UNEMPLOYMENT AND WELFARE CONSEQUENCES OF INTERNATIONAL OUTSOURCING UNDER MONOPOLISTIC COMPETITION

Richard A. Brecher and Zhiqi Chen
Carleton University

October 2011

CARLETON ECONOMIC PAPERS
UNEMPLOYMENT AND WELFARE CONSEQUENCES OF
INTERNATIONAL OUTSOURCING UNDER MONOPOLISTIC COMPETITION

Richard A. Brecher and Zhiqi Chen†

Department of Economics
Carleton University
1125 Colonel By Drive
Ottawa ON K1S 5B6
Canada

October 2011

Abstract

This paper challenges the conventional academic view that international outsourcing is just another form of gainful trade. Contrary to this view, we show that labor-service outsourcing can reduce the high-wage country's welfare even when product-market trade is beneficial, within a model that combines involuntary unemployment and monopolistic competition. Outsourcing's impact on welfare is worsened by a definite loss of jobs and a possible contraction in the range of varieties produced worldwide. While owners of capital benefit from outsourcing under certain conditions, labor's welfare always falls.

Key Words: Outsourcing, Offshoring, Unemployment, Welfare, Variety

JEL Classification Codes: F12, F16, F20

†Department of Economics, Carleton University, 1125 Colonel By Drive, Ottawa ON K1S 5B6, Canada. E-mail: richard_brecher@carleton.ca, z_chen@carleton.ca. Telephone: 613-520-2600, extension 3765 (Brecher), 7456 (Chen). Fax: 613-520-3906. Chen’s research was financially supported by the Social Sciences and Humanities Research Council of Canada.
1. Introduction

This paper challenges the conventional academic wisdom in favor of international outsourcing, defined as cross-border supply of labor services without physical movement of workers across national borders.¹ Professional economists typically argue that outsourcing is socially beneficial because it is essentially just another type of international trade.² This argument is unconvincing for two reasons. First, in accordance with a well-established literature, trade can be harmful in the presence of domestic distortions that are difficult or impossible to correct.³ Second, even if such a distortion does not prevent gains from trade in goods, the subsequent introduction of outsourcing can still reduce national welfare, as our paper shows.

The present analysis incorporates a distorted labor market with involuntary unemployment, because concerns over job loss are at the heart of public opposition to outsourcing. To ensure that trade (in goods) is beneficial despite the existence of jobless workers, we develop a simple one-sector model similar to that of Matusz (1998), who assumes monopolistic competition and efficiency-wage unemployment.⁴ As he shows, (intra-industry) trade expands the range of product varieties, thus raising the real wage and relaxing the efficiency-wage constraint.

¹This definition corresponds to “mode 1” trade of services in the terminology of the World Trade Organization, as discussed by Bhagwati, Panagariya and Srinivasan (2004). The same phenomenon is also known as “offshoring”, “offshore outsourcing” or simply “outsourcing”.

²A prominent statement of this view is provided by Mankiw and Swagel (2006). Incidentally, their paper also documents how its first author ignited a “political firestorm” in 2004 when he, as chair of the (U.S.) Council of Economic Advisors, publicly expressed this view of outsourcing.

³An early demonstration of this long-known point appears in the classic article by Haberler (1950).

⁴Whereas his model has a single input (labor), ours includes an additional one (capital), so that relative factor intensities create an incentive for outsourcing.
relaxation increases the number of jobs and magnifies the benefits of trade, as the additional employment further expands the variety range.\footnote{Outsourcing in the presence of unemployment is analyzed also by Brecher and Chen (2010), Brecher, Chen and Yu (2011), Davidson, Matusz and Shevchenko (2008), Egger and Kreickemeir (2008), Koskela and Stenbacka (2010), Mitra and Ranjan (2010) and Zhang (2011). However, in these studies (unlike the present one), trade either is absent or has uncertain effects on welfare.}

Thus, it is tempting to conjecture that outsourcing (as a form of labor-service trade) would similarly lead to gains in employment and welfare. As workers participate in the production process of another country where their marginal productivity is higher, this improved allocation of labor tends to raise world output, and hence increase the number of product varieties. Such an increase in variety would (as before) relax the efficiency-wage constraint and expand employment. More variety and employment would mean higher welfare.

However, we show that this reasoning is incomplete, because it neglects an important first-order effect of employment adjustment. More specifically, an immediate consequence of outsourcing is to lower (raise) the marginal product of labor in the high-wage (low-wage) country, which reduces (increases) national employment there. Such an employment change is clearly detrimental to welfare of workers in the high-wage country, given the existence of involuntary unemployment.

