Abstract

This paper analyzes optimal taxation in an efficiency-wage economy with involuntary unemployment, thereby extending Chamley’s (1986) optimal-tax analysis of the standard full-employment case. For this purpose, we introduce optimal savings into the shirking-unemployment model of Shapiro and Stiglitz (1984), and go beyond their exclusive focus on steady-state equilibrium. Our most surprising result is that—despite the presence of jobless workers—the government should impose a positive tax (rather than subsidy) on wage income in the long run, if the labor market is sufficiently distorted.
1. Introduction

A prominent explanation of unemployment is based on the efficiency-wage hypothesis, according to which labor productivity (or efficiency) is positively related to the real wage rate. Shapiro and Stiglitz (1984) use an optimizing framework to develop an important version of this explanation, in which firms set the wage rate to prevent shirking, because they cannot perfectly monitor labor effort. The no-shirking wage is greater than the marginal social cost of labor effort, and thus the efficiency-wage equilibrium produces a distortion in the labor market. Shapiro and Stiglitz show that the optimal policy response to this distortion in their model—with its fixed stock of capital—is to tax away all capital income and use this revenue to subsidize wages.

Phelps (1994, chap. 15) extends the Shapiro-Stiglitz (1984) model to allow for capital accumulation via optimal saving. In his extended model, however, a tax on capital income would introduce an intertemporal distortion, and it is not clear to what extent (if any) capital should be taxed to subsidize labor. One basic problem in designing optimal tax policy for a shirking-unemployment economy with optimal savings is that the no-shirking wage depends on wealth, which varies across individuals because of their different employment/unemployment histories. Thus, the distribution of wealth needs to be pinned down, in order to derive a tractable relation for determining the no-shirking wage. Brecher, Chen and Choudhri (2002) obtain such a relation for steady state, but their analysis does not cover transitional periods.

The present paper simplifies the determination of the no-shirking wage under optimal savings, by the assuming that each household makes saving and shirking decisions for its many members, using an aggregate (household) utility function. This
simplification suppresses the role of wealth differences across individuals, and enables us to derive a no-shirking wage relation for the entire transitional path (including steady state), thereby nesting the Shapiro-Stiglitz (1984) analysis as a special case. Our model also makes it possible to use a representative-agent framework to analyze optimal policy in the presence of efficiency-wage unemployment.

For the full-employment case, a well-known analysis by Chamley (1986) lets the government optimally choose wage and interest taxes to deal with a distortion (caused, in his model, by an exogenous stream of government expenditures). We consider this fiscal setting to explore optimal taxation in the presence of efficiency-wage unemployment. Although our labor-market dynamics differ from Chamley’s, we still obtain his key result that there are two interest-tax regimes—namely, a short-run regime in which interest income is taxed at a rate of 100%, and a long-run regime with no taxation of interest.

This long-run result is a sharp departure from the Shapiro-Stiglitz (1984) prescription for a maximal tax on capital income at all times. There are also notable departures regarding the taxation of labor. For example, in the short run, rather than spend all interest-tax revenue on labor subsidies, it might be better for our government to use some of this revenue to accumulate assets (negative debt) that could finance a subsidy on labor in the long run. More surprisingly, we find that it might even be optimal to tax (rather than subsidize) labor in steady state.

Sections 2 and 3, respectively, derive the fundamental no-shirking condition and the related equilibrium wage. As a preliminary analysis to elucidate the basic workings of the model, section 4 investigates the steady-state equilibrium and dynamic stability of our efficiency-wage economy in the laissez-faire case, free from government
intervention. Then, section 5 tackles the more complex problem of optimal taxation by an active government. We offer some concluding remarks in section 6.

