Phillips Curves?, *The Quarterly Journal of Economics* **139**(1), 121–186.

Bakhshi, H., Burriel-Llombart, P., Khan, H. and Rudolf, B.: 2007, The New Keynesian Phillips curve under trend inflation and strategic complementarity, *Journal of Macroeconomics* **29**(1), 37–59.

Bakhshi, H., Khan, H. and Rudolf, B.: 2007, The Phillips curve under state-dependent pricing, *Journal of Monetary Economics* **54**, 2321–2345.

Ball, L. M. and Mazumder, S.: 2011, Inflation Dynamics and the Great Recession, *NBER Working Paper* **17044**.

Ball, L. and Romer, D.: 1990, Real rigidities and the non-neutrality of money, *The Review of Economic Studies* **57**(2), 183.

Basu, S.: 1995, Intermediate goods and business cycles: Implications for productivity and welfare, *The American Economic Review* **85**(3), 512–531.

Beck, G. W. and Lein, S. M.: 2015, Microeconometric evidence on demand-side real rigidity and implications for monetary non-neutrality, *Faculty of Business and Economics, University of Basel, Working Paper* **2015/13**.

Bils, M. and Klenow, P. J.: 2004, Some evidence on the importance of sticky prices, *Journal of Political Economy* **112**(5), 947–985.

Blanchard, O. and Gali, J.: 2007, Real wage rigidities and the new keynesian model, *Journal of Money, Credit and Banking* **39**, 35–65.

Blanco, A., Boar, C., Jones, C. J. and Midrigan, V.: 2024, The Inflation Accelerator, *NBER Working Paper* **32531**.

Bleaney, M. F.: 1999, The Disappearing Openness-Inflation Relationship: A Cross-Country Analysis of Inflation Rates, *IMF Working Papers* .

Bredemeier, C., Gravert, J. and Juessen, F.: 2023, Accounting for limited commitment between spouses when estimating labor-supply elasticities, *Review of Economic Dynamics* **51**, 547–578.

Broda, C. and Weinstein, D. E.: 2006, Globalization and the Gains From Variety, *The Quarterly Journal of Economics* **121**(2), 541–585.

Burya, A. and Mishra, S.: 2022, Variable markups, demand elasticity and pass-through of marginal costs into prices, *Columbia University, Job Market Paper* .

Calvo, G. A.: 1983, Staggered prices in a utility-maximizing framework, *Journal of Monetary Economics* **12**(3), 383–398.

Canova, F. and Forero, F. P.: 2024, Does the Transmission of Monetary Policy Shocks Change when Inflation is High?, *CEPR Discussion Paper* **18993**.

Chan, J. C., Clark, T. E. and Koop, G.: 2018, A New Model of Inflation, Trend Inflation, and Long-Run Inflation Expectations, *Journal of Money, Credit and Banking* **50**(1), 5–53.

Chetty, R., Guren, A., Manoli, D. and Weber, A.: 2011, Are micro and macro labor supply elasticities consistent? a review of evidence on the intensive and extensive margins, *American Economic Review* **101**(3), 471–75.

Cogley, T. and Sbordone, A. M.: 2005, A search for a structural phillips curve, *Federal Reserve Bank of New York, Staff Reports* **203**.

Cogley, T. and Sbordone, A. M.: 2008, Trend Inflation, Indexation, and Inflation Persistence in the New Keynesian Phillips Curve, *American Economic Review* **98**(5), 2101–2126.

Coibion, O. and Gorodnichenko, Y.: 2011, Monetary policy, trend inflation, and the great moderation: An alternative interpretation, *American Economic Review* **101**(1), 341–70.

Cooley, T. F. and Prescott, E. C.: 1995, Economic growth and business cycles, *Frontiers of Business Cycles Research*, Princeton University Press, pp. ed. Thomas Cooley, Chapter 1, 1–38.

Costain, J. and Nakov, A.: 2011, Distributional dynamics under smoothly state-dependent pricing, *Journal of Monetary Economics* **58**(6), 646–665.

Costain, J. and Nakov, A.: 2023, Price adjustments in a general model of state-dependent pricing, *Journal of Money, Credit and Banking* **43**(2), 385–406.

Dixit, A. K. and Stiglitz, J. E.: 1977, Monopolistic competition and optimum product diversity, *American Economic Review* **67**(3), 297–308.

Dossche, M., Heylen, F. and Van Den Poel, D.: 2010, The kinked demand curve and price rigidity: Evidence from scanner data*, *The Scandinavian Journal of Economics* **112**(4), 723–752.

Dotsey, M. and King, R. G.: 2005, Implications of state-dependent pricing for dynamic macroeconomic models, *Journal of Monetary Economics* **52**(1), 213–242.

Dotsey, M., King, R. G. and Wolman, A. L.: 1999, State-Dependent Pricing and the General Equilibrium Dynamics of Money and Output, *The Quarterly Journal of Economics* **114**(2), 655–690.

Gagliardone, L., Gertler, M., Lenzu, S. and Tielens, J.: 2023, Anatomy of the Phillips Curve: Micro Evidence and Macro Implications, *NBER Working Paper* **31382**.

Galí, J.: 2015, *Monetary Policy, Inflation, and the Business Cycle: An Introduction to the New Keynesian Framework and Its Applications*, 2nd edn, Princeton University Press, Princeton, NJ.

Gautier, E. and Le Bihan, H.: 2022, Shocks versus Menu Costs: Patterns of Price Rigidity in an Estimated Multisector Menu-Cost Model, *The Review of Economics and Statistics* **104**(4), 668–685.

Golosov, M. and Lucas, R. E.: 2007, Menu costs and phillips curves, *Journal of Political Economy* **115**(2), 171–199.

Hahn, V.: 2022, Price dispersion and the costs of inflation, *Journal of Money, Credit and Banking* **54**(2), 459–491.

Harding, M., Linde, J. and Trabandt, M.: 2022, Resolving the missing deflation puzzle, *Journal of Monetary Economics* **126**, 15–34.

Hazell, J., Herreno, J., Nakamura, E. and Steinsson, J.: 2022, The Slope of the Phillips Curve: Evidence from U.S. States, *The Quarterly Journal of Economics* **137**(3), 1299–1344.

Kaplan, G. and Menzio, G.: 2015, The morphology of price dispersion, *International Economic Review* **56**(4), 1165–1206.

Karadi, P., Nakov, A., Barrau, G. N., Pasten, E. and Thaler, D.: 2024, Strike while the iron is hot: optimal monetary policy with a nonlinear Phillips curve, *BIS Working Paper* **1203**.

Keane, M. and Rogerson, R.: 2012, Micro and macro labor supply elasticities: A reassessment of conventional wisdom, *Journal of Economic Literature* **50**(2), 464–476.

Khan, H., Phaneuf, L. and Victor, J.-G.: 2020, Rules-based monetary policy and the threat of indeterminacy when trend inflation is low, *Journal of Monetary Economics* **114**, 317–333.

Kimball, M. S.: 1995, The quantitative analytics of the basic neomonetarist model, *Journal of Money, Credit and Banking* **27**(4), 1241–1277.

King, R. G. and Wolman, A. L.: 1996, Inflation targeting in a st. louis model of the 21st century, *Review, Federal Reserve Bank of St. Louis* pp. 83–107.

Klenow, P. J. and Kryvtsov, O.: 2008, State-dependent or time-dependent pricing: Does it matter for recent u.s. inflation?, *The Quarterly Journal of Economics* **123**(3), 863–904.

Klenow, P. J. and Malin, B. A.: 2010, Microeconomic evidence on price-setting, *NBER Working Paper* **15826**.

Klenow, P. J. and Willis, J. L.: 2016, Real rigidities and nominal price changes, *Economica* **83**(331), 443–472.

Kurozumi, T. and Van Zandweghe, W.: 2016, Kinked demand curves, the natural rate hypothesis, and macroeconomic stability, *Review of Economic Dynamics* **20**, 240–257.

Kurozumi, T. and Van Zandweghe, W.: 2024, Output-Inflation Trade-offs and the Optimal Inflation Rate, *Journal of Economic Dynamics and Control* **164**.

Levin, A., Lopez-Salido, J. D. and Yun, T.: 2007, Strategic complementarities and optimal monetary policy, *CEPR Discussion Papers* **6423**.

Levin, A. T., Onatski, A., Williams, J. and Williams, N. M.: 2005, Monetary policy under uncertainty in micro-founded macroeconometric models, *NBER Macroeconomics Annual* **20**, 229–287.

Michaillat, P. and Saez, E.: 2015, Aggregate Demand, Idle Time, and Unemployment, *The Quarterly Journal of Economics* **130**(2), 507–569.

Nakamura, E. and Steinsson, J.: 2008, Five facts about prices: A reevaluation of menu cost models, *The Quarterly Journal of Economics* **123**(4), 1415–1464.

Nakamura, E., Steinsson, J., Sun, P. and Villar, D.: 2018, The elusive costs of inflation: Price dispersion during the u.s. great inflation, *The Quarterly Journal of Economics* **133**(4), 1933–1980.

Orphanides, A. and Williams, J. C.: 2007, Robust monetary policy with imperfect knowledge, *Journal of Monetary Economics* **54**(5), 1406–1435.

Peterman, W. B.: 2016, Reconciling micro and macro estimates of the frisch labor supply elasticity, *Economic Inquiry* **54**(1), 100–120.

Romer, D. H., Ball, L. and Mankiw, N. G.: 1988, The New Keynesian Economics and the Output-Inflation Trade-Off, *Brookings Papers on Economic Activity* **1**.

Schmitt-Grohe, S. and Uribe, M.: 2007, Optimal simple and implementable monetary and fiscal rules, *Journal of Monetary Economics* **54**(6), 1702–1725.

Sheremirov, V.: 2020, Price dispersion and inflation: New facts and theoretical implications, *Journal of Monetary Economics* **114**, 59–70.

Smets, F. and Wouters, R.: 2007, Shocks and frictions in US business cycles: A bayesian DSGE approach, *American Economic Review* **97**(3), 586–606.

Walsh, C.: 2003, Speed limit policies: The output gap and optimal monetary policy, *American Economic Review* **93**(1), 265–278.

Walsh, C. E.: 2017, *Monetary Theory and Policy*, Fourth ed., The MIT Press.

Wang, O. and Werning, I.: 2022, Dynamic Oligopoly and Price Stickiness, *American Economic Review* **112**(8), 2815–2849.

Woodford, M.: 2003, *Interest and Prices: Foundations of a Theory of Monetary Policy*, Princeton University Press.

Yun, T.: 2005, Optimal monetary policy with relative price distortions, *American Economic Review* **95**(1), 89–109.

Online Appendix

A Real Rigidity Under General Demand Functions

Here, we assess what the general demand function $\frac{y_t(z)}{Y_t} = f\left(\wp_t(z), \wp_{s,t}\right)$ implies for real rigidity in price setting. In the natural equilibrium, i.e. flexible prices ($\alpha = 0$), system (4) simplifies into

$$f\left(\wp_t^n, \wp_{s,t}^n\right) + \left(\wp_t^n\right) f_1\left(\wp_t^n, \wp_{s,t}^n\right) = \left(1 - \frac{1}{\xi^n}\right) f_1\left(\wp_t^n, \wp_{s,t}^n\right) \left[f\left(\wp_t^n, \wp_{s,t}^n\right)\right]^{\omega} \left(X_t\right)^{(\sigma+\omega)}$$

Using the fact that $f(1,1) = 1$, the last relation is easily log-linearized about the economy steady-state with flexible prices, in which $\overline{\wp}^n = \overline{\wp}_s^n = 1$:

$$\widehat{\wp}_t^n = -\frac{\left[\frac{f_2(1,1)+f_{12}(1,1)}{1+f_1(1,1)} - \frac{f_{12}(1,1)}{f_1(1,1)} - \omega f_2(1,1)\right]}{\left[\frac{2f_1(1,1)+f_{11}(1,1)}{1+f_1(1,1)} - \frac{f_{11}(1,1)}{f_1(1,1)} - \omega f_1(1,1)\right]} \widehat{\wp}_{s,t} + \frac{(\sigma+\omega)}{\left[\frac{2f_1(1,1)+f_{11}(1,1)}{1+f_1(1,1)} - \frac{f_{11}(1,1)}{f_1(1,1)} - \omega f_1(1,1)\right]} \hat{x}_t$$

where $\widehat{\wp}_t^n$, $\widehat{\wp}_{s,t}$ and $\hat{x}_t$ are log-deviations from steady state levels.

Following Ball and Romer (1990) approach, we compute the content $\psi_{\text{real}} \equiv \frac{1}{\kappa_{\text{real}}}$ of real rigidities in this model, where $\kappa_{\text{real}} \equiv \frac{\partial \widehat{\wp}_t^n}{\partial \hat{x}_t}$ is the pass-through from output gap to prices. If prices are rigid ($\alpha > 0$), κ_{real} is part of the output-gap coefficient in the Phillips curve. Evaluating equations (3) in the steady-state equilibrium with flexible prices, we obtain a simple result to general demand-driven real rigidities[22] as a function of the natural elasticity ξ^n and superelasticity η^n. These expressions are:[23]

$$\kappa_{\text{real}} = \frac{(\sigma+\omega)}{1+\frac{\eta^n}{(\xi^n-1)}+\omega\xi^n} \quad ; \ \psi_{\text{real}} = \frac{1}{\kappa_{\text{real}}} \tag{A.1}$$

Therefore, the demand structure is a relevant source of real rigidities. As for the role of changes in ξ^n and η^n, notice that $\frac{\partial(\psi_{\text{real}})}{\partial \eta^n} = \frac{1}{(\sigma+\omega)} \frac{1}{(\xi^n-1)}$ and $\frac{\partial(\psi_{\text{real}})}{\partial \xi^n} = -\frac{1}{(\sigma+\omega)} \left(\frac{\eta^n}{(\xi^n-1)^2} - \omega\right)$. We conclude that, no matter the form of the demand function, there must be the case that: (i) increases in natural superelasticity η^n leads to larger (smaller) real rigidity ψ_{real} if natural elasticity ξ^n is larger (smaller) than unity; and (ii) increases in natural elasticity ξ^n leads to larger (smaller) real rigidity ψ_{real} if natural superelasticity η^n is smaller (larger) than $\omega\left(\xi^n - 1\right)^2$.

