Financing Constraints, Firm Dynamics, and International Trade

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FINANCING CONSTRAINTS, FIRM DYNAMICS, AND INTERNATIONAL TRADE*

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Abstract

This paper studies the impact of financial constraints on exporter dynamics, and the role of financial intermediation in international trade. We propose a two-country general equilibrium model economy in which entrepreneurs and lenders engage in long-term credit relationships. Financial markets are endogenously incomplete because of private information, and financial constraints arise as a consequence of optimal financial contracts. In equilibrium, competitive financial intermediaries actively channel individuals’ short-term deposits to fund a diversified portfolio of long-term risky firms. Young and small firms operate below their efficient level, and their financial constraint is relaxed as the entrepreneur’s claim to future cash-flows increases. Consistent with empirical regularities, there is a substantial year-to-year transition in and out of export markets for smaller firms, and new exporters account only for a small share of total exports. Established exporters are less likely to exit export markets and tend to experience slower, albeit more stable growth.

Keywords: private information, dynamic optimal contracts, exporter dynamics, financial intermediation

JEL classifications: F10, D82, L14

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1 Introduction

There is substantial empirical evidence that the financial conditions faced by small and young firms play an important role in shaping their growth. This is widely interpreted as indirect evidence of frictions in financial markets, since small- and medium-sized firms tend to be more reliant on external financing, which is mostly debt.¹

Exporters are rare, and most of them are small. In the United States, for example, less than 5 percent of all firms exported some of their production in 2010. More than 97 percent of these exporters were small- and medium-sized firms (500 employees or less), which only accounted for about 34 percent of total exports.² In contrast to large exporters, there is substantial year-to-year transition in and out of export markets for smaller firms. New exporters are typically small relative to the average exporter and frequently stop exporting after one year. New exporters are more likely to exit export markets and to experience faster, but also more volatile growth than established exporters (Bernard and Jensen (2004), Ruhl and Willis (2008)).

Hysteresis in export markets suggests the presence of fixed costs of entry (e.g., Das, Roberts, and Tybout (2007), Paravisini, Rappoport, Schnabl, and Wolfenzon (2011)), thereby suggesting that participation in international trade may require even greater access to external financing. This in turn could imply that the export decisions of small- and medium-sized firms are sensitive to the availability of credit; and the widespread use of relatively expensive trade credit among exporters could suggest that many face binding credit constraints (Antras and Foley (2011)).³

This paper seeks answers to three questions: How does access to credit affect firms’

¹See Hubbard (1998) and Stein (2003) for surveys.
²http://www.census.gov/foreign-trade/Press-Release/edb/2010/text.txt
³Minetti and Zhu (2011) find evidence that access to credit has an impact on firm export status, and Amiti and Weinstein (2011) find that access to credit has an impact on the intensity of exports. Other studies find a positive association between a firm’s financial health and its export status (e.g., Greenaway and Kneller (2007), Muuls (2008) and Manova, Wei, and Zhang (2011), and Bellone, Musso, Nesta, and Schiavo (2010)). An important challenge in any such analysis stems from difficulties in directly measuring the extent of firm financial constraints and in separating the effect of credit supply from credit demand on export decisions. For example, it could also be the case that firms’ export status has an important impact on their financial health by giving them access to a larger market and further risk diversification.
decisions to export? How do credit relationships shape the growth of new exporters? And what is the role of financial intermediaries in allocating capital to finance international trade? We argue that financial frictions that disproportionately affect small and young firms are likely to play an important role in shaping exporter dynamics, and that the allocation of credit in the economy depends on the distribution of firm characteristics. Accordingly, we propose a theory in which firm financial constraints arise as a consequence of endogenously incomplete financial markets, and in which the allocation of capital in the economy is consistent with the aggregate effects of all individual decisions.

Small and young firms are generally more opaque to external scrutiny. This opacity creates an informational asymmetry between lenders and entrepreneurs leading to adverse selection and moral hazard problems. As a result, competitive banks may choose to either ration the supply of credit to young and small firms instead of increasing the price of credit to clear the market (e.g., Stiglitz and Weiss (1981)), or to reduce the impact of private information through repeated interaction and monitoring of firms (e.g., Diamond (1984, 1991), Rajan (1992), and Allen and Gale (1999)). Quadrini (2004) and Clementi and Hopenhayn (2006) show that long-term financial contracts that are constrained efficient under private information can help account for some of the empirical regularities on firm dynamics – firm entry and exit, and the mean and variance of firm growth.

Building on this research and Dixit and Stiglitz’s (1977) model of monopolistic competition, we propose a general equilibrium two-country model economy in which entrepreneurs and lenders enter into dynamic lending relationships that are constrained efficient under private information. Endogenous borrowing constraints arise as the outcome of the optimal lending contract. Our general equilibrium framework provides a novel link between industry dynamics, the balance sheets of lenders, and aggregate conditions, thereby relating financial intermediation to international trade.

Our theory lies at the intersection of corporate finance, international trade, and macroeconomics. The effect of financing constraints due to private information on firm selection into export markets is also studied by Feenstra, Li, and Yu (2011), who find
strong support for the theory in Chinese plant-level data. Related research on exporter dynamics has modeled exporter dynamics as the outcome of learning as in Eaton, Eslava, Jinkins, Krizan, and Tybout (2012), investment in risky R&D as in Atkeson and Burstein (2010), and persistent idiosyncratic shocks to productivity (e.g., Ruhl and Willis (2008), Arkolakis (2011), Alessandria and Choi (2011), and Kohn, Leibovici, and Szkup (2011)). Models of monopolistic competition in which firms face uncertainty about their productivity and fixed export costs account well for the cross-section of exporters and the pattern of intra-industry trade (e.g., Melitz (2003) and Chaney (2008)). Dynamic extensions of this class of heterogeneous firm models in which firms experience persistent productivity shocks are, however, less successful in accounting for exporter dynamics. In particular, these models cannot account well for the facts that sales of new exporters are more volatile than those of established exporters, and that the survival rate of continuing exporters rises over time. Additionally, the large market entry costs proposed in these models are at odds with the observation that many firms export for short periods on a very small scale (Ruhl and Willis (2008), Eaton, Eslava, Krizan, Kugler, and Tybout (2011)).

A key difference in our framework is that selection into export markets does not depend on a firm’s (expected) productivity as in Melitz (2003)—expected productivity is constant in our model. Rather, selection into export markets depends on a firm’s present value of expected discounted cash flows, whose evolution is governed by the financial contract and the firm’s performance over time.\textsuperscript{4}

In the model, entrepreneurs are born with a blueprint to start a long-lived monopolistic firm. A firm requires an initial fixed investment to start, and working capital to pay for factor inputs and trade costs before production takes place. New entrepreneurs do not have sufficient wealth to start a firm, and must seek financing from competitive financial intermediaries. Financial frictions arise because financial intermediaries cannot directly observe the revenue generated by the firms they are financing, and must

\textsuperscript{4}There is widespread empirical evidence that exporters are more productive than domestic firms, and we abstract from productivity heterogeneity to focus on the role of financial constraints. In Section \textit{7}, we discuss how firm productivity and financial market incompleteness may jointly shape exporter dynamics.
instead rely on reports from creditor entrepreneurs. Financial intermediaries mitigate the moral hazard by offering new entrepreneurs a long-term financing contract designed to induce truthful reporting. The financial arrangement in our model is closely related to that in Clementi and Hopenhayn (2006), and a financing constraint emerges as an outcome of the optimal contract. Financial intermediaries actively engage in maturity and risk transformation in a competitive financial market using workers’ and entrepreneurs’ short-term deposits to fund a diversified portfolio of long-term risky projects.

In equilibrium, new firms operate below their efficient level, and the financing constraint is relaxed as the entrepreneur’s claim to future cash-flows increases. Firms that are able to service their debt for a sufficiently long time may borrow enough to pay the trade costs and expand into international markets. New exporters are less financially constrained than domestic firms, but their growth continues to depend on their performance each period until they become fully unconstrained. Importantly, poor performance leads to a decline of the firm and may force it to exit export markets.

Consistent with empirical regularities on firm dynamics (e.g., Cooley and Quadrini (2001)), the model implies that older and larger firms have on average lower and more stable growth rates, and are more likely to survive; that smaller and younger firms pay fewer dividends, and borrow and invest more; and that the investment of small firms is more sensitive to cash flows. Consistent with empirical studies on exporters (e.g., Eaton, Eslava, Kugler, and Tybout (2007) and Ruhl and Willis (2008)), our model implies that exporters are larger in terms of sales and employment. New exporters account only for a small share of total exports, and a large fraction of new exporters does not continue to export in the following year. Continuing exporters are less likely to exit export markets the longer they export, have larger and more stable sales, and generally reach their efficient size in a few years.

The rest of the paper is organized as follows: Section 2 presents the model and Section 3 describes the financial arrangement between investors and entrepreneurs and Section 4 derives the properties of the optimal contract. Section 5 defines the general equilibrium, and section 6 analyzes the model numerically. Section 7 discusses the main
result and its relation to productivity and the effect of other sources of financial market incompleteness. In particular, we disentangle the effect of moral hazard relative to productivity by discussing an extension of our model in which firms face persistent idiosyncratic shocks to their productivity. We also show how our mechanism differs importantly from other source of financial market incompleteness, such as limited contract enforcement. Section 8 concludes by presenting new facts on exporters’ finances and lending relationships using the 2003 Survey of Small Business Finances with firm export status from matched firms in the National Establishment Time-Series database. Proofs of propositions and derivations are relegated to the Appendix.5

2 Model

Time is discrete, infinite, and each period is indexed by \( t \). The world is comprised of two, possibly symmetric, countries, each populated by a measure of workers and entrepreneurs. The agents’ career path is an endowment and cannot be altered. Countries may trade intermediate goods, but capital can only be invested domestically.

2.1 Workers

Workers are born without wealth, and are endowed with one unit of time each period. Following Blanchard (1985), workers survive into the next period with constant probability \((1 - \gamma_w)\), and are instantly replaced by newborns upon death. Workers discount the future at rate \((1 - \gamma_w)\beta\), and allocate their time between labor \( h_t \) and leisure. Labor is paid a wage \( w_t \), and workers use their income to buy the numéraire consumption good \( c_t \), and to insure against mortality risk by purchasing annuities \( d_{t+1} \) priced at \( p^a_t \).6 Workers do not value bequests, and thus place all their savings in these claims. Workers

5Details of derivations and two extensions of the model are contained in an online appendix available at: https://sites.google.com/site/veranistephane/home/working-papers/Gross_Verani_Trade_OnlineAppendix.pdf

6These annuities pay \((1 + r_t)\) units of consumption in the next period if the agent is alive, and zero otherwise.
assess their consumption-leisure decision according to

$$E_0 \sum_{t=0}^{\infty} [(1 - \gamma_w) \hat{\beta}]^t u(c_t, 1 - h_t), \quad (1)$$

which they maximize subject to a budget constraint

$$c_t + p_t^a d_{t+1} \leq (1 + r_t) d_t + w_t h_t, \quad (2)$$

and $d_{t+1} \geq \epsilon$, where $\epsilon$ is the workers' natural borrowing limit.

