Does Aggregate Government Size Effect Private Economic Performance in Canada?

J. Stephen Ferris
(steve.ferris@carleton.ca)
and
Marcel-Cristian Voia
(marcel-cristian.voia@carleton.ca)
November 10, 2014

Abstract

This paper asks whether comprehensive, non-interest government size has an inverted U-shaped effect on private economic output in Canada and whether its current size is too large relative to the estimated tipping point. Using data from 1929 through 2011 and controlling for both correlations arising among the independent variables across time and endogeneity in the relationship between size and performance, we find evidence consistent with size producing an inverted U-shaped effect on private output that peaks in the range of 30 to 34 percent of GDP. Use of graphic nonparametric methods, conditional on the same control and instrument variables, reinforces the parametric estimates of threshold and quadratic models while illustrating visually the effect of controlling for endogeneity through instrumented variables.

Key Words: Government Size, nonlinear time series, nonparametric methods, tipping point, endogeneity correction.

JEL numbers: H21, H23, C22, C26

Department of Economics
Carleton University
Ottawa, Ontario, Canada K1S 5B6
1. Introduction

There is an ever growing literature on the many ways that government size interacts with the private economy. Early empirical contributions focused on cross country comparisons and include such classic papers as Landau (1983), Kormendi and Maguire (1985), Ram (1986) and Grossman (1987). More recent contributions that relate government size and output specifically to Canada include Scully (1989), Afxentiou and Serletis, (1991), Chao and Grubel (1998), Petry, Imbeau, Crete and Clavet (2000), Voia and Ferris (2013), and Ferris and Voia (2014). While much of that literature examines how government size responds to income and output—what is called Wagner's Law—the strand of the literature we are interested in this paper reverses that causality to ask whether government size is complementary with or a substitute for private economic performance. The primary reason for the latter interest is because virtually all developed economies have experienced long periods of growth in the size and scope of government so that recent periods of economic contraction have triggered concern that size may be excessive, unduly constraining private economic performance. In part in response to the development of endogenous growth theory, the focus of analysis has narrowed to the effect of size on economic growth (Barro, 1990; Armey, 1995). Here the consensus view is that government size has a negative effect on growth (at least within developed economies). For example, Afonso and Furceri (2010, p. 527) investigate the effect of government size and its volatility on economic growth in OECD and European Union countries and conclude that "both dimensions tend to hamper growth." Similarly Bergh and Henrekson (2011, p.1) conclude that "most recent studies typically find a negative correlation between total government size and economic growth". Finally, Facchini and Melki (2013, p.2) survey sixty investigations of the relation between government size and economic outcomes and find that "66.6% of the studies find a negative effect from government size while only 8.3% find the opposite effect and 25.1% are inconclusive."

While the majority of studies find a negative relationship, the not infrequent finding of insignificant or even positive effects for government size has led to a search for reasons for this ambiguity (Ciccone and Jarocinski, 2010). Most often researchers have focused on compositional differences in government spending/taxation across countries (the type of expenditure or tax matters more than size itself) or the possibility that government size has had a nonlinear, inverted U-shaped effect on private output (bigger size has a bigger negative effect). In earlier work we examined the inverted U-shaped effect in relation to the size of the federal government in Canada (Ferris and Voia, 2014). This analysis had the advantage of using an exceptionally long time series of data—reliable data on aggregate federal government expenditures can be found from shortly after Confederation (1870) to the present (2011). It also allowed for a natural separation between the long run effect of government size and the shorter run effect of the changes in size associated with the second significant activity of federal government—the countercyclical use of fiscal policy in relation to the business cycle. The disadvantage of focusing solely on federal government size, however, was that it did not account for the possibility that government

---

1 A representative finding is given by Chao and Grubel (1998) who, following Scully’s methodology, find the optimal long run size of government in Canada to be about 34% of GDP.
2 Afonso and Furceri (2010) find that a 1% increase in government size decreases output growth by .12% for OECD countries and .13% for European Union countries.
activities may be redistributed among the different levels of government over time. For this reason we re-examine the relationship between government size and private output performance in Canada where our expenditure measure of size is now comprehensive, that is, the sum of federal, provincial and municipal government spending over the 1929 to 2011 time period. A focus on comprehensive government size, net of inter-governmental transfers and interest payments, has the additional advantage of producing a perhaps more appropriate definition of purely private economic activity (measured as GDP – aggregate government spending).