[22] For that, we easily compute $\kappa_{\text{real}} \equiv \frac{\partial \widehat{\wp}_t^n}{\partial \hat{X}_t} = \frac{(\sigma+\omega)}{\frac{2f_1(1,1)+f_{11}(1,1)}{1+f_1(1,1)} - \frac{f_{11}(1,1)}{f_1(1,1)} - \omega f_1(1,1)}$, and apply the definitions in (3).

[23] In Burya and Mishra (2022), the authors derive a similar but simpler result in a model with linear production function, log utility to consumption and no disutility to work, which implies $\omega = 0$. The authors show the pass-through $\kappa_p \equiv \frac{\kappa_{\text{real}}}{(\sigma+\omega)}$ from marginal costs to prices. When $\omega = 0$, their inverse real rigidity metrics is then $\kappa_{pt} = \frac{(\xi^n-1)}{(\xi^n-1)+\eta^n}$.

B Kimball Aggregator

In Kimball (1995), consumption over all differentiated goods $c_t(z)$ are aggregated into a bundle C_t, according to $1 = \int_0^1 G\left(\frac{c_t(z)}{C_t}\right) dz$, where function $G(\varkappa)$ satisfies $G(1) = 1$, $G'(\varkappa) > 0$, and $G''(\varkappa) < 0$, for all $\varkappa \geq 0$. In this context, Dotsey and King (2005) propose the particular functional form $G\left(\frac{c_t(z)}{C_t}\right) = \frac{\mathfrak{m}}{1+\varphi}\left[(1+\varphi)\frac{c_t(z)}{C_t} - \varphi\right]^{\frac{1}{\mathfrak{m}}} + 1 - \frac{\mathfrak{m}}{1+\varphi}$, where $\mathfrak{m} \equiv \frac{\mu_k(1+\varphi)}{(1+\mu_k\varphi)}$ is a composite parameter, $\mu_k \geq 1$ is the elasticity parameter, which matches the implicit markup rate μ under flexible prices, and $\varphi \leq 0$ sets the aggregation curvature. If $\varphi = 0$, $G(\cdot)$ simplifies into the standard Dixit and Stiglitz (1977) aggregation form. Allowing for smooth-kinked demand function, it has also been used by Levin et al. (2007), Harding et al. (2022) and Kurozumi and Van Zandweghe (2024).

According the notation used in Section 3.1, this model sets $\delta = 0$. The literature typically derives the utility-based demand function by choosing $c_t(z)$ to minimize expenditure $P_tC_t \equiv \int_0^1 p_t(z)\,c_t(z)\,dz$, subject to only one restriction, the Kimball aggregation $1 = \int_0^1 G\left(\frac{c_t(z)}{C_t}\right) dz$. The implied demand function and implied price aggregation are:

$$\frac{c_t(z)}{C_t} = f(\wp_t(z), \wp_{s,t}) = \begin{cases} \frac{1}{(1+\varphi)}\left(\frac{\wp_t(z)}{\wp_{k,t}}\right)^{\varpi} + \frac{\varphi}{(1+\varphi)} & ;\text{if } \left(\frac{\wp_t(z)}{\wp_{k,t}}\right) \leq (-\varphi)^{\frac{1}{\varpi}} \\ 0 & ;\text{if } \left(\frac{\wp_t(z)}{\wp_{k,t}}\right) > (-\varphi)^{\frac{1}{\varpi}} \end{cases} \tag{B.2}$$
$$\wp_{k,t} \equiv (1+\varphi) - \varphi\wp_{s,t} \quad ;\ P_{s,t} \equiv \int_0^1 p_t(z)\,dz \quad ;\ 1 = \int_0^1 \left(\frac{\wp_t(z)}{\wp_{k,t}}\right)^{(1+\varpi)} dz$$

where $\varpi \equiv \frac{\mu_k(1+\varphi)}{(1-\mu_k)} = -\frac{\mathfrak{m}}{(\mathfrak{m}-1)}$, $P_{k,t}$ is an auxiliary composite price aggregate, $P_{s,t}$ is the average price. Paralleling the notation used in Section 3.1.1, we define $\wp_t(z) \equiv \frac{p_t(z)}{P_t}$ as the relative price of firm z, and $\wp_{s,t} = \frac{P_{s,t}}{P_t}$ as the average relative price. We also set $\wp_{k,t} \equiv \frac{P_{k,t}}{P_t}$ as the auxiliary composite relative price. In addition, it is straightforward to verify that the price aggregation $1 = \int_0^1 \left(\frac{\wp_t(z)}{(1+\varphi)-\varphi\wp_{s,t}}\right)^{(1+\varpi)} dz$ is equivalent to $P_t = \int_0^1 p_t(z)\,f(\wp_t(z), \wp_{s,t})\,dz$.

Under this type of Kimball aggregation, the firm z's price $p_t(z)$ elasticity and superelasticity are: (i) $\xi_t(z) = -\varpi\left(\frac{\wp_t(z)}{\wp_{k,t}}\right)^{\varpi}\left[\left(\frac{\wp_t(z)}{\wp_{k,t}}\right)^{\varpi} + \varphi\right]^{-1}$ and $\eta_t(z) = \varpi\varphi\left[\left(\frac{\wp_t(z)}{\wp_{k,t}}\right)^{\varpi} + \varphi\right]^{-1}$, if $\left(\frac{\wp_t(z)}{\wp_{k,t}}\right) \leq (-\varphi)^{\frac{1}{\varpi}}$; or (ii) $\xi_t(z) = 0$ and $\eta_t(z) = 0$, if $\left(\frac{\wp_t(z)}{\wp_{k,t}}\right) > (-\varphi)^{\frac{1}{\varpi}}$.

In macroeconomic models, the way to generate empirically observed persistent non-neutrality in aggregate output is to combine real and nominal rigidities. However, empirical evidence suggest that price stickiness is not so large.[24] Therefore, macroeconomists tend to use theoretical models with large real rigidities (see e.g. Ball and Romer (1990), Basu (1995), Blanchard and Gali (2007)). In this regard, Kimball's implied real rigidity can be easily computed using $(A.1)$, evaluated in the steady

[24] As in e.g. Bils and Klenow (2004) and Nakamura and Steinsson (2008), estimated median duration between price changes ranges from about 4.5 months, when sales are included, to 10 months, when they are excluded.

state equilibrium with flexible prices:[25]

$$\kappa_{\text{real}}^{Kimball} = \frac{(\sigma+\omega)}{1-\mu\varphi+\frac{\mu}{(\mu-1)}\omega} \quad ; \psi_{\text{real}}^{Kimball} = \frac{1}{\kappa_{\text{real}}}$$

Therefore, for a given preferences/production structure represented by σ and ω, a large degree of large real rigidity $\psi_{\text{real}}^{Kimball}$ can be achieved with a convenient balance between a large demand curvature ($\varphi << 0$) and a appropriate markup $\mu > 1$.

B.1 Kimball NK Model

In general equilibrium, based on the generic model shown in Section 3, we have:

$$\begin{aligned}
1 = &\ \beta E_t \left(\frac{\epsilon_{t+1}}{\epsilon_t} \left(\frac{Y_t}{Y_{t+1}} \right)^{\sigma} \frac{I_t}{\Pi_{t+1}} \right) \\
q_t = &\ \beta \frac{\epsilon_t}{\epsilon_{t-1}} \left(\frac{Y_{t-1}}{Y_t} \right)^{\sigma} \frac{1}{\Pi_t} \\
\left(\frac{I_t}{\bar{I}} \right) = &\ \epsilon_{\mathrm{i},t} \left(\frac{I_{t-1}}{\bar{I}} \right)^{\phi_{\mathrm{i}}} \left[\left(\frac{\Pi_t}{\bar{\Pi}} \right)^{\phi_\pi} \left(\frac{X_t}{\bar{X}} \right)^{\phi_x} \left(\frac{Y_t}{Y_{t-1}} \right)^{\phi_{gy}} \left(\frac{Y_t}{\bar{Y}} \right)^{\phi_y} \right]^{(1-\phi_{\mathrm{i}})} \\
(Y_t^n)^{(\sigma+\omega)} = &\ \frac{1}{\mu} \frac{\varepsilon}{\chi} \epsilon_t \left(\mathcal{A}_t \right)^{(1+\omega)} \\
X_t = &\ \frac{Y_t}{Y_t^n} \quad ; \; q_{t,t+j} = q_{t+1} q_{t+1,t+j} \quad \text{for } j \geq 1 \text{ and } q_{t,t} = 1 \\
\mathcal{G}_t = &\ \frac{Y_t}{Y_{t-1}} \quad ; \; \Pi_{t,t+j} = \Pi_{t+1} \Pi_{t+1,t+j} \quad \text{for } j \geq 1 \text{ and } \Pi_{t,t} = 1 \\
\Pi_t^{ind} = &\ \Pi_{t-1}^{\gamma} \quad ; \; \Pi_{t,t+j}^{ind} = \Pi_{t+1}^{ind} \Pi_{t+1,t+j}^{ind} \quad \text{for } j \geq 1 \text{ and } \Pi_{t,t}^{ind} = 1 \\
&\ \quad ; \; \mathcal{G}_{t,t+j} = \mathcal{G}_{t+1} \mathcal{G}_{t+1,t+j} \quad \text{for } j \geq 1 \text{ and } \mathcal{G}_{t,t} = 1
\end{aligned}$$

[25]Considering that μ here is the gross markup rate, the component $(1 - \mu\varphi)$ is the same found in Levin, Lopez-Salido and Yun (2007) and Harding, Linde and Trabandt (2022), as their models implies $\omega = 0$.

$$\wp_{s,t} = (1-\alpha)\,\wp_t^* + \alpha \frac{\Pi_t^{ind}}{\Pi_t} \wp_{s,t-1}$$

$$(1+\varphi) = \wp_{k,t} + \varphi \wp_{s,t}$$

$$\left(\wp_{k,t}\right)^{-\frac{1}{(\mathfrak{m}-1)}} = (1-\alpha)\left(\wp_t^*\right)^{-\frac{1}{(\mathfrak{m}-1)}} + \alpha \left(\frac{\Pi_t^{ind}}{\Pi_t}\right)^{-\frac{1}{(\mathfrak{m}-1)}} \left(\wp_{k,t-1}\right)^{-\frac{1}{(\mathfrak{m}-1)}}$$

$$\wp_t^* = \varphi\,(\mathfrak{m}-1)\left(\wp_t^*\right)^{\left(1+\frac{\mathfrak{m}}{(\mathfrak{m}-1)}\right)} \frac{N_{1,t}}{D_t} + \frac{\mathfrak{m}}{\mu}\frac{N_{2,t}}{D_t}$$

$$D_t = \left(\wp_{k,t}\right)^{\frac{\mathfrak{m}}{(\mathfrak{m}-1)}} + \alpha E_t q_{t+1} \mathcal{G}_{t+1} \Pi_{t+1} \left(\frac{\Pi_{t+1}}{\Pi_{t+1}^{ind}}\right)^{\frac{1}{(\mathfrak{m}-1)}} D_{t+1}$$

$$N_{1,t} = 1 + \alpha E_t q_{t+1} \mathcal{G}_{t+1} \Pi_{t+1}^{ind} N_{1,t+1}$$

$$N_{2,t} = \mu E_t \sum_{j=0}^{\infty} q_{t,t+j} \alpha^j \mathcal{G}_{t,t+j} \Pi_{t,t+j} \left(\frac{\Pi_{t,t+j}}{\Pi_{t,t+j}^{ind}}\right)^{\frac{\mathfrak{m}}{(\mathfrak{m}-1)}} \left(\wp_{k,t+j}\right)^{\frac{\mathfrak{m}}{(\mathfrak{m}-1)}} mc_{t,t+j}^*$$

$$mc_{t,t+j}^* = \frac{1}{\mu}\left(X_{t+j}\right)^{(\sigma+\omega)} \left[\frac{1}{(1+\varphi)}\left(\frac{\Pi_{t,t+j}^{ind}}{\Pi_{t,t+j}} \frac{\wp_t^*}{\wp_{k,t+j}}\right)^{-\frac{\mathfrak{m}}{(\mathfrak{m}-1)}} + \frac{\varphi}{(1+\varphi)}\right]^{\omega}$$

Since power ω in the equation for $mc_{t,t+j}^*$ is not a positive integer, we cannot write $N_{2,t}$ in a finite recursive way. Therefore, simulations are to be carried out using the same approximation we use for the price wedge model.

