Imitation and Price Dynamics

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Abstract

We propose a theory of sticky prices that generalizes Calvo pricing and can generate realistic price and output dynamics. Firms are given the opportunity to adopt the successful pricing strategies of other firms. We analyze the transition dynamics between steady-states and find that the probability that a firm changes its price is increasing over time.

[PRELIMINARY. PLEASE DO NOT CITE.]

JEL Classification Codes: E31, E32, L16

1 Introduction

Confucius declares[1], "Man has three ways of acting wisely. First, on meditation: that is the noblest. Second, on imitation: that is the easiest. Thirdly, on experience: that is the bitterest." Standard macroeconomic theory assumes that firms do the first and optimize profits. This paper develops a model in which firms may imitate the successful strategies of other firms.[2]

The main result is that imitation and imperfect information lead to sticky prices and sticky inflation. Firms that imitate only change prices when other firms have a better history of pricing. The probability of such an event is generally less than one. The result is that the probability of changing one's price is higher when more firms are playing more profitable prices.

The result has empirical implications. The theoretical impulse response functions closer match those estimated with U.S data by Christiano et al. (1999). Our model predicts that the rate at which aggregate prices change depends on how many firms have changed prices in the past, implying that the rate is non-constant over time. The result is an "S"-shaped impulse response function that resembles diffusion processes.

There are two major literatures that we contribute to. The first is the macroeconomics literature on sticky prices. An early theoretical model of staggered prices is Taylor (1980) which had firms exogenously locked into a price for $n > 1$ periods. The contracts of different firms are set in different periods leading to a series of overlapping contracts. Calvo (1983) has firms stochastically able to change prices with probability θ. The result is that firms spend an average of $\frac{1}{\theta}$ periods locked into a price.

A second class of models has focused on the firm's choice of whether to reprice or not. Rotemberg (1982), Caplin and Leahy (1991), Rotemberg and Woodford (1997)) analyzes firms that face costs to changing prices, called *menu costs*. A firm will change the price when the benefits outweigh the menu costs, both of which depend on the aggregate and idiosyncratic state of the economy.

The empirical performance of sticky price models has been mixed. Christiano et al. (1999) estimate a model of sticky-prices using U.S. data using structural VAR methods. A major result is that exogenous changes in monetary policy lead to a "hump" shaped impulse response function for output, and an "S" shaped impulse response function for aggregate prices. The result contradicts the implications of Calvo pricing or staggered pricing. Christiano et al. (2005) attempt to reconcile the theory with the evidence by including non-time separable preferences, adjustment costs in investment, and variable capital utilization.

[1] Translation by Muller (1990).

[2] Here we do not analyze learning from experience. Presumably this model can incorporate learning by extending the memory of a firm.

The second literature we contribute to is the industrial organization literature on imitation amongst firms. Vega-Redondo (1997) analyzes imitation in a Cournot framework and finds the Walrasian outcome is stochastically stable. Bergin and Bernhardt (2009) add long memory to the former model and find the cooperative outcome to be stochastically stable. Selten and Ostmann (2001) analyze local imitation (in a network) of Cournot competitive firms by providing a solution concept of *imitation equilibrium*. Selten and Apesteguia (2005) provide experimental evidence of competition on a circle and find support for imitative behaviour.

Our work is related to Bass (1969), which has consumers adopting a new product randomly or by imitating other consumers.

This chapter is organized as follows. First we present the New Keynesian model of Galí (2008). The model incorporates the pricing dynamics from Calvo (1983), in which every firm changes its price with some probability θ and maintains its current price otherwise. So-called *Calvo pricing* is a standard way of incorporating sticky prices in a monopolistically competitive framework. Second, we augment Galí's model by adding imitation and analyze the inflation dynamics. Third, we characterize the equilibrium and how it is affected by imitation. The final section is the discussion.

2 Model

In this section we introduce a basic New Keynesian model which closely follows Galí (2008) and use it as a benchmark model, which is a dynamic version of Dixit and Stiglitz (1977). The presentation includes an exposition of Calvo pricing, where we highlight our innovation.