In the following section we begin our analysis by asking what measures of private performance can be related meaningfully to government size over the long run. Here time series considerations suggest that it is the level rather than growth rate of economic performance that should be related to government size. We follow this empirical start by presenting a survey of the public choice theory that suggests that the form of this interrelationship should be nonlinear, perhaps featuring a tipping point. After controlling for a number of other determinants of private performance, we explore the empirical question of which parametric shape best describes the empirical relationship arising between size and performance. Making adjustments for correlations arising among the covariates across time and particularly for the likelihood that endogeneity arises between government size and private output, we find both threshold and quadratic models of the relationship perform well, producing measures of a tipping point that are broadly consistent. Differences in the implied shape of the function surrounding the tipping point and the size of the confidence interval led us to adopt nonparametric modeling methods that do not require a discontinuity in the relationship nor symmetry about the tipping point.

Our nonparametric method uses the spline-based method developed by Ma, Racine and Yang (2011), Nie and Racine (2012) and Ma and Racine (2013) to describe the form of the relationships arising in the data. They allow the unconstrained patterns of response to different control variables to be illustrated in a convenient graphical way and in a form that allows for the incorporation of endogenous regressors through the generation of instrumental variable (IV) nonparametric plots. The enhancement of the analysis of the tipping point by surrounding the point estimate with an appropriate confidence interval allows assessment of whether or not the peak estimates generated by the threshold and quadratic models are meaningful and thus relevant for policy analysis. Because time series issues are not as serious a concern under the nonparametric approach, the ability to control for endogeneity in the size-performance relationship and reveal the inverted U-shaped relationship makes outcomes found with the parametric analysis more convincing. The graphs of the nonparametric analysis illustrate visually the conditional effect of the interdependent variables and how controlling for endogeneity brings out more precisely the inverted U-shaped effect of government size on private performance.

2. Time series and endogeneity concerns with government size and economic performance

The time series issue posed by the long run relationship between government size and private economic performance can be seen in the following set of diagrams. In Figure 1 below we show government size,

---

3 The starting date, 1929, was chosen as the earliest date for which a comprehensive measure of government size was available (see Ferris and Winer, 2007). The ending date is, at the time of writing, the latest for which reliable information could be obtained for our entire set of variables.
measured as the logarithm of the ratio of aggregate non-interest government expenditure to GNP (lnGshare), in relation to two alternative measures of private economic performance. First, private performance is measured as the logarithm of private output per capita (lnPYPC), and second, performance is measured as the rate of growth of private output per capita (PCGROWTH). As can be seen from the second diagram in Figure 1, lnPYPC rises more or less continuously following the Great Depression in Canada whereas in the bottom diagram, PCGROWTH, does not increase, varying more or less randomly about a constant mean of 1.48% per year. In econometric terms, the level of private economic activity is non-stationary or integrated of order one, I(1), while its rate of growth is stationary or integrated of order zero, I(0). When we look to the top diagram to examine long run government size, lnGshare, it is apparent that abstracting from the spike associated with WW2 expenditures, aggregate government size has increased continuously since 1929. Beginning from the low level of 11.8 percent of GDP in 1929, aggregate government size increased to over 45 percent of GDP by the early 1990’s, before falling back to 36 percent of GDP in the period just before the recent recession. In econometric terms, lnGshare is I(1) or non-stationary.

The significance of the time series issue highlighted above is that when variables of different order are regressed together, the resulting coefficient estimates are often interpreted erroneously. For example, the finding that there exists no relationship between the stochastically growing level of government size and a stationary performance growth rate may lead to the premature rejection of the possibility that a meaningful relationship could arise between the two levels. On the other hand, finding that nonstationary variables simply trend together through time is often misinterpreted as implying causality. Finally, a relationship arising between stationary growth variables is often misinterpreted as implying a permanent rather than transient or purely short run relationship arising between levels. This suggests that when putting together long time series in a hypothesis test, one should regress together variables of the same order of integration and, if relating I(1) variables, look for evidence of cointegration among these variables. In our case we begin by exploring the reasoning that would link together the two I(1) variables: government size and the level of private output per capita.