2. To Shirk or Not to Shirk

This section discusses optimization by households, and shows that there will be no shirking if and only if a certain condition is satisfied. For this purpose, we set up a one-good model that incorporates key features of the Shapiro-Stiglitz (1984) efficiency-wage framework, but allows also for asset accumulation and transitional dynamics. At time $t$, the employment status of individual $i$ is represented by $z_i(t)$, which equals 1 or 0 as the individual is respectively employed or unemployed. The shirking behavior of an employee is given by $S_i(t)$, which (for simplicity) equals 1 if there is shirking or 0 otherwise. Thus, the individual supplies a quantity of effort equal to $z_i(t)[1 - S_i(t)]$.

Letting $C_i(t)$ denote the amount consumed at time $t$, assume that the utility function of an individual takes the following standard form:

$$U_i = \int_0^\infty e^{-\rho t} \{ C_i^{1-\theta} / (1-\theta) - l[z_i(1-S_i)] \} dt,$$

where $\rho$ is the constant rate of time preference; $\theta > 0$; $l$ is an increasing function; and the time argument is suppressed henceforth, unless needed for clarity.

For non-shirking employees, the fixed probability of being exogenously separated from a job is $b$ per unit time. Shirkers face a higher probability $b + q$ of job loss, where $q$ represents the fixed probability per unit time of being detected shirking. Unemployed workers, on the other hand, have a probability $a(t)$ per unit time of acquiring a job. To abstract from complications due to wealth differences among agents, assume that the
economy has many identical households, each consisting of numerous individuals. Each household chooses the same consumption level and shirking behavior for all of its members, to maximize a utility function that is an average of the utilities of individual members.

More specifically, assume a continuum of households in the unit interval, and similarly let each household have a continuum of individuals indexed by \( i \in [0,1] \). The utility function of a typical household is then given by \( U = \int_0^1 U_i di \). Using this function and (1), while setting \( C_i = C \) and \( S_i = S \), obtain

\[
U = \int_0^\infty e^{-\delta t} [C^{1-\theta} / (1-\theta) - \delta Z(1-S)] dt ,
\]

where \( \delta \equiv l(1) - l(0) \); \( l(0) = 0 \) by normalization; and \( Z \equiv \int_0^1 z_i di \), which equals the proportion of members who are employed.

Let \( w(t) \) and \( r(t) \) represent the pre-tax wage and interest rates. The government levies proportional taxes on wage and interest incomes at the rates \( \tau_w(t) \leq 1 \) and \( \tau_r(t) \leq 1 \), respectively. (If a tax rate is negative, it represents a positive subsidy.)

Households choose \( C \) and \( S \) to maximize \( U \) in (2), subject to the following constraints:

\[
\dot{X} = \tilde{r}X + \tilde{w}Z - C ,
\]

\[
\dot{Z} = a(1 - Z) - (b + qS)Z ,
\]

\[
S(1 - S) = 0 ,
\]

\[
X(0) = X_0 , \quad Z(0) = Z_0 .
\]

__1__ For examples of this type of specification, see Alexopoulos (2006) and Shi (1997).
where \( X \) is household wealth, while \( \tilde{w} \equiv w(1 - \tau_w) \) and \( \tilde{r} \equiv r(1 - \tau_r) \) are the take-home wage and interest rates. Constraints (3) and (4) represent wealth accumulation and employment dynamics; \(^2\) (5) restricts the choice of \( S \) to its two possible values (0 and 1); and (6) gives the initial conditions.

The Lagrangean function for the household is

\[
L^h = C^{1-\theta}/(1-\theta) - \delta Z(1-S) + \mu(\tilde{r}X + \tilde{w}Z - C) + m[a(1-Z) - (b + qS)Z] + \beta S(1-S),
\]

where \( L^h - \beta S(1-S) \) represents the current-value Hamiltonian; \( \mu \) and \( m \) are co-state variables, which can be interpreted as shadow values of wealth and employment, respectively; and \( \beta \) is a Lagrange multiplier. The necessary conditions for the household’s maximization problem are

\[
\frac{\partial L^h}{\partial C} = C^{-\theta} - \mu = 0, \quad \frac{\partial L^h}{\partial S} = (\delta - mq)Z + \beta(1-2S) = 0, \quad \frac{\partial L^h}{\partial X} = \mu(\rho - \tilde{r}), \quad \frac{\partial L^h}{\partial Z} = (\rho + a + b + qS)m + \delta(1-S) - \tilde{w}\mu,
\]

\[
\lim_{t \to \infty} \mu X e^{-\rho t} = \lim_{t \to \infty} m Ze^{-\rho t} = 0,
\]

as well as constraints (3) – (6). \(^3\)