2.1 Households

The households in this paper behave exactly as in Galí (2008). More specifically, there is a representative infinitely lived household that maximizes the following problem:

$$E_0 \sum_{t=0}^{\infty} \beta^t U(C_t, N_t) \tag{1}$$

s.t.

$$\int_0^1 P_{i,t} C_{i,t} di + Q_t B_t \leq B_{t-1} + W_t N_t + T_t \tag{2}$$

where $C_{i,t}$ is the amount of good i consumed by the household at time t and produced by firm i.[3] We assume that these goods are indexed by the interval $[0, 1]$. Therefore,

[3]Each firm produces a different good, and each good is substitutable.

given this set up, the households have preferences for variety. $P_{i,t}$ is the price of good i, B_t is the number of one-period bonds purchased (at price Q_t), and T_t is the lump-sum component of income. Also, $\lim_{T\to\infty} E_t\{B_T\} \geq 0$ must be satisfied for all t (No-Ponzi scheme condition). C_t is a consumption index:

$$C_t = \left(\int_0^1 C_{i,t}^{\frac{\epsilon-1}{\epsilon}}\right)^{\frac{\epsilon}{\epsilon-1}} \tag{3}$$

It can be shown that the budget constraint can be written as:[4]

$$P_t C_t + Q_t B_t \leq B_{t-1} + W_t N_t + T_t \tag{4}$$

where P_t, similarly to C_t, is an aggregate price index characterized by the following equation:

$$P_t \equiv \left[\int_0^1 P_{i,t}^{1-\epsilon} di\right]^{\frac{1}{1-\epsilon}} \tag{5}$$

The demand equation for good i is:

$$C_{i,t} = \left(\frac{P_{i,t}}{P_t}\right)^{-\epsilon} C_t \tag{6}$$

Finally, let's assume that the household has the following preferences over labour and consumption:

$$U(C_t, N_t) = \frac{C_t^{1-\sigma}}{1-\sigma} - \frac{N_t^{1+\varphi}}{1+\varphi} \tag{7}$$

The first order conditions are:

$$-\frac{U_{n,t}}{U_{c,t}} = C_t^{\sigma} N_t^{\varphi} = \frac{W_t}{P_t} \tag{8}$$

$$Q_t = \beta E_t \frac{U_{c,t+1}}{U_{c,t}} \frac{P_t}{P_{t+1}} = \beta E_t \left\{ \left(\frac{C_{t+1}}{C_t}\right)^{-\sigma} \frac{P_t}{P_{t+1}} \right\} \tag{9}$$

The linearized version of the labour supply, (8), is:

$$w_t - p_t = \sigma c_t + \varphi n_t \tag{10}$$

[4] See Appendix 3.1 of Galí (2008)

where lower-case letters denote variables in log (i.e. $x_t \equiv \log(X_t)$). The log-linear approximation of the Euler equation, (9), around a steady state is:

$$c_t = E_t\{c_t\} - \frac{1}{\sigma}(i_t - E_t\pi_{t+1} - \rho) \tag{11}$$

where $i_t \equiv -\log(Q_t)$, $\rho \equiv -\log(\beta)$, and $\pi_t \equiv p_t - p_{t-1}$. Equation (11) is called IS curve.

2.2 Firms

While the households are identical to Galí (2008), firms are different in an important respect. Here we introduce the imitation mechanism in an easily accessible framework.

For simplicity we assume that there is no capital, and that each firm produces according to the following production function:

$$Y_t(i) = A_t N_{i,t}^{1-\alpha} \tag{12}$$

where $Y_t(i)$ is the output of firm i at time t, $N_{i,t}$ is the labour input, A_t is productivity/technology common to all firms, and α is the output elasticity of labour. All firms face the same demand function, (6), and take the aggregate consumption index, (3), and the aggregate price index, (5), as given.

Now consider a discrete time environment in which a unit interval of firms each set the price of their good, $P_t(i)$. With some probability $\theta \in [0, 1]$ firm i cannot update its price while with probability $1 - \theta$ the firm may change its price. This exogenous pricing probability is called Calvo pricing. Notice that $\theta = 0$ is the special case of flexible prices.