This negative employment effect on welfare could be offset if the total number of product varieties rises with outsourcing. But such a rise is not guaranteed, because the world levels of employment and output might actually decrease, as jobs shift to workers in the low-wage country. Interestingly, this shift might even cause unemployment in the high-wage country to increase by more than the number of jobs outsourced. We also show that the owners of capital in
the high-wage country benefit as long as outsourcing not only increases the aggregate amount of labor used by firms located there, but also expands the total number of varieties produced worldwide.

The rest of this paper is organized as follows. Section 2 sets up the model, which section 3 uses to examine the effects of outsourcing. Section 4 concludes.

2. Basic Model

To analyze outsourcing, we construct a framework that combines a monopolistic-competition model of intra-industry trade and an efficiency-wage model of unemployment. Two countries, home and foreign, use capital and labor to produce a single differentiated good. All varieties of this good are freely traded internationally. Although neither input is able to move physically across national borders, outsourcing permits the services of one country’s labor to be used in the other economy’s production. In each country, involuntary unemployment persists as firms keep the wage above the full-employment level to discourage shirking in the absence of perfect monitoring. Individuals maximize the expected value of lifetime utility over an infinite horizon, with neither borrowing/lending nor saving/investment.

The only assumed difference between the two countries lies in their endowments of labor. To be more specific, suppose that \(\bar{L} < \bar{L}^*\); where \(\bar{L}\) and \(\bar{L}^*\) denote the labor endowments of the home and foreign countries, respectively. The sizes of capital endowments, denoted by \(\bar{K}\) and \(\bar{K}^*\), are the same in the two countries. Without loss of generality, let \(\bar{K} = \bar{K}^* = 1\) by choice of units. Within each country, every worker owns an equal share of the capital stock.
2.1. Product Market

Each of the \( L \) workers at home has the following utility function:

\[
u = \left( \sum_{i=1}^{I} c_i^{1-1/\sigma} \right)^{\sigma/(\sigma-1)} - e,
\]

where \( I \) denotes the number of varieties consumed, \( c_i \) represents consumption of variety \( i \), and \( e \) stands for effort. The value of \( e \) is a positive constant \( \bar{\sigma} \) for employed non-shirkers, but 0 for shirking employees and unemployed workers. It is well known that \( \sigma \), a constant assumed to be greater than unity, is equal to the elasticity of substitution between each pair of varieties. Except for the inclusion of \( e \) to accommodate the Shapiro-Stiglitz (1984) focus on shirking, our utility function in (1) is the love-of-variety type introduced by Spence (1976) and Dixit and Stiglitz (1977). Alternatively stated, our (1) extends Shapiro and Stiglitz to allow for product differentiation.

Let \( p_i \) denote the price of variety \( i \), and \( y \) represent the consumer’s income. The individual’s demand for variety \( i \) can be derived from (1), and expressed as follows:

\[
c_i = \frac{y}{p_i^\sigma q},
\]

where \( q \equiv \sum_{j=1}^{I} p_j^{-\sigma} \). Let \( N \) and \( N^* \) be the number of varieties produced at home and abroad, respectively, implying that \( I = N + N^* \). We assume that the equilibrium value of \( I \) is large enough to ensure that each individual producer of a single variety treats \( q \) as a parameter. Accordingly, the perceived elasticity of world demand with respect to price is \( \sigma \) for every firm.

\(^{6}\text{See, for example, Helpman and Krugman (1985, p. 118).}\)
On the production side, each variety of the consumption good is manufactured at a plant using capital and labor. Furthermore, production and sales of each variety require one unit of headquarter services for the purpose of administration and marketing. For simplicity, we assume that headquarter services and plant output use the same production function, which is identical for all varieties. This production function satisfies the Inada conditions, and exhibits constant returns to scale, with positive but diminishing marginal products.