B.2 Steady state

Given an exogenous level of trend inflation $\bar{\Pi}$, the steady state levels can be numerically obtained as follows. First, we compute $\bar{I}$, $\bar{q}$, and $\bar{Y}^n$:

$$\bar{I} = \frac{\bar{\Pi}}{\beta} \quad ; \; \bar{q} = \frac{\beta}{\bar{\Pi}} \quad ; \; \left(\bar{Y}^n\right)^{(\sigma+\omega)} = \frac{1}{\mu}\frac{\varepsilon}{\chi}\bar{\epsilon}\left(\overline{\mathcal{A}}\right)^{(1+\omega)}$$

Next, we use a numerical code to solve the following non-linear system for relative prices $\bar{\wp}^*$, $\bar{\wp}_s$, and $\bar{\wp}_k$:

$$\bar{\wp}^* = \frac{(1+\varphi)}{\left[\left(\frac{1-\alpha}{1-\bar{\alpha}_{k1}}\right)^{-(\mathfrak{m}-1)} + \varphi \frac{(1-\alpha)}{(1-\bar{\alpha}_{k3})}\right]} \quad ; \; \bar{\wp}_s = \frac{(1-\alpha)}{(1-\bar{\alpha}_{k3})}\bar{\wp}^* \quad ; \; \bar{\wp}_k = (1+\varphi) - \varphi\bar{\wp}_s$$

where

$$\bar{\alpha}_{k1} = \alpha\left(\bar{\Pi}\right)^{\frac{(1-\gamma)}{(\mathfrak{m}-1)}} \quad ; \; \bar{\alpha}_{k2} = \alpha\bar{\Pi}^{\frac{\mathfrak{m}(1-\gamma)}{(\mathfrak{m}-1)}} \quad ; \; \bar{\alpha}_{k3} = \alpha\bar{\Pi}^{-(1-\gamma)}$$

The following step is to find the gross output gap $\bar{X}$:

$$\bar{S}_d \equiv \sum_{j=0}^{\infty}\left(\bar{\alpha}_{k2}\beta\right)^j \left[\frac{1}{(1+\varphi)}\left(\frac{\bar{\alpha}_{k2}}{\alpha}\right)^j \left(\frac{\bar{\wp}^*}{\bar{\wp}_k}\right)^{-\frac{\mathfrak{m}}{(\mathfrak{m}-1)}} + \frac{\varphi}{(1+\varphi)}\right]^{\omega} \quad ; \; \bar{D} = \frac{\left(\bar{\wp}_k\right)^{\frac{\mathfrak{m}}{(\mathfrak{m}-1)}}}{(1-\bar{\alpha}_{k1}\beta)} \quad ; \; \bar{N}_1 = \frac{1}{(1-\bar{\alpha}_{k3}\beta)}$$

$$\bar{N}_2 = \frac{\mu}{\mathfrak{m}}\left(\bar{\wp}^*\right)\left(\bar{D} - \varphi\,(\mathfrak{m}-1)\left(\bar{\wp}^*\right)^{\frac{\mathfrak{m}}{(\mathfrak{m}-1)}}\bar{N}_1\right) \quad ; \; \left(\bar{X}\right)^{(\sigma+\omega)} = \frac{\bar{N}_2}{\left(\bar{\wp}_k\right)^{\frac{\mathfrak{m}}{(\mathfrak{m}-1)}}}\frac{1}{\bar{S}_d} \quad ; \; \bar{Y} = \bar{X}\bar{Y}^n$$

If $\frac{\bar{\alpha}_{k2}}{\alpha} \leq 1$ the infinite sum $\bar{S}_d$ converges when $\left(\bar{\alpha}_{k2}\beta\right) < 1$. If $\frac{\bar{\alpha}_{k2}}{\alpha} > 1$, it converges when $\left(\bar{\alpha}_{k2}\beta\right)\left(\frac{\bar{\alpha}_{k2}}{\alpha}\right)^{\omega} <$

1.

The infinite sum $\bar{S}_d$ is generally numerically retrieved by using a finite sum in $j = \{0, 1, ..., J\}$ for a large J. In this paper, we use $J = 10000$. For numerical stability when $\frac{\bar{\alpha}_{k2}}{\alpha} > 1$, $\bar{S}_d$ is better computed using $\bar{S}_d = \sum_{j=0}^{\infty} \left(\bar{\alpha}_{k2}\beta\left(\frac{\bar{\alpha}_{k2}}{\alpha}\right)^{\omega}\right)^j \left[\frac{1}{(1+\varphi)}\left(\frac{\bar{\wp}^*}{\bar{\wp}_k}\right)^{-\frac{m}{(m-1)}} + \frac{\varphi}{(1+\varphi)\left(\frac{\bar{\alpha}_{k2}}{\alpha}\right)^j}\right]^{\omega}$.

Alternatively, if ω is a positive integer, it is feasible to derive an exact closed form solution for $\bar{S}_d$. For that, we only need to expand the term in brackets and obtain a couple of infinite sums that allow for closed form solutions. For instance, if $\omega = 2$, $\frac{(\bar{\alpha}_{k2})^2}{\alpha^2}\beta < 1$ and $\bar{\alpha}_{k2}\beta < 1$, we obtain:

$$
\begin{aligned}
S \equiv & \sum_{j=0}^{\infty} (\bar{\alpha}_{k2}\beta)^j \left[\frac{1}{(1+\varphi)}\left(\frac{\bar{\alpha}_{k2}}{\alpha}\right)^j \left(\frac{\bar{\wp}^*}{\bar{\wp}_k}\right)^{-\frac{m}{(m-1)}} + \frac{\varphi}{(1+\varphi)}\right]^2 \\
= & \sum_{j=0}^{\infty} (\bar{\alpha}_{k2}\beta)^j \left[\frac{1}{(1+\varphi)^2}\left(\frac{\bar{\wp}^*}{\bar{\wp}_k}\right)^{-\frac{2m}{(m-1)}} \left(\left(\frac{\bar{\alpha}_{k2}}{\alpha}\right)^2\right)^j + 2\frac{1}{(1+\varphi)}\left(\frac{\bar{\wp}^*}{\bar{\wp}_k}\right)^{-\frac{m}{(m-1)}}\left(\frac{\bar{\alpha}_{k2}}{\alpha}\right)^j + \frac{\varphi^2}{(1+\varphi)^2}\right] \\
= & \frac{(\bar{\wp}^*/\bar{\wp}_k)^{-\frac{2m}{(m-1)}}}{(1+\varphi)^2} \sum_{j=0}^{\infty}\left(\frac{(\bar{\alpha}_{k2})^2}{\alpha^2}\beta\right)^j + \frac{2(\bar{\wp}^*/\bar{\wp}_k)^{-\frac{m}{(m-1)}}}{(1+\varphi)} \sum_{j=0}^{\infty}\left(\frac{(\bar{\alpha}_{k2})^2}{\alpha}\beta\right)^j + \frac{\varphi^2}{(1+\varphi)^2}\sum_{j=0}^{\infty}(\bar{\alpha}_{k2}\beta)^j \\
= & \frac{1}{(1+\varphi)^2}\frac{(\bar{\wp}^*/\bar{\wp}_k)^{-\frac{2m}{(m-1)}}}{\left(1-\frac{(\bar{\alpha}_{k2})^2}{\alpha^2}\beta\right)} + \frac{2}{(1+\varphi)}\frac{(\bar{\wp}^*/\bar{\wp}_k)^{-\frac{m}{(m-1)}}}{\left(1-\frac{(\bar{\alpha}_{k2})^2}{\alpha}\beta\right)} + \frac{\varphi^2}{(1+\varphi)^2}\frac{1}{(1-\bar{\alpha}_{k2}\beta)}
\end{aligned}
$$

B.3 Constrained Demand

Given the extra demand kink at $\left(\frac{p_t(z)}{P_{k,t}}\right) = (-\varphi)^{\frac{1}{\omega}}$, this particular case of Kimball's aggregation implies firms will typically not set any price $p_t(z)$ larger than $(-\varphi)^{\frac{1}{\omega}}[(1+\varphi)P_t - \varphi P_{s,t}]$, as would lead to zero demand. If $\varphi = 0$, in particular, the threshold $(-\varphi)^{\frac{1}{\omega}}$ is infinity. And so the restriction $\frac{c_t(z)}{C_t} \geq 0$ is never biding under Dixit and Stiglitz (1977) aggregation. If $\varphi < 0$, however, we argue that the condition for non-zero demand $\left(\frac{p_t(z)}{P_{k,t}}\right) < (-\varphi)^{\frac{1}{\omega}}$ might not always hold with real data. That is, an empirical test for this type of aggregation is to verify whether empirical values of relative prices $\frac{p_t(z)}{P_{k,t}}$ are smaller than $(-\varphi)^{\frac{1}{\omega}}$.

Here, as it is not the main scope of this paper, we do not propose a sophisticated formal econometric test. Rather, we propose a simple approach. So, the question is whether we can find a way to compute P_t and $P_{s,t}$ using typical moments from price samples, which would provide us with an estimate for $P_{k,t} \equiv [(1+\varphi)P_t - \varphi P_{s,t}]$. And here lies a slight caveat. While $P_{s,t} \equiv \int_0^1 p_t(z)\,dz$ is the simple average price, which can be easily estimated using sample price means, P_t has no obvious empirical counterpart. Therefore, P_t is not easily empirically retrievable without relying on a general equilibrium model.

In order to tackle this issue, we propose a second-order approximation approach. Consider the typical price aggregation into P_t, abstracting from the relative price threshold. It can be written as

$(P_{k,t})^{-\frac{1}{(\mathfrak{m}-1)}} = \int_0^1 (p_t(z))^{-\frac{1}{(\mathfrak{m}-1)}} dz$. Note that a second-order approximation of $(p_t(z))^{-\frac{1}{(\mathfrak{m}-1)}}$, about the average price $P_{s,t} \equiv \int_0^1 p_t(z)\, dz$, is:

$$(p_t(z))^{-\frac{1}{(\mathfrak{m}-1)}} \approx (P_{s,t})^{-\frac{1}{(\mathfrak{m}-1)}} - \frac{(P_{s,t})^{-\frac{\mathfrak{m}}{(\mathfrak{m}-1)}}}{(\mathfrak{m}-1)} (p_t(z) - P_{s,t}) + \frac{1}{2} \frac{\mathfrak{m}(P_{s,t})^{-\frac{2\mathfrak{m}-1}{(\mathfrak{m}-1)}}}{(\mathfrak{m}-1)^2} (p_t(z) - P_{s,t})^2$$

It implies that

$$(P_{k,t})^{-\frac{1}{(\mathfrak{m}-1)}} = \int_0^1 (p_t(z))^{-\frac{1}{(\mathfrak{m}-1)}} dz \approx (P_{s,t})^{-\frac{1}{(\mathfrak{m}-1)}} \left[1 + \frac{1}{2} \frac{\mathfrak{m}}{(\mathfrak{m}-1)^2} \int_0^1 \left[\frac{p_t(z)}{P_{s,t}} - P_{s,t}\right]^2 dz\right]$$

Therefore, we obtain the following relation between $P_{k,t}$ and $P_{s,t}$:

$$\begin{aligned} P_{k,t} \approx & \left[1 + \frac{1}{2} \frac{\mathfrak{m}}{(\mathfrak{m}-1)^2} s_{s,t}^2\right]^{-(\mathfrak{m}-1)} P_{s,t} \\ s_{s,t}^2 \equiv & \int_0^1 \left[\frac{p_t(z)}{P_{s,t}} - 1\right]^2 dz \end{aligned} \tag{B.3}$$

where $P_{k,t} \equiv [(1+\varphi) P_t - \varphi P_{s,t}]$. Since $P_{s,t} \equiv \int_0^1 p_t(z)\, dz$ and $\int_0^1 \frac{p_t(z)}{P_{s,t}} dz = 1$, $s_{s,t}$ is the cross-section standard deviation of relative prices $\frac{p_t(z)}{P_{s,t}}$, which is a measure of relative price dispersion.

Recalling that $\mathfrak{m} \equiv \frac{\mu_k(1+\varphi)}{(1+\mu_k\varphi)}$, it is not hard to verify that: (i) $\frac{p_t(z)}{P_{s,t}} < \frac{p_t(z)}{P_{k,t}} < \frac{p_t(z)}{P_t}$, if $\varphi \in (-1, 0]$; and (ii) $\frac{p_t(z)}{P_{k,t}} \leq \frac{p_t(z)}{P_{s,t}} < \frac{p_t(z)}{P_t}$, if $\varphi \leq -1$. In both cases, all three relative prices are very close to each other whenever price dispersion is $s_{s,t}$ small.

Since the cross section average relative price is unity, i.e. $\int_0^1 \frac{p_t(z)}{P_{s,t}} dz = 1$, we can reasonable conclude that $\int_0^1 \frac{p_t(z)}{P_{k,t}} dz$ is also close to unity. And there lies a potential empirical issue with this type Kimball's demand function. Recall that its relative price constraint for non-zero demand is $\left(\frac{p_t(z)}{P_{k,t}}\right) \leq (-\varphi)^{\frac{1}{\varpi}}$, for $\varpi \equiv \frac{\mu_k(1+\varphi)}{(1-\mu_k)}$.

In the literature, the upper limit $(-\varphi)^{\frac{1}{\varpi}}$ for relative prices is generally very close to unity when φ is set, or implied, using common values estimated or calibrated for the US. In order to verify this property, consider first that μ_k matches the implicit markup rate μ under flexible prices. In this case, some typical calibrations for the US are the following ones: (i) $\mu = 1.10$, $\varphi = -12.2$ and $(-\varphi)^{\frac{1}{\varpi}} = 1.021$ in Harding et al. (2022); (ii) $\mu = 1.17$, $\varphi = -8$ and $(-\varphi)^{\frac{1}{\varpi}} = 1.043$ in Levin et al. (2007);[26] and (iii) $\mu = 1.61$ (estimated), $\varphi = -3.79$ and $(-\varphi)^{\frac{1}{\varpi}} = 1.198$ in Smets and Wouters (2007).[27] In addition, obtaining a better marginal likelihood statistics for model comparison,[28] Harding et al. (2022) re-estimate

[26] In Levin et al. (2007), the elasticity of substitution between goods ϵ can be mapped into our notation as $\mu = \frac{\epsilon}{(\epsilon-1)}$. The authors calibrated $\epsilon = 7$, and so $\mu = \frac{7}{6}$.

[27] In Smets and Wouters (2007), the demand's curvature parameter ϵ_p can be mapped into our notation as $\epsilon_p = -\frac{\mu\varphi}{(\mu-1)}$. The authors calibrated $\epsilon_p = 10$ and estimated the gross markup rate at $\mu = 1.61$.

[28] The authors obtain a marginal likelihood gain of 5 log points.

Smets and Wouters (2007) model with a different prior distribution. Their new posterior modes imply $\mu = 1.34$ (estimated), $\varphi = -16.37$ and $(-\varphi)^{\frac{1}{\omega}} = 1.047$.

As for the empirical dispersion of relative prices, we consider the Kaplan and Menzio (2015) results described in Section 2. In particular, we make a conservative choice by considering the authors' Brand Aggregation, in which products have at least the same features and the same size, and so are in line with what economists usually think about commodity goods. Under this aggregation, the authors find that the empirical standard-deviation of relative prices, relatively to the sample average price $P_{a,t}$, is 0.25.[29] Notice that, under this type of Kimball aggregation, $P_s = P_a$. As depicted in Section 2, the authors' findings imply that a 80% confidence interval for empirical relative prices in the US are at least ranging from $\left(\frac{p(z)}{P_s}\right)_{0.10} = 0.68$ to $\left(\frac{p(z)}{P_s}\right)_{0.90} = 1.38$.