\(^2\) Kimball (1994) extends the model of Shapiro and Stiglitz (1984) to analyze labor-market dynamics outside steady state, but (like them) does not allow for savings.

\(^3\) Throughout this paper, in stating transversality conditions like (12), we follow the general approach taken by Barro and Sala-i-Martin (2004, p. 615) for the infinite-horizon case.
The second-order condition with respect to $S$ is $\delta^2 L / \partial S^2 = -2\beta \leq 0$, which implies that $\beta \geq 0$. Clearly, $\beta > 0$ to satisfy (9) if $\delta \neq mq$ (and $Z > 0$), in which case $S$ equals 0 or 1 as $\delta$ is respectively less or greater than $mq$. On the other hand, if $\delta = mq$, $\beta = 0$ to satisfy (9), which implies that $S$ equals either 0 or 1 (i.e., households are indifferent between shirking and not shirking). In this case of indifference, adopt the Shapiro-Stiglitz (1984) convention that households choose $S = 0$. To summarize, we have the following result.

**Proposition 1.** There is no shirking at time $t$ if and only if

$$m(t) \geq \delta / q,$$

(13)

where the equality implies indifference between shirking and not shirking, while the inequality indicates a definite preference for not shirking.

To interpret this no-shirking condition, note first that $\delta$ represents the household’s gain from shirking, by which effort is being withdrawn. Since $m$ is the shadow value of employment, moreover, $qm$ is the expected loss from shirking that leads to firing if detected. Condition (13) thus simply requires that the gain ($\delta$) from shirking not exceed the expected loss ($qm$) from this behavior.

3. **No-Shirking Wage**

Assuming a large number of identical firms, we now derive a formula for the equilibrium wage at each point in time. In accordance with the conventional story, the forces of competition drive the market wage to the level that keeps households indifferent between shirking and not shirking.\(^4\) Thus, by proposition 1,

\(^4\) For a lucid account of this process, see Phelps (1994, chap. 1).
\[ m = \frac{\delta}{q} \quad \text{for all } t, \quad (14) \]

in competitive equilibrium.

If the market wage were such that \( m > \frac{\delta}{q} \) throughout an interval, each firm would be tempted to reduce its wage slightly in the interval. This reduction would not cause the firm’s employees to shirk, given a market wage better than needed to prevent shirking. (They would not quit for higher-paying jobs, because separation from the firm entails an immediate spell of unemployment.) Therefore, any market wage inconsistent with (14) would be unsustainable.\(^5\)

To derive the equilibrium wage, note that (14) implies \( \dot{m} = 0 \), substitute these two equalities into (11), set \( S = 0 \), and obtain the following result.

**Proposition 2.** At each point in time, the take-home wage is given by

\[ \tilde{w} = \frac{(\rho + a + b + q)\delta}{q\mu}, \quad (15) \]

which represents the no-shirking wage that leaves households indifferent between shirking and not shirking.

In light of this result, set \( S = 0 \) henceforth.

Equation (15) simplifies to the no-shirking wage of Shapiro and Stiglitz (1984) in the following special case: \( \theta = 0 \), given that their utility function is linear in consumption; and \( \dot{X} \equiv 0 \), because they do not allow for saving. (With this last restriction, \( \mu \) becomes a Lagrange multiplier instead of a co-state variable.) Under these assumptions, the equilibrium wage is still given by (15), but with \( \mu \equiv 1 \).