--- insert Figure 1 about here ---

---

4 Private output is defined as GDP minus aggregate non-interest government expenditures net of intergovernmental transfers.
5 The order of integration refers to the number of times a time series must be differenced before finding stationarity. The adjusted Dickey Fuller (ADF) test statistic for lnPYPC is -0.312 (constant) and -6.13 (constant) for PCGROWTH. The corresponding MacKinnon 1% critical value of -3.51 allows rejection of the hypothesis that the growth rate is nonstationary.
6 The ADF test statistics for lnGshare are -2.19 (constant) and -3.17 (constant and trend), with the latter only marginally smaller than the weakest MacKinnon critical value of -3.16 (for 10%). The ADF statistic for its first difference is -6.13 (constant) considerably smaller than the corresponding MacKinnon 1% critical value of -3.52.
7 That is, it implies a permanent relationship between the changes in levels, where the changes in both levels are stationary and hence transitory about their long run values.
8 While the analysis could begin by linking first differences, doing so loses any information arising from a relationship existing between the two levels. In our case, because the business cycle is stationary over time transitory changes in government size that reflect purely countercyclical intervention (or automatic cyclical response) may dominate the fewer permanent changes in government size that are of interest to this analysis.
The second significant econometric issue to be faced is endogeneity. That is, while cointegration does provide evidence of a long run equilibrium relationship, it says nothing about directional causality. Moreover, while our interest is on how government size affects private output, the literature investigating Wagner's Law argues that the level of government size can be explained by the scale and complexity of the private economy. It follows that the ability to interpret the correlation between government size and per capita output as a measure of government’s effect on private output is problematic. To be more precise about any one of these causal routes, the analysis must control for the feedback that can come from induced changes to the other side. This we discuss at length in Section 4 below. But before turning to these empirical issues, we motivate the hypothesis of an inverted U-shaped of the effect of government size on private economic performance from public choice theory.

3. Public choice and the effect of federal government size on the private economy

Broadly speaking public choice analysis views increases in government size as producing two opposing effects on the output of the private sector. First in terms of size generating benefits, initial levels of government spending are viewed as providing basic levels of security that keep individuals safe from physical threat (through collective policing, national defense and diplomatic services) and their property rights secure from arbitrary seizure (through the administration of justice as codified in criminal and commercial law). Further expansion of these roles allow higher levels and better qualities of policing and legal services that in turn permit individuals greater predictability in their dealing with strangers thus realize larger levels of production, trade and welfare (Coase, 1960; Becker, 1983; Wittman, 1995). To the extent that communal services such as health, sanitation, social welfare, education, and research and development are provided by government revenues, larger levels of government spending can increase the quality of productive inputs and through this the output of the private sector (Dahlman, 1991; Thomson and Jensen, 2013). Even more directly, the government provision and monitoring of transportation infrastructure provides inputs that complement private capital and enhance private output (Karras, 1997; Sturm, Kuper and de Haan, 1998). It follows that if government expands by adopting projects in order of their social merit, the social marginal product of government's involvement in the private economy would begin positively and then fall as government size increases. This is illustrated as the concave curve in upper diagram of Figure 2.

The opposing channel of influence focuses on the cost of government intervention. The necessity of funding government activity means that as government size grows, larger levels of resources must be obtained from the private sector. This loss reduces directly the private consumption possibilities of the private sector. In addition, however, the acquisition of these resources through taxation—through both higher levels and often differential rates—further decreases private output by discouraging the supply of productive inputs and distorting the cost of private provision (Stuart, 1984; Usher, 1986). Thus as the size of government grows, the tax price of government services increases. This has led writers such as

Hence by initially looking for cointegration among levels we get a cleaner measure of the long run relationships (with the cyclical effects remaining in the residuals). See Ferris (2014) for an expansion of this idea in relation to government size in New Zealand.
Grossman (1987), Scully (1989, 2000), Armey (1995) and Facchini and Melki (2013) to argue that on net, larger government size must eventually encounter diminishing returns that at some point may reach the point where further increases in size reduce rather than increase private output. The increasing cost of government size on private per capita output is illustrated as the upward sloping curve in the upper diagram of Figure 2. Finally the hypothesized net effect of government size and the level of size that maximizes per capita private output (referred to in the literature as the tipping point) is illustrated in the second diagram of Figure 2. Posed as a regression equation, this can be tested for empirically (if the nonlinear effect is assumed to be quadratic) as

\[
\ln PYP_{it} = c_0 + c_1 \ln Gshare_{it} + c_2 (\ln Gshare_{it})^2 + c_3 Controls_{it} + e_t
\]

where the analysis predicts that \(c_1 > 0\), \(c_2 < 0\) and \(\tau = \exp\left(-\frac{c_2^2}{2c_1}\right)\) is the estimate of the tipping point shown in Figure 2. In (1) \(e_t\) is a random error.