As defined above, outsourcing is the use of foreign labor services in home production, without workers moving physically across national borders. In the absence of outsourcing, the wage rate would be higher at home than abroad, as shown below. Accordingly, when outsourcing is allowed, home firms will have an incentive to employ foreign labor. Let $S$ denote the amount of outsourcing (i.e., the amount of foreign labor used in home production). Given our assumption that all plants and headquarters use the same constant-returns-to-scale technology, the aggregate level of home output can be expressed as $X = F(K, L + S)$—of which $N$ units are headquarter services, while the remaining $X - N$ are units of the differentiated good sold and consumed domestically and abroad. Using a familiar property of constant returns to scale, we can rewrite aggregate output as

$$X = \bar{K}F\left(1, \frac{L+S}{K}\right) = \bar{K}f\left(\frac{L+S}{K}\right) = f(L+S),$$ (3)

---

7For simplicity of exposition, we treat $S$ as an exogenous parameter, as in the case (for instance) of an outsourcing quota given to foreign workers without charge. Nevertheless, our analysis is easily extendable to make $S$ endogenous. Specifically, we could assume that outsourcing proceeds to the point at which the home wage equals the foreign wage multiplied by $1+T$; where $T$ is an ad valorem cost of transaction, which home firms face when using foreign labor. An exogenous reduction in the parameter $T$ would lead to an endogenous increase in $S$, thereby reproducing all of our results derived below.
recalling that $K = 1$; where $f'' > 0 > f'''$ for marginal productivity that is positive but diminishing.

Since the production of an additional variety involves a fixed cost in the form of a unit of headquarter services, each firm produces at most one variety in equilibrium. Constant returns to scale imply that $m$—denoting the marginal and average cost of (plant or headquarter) output for every variety—is independent of the quantity produced. To maximize profits, a firm sets marginal cost ($m$) equal to marginal revenue, given (as usual) by $p_i(1-1/\sigma)$. Thus, every firm has the same price, denoted by

$$p = \frac{m}{1-1/\sigma}. \quad (4)$$

This implies that in equilibrium, each of the $N$ home firms sells the same quantity $(X - N)/N$ of the differentiated product, since they face identical demand curves in world markets.

Free entry and exit drive the profits of each firm to zero in equilibrium. Noting that the firm earns revenue $p(X - N)/N$ and incurs total costs $mX/N$, the zero-profit condition ensures that $p(X - N) = mX$. Using this equality and (4), we determine the equilibrium quantity sold by each home firm to be

$$\frac{X - N}{N} = \sigma - 1. \quad (5)$$

This equation implies that $N = X / \sigma$, which means that the number of varieties produced in equilibrium is proportional to the level of output.

Since each country has the same $\sigma$, (5) and its foreign counterpart imply that all firms in the
world sell the same amount $\sigma - 1$. Given that they also face the same demand schedule in world markets, the price of each foreign variety also equals $p$ (the price charged by home firms).

Demand equation (2) now simplifies to

$$c_i = \frac{y}{p(N + N^*)}. \quad (6)$$

From (6), (1) and the fact that $I = N + N^*$, indirect utility is given by

$$u = \frac{y}{p}(N + N^*)^{1/(\sigma - 1)} - e. \quad (7)$$

Thus, $u$ rises with the individual’s real income ($y/p$) and the total number of varieties ($N + N^*$), but falls with effort ($e$).

2.2 Labor Market

The value of $y$ equals $w + r/\bar{L}$ or $r/\bar{L}$, respectively, as the worker is employed or jobless; where $w$ is the nominal wage rate of labor; and $r$ denotes the nominal rental rate of capital, as well as the national earnings of this factor (given the above normalization that $\bar{K} = 1$). Thus, utility in (7) is $(N + N^*)^{1/(\sigma - 1)}(w + r/\bar{L})/p - \bar{e}$ for a non-shirking employee,

$(N + N^*)^{1/(\sigma - 1)}(w + r/\bar{L})/p$ for an employed shirker, and $(N + N^*)^{1/(\sigma - 1)}r/\bar{L}p$ for a jobless worker.

Within the tradition of efficiency-wage theories of unemployment, we follow the familiar approach of Shapiro and Stiglitz (1984). Accordingly, firms can monitor workers only

---

8To simplify the exposition, we assume that there are no unemployment benefits.
imperfectly, and an employee who shirks will be caught and fired with a fixed probability of $q$ per unit time. A worker may also be separated from employment for reasons other than shirking, at a constant rate of $b$ per unit time. Let $\rho$ denote the fixed rate of time preference. By equating the expected life-time utilities of a non-shirker and a shirker à la Shapiro and Stiglitz, we derive the following equation for the wage that firms must offer to prevent shirking in equilibrium$^9$:

$$
\frac{w}{p} (N + N^*)^{1/(\sigma - 1)} = \bar{\sigma} + \frac{\bar{\tau}}{q} \left( \frac{b}{1-L/L} + \rho \right) \equiv \omega(L/L),
$$

(8)

where the function $\omega$ gives (a utility-based measure of) the no-shirking wage, defined as the lowest wage consistent with provision of effort.$^{10}$ For any given value of $w/p$, an increase in the total number of varieties would relax efficiency-wage constraint (8), and hence allow employment to rise because we clearly have $\omega' > 0$.