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\(^5\) To formalize this heuristic argument, we could extend the model to let each household’s shirking decision vary across firms if they offer different wage rates. Such an extension would complicate the exposition, without affecting our results.
Aggregate output is given by $F(K, Z)$; where $F$ is a neoclassical production function that exhibits homogeneity of degree one, strict quasi-concavity and the Inada conditions; $K$ represents the total stock of (fully utilized) capital; and, under our continuum-of-households assumption above, the aggregate level of employment equals $Z$. Output can be consumed or added to the capital stock, which is consumable but not otherwise subject to depreciation. Profit maximization yields the usual marginal-productivity conditions, which are

$$r = \frac{\partial F(K, Z)}{\partial K} = f'(K/Z), \quad (16)$$

$$w = \frac{\partial F(K, Z)}{\partial Z} = f(K/Z) - (K/Z) f'(K/Z), \quad (17)$$

where $f(K/Z) \equiv F(K/Z, l) = F(K, Z)/Z$.

### 4. Laissez-Faire Economy

For the preliminary case without government intervention, this section derives the steady-state equilibrium of our efficiency-wage economy, and shows that the dynamic system of the model is saddle-path stable. In this laissez-faire case, $\tau_r \equiv \tau_w \equiv 0$, $\tilde{r} \equiv r$, $\tilde{w} \equiv w$ and $X \equiv K$.

Thus, given (16) and (17), rewrite (3), (4) and (10) as

$$\dot{K} = Zf'(K/Z) - \mu^{-1/\theta}, \quad (18)$$

$$\dot{Z} = [(f(K/Z) - (K/Z)f'(K/Z)]\mu q / \delta - \rho - b - q)(1 - Z) - bZ, \quad (19)$$

$$\dot{\mu} = \mu[\rho - f'(K/Z)], \quad (20)$$

using (8) and (15) to substitute for $C$ and $a$, respectively, while recalling that $S = 0$.

Equations (18) – (20) describe the dynamic system for $K$, $Z$ and $\mu$. 
To find the steady-state equilibrium, set \( \dot{K} = \dot{Z} = \dot{\mu} = 0 \). Then, from (20), solve for \( K/Z = \bar{k} \), where overbars indicate steady-state values. Next, substitute this solution into (18) and (19) to solve simultaneously for \( \mu = \bar{\mu} \) and \( Z = \bar{Z} \). Thus, \( K = \bar{K} = k\bar{Z} \).

To show that the dynamic system is saddle-path stable, linearize (18) – (20) in the neighborhood of steady state, using a Taylor approximation. The resulting set of equations is

\[
\begin{pmatrix}
\dot{K} \\
\dot{Z} \\
\dot{\mu}
\end{pmatrix} =
\begin{pmatrix}
p_{11} & p_{12} & p_{13} \\
p_{21} & p_{22} & p_{23} \\
p_{31} & p_{32} & p_{33}
\end{pmatrix}
\begin{pmatrix}
K - \bar{K} \\
Z - \bar{Z} \\
\mu - \bar{\mu}
\end{pmatrix},
\]

where \( p_{11} = \rho, \ p_{12} = \bar{w}, \ p_{13} = \theta^{-1}\bar{\mu}^{-1/\theta-1}, \ p_{21} = -\bar{k}f^*(\bar{k})\bar{\mu}q(1 - \bar{Z})/\bar{Z}\delta, \)

\( p_{22} = -\bar{k}p_{21} - \bar{a} - b, \ p_{23} = (1 - \bar{Z})\bar{w}q/\delta, \ p_{31} = -\bar{\mu}f^*(\bar{k})/\bar{Z}, \ p_{32} = -\bar{k}p_{31}, \) and \( p_{33} = 0 \). The expressions for \( p_{11} \) and \( p_{33} \) follow from the fact that \( \bar{\rho} = \rho \) by (20). To sign \( p_{21} \) and \( p_{31} \), note that \( f^*(\bar{k}) < 0 \) by diminishing marginal productivity.