Ideally we would like to include as controls factors of production such as the capital to labour ratio and measures of human capital. However in Canada good estimates of the aggregate capital stock exist only as far back as 1955 and aggregate education variables (as provincial in responsibility) are similarly lacking over the long run. Hence long run studies of output levels in Canada have used what data is available as proxies for these variables (see, for example, Winer and Ferris, 2008). In our analysis we use four control variables including: LnAgric, where Agric is the share of the labour force in agriculture (to capture changes in the composition of industrial output and urbanization across time); Lnimratio, where Imratio measures immigration relative to population size (particularly important in the pre-depression and post-world war two time periods); LnYoung, where Young is the proportion of the population sixteen and below (to capture changing demographics); and LnUSiip, where USiip is the U. S. index of industrial production. The very close integration of the Canadian and US economies means that the US index of industrial production will capture common improvements in productivity and performance arising across the two countries. As importantly, the small size of the Canadian economy relative to the US means that we can treat LnUSiip as exogenous to the performance of the Canadian economy.

Table 1 provides descriptive statistics for these variables together with LnPYPC, LnGshare and LnGshare_sq and the Data Appendix provides more detail on their sources. The key point to be noted is that all variables except for one of the political variables (to be introduced later) are I(1).

4. Testing for the linear effect of government size on output versus a threshold effect

We begin our search for the effect of government size on private output per capita by first asking whether there is evidence that the relationship arising among the two potentially endogenous variables and the control variables is strictly linear. That is finding that the linear version of equation (1) is cointegrated would provide strong evidence of a linear long run equilibrium relationship that would negate the need to search further for nonlinearity. The test for the presence of a such a long run linear

\footnote{Theories directed at explaining why government size may become too large include Niskanen's (1968) theory of the bureau, Meltzer and Richard's (1981) median voter theory, Brennen and Buchanan's (1980) Leviathan model and Buchanan and Tullock's (1962) emphasis of the common pool problems that arise in modern democracies.}
relationship requires evidence of the stationarity of the residuals of an OLS regression of equation (1) with $c_2 = 0$. The results of this test are presented as column (1) in Table 2.

While an initial view of column (1) suggests that there is a long run negative relationship between the logarithms of government size and private output, the large value of the $R^2$ term (.993) and the low size of the Durbin-Watson statistic (0.869) provide the classic sign that the estimated relationship may be spurious. As is well known, such a conclusion can be rejected if the residuals of the OLS equation are stationary. In our case, however, the Engle-Granger test for stationarity among the residuals provides only weak evidence of cointegration—that is, the Adjusted Dickey-Fuller (ADF) test statistic (-4.78) allows us to reject the hypothesis of a unit root in the residuals only at the ten percent level. Similarly the inverse Kwiatkowski-Phillips-Schmidt-Shin test statistic for stationarity in the residuals (0.057) falls well below the ten percent critical value (0.347) for stationarity. Neither test generates confidence in the hypothesis that there is a long run (linear) equilibrium relationship between government size and private performance.

The relatively weak evidence of linearity led us to question whether a threshold model might, as Hansen (2000, 575) suggests, “yield parsimonious evidence of an underlying nonlinearity”. In our case we look for evidence of a downward kink in the slope of an otherwise linear relationship arising between size and performance. More explicitly, then, we test whether Bruce Hansen’s threshold model (2000) allows for a better representation of the data. In columns (2a) and (2b) we present the results of this test in the form of the two estimated equations and the estimated breakpoint. The data does suggest a downward shift in the estimate of the government size coefficient (from -.130 to -.965) around 1980, where the kink in the otherwise linear relationship takes place at a government size of roughly 37 percent of GDP. The accompanying test of the hypothesis of no threshold against the alternative of having a threshold (allowing for White corrected heteroskedastic errors and 5000 bootstrap replications) yields a P-value of .0006.