Our environment differs from Calvo pricing in that a firm that is allowed to change its price might not optimize. Assume that with probability $\gamma \in [0, 1]$ firm i is be able to choose the price to maximize its profit while with probability $1 - \gamma$ it is matched with another firm and might imitate its price. In other words, firms randomly do one of three things: optimize prices, imitate prices, or leave prices unchanged. We assume that a firm will only imitate the other firm if the latter has a better pricing history (in terms of previous period profits). Therefore, our environment is a generalization of Calvo pricing.

For the time being we need to introduce some notation before solving the optimization problem. Let $P_t^*(\theta, \gamma)$ be the optimal price for firms re-optimizing at time t with fixed parameters θ and γ, $P_t(\theta, \gamma)$ be the price level at time t, and $f_t(x)$ be the number of firms with price x at time t.[5]

[5] It can be show that all firms optimizing will choose the same price.

For example, suppose that at $t = 0$ all firms are playing the optimal price $P_0^*(\theta, \gamma)$, which means $f_0(P_0^*(\theta, \gamma)) = 1$. When $\theta = 0$ and $\gamma = 1$ (flexible prices) then this will always be the case. Furthermore, given that all firms have the same production function and information set they will all play the same optimal price every period $f_t(P_t^*(0, 1)) = 1$, $\forall t$. Note that Calvo pricing is a special case in which $\gamma = 1$ and $\theta \in [0, 1]$.

Consider the simple scenario in which the parameters of the model are constant (non-stochastic), every firm plays the same initial price ($f_0(P_0) = 1$), and the long run optimal price is larger than the initial price ($\lim_{t\to\infty} P_t^* > P_0$). This simple environment is easy to analyze and clearly demonstrates our mechanism at work. The analysis presented here is restricted to these transitory dynamics, though symmetric results hold for the case of $\lim_{t\to\infty} P_t^* < P_0$.

Proposition 1 describes the probability of a price change for a given firm.

Proposition 1. *Let $\gamma < 1$ and $\nu_j(P_{i,t}(\theta, \gamma))$ be the probability that a firm that last set its price at t to price $P_{i,t}(\theta, \gamma)$ will change its price at $t + j$ for $j \geq 1$.*

(i) $\nu_j(P_{i,t}(\theta, \gamma)) = (1 - \theta)(\gamma + (1 - \gamma)(1 - F_{t+j-1}(P_{i,t}(\theta, \gamma))))$ *for* $t \geq 1$.

(ii) $\nu_{j+1}(P_{i,t}(\theta, \gamma)) > \nu_j(P_{i,t}(\theta, \gamma))$

(iii) $\nu_1(P_{i,t}(\theta, \gamma)) = (1 - \theta)\gamma$ *and* $\lim_{j\to\infty} \nu_j(P_{i,t}(\theta, \gamma)) = 1 - \theta$

The proof of (i) follows directly from the definitions, and (ii) and (iii) follow almost immediately from (i). Therefore, the proof is omitted.

The proposition discusses the probability of changing one's price j periods after the last price change. The probability varies as the distribution of prices varies. The probability of changing one's price increases over time, in contrast to Calvo pricing under which the probability of changing the price remains constant.

2.3 Price Dynamics

In this section, we analyze aggregate price dynamics in the New Keynesian framework of Galí (2008) with imitation.

As discussed above, prices are aggregated with the following index:

$$P_t \equiv \left[\int_0^1 P_{i,t}(\theta, \gamma)^{1-\epsilon} di \right]^{\frac{1}{1-\epsilon}} \tag{5}$$

In the Calvo pricing case, when $\gamma = 1$, it can be shown that the aggregate price dynamics are described by:[6]

$$P_t(\theta, 1) = \left[(1-\theta)P_t^*(\theta, 1)^{1-\epsilon} + \theta P_{t-1}(\theta, 1)^{1-\epsilon}\right]^{\frac{1}{1-\epsilon}} \tag{13}$$

Every firm that is re-optimizing is choosing the same price, $P_t^*(\theta, 1)$, while the mass of firms that do not re-optimize correspond to the distribution of prices from the previous period.