Since the sum of the eigenvalues of a square matrix equals the trace of the matrix,

\[
E_1 + E_2 + E_3 = \rho - \bar{a} - b + f^*(\bar{k})\bar{k}^2\bar{\mu}q(1 - \bar{Z})/\bar{Z}\delta,
\]

where \( E_1, E_2 \) and \( E_3 \) denote the eigenvalues of the (square) matrix in (21). Because the product of the eigenvalues of a square matrix equals the determinant of the matrix,

\[
E_1E_2E_3 = -f^*(\bar{k})\bar{\mu}[\bar{w}f(\bar{k})(1 - \bar{Z})q/\delta + (\bar{a} + b)\theta^{-1}\bar{\mu}^{-1/\theta-1}]/\bar{Z}.
\]

If not for the presence of \( \rho \) in (22), the right-hand side of this equation would always be negative. To ensure this outcome, simply assume that \( \rho < b \), although this assumption is clearly stronger than required. In this case, (22) implies that at least one
eigenvalue is negative. Then, observing that the right-hand side of (23) is always positive, we know that exactly two eigenvalues are negative.

With two negative eigenvalues, the dynamic system is saddle-path stable, and the stable arm (manifold) has two dimensions. We therefore have the following result.

**Proposition 3.** Starting from any arbitrary combination of \(K_0\) and \(Z_0\) close to \(\bar{K}\) and \(\bar{Z}\), \(\mu(0)\) takes the value needed to reach the stable arm, along which the economy converges to the steady-state equilibrium.

5. **Optimal Taxation of Capital and Labor Incomes**

Now let the government tax (subsidize) wage and interest incomes optimally, by choosing \(\tilde{w}\) and \(\tilde{r}\). This choice is equivalent to setting \(\tau_w\) and \(\tau_r\), since \(w\) and \(r\) are determined by (17) and (16), respectively. Thus, the government is constrained to set

\[
\tilde{r} \geq 0,
\]

as interest income cannot be taxed at a rate of more than 100%. A similar restriction on \(\tilde{w}\) is redundant, since the no-shirking wage is positive by (15).

The government can also issue debt, which is a perfect substitute for claims on capital. With \(D(t)\) denoting the stock of this debt at time \(t\),

\[
\dot{D} = \tilde{r}(D + K) + \tilde{w}Z - F(K, Z),
\]

which represents the deficit in the government’s instantaneous budget.\(^6\) If this deficit is negative (indicating a surplus), public wealth \((-D)\) is accumulating.

\(^6\) Without significantly affecting the results below, we could easily incorporate an exogenous flow of government expenditure that absorbs output but does not enter the utility function.
In addition to initial conditions (6), we have
\[ K(0) = K_0, \quad D(0) = X_0 - K_0, \tag{26} \]
where the second equality follows from the fact that \( X \equiv D + K \). In light of this identity, (25), (18) and (8) imply (3).

The optimal-policy problem is to maximize the representative household’s utility in (2), by choosing a combination of \( \bar{w} \) and \( \bar{r} \) for each \( t \), subject to the following constraints: transition equations (4), (18) and (25) for state variables \( Z, K \) and \( D \); household-optimization conditions (8) and (10) involving \( \mu \), which the government treats as a state variable; formula (15) for the no-shirking wage; non-negativity restriction (24) on the interest rate; and initial conditions (6) and (26). Write the Lagrangean for this problem as

\[
L^g = (\mu^{-\theta})^{1-\theta} [(1-\theta) - \delta Z + \eta(F(K, Z) - \mu^{-\theta}) + \nu[(\bar{w} \mu + \delta - \rho - q)(1-Z) - bZ] + \pi(\bar{r}(D + K) + \bar{w}Z - F(K, Z)] + \gamma(\rho - \bar{r}) + \psi \bar{r}, \tag{27}
\]

where \( \eta, \nu, \pi \) and \( \gamma \) are co-state variables; \( \psi \) is a Lagrange multiplier; and \( L^g - \psi \bar{r} \) is the current-value Hamiltonian.