### 2.2 Entrepreneurs

Entrepreneurs are born with a blueprint to produce an intermediate good $\omega \in \tilde{\Omega}$. Entrepreneurs, like workers, are born without wealth, survive into the next period with probability $(1 - \gamma_e)$, and are instantly replaced upon exit. Entrepreneurs are risk-neutral, and discount the future at the rate $\beta$. We assume entrepreneurs do not make labor-leisure decisions, and instead devote a fixed fraction of their time to supervising their firm. Entrepreneurs assess their consumption decision according to

$$E_0 \sum_{t=0}^{\infty} \beta^t c_t, \quad (3)$$

where $\beta = \left( \frac{1 - \gamma_e}{1 + r} \right)^7$. Entrepreneurs do not take part in the annuity market, and consume all their period income.

### 2.3 Financial intermediation

Financial intermediaries are risk-neutral and discount the future at the same rate as entrepreneurs. Financial intermediaries maximize profit using short-term deposits from workers and entrepreneurs to finance a portfolio of long-lived intermediate good producers. Perfect competition and constant returns to scale in the financial intermediation

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7We are anticipating that the risk free rate $r_t$ is constant in the stationary equilibrium.
sector imply that intermediaries can be own by any coalition of agents, letting us focus on a representative financial intermediary without loss of generality.

2.4 Intermediate good production

Producing an intermediate good $\omega \in \tilde{\Omega}$ requires starting a firm. A firm requires an initial investment $I_0$ that is sunk, and per period working resources $R_t$ to hire labor and rent capital. The $\omega$-th firm produces the $\omega$-th good according to a neo-classical production function $G(k_t, n_t)$, where $k_t$ is the capital input and $n_t$ is the labor input. We assume that the capital used in production in one period is fully depreciated at the end of the period. An entrepreneur wishing to export must pay a fixed export cost $I_E$ before production begins, and chooses the quantity $q_t$ and $q^*_t$ of goods to sell domestically and abroad, respectively. The allocation of period working resources $R$ must satisfy:

$$k + nw + 1_{(q^* > 0)}I_E \leq R ,$$

where $1_{(q^* > 0)}$ is an indicator function that is equal to 1 when $q^* > 0$. Furthermore, a firm must ship $(1 + I_T)$ units of this good to sell one unit of goods abroad. This is a standard iceberg cost, and implies that the allocation of output between domestic and export sales must satisfy:

$$q + q^*(1 + I_T) \leq G(k, n) .$$

The $\omega$-th firm is a monopolist for its differentiated product, and takes the inverse demand function for its product $p(q)$ – price as a function of quantity – as given. Firm status is indexed by $i \in \{D, E\}$, where $D$ and $E$ indicate that a firm sells to the domestic market only, or to both the domestic and export market, respectively. The maximum

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8In what follows, variables marked with an asterisk denote exported goods’ quantities and prices.
revenue a firm can generate with resources $R$ is:

$$F_i(R) = \max_{q,q^*,k,n} p(q)q + 1_{(i=E)}p^*(q^*)q^*$$

s.t. $q + q^*(1 + I_T) \leq G(k,n)$

$$k + nw + 1_{(q^* > 0)}I_E \leq R.$$  \hspace{1cm} (6)

We assume there exists a unique level of resources past which a firm can only maximize its revenue by exporting some of its production. That is, we assume that there exists $R_{dx}$ such that $F_D(R) > F_E(R)$ for all $R < R_{dx}$ and $F_D(R) < F_E(R)$ for all $R > R_{dx}$. If no such level exists, trade is never profitable and the two countries do not trade. In the appendix, we show that if $R_{dx}$ exists, then the crossing point of $F_D(R)$ and $F_E(R)$ is unique. A firm is terminated when the entrepreneur dies.\footnote{This assumption is convenient to capture other sources of exit not modeled explicitly and is sufficient to obtain a stationary distribution of firms. See for instance Cooley and Quadrini (2001), Cooley, Marimon, and Quadrini (2004), and Smith and Wang (2006).}

### 2.5 Final good production

The final good is assembled by a large number of perfectly competitive firms using domestically produced and imported intermediate goods, and a constant returns to scale technology. Intermediate goods are imperfect substitutes, and final good producers maximize profit taking the price of intermediates as given. As with financial intermediaries, constant returns to scale and perfect competition in the final good market imply zero-profits, and lets us concentrate on a representative final good producer.

### 2.6 Information

Revenues from the production of intermediate goods are subject to a sequence of independent and identically distributed idiosyncratic revenue shocks $(\theta_t)_{t \geq 0}$, so that entrepreneurs receive $\theta_t F_i(R_t)$ for $i = \{D,E\}$ after production takes place. We assume that $Pr(\theta_t = 1) = 1 - Pr(\theta_t = 0) = \pi$, so that the expected revenue in any period
given a firm has resources $R$ at its disposal is $\pi_{Fi}(R_t)$ for $i = \{D, E\}$. A friction arises because the realization of revenues are only observed by entrepreneurs, and lenders can only learn about the firm performance and the realizations of the revenue shocks $\theta_t$ through entrepreneurs’ reports, $\hat{\theta}_t$.

### 3 The optimal long-term financial contract

Denote the history of reports up to period $t$ by $h^t = (\hat{\theta}_1, \ldots, \hat{\theta}_t)$. A contract is a set of rules $\kappa_t = \{\ell_t(h^{t-1}), e_t(h^{t-1}), Q_t(h^{t-1}), R_t(h^{t-1}), \tau_t(h^{t-1}, \hat{\theta}_t)\}$ that depends on the history and the current report from the entrepreneur. Conditional on surviving, a firm is either liquidated, $\ell_t(h^{t-1})$, in which case the entrepreneur receives $Q_t(h^{t-1})$ and the financial intermediary receives $S - Q(h^{t-1})$, where $S \leq I_0$ is the salvage value, or it remains in operation. If a firm is kept in operation, the contract specifies whether or not the firm exports, $e_t(h^{t-1})$, and the size of the loan, $R_t(h^{t-1})$. After production takes place and revenues are realized, an entrepreneur makes a repayment $\tau(h^{t-1}, \hat{\theta}_t)$ to the financial intermediary conditional on his ex-post report $\hat{\theta}_t$.

A reporting strategy for an entrepreneur is a sequence of reports $\hat{\theta} = \{\hat{\theta}_t(\theta^t)\}$, where $\theta^t = (\theta_1, \ldots, \theta_t)$ is the true history of realizations of revenue shocks. After every history $h^{t-1}$, the pair $(\kappa, \hat{\theta})$ implies an expected discounted cash flow $V_t(\kappa, \hat{\theta}, h^{t-1})$ and $B_t(\kappa, \hat{\theta}, h^{t-1})$ for the entrepreneur and the financial intermediary, respectively. A feasible and incentive compatible contract is optimal if it maximizes $B_t(V_t)$ for every possible $V_t$. Following Clementi and Hopenhayn (2006), we refer to $V_t$ and $B_t$ as equity and debt, respectively, so that the joint surplus $W(V) = B(V) + V$ is the value of the firm.

Using the method of Abreu, Pearce, and Stacchetti (1990), the optimal contract can be written recursively by using $V_t$ as a state variable and by defining $V_t^H$ and $V_t^L$ as promised continuation values. It follows that equity must satisfy the following accounting

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10 This revenue shock could for instance be interpreted as a demand shock (whether there is demand for the product or not) or a productivity shock (whether productivity is 0 or 1).

11 We refer the reader to Clementi and Hopenhayn (2006) for a more formal characterization of the set of contracts.
identity:

\[ V = \pi F_i(R) - \tau(\hat{\theta}) + (R - k - nw - 1_{(q^{*} > 0)}I_E) + \beta[\pi V^H + (1 - \pi)V^L], \quad (7) \]

which states that current period equity is equal to the expected net cash flow this period plus the discounted expected equity next period. In order to induce truthful reporting, incentive compatibility constraints are required. Since entrepreneurs who receive a low shock do not have an incentive to report a high shock, there is only one binding constraint:

\[ F_i(R) - \tau(\hat{\theta} = 1) + \beta V^H \geq F_i(R) - \tau(\hat{\theta} = 0) + \beta V^L. \quad (8) \]

Limited liability requires entrepreneurs’ dividends to be non-negative, so that

\[ \tau(\hat{\theta}) \leq 1_{\theta = 1}F_i(R) + (R - k - nw - 1_{(q^{*} > 0)}I_E). \quad (9) \]

In equilibrium, the financial intermediary provides resources to an intermediate good firm such that \( R = k - nw - 1_{(q^{*} > 0)}I_E \). This implies that \( \tau(\hat{\theta} = 0) = 0 \), and we simplify the notation by letting \( \tau(\hat{\theta} = 1) = \tau \). Conditional on surviving, the value of an \( i \)-type firm is given by,

\[
\bar{W}_i(V) = \max_{\tau, R, V^H, V^L} \pi F_i(R) - (1 + r)R + \beta[(1 - \pi)W(V^L) + \pi W(V^H)] \\
\text{s.t.} \quad (7), (8), \text{ and } (9) \\
V^H, V^L \geq 0.
\]

There exists a region \([V_D, V_E]\) within which a greater value of the joint surplus can be
reached by allowing for a lottery on the export decision:\(^\text{12}\)

\[
\hat{W}(V) = \max_{\delta \in [0,1], V_D, V_E} \delta \hat{W}_E(V_E) + (1 - \delta) \hat{W}_D(V_D)
\]

\[
\text{s.t. } \delta V_E + (1 - \delta) V_D = V, \quad V_D, V_E \geq 0
\]

(11)

where \(\delta\) is the probability of entering (remaining in) export markets, and \(V_D\) and \(V_E\) are the respective continuation values (within the same period) if the firm sells purely domestically or also exports. Last, \(W(V)\) takes into account the option of liquidating the firm

\[
W(V) = \max_{\alpha \in [0,1], Q, V_r} \alpha S + (1 - \alpha) \hat{W}(V_r)
\]

\[
\text{s.t. } \alpha Q + (1 - \alpha) V_r = V, \quad Q, V_r \geq 0
\]

(12)

where \(\alpha\) is the probability of liquidation and \(V_r\) is the continuation value when the firm is not liquidated.\(^\text{13}\) Figure 1 summarizes the timing of events within one period, and Proposition 3.1 summarizes some basic properties of the value function.