Using approximation (*B*.3), with standard deviation $s_s = 0.25$, and considering the authors' different calibration options for μ and φ, we are able to compute the implied 80% confidence intervals for $\left(\frac{p(z)}{P_k}\right)$ as follows:

$$\left(\frac{p(z)}{P_k}\right)_{0.10} = \left(\frac{P_s}{P_k}\right)\left(\frac{p(z)}{P_s}\right)_{0.10} \quad ; \quad \left(\frac{p(z)}{P_k}\right)_{0.90} = \left(\frac{P_s}{P_k}\right)\left(\frac{p(z)}{P_s}\right)_{0.90}$$

Therefore, considering different calibration options for μ and φ, Table 1 verifies whether the implied 80% confidence intervals for $\left(\frac{p(z)}{P_k}\right)$ are at least totally included in the feasibility region $\left(\frac{p(z)}{P_k}\right) \leq (-\varphi)^{\frac{1}{\omega}}$. Of course, this back-of-the-envelope analysis is by no means meant to be a formal hypothesis test, but the fact that all 90% quantiles surpass the theoretical Kimball's upper limit $(-\varphi)^{\frac{1}{\omega}}$ strongly suggests that an important fraction of relative prices are larger than the implied Kimball's upper limit for relative prices $(-\varphi)^{\frac{1}{\omega}}$. This conclusion is specially so for cases in which $(-\varphi)^{\frac{1}{\omega}}$ is very close to unity. This result is in line with simulations carried out by Klenow and Willis (2016), who find that about 15% of goods end up with zero relative demand when the demand function is Kimball-based with large curvature.

[29] Here, we are abstracting from frequency considerations the authors dealt with when using empirical data.

Table 1: Kimball's Relative Prices - Empirical confidence intervals and Kimball's upper limit

Authors	μ	φ	$\left(\frac{P_s}{P_k}\right)$	$(-\varphi)^{\frac{1}{\varpi}}$	$\left(\frac{p(z)}{P_k}\right)_{0.90}$
Harding et al. (2022)[a]	1.10	−12.2	0.95	1.02	1.31
Harding et al. (2022)[b]	1.34	−16.4	0.93	1.05	1.28
Levin et al. (2007)	1.17	−8.0	0.92	1.04	1.27
Smets and Wouters (2007)	1.61	−3.8	0.88	1.20	1.21

Notes: The empirical relative price 90% quantile is computed using Equation (B.3) and Kaplan and Menzio (2015) estimates. Kimball's relative price upper bound is $(-\varphi)^{\frac{1}{\varpi}}$. Harding et al. (2022) first (a) calibrates $\mu = 1.10$ and $\varphi = -12.2$; and then (b) estimates $\mu = 1.34$ and $\varphi = -16.4$ using Smets and Wouters (2007) model with a different prior distribution. Again, ϖ is defined as $\varpi \equiv \frac{\mu_k(1+\varphi)}{(1-\mu_k)}$, and $\mu_k = \mu$.

C Price Wedge Model

The household pays $(p_t(z) + \delta P_t)$ for each unit of good z. The aggregate price definition is $(1+\delta) P_t C_t \equiv \int_0^1 (p_t(z) + \delta P_t) c_t(z) dz$. Firm z revenue is $(p_t(z) + \delta P_t) y_t(z)$.

C.1 General Equilibrium - Dixit-Stiglitz with Price Wedges

The composite parameters are:

$$\omega \equiv \frac{(1+\nu)}{\varepsilon} - 1 \quad ; \theta_1 \equiv \theta(1+\omega) \quad ; \mu_k \equiv \frac{\theta}{(\theta-1)} \quad ; \mu = \frac{\mu_k}{(1+\delta)}$$

The dynamic equations are:

$$\begin{aligned}
1 &= \beta E_t \left(\frac{\epsilon_{t+1}}{\epsilon_t} \left(\frac{Y_t}{Y_{t+1}} \right)^{\sigma} \frac{I_t}{\Pi_{t+1}} \right) \\
q_t &= \beta \frac{\epsilon_t}{\epsilon_{t-1}} \left(\frac{Y_{t-1}}{Y_t} \right)^{\sigma} \frac{1}{\Pi_t} \\
\left(\frac{I_t}{\bar{I}} \right) &= \epsilon_{\mathrm{i},t} \left(\frac{I_{t-1}}{\bar{I}} \right)^{\phi_{\mathrm{i}}} \left[\left(\frac{E_t \Pi_{t+1}}{\bar{\Pi}} \right)^{\phi_{\pi f}} \left(\frac{X_t}{X_{t-1}} \right)^{\phi_{gx}} \right]^{(1-\phi_{\mathrm{i}})} \\
(Y_t^n)^{(\sigma+\omega)} &= \frac{1}{(1+\delta)\mu} \frac{\varepsilon}{\chi} \epsilon_t (\mathcal{A}_t)^{(1+\omega)}
\end{aligned}$$

$$\begin{aligned}
X_t &= \frac{Y_t}{Y_t^n} &; \quad q_{t,t+j} &= q_{t+1} q_{t+1,t+j} && \text{for } j \geq 1 \text{ and } q_{t,t} = 1 \\
\mathcal{G}_t &= \frac{Y_t}{Y_{t-1}} &; \quad \Pi_{t,t+j} &= \Pi_{t+1} \Pi_{t+1,t+j} && \text{for } j \geq 1 \text{ and } \Pi_{t,t} = 1 \\
\Pi_t^{ind} &= \Pi_{t-1}^{\gamma} &; \quad \Pi_{t,t+j}^{ind} &= \Pi_{t+1}^{ind} \Pi_{t+1,t+j}^{ind} && \text{for } j \geq 1 \text{ and } \Pi_{t,t}^{ind} = 1 \\
& &; \quad \mathcal{G}_{t,t+j} &= \mathcal{G}_{t+1} \mathcal{G}_{t+1,t+j} && \text{for } j \geq 1 \text{ and } \mathcal{G}_{t,t} = 1
\end{aligned}$$

$$1 = \quad (1-\alpha)\sum_{j=0}^{\infty}\alpha^j\left(\frac{z^*_{t-j,t}}{1+\delta}\right)^{-(\theta-1)}$$

$$(\wp_{\delta,t})^{-\theta} = \quad (1-\alpha)\sum_{j=0}^{\infty}\alpha^j\left(\frac{\Pi^{ind}_{t-j,t}}{\Pi_{t-j,t}}\wp^*_{t-j}+\delta\wp_{s,t}\right)^{-\theta}$$

$$1 = \quad \mu_k\frac{\mathcal{N}_{1,t}}{\mathcal{D}_{1,t}}$$

$$\mathcal{N}_{1,t} = \quad \frac{1}{\mu}E_t\sum_{j=0}^{\infty}\alpha^j q_{t,t+j}\Pi^{ind}_{t,t+j}\mathcal{G}_{t,t+j}\left(\frac{z^*_{t,t+j}}{1+\delta}\right)^{-\left(1+\bar{\theta}_1\right)}\frac{\left(X_{t+j}\right)^{(\sigma+\omega)}}{(1+\delta)}$$

$$\mathcal{D}_{1,t} \equiv \quad E_t\sum_{j=0}^{\infty}\alpha^j q_{t,t+j}\Pi^{ind}_{t,t+j}\mathcal{G}_{t,t+j}\left(\frac{z^*_{t,t+j}}{1+\delta}\right)^{-\theta}$$

$$z^*_{t,t+j} = \quad \frac{\Pi^{ind}_{t,t+j}}{\Pi_{t,t+j}}\wp^*_t+\delta$$

$$z^*_{t-j,t} = \quad \frac{\Pi^{ind}_{t-j,t}}{\Pi_{t-j,t}}\wp^*_{t-j}+\delta$$

As for the remaining aggregates and welfare measures, they are:

$$\mathcal{W}_t = u_t - v_t \qquad ;\ u_t = \epsilon_t\frac{(Y_t)^{(1-\sigma)}-1}{(1-\sigma)}$$

$$h_t = \left(\mathfrak{d}_{y,t}\frac{Y_t}{\mathcal{A}_t}\right)^{\frac{1}{\varepsilon}} \qquad ;\ v_t = \frac{\chi}{(1+\nu)}\mathfrak{d}_{v,t}\left(h_t\right)^{(1+\nu)}$$

$$\mathfrak{d}_{y,t} = (1+\delta)^{\theta}\left(\Lambda_{y,t}\right)^{\varepsilon} \qquad ;\ \mathfrak{d}_{v,t} = \frac{\Lambda_t}{\left(\Lambda_{y,t}\right)^{(1+\nu)}}$$

$$\Lambda_{y,t} = (1-\alpha)\sum_{j=0}^{\infty}\alpha^j\left(z^*_{t-j,t}\right)^{-\frac{\theta}{\varepsilon}} \qquad ;\ \Lambda_t = (1-\alpha)\sum_{j=0}^{\infty}\alpha^j\left(z^*_{t-j,t}\right)^{-\theta_1}$$

$$\mathcal{W}^n_t = u^n_t - v^n_t \qquad ;\ u^n_t = \epsilon_t\frac{(Y^n_t)^{(1-\sigma)}-1}{(1-\sigma)}$$

$$h^n_t = \left(\frac{Y^n_t}{\mathcal{A}_t}\right)^{\frac{1}{\varepsilon}} \qquad ;\ v^n_t = \frac{\chi}{(1+\nu)}\left(h^n_t\right)^{(1+\nu)}$$

$$\mathcal{W}_t = \epsilon_t\frac{\left(Y^{eq}_t\right)^{(1-\sigma)}-1}{(1-\sigma)} - \frac{\chi}{(1+\nu)}\left(\frac{Y^{eq}_t}{\mathcal{A}_t}\right)^{(1+\omega)} \qquad ;\ X^{eq}_t \equiv \frac{Y^{eq}_t}{Y^n_t}$$

C.2 Steady State

For any variable χ_t, its steady state level is defined as $\bar{\chi}$. The steady state equilibrium can be numerically obtained as follows. First, we compute $\bar{I}$, $\bar{q}$, and $\bar{Y}^n$:

$$\bar{I} = \frac{\bar{\Pi}}{\beta} \quad ;\ \bar{q} = \frac{\beta}{\bar{\Pi}} \quad ;\ (\bar{Y}^n)^{(\sigma+\omega)} = \frac{1}{(1+\delta)\mu}\frac{\varepsilon}{\chi}\bar{\epsilon}\left(\overline{\mathcal{A}}\right)^{(1+\omega)}$$

Next, we use a numerical code to solve the following non-linear system for the resetting relative price $\bar{\wp}^*$, in which the infinite sum is retrieved by using finite sums in $j = \{0, 1, ..., J\}$ for a large J. In particular, we consider $J = 10000$:

$$1 = (1+\delta)^{(\theta-1)} (1-\alpha) \sum_{j=0}^{\infty} \alpha^j \left(\frac{\bar{\wp}^*}{\bar{\Pi}^{(1-\gamma)j}} + \delta \right)^{-(\theta-1)}$$

After computing the relative prices, we pin down the following composite parameters:

$$\Sigma_{N1} \equiv \sum_{j=0}^{\infty} (\bar{\alpha}_1 \beta)^j \left(\bar{z}_j^* \right)^{-(1+\bar{\theta}_1)} \qquad \Sigma_{D1} \equiv \sum_{j=0}^{\infty} (\bar{\alpha}_1 \beta)^j \left(\bar{z}_j^* \right)^{-\theta}$$

$$\Sigma_y \equiv \textstyle\sum_{j=0}^{\infty} \alpha^j \left(\bar{z}_j^* \right)^{-\frac{\theta}{\varepsilon}} \qquad \Sigma_\Lambda \equiv \textstyle\sum_{j=0}^{\infty} \alpha^j \left(\bar{z}_j^* \right)^{-\theta_1}$$

where $\bar{z}_j^* = \left(\frac{\bar{\wp}^*}{(\bar{\Pi})^{(1-\gamma)j}} + \delta \right)$, $\bar{\alpha}_1 \equiv \frac{\alpha}{\bar{\Pi}^{(1-\gamma)}}$ and $\bar{\alpha}_2 \equiv \frac{\alpha}{\bar{\Pi}^{2(1-\gamma)}}$.

Since the price wedge demand function satisfies the conditions of Theorem 1, we know that the infinite sums converge. Therefore, we can retrieve them numerically, by considering finite sums up to a very large horizon J, i.e. $j \in \{0, 1, ..., J\}$. Again, we use $J = 10000$. For avoiding numerical issues arising from dealing with very large numbers when $\delta > 0$, we proceed as follows. First, for each infinite sum of the form $\Sigma_\varphi \equiv \sum_{j=0}^{\infty} (ß)^j \left(\bar{z}_j^* \right)^{-\varphi}$, where ß<1, we define its normalized peer $\tilde{\Sigma}_\varphi \equiv \sum_{j=0}^{\infty} (ß)^j \left(\tilde{z}_j^* \right)^{-\varphi}$, where $\tilde{z}_j^* \equiv \frac{\bar{z}_j^*}{\delta \bar{\wp}_s} = \left(1 + \frac{1}{(\bar{\Pi})^{(1-\gamma)j}} \frac{\bar{\wp}^*}{\delta} \right)$. Therefore, whenever $\delta > 0$, we can accurately approximate $\tilde{\Sigma}_\varphi$ using $\sum_{j=0}^{J} (ß)^j \left(\tilde{z}_j^* \right)^{-\varphi}$. After retrieving $\tilde{\Sigma}_\varphi$, we compute $\Sigma_\varphi = (\delta)^{-\varphi} \tilde{\Sigma}_\varphi$.