The necessary conditions (additional to the relevant constraints) for the government’s problem are

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7 To avoid the problem of time inconsistency that Turnovsky (2000, p.409) addresses, make the usual assumption that the government is fully and credibly committed to honor forever the tax choices that it announces at time 0.

8 Because we are implicitly assuming an interior solution at every \( t \), it is not necessary to include explicitly the following additional constraints: \( Z \leq 1; a \geq 0 \); and \( X \geq 0 \), to avoid the moral hazard of unsecured private borrowing.
\[ \frac{\partial L^g}{\partial \tilde{w}} = \pi Z + \nu \mu (1 - Z) q / \delta = 0, \]  
(28)

\[ \frac{\partial L^g}{\partial \tilde{r}} = \pi (D + K) - \gamma \mu + \psi = 0, \ \psi \geq 0, \ \psi \tilde{r} = 0, \]  
(29)

\[ \dot{\eta} = \rho \eta - \frac{\partial L^g}{\partial K} = \eta (\rho - r) - \pi (\tilde{r} - r), \]  
(30)

\[ \dot{v} = \rho v - \frac{\partial L^g}{\partial Z} = v (\rho + a + b) + \delta - \eta w - \pi (\tilde{w} - w), \]  
(31)

\[ \dot{\pi} = \rho \pi - \frac{\partial L^g}{\partial D} = \pi (\rho - \tilde{r}), \]  
(32)

\[ \dot{\gamma} = \rho \gamma - \frac{\partial L^g}{\partial \mu} = \dot{\gamma} - (\eta - \mu) \mu^{-1/\theta - 1} / \theta - v \tilde{w} (1 - Z) q / \delta, \]  
(33)

\[ \lim_{t \to \infty} \gamma e^{-\rho t} = \lim_{t \to \infty} \nu Z e^{-\rho t} = \lim_{t \to \infty} \pi D e^{-\rho t} = \lim_{t \to \infty} \gamma w e^{-\rho t} = \gamma (0) = 0, \]  
(34)

where \( a, r \) and \( w \), satisfy (15), (16) and (17), respectively. The last equality in transversality conditions (34) is needed because \( \mu(0) \) is unconstrained.

An important implication of (10) and (32) is that

\[ \dot{\phi} = 0 \]  
(35)

where \( \phi = \pi / \mu \). Since \( \pi \) (the shadow value of public debt) can be interpreted as the marginal excess burden of using distortionary (rather than lump-sum) taxation,\(^9\) \( \phi \) is a measure of this social burden in units of household wealth (whose shadow price is \( \mu \)). This measure of the distortion is constant along the optimal path, according to (35).

Given the state of involuntary unemployment, it is natural to focus on the case in which the initial employment level \( Z_0 \) is less than socially optimal. Thus, assume that

\[ \nu (0) > 0, \]  
(36)

which means that an instantaneous (hypothetical) rise in employment at time 0 would

\[^9\] For this interpretation, see Atkinson and Stern (1974).
increase the present discounted value of lifetime utility.\textsuperscript{10} From (36) and (28), $\pi(0) < 0$.

Therefore, in light of (35),

\[ \pi < 0 \quad \text{for all } t. \quad (37) \]

Conditions (37) and (28) imply that

\[ \nu > 0 \quad \text{for all } t. \quad (38) \]

Thus, ceteris paribus, an increase in employment would always be socially desirable.

Nevertheless, stimulating employment by raising $\bar{w}$ would also increase $D$ [via (25)], whose shadow value ($\pi$) is negative [by (37)]. For this reason, the government would refrain from expanding employment beyond the level consistent with (28). In summary, we have the following result.