Re-arranging gives us a formula in terms of the inflation rate, Π_t, between period $t-1$ and t:

$$\left(\frac{P_t(\theta, 1)}{P_{t-1}(\theta, 1)}\right)^{1-\epsilon} = \Pi_t^{1-\epsilon} = \theta + (1-\theta)\left(\frac{P_t^*(\theta, 1)}{P_{t-1}(\theta, 1)}\right)^{1-\epsilon} \tag{14}$$

Where its log-linear approximation around a zero inflation steady state is:

$$\pi_t = (1-\theta)(p_t^*(\theta, 1) - p_{t-1}(\theta, 1)) \tag{15}$$

However, in our setup, with $\gamma < 1$, aggregate prices also depends on the probability of being matched with a firm that has a better pricing history. In general, this probability will depend on the distribution of prices at a given point in time, denoted $F_t(x)$. For ease of notation, when we are talking about the general case (i.e. $\gamma \in (0, 1)$ and $\theta \in (0, 1)$), the θ and γ are dropped such that $P_t \equiv P_t(\theta, \gamma)$, $P_{i,t} \equiv P_{i,t}(\theta, \gamma)$, and $P_t^* \equiv P_t^*(\theta, \gamma)$. It is straightforward to demonstrate that the the index is equivalent to:

$$P_t(\theta, \gamma) \equiv$$
$$\left[\theta(P_{t-1})^{1-\epsilon} + (1-\theta)\gamma(P_t^*)^{1-\epsilon} + 2(1-\theta)(1-\gamma)\sum_{k=0}^{t-1} f_{t-1}(P_k^*)F_{t-1}(P_k^*)(P_k^*)^{1-\epsilon}\right]^{\frac{1}{1-\epsilon}} \tag{16}$$

The following proposition describes the evolution of logged aggregate prices.

Proposition 2. *The evolution of optimal prices toward a long-run stead-state is described by the following equation:*

$$\pi_t = (1-\theta)\gamma(p_t^* - p_{t-1}) + (1-\theta)(1-\gamma)\sum_{k=0}^{t-1} \frac{2\gamma(1-\theta)e^{-2(1-\theta)(t-k-1)}}{(\gamma + (1-\gamma)e^{-(1-\theta)(t-k-1)})^3}(p_k^* - p_{t-1}) \tag{17}$$

[6] See Galí (2008).

Proof:
The price index can be decomposed as:

$$P_t = (\int_0^1 P_{i,t}^{1-\epsilon} di)^{\frac{1}{1-\epsilon}}$$
$$= ((1-\theta)\gamma P_t^{*1-\epsilon} + \theta P_{t-1}^{1-\epsilon} + 2(1-\theta)(1-\gamma)\sum_{k=0}^{t-1} f_{t-1}(P_k^*)F_{t-1}(P_k^*)P_k^{*1-\epsilon})^{\frac{1}{1-\epsilon}}$$

Rearranging gives:

$$\Pi_t^{1-\epsilon} = ((1-\theta)\gamma X_t^{1-\epsilon} + \theta + 2(1-\theta)(1-\gamma)\sum_{k=0}^{t-1} f_{t-1}(P_k^*)F_{t-1}(P_k^*)X_k^{1-\epsilon})$$

where $\Pi_t = \frac{P_t}{P_{t-1}}$ and $X_k = \frac{P_k^*}{P_{t-1}}$.
Log-linearizing around a steady-state of $\Pi_t = X_1 = X_2 = ... = X_t = 1$, while keeping the distributions unchanged, gives:

$$\pi_t = (1-\theta)\gamma(p_t^* - p_{t-1}) + 2(1-\theta)(1-\gamma)\sum_{k=0}^{t-1} f_{t-1}(P_k^*)F_{t-1}(P_k^*)(p_k^* - p_{t-1})$$