After pinning down the gross output gap $\bar{X} = \left[(1+\delta)^{-\theta\omega} \frac{\mu}{\mu_k} \frac{\Sigma_{D1}}{\Sigma_{N1}} \right]^{\frac{1}{(\sigma+\omega)}}$, we compute the aggregate output $\bar{Y} = \bar{X} \bar{Y}^n$. As for the remaining aggregates and welfare measures, they are:

$$\bar{\mathcal{W}}^n = \bar{u}^n - \bar{v}^n \qquad \bar{\mathcal{W}} = \bar{u} - \bar{v} \qquad ;\ \bar{\Lambda} = (1-\alpha) \Sigma_\Lambda$$

$$\bar{h}^n = \left(\frac{\bar{Y}^n}{\mathcal{A}} \right)^{\frac{1}{\varepsilon}} \qquad \bar{h} = \left(\eth_y \frac{\bar{Y}}{\mathcal{A}} \right)^{\frac{1}{\varepsilon}} \qquad ;\ \bar{\Lambda}_y = (1-\alpha) \Sigma_y$$

$$\bar{u}^n = \bar{\epsilon} \frac{(\bar{Y}^n)^{(1-\sigma)} - 1}{(1-\sigma)} \qquad ;\ \bar{u} = \bar{\epsilon} \frac{(\bar{Y})^{(1-\sigma)} - 1}{(1-\sigma)} \qquad ;\ \eth_y = (1+\delta)^\theta \left(\bar{\Lambda}_y \right)^\varepsilon$$

$$\bar{v}^n = \frac{\chi}{(1+\nu)} \left(\bar{h}^n \right)^{(1+\nu)} \qquad ;\ \bar{v} = \frac{\chi}{(1+\nu)} \eth_v \left(\bar{h} \right)^{(1+\nu)} \qquad ;\ \eth_v = \frac{\bar{\Lambda}}{\left(\bar{\Lambda}_y \right)^{(1+\nu)}}$$

As for the consumption-equivalent welfare metrics, we use numerical methods to solve the following non-linear equation:

$$\bar{\epsilon} \frac{(\bar{Y}^{eq})^{(1-\sigma)} - 1}{(1-\sigma)} - \frac{\chi}{(1+\nu)} \left(\frac{\bar{Y}^{eq}}{\mathcal{A}} \right)^{(1+\omega)} = \bar{\mathcal{W}}$$

After that, we compute $\bar{X}^{eq} = \frac{\bar{Y}^{eq}}{\bar{Y}^{n}}$ as the consumption-equivalent output gap.

In the particular case of $(\bar{\Pi})^{(1-\gamma)} = 1$, it is possible to obtain a closed form solution:

$$\bar{\alpha}_1 = \bar{\alpha}_2 = \alpha \quad ; \bar{\wp}^* = \bar{X} = 1 \quad ; \bar{z}^*_j = (1+\delta)$$

The remaining steady state levels, when $(\bar{\Pi})^{(1-\gamma)} = 1$, are then easily retrieved using the same relations previously detailed.

C.3 Log-Linearized Model

For computing the model loglinearized equilibrium, we also need the augment the set of composite parameters:

$$\begin{aligned} &\omega \equiv \frac{(1+\nu)}{\varepsilon} - 1 \quad ; \mu_k \equiv \frac{\theta}{(\theta-1)} \quad ; \bar{\alpha}_1 \equiv \frac{\alpha}{\bar{\Pi}^{(1-\gamma)}} \\ &\bar{\theta}_1 \equiv \theta\,(1+\omega) \quad ; \mu = \frac{\mu_k}{(1+\delta)} \quad ; \bar{\alpha}_2 \equiv \frac{\alpha}{\bar{\Pi}^{2(1-\gamma)}} \end{aligned}$$

Given an exogenous level of trend inflation $\bar{\Pi}$, we start by defining $\bar{z}^*_j \equiv \left(\frac{\bar{\wp}^*}{(\bar{\Pi})^{(1-\gamma)j}} + \delta\right)$ and the following composite parameters:

$$\begin{aligned} &\Sigma_{N1} \equiv \sum_{j=0}^{\infty} (\bar{\alpha}_1\beta)^j \left(\bar{z}^*_j\right)^{-\left(1+\bar{\theta}_1\right)} &&; \Sigma_{N2} \equiv \sum_{j=0}^{\infty} (\bar{\alpha}_2\beta)^j \left(\bar{z}^*_j\right)^{-\left(2+\bar{\theta}_1\right)} &&; \Sigma_{\wp} \equiv \sum_{j=0}^{\infty} \alpha^j \left(\bar{z}^*_j\right)^{-(\theta-1)} \\ &\Sigma_{D1} \equiv \sum_{j=0}^{\infty} (\bar{\alpha}_1\beta)^j \left(\bar{z}^*_j\right)^{-\theta} &&; \Sigma_{D2} \equiv \sum_{j=0}^{\infty} (\bar{\alpha}_2\beta)^j \left(\bar{z}^*_j\right)^{-(1+\theta)} &&; \Sigma_{c} \equiv \sum_{j=0}^{\infty} (\bar{\alpha}_1)^j \left(\bar{z}^*_j\right)^{-\theta} \\ &\Theta_{N1} \equiv (\bar{\wp}^* + \delta)^{-\left(1+\bar{\theta}_1\right)} &&; \Theta_{N2} \equiv (\bar{\wp}^* + \delta)^{-\left(2+\bar{\theta}_1\right)} &&; \Theta_{\wp} \equiv (\bar{\wp}^* + \delta)^{-(\theta-1)} \\ &\Theta_{D1} \equiv (\bar{\wp}^* + \delta)^{-\theta} &&; \Theta_{D2} \equiv (\bar{\wp}^* + \delta)^{-(1+\theta)} &&; \Theta_{c} \equiv (\bar{\wp}^* + \delta)^{-\theta} \end{aligned} \tag{C.4}$$

Since the price wedge demand function satisfies the conditions of Theorem 1, we know that the infinite sums converge. When $\delta > 0$, we proceed as in Appendix C.2 and accurately approximate $\tilde{\Sigma}_{\varphi}$ using $\sum_{j=0}^{J} (ß)^j \left(\tilde{z}^*_j\right)^{-\varphi}$, where $J = 10000$. After retrieving $\tilde{\Sigma}_{\varphi}$, we compute $\Sigma_{\varphi} = (\delta)^{-\varphi}\,\tilde{\Sigma}_{\varphi}$.

For $k \in \{0, 1, 2, ...\}$ and defining $\bar{\wp}^*_k \equiv \frac{\bar{\wp}^*}{\bar{\Pi}^{(1-\gamma)k}}$ and $\bar{z}^*_{j+k} = \frac{\bar{\wp}^*_k}{\bar{\Pi}^{j(1-\gamma)}} + \delta$, consider the following sequence of composite parameters:

$$\Sigma_{k,N1} \equiv \sum_{j=0}^{\infty} (\bar{\alpha}_1\beta)^j \left(\bar{z}^*_{j+k}\right)^{-\left(1+\bar{\theta}_1\right)} \quad ; \Sigma_{k,N2} \equiv \sum_{j=0}^{\infty} (\bar{\alpha}_2\beta)^j \left(\bar{z}^*_{j+k}\right)^{-\left(2+\bar{\theta}_1\right)} \quad ; \Sigma_{k,\wp} \equiv \sum_{j=0}^{\infty} \alpha^j \left(\bar{z}^*_{j+k}\right)^{-(\theta-1)}$$

$$\Sigma_{k,D1} \equiv \sum_{j=0}^{\infty} (\bar{\alpha}_1\beta)^j \left(\bar{z}^*_{j+k}\right)^{-\theta} \quad ; \Sigma_{k,D2} \equiv \sum_{j=0}^{\infty} (\bar{\alpha}_2\beta)^j \left(\bar{z}^*_{j+k}\right)^{-(1+\theta)} \quad ; \Sigma_{k,c} \equiv \sum_{j=0}^{\infty} (\bar{\alpha}_1)^j \left(\bar{z}^*_{j+k}\right)^{-\theta}$$

$$\Theta_{k,N1} \equiv \left(\bar{\wp}^*_k + \delta\right)^{-\left(1+\bar{\theta}_1\right)} \quad ; \Theta_{k,N2} \equiv \left(\bar{\wp}^*_k + \delta\right)^{-\left(2+\bar{\theta}_1\right)} \quad ; \Theta_{k,\wp} \equiv \left(\bar{\wp}^*_k + \delta\right)^{-(\theta-1)}$$

$$\Theta_{k,D1} \equiv \left(\bar{\wp}^*_k + \delta\right)^{-\theta} \quad ; \Theta_{k,D2} \equiv \left(\bar{\wp}^*_k + \delta\right)^{-(1+\theta)} \quad ; \Theta_{k,c} \equiv \left(\bar{\wp}^*_k + \delta\right)^{-\theta}$$

Consider now each k-infinite sum $\Sigma_{k,\varphi}$ and k-parameter $\Theta_{k,\varphi}$, for $\varphi \in \{N1, D1, N2, D2, \wp, c\}$. Note that $\Sigma_{0,\varphi} = \Sigma_\varphi$ and $\Theta_{0,\varphi} = \Theta_\varphi$. In this case, the k-infinite sums are easier computed using the following recursions for $k \geq 1$, derived in Appendix C.3.2:

$$\begin{array}{ll}
\Sigma_{k,N1} = \frac{1}{\bar{\alpha}_1\beta}\left[\Sigma_{(k-1),N1} - \Theta_{(k-1),N1}\right] & ; \Sigma_{k,N2} = \frac{1}{\bar{\alpha}_2\beta}\left[\Sigma_{(k-1),N2} - \Theta_{(k-1),N2}\right] \\
\Sigma_{k,D1} = \frac{1}{\bar{\alpha}_1\beta}\left[\Sigma_{(k-1),D1} - \Theta_{(k-1),D1}\right] & ; \Sigma_{k,D2} = \frac{1}{\bar{\alpha}_2\beta}\left[\Sigma_{(k-1),D2} - \Theta_{(k-1),D2}\right] \\
\Sigma_{k,\wp} = \frac{1}{\alpha}\left[\Sigma_{(k-1),\wp} - \Theta_{(k-1),\wp}\right] & ; \Sigma_{k,c} = \frac{1}{\bar{\alpha}_1}\left[\Sigma_{(k-1),c} - \Theta_{(k-1),c}\right]
\end{array}$$

The full log-linearized aggregate supply curve ultimately depends on the following ratios:

$$\begin{array}{lll}
\Omega_{k,N1} \equiv \frac{\Theta_{k,N1}}{\Sigma_{k,N1}} & ; \Omega_{k,N2} \equiv \frac{\Theta_{k,N2}}{\Sigma_{k,N2}} & ; \Omega_{k,\wp} \equiv \frac{\Theta_{k,\wp}}{\Sigma_{k,\wp}} \\
\Omega_{k,D1} \equiv \frac{\Theta_{k,D1}}{\Sigma_{k,D1}} & ; \Omega_{k,D2} \equiv \frac{\Theta_{k,D2}}{\Sigma_{k,D2}} & ; \Omega_{k,c} \equiv \frac{\Theta_{k,c}}{\Sigma_{k,c}}
\end{array}$$

Using the recursion equations for the k-infinite sums, we obtain:

$$\begin{array}{ll}
\Omega_{k,N1} = \bar{\alpha}_1\beta\frac{\Theta_{k,N1}}{\Theta_{(k-1),N1}}\frac{\Omega_{(k-1),N1}}{\left[1-\Omega_{(k-1),N1}\right]} & ; \Omega_{k,N2} = \bar{\alpha}_2\beta\frac{\Theta_{k,N2}}{\Theta_{(k-1),N2}}\frac{\Omega_{(k-1),N2}}{\left[1-\Omega_{(k-1),N2}\right]} \\
\Omega_{k,D1} = \bar{\alpha}_1\beta\frac{\Theta_{k,D1}}{\Theta_{(k-1),D1}}\frac{\Omega_{(k-1),D1}}{\left[1-\Omega_{(k-1),D1}\right]} & ; \Omega_{k,D2} = \bar{\alpha}_2\beta\frac{\Theta_{k,D2}}{\Theta_{(k-1),D2}}\frac{\Omega_{(k-1),D2}}{\left[1-\Omega_{(k-1),D2}\right]} \\
\Omega_{k,\wp} = \alpha\frac{\Theta_{k,\wp}}{\Theta_{(k-1),\wp}}\frac{\Omega_{(k-1),\wp}}{\left[1-\Omega_{(k-1),\wp}\right]} & ; \Omega_{k,c} = \bar{\alpha}_1\frac{\Theta_{k,c}}{\Theta_{(k-1),c}}\frac{\Omega_{(k-1),c}}{\left[1-\Omega_{(k-1),c}\right]}
\end{array} \tag{C.5}$$

In general, for any variable χ_t, its log-linearized version is defined as $\hat{\chi}_t \equiv \log\left(\frac{\chi_t}{\bar{\chi}}\right)$, keeping the same case as in the original variable, e.g. $\hat{Y}_t = \log\left(\frac{Y_t}{\bar{Y}}\right)$. For gross rates, though, we represent its loglinearized version in lower cases, e.g. $\hat{\pi}_t = \log\left(\frac{\Pi_t}{\bar{\Pi}}\right)$. Usual loglinearizations from the general part of the model, i.e. comprising equations independent of pricing structure, leads to the following system:

$$\begin{aligned}
\hat{Y}_t =& \quad E_t\hat{Y}_{t+1} - \tfrac{1}{\sigma}E_t\left[(\hat{\imath}_t - \hat{\pi}_{t+1}) + (\hat{\epsilon}_{t+1} - \hat{\epsilon}_t)\right] \\
\hat{q}_t =& \quad \sigma\left(\hat{Y}_{t-1} - \hat{Y}_t\right) - \hat{\pi}_t + (\hat{\epsilon}_t - \hat{\epsilon}_{t-1}) \\
\hat{\imath}_t =& \quad \phi_{\mathrm{i}}\hat{\imath}_{t-1} + (1-\phi_{\mathrm{i}})\left[\phi_{\pi f}E_t\hat{\pi}_{t+1} + \phi_{gx}\left(\hat{x}_t - \hat{x}_{t-1}\right)\right] + \hat{\epsilon}_{\mathrm{i},t} \\
(\sigma+\omega)\,\hat{Y}^n_t =& \quad \hat{\epsilon}_t + (1+\omega)\,\widehat{\mathcal{A}}_t \\
\hat{\pi}^{ind}_t =& \quad \gamma\hat{\pi}_{t-1} \quad ; \quad \hat{x}_t = \hat{Y}_t - \hat{Y}^n_t \quad ; \quad \widehat{\mathfrak{g}}_t = \hat{Y}_t - \hat{Y}_{t-1}
\end{aligned}$$