Proposition 3.1 The value function \(W(V)\) is increasing and concave. Furthermore, there exist values \(0 < V_r < V_D < V_E < \tilde{V}\) such that:

- \(W(V)\) is linear for \(V \in [0, V_r] \cup [V_D, V_E]\), equal to \(\hat{W}\) when \(V = \tilde{V}\) and strictly increasing when \(V \in [V_r, V_D] \cup [V_E, \tilde{V}]\)

- The firm is liquidated with probability \(\alpha(V) = (V_r - V)/V_r\) if \(V \in [0, V_r]\), and 0 otherwise

- The firm exports with probability 1 if \(V \in [V_E, \tilde{V}]\), \(\delta(V) = (V - V_D)/(V_E - V_D)\) when \(V \in (V_D, V_E)\), and 0 otherwise

\(^\text{12}\)See Proof of Proposition 3.1 in the appendix for more details.

\(^\text{13}\)We implicitly assume that \(V_D > V_r\), which is is a necessary condition to obtain non-exporting firms.
4 Properties of the optimal contract

Before proceeding to the definition of the general equilibrium, it is instructive to first study the behavior of the firms and financial intermediary in partial equilibrium. Panel (a) of Figure 2 plots the optimal value of the firm, $W(V)$, and the value to the intermediary, $B(V)$, as a function of equity, $V$. A firm faces a binding borrowing constraint whenever its equity is below $\tilde{V} = \pi F_E(\tilde{R})/(1 - \beta)$, where $\tilde{R}$ is the unconstrained level of resources. That is, $\tilde{R}$ is the level of resources that solves the static profit maximization of the firm such that $\tilde{R} = \operatorname{argmax}_R (\pi F_E(R) - R(1 + r))$. New firms start at $V_0 \leq \tilde{V}$ (which is pinned down in general equilibrium), so that expected profits of the intermediary $B(V_0)$ cover the cost of the initial investment $I_0(1 + r)$. Smaller firms take on more debt than larger firms, and firms with equity less than $V_D$ cannot borrow enough, or do not find it profitable, to pay the trade costs.

Panel (b) of Figure 2 summarizes the evolution of firm equity following a particular sequence of revenue shocks. Small domestic firm may start exporting after receiving a finite sequence of positive revenue shocks. New exporters are less financially constrained than domestic firms, but their growth continues to be shaped by the optimal contract.

\[ V^L(V) = \begin{cases} \frac{V - \pi F_D(R(V))}{\beta} & \text{if } V \in [V_r, V_D] \\ \frac{V - \pi F_E(R(V))}{\beta} & \text{if } V \in [V_E, \tilde{V}] \end{cases} \quad (13) \]

and

\[ V^H(V) = \begin{cases} \frac{V + (1 - \pi) F_D(R(V))}{\beta} & \text{if } V \in [V_r, V_D] \\ \min \left\{ \tilde{V}, \frac{V + (1 - \pi) F_E(R(V))}{\beta} \right\} & \text{if } V \in [V_E, \tilde{V}] \end{cases} \quad (14) \]

Panel (b) of Figure 2 summarizes the evolution of firm equity following a particular sequence of revenue shocks. Small domestic firm may start exporting after receiving a finite sequence of positive revenue shocks. New exporters are less financially constrained than domestic firms, but their growth continues to be shaped by the optimal contract.
and the revenue shocks as long as $V_E < \tilde{V}$. And from Proposition 4.1, firms grow on average.

**Proposition 4.1** *Conditional on surviving, a firm grows on average. That is \( \{V'\}_{t \geq 0} \) is a sub-martingale so that \( E(V'|V) \geq V \).*

![Figure 3 About Here]

Figure 3 plots the decision rules for loans, repayments, and dividends as a function of equity. Due to risk-neutrality, joint surplus is maximized when equity grows fastest, so dividends to the entrepreneur are optimally zero until the firm can no longer grow faster by postponing dividends, which is when $V^H(V) = \tilde{V}$. This implies that it is optimal for the financial intermediary to set the entrepreneur’s repayments to $\tau(V) = F_i(R(V))$ for $i = \{D, E\}$ whenever $V^H(V) < \tilde{V}$ as it allows for the fastest accumulation of equity toward the unconstrained level. Furthermore, the optimization problem takes place on the convex set $[0, \tilde{V}]$, which implies $V^H(V) = \tilde{V}$ whenever $(V + (1 - \pi)F_i(R(V))/\beta > \tilde{V}$. From constraints (7) and (8):

$$\tau(V) = \begin{cases} 
F_i(R(V)) & \text{if } V^H(V) < \tilde{V} \\
\beta(\tilde{V} - V^L(V)) & \text{if } V^H(V) = \tilde{V} 
\end{cases}, \quad (15)$$

which implies that conditional on a high revenue shock, resource advancement $R(V)$ and repayment $\tau(V)$ increase with firm equity up until $V^H(V) = \tilde{V}$. Past this threshold, repayments start declining and dividend payments start increasing until they eventually reach 0 and $F(R)$, respectively. At this size, firm equity no longer changes, and the borrowing constraint ceases forever. The value of an unconstrained firm is

$$W(\tilde{V}) = \tilde{V} + B(\tilde{V}) = \frac{\pi F_E(\tilde{R})}{1 - \beta} - \frac{\tilde{R}(1 + r)}{1 - \beta}. \quad (16)$$

Therefore, a firm is financially unconstrained when its entrepreneur has accumulated enough capital through its repayments to the financial intermediary to finance the firm.
operation at full scale in every period and under all contingencies at the current interest rate.

Panel (c) of Figure 3 plots the investment rate conditional on receiving a high and low revenue shock as a function of equity. Investment by constrained firms is always positive after receiving a high shock, and always negative after receiving a low shock for constrained firms. The investment of small firms, and therefore their cash flow, is also more sensitive to revenue shocks than that of larger firms. Furthermore, there is a large increase in investment once the firm becomes an exporter, with subsequent very high possible disinvestment should the firm receive a low shock and exit export markets.

5 General equilibrium

The workers’ problem can be written recursively as

\[ U(d) = \max_{d',c,h} u(c, h) + (1 - \gamma_w) \beta U(d') \]
\[ \text{s.t. } c + (1 - \gamma_w)d' = (1 + r)d + wh \]
\[ d' \geq -\epsilon \]  

(17)

in which our assumptions on worker characteristics implies a stationary demographic, and that the actuarially fair annuities are priced at the survival rate in equilibrium. Let \(d_j\) and \(h_j\) be the deposits and hours worked of a \(j\)-period old worker. Normalizing the mass of workers to one and given our demographic assumption, it follows that aggregate net-deposits \(D\) and hours worked \(H\) by workers each period are given by

\[ D = \gamma_w \sum_{j=1}^{\infty} (1 - \gamma_w)^j d_j, \text{ and } H = \gamma_w \sum_{j=0}^{\infty} (1 - \gamma_w)^j h_j. \]  

(18)

Perfect competition in the financial sector implies that the representative financial

\[ ^{15}\text{Analogous conditions must hold at home and abroad.}\]
intermediary breaks even on new contracts, such that

\[ V_0 = \sup_v \{ B(V) = (1 + r)I_0 \} . \quad (19) \]

New firms starts with equity \( V_0 \), which then evolves according to the state contingent contract rules \( \kappa(V) = \{ \alpha(V), R(V), \tau(V), V^L(V), V^H(V) \} \), and the sequence of revenue and death shocks. We focus our analysis on the steady state of the economy, which is characterized by a unique stationary distribution of firms \( \mu \) from Proposition 5.1.\(^{16}\)

**Proposition 5.1** \( \exists \) a unique and ergodic stationary distribution of firms \( \mu \).

Constrained entrepreneurs increase their stake in their firm’s future cash flow by making positive payments to the intermediary, while unconstrained entrepreneurs have accumulated enough equity through repayments to self-finance their firm. The representative intermediary holds a portfolio of claims to future cash flows (when \( B(V) > 0 \)) and obligations to finance firms in the future (when \( B(V) < 0 \)) worth \( \int B(V)d\mu \) in present value terms. The financial intermediary makes zero profits because of perfect competition, which implies that the net-present value of her assets is also zero. In the aggregate, \( \int B(V)d\mu + Z = 0 \), where \( Z \) can be positive, negative or zero. A positive \( Z \) implies that the intermediary uses entrepreneur deposits (i.e., retained earnings) from her portfolio of firms to finance workers’ debt. A negative \( Z \) implies that the intermediary raises more deposits from workers to finance her portfolio of firms.\(^{17}\)

The financial intermediary’s budget must be balanced each period, which implies that aggregate entrepreneurs’ deposits evolves according to the following law of motion:

\[ Z' = (1 + r)Z + \int \pi \tau(V)d\mu + \Gamma_bS - (1 + r) \left( \int R(V)d\mu + \Gamma I_0 \right) . \quad (20) \]

\(^{16}\)The existence and ergodicity results for \( \mu \) depend on the property of the optimal decision rule for period loan \( R(V) \), which is not a monotone function of \( V \) because of the randomization regions. We discuss how to deal with the non-monotonicity of \( R(V) \) in the appendix.