Standard cost-minimization conditions of a firm imply that

$$
w = mf'(L + S),
$$

(9)

$$
r = m[ f(L + S) - (L + S)f'(L + S) ],
$$

(10)

where $f'(L + S)$ and $f(L + S) - (L + S)f'(L + S)$ are the marginal products of labor and capital, respectively.

From (3) – (5), (8), (9) and their foreign counterparts, we have

---

$^9$This equation is a straightforward extension of the Shapiro-Stiglitz (binding) constraint (11).

$^{10}$The no-shirking wage does not depend on $r/L$, because this income (from capital) is earned by employed and unemployed workers alike.
\[
\phi(L, L^*, S) = \frac{\sigma^{-1}}{\sigma/(\sigma-1)} f'(L + S)[f(L + S) + f(L^* - S)]^{1/(\sigma-1)} - \omega \left( \frac{L}{L} \right) = 0, \tag{11}
\]
\[
\phi^*(L, L^*, S) = \frac{\sigma^{-1}}{\sigma/(\sigma-1)} f'(L^* - S)[f(L + S) + f(L^* - S)]^{1/(\sigma-1)} - \omega \left( \frac{L^*}{L^*} \right) = 0, \tag{12}
\]

where \( L^* \) is the level of foreign employment. Because \( S \) units of employment are outsourced by home firms to foreign workers, \( L^* - S \) units of labor are used in foreign production. With \( S \) treated as a parameter, (11) and (12) simultaneously determine the home and foreign employment levels (\( L \) and \( L^* \)) and unemployment rates (\( 1 - L / L \) and \( 1 - L^* / L^* \)).

In the pre-outsourcing equilibrium where \( S = 0 \), (11) and (12) yield
\[
\frac{f'(L)}{f'(L^*)} = \frac{\omega(L/L)}{\omega(L^*/L^*)}. \tag{13}
\]

If \( L^* \) were equal to \( L \), (13) would clearly imply that \( L = L^* \), and hence that \( w = w^* \) by (9) and its foreign counterpart together with (4); where \( w^* \) is the foreign wage. In this case, there would be no incentive for outsourcing. On the other hand, under our assumption that \( L^* > L \), we have \( w > w^* \) (given \( f'' < 0 < \omega' \)) before the introduction of outsourcing. Moreover, this wage inequality—like the corresponding marginal-product inequality \( f'(L + S) > f'(L^* - S) \)—persists as long as \( S \) is below its free-outsourcing level. Hence, \( L + S < L^* - S \) (by \( f'' < 0 \)) and \( L / L > L^* / L^* \) [by (8), (9) and their foreign counterparts], for \( S \) in the relevant range.

As shown in the Appendix, the \( \phi \) and \( \phi^* \) functions satisfy the properties that \( \phi_{L^*} = \phi^*_L > 0 ; \) where subscripts of functions indicate partial derivatives (e.g., \( \phi_{L^*} \equiv \partial \phi / \partial L^* \)). The Appendix
also shows that \( \phi_L < 0, \phi_L^* < 0 \) and \( \Delta = \phi_L \phi_L^* - \phi_L^* \phi_L^* > 0 \) under the following assumption (whose plausibility is established at the end of the present section):

\[
\sigma > 1 - \frac{[f'(\lambda)]^2}{f(\lambda)f''(\lambda)},
\]

(14)

for \( \lambda = L + S, \ L^* - S \). We make this assumption because its above-mentioned implications (for \( \phi_L \), \( \phi_L^* \) and \( \Delta \)) guarantee existence and uniqueness of equilibrium, as follows.