We must find F_{t-1} and f_{t-1}. Notice that $F_t = 1$ for $t < k$ and for $t \geq k$ it is described recursively by:

$$F_t = F_{t-1} - (1-\theta)(1-\gamma)F_{t-1} - (1-\theta)\gamma(1-F_{t-1})F_{t-1}$$

This is a difference equation. The analogous continuous time differential equation is:

$$\dot{F}_t = -(1-\theta)(1-\gamma)F_t - (1-\theta)\gamma(1-F_t)F_t$$

The solution is given by Bass (1969) as:

$$F_t(P_k) = \frac{e^{-(1-\theta)(t-k)}}{\gamma + (1-\gamma)e^{-(1-\theta)(t-k)}}$$

As f_t is just the derivative of F_t with respect to k, we get:

$$f_t(P_k) = \frac{\gamma(1-\theta)e^{-(1-\theta)(t-k)}}{(\gamma + (1-\gamma)e^{-(1-\theta)(t-k)})^2}$$

Therefore, inflation can be expressed as:

$$\pi_t = (1-\theta)\gamma(p_t^* - p_{t-1}) + (1-\theta)(1-\gamma)\sum_{k=0}^{t-1}\frac{2\gamma(1-\theta)e^{-2(1-\theta)(t-k-1)}}{(\gamma + (1-\gamma)e^{-(1-\theta)(t-k-1)})^3}(p_k^* - p_{t-1})$$

□

The proposition provides a few insights. First, past prices enter the inflation equation directly, and not simply through p_{t-1}. Second, the distribution has a closed-form solution. Also note that with $\gamma = 1$, equation (17) collapses to (15), the inflation dynamics equation without imitation.

2.4 Optimal Pricing

The sequence of optimal prices has thus far been taken as parametric. To discuss the full effect of imitation we must solve for the monopolistic competition equilibrium and sequence of optimal prices. We begin examining the firm's optimization problem, and comparing our approach to Galí (2008).

In the Calvo pricing case, a typical firm faces the following pricing problem where every firm adjusting its price in t will choose the same price $P_t(i) = P_t^* \; \forall i$:

$$\max_{P_t^*(\theta,1)} \sum_{k=t}^{\infty} \theta^k E_t \left\{ Q_{t,t+k}(P_t^*(\theta,1) Y_{t+k|t} - \Psi_t(Y_{t+k|t})) \right\} \tag{18}$$

where $Q_{t,t+k} \equiv \beta^k (C_{t+k}/C_t)^{-\sigma} P_t/P_{t+k}$ is the stochastic discount factor for nominal payoffs, $\Psi_t(\cdot)$ is the cost function in nominal term, and $Y_{t+k|t}$ is the output in period $t+k$ for a firm that reset its price in period t.

The first order condition is:

$$\sum_{k=0}^{\infty} \theta^k E_t \left\{ Q_{t,t+k}(P_k^* - \mathcal{M}\psi_{t+k|t}) \right\} = 0 \tag{19}$$

where $\psi_{t+k|t} \equiv \Psi'(Y_{t+k|t})$ is the marginal cost in nominal term in period $t+k$ for a firm that reset its price in period t, and $\mathcal{M} \equiv \frac{\epsilon}{\epsilon-1}$. Let $MC_{t+k|t} = \psi_{t+k|t}/P_{t+k}$ be the real marginal cost at time $t+k$ of a firm that reset its price in period t.

Equation (19) can be re-arrange as:

$$P_t^* = \mathcal{M} \frac{E_t \sum_{k=0}^{\infty} \theta^k Q_{t,t+k} Y_{t+k|t} P_{t+k} MC_{t+k|t}}{E_t \sum_{k=0}^{\infty} \theta^k Q_{t,t+k} Y_{t+k|t}} \tag{20}$$

Galí (2008) demonstrates that the log-linear approximation of the first-order condition can be written as:

$$p_t^* = m + (1-\beta\theta) \sum_{k=t}^{\infty} \theta^k \beta^k E_t \left\{ mc_{t+k|t} + p_{t+k} \right\} \tag{21}$$

where $m = \log(\mathcal{M})$.