\(^{17}\)Note that since both workers and entrepreneurs face a stochastic death, efficient intermediation of funds from savers to borrowers requires an infinitely lived institution for record keeping, thereby providing a rational for the existence of a financial intermediary.
That is, $Z'$ is equal to returns on aggregate entrepreneur deposits $(1+r)Z$ plus aggregate net-payments from entrepreneurs $\int \pi \tau(V)d\mu$ and the scrap value of all liquidated firms $\Gamma_b S$ net of the cost of financing firms’ operation $(1+r)\int R(V)d\mu$ and start-up costs of new firms $(1+r)\Gamma I_0$. Aggregate entrepreneur deposits is constant in the steady state, which implies that $Z' = Z$ and the balanced budget equation simplifies to:\textsuperscript{18}

\begin{equation}
(1 + r) \left( \int R(V)d\mu + \Gamma I_0 \right) - \int \pi \tau(V)d\mu - \Gamma_b S = rZ ,
\end{equation}

which states that the return on aggregate entrepreneur deposits should be just enough to offset any mismatch between aggregate payments from and to entrepreneurs. It follows that the aggregate demand for capital is equal to the aggregate resources required by all firms plus the sum of fixed costs to start new firms. The aggregate supply of capital is equal to workers and entrepreneurs’ aggregate net-deposits. It follows that the capital market clears when

\begin{equation}
\int R(V)d\mu + \Gamma I_0 = D + Z .
\end{equation}

The labor market clears when aggregate labor hours supplied by workers equal to the demand for labor by firms, so that:

\begin{equation}
H = \int nd\mu .
\end{equation}

The intermediate goods markets at home clears when the quantity of intermediates demanded by the representative final good producer is equal to the quantity supplied by intermediate good firms:

\begin{equation}
y(\omega) = q(\omega) \quad \forall \omega \in \Omega , \text{ and } y(\omega_f) = q(\omega_f) \quad \forall \omega_f \in \Omega_f ,
\end{equation}

where $y(\omega)$ and $y(\omega_f)$ are the quantities of domestic and imported goods demanded by the final good producer, and $\Omega$ and $\Omega_f$ are the sets of intermediate goods available from

\textsuperscript{18}Note that $Z$ is necessarily finite since $Z \leq B(\tilde{V})$ and the optimal firm size is finite.
domestic and foreign producers, respectively – similar conditions must also hold abroad. Trade between countries must be balanced, so that the total value of imports is equal to the total value of exports times the exchange rate $X$:\footnote{Note that a condition concerning arbitrage between the home and foreign final good is not necessary as the final good cannot be traded.}

$$\int_{\Omega_f} y(\omega_f)p(\omega_f)d\omega_f = X\int_{\Omega^*} y(\omega^*)p(\omega^*)d\omega^*, \quad (25)$$

where $p(\omega)$ and $p(\omega_f)$ are the prices of domestic and imported intermediates goods, respectively. The final good market clears when total production equals aggregate consumption of workers and entrepreneurs plus investment:\footnote{Refer to the on-line appendix for a proof that Walras’s law holds in this economy.} That is,

$$Y = C_w + C_e + K, \quad (26)$$

where total capital expenditure $K$, aggregate consumption by workers, $C_w$, and by entrepreneurs, $C_e$, are given by

$$K = \int k\mu + \Gamma_eI_E + \Gamma I_0 - \Gamma bS, \quad C_w = rD + wH, \quad \text{and} \quad C_e = \pi \int F(R)d\mu - \pi \int \tau d\mu .$$

The definition of the worldwide stationary equilibrium follows:

**Definition 5.2 (Worldwide stationary equilibrium)** A worldwide stationary equilibrium consists of decision rules $\{h(d),c_w(d),d'(d)\}$ for workers, a contract $\kappa(V)$ between entrepreneurs and the financial intermediary, an initial contract state $V_0$, wage rates $\{w,w^*\}$, interest rates $\{r,r^*\}$, prices for intermediate goods $\{p(\omega),p_f(\omega)\}$ and $\{p(\omega^*),p_f(\omega^*)\}$; and an exchange rate $X$, such that in each country

1. $h(d)$, $c_w(d)$, and $d'(d)$ maximizes the workers’ value function $U(d)$

2. $\kappa(V)$ maximizes the value of the firm $W(V)$

3. the financial intermediary breaks even on new contracts (equation 19)
4. the financial intermediary’s budget is balanced every period (equation 20)

5. the labor and capital markets clear (equations 23 and 22)

6. the intermediate good markets clear (equation 24)

7. trade is balanced (equation 25)

Proposition 5.3 \( \exists \) a worldwide stationary equilibrium.

6 Numerical analysis

Once the value function \( W(V) \) and the decision rule for loan size \( R(V) \) are known, the remaining decision rules can be expressed in closed form as functions of \( R(V) \). Given the initial firm size \( V_0 \) and the law of motion for \( V \), we can simulate the life-cycle of a large number of firms to estimate the stationary distribution of firms.\(^{21}\)

6.1 Parameterization

Let the instantaneous utility function for the workers be \( u(c, h) = \ln(c) + \lambda \ln(1-h) \).\(^{22}\) We simplify the analysis by considering a world of symmetric countries in which intermediate goods are produced with constant returns to scale Cobb-Douglas production technology:

\[
G(k, n) = k^{\eta_k} n^{\eta_n}, \text{ with } \eta_n = 1 - \eta_k.
\]

The final good is assembled using a constant elasticity of substitution (CES) production function, also with constant returns to scale. The final good producer maximizes its profit given by:

\[
\left( \int_{\Omega} y(\omega)^{\frac{\sigma-1}{\sigma}} d\omega + \int_{\Omega_f} y(\omega_f)^{\frac{\sigma-1}{\sigma}} d\omega_f \right)^{\frac{\sigma}{\sigma - 1}} - \int_{\Omega} y(\omega)p(\omega)d\omega - X \int_{\Omega_f} y(\omega_f)p(\omega_f)d\omega_f \quad (27)
\]

\(^{21}\)The code to solve and simulate the model is written in object-oriented Python using the Scipy library, and is available from the authors.

\(^{22}\)This functional form implies closed-form solutions for the aggregate supply of labor and aggregate deposits given the workers’ demographic assumption, which simplifies the numerical implementation and reduces the computational burden—see Smith and Wang (2006) for more details.
where $\sigma > 1$ is the elasticity of substitution between varieties; and the assumption that countries are symmetric implies that the exchange rate $X$ is equal to 1.

A period in the model is 1 year. We begin by fixing five parameters: The worker death rate $\gamma_w$ is chosen so that the average working life of workers is 50 years. The iceberg cost of exporting is set to 40 percent, which is in line with previous studies such as Anderson and van Wincoop (2004). The probability of a high revenue shock $\pi$ is 0.5, which produces investment volatility roughly in line with studies of firm dynamics such as Cooper and Haltiwanger (2006); and the salvage value $S$ is set to 80 percent of the set-up cost $I_0$. We set the elasticity of substitution between intermediates to $\sigma = 6$, which is consistent with Broda and Weinstein (2006).  

Given the above, the six remaining parameters are jointly chosen to match the following six moments: a labor income share of 60 percent, an average working time of 35 percent, an interest rate of 4 percent, an exit and entry rate of 6.3 percent (in line with Lee and Mukoyama (2008)), a share of exporters of 27 percent in line with Bernard, Jensen, Redding, and Schott (2007), and an exporter start rate of 3 percent (in line with Ruhl and Willis (2008)).  

Table 1 summarizes the parametrization. After solving the model, we simulate the life of 310,000 firms from which we compute the statistics reported in Table 2 and the figures discussed in the next sub-sections.

6.2 Aggregate statistics

Table 2 shows that, in the aggregate, the consumption-to-output and investment-to-output ratios are roughly in line with data, which principally follows from targeting the labor income share and labor hours. The export-to-output ratio is 8.3 percent, which is

---

23 The study by Simonovska and Waugh (2010) suggests a lower value, around 4, for the elasticity of substitution, and results for an economy with $\sigma = 4$ are qualitatively comparable.

24 The exporter start rate is the percentage of non-exporting firms starting to export in the next period.
in line with the US experience over the last four decades.\textsuperscript{25}

Table 2 shows that the average exporter is four times larger (in terms of labor and capital) than the average domestic firm. This is due to two effects: First, the contract requires that entrepreneurs have a large enough stake in their firms to obtain a large enough loan to pay the export costs—i.e., a firm must be in good financial health to export. Second, entrepreneurs’ access to credit increases after export begins. Figure 3 shows that there is a large increase in the loan size $R(V)$ once firms export, which implies that the financial conditions faced by firms improve after they start exporting. The causality between firm access to credit and export status goes in both directions in our model, highlighting the difficulty of disentangling these effects empirically.

6.3 Firm and exporter dynamics

Our results on firm dynamics are qualitatively similar to those from Quadrini (2004), and Clementi and Hopenhayn (2006). Panel (a) of Figure 4 shows that the hazard rate of exit is high for young firms and then decreases with firm age. On average, 1.2 percent of all firms are liquidated every period, which accounts for about 20 percent of all exiting firms. Panels (b) and (c) of Figure 4 plot the mean and standard deviation of investment for firms of a given age, and show that younger firms experience faster albeit more volatile growth than older ones.\textsuperscript{26}

We begin the discussion of exporter dynamics with an example. Figure 5 plots the life-cycle of three firms taken from our sample of simulated firms. Firm 1 gains access to export markets at age 17, from which time it continues to grow until it reaches its

\textsuperscript{25}The export share is affected by the elasticity of substitution for intermediate goods. Using the same targets, we obtain an export to output ratio that is about 1.5 times larger for an economy with $\sigma = 4$.

\textsuperscript{26}Given the full depreciation assumption, we define firm investment as the change in loan size from one period to the next, $R_{t+1}/R_t$. 

TABLE 2 ABOUT HERE

FIGURE 4 ABOUT HERE
efficient size at age 25 and finally exogenously exits at age 27. It takes 14 years for Firm 2 to accumulate enough equity to start exporting, but exits export markets after two years. Firm 3 never grows nearly large enough to export, and is liquidated at age 23.

[FIGURE 5 ABOUT HERE]

The long-term financial contract plays a critical role in shaping both the extensive and intensive margins of trade. Consider first the extensive margin: 27 percent of all firms export, and about 3 percent of domestic firms begin exporting each period. New exporters, which represent about 10 percent of all exporters (Panel (a) of Figure 6), are about a third of the size of incumbents (Table 2); and a large fraction of these new exporters—almost 35 percent—stop exporting after one year. The remaining 65 percent are less likely to exit during their second year exporting, and their conditional survival rate in international markets continues to rise until they only face the exogenous exit rate (Panel (b) of Figure 6).

These results follow from the property that new exporters are close to the export lottery region, so that exit from export markets following a low revenue shock is very likely. But since firms grow on average (the sub-martingale property of $V$), older exporters tend to be larger in terms of equity, and are therefore further away from the export lottery region. Ultimately, unconstrained exporters only exit exogenously. The high (endogenous) exit rate of young exporters also implies that the average sales of exiting and new exporters are approximately the same—their size differs only by 3.8 percent in terms of resources $R$. This is also apparent from Figure 3, showing that equity of both new exporters and exiting ones is close to $V_E$.