While the threshold model does provide evidence of a non-linearity in the relationship between government size and private per capita output, the coefficient estimates are likely to include the effects of endogeneity. Therefore we correct for the presence of Wagner Law type endogeneity running from private output to government size by using a set of variables that affect private per capita output only through their effect on government size. An OLS regression of these variables on government size is then used to generate a prediction for government size that will be independent of feedback. Thus replacing the actual with the predicted value of government size in the threshold model allows us to derive a new set of estimated size effects that are less likely to incorporate this form of endogeneity. The instruments used as explanatory variables include three purely political determinants of government size: the proportion of the population that is registered to vote; the proportion of registered voters who actually voted in federal elections (with larger voting participation—a bigger

---

Note that the results suggest that immigration played a significant positive role in relation to private output per capita only in the earlier, pre-1980 time period. The immigration rate has had no significant effect on output in the most recent time period.
franchise and larger turnout expected to increase government size); and the degree of political competition (measured as the average size of the winning margin in federal constituencies and expected to decrease government size). We also followed Rodrik (1998) in including the increasing openness of trade as a metric that could explain the growth of government size in Canada.  

The results are shown as columns (3a) and (3b) of Table 2. Three things are of particular interest. First, the coefficient estimates of government size now indicate a peak rather than just a kink in the effect of government size at the threshold point. The result is then consistent with the hypothesis of government size exerting an inverted V shaped effect on private performance. Second, the estimated tipping point in the threshold model of column (3a and b) is smaller than that estimated in column (2a and b). This implies that the correction for endogeneity has lowered the estimate of the level at which government size begins to reduce private per capita output. Third while the correction for endogeneity has had little effect on the size of the coefficient estimates of LnAgric and LnUSiip, the separation into two distinct time periods does bring out more distinctly a change in the estimated effect of immigration and youth on private per capita output. In the period between 1929 and 1985 a higher immigration rate and a lower proportion of the population 16 or younger is associated with higher private per capita output. In the later 1985 to 2011 time period, however, the opposite holds true. Finally, the accompanying test of the hypothesis that there is no threshold against the alternative of having a threshold (allowing for White corrected heteroskedastic errors and 5000 bootstrap replications) yields a P-value of .0096. This implies that the threshold model presents a better fit with the data than does the simple linear version of column (1).

5. From a Linear to a Quadratic effect of Government Size on Per Capita Private Output

The improvement of the threshold model over the linear model allows for the possibility that further insight can be gained by loosening the form of the constraint on the nonlinear relationship. This leads us in Table 3 to present three sets of estimates of the hypothesized quadratic form discussed earlier as equation (1). Here the progression of the three tests from column (1) through column (3) corresponds to corrections made to account first for correlations arising among the independent variables across time and then for endogeneity in the relationship between government size and private performance.

--- insert Table 3 about here ---

The fact that the standard errors of the coefficient estimates in (1) are likely biased by correlations arising among the independent variables across time leads to the correction embodied in dynamic ordinary least squares (DOLS) estimation. In column (2) we present these DOLS results. The results here reinforce the significance of the inverted U-shaped form suggested by column (1) and imply an increase in both the curvature of the hypothesized quadratic relationship and the estimate of the size of the tipping point (from 11.5 to 21 percent). As was the case in the threshold model estimated earlier, the inverted U-shaped relationship now indicated by the coefficient signs is consistent with a long run

---

11 Openness is defined as the ratio of exports plus imports divided by GDP. As the estimated equation in the footnote to Table 2 indicates, greater openness is associated with a larger sized government as predicted by Rodrik (1998). The positive correlation estimated, however, is not strongly significant.
equilibrium relationship, but not necessarily with causality running from government size to per capita private output. In column (3) we then present the results that correct simultaneously for both the presence of intertemporal correlations and the likelihood of endogeneity. This is done by retaining the DOLS formulation and then instrumenting government size. In controlling for endogeneity—the feedback from greater private performance to a larger sized government—we use the same instruments for government size as before: the set of political determinants of government size and the degree of openness to foreign trade.

What is apparent from the results in column (3) is that instrumenting LnGshare and LnGshare_sq does succeed in removing much of the endogeneity—the probability that endogeneity remains in the regression estimates is now only 0.008. As was the case for the threshold model, the coefficient correction increases the curvature of the function relating government size to private output per capita but unlike the threshold case, results in an estimate of the tipping point that is higher. The adjusted quadratic model now implies a peak in private output per capita when government size is roughly 32 percent of GDP and value much closer to the tipping point estimated earlier for the instrumented threshold model. In terms of the control variables, LnUSiip is an exception in that all other control variables retain their sign and significance with the absolute size of their estimated effect tending to rise.