\textit{Proposition 4. Employment is always below the level that is socially optimal for current values of other state variables.}

Despite the presence of unemployment, the optimal path of the tax on interest income is essentially the same as in Chamley’s (1986) full-employment model. For example, given our last equality in (34), conditions (29) and (37) imply that

$\psi(0) > 0 = \bar{r}(0)$, which means that interest income should be taxed at a rate of 100\% in some initial interval. On the other hand, interest should not be taxed in steady state (where $\dot{\mu} = \dot{\pi} = 0$), since $\bar{r} = r$ from (10), (30) and (37).\textsuperscript{11} More generally, we have the following result.

\textsuperscript{10} This is analogous to the Shapiro-Stiglitz (1984) assumption that the marginal product of labor exceeds the disutility of effort in their no-savings model.

\textsuperscript{11} We are implicitly assuming the normal case in which $\eta > 0$. Otherwise, paradoxically, the government would not regret a natural disaster that instantly destroys part of the capital stock.
Proposition 5. There exists a time $t^* (> 0)$ such that interest income should be maximally taxed $\left( i.e., \tilde{r} = 0 \right)$ for $t < t^*$, but completely untaxed $\left( i.e., \tilde{r} = r \right)$ for $t > t^*$.\(^{12}\)

Thus, interest should be taxed only in the short run (at 100%), even though the labor-market distortion (indicated by $\nu > 0$) persists along the optimal path in its entirety, including the long run.

The difference between short- and long-run optimality is not so clear-cut for the wage tax. In fact, it would be reasonable to conjecture that the government should subsidize employment at every $t$, because $\nu > 0$ always. Surprisingly, however, we now show that the tax on wage income might actually be positive in steady state.

For this purpose, several features of steady-state equilibrium are used. First, we can express $Z$ as a function $\tilde{Z}(\tilde{w}\mu)$, where $\tilde{Z}'(\tilde{w}\mu) > 0$.\(^{13}\) It is also possible to write $\mu$ as a function $\tilde{\mu}(\tilde{w})$, where $-\mu/\tilde{w} < \tilde{\mu}'(\tilde{w}) < 0$.\(^{14}\) Finally, $\eta = (\phi\theta + 1)\mu$.\(^{15}\)

In light of these features, rewrite (28) as
\[
\phi\tilde{Z}[\tilde{w}\tilde{\mu}(\tilde{w})] + \left\{1 - \tilde{Z}[\tilde{w}\tilde{\mu}(\tilde{w})]\right\}wq / \delta = 0, \tag{39}
\]
where

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\(^{12}\) To verify this result, follow (mutatis mutandis) the method that Chamley uses to prove Theorem 2, but see Brecher and Chen (2006) who identify and fill a serious gap in his proof.

\(^{13}\) Set $\dot{Z} = 0$ $(= S)$ in (4) to solve for $a$, substitute this solution into (15), and obtain $Z = 1 - b / [(\tilde{w}\mu - \delta)q / \delta - \rho]$.

\(^{14}\) Set $\dot{K} = 0$ in (18) to obtain $(\bar{w} + \rho k)\tilde{Z}(\tilde{w}\mu) = \mu^{-1/\theta}$, where $k$ and $\bar{w}$ are the same as in the laissez-faire case.

\(^{15}\) Set $\dot{\gamma} = 0$ in (33); use (28), (29), and the fact that $\psi = 0$ (given $t > t^*$); set $\dot{D} = 0$ in (25), and use the resulting solution for $D + K$; observe that $\tilde{r} = \rho$ when $\tilde{\mu} = 0$ in (10); and note that $F(K, Z) = \mu^{-1/\theta}$, from (18) with $\dot{K} = 0$. 

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in steady state.\footnote{To derive (40), also set $\dot{v} = 0$ in (31), and eliminate $\rho + a + b$ via substitution from (15).} Substituting (40) into (39), solve for $\phi$ as a function $\tilde{\phi}(\tilde{w})$. It is straightforward (though tedious) to verify that $\tilde{\phi}(\tilde{w}) < 0 < \tilde{\phi}'(\tilde{w})$,\footnote{Verification relies on the properties of functions $\tilde{Z}$ and $\tilde{\mu}$, and on the fact that $\tilde{w} > \delta / \tilde{\mu}(\tilde{w})$ by (15), since $\tilde{\mu}(\tilde{w})$ equals the laissez-faire $\mu$.} which clearly implies that $\tilde{w} = \bar{w}$ as $\phi = \bar{\phi}(\bar{w})$. Thus, defining $\phi^* \equiv \bar{\phi}(\bar{w})$, we have the following result.