[FIGURE 6 ABOUT HERE]

Now consider the intensive margin: Young exporters are relatively smaller (Panel (a) of Figure 7), and grow faster than established ones, but their growth is more volatile. Panels (c) and (d) of Figure 7 plot the mean and standard deviation of investment of exporters conditional on the length of their export spell. The average growth rate of a
new exporter is 12 percent and is close to zero after ten years. The standard deviation of investment, however, is almost two times higher for a new exporter than a ten year old one. This implies that the investment of established exporters is much less sensitive to their cash-flows. Most exports are produced by large and financially unconstrained exporters. Panel (a) of Figure 7 plots the exports of the different age groups as a percentage of unconstrained firms’ exports; and panel (b) of Figure 7 shows that new exporters (up to six years old) only account for 30 percent of total exports.

[FIGURE 7 ABOUT HERE]

In sum, the impact of financial constraints on international trade in our model stems from the properties of the optimal financial contract that shapes firm dynamics. Changes in the economic environment, such as a lowering of trade barriers, have a non-trivial impact on both the extensive and intensive margin of trade by changing the contract terms between investors and entrepreneurs; and on the aggregate through the general equilibrium. For example, we find that a 20 percent decrease in the variable trade cost increases the share of exporters and exports as a fraction of total output. Following the reform, firms grow faster, begin exporting at a younger age (and at a smaller size), the survival of young exporters in international market is lower, and aggregate output is about one percentage point higher.

7 Discussion

Much of the recent focus in the international trade literature has been on exporter dynamics, which remains an area marked by important puzzles. Most notably, a Melitz model in which firms face persistent productivity shocks and sunk cost of exporting—in the spirit of Hopenhayn (1992)—cannot account simultaneously for firms’ low entry rate into export markets together with the higher exit rate of young exporters (Ruhl and Willis (2008), and Arkolakis (2010)). It is also at odds with the empirical observation that average sales of firms exiting from, and entering into international markets are approximately the same (Eaton et al., 2011).
These inconsistencies arise because in a model such as the one in Ruhl and Willis (2008), the export sunk cost must be paid every time a firm exits and subsequently wishes to re-enter export markets.\(^{27}\) That is, a firm starts exporting when it receives a high enough productivity shock and pays the sunk export cost. When productivity reverts to its original level, the firm continues exporting to avoid paying the sunk costs in the near future in case it receives another high productivity shock. This creates a certain momentum after entry into export markets and implies that the hazard rate of survival decreases with the export spell. Another implication is that exiting exporters must have a much lower productivity (and therefore size) than new exporters.

Although there is widespread empirical evidence that exporters tend to be more productive than domestic firms, productivity may not be the only source of selection into export markets. Not all highly productive firms, especially the young ones, export. Our model abstracts from the effects of firm productivity differentials on firm selection into export markets, and focuses on a different selection mechanism: the effects of financial constraints that arise when financial markets are incomplete because of private information.

To better understand the role of private information in shaping exporter dynamics, consider an extension of our model in which the production function of intermediate goods firms is \(\exp(z)G(k, n)\).\(^ {28}\) Assume that firm productivity \(z\) is publicly observable and follows a time-invariant Markov process with transition probability \(\Phi(z', z)\), such that \(z \in [z_{\text{min}}, z_{\text{max}}]\) and \(|z_{\text{min}}|, z_{\text{max}} < \infty\). New firms draw their productivity from an initial distribution \(\Phi_0(z)\).\(^ {29}\)

Let \(V_{z'}\) be the state-contingent continuation value to the entrepreneur, and rewrite

\(^{27}\)A sunk cost is required in this model to quantitatively matched the fraction of firms that export.

\(^{28}\)In this version of the model, the idiosyncratic revenue shock \(\theta\) could then be interpreted as a demand shock (e.g., Arkolakis (2010)).

\(^{29}\)Note that in our baseline model, firms face a publicly observed binary idiosyncratic productivity shock (the death shock), such that \(z' = 1\) and \(z' = 0\) with probability \(1 - \gamma_e\) and \(\gamma_e\), respectively, conditional on \(z = 1\) in the previous period; conditional on \(z = 0\) in the previous period, \(z' = 0\) with probability one.
the value function of a \( i \)-type firm conditional on \( z \neq 0 \) as

\[
\hat{W}_i(V, z) = \max_{\tau, R, V^H_z, V^L_z} \pi F_i(R, z) - (1 + r)R + \beta E[(1 - \pi)W(V^L_z) + \pi W(V^H_z)|z] 
\]

s.t. \( V = \pi(F_i(R, z) - \tau) + \beta E[\pi V^H_z + (1 - \pi)V^L_z|z] \)

\[
F_i(R, z) - \tau + \beta E(V^H_z|z) \geq F_i(R, z) + \beta E(V^L_z|z) 
\]

\[
\tau \leq F_i(R, z) 
\]

\[
V^H_z, V^L_z \geq 0. 
\]

where \( i \in \{D, E\} \) and \( \beta = 1/(1 + r) \). The optimal continuation values \( V'(\hat{\theta}, z', V, z) \) to the entrepreneur are now functions of the current productivity \( z \), equity \( V \), report \( \hat{\theta} \) and the realization of next period’s shock \( z' \). Our previous results are largely unchanged simply replacing \( V^L \) and \( V^H \) by \( E[V^L_z|z] \) and \( E[V^H_z|z] \), respectively.\(^{30}\)

The effect of financial frictions relative to productivity shocks on exporter dynamics depends on the volatility (and persistence) of each shock. Clearly, when \( z_{\text{min}} \to z_{\text{max}} \), only financial frictions matter, as in our baseline model. When \( \theta^H \to \theta^L \), financial frictions play no role and the productivity process determines exporter dynamics. Access to credit and productivity matter when (private) revenue and (public) productivity shocks have a wide enough support.\(^{31}\) A firm stops exporting as \( V \to 0 \), no matter how productive it is. Similarly, even an unconstrained firm stops exporting as \( z \to 0 \).

To focus on the role of productivity shocks, assume that \( \theta^H \to \theta^L \), so that there are no financial frictions. Assume further that the Markov process for \( z \) is the discrete approximation of the auto-regressive process for \( \tilde{z} \), such that \( \tilde{z}' = (1 - \rho)\mu + \rho \tilde{z} + \epsilon \), where \( \epsilon \sim N(0, \sigma^2_{\epsilon}) \) and \( \mu \) shifts the mean. The degree of persistence for firm productivity is thus captured by \( \rho \). When the productivity process is IID (\( \rho = 0 \)), the probability of exiting export markets and the average productivity of exporters are independent of

\(^{30}\)Once \( E[V^L_z|z] \) and \( E[V^H_z|z] \) are known, \( V^L_z \) and \( V^H_z \) for all \( z \in [z_{\text{min}}, z_{\text{max}}] \) can be determined by maximizing \( E[W(V^L_z)|z] \) and \( E[W(V^H_z)|z] \) holding constant the respective expected continuation values to the entrepreneur. Since \( W(V, z) \) is strictly concave in \( V \), there is a unique interior solution; and given \( \hat{W}_i(V, z) \), the export and liquidation decisions are exactly as before.

\(^{31}\)We assume that \( \rho \) is less than 1 when firms are also subject to revenue shocks.
exporter age. With a unit root process \( (\rho = 1) \), firm productivity follows a random walk. Since all firms below the export threshold exit export markets, this implies that the probability of exiting export markets is decreasing, and the average productivity of exporters is increasing in exporter age. Thus, with a persistent productivity process \( (0 < \rho < 1) \), the probability of exiting export markets is decreasing and the average productivity is increasing in exporter age. Moreover, the rate at which average productivity increases is also decreasing in exporter age. However, it is well-known that if firms also face sunk costs – when the cost of exporting is higher for new exporters than for established ones –, the above conclusions can be reversed depending on the persistence of the shock and the size of the sunk cost \((\text{Ruhl and Willis (2008)})\).\(^{32}\)

Two conclusions emerge: First, persistent (publicly observable) productivity shocks reinforce the main mechanism driving exporter dynamics derived from our model with moral hazard. Second, conditional on productivity (age) older (more productive) firms are larger in terms of sales, more likely to export, and less likely to become liquidated or exit export markets. Taken together, our findings and the above discussion suggest that firm productivity and lending relationships could be jointly important in shaping exporter dynamics. This echoes the conclusions of Cooley and Quadrini (2001), who show that models combining persistent shocks and financial frictions can account for firm dynamics.\(^{33}\)

Financial markets could also be incomplete because of limited enforcement such as in Kiyotaki and Moore (1997) and Albuquerque and Hopenhayn (2004). Consider now a version of our model with long-term financial contracts that are constrained efficient under limited enforcement.\(^{34}\) In this model, revenue shocks are observable by everyone.

\(^{32}\)Kohn et al. (2011) quantify the effect of financial frictions on new exporter dynamics by contrasting a model in which firms face productivity shocks and an (exogenous) collateral requirement for their one period debt to a model in which firm face productivity shocks and sunk costs. Kohn et al. (2011) show that their model accounts for the downward sloping hazard rate of exit from export markets and positive, downward-sloping sales growth. However, the impact of financial constraints is unclear. As pointed out above, the productivity process alone already accounts for these facts in the absence of sunk costs.

\(^{33}\)In particular, the simultaneous dependence of firm dynamics on size conditional on age, and on age conditional on size.

\(^{34}\)This contract is similar to Albuquerque and Hopenhayn (2004) and studied in the context of trade
Without loss of generality, assume that the contract can be repudiated at no cost, and that a defaulting entrepreneur can divert the current revenue, but will in that case be excluded from financial markets in the future. The outside value option of default is then simply \( F_i(R) \) for \( i \in \{D, E\} \) after a high revenue shock, and 0 otherwise. The optimal contract maximizes the value of the joint surplus subject to a no-default \( F_i(R) - \tau + \beta V^H \geq F_i(R) \) rather than an incentive compatibility constraints. In equilibrium, no entrepreneur chooses to repudiate the contract, and it can be shown than \( V^L(V) = V < V^H(V) \), which implies that firm size never decreases. Thus, conditional on productivity, limited enforcement cannot alone account for the higher exit rate of young exporters. Therefore, our results highlight the potentially important role moral hazard plays in shaping exporters dynamics.