Note that $E_t\hat{q}_{t+1} = -\hat{\imath}_t$. Lastly, as we show in Appendix C.3.2, the price setting equations presented in systems (6) and (7) imply the following log-linearized system describing the full price-wedge NKPC (supply curve) under trend inflation:

$$\begin{aligned}
\Omega_{0,c}\hat{\wp}^*_t =& \quad (1-\Omega_{0,c})\left(\hat{\pi}_t - \hat{\pi}^{ind}_t\right) - (1-\Omega_{0,c})\,\widehat{\mathfrak{s}}_{1,c,t-1} \\
\left[\tfrac{\Sigma_{D1}}{\Sigma_{N1}} - \tfrac{\theta}{\left(1+\bar{\theta}_1\right)}\tfrac{\Sigma_{D2}}{\Sigma_{N2}}\right]\hat{\wp}^*_t =& \quad \tfrac{1}{\left(\bar{\wp}^*\right)\left(1+\bar{\theta}_1\right)}\tfrac{\Sigma_{D1}}{\Sigma_{N2}}\left[\widehat{\mathfrak{s}}_{0,N1,t} - \widehat{\mathfrak{s}}_{0,D1,t}\right] + \left[\tfrac{\Sigma_{D1}}{\Sigma_{N1}}\widehat{\mathfrak{s}}_{0,N2,t} - \tfrac{\theta}{\left(1+\bar{\theta}_1\right)}\tfrac{\Sigma_{D2}}{\Sigma_{N2}}\widehat{\mathfrak{s}}_{0,D2,t}\right]
\end{aligned} \tag{C.6}$$

where, for $k \in \{0, 1, 2, ...\}$, the k-ancillary variables $\widehat{\mathfrak{s}}_{k,c,t}$, $\widehat{\mathfrak{s}}_{k,N1,t}$, $\widehat{\mathfrak{s}}_{k,D1,t}$, $\widehat{\mathfrak{s}}_{k,N2,t}$, and $\widehat{\mathfrak{s}}_{k,D2,t}$ evolve according to k-dependent recursive equations:

$$\begin{aligned}
\widehat{\mathfrak{s}}_{k,c,t} =& \quad \Omega_{k,c}\hat{\wp}^*_t - (1-\Omega_{k,c})\left[\left(\hat{\pi}_t - \hat{\pi}^{ind}_t\right) - \widehat{\mathfrak{s}}_{(k+1),c,t-1}\right] \\
\widehat{\mathfrak{s}}_{k,N1,t} =& \quad \Omega_{k,N1}(\sigma+\omega)\,\hat{x}_t - (1-\Omega_{k,N1})\,E_t\left[\left(\hat{\pi}_{t+1} - \hat{\pi}^{ind}_{t+1}\right) + (\sigma-1)\,\widehat{\mathfrak{g}}_{t+1} - (\hat{\epsilon}_{t+1} - \hat{\epsilon}_t) - \widehat{\mathfrak{s}}_{(k+1),N1,t+1}\right] \\
\widehat{\mathfrak{s}}_{k,D1,t} =& \quad -(1-\Omega_{k,D1})\,E_t\left[\left(\hat{\pi}_{t+1} - \hat{\pi}^{ind}_{t+1}\right) + (\sigma-1)\,\widehat{\mathfrak{g}}_{t+1} - (\hat{\epsilon}_{t+1} - \hat{\epsilon}_t) - \widehat{\mathfrak{s}}_{(k+1),D1,t+1}\right] \\
\widehat{\mathfrak{s}}_{k,N2,t} =& \quad (1-\Omega_{k,N2})\left[E_t\left(\hat{\pi}_{t+1} - \hat{\pi}^{ind}_{t+1}\right) + \widehat{\mathfrak{s}}_{(k+1),N2,t+1}\right] \\
\widehat{\mathfrak{s}}_{k,D2,t} =& \quad (1-\Omega_{k,D2})\left[E_t\left(\hat{\pi}_{t+1} - \hat{\pi}^{ind}_{t+1}\right) + \widehat{\mathfrak{s}}_{(k+1),D2,t+1}\right]
\end{aligned} \tag{C.7}$$

In our simulations described in Section (6), we truncate the infinite recursive system of ancillary variables at $\bar{k} = 40$. With this approximation, we substitute $\widehat{\mathfrak{s}}_{40,c,t-1}$ for $\widehat{\mathfrak{s}}_{41,c,t-1}$, $E_t\widehat{\mathfrak{s}}_{40,N1,t+1}$ for $E_t\widehat{\mathfrak{s}}_{41,N1,t+1}$, $E_t\widehat{\mathfrak{s}}_{40,D1,t+1}$ for $E_t\widehat{\mathfrak{s}}_{41,D1,t+1}$, $E_t\widehat{\mathfrak{s}}_{40,N2,t+1}$ for $E_t\widehat{\mathfrak{s}}_{41,N2,t+1}$, and $E_t\widehat{\mathfrak{s}}_{40,N2,t+1}$ for $E_t\widehat{\mathfrak{s}}_{41,N2,t+1}$.

C.3.1 The approximated price-wedge NKPC under trend inflation

All infinite sums converge if $\delta > 0$. Therefore, each one of them, we can compute gross trend inflation equivalents $\bar{\Pi}_{N1}$, $\bar{\Pi}_{D1}$, $\bar{\Pi}_{N2}$, $\bar{\Pi}_{D2}$, $\bar{\Pi}_{\wp}$, and $\bar{\Pi}_c$ implicitly defined as follows:

$$\begin{aligned}
\Sigma_{N1} &= \tfrac{\Theta_{N1}}{(1-\bar{\alpha}_1\beta\bar{\Phi}_{N1})} \quad ; \Sigma_{N2} = \tfrac{\Theta_{N2}}{(1-\bar{\alpha}_2\beta\bar{\Phi}_{N2})} \quad ; \Sigma_{\wp} = \tfrac{\Theta_{\wp}}{(1-\alpha\bar{\Phi}_{\wp})} \\
\Sigma_{D1} &= \tfrac{\Theta_{D1}}{(1-\bar{\alpha}_1\beta\bar{\Phi}_{D1})} \quad ; \Sigma_{D2} = \tfrac{\Theta_{D2}}{(1-\bar{\alpha}_2\beta\bar{\Phi}_{D2})} \quad ; \Sigma_c = \tfrac{\Theta_c}{(1-\bar{\alpha}_1\bar{\Phi}_c)}
\end{aligned}$$

where

$$\bar{\Phi}_{N1} \equiv \left(\bar{\Pi}_{N1}\right)^{\left(1+\bar{\theta}_1\right)(1-\gamma)} \quad ; \bar{\Phi}_{N2} \equiv \left(\bar{\Pi}_{N2}\right)^{\left(2+\bar{\theta}_1\right)(1-\gamma)} \quad ; \bar{\Phi}_{\wp} \equiv \left(\bar{\Pi}_{\wp}\right)^{(\theta-1)(1-\gamma)}$$
$$\bar{\Phi}_{D1} \equiv \left(\bar{\Pi}_{D1}\right)^{\theta(1-\gamma)} \quad ; \bar{\Phi}_{D2} \equiv \left(\bar{\Pi}_{D2}\right)^{(1+\theta)(1-\gamma)} \quad ; \bar{\Phi}_{c} \equiv \left(\bar{\Pi}_{c}\right)^{\theta(1-\gamma)}$$

In addition, we can truncate he infinite recursive system short at $\bar{k} = 0$. With this approximation, we substitute $\widehat{\mathfrak{s}}_{0,c,t-1}$ for $\widehat{\mathfrak{s}}_{1,c,t-1}$, $E_t\widehat{\mathfrak{s}}_{0,N1,t+1}$ for $E_t\widehat{\mathfrak{s}}_{1,N1,t+1}$, $E_t\widehat{\mathfrak{s}}_{0,D1,t+1}$ for $E_t\widehat{\mathfrak{s}}_{1,D1,t+1}$, $E_t\widehat{\mathfrak{s}}_{0,N2,t+1}$ for $E_t\widehat{\mathfrak{s}}_{1,N2,t+1}$, and $E_t\widehat{\mathfrak{s}}_{0,N2,t+1}$ for $E_t\widehat{\mathfrak{s}}_{1,N2,t+1}$. Droping the index notation for $k = 0$, the approximated system is:

$$\begin{aligned}
\Omega_c \widehat{\wp}_t^* \approx& \quad (1-\Omega_c)\left(\hat{\pi}_t - \hat{\pi}_t^{ind}\right) - (1-\Omega_c)\,\widehat{\mathfrak{s}}_{c,t-1} \\
\left[\frac{\Sigma_{D1}}{\Sigma_{N1}} - \frac{\theta}{\left(1+\bar{\theta}_1\right)}\frac{\Sigma_{D2}}{\Sigma_{N2}}\right]\widehat{\wp}_t^* =& \quad \frac{1}{\left(\bar{\wp}^*\right)\left(1+\bar{\theta}_1\right)}\frac{\Sigma_{D1}}{\Sigma_{N2}}\left[\widehat{\mathfrak{s}}_{N1,t} - \widehat{\mathfrak{s}}_{D1,t}\right] + \left[\frac{\Sigma_{D1}}{\Sigma_{N1}}\widehat{\mathfrak{s}}_{N2,t} - \frac{\theta}{\left(1+\bar{\theta}_1\right)}\frac{\Sigma_{D2}}{\Sigma_{N2}}\widehat{\mathfrak{s}}_{D2,t}\right]
\end{aligned}$$

$$\begin{aligned}
\widehat{\mathfrak{s}}_{c,t} \approx& \quad \Omega_c\widehat{\wp}_t^* - (1-\Omega_c)\left[\left(\hat{\pi}_t - \hat{\pi}_t^{ind}\right) - \widehat{\mathfrak{s}}_{c,t-1}\right] \\
\widehat{\mathfrak{s}}_{N1,t} \approx& \quad \Omega_{N1}(\sigma+\omega)\,\hat{x}_t - (1-\Omega_{N1})\,E_t\left[\left(\hat{\pi}_{t+1} - \hat{\pi}_{t+1}^{ind}\right) + (\sigma-1)\,\widehat{\mathfrak{g}}_{t+1} - \left(\hat{\epsilon}_{t+1} - \hat{\epsilon}_t\right) - \widehat{\mathfrak{s}}_{N1,t+1}\right] \\
\widehat{\mathfrak{s}}_{D1,t} \approx& \quad -(1-\Omega_{D1})\,E_t\left[\left(\hat{\pi}_{t+1} - \hat{\pi}_{t+1}^{ind}\right) + (\sigma-1)\,\widehat{\mathfrak{g}}_{t+1} - \left(\hat{\epsilon}_{t+1} - \hat{\epsilon}_t\right) - \widehat{\mathfrak{s}}_{D1,t+1}\right] \\
\widehat{\mathfrak{s}}_{N2,t} \approx& \quad (1-\Omega_{N2})\,E_t\left[\left(\hat{\pi}_{t+1} - \hat{\pi}_{t+1}^{ind}\right) + \widehat{\mathfrak{s}}_{N2,t+1}\right] \\
\widehat{\mathfrak{s}}_{D2,t} \approx& \quad (1-\Omega_{D2})\,E_t\left[\left(\hat{\pi}_{t+1} - \hat{\pi}_{t+1}^{ind}\right) + \widehat{\mathfrak{s}}_{D2,t+1}\right]
\end{aligned}$$

And so, $\widehat{\mathfrak{s}}_{c,t} \approx \widehat{\mathfrak{s}}_{c,t-1} \approx 0$. Using the trend inflation equivalents, we obtain:

$$\left[1 - \frac{\theta\bar{\varrho}}{\left(1+\bar{\theta}_1\right)}\right]\frac{\bar{\alpha}_1\bar{\Phi}_c}{(1-\bar{\alpha}_1\bar{\Phi}_c)}\left(\hat{\pi}_t - \hat{\pi}_t^{ind}\right) \approx \quad \frac{\left(\bar{\wp}^*+\delta\right)}{\left(\bar{\wp}^*\right)\left(1+\bar{\theta}_1\right)}\frac{(1-\bar{\alpha}_2\beta\bar{\Phi}_{N2})}{(1-\bar{\alpha}_1\beta\bar{\Phi}_{N1})}\left[\widehat{\mathfrak{s}}_{N1,t} - \widehat{\mathfrak{s}}_{D1,t}\right] + \widehat{\mathfrak{s}}_{N2,t} - \frac{\theta\bar{\varrho}}{\left(1+\bar{\theta}_1\right)}\widehat{\mathfrak{s}}_{D2,t}$$

$$\begin{aligned}
\widehat{\mathfrak{s}}_{N1,t} \approx& \quad (1-\bar{\alpha}_1\beta\bar{\Phi}_{N1})(\sigma+\omega)\,\hat{x}_t - \bar{\alpha}_1\beta\bar{\Phi}_{N1}E_t\left[\left(\hat{\pi}_{t+1} - \hat{\pi}_{t+1}^{ind}\right) + (\sigma-1)\,\widehat{\mathfrak{g}}_{t+1} - \left(\hat{\epsilon}_{t+1} - \hat{\epsilon}_t\right) - \widehat{\mathfrak{s}}_{N1,t+1}\right] \\
\widehat{\mathfrak{s}}_{D1,t} \approx& \quad -\bar{\alpha}_1\beta\bar{\Phi}_{D1}E_t\left[\left(\hat{\pi}_{t+1} - \hat{\pi}_{t+1}^{ind}\right) + (\sigma-1)\,\widehat{\mathfrak{g}}_{t+1} - \left(\hat{\epsilon}_{t+1} - \hat{\epsilon}_t\right) - \widehat{\mathfrak{s}}_{D1,t+1}\right] \\
\widehat{\mathfrak{s}}_{N2,t} \approx& \quad \bar{\alpha}_2\beta\bar{\Phi}_{N2}E_t\left[\left(\hat{\pi}_{t+1} - \hat{\pi}_{t+1}^{ind}\right) + \widehat{\mathfrak{s}}_{N2,t+1}\right] \\
\widehat{\mathfrak{s}}_{D2,t} \approx& \quad \bar{\alpha}_2\beta\bar{\Phi}_{D2}E_t\left[\left(\hat{\pi}_{t+1} - \hat{\pi}_{t+1}^{ind}\right) + \widehat{\mathfrak{s}}_{D2,t+1}\right]
\end{aligned}$$