**Proposition 6.** The steady-state tax on wage income is greater than, equal to or less than zero as $\phi$ is respectively less than, equal to or greater than $\phi^*$.

For insight into this result, use (28) and (35) to obtain $\phi = -\nu(0)(1-Z_0)q / \delta Z_0$. Thus, if $Z_0$ is small enough, $\phi$ is less than $\phi^*$,\footnote{Note that $\lim_{Z_0 \to 0} \phi = -\infty$} and proposition 6 then implies that the wage tax is positive in steady state. Intuitively, if the initial level of employment is much less than socially optimal, the government wants to create jobs quickly, paying wage subsidies that exceed the interest-tax revenues in the short run. Then, to finance the resulting debt in the long run, a wage tax is eventually required.\footnote{The entire path of $\tilde{w}$ can be derived as follows. Differentiate (28) with respect to $t$, and solve for $\dot{\nu}$; substitute this solution into (31); use (4), (15) and (17) to substitute for $\tilde{Z}$, $\tilde{w}$ and $w$; simplify to eliminate $a$; and obtain an equation in $K$, $Z$, $\mu$, $\eta$ and $\phi$ (as well as parameters). Differentiate this equation with respect to $t$; use (4), (10), (18), (30) and (35); replace $\tilde{r}$ by 0 or $f'(K/Z)$ as $t$ is respectively less or greater than $t^*$; and solve for $a$ in terms of $K$, $Z$, $\mu$, $\eta$ and $\phi$. Substitute this solution into (15); and solve for $\tilde{w}$ as a function of $K$, $Z$, $\mu$, $\eta$ and $\phi$.}

Thus, in the long run, proposition 6 clearly rejects the Shapiro-Stiglitz (1984) prescription for a take-home wage equal to the average product of labor. On the contrary, in
steady state where $\dot{D} = 0$ and $\bar{r} = r$, (25) implies that $\bar{w} - F(K, Z) / Z = -r(K + D) / Z < 0$.

Even in the short run with $\bar{r} = 0$ and $\bar{w} - F(K, Z) / Z = \dot{D} / Z$, the Shapiro-Stiglitz prescription holds if and only if it is optimal for the government to maintain a balanced budget (with $\dot{D} = 0$ for all $t < t^*$).

6. Conclusion

Referring to employees in the Shapiro-Stiglitz (1984) model, Blanchard and Fischer (1989, p. 458) observe that “once a shirker, always a shirker”, because of the exclusive focus on steady state. This observation does not apply to our model, which deals also with transitional dynamics. Outside steady state, the no-shirking wage is affected by changes in the unemployment rate, via adjustments in the job-acquisition probability. Because we introduce asset accumulation, moreover, this wage is also affected by the shadow price of private wealth. Thus, the propensity to shirk can vary over time, and the equilibrium wage must consequently follow a path that (just barely) extinguishes shirking at each instant.

Since our efficiency-wage model of unemployment is not restricted to steady-state analysis, we can investigate the optimal path of factor-income taxes, as Chamley (1986) does for the standard full-employment case. Our investigation not only reconfirms his results on taxation of capital income, but also challenges the policy prescriptions of Shapiro and Stiglitz (1984). Most surprisingly, we find that employment should be taxed (not subsidized) in the long run, if the labor-market distortion is large enough.

20 Under a balanced-budget rule requiring $D = 0$ for all $t$, we could readily establish the following implication for the long run. In steady state, if the interest tax is set at its optimal level (still zero), the wage tax must equal zero.
References