Since \( \phi_L < 0 \), a unique value of \( L \) satisfies (11) for any given combination of \( L^* \) and \( S \).\(^{11}\)

Similarly, because \( \phi_L^* < 0 \), a unique value of \( L^* \) satisfies (12) for any given combination of \( L \) and \( S \). In view of the fact that \( \Delta > 0 \), the model has a unique equilibrium. To see this, consider the two (generally nonlinear) curves in Figure 1.\(^{12}\) The home curve shows combinations of \( L \) and \( L^* \) that satisfy (11) for a given value of \( S \). The slope of this curve is \( -\phi_L / \phi_L^* \), which is positive because of (14). The horizontal intercept of the home curve is also positive, since (11) can be solved for \( L > 0 \) even if \( L^* = 0 \). Similarly, the slope \( (-\phi_L^* / \phi_L^*) \) and vertical intercept of the foreign curve corresponding to (12) are both positive. A unique equilibrium (at a single point of intersection of the two curves) is assured if the home curve is steeper than the foreign one, which is true because \( \Delta > 0 \).

\(^{11}\)Note that \( \lim_{L \to L^*} \phi = -\infty \) (since \( \lim_{L \to L^*} \omega = \infty \)), and \( \lim_{L \to 0} \phi = \infty \) (because \( \lim_{L \to 0} f' = \infty \) by the Inada conditions). Therefore, by continuity, there exists exactly one value of \( L \) for which \( \phi = 0 > \phi_L \).

\(^{12}\)This diagram is similar to Matusz’s (1998) Figure 3.
Condition (14) requires that the elasticity of substitution between each pair of varieties exceeds a certain threshold determined by properties of the production function. In the special case of Cobb-Douglas technology, (14) simplifies to

\[ \alpha \sigma > 1, \]

where \( \alpha \) represents the ratio of non-wage earnings to total income. Typical estimates of \( \alpha \) place it in the neighborhood of 1/3 for countries like the United States. In Matusz’s (1998) calibration of free trade’s effects on the U.S. economy, \( \sigma \) takes on values between 10/3 and 10. In light of these numbers, our condition (15) seems plausible, at least for some countries.

3. Effects of Outsourcing

We now investigate the effects of outsourcing by conducting comparative statics with respect to \( S \). To abbreviate some lengthy mathematical expressions, let \( f, f^*, \omega \) and \( \omega^* \) serve as short-hand notation for \( f(L + S), f(L^* - S), \omega(L / L) \) and \( \omega(L^* / L^*) \), respectively. Similar notational abbreviations apply to the derivatives of these four functions.

Start by considering outsourcing’s effects on home and foreign employment, with the aid of (11) and (12). If \( L \) and \( L^* \) are held constant temporarily, a small rise in \( S \) reduces the (utility of) labor’s marginal product below the no-shirking wage at home, while doing the opposite abroad. Thus, \( L \) tends to fall and \( L^* \) rise, to restore (11) and (12), respectively.

But this direct effect on employment levels is not the end of the story. The changes in \( S, L \) and \( L^* \) cause the total number of varieties—proportional to \( f(L + S) + f(L^* - S) \) in (11) and (12)—to increase or decrease (as elaborated below). Such an increase (decrease) tends to raise
(lower) the real wage above (below) the no-shirking wage, thereby having a positive (negative) effect on $L$ and $L^*$. If this variety effect is positive (negative), it opposes (reinforces) the direct effect on home (foreign) employment. But the variety effect is not sufficient to outweigh the direct effect, as established by the following proposition.

**Proposition 1:** An increase in outsourcing reduces home employment but raises foreign employment.

**Proof:** Totally differentiate (11) and (12) with respect to $S$, and solve the resulting equations simultaneously to obtain

$$
\frac{dL}{dS} = \frac{\beta}{\Delta} \left[ \left( f^* + \frac{(f')^2 - f' f^*}{(\sigma - 1)(f + f^*)} \right) \omega^* - \frac{f^- f^* f^* + (f')^2 f^* + (f^*)^2 f^*}{(\sigma - 1)(f + f^*)} \right] < 0, 
$$

(16)

$$
\frac{dL^*}{dS} = \frac{\beta}{\Delta} \left[ \beta \left[ f^* f^* + \frac{(f')^2 f^* f^* + (f^*)^2 f^*}{(\sigma - 1)(f + f^*)} \right] - \frac{f^* f^* - (f' f^*) f^*}{(\sigma - 1)(f + f^*)} \right] \omega^* > 0,
$$

(17)

where $\beta \equiv \left[ \frac{(\sigma - 1)}{\sigma^{1/(\sigma - 1)}} \right] (f + f^*)^{1/(\sigma - 1)}$. The inequalities in (16) and (17) are determined by (14) and the fact that $f' > f^*$. QED