where $\bar{\varrho} \equiv \frac{(1-\bar{\alpha}_1\beta\bar{\Phi}_{D1})}{(1-\bar{\alpha}_1\beta\bar{\Phi}_{N1})}\frac{(1-\bar{\alpha}_2\beta\bar{\Phi}_{N2})}{(1-\bar{\alpha}_2\beta\bar{\Phi}_{D2})}$. Using the lead ($L^{-1}$) operator, the recursive system of ancillary variables can be written as follows:

$$\begin{aligned}
\widehat{\mathfrak{s}}_{N1,t} \approx& \quad \frac{\bar{\alpha}_1\beta\bar{\Phi}_{N1}}{(1-\bar{\alpha}_1\beta\bar{\Phi}_{N1}L^{-1})}E_t\left[-\left(\hat{\pi}_{t+1} - \hat{\pi}_{t+1}^{ind}\right) + (1-\sigma)\,\widehat{\mathfrak{g}}_{t+1} + \left(\hat{\epsilon}_{t+1} - \hat{\epsilon}_t\right)\right] + \frac{(1-\bar{\alpha}_1\beta\bar{\Phi}_{N1})(\sigma+\omega)}{(1-\bar{\alpha}_1\beta\bar{\Phi}_{N1}L^{-1})}\hat{x}_t \\
\widehat{\mathfrak{s}}_{D1,t} \approx& \quad \frac{\bar{\alpha}_1\beta\bar{\Phi}_{D1}}{(1-\bar{\alpha}_1\beta\bar{\Phi}_{D1}L^{-1})}E_t\left[-\left(\hat{\pi}_{t+1} - \hat{\pi}_{t+1}^{ind}\right) + (1-\sigma)\,\widehat{\mathfrak{g}}_{t+1} + \left(\hat{\epsilon}_{t+1} - \hat{\epsilon}_t\right)\right] \\
\widehat{\mathfrak{s}}_{N2,t} \approx& \quad \frac{\bar{\alpha}_2\beta\bar{\Phi}_{N2}}{(1-\bar{\alpha}_2\beta\bar{\Phi}_{N2}L^{-1})}E_t\left(\hat{\pi}_{t+1} - \hat{\pi}_{t+1}^{ind}\right) \\
\widehat{\mathfrak{s}}_{D2,t} \approx& \quad \frac{\bar{\alpha}_2\beta\bar{\Phi}_{D2}}{(1-\bar{\alpha}_2\beta\bar{\Phi}_{D2}L^{-1})}E_t\left(\hat{\pi}_{t+1} - \hat{\pi}_{t+1}^{ind}\right)
\end{aligned}$$

Therefore, the systen is represented as follows:

$$\begin{aligned}\left(\hat{\pi}_t-\hat{\pi}_t^{ind}\right)\approx\quad & \tfrac{1}{(1-\bar{\alpha}_1\beta\bar{\Phi}_{N1}L^{-1})}\bar{\kappa}\hat{x}_t \\ + \quad & \tfrac{1}{(1-\bar{\alpha}_1\beta\bar{\Phi}_{N1}L^{-1})}\tfrac{\bar{\kappa}\bar{\alpha}_1\beta\bar{\Phi}_{N1}}{(\sigma+\omega)(1-\bar{\alpha}_1\beta\bar{\Phi}_{N1})}E_t\left[-\left(\hat{\pi}_{t+1}-\hat{\pi}_{t+1}^{ind}\right)+(1-\sigma)\hat{\mathfrak{g}}_{t+1}+(\hat{\epsilon}_{t+1}-\hat{\epsilon}_t)\right] \\ - \quad & \tfrac{1}{(1-\bar{\alpha}_1\beta\bar{\Phi}_{D1}L^{-1})}\tfrac{\bar{\kappa}\bar{\alpha}_1\beta\bar{\Phi}_{D1}}{(\sigma+\omega)(1-\bar{\alpha}_1\beta\bar{\Phi}_{N1})}E_t\left[-\left(\hat{\pi}_{t+1}-\hat{\pi}_{t+1}^{ind}\right)+(1-\sigma)\hat{\mathfrak{g}}_{t+1}+(\hat{\epsilon}_{t+1}-\hat{\epsilon}_t)\right] \\ + \quad & \tfrac{(1-\bar{\alpha}_1\bar{\Phi}_c)}{\bar{\alpha}_1\bar{\Phi}_c}\tfrac{\left(1+\bar{\theta}_1\right)}{[(1+\theta\omega)+\theta(1-\bar{\varrho})]}\left[\tfrac{\bar{\alpha}_2\beta\bar{\Phi}_{N2}}{(1-\bar{\alpha}_2\beta\bar{\Phi}_{N2}L^{-1})}-\tfrac{\theta\bar{\varrho}}{\left(1+\bar{\theta}_1\right)}\tfrac{\bar{\alpha}_2\beta\bar{\Phi}_{D2}}{(1-\bar{\alpha}_2\beta\bar{\Phi}_{D2}L^{-1})}\right]E_t\left(\hat{\pi}_{t+1}-\hat{\pi}_{t+1}^{ind}\right)\end{aligned}$$

where $\bar{\kappa}\equiv\left(1+\frac{\delta}{\bar{\wp}^*}\right)\frac{(1-\bar{\alpha}_1\bar{\Phi}_c)(1-\bar{\alpha}_2\beta\bar{\Phi}_{N2})}{\bar{\alpha}_1\bar{\Phi}_c}\frac{(\sigma+\omega)}{[(1+\theta\omega)+\theta(1-\bar{\varrho})]}$.

Multiplying the system by $\left(1-\bar{\alpha}_1\beta\bar{\Phi}_{D1}L^{-1}\right)$, and rearranging the terms, we obtain:

$$\begin{aligned}\left(\hat{\pi}_t-\hat{\pi}_t^{ind}\right)\approx\quad & \beta E_t\left(\hat{\pi}_{t+1}-\hat{\pi}_{t+1}^{ind}\right)+\bar{\kappa}\hat{x}_t+\left(\tfrac{\bar{\Phi}_{D1}}{\bar{\Phi}_c}-1\right)\beta E_t\left(\hat{\pi}_{t+1}-\hat{\pi}_{t+1}^{ind}\right) \\ + \quad & \left(\tfrac{\bar{\Phi}_{N1}}{\bar{\Phi}_{D1}}-1\right)\bar{\kappa}\tfrac{\bar{\alpha}_1\bar{\Phi}_{D1}\beta}{(1-\bar{\alpha}_1\beta\bar{\Phi}_{N1}L^{-1})}E_t\left\{\hat{x}_{t+1}-\bar{\lambda}_1\left[\left(\hat{\pi}_{t+1}-\hat{\pi}_{t+1}^{ind}\right)+(\sigma-1)\hat{\mathfrak{g}}_{t+1}-(\hat{\epsilon}_{t+1}-\hat{\epsilon}_t)\right]\right\} \\ + \quad & \left(\tfrac{\bar{\Phi}_{N2}}{\bar{\Phi}_{D2}}-1\right)\bar{\kappa}\tfrac{\bar{\alpha}_2\bar{\Phi}_{D2}\beta}{(1-\bar{\alpha}_2\beta\bar{\Phi}_{N2}L^{-1})}\bar{\lambda}_2E_t\left(\hat{\pi}_{t+1}-\hat{\pi}_{t+1}^{ind}\right) \\ + \quad & \left(\tfrac{\bar{\alpha}_1\bar{\Phi}_{D1}}{\bar{\alpha}_2\bar{\Phi}_{D2}}-1\right)\bar{\kappa}\left(\tfrac{\bar{\alpha}_2\bar{\Phi}_{D2}\beta}{(1-\bar{\alpha}_2\beta\bar{\Phi}_{D2}L^{-1})}\bar{\lambda}_3-\tfrac{\bar{\alpha}_2\bar{\Phi}_{D2}\beta}{(1-\bar{\alpha}_2\beta\bar{\Phi}_{N2}L^{-1})}\bar{\lambda}_2\right)E_t\left(\hat{\pi}_{t+1}-\hat{\pi}_{t+1}^{ind}\right)\end{aligned}$$

where

$$\bar{\lambda}_1\equiv\frac{1}{(\sigma+\omega)(1-\bar{\alpha}_1\beta\bar{\Phi}_{N1})}\quad;\ \bar{\lambda}_2\equiv\frac{(\bar{\wp}^*)}{(\bar{\wp}^*+\delta)}\frac{\left(1+\bar{\theta}_1\right)}{(\sigma+\omega)(1-\bar{\alpha}_2\beta\bar{\Phi}_{N2})}\quad;\ \bar{\lambda}_3\equiv\frac{(\bar{\wp}^*)}{(\bar{\wp}^*+\delta)}\frac{\theta\bar{\varrho}}{(\sigma+\omega)(1-\bar{\alpha}_2\beta\bar{\Phi}_{N2})}$$

Therefore, we can write the approximated price-wedge NKPC under trend inflation as follows:

$$\begin{aligned}\left(\hat{\pi}_t-\hat{\pi}_t^{ind}\right)\approx\quad & \beta E_t\left(\hat{\pi}_{t+1}-\hat{\pi}_{t+1}^{ind}\right)+\bar{\kappa}\hat{x}_t+\left(\tfrac{\bar{\Phi}_{N1}}{\bar{\Phi}_{D1}}-1\right)\bar{\kappa}\bar{\alpha}_1\bar{\Phi}_{D1}\beta E_t\hat{\omega}_{1,t+1}+\left(\tfrac{\bar{\Phi}_{N2}}{\bar{\Phi}_{D2}}-1\right)\bar{\kappa}\bar{\alpha}_2\bar{\Phi}_{D2}\beta E_t\hat{\omega}_{2,t+1} \\ + \quad & \left(\tfrac{\bar{\alpha}_1\bar{\Phi}_{D1}}{\bar{\alpha}_2\bar{\Phi}_{D2}}-1\right)\bar{\kappa}\bar{\alpha}_2\bar{\Phi}_{D2}\beta E_t\left(\hat{\omega}_{3,t+1}-\hat{\omega}_{2,t+1}\right)+\left(\tfrac{\bar{\Phi}_{D1}}{\bar{\Phi}_c}-1\right)\beta E_t\left(\hat{\pi}_{t+1}-\hat{\pi}_{t+1}^{ind}\right)\end{aligned}$$

where $\hat{\omega}_{1,t}$, $\hat{\omega}_{2,t}$ and $\hat{\omega}_{3,t}$ are three ancillary variables, which evolve according to:

$$\begin{aligned}\hat{\omega}_{1,t}&=\bar{\alpha}_1\bar{\Phi}_{N1}\beta E_t\hat{\omega}_{1,t+1}+\hat{x}_t-\bar{\lambda}_1\left[\left(\hat{\pi}_t-\hat{\pi}_t^{ind}\right)+(\sigma-1)\hat{\mathfrak{g}}_t-(\hat{\epsilon}_t-\hat{\epsilon}_{t-1})\right] \\ \hat{\omega}_{2,t}&=\bar{\alpha}_2\bar{\Phi}_{N2}\beta E_t\hat{\omega}_{2,t+1}+\bar{\lambda}_2\left(\hat{\pi}_t-\hat{\pi}_t^{ind}\right) \\ \hat{\omega}_{3,t}&=\bar{\alpha}_2\bar{\Phi}_{D2}\beta E_t\hat{\omega}_{3,t+1}+\bar{\lambda}_3\left(\hat{\pi}_t-\hat{\pi}_t^{ind}\right)\end{aligned}$$

Again, the composite parameters are

$$\bar{\varrho}\equiv\frac{(1-\bar{\alpha}_1\beta\bar{\Phi}_{D1})}{(1-\bar{\alpha}_1\beta\bar{\Phi}_{N1})}\frac{(1-\bar{\alpha}_2\beta\bar{\Phi}_{N2})}{(1-\bar{\alpha}_2\beta\bar{\Phi}_{D2})}\quad;\ \bar{\kappa}\equiv\left(1+\frac{\delta}{\bar{\wp}^*}\right)\frac{(1-\bar{\alpha}_1\bar{\Phi}_c)(1-\bar{\alpha}_2\beta\bar{\Phi}_{N2})}{\bar{\alpha}_1\bar{\Phi}_c}\frac{(\sigma+\omega)}{[(1+\theta\omega)+\theta(1-\bar{\varrho})]}$$

$$\bar{\lambda}_1\equiv\frac{1}{(\sigma+\omega)(1-\bar{\alpha}_1\beta\bar{\Phi}_{N1})}\quad;\ \bar{\lambda}_2\equiv\frac{(\bar{\wp}^*)}{(\bar{\wp}^*+\delta)}\frac{\left(1+\bar{\theta}_1\right)}{(\sigma+\omega)(1-\bar{\alpha}_2\beta\bar{\Phi}_{N2})}\quad;\ \bar{\lambda}_3\equiv\frac{(\bar{\wp}^*)}{(\bar{\wp}^*+\delta)}\frac{\theta\bar{\varrho}}{(\sigma+\omega)(1-\bar{\alpha}_2\beta\bar{\Phi}_{N2})}$$