6. Confidence Intervals and Nonparametric Estimation

If we think of the results in column (3) of Tables 2 and 3 as the versions of the threshold and quadratic models that best capture the effect of government size on private per capita output, can we say anything about whether the implied tipping points in these models are consistent with each other? One way of making this assessment is to see if the point estimate of one tipping point lies within the confidence interval of the other point estimate. To do so we use the Delta method (see Oehlert 1992) to establish Wald-type confidence intervals about the tipping point for the quadratic case in Table 3. After adopting a 95% confidence criterion, Delta lower and upper bounds can be established around the 31.8 percent point estimate of the tipping point in the quadratic case as, respectively, 28.6 and 34.9 percent. This then encompasses point estimate of the tipping point in the threshold model of 34.5 percent. In this sense the alternative parametric estimates are consistent.

While the estimates of the two parametric models are not dissimilar, the parametric representations of threshold and the quadratic models impose relatively strong restrictions on the possible shapes of the underlying functional relationship to be estimated. The threshold model, for example, allows the effect of government size on private output to differ on either side of the tipping point but allows only a single discontinuity in the linear slope of that relationship. The quadratic form, on the other hand, allows for more continuous variation in the size of the estimated effect but imposes symmetry on either side of the tipping point. Both restrictions are likely to misidentify the tipping point should the underlying relationship not exhibit either convenient feature.

---

12 See the following section for a more detailed comparison.
13 For details on this process see Ferris and Voia (2014, Appendix B).
To assess whether either parametric form may result in a misidentification of the tipping point we adopt
the nonparametric spline-based method of Ma, Racine and Yang (2011), Nie and Racine (2012) and Ma
and Racine (2013). In doing so we assume that the conditional mean of LNPYPC depends on government
size, the controls adopted earlier, and follows a non-linear, unknown function approximated by the
best-fit B-splines that allows for heteroskedasticity of an unknown form. That is, we assume
\[
\text{LNPYPC}_t = f(\text{LnGshare}_t, \text{Controls}_t) + \sigma(\text{Gshare}, \text{Controls}_t) \omega_t
\]
(2)
where \(\sigma\) and \(\omega_t\) are unknown and estimation is conducted assuming exogenous and then possibly
endogenous covariates. Readers interested in a more technical description of the method used and the
generation of the graphical representations of the fitted function and the partial effects associated with
each covariate are referred to Ferris and Voia (2014, Appendix C). It also details the derivation of the
point-wise confidence bands. To control for endogeneity, we estimate a nonparametric instrumental
variables model following a regression spline methodology developed by Horowitz (2011) where the
instruments used are the same as those used in the earlier parametric regressions. Once again readers
interested in the technical details are referred to Ferris and Voia (2014, Appendix D).

--Figures 3 and 4 about here--

The results of using these nonparametric methods are illustrated in Figures 3 and 4, where the
difference between these two sets of figures arises because of instrumenting government size for
endogeneity. What the nonparametric results make more apparent visually is how controlling for the
feedback from private output to government size brings out the inverted U-shaped effect of
government size on private per capita output. This is particularly important since the nonparametric
method is less susceptible to misspecification by time series issues. Thus the graphic method illustrates
how the flatness of the estimated effect of government size in the neighborhood surrounding the
estimated tipping point and its asymmetric shape can result in the varied estimates of the tipping point
found by the threshold and quadratic models. However by calculating the derivatives of the functions
estimated in Figure 4 and solving for the log-value of government size at which its derivative becomes
zero, we can more precisely determine the log-value of the tipping point. This is found as 3.4, a log-
value that corresponds to an actual government size of 29.5 percent.\(^{14}\) The upper and lower bounds of
the 95 percent confidence interval can also be determined and established, respectively, as 31.7 and
27.2 percent.\(^ {15}\) Hence although the earlier parametric estimates and their confidence intervals suggest
that the effect of government size could peak somewhere in the 27 to 35 percent range, the
nonparametric method finds that the quadratic form does a better job than does the threshold model in
capturing not only the shape of the underlying function in the neighborhood of the tipping point but
also the location of its peak. As a bonus the nonparametric control diagrams in Figure 4 also illustrate

\(^{14}\) The diagrams in Figure 4b show the slope of the corresponding functions in Figure 4a. Use of the first derivative
allows for a more precise determination of the tipping point as a function of government size (variable \(z\) in the first
diagrams) when the function itself is very flat in the neighborhood of its peak.

