COMMENT ON CHAMLEY’S (1986) OPTIMAL TAXATION OF CAPITAL

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08 September 2006

A central result in Chamley’s (1986) influential article on optimal taxation of capital is Theorem 2, which characterizes the trajectory of the optimal tax rate. The present comment identifies, and then fills, a serious gap in the proof of this theorem.

In the last paragraph of the proof, Chamley considers the case in which \( v = 0 < \dot{v} \) at time \( t_1 \), and shows that this case leads to a contradiction. To complete the proof, it is also necessary to rule out the following alternative case, which he ignores: \( v = \dot{v} = 0 \) at \( t_1 \), but \( v > 0 \) and \( \dot{v} > 0 \) immediately after \( t_1 \) (since \( \overline{r} = 0 \) whenever \( t_1 < t < t_2 \)).

In this alternative case, (32) implies that \( Z = 0 \) at \( t_1 \), and (33) then implies that the following two conditions hold immediately after \( t_1 \). First, \( \dot{Z} > 0 \) because, by continuity, \( Z \) is close to 0. (Recall that \( \lambda - \mu > 0 \) in a second-best solution.) Consequently, \( Z > 0 \) as the second condition. Under these two conditions, (32) and (33) imply that for all \( t > t_1 \), \( v > 0 \) and thus \( \overline{r} = 0 \). This implication, however, is absurd (as Chamley explains). Therefore, the alternative case is also impossible.