With imitation the pricing problem of the firms is slightly different from equation (18). Firms that can optimize must take into account the impact of their price on the probability of imitating into a better price in the future. In this case firms are facing the following maximization problem:

$$\max_{P_t^*} \sum_{k=0}^{\infty} \prod_{l=0}^{k} (1 - \nu_l(P_t^*)) E_t \left\{ Q_{t,t+k} (P_t^* Y_{t+k|t} - \Psi_t(Y_{t+k|t})) \right\} \tag{22}$$

where $\prod_{l=0}^{k} (1 - \nu_l(P_{i,t}))$ is the probability that a firm with price $P_{i,t}$ will not be able to change its price at $t+k$.

In general, the first order conditions will be different under imitation and difficult to solve. However, we are interested in a special case. We care about the optimization when all firms begin at the same initial price and move towards a long-run steady-state. In this case the distribution of prices will be discrete in equilibrium.

The key to our next proposition is that, in the special case of transitory dynamics, the firm can ignore the impact of their prices on the probability of imitating in the future. The idea is that when optimal prices increase in average prices then a firm will never find it optimal to "jump" a future optimal price.

Proposition 3. *Suppose that* $\frac{\partial P_k^*}{\partial P_k} > 0$*. The optimal price characterized by*

$$P_t^* = \frac{\epsilon}{\epsilon - 1} \frac{\sum_{k=0}^{\infty} \prod_{l=0}^{k} (1 - \nu_l(P_t^*))) E_t \left\{ Q_{t,t+k} Y_{t+k} P_{t+k} MC_{t+k|t} \right\}}{\sum_{k=0}^{\infty} \prod_{l=0}^{k} (1 - \nu_l(P_t^*)) E_t \left\{ Q_{t,t+k} Y_{t+k} \right\}} \tag{23}$$

Proof

The proof involves demonstrating that the first order condition of the maximization problem is essentially unchanged (except for the probabilities). It must be shown that the influence of a firm's price on the probability of imitating is ignored because it will never increase profits.

Consider the sequence of optimal prices $\{P_k^*\}_{k=0,1,\ldots}$ and suppose that the optimal price increases in the average price. Or

$$\frac{\partial P_t^*}{\partial P_t} \geq 0$$

$\forall t$. Then for $P_0 < P_\infty^*$ the sequence of prices is always increasing, or $P_t^* < P_{t+1}^*$ as average prices continue to increase.

The probability of a firm imitating is unchanged for $P_t^* \in [P_{t-1}^*, P_{t+1}^*]$. If P_t^* is outside this interval then either P_t is not monotonic in t or P_{t+1}^* (or P_{t-1}^*) cannot be optimal, which is a contradiction.

Given that this is true the P_t^* is solved for, and it is increasing in P_t.

□

The result essentially says that the optimal price under imitation is the optimal price under Calvo pricing, with the probabilities changing over time according to the probability of imitating. The significance of the result is that the firm's decision is not affected by its ability to change the probability of imitation. Again, with $\gamma = 1$, it can be show that the optimal pricing equation with imitation, (23), collapses to equation (21).

The log-linear approximation of equation (23), is:

$$p_t^* = m + (1 - \Omega\beta) \sum_{k=0}^{\infty} \Omega^k \beta^k E_t \left\{ mc_{t+k|t} + p_{t+k} \right\} \tag{24}$$

where $\Omega = 1 - (1 - \theta)\gamma$. This result involves the same arguments made in the proof of proposition 3.

3 Discussion

We have identified a mechanism that generates sticky inflation by generalizing a commonly used sticky price model. We demonstrate the performance of the mechanism analytically.

The model leaves open several avenues for future research. First, the model is not stochastic. Future models should allow changes in the parameters (such as money supply or productivity) to vary stochastically. Second, the imitative behaviour lacks a foundation in rational optimization. Suppose that a firm faces uncertainty, and may decide whether to imitate instead of optimize based on the expected value of observing the true state of the world. We leave this for future research.

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