Proposition 1 indicates that outsourcing causes employment to shift from home to foreign workers. This shift clearly exacerbates the unemployment problem at home. (At the same time, the expansion of employment abroad helps to compensate the foreign country for some of the production activities that it loses when $S$ units of its labor services are shifted to home production.) This secondary effect raises the intriguing possibility that the total amount of labor used at home, $L + S$, might not rise with $S$. In other words, home unemployment might increase by more than the number of jobs outsourced. Such a possibility could arise if the efficiency wage is not sufficiently responsive to changes in employment, in accordance with the following
Proposition 2: An increase in outsourcing has an ambiguous effect on the total amount of labor used in home production, \( L + S \). This increase raises \( L + S \) if the efficiency wage’s elasticity with respect to the employment rate is large enough to ensure that \( \left( \frac{\omega'}{\omega} \right) \left( \frac{L}{\bar{L}} \right) > L f^* / (\sigma - 1)(f + f^*) \).

Proof: Use (16) to derive

\[
\frac{d(L + S)}{dS} = \frac{1}{\Delta} \left[ \frac{\omega'}{L} - \frac{\omega f^*}{(\sigma - 1)(f + f^*)} \right] \frac{\omega^*}{L} - \beta \left[ f^* + \frac{(f^*)^2}{(\sigma - 1)(f + f^*)} \right] \frac{\omega'}{L}.
\]  

(18)

The sum of terms in this equation’s second pair of square brackets is negative because of (14). Hence, the right-hand side of (18) is positive if the sum of terms in the first pair of square brackets exceeds zero, which will be the case if \( \left( \frac{\omega'}{\omega} \right) \left( \frac{L}{\bar{L}} \right) > L f^* / (\sigma - 1)(f + f^*) \). QED

Now returning to outsourcing’s impact on the total number of varieties, note that

\[
\frac{d(N + N^*)}{dS} = \frac{1}{\sigma} \frac{d(f + f^*)}{dS} = \frac{1}{\sigma} \left( f' - f'^* \right) + \frac{1}{\sigma} \left( f' \frac{dL}{dS} + f^* \frac{dL^*}{dS} \right),
\]  

(19)

from (3), (5) and their foreign counterparts. Since \( f' - f'^* > 0 \), (19) implies that the variety range would expand if employment levels in both countries were held fixed (i.e., if \( dL / dS = dL^* / dS = 0 \)). However, when the endogenous adjustments in employment are taken into account, the reduction in \( L \) creates ambiguity for the sign of \( d(N + N^*) / dS \), according to the following proposition.

Proposition 3: An increase in outsourcing generally has an ambiguous effect on the total number of varieties produced and consumed in the world. However, the number of varieties increases if
the elasticity of labor’s marginal product—namely, $-\lambda f''(\lambda) / f'(\lambda)$ — is a non-decreasing function of $\lambda$ in the relevant range.

**Proof:** Use (16), (17) and (19) to obtain the following result:

$$\frac{d(N + N^*)}{dS} = \frac{1}{\sigma_\Delta} \left[ \left( f' - f^* \right) \frac{\omega' \omega^*}{L^L} + \beta f'' f^* \left( \frac{\omega^*}{L^*} - \frac{f^*}{L^*} \right) \right].$$ (20)

Since $f' - f^* > 0$, the right-hand side of (20) is positive if the expression in the second pair of parentheses is positive. From (8) and its foreign counterpart, we know that

$$\omega' = \bar{\omega} b / q(1 - L / \bar{L})^2 > \bar{\omega} b / q(1 - L^* / \bar{L}^*)^2 = \omega^*,$$ recalling (from section 2) that $L / \bar{L} > L^* / \bar{L}^*$.

Thus, since $(L + S) / \bar{L} > (L^* - S) / \bar{L^*}$, the right-hand side of (20) is positive if

$$-(L + S) f''(L + S) / f'(L + S) \leq -(L^* - S) f''(L^* - S) / f'(L^* - S).$$ The latter inequality is satisfied if $-\lambda f''(\lambda) / f'(\lambda)$ is a non-decreasing function of $\lambda$, when we recall (from section 2) that $L + S < L^* - S$. QED

In the special case of a Cobb-Douglas production function, the elasticity of labor’s marginal product is constant. In this particular case, Proposition 3 implies that the variety effect on home employment is unambiguously positive.