C.3.2 Deriving the Log-Linearized Supply System

Direct loglinearization of the pricing systems (6) and (7) initially gives the following equations:

$$0 = \sum_{j=0}^{\infty} (\bar{\alpha}_1)^j \left(\bar{z}_j^*\right)^{-\theta} \left[\hat{\wp}_{t-j}^* - \left(\hat{\pi}_{t-j,t} - \hat{\pi}_{t-j,t}^{ind}\right)\right]$$

$$0 = \mu_k \bar{\mathcal{N}}_1 \hat{\mathcal{N}}_{1,t} - \bar{\mathcal{D}}_1 \hat{\mathcal{D}}_{1,t}$$

$$\begin{aligned}
\bar{\mathcal{N}}_1 \hat{\mathcal{N}}_{1,t} = & -\tfrac{1}{\mu}(1+\delta)^{\bar{\theta}_1} (\bar{X})^{(\sigma+\omega)} \left(1+\bar{\theta}_1\right) \Sigma_{N2}(\bar{\wp}^*) \hat{\wp}_t^* \\
& + \tfrac{1}{\mu}(1+\delta)^{\bar{\theta}_1} (\bar{X})^{(\sigma+\omega)} E_t \sum_{j=0}^{\infty} (\bar{\alpha}_1 \beta)^j \left(z_j^*\right)^{-\left(1+\bar{\theta}_1\right)} \left[\hat{q}_{t,t+j} + \hat{\pi}_{t,t+j}^{ind} + \hat{\mathfrak{g}}_{t,t+j} + (\sigma+\omega)\hat{x}_{t+j}\right] \\
& + \tfrac{1}{\mu}(1+\delta)^{\bar{\theta}_1} (\bar{X})^{(\sigma+\omega)} \left(1+\bar{\theta}_1\right) (\bar{\wp}^*) E_t \sum_{j=0}^{\infty} (\bar{\alpha}_2 \beta)^j \left(z_j^*\right)^{-\left(2+\bar{\theta}_1\right)} \left(\hat{\pi}_{t,t+j} - \hat{\pi}_{t,t+j}^{ind}\right)
\end{aligned}$$

$$\begin{aligned}
\bar{\mathcal{D}}_1 \hat{\mathcal{D}}_{1,t} = & -(1+\delta)^{\theta} \theta \Sigma_{D2}(\bar{\wp}^*) \hat{\wp}_t^* \\
& + (1+\delta)^{\theta} E_t \sum_{j=0}^{\infty} (\bar{\alpha}_1 \beta)^j \left(z_j^*\right)^{-\theta} \left[\hat{q}_{t,t+j} + \hat{\pi}_{t,t+j}^{ind} + \hat{\mathfrak{g}}_{t,t+j}\right] \\
& + (1+\delta)^{\theta} \theta (\bar{\wp}^*) E_t \sum_{j=0}^{\infty} (\bar{\alpha}_2 \beta)^j \left(z_j^*\right)^{-(1+\theta)} \left(\hat{\pi}_{t,t+j} - \hat{\pi}_{t,t+j}^{ind}\right)
\end{aligned}$$

Since the discounted sums do not allow for finite recursive representations, we use the following Lemmas to help us obtain simpler expressions, used to deriving the system of equations in (C.6) and (C.7).

Lemma 1 *Consider generic forward and backward equations* $\hat{S}_t^f \equiv E_t \sum_{j=0}^{\infty} (\beta)^j \left(\bar{z}_j^*\right)^{-\phi} \left(\hat{\varkappa}_{t,t+j}^a + \hat{\varkappa}_{t+j}^b\right)$ *and* $\hat{S}_t^l \equiv \sum_{j=0}^{\infty} (\beta)^j \left(\bar{z}_j^*\right)^{-\phi} \left(\hat{\varkappa}_{t-j,t}^a + \hat{\varkappa}_{t-j}^b\right)$, *where* $\beta \in (0,1)$ *is a discounting parameter,* $\hat{\varkappa}_{\tau_1,\tau_2}^a$ *is a cumulative variable from* τ_1 *to* τ_2, *while* $\hat{\varkappa}_\tau^b$ *is a spot variable at period* τ. *Since* $\hat{\varkappa}_{t,t+j}^a = \hat{\varkappa}_{t+1}^a + \hat{\varkappa}_{t+1,t+j}^a$, $\hat{\varkappa}_{t-j,t}^a = \hat{\varkappa}_t^a +$

$\hat{\varkappa}^a_{t-j,t-1}$, and $\hat{\varkappa}^a_{t,t}=0$, the infinite sums lead to the following infinite recursive systems, for $k=\{0,1,2,...,\infty\}$:

$$
\begin{aligned}
\hat{S}^f_t &= \hat{S}^f_{0,t} \\
\hat{S}^f_{k,t} &= \left(\Sigma_{k,\phi}-\Theta_{k,\phi}\right)E_t\hat{\varkappa}^a_{t+1}+\Theta_{k,\phi}\hat{\varkappa}^b_t+\text{ß}E_t\hat{S}^f_{(k+1),t+1} \\
\hat{S}^l_t &= \hat{S}^l_{0,t} \\
\hat{S}^l_{k,t} &= \left(\Sigma_{k,\phi}-\Theta_{k,\phi}\right)\hat{\varkappa}^a_t+\Theta_{k,\phi}\hat{\varkappa}^b_t+\text{ß}\hat{S}^l_{(k+1),t-1} \\
&\textit{where} \\
\hat{S}^f_{k,t} &\equiv E_t\sum_{j=0}^{\infty}(\text{ß})^j\left(\bar{z}^*_{j+k}\right)^{-\phi}\left(\hat{\varkappa}^a_{t,t+j}+\hat{\varkappa}^b_{t+j}\right) \\
\hat{S}^l_{k,t} &\equiv \sum_{j=0}^{\infty}(\text{ß})^j\left(\bar{z}^*_{j+k}\right)^{-\phi}\left(\hat{\varkappa}^a_{t-j,t}+\hat{\varkappa}^b_{t-j}\right) \\
\Theta_{k,\phi} &\equiv \left(\bar{z}^*_k\right)^{-\phi} \quad , \quad \Sigma_{0,\phi}=\Sigma_\phi\equiv\sum_{j=0}^{\infty}(\text{ß})^j\left(\bar{z}^*_j\right)^{-\phi} \\
\Sigma_{k,\phi} &\equiv \sum_{j=0}^{\infty}(\text{ß})^j\left(\bar{z}^*_{j+k}\right)^{-\phi}=\frac{1}{\text{ß}}\left[\Sigma_{(k-1),\phi}-\Theta_{(k-1),\phi}\right]
\end{aligned}
$$

Proof. As for $\Sigma_{k,\phi}\equiv\sum_{j=0}^{\infty}(\text{ß})^j\left(\bar{z}^*_{j+k}\right)^{-\phi}$, note that:

$$
\begin{aligned}
\Sigma_{k,\phi}\equiv &\sum_{j=0}^{\infty}(\text{ß})^j\left(\bar{z}^*_{j+k}\right)^{-\phi}=\sum_{j=-1}^{\infty}(\text{ß})^j\left(\bar{z}^*_{j+k}\right)^{-\phi}-(\text{ß})^{-1}\left(\bar{z}^*_{k-1}\right)^{-\phi} \\
= &\sum_{j=0}^{\infty}(\text{ß})^{j-1}\left(\bar{z}^*_{j-1+k}\right)^{-\phi}-(\text{ß})^{-1}\left(\bar{z}^*_{k-1}\right)^{-\phi}=\frac{1}{\text{ß}}\left[\sum_{j=0}^{\infty}(\text{ß})^j\left(\bar{z}^*_{j+k-1}\right)^{-\phi}-\left(\bar{z}^*_{k-1}\right)^{-\phi}\right]
\end{aligned}
$$

For the forward infinite sum, we obtain:

$$
\begin{aligned}
\hat{S}^f_{k,t}\equiv\ & E_t\sum_{j=0}^{\infty}(\text{ß})^j\left(\bar{z}^*_{j+k}\right)^{-\phi}\left(\hat{\varkappa}^a_{t,t+j}+\hat{\varkappa}^b_{t+j}\right) \\
=\ & E_t\sum_{j=0}^{\infty}(\text{ß})^j\left(\bar{z}^*_{j+k}\right)^{-\phi}\left(\hat{\varkappa}^a_{t,t+j}\right)+E_t\sum_{j=0}^{\infty}(\text{ß})^j\left(\bar{z}^*_{j+k}\right)^{-\phi}\left(\hat{\varkappa}^b_{t+j}\right) \\
=\ & E_t\sum_{j=1}^{\infty}(\text{ß})^j\left(\bar{z}^*_{j+k}\right)^{-\phi}\left(\hat{\varkappa}^a_{t,t+j}\right)+E_t\sum_{j=0}^{\infty}(\text{ß})^j\left(\bar{z}^*_{j+k}\right)^{-\phi}\left(\hat{\varkappa}^b_{t+j}\right) \\
=\ & E_t\sum_{j=1}^{\infty}(\text{ß})^j\left(\bar{z}^*_{j+k}\right)^{-\phi}\left(\hat{\varkappa}^a_{t+1}+\hat{\varkappa}^a_{t+1,t+j}\right)+\left(\bar{z}^*_k\right)^{-\phi}\hat{\varkappa}^b_t+E_t\sum_{j=1}^{\infty}(\text{ß})^j\left(\bar{z}^*_{j+k}\right)^{-\phi}\left(\hat{\varkappa}^b_{t+j}\right) \\
=\ & \left(\sum_{j=1}^{\infty}(\text{ß})^j\left(\bar{z}^*_{j+k}\right)^{-\phi}\right)E_t\hat{\varkappa}^a_{t+1}+\left(\bar{z}^*_k\right)^{-\phi}\hat{\varkappa}^b_t+E_t\sum_{j=1}^{\infty}(\text{ß})^j\left(\bar{z}^*_{j+k}\right)^{-\phi}\left(\hat{\varkappa}^a_{t+1,t+j}+\hat{\varkappa}^b_{t+j}\right) \\
=\ & \left(\Sigma_{k,\phi}-\left(\bar{z}^*_k\right)^{-\phi}\right)E_t\hat{\varkappa}^a_{t+1}+\left(\bar{z}^*_k\right)^{-\phi}\hat{\varkappa}^b_t+\text{ß}E_t\sum_{j=0}^{\infty}(\text{ß})^j\left(\bar{z}^*_{j+k+1}\right)^{-\phi}\left(\hat{\varkappa}^a_{t+1,t+1+j}+\hat{\varkappa}^b_{t+1+j}\right) \\
=\ & \left(\Sigma_{k,\phi}-\Theta_{k,\phi}\right)E_t\hat{\varkappa}^a_{t+1}+\Theta_{k,\phi}\hat{\varkappa}^b_t+\text{ß}E_t\hat{S}^f_{(k+1),t+1}
\end{aligned}
$$

And lastly, for the backward sum, we obtain:

$$
\begin{aligned}
\hat{S}^{l}_{k,t} \equiv & \sum_{j=0}^{\infty} (ß)^{j} \left(\bar{z}^{*}_{j+k}\right)^{-\phi} \left(\hat{\varkappa}^{a}_{t-j,t} + \hat{\varkappa}^{b}_{t-j}\right) = \sum_{j=0}^{\infty} (ß)^{j} \left(\bar{z}^{*}_{j+k}\right)^{-\phi} \left(\hat{\varkappa}^{a}_{t-j,t}\right) + \sum_{j=0}^{\infty} (ß)^{j} \left(\bar{z}^{*}_{j+k}\right)^{-\phi} \left(\hat{\varkappa}^{b}_{t-j}\right) \\
= & \sum_{j=1}^{\infty} (ß)^{j} \left(\bar{z}^{*}_{j+k}\right)^{-\phi} \left(\hat{\varkappa}^{a}_{t-j,t}\right) + \sum_{j=0}^{\infty} (ß)^{j} \left(\bar{z}^{*}_{j+k}\right)^{-\phi} \left(\hat{\varkappa}^{b}_{t-j}\right) \\
= & \sum_{j=1}^{\infty} (ß)^{j} \left(\bar{z}^{*}_{j+k}\right)^{-\phi} \left(\hat{\varkappa}^{a}_{t} + \hat{\varkappa}^{a}_{t-j,t-1}\right) + \left(\bar{z}^{*}_{k}\right)^{-\phi} \hat{\varkappa}^{b}_{t} + \sum_{j=1}^{\infty} (ß)^{j} \left(\bar{z}^{*}_{j+k}\right)^{-\phi} \left(\hat{\varkappa}^{b}_{t-j}\right) \\
= & \left(\sum_{j=1}^{\infty} (ß)^{j} \left(\bar{z}^{*}_{j+k}\right)^{-\phi}\right) \hat{\varkappa}^{a}_{t} + \left(\bar{z}^{*}_{k}\right)^{-\phi} \hat{\varkappa}^{b}_{t} + \sum_{j=1}^{\infty} (ß)^{j} \left(\bar{z}^{*}_{j+k}\right)^{-\phi} \left(\hat{\varkappa}^{a}_{t-j,t-1} + \hat{\varkappa}^{b}_{t-j}\right) \\
= & \left(\Sigma_{k,\phi} - \left(\bar{z}^{*}_{k}\right)^{-\phi}\right) \hat{\varkappa}^{a}_{t} + \left(\bar{z}^{*}_{k}\right)^{-\phi} \hat{\varkappa}^{b}_{t} + ß \sum_{j=0}^{\infty} (ß)^{j} \left(\bar{z}^{*}_{j+k+1}\right)^{-\phi} \left(\hat{\varkappa}^{a}_{t-1-j,t-1} + \hat{\varkappa}^{b}_{t-1-j}\right) \\
= & \left(\Sigma_{k,\phi} - \Theta_{k,\phi}\right) \hat{\varkappa}^{a}_{t} + \Theta_{k,\phi} \hat{\varkappa}^{b}_{t} + ß \hat{S}^{l}_{(k+1),t-1}
\end{aligned}
$$

■

The recursive systems are infinite, for $\hat{S}^{l}_{k,t}$ $\left(\hat{S}^{f}_{k,t}\right)$ depends on $\hat{S}^{l}_{(k+1),t-1}$ $\left(E_t \hat{S}^{f}_{(k+1),t+1}\right)$, instead of $\hat{S}^{l}_{k,t-1}$ $\left(E_t \hat{S}^{f}_{k,t+1}\right)$, for $k = \{0, 1, 2, ..., \infty\}$. However, since coefficients $\left(\Sigma_{k,\phi} - \Theta_{k,\phi}\right)$ and $\Theta_{k,\phi}$ converges asymptotically as k rises, the equations at a conveniently chosen large level $\bar{k}$ can be approximated by finite recursions, using $\hat{S}^{l}_{k,t-1}$ $\left(E_t \hat{S}^{f}_{k,t+1}\right)$, instead of $\hat{S}^{l}_{(k+1),t-1}$ $\left(E_t \hat{S}^{f}_{(k+1),t+1}\right)$. In this paper, we set $\bar{k} = 40$.