\(^{15}\) These calculations and diagrams are available on request.
why the coefficient signs on lnimratio and lnyoung depend on the time interval (via their growing size) over which the separate parts of the threshold model is estimated. In this sense the nonparametric methods bring together in one set of diagrams the separate aspects of the relationship appearing earlier in the different parametrically estimated models.

7. Policy Significance

If we use the above analyses and its conclusion that government size peaks in its positive effect on private per capita output at closer to 32 than to 35 percent of GDP, then this criterion implies that government size is currently too large in Canada. Inspection of Figure 5 shows that the expenditure share of consolidated government in GDP has exceeded forty percent in each of the last three years of our sample. On the other hand, some portion of both provincial and federal government spending in these years will reflect higher than normal levels of expenditure arising in response to the 2008/9 recession. In that sense, some portion of the more recent increase in government size will be purely transitory. Hence if we use the pre-recession period instead as the guideline for what has been the current normal size of government, consolidated government has somewhat smaller, averaging 36.6 percent over the 2000 to 2006 time period. Using this metric, actual government size has been 4.6 percentage points or 15 percent higher than the output maximizing size of government. It follows that if the policy objective of the community were to maximize the effect of government size on private per capita output, this analysis suggests that government size should be smaller. As we have seen, however, the function describing the effect of government size on private per capita output is virtually flat over the 30 to 35 percent range. This suggests that the loss in foregone private output per capita has been relatively marginal.

It is also important recognize that while the government’s ability to enhance the output performance of the private sector is a commendable objective, it is not the only objective of government. Indeed the variation in government size that takes place over the business cycle is a clear indication that counter cyclical spending, particularly in times of recession is a valued activity of government. Somewhat more generally, government performs a number of redistributive functions that imply a willingness to accept the need to sacrifice some amount of efficiency for greater equity. Hence if one admits a legitimate redistributive role for government, the optimal size of government size would be expected to be somewhat larger than the level that would maximize private output performance.

Finally while we have focused on aggregate government size, it may be of interest to ask whether expenditure levels have risen equally across the different levels of government or were concentrated at one level of government. This comparison can be seen in Figure 5 where we plot both non-interest federal and consolidated government spending as a percentage of GDP. As that figure indicates, while both curves rose through 1974/5, the distance between the two curves gradually widened, reflecting the growth in provincial and local government expenditure relative to the federal government. In the period following 1975, aggregate government size itself did not increase, varying both above and below
40 percent. However, in terms of its subcomponents federal government size slowly fell from 20 percent to 14 percent of GDP implying that provincial and local government size increased both relative to the federal government and in absolute size (from 18 to 27 percent). In part this reflects the growth in the importance of health and education expenditures which are both areas of provincial responsibility.

From this it is tempting to argue that if a smaller sized government is desired, the relative growth of provincial and local government spending would place the onus of adjustment on the provinces rather than the federal government. However, such a conclusion is well beyond the scope of this paper and would require a much more intensive analysis of the evolving roles of government within the Canadian federation (see also Kneebone, 1992). Moreover a point of caution is suggested by the findings of our earlier analysis of federal government size (Ferris and Voia, 2014). That is, even though federal government size has been falling, that analysis did suggest that federal government size was proportionally higher than its output maximizing level than consolidated government size was found to be here.

8. Conclusion

In this paper we test the hypothesis that aggregate government size in Canada has exerted an inverted U shaped effect on private output per capita and find evidence consistent with that hypothesis. No matter whether the form of that relationship is viewed as linear with a kink (as in the threshold analysis) or as a more smoothly symmetric function (as in the quadratic model), our parametric estimates suggest a peaking in the positive effect of government size in the range of between 27 and 35 percent of GDP. Our use of nonparametric methods to assess the robustness of these restricted forms reinforces the parametric findings generally but does suggest that the output maximizing size of government will lie closer to the 32 percent level found using the quadratic form. The graphical techniques of the nonparametric method also illustrated diagrammatically the importance of controlling for endogeneity and its role in reconciling the findings across the three major methods used for estimation.