Turning to outsourcing’s effects on home welfare, let $V^E$ and $V^U$ denote the present discounted value of lifetime utility for an employed (non-shirking) worker and an unemployed individual, respectively. Then, welfare of the home country can be measured by the national average value of lifetime utility, given by
\[ V \equiv V^E \frac{L}{\bar{L}} + V^U \left( 1 - \frac{L}{\bar{L}} \right). \]  

(21)

Following the Shapiro-Stiglitz (1984) method of derivation—modified obviously for the present generalization of their utility function—use our efficiency wage constraint (8), and rearrange terms to obtain the following two equations: \(^{13}\)

\[ V^E = \frac{1}{\rho + b} \left[ \frac{w + r}{p} \left( N + N^* \right)^{1/(\sigma - 1)} - \bar{e} + bV^U \right], \]

(22)

\[ V^U = \frac{1}{\rho} \left[ \frac{w + r}{p} \left( N + N^* \right)^{1/(\sigma - 1)} - \bar{e} - \frac{(\rho + b)\bar{e}}{q} \right]. \]

(23)

Substitute (22) and (23) into (21) to yield

\[ V = \frac{1}{\rho L} \left[ \frac{wL + r}{p} \left( N + N^* \right)^{1/(\sigma - 1)} - \bar{e}L \right]. \]

(24)

Using (4), (9) and (10) to replace \( w \) and \( r \) in (24), we find that.

\[ V = \frac{1}{\rho L} \left[ \left( 1 - \frac{1}{\sigma} \right) [f(L + S) - Sf'(L + S)](N + N^*)^{1/(\sigma - 1)} - \bar{e}L \right] \equiv \tilde{V}(L, N + N^*, S). \]

(25)

The \( \tilde{V} \) function is strictly increasing in each of its first two arguments and non-decreasing in the third, according to the following conditions:

\[ \frac{\partial \tilde{V}}{\partial L} = \frac{1}{\rho L} \left[ \frac{w}{p} \left( N + N^* \right)^{1/(\sigma - 1)} - \bar{e} - \left( 1 - \frac{1}{\sigma} \right) (Sf'(N + N^*)^{1/(\sigma - 1)} - \bar{e} \right] > 0, \]

(26)

\(^{13}\)These two equations are straightforward generalizations of the Shapiro-Stiglitz conditions (2) and (5), respectively, when the no-shirking constraint is binding.
\[
\frac{\partial \tilde{V}}{\partial(N + N^*)} = \frac{(f - Sf')(N + N^*)^{(2-\sigma)/(\sigma-1)}}{\sigma \tilde{\rho} \tilde{L}} > 0 , \tag{27}
\]

\[
\frac{\partial \tilde{V}}{\partial S} = \frac{1}{\rho \tilde{L}} \left( 1 - \frac{1}{\sigma} \right) (-Sf^*)(N + N^*)^{1/(\sigma-1)} \geq 0 . \tag{28}
\]

To verify (26), reuse (4) and (9), while noting that \((w/p)(N + N^*)^{1/(\sigma-1)} > \tilde{e}\) in light of (8).

Condition (27) holds because \(f - Sf^*(L + S) > 0\). The weak inequality in (28) follows from the assumption that \(S \geq 0\).

In the event of an increase in outsourcing, its overall impact on home welfare can now be conceptually decomposed into three components, as follows. Differentiate (25) totally with respect to \(S\), to obtain

\[
\frac{dV}{dS} = \frac{\partial \tilde{V}}{\partial L} \frac{dL}{dS} + \frac{\partial \tilde{V}}{\partial(N + N^*)} \frac{d(N + N^*)}{dS} + \frac{\partial \tilde{V}}{\partial S} . \tag{29}
\]

The first product on the right-hand side of this equation represents the employment effect, which is negative because \(\frac{\partial \tilde{V}}{\partial L} > 0\) and \(\frac{dL}{dS} < 0\) from (26) and (16), respectively. The second product is the variety effect, which has an ambiguous sign, in light of (27) and Proposition 3. The remaining term in (29) represents the direct effect, which is non-negative by (28). We thus have the following proposition.

**Proposition 4**: An increase in outsourcing generally has an ambiguous effect on home welfare. However, if outsourcing’s initial level is zero and its subsequent increase is small, welfare unambiguously falls if the total number of varieties fails to rise.

**Proof**: The results follow from (26) – (29), together with Propositions 1 and 3. QED
An alternative way to assess the welfare effects of outsourcing is to examine its impact on owners of capital and labor. To do so, rewrite (24) as

$$V = \left[ \omega \left( \frac{L}{L} \right) - \bar{\sigma} \right] \frac{L}{L \rho} + \left( 1 - \frac{1}{\sigma} \right) (N + N^*)^{1/(\sigma-1)} \frac{f(L + S) - (L + S) f'(L + S)}{L \rho},$$

(30)
after using (3) – (5), (8) and (10).