Last, our model shares the same limitation as the Melitz model in that it predicts a constant export to total sales ratio. This implication is at odds with data (Berman and Héricourt, 2010; Iacovone and Javorcik, 2010), and is due to the assumption of a CES aggregator with iceberg-style variable trade costs. For instance, our model implies that

\[
\frac{q^*}{q} = (1 + I_T)^{-\sigma},
\]

and although new exporters increase their production and exports as they spend more years in international markets, the composition of their sales remains unchanged. However, extending our model to include a per-unit cost \( \hat{I}_T \) of export implies that

\[
\frac{q^*}{q} = \left(1 + I_T + \frac{\mu}{\lambda} \hat{I}_T\right)^{-\sigma},
\]

where \( \mu \) and \( \lambda \) are the Lagrangian multipliers associated with the resource and production constraints (equation (4) and (5)) in the static profit maximization problem solved by the intermediate producers (problem (6)), respectively.\(^{35}\) It can be shown that \( \mu/\lambda \) is increasing in \( V \) when intermediate good producers have access to a decreasing return to

\(^{35}\)With a per unit cost, the firm resource constraints becomes \( k + nw + 1_{(q^*>0)}(I_E + q^* \hat{I}_T) \leq R(V) \).

scale technology, which implies that their export to total sales ratio increases the longer they export.\(^{36}\) Alternative rationales proposed to account for the increasing export to total sales ratio include learning both from the demand side (Rauch and Trindade, 2003) and the supply side (Ruhl and Willis, 2008), as well as convex costs of advertising, which force exporters to slowly build market share (Arkolakis, 2010).

\section{Concluding remarks}

There is widespread empirical evidence that financial frictions play an important role in shaping the growth of young and small firms. There is also growing empirical evidence that the export decisions of firms are sensitive to the availability of credit. This paper investigates how private information and the long-term financial arrangements that arise in consequence affect firms' entry and growth in international markets in general equilibrium.

Our model predicts that new exporters account only for a small share of total exports, in line with recent empirical studies. A large fraction of new exporters stop exporting after one year, while continuing exporters are less likely to exit. Young exporters experience faster and more volatile growth than established exporters.

Although testing the effect of credit relationships on firm export decisions is beyond the scope of this paper, we provide new facts in Table 3 about manufacturing firm finances from the 2003 Survey of Small Business Finances. Consistent with our theory, we find evidence that exporters are fewer, larger, and less leveraged than non-exporters. Younger exporters tend to have longer lending relationships with their primary lenders, and almost all exporters operate under a limited liability status. In contrast, about 60\% of domestic firms operate under limited liability. We also find that nearly all young exporters use trade credit, and tend to use it more intensively than young domestic exporters.

\footnote{Per-unit costs of trade are in terms of resources, whereas iceberg costs are in terms of production. With decreasing returns to scale, the shadow value of resources is diminishing faster than the shadow value of production. This implies that per-unit trade costs play a smaller role in determining exports as firms grow and have more resources at their disposal. A more general discussion is contained in the on-line appendix.}
firms.

Our theory abstracts from the effects of firm productivity on selection into export markets. Assessing the joint role of financial market incompleteness and firm productivity remains an important empirical and theoretical question. Our theory also abstracts from other forms of credit relationships, such as trade credit with suppliers. Given the pervasive use of the relatively expensive trade credit among exporters, it is likely that trade credit is an important alternative source of credit for financially constrained exporters.

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A Miscellaneous proofs

Proposition A.1 There exists a point \( V_{dx} \) such that \( \hat{W}_D(V) > \hat{W}_E(V) \) for all \( V \in [0, V_{dx}) \), and \( \hat{W}_D(V) < \hat{W}_E(V) \) for all \( V \in (V_{dx}, \tilde{V}] \).

From our assumptions on \( F_i(\cdot) \) for \( i \in \{D, E\} \), it can be shown that \( F_D(R) = f(R)^{1-1/\sigma}A \) and \( F_E(R) = f(R - I_E)^{1-1/\sigma}A^* \) which are both strictly increasing and concave as long as the span of control parameter is less then \( \sigma/(\sigma - 1) \). This implies that the current period expected cash flow \( \pi F_i(R) - (1+r)R \) is also a strictly increasing, strictly concave function on \( [0, \tilde{R}] \) for \( i \in \{D, E\} \). It follows that \( \hat{W}_D(V) \) defined over \( [0, \tilde{V}_D] \) and \( \hat{W}_E(V) \) defined over \( [0, \tilde{V}] \) are both strictly increasing and concave. It remains to be shown that \( \hat{W}_E(V) \) is lower than \( \hat{W}_D(V) \) for all \( V < V_{dx} \) and vice versa – i.e., there is a unique crossing point \( V_{dx} \). Without loss of generality, let \( \tilde{W}_D(V) = \hat{W}_D(\tilde{V}) \forall V > \tilde{V}_D \). Assume that participating in international trade is profitable, so that \{\( F_E(\tilde{R}) - (1+r)\tilde{R} \} > \{F_D(\tilde{R}_D) - (1+r)\tilde{R}_D \} \), which implies that \( \hat{W}_E(\tilde{V}) > \hat{W}_D(\tilde{V}) \). Given the participation constraint (equation 7) and the requirement that \( V^L(V) \) and \( V^H(V) \) be non-negative,
$V^L(V)$ and $V^H(V)$ force the continuation tend to zero, as $V$ tends to 0. It follows that $V_H - V_L$ also tends to 0 as $V$ tends to 0, which reduced $R(V)$ to maintain incentive compatibility (equation 8). This implies that $\hat{W}_D(V)$ tends to $\beta S$ as $V$ tends to 0. Using a similar argument for exporters, $R(V)$ tends to $I_E$ as $V$ tends to 0, which implies that $\hat{W}_E(V)$ tends to $\beta S - I_E$ as $V$ tends to 0. The property that $\hat{W}_D(V)$ and $\hat{W}_E(V)$ are increasing and concave together with the above yields the results.

Proof of Proposition 3.1: Partitions of the domain for $W(V)$: From Proposition A.1, $\hat{W}(0) < S$ and $\hat{W}(V) > S$ for any $0 < V < \tilde{V}$ (otherwise the contract would not be feasible), so that $V_r > 0$. Furthermore, $\hat{W}_E'(V) > \hat{W}_D'(V) \forall V < \tilde{V}$, and it follows that $V_D < V_E$ by the strict concavity of $\hat{W}_i(V)$ for $i \in \{D,E\}$. Finally, because $\hat{W}_E(\tilde{V}) = 0$, it must be the case that $V_E < \tilde{V}$. We assume that $V_D > V_r$, which must hold if there is to be any non-exporting firms in the economy.

Export lottery: $\hat{W}_E(V_{dx}) = \hat{W}_D(V_{dx})$ from Proposition A.1, which, together with the concavity of $\hat{W}_D$ and the strict concavity of $\hat{W}_E$ on $[0,\tilde{V}]$ imply that $V_D < V_{dx} < V_E$. It follows that the function max{$\hat{W}_D(V),\hat{W}_E(V)$} defined over $[0,\tilde{V}]$ is convex on $[V_D,V_E] \subset (0,\tilde{V})$. This implies that the joint surplus can be improved by making the export decision random. That is, whenever a firm reaches a size $V \in [V_D,V_E]$, a risk neutral entrepreneur accepts a lottery with which her firm reaches $V_E$ with probability $\delta(V) = (V - V_D)/(V_E - V_D)$, or $V_D$ with probability $(1 - \delta(V))$. The boundaries of this export region, $[V_D,V_E]$, are determined such that the tangent of $\hat{W}_D(V)$ at $V_D$ is equal to the tangent of $\hat{W}_E(V)$ at $V_E$.

Liquidation lottery: A positive scrap value $S$ implies that the financial intermediary may have an incentive to liquidate the firm if its value falls below a threshold level $V_r$. Following the same argument as for the export lottery above, the function max{$S,\hat{W}_D(V)$} defined over $[0,V_D]$ is convex. The joint surplus can be improved by randomizing the liquidation decision. Optimally, $V_r = \text{sup}_V\{\hat{W}(V) - S = V\hat{W}'(V)\}$, such that whenever the equity of a firm falls below $V_r$, the financial intermediary offers the entrepreneur a lottery with which her firm is liquidated with probability $\alpha(V) = (V_r - V)/V_r$ or kept in operation with probability $1 - \alpha$. 

30
Concavity of $W(V)$: Since $\hat{W}_i(V)$ for $i \in \{D, E\}$ are each increasing and concave on $[0, V_i]$, any convex combination of the two functions and $S$ on $[0, \tilde{V}]$ is also increasing and concave, which implies that $W(V)$ is increasing and concave. Furthermore, from the definition of the liquidation and export lotteries, $W(V)$ is strictly increasing and linear on $[0, S] \cup [V_D, V_E]$, and strictly increasing and concave on $(V_r, V_D) \cup (V_E, \tilde{V})$.

Proof of Proposition 4.1: Partition the domain of the contract $[0, \tilde{V}]$ in five parts $[0, V_r) \cup [V_r, V_D] \cup (V_D, V_E) \cup [V_E, \tilde{V}) \cup \{\tilde{V}\}$. Given the expression for $V^L(V)$ and $V^H(V)$, it follows that $E(V'|V) = V/\beta$ when $V \in [V_r, V_D] \cup [V_E, \tilde{V})$.\footnote{When $V = \tilde{V}$, the firm is unconstrained and there is no need to provide any incentives to report the truth (as all the revenue go to the entrepreneur), and hence $V^L = V^H = \tilde{V}$, so $E(V'|\tilde{V}) = \tilde{V}$. Whenever $V^H(V) \in (V_D, V_E)$ or $V^L(V) \in (V_D, V_E)$, optimality of the lottery implies that the entrepreneur is indifferent between her current level $V$ and the expected payoff of the lottery next period. It follows that $E(V'|V \in [0, V_r)) = E(V'|V \in (V_D, V_E)) = V/\beta$.}

B  Existence of a general equilibrium

Given interest rate $r$ and wage $w$, perfect competition in the financial market implies that new firms start with equity $V_0$. Consider the sequence $(X_t)_{t \geq 0}$ of equity levels from a single firm indefinitely replaced by a new one upon liquidation or exogenous exit, with $X_0 = V_0$. It is clear $(X_t)_{t \geq 0}$ is a sequence of random variables, and its evolution depends on the properties of the optimal contracts and on the sequence of shocks – liquidation lottery, export lottery, revenue shock, and exogenous exit.

Our proof of existence consists of four parts. The first part show that $X = (X_t)_{t \geq 0}$ is a time-homogeneous Markov chain such that

$$X_{t+1} = T_\omega(X_t, \epsilon_t), (\epsilon_t)_{t \geq 0} \sim \phi_\omega \in \mathcal{P}(Z), X_0 = V_0 \in S$$

\footnote{Here we assume that repayments $\tau$ are equal to all revenues $F_i(R(V))$ for all $V < \tilde{V}$. In our numerical analysis we have $\tau < F_i(R(V))$ for some $V$ with $V^H(V) = \tilde{V}$, but the sub-martingale property still holds.}
where $T_\omega : S \times Z \rightarrow S$ is a collection of measurable functions indexed by $\omega \in \Omega$ the parameter space, $(\epsilon_t)_{t=1}^\infty$ is a sequence of independent random shocks with (joint) distribution $\phi_\omega$, and $S$ and $Z$ are the state space and the probability space respectively.

The second part establishes that the Markov chain has a unique distribution of firms, which can be attained in a finite number of periods starting from any initial distribution.

The third part establish that the stationary distribution of firms is continuous in $\omega$.

The last part defines a continuous mapping of $\Omega$ on itself and apply Schauder Fixed-Point Theorem, which, together with the first three results and the condition that $\Omega$ be compact and convex set, implies that this mapping admits at least one fixed point.\footnote{Our proof of existence is similar to the one in Verani (2013) who studies a closed economy version of this model.}

**Proposition B.1 (Step 1)** $X$ is a time-homogeneous Markov chain on a general state space.

 Equip the state space $S$ with a boundedly compact, separable, metrizable topology $\mathcal{B}(S)$. Let $(Z, \mathcal{Z})$ be the measure space for the shocks. Let $A$ be any subset of $\mathcal{B}(S)$. It follows for any $x \in \{x : V_r < x < V_D \text{ and } V^L(x) < V_r\}$

$$P(x, A) = \begin{cases} 
(1 - \gamma)(1 - \pi)\alpha(V^L(x)) + \gamma & \text{if } A = \{V_0\} \\
(1 - \gamma)(1 - \pi)(1 - \alpha(V^L(x))) & \text{if } A = \{V_r\} \\
(1 - \gamma)\pi & \text{if } A = \{V^H(x)\} \\
0 & \text{otherwise}
\end{cases}$$

(36)

For any $x \in \{x : V_r < x < V_D \text{ and } V_r \leq V^L(x) \leq V_D \text{ and } V^H(x) \leq V_D\}$

$$P(x, A) = \begin{cases} 
\gamma & \text{if } A = \{V_0\} \\
(1 - \gamma)(1 - \pi) & \text{if } A = \{V^L(x)\} \\
(1 - \gamma)\pi & \text{if } A = \{V^H(x)\} \\
0 & \text{otherwise}
\end{cases}$$

(37)
For any $x \in \{x : V_r < x < V_D$ and $V_r \leq V^L(x) \leq V_D$ and $V^H(x) \geq V_D\}$

$$P(x, A) = \begin{cases} 
\gamma & \text{if } A = \{V_0\} \\
(1 - \gamma)(1 - \pi) & \text{if } A = \{V^L(x)\} \\
(1 - \gamma)\pi \delta(V^H(x)) & \text{if } A = \{V_E\} \\
(1 - \gamma)(1 - \delta(V^H(x))) & \text{if } A = \{V_D\} \\
0 & \text{otherwise}
\end{cases} \quad (38)$$

For any $x \in \{x : V_E < x < \tilde{V}$ and $V^L(x) \leq V_E$ and $V_E \leq V^H(x) < \tilde{V}\}$

$$P(x, A) = \begin{cases} 
\gamma & \text{if } A = \{V_0\} \\
(1 - \gamma)(1 - \pi)\delta(V^L(x)) & \text{if } A = \{V_E\} \\
(1 - \gamma)(1 - \pi)(1 - \delta(V^L(x))) & \text{if } A = \{V_D\} \\
(1 - \gamma)\pi & \text{if } A = \{V^H(x)\} \\
0 & \text{otherwise}
\end{cases} \quad (39)$$

For any $x \in \{x : V_E < x < \tilde{V}$ and $V_E \leq V^L(x) \leq \tilde{V}$ and $V^H(x) \geq \tilde{V}\}$

$$P(x, A) = \begin{cases} 
\gamma & \text{if } A = \{V_0\} \\
(1 - \gamma)(1 - \pi) & \text{if } A = \{V^L(x)\} \\
(1 - \gamma)\pi & \text{if } A = \{\tilde{V}\} \\
0 & \text{otherwise}
\end{cases} \quad (40)$$

And for $x = \{\tilde{V}\}$

$$P(x, A) = \begin{cases} 
\gamma & \text{if } A = \{V_0\} \\
(1 - \gamma) & \text{if } A = \{\tilde{V}\} \\
0 & \text{otherwise}
\end{cases} \quad (41)$$

For each $A \in \mathcal{B}(S)$, $P(\cdot, A)$ is a non-negative function on $\mathcal{B}(S)$, and for each $x \in S$, $P(x, \cdot)$ is a probability measure on $\mathcal{B}(S)$. Therefore, for any initial distribution $\psi$, the stochastic process $X$ defined on $S^\infty$ is a time-homogeneous Markov chain. \qed
Proposition B.2 (Step 2) \( X \) is globally stable.

Let \( M \) denote the corresponding Markov operator, and let \( \mathcal{P}(S) \) denote the collection of firms distribution generated by \( M \) for a given initial distribution. \(^{39}\)

Write the stochastic kernel \( P \) with the density representation \( p \) so that \( P(x,dy) = p(x,y)dy \) for all \( x \in S \). The Dobrushin coefficient \( \alpha(p) \) of a stochastic kernel \( p \) is defined by

\[
\alpha(p) := \min \left\{ \int p(x,y) \land p(x',y)dy : (x,x') \in S \times S \right\}
\]

(\( \mathcal{P}(S), M \)) is globally stable if \( (\psi^t M^t)_{t \geq 0} \rightarrow \psi^* M \) where \( \psi^* \in \mathcal{P}(S) \) is the unique fixed point of \( (\mathcal{P}(S), M) \). This occurs if the Markov operator is a uniform contraction of modulus \( 1 - \alpha(p) \) on \( \mathcal{P}(S) \) whenever \( \alpha(p) > 0 \). A firm dies with a fixed, exogenous and independent probability \( \gamma \) each period, and is instantaneously replaced by a new one of size \( V_0 \). Therefore,

\[
P(x,\{V_0\}) \geq 0 \quad \forall \ x \in S.
\]

Equation (11.15) and Exercise (11.2.24) in Stachurski (2009) yield \( \alpha(p) > \gamma \). By Stachurski (2009, Th. 11.2.21), this implies

\[
||\psi M - \psi' M||_{TV} \leq (1 - \gamma)||\psi - \psi'||
\]

for every pair \( \psi, \psi' \) in \( \mathcal{P}(Z) \), and where \( TV \) indicates the total variation norm.

Proposition B.3 (Step 3) \( \psi^* \) is continuous in \( \omega \).

\(^{39}\)Note that Stokey, Lucas, and Prescott (1989, Theorem 12.12) fails to apply in this case because the stochastic kernel is not monotone on \([V_r,a]\) where \( a \) is such that \( V^L(a) = V_r \). For instance, consider the function \( f(x) = x \). From the above,

\[
\int_{[V_r,a]} P(x,dy)\gamma = (1 - \gamma)(1 - p)\gamma V_0 + (1 - \alpha(V^L(x))V_r + pV^H(x) + \gamma e V_0
\]

\[
= (1 - \gamma)(1 - p)\gamma V_0(V_r - V^L(x)/V_r) + (1 - p)V^L(x) + pV^H(x) + \gamma e V_0
\]

\[
= (1 - \gamma)(1 - p)\gamma V_0(V^L(x) + x/\beta) + \gamma e V_0
\]

Which is generally not increasing. When \( x \) falls below \( a \), the probability of liquidation \( \alpha(x) \) becomes non-zero in case of a \( \theta = 0 \). The liquidation rule sends \( x \) to \( V_0 \) (as the firm regenerate), which can be larger than \( V_r \) and \( V^H(x) \) so that the lower the \( x \), the higher the expected value of next period \( x \).
Parametric continuity of $X$ follows if the conditions of LeVan and Stachurski (2007, Proposition 2) are satisfied. Without loss of generality, we only consider the case of symmetric countries in which case the exchange rate is 1, and $\omega = \{r, w\}$. Consider again the state space $S$ equipped with a boundedly compact, separable, metrizable topology, $(Z, \mathcal{Z})$ a measure space, and $\mathcal{P}(Z)$ the collection of probabilities on $(Z, \mathcal{Z})$. From the above, the model can be written as

$$X_{t+1} = T_\omega(X_t, \epsilon_t), \text{ where } \epsilon_t \sim \psi_\omega \in \mathcal{P}(Z), \ \forall t \in \mathbb{N}$$

where $(\epsilon_t)_{t=1}^\infty$ independently distributed, and $T_\omega : S \times Z \to S$ is measurable.

Given prices $\omega$, the stochastic kernel can be written as $P_\omega(x, B) := \psi_\omega \{z \in Z : T_\omega(x, z) \in B\}$, and given the parameter space $\Omega$, the family of stochastic kernel is $\{P_\omega : \omega \in \Omega\}$. Let $N$ be any subspace of $\Omega$, and define $\Lambda(\omega) := \{\mu \in \mathcal{P}(S) : \mu = \mu P_\omega\}$ the collection of invariant distribution corresponding indexed by $\omega$.

**Proposition B.4 (LeVan and Stachurski (2007))** If $\Lambda(\omega) = \{\mu_\omega\}$, then $\omega \mapsto \mu_\omega$ is continuous on $N$ if the following four conditions are satisfied:

1. the map $N \ni \omega \mapsto T_\omega(x, z) \in S$ is continuous for each pair $(x, z) \in S \times Z$

2. for each compact $C \subset S$, there is a $K < \infty$ with

$$\int d(T_\omega(x, z), T_\omega(x', z)) \psi_\omega(dz) \leq K d(x, x'), \forall x, x' \in C, \forall \omega \in N$$

3. $\exists$ a Lyapunov function $V \in \mathcal{L}(S)$, $\lambda \in (0, 1)$, and $L \in [0, \infty)$ s.t. $\forall \omega \in N$

$$P_\omega V(x) := \int V(T_\omega(x, z)) \psi_\omega(dz) \leq \lambda V(x) + L \ \forall x \in S$$

---

40LeVan and Stachurski (2007, Proposition 2) is an application of LeVan and Stachurski (2007, Theorem 1) of which Stokey et al. (1989, Theorem 12.13) is a special case.

41Here, $(\epsilon_t)_{t=1}^\infty = \{\{D_{\gamma,t}, D_{\pi,t}, D_{\alpha,t}, D_{\delta,t}\}\}_{t=1}^\infty$ where each $D_i$ for $i \in \{\gamma, \pi, \alpha, \delta\}$ is a binary i.i.d. random variable corresponding to the death, revenue, and liquidation and export lotteries, respectively.