The terms in the first and second pairs of curly brackets on the right-hand side of (30) represent the respective contributions that labor and capital make to average lifetime utility of the home country. The first of these contributions depends positively on $L$ (given that $\omega' > 0$ and $\omega - \bar{\sigma} > 0$). The second contribution depends positively on both $N + N^*$ and $L + S$ (since $f'' < 0$). Thus, we have the following proposition.

**Proposition 5:** An increase in outsourcing reduces labor’s but ambiguously affects capital’s contribution to home welfare. The contribution of capital rises if there is an increase in both the aggregate amount of labor $(L + S)$ used in home production and the total number of varieties $(N + N^*)$.

**Proof:** The results follow from (30). QED

If we abandon our assumption that ownership of capital is equally distributed, and assume instead that workers own no assets, the labor contribution in (30) is equivalent to the average lifetime utility of (employed and jobless) workers as a whole. Then, an outsourcing-induced fall in $L$ implies a definite drop in labor’s welfare, regardless of a possible increase in the worldwide total of available varieties.

Note that the presence of involuntary unemployment plays a crucial role in the ambiguous
welfare effects within our model. Alternatively, if both countries were instead assumed to be in a perpetual state of full employment (with \( dL / dS = dL^* / dS = 0 \)), the total number of varieties would always rise with international outsourcing, as implied by (19). In (29), the variety effect would thus be positive, and the employment effect would be zero (by assumption). In other words, in a world of full employment, the home country would unambiguously gain from outsourcing.

It is also worth emphasizing an important difference between the effects of (product-market) trade and (labor-service) outsourcing, in the presence of monopolistic competition and efficiency-wage unemployment. As Matusz (1998) shows, trade increases the number of varieties and hence relaxes the efficiency-wage constraint, thereby leading to employment gains in both countries. These gains magnify the benefits of trade, by enabling the production of even more varieties. On the other hand, we demonstrate that outsourcing has a negative direct effect on home employment, possibly causing a welfare-reducing contraction in the number of varieties.

4. Conclusion

Our analysis identifies three key effects that outsourcing has on welfare of the high-wage country. First, there is the employment effect, which is welfare-reducing via an exacerbation of the unemployment problem. Second, we have the variety effect, which has ambiguous implications for welfare, as the worldwide range of varieties can either contract or expand. Third, there is the direct effect, which represents the welfare improvement that would arise if the employment level and variety range were constant. If all workers were to remain fully employed, the employment effect would be absent and the variety effect would be favorable, in
which case outsourcing would unambiguously improve welfare. But in a world with involuntary unemployment, one can no longer assert that outsourcing is just another type of gainful trade.
Appendix

Partially differentiate (11) and (12) to obtain the following results:

\[
\phi_{L*} = \phi_{L} = \beta \frac{f' f^{*'}}{(\sigma - 1)(f + f^*)},
\]

\[
\phi_L = \beta \left[ f'' + \frac{(f')^2}{(\sigma - 1)(f + f^*)} \right] - \frac{\omega'}{L},
\]

\[
\phi_{L*} = \beta \left[ f^{*''} + \frac{(f^*)^2}{(\sigma - 1)(f + f^*)} \right] - \frac{\omega^*}{L},
\]

\[
\Delta = \beta^2 \left[ f'' f^{*''} + \frac{(f')^2 (f^{*'})^2}{(\sigma - 1)(f + f^*)} \right] + \frac{\omega' \omega^*}{L L^*}
\]

\[
-\beta \left[ f'' + \frac{(f')^2}{(\sigma - 1)(f + f^*)} \right] \frac{\omega^*}{L^*} - \beta \left[ f^{*''} + \frac{(f^*)^2}{(\sigma - 1)(f + f^*)} \right] \frac{\omega'}{L}.
\]

The expressions for \( \phi_{L*} \) and \( \phi_L \) are both positive, because \( f' > 0 \) and \( f^{*'} > 0 \). Given condition (14) along with the facts that \( \omega' > 0 \) and \( \omega^{*'} > 0 \), we also have \( \phi_L < 0 \), \( \phi_{L*} < 0 \) and \( \Delta > 0 \).
References


Figure 1. Uniqueness of Equilibrium