42This is another abuse of notation. It is customary to use $V(x)$ to denote a Lyapunov function.
4. $\omega \mapsto \psi_\omega$ is continuous in total variation norm.

That $\Lambda(\omega)$ is nonempty, and $\Lambda(\omega) = \{\mu_\omega\}$ for each $\omega \in N$ follows from Proposition B.2. Condition (1) requires the optimal value function $W(x)$ to be continuous in $\omega$ which follows from Berge’s theorem. Condition (4) holds as the shocks are independent and the probability of liquidation is $\alpha(x) = (x - V_r)/V_r$ and export $\delta(x) = (x - V_D)/(V_E - V_D)$ are both continuous in $\omega$ since $V_r$, $V_D$, and $V_E$ are continuous in $\omega$ from condition (1).

To show condition (2) holds, define again $a \ni V_L(x) = V_r$, and $b \ni V_H(b) \geq \tilde{V}$. Pick any $x, x' \in C \subset [V_r, a)$. Without loss of generality assume $x > x'$ so that $\alpha(V_L(x')) > \alpha(V_L(x))$. By noting that $\alpha(V_L(x')) - \alpha(V_L(x)) = (V_L(x') - V_L(x))/V_r$, and $x = \beta[pV_H(x) + (1 - p)V_L(x)]$ at optimum, it follows easily that

$$\int d(T_\omega(x, z), T_\omega(x', z))\psi_\omega(dz) < |pV_H(x) + (1 - p)V_L(x' - pV_H(x' - (1 - p)V_L(x'))|$$

$$= \frac{1}{\beta}|x - x'| = \frac{1}{\beta} d(x, x').$$

The above inequality also holds for any $x, x' \in C \subset [a, b)$. Last, recall that $V_L(x) = (x - pF(R(x)))/\beta$. It follows for any $x, x' \in C \subset [b, \tilde{V})$

$$\int d(T_\omega(x, z), T_\omega(x', z))\psi_\omega(dz) < (1 - p)|V_L(x) - V_L(x')|$$

$$= \frac{(1 - p)}{\beta}|x - x'| = \frac{(1 - p)}{\beta} d(x, x').$$

It remains to show condition (3) holds. Pick $V(x) = x$ which is a Lyapunov function (since $S$ is boundedly compact). Then,

$$P_\omega x = \begin{cases} (1 - \gamma_e) \{ (1 - p)[\alpha(V_L(x))V_0 + (1 - \alpha(V_L(x)))V_r] + pV_H(x) \} 1_{[V_r, a)}(x) \\
+ [pV_H(x) + (1 - p)V_L(x)] 1_{[a, b)}(x) \\
+ \tilde{V} 1_{(x = \tilde{V})}(x) \} + \gamma_e V_0 
\end{cases}$$

(48)
Pick any $x \in [V_r, a)$ so that $V_r \leq x \leq V^H(x)$. Then,

$$
P_\omega x < (1 - p)(1 - \alpha(V^L(x)))V_r + pV^H(x) + [(1 - p)\alpha(V^L(x)) + \gamma_e)V_0
\leq (1 - p)V_r + pV^H(x) + V_0
\leq \lambda x + \sup_{\omega \in \mathcal{N}} V_0 = \lambda V(x) + L
$$

The same inequality holds for any $x \in [a, b)$, since $V^L(x) < x < V^H(x)$. Last, when $x = \tilde{V}$,

$$
P_\omega x = (1 - \gamma_e)\tilde{V} + \gamma_e V_0
\leq \lambda x + \sup_{\omega \in \mathcal{N}} V_0
$$

Proposition B.5 (Step 4) There exists an equilibrium

Using equations 22, 23, and 26, define the mapping

$$
f(\omega) = \begin{bmatrix}
\int R(V, \omega)d\mu(\omega) + \Gamma(\omega)I_0 - D(\omega) - Z(\omega) \\
\int n(\omega)d\mu(\omega) - H \\
\pi F(R(V, \omega), \omega) - C_w(\omega) - C_e(\omega) - K(\omega)
\end{bmatrix}
$$

such that $f : \Omega \mapsto \mathbb{R}^3$, and where

$$
\begin{align*}
Z(\omega) &= -\int B(V, \omega)d\mu(\omega) - \Gamma_b(\omega)S \\
K(\omega) &= \int k(\omega)d\mu(\omega) + \Gamma_e(\omega)I_E + \Gamma(\omega)I_0 - \Gamma_b(\omega)S \\
C_w(\omega) &= D(\omega)r(\omega) + H(\omega)w(\omega) \\
C_e(\omega) &= \pi \int F(R(V, \omega), \omega)d\mu(\omega) - \pi \int \tau(\omega)d\mu(\omega)
\end{align*}
$$

Prices $r$ and $w$ must each be positive and greater than zero. Without loss of generality, assume that $r$ and $w$ are bounded above by arbitrarily large but finite numbers $\bar{r}$and $\bar{w}$.

It follows that the set $\Omega$ is compact and convex. Define the mapping $\Phi : \Omega \mapsto \Omega$ such
that
\[ \Phi(\omega) = \arg\max_{\omega \in \Omega} -||f(\omega)||^2 \] (49)

From Proposition B.2 and Proposition B.5, the maximand is continuous in \( \omega \) so that the correspondence \( \Phi \) is also continuous. Applying the Schauder Fixed-Point (Stokey et al., 1989, Theorem 17.4) yields the results.  

References


### Table 1: Parameter values

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma$ Elasticity of substitution</td>
<td>6</td>
</tr>
<tr>
<td>$\beta$ Workers’ discount rate</td>
<td>0.96</td>
</tr>
<tr>
<td>$\lambda$ Elasticity of leisure</td>
<td>2.35</td>
</tr>
<tr>
<td>$\gamma_w$ Workers’ death probability</td>
<td>0.02</td>
</tr>
<tr>
<td>$\eta_k$ Capital share</td>
<td>0.20</td>
</tr>
<tr>
<td>$I_0$ Setup investment</td>
<td>0.278</td>
</tr>
<tr>
<td>$S$ Salvage value</td>
<td>$0.8 \times I_0$</td>
</tr>
<tr>
<td>$I_T$ Variable export cost</td>
<td>0.40</td>
</tr>
<tr>
<td>$I_E$ Fixed export cost</td>
<td>0.013</td>
</tr>
<tr>
<td>$\pi$ Probability of high/low shock</td>
<td>0.5</td>
</tr>
<tr>
<td>$\gamma_e$ Firm exogenous exit rate</td>
<td>0.048</td>
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Table 2: Steady state moments (in percent)

<table>
<thead>
<tr>
<th>Targeted:</th>
<th>Not targeted:</th>
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<tbody>
<tr>
<td>Interest rate</td>
<td>4</td>
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<tr>
<td>Hours worked</td>
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<tr>
<td>Labor income share</td>
<td>59.7</td>
</tr>
<tr>
<td>Entry/exit rate</td>
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<tr>
<td>Entry rate in export market</td>
<td>3.1</td>
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<tr>
<td>Share of exporters</td>
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<tr>
<td>Consumption/Output</td>
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<td>Investment/Output</td>
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<td>Export/Output</td>
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<td>Relative size of entrants</td>
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<tr>
<td>Exit rate from export market after 1 year</td>
<td>32.5</td>
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<tr>
<td>Domestic firm size relative to exporters</td>
<td>27</td>
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Table 3: Small and medium size exporter finances

<table>
<thead>
<tr>
<th></th>
<th>Exporters</th>
<th>Non-Exporters</th>
<th>All firms</th>
<th>Young firms</th>
</tr>
</thead>
<tbody>
<tr>
<td>Assets (millions)</td>
<td>2.36***</td>
<td>0.91</td>
<td>0.438</td>
<td>1.87**</td>
</tr>
<tr>
<td>Employment</td>
<td>33.53***</td>
<td>17</td>
<td>5.779</td>
<td>34.67*</td>
</tr>
<tr>
<td>Sales (millions)</td>
<td>4.68***</td>
<td>1.78***</td>
<td>0.854</td>
<td>3.33*</td>
</tr>
<tr>
<td>Debt / Assets</td>
<td>0.43*</td>
<td>0.21</td>
<td>0.11</td>
<td>0.74</td>
</tr>
<tr>
<td>Average lending relationship</td>
<td>9.47</td>
<td>6</td>
<td>1.126</td>
<td>6.49*</td>
</tr>
<tr>
<td>Longest lending relationship</td>
<td>11.7</td>
<td>10</td>
<td>1.242</td>
<td>10.51***</td>
</tr>
<tr>
<td># of lending institutions</td>
<td>3.02***</td>
<td>3</td>
<td>0.267</td>
<td>3.24***</td>
</tr>
<tr>
<td>Firms using trade credit (%)</td>
<td>94.22</td>
<td>99.20</td>
<td>76.43</td>
<td>63.42</td>
</tr>
<tr>
<td>Purchases using trade credit (%)</td>
<td>91.25</td>
<td>95*</td>
<td>2.063</td>
<td>83.38**</td>
</tr>
<tr>
<td>Limited liability firms (%)</td>
<td>94.91</td>
<td>98.98</td>
<td>60.18</td>
<td>59.12</td>
</tr>
<tr>
<td>Number of firms (estimate)</td>
<td>36,540</td>
<td>8,120</td>
<td>322,127</td>
<td>135,653</td>
</tr>
</tbody>
</table>

Data are of manufacturing firms (SIC codes between 200 and 399) from the 2003 Survey of Small Business Finances (SSBF), with firm export status from matched firms in the National Establishment Time-Series (NETS) database. The average and longest lending relationships are computed using firms’ relationships with commercial banks, savings banks, savings and loan associations, credit unions, and finance companies. Young firms are firms that are less than 10 years old. We test the hypotheses that the mean and the median for each variable are the same for exporters and non-exporters. We use a t-test for means and the Wilcoxon rank sum test for distributions. The level of significance **, **, and *** represent the 10%, 5% and 1% significance level respectively.
Figure 1: Timing

Figure 2: The optimal financial contract
Figure 3: Financial characteristics of the contract

(a) Period loan, $R(V)$  
(b) Debt and dividend  
(c) Conditional investment

Figure 4: Firm dynamics

(a) Hazard rate of exit  
(b) Mean investment  
(c) Std. Dev. investment

Figure 5: The life cycle of firms
Figure 6: The extensive margin of exporter dynamics

(a) Fraction of exporters by age
(b) Export markets survival rate

Figure 7: The intensive margin of exporter dynamics

(a) Relative size of exporters
(b) Share of total exports
(c) Mean investment rate
(d) Investment std. dev.