In terms of its policy significance, the results are broadly reassuring in their implication that current aggregate government size in Canada is not particularly excessive. That is, while the current ‘normal’ size of Canada’s consolidated government is somewhat larger than the estimated tipping point, the gap between the quadratic estimated peak and the actual is relatively small (only 4.6 percentage points above the quadratic estimate of 32 percent) with the marginally negative effect of larger size is estimated to be small in the neighborhood of the tipping point. Moreover it is not clear that maximizing private output per capita is the sole or even overriding objective of government policy. When one takes into account the collective desire by the community to use government to improve equity through redistributive programs, the implied efficiency cost means that the optimal size of government will be somewhat above the tipping point of our analysis. On the other hand, what the analysis does say forcefully is that government size does have a cost in terms of foregone private output. Given the scale of current government size in Canada the analysis suggests that there is no free lunch associated with the further expansion of government size. As with all other economic choices, the operative question for government is always whether the proposed lunch is worth its cost.
References


Data Appendix: Economic and Political Variables

Economic Variables:


GOV_COMP = consolidate non-interest government expenditure =GCI + GTR + GSUB


Gshare = GOV_COMP/GDP


IMRATIO = Immigration/POP

LnX = log(X)

OPEN = EXPORTS + IMPORTS/GDP


PYPC = (GDP – GOV_COMP)/(P*POP).

Young = percentage of the population 16/17 and younger; 1929-1970 Cansim C892547; 1971-2011 Cansim II v466965.

Political Variables:

Registered = fraction of the population registered to vote. Source: Elections Canada web site, www.elections.ca/past elections/A History of the Vote in Canada: Appendix


Winningmargin = the percentage of the votes won the winning party minus the percentage won by its closest rival multiplied by 1/election distance (where election distance is the number of years from the closest election). Source: Electoral results by Party, 1867 to Date. Web site of the Parliament of Canada: www.parl.gc.ca.
Figures and Tables
Figure 1: The logarithms of Government Size, Private Output per capita and the Growth Rate

LNGSHARE

LNPYPC

PCGROWTH
Figure 2 The Effects of Government Size and Tipping Point
Table 1: Descriptive Statistics: Canada 1929 - 2011

<table>
<thead>
<tr>
<th>Variable name</th>
<th>Mean</th>
<th>Maximum</th>
<th>Minimum</th>
<th>Standard Deviation</th>
<th>ADF Statistic -- level (constant)</th>
<th>ADF Statistic 1st difference (constant)</th>
</tr>
</thead>
<tbody>
<tr>
<td>LnPYPC</td>
<td>9.01</td>
<td>9.73</td>
<td>7.99</td>
<td>0.480</td>
<td>-0.312</td>
<td>-6.13***</td>
</tr>
<tr>
<td>LnGshare</td>
<td>3.40</td>
<td>3.83</td>
<td>2.47</td>
<td>0.335</td>
<td>-2.19</td>
<td>-6.30***</td>
</tr>
<tr>
<td>(LnGshare)^2</td>
<td>11.70</td>
<td>14.67</td>
<td>6.09</td>
<td>2.19</td>
<td>-2.11</td>
<td>-6.46***</td>
</tr>
<tr>
<td>LnAgric</td>
<td>2.04</td>
<td>3.59</td>
<td>0.545</td>
<td>1.00</td>
<td>-0.198</td>
<td>-6.58***</td>
</tr>
<tr>
<td>LnYoung</td>
<td>3.41</td>
<td>3.67</td>
<td>3.00</td>
<td>0.210</td>
<td>-1.34</td>
<td>-1.21</td>
</tr>
<tr>
<td>LnImratio</td>
<td>-0.677</td>
<td>0.529</td>
<td>-2.679</td>
<td>0.756</td>
<td>-2.29</td>
<td>-7.27***</td>
</tr>
<tr>
<td>LnUSiip</td>
<td>8.55</td>
<td>7.74</td>
<td>4.54</td>
<td>0.898</td>
<td>-0.935</td>
<td>-7.40</td>
</tr>
<tr>
<td>LnOpen</td>
<td>3.92</td>
<td>4.45</td>
<td>3.46</td>
<td>0.237</td>
<td>-1.76</td>
<td>-6.76***</td>
</tr>
<tr>
<td>LnRegistered</td>
<td>4.15</td>
<td>4.32</td>
<td>3.96</td>
<td>0.109</td>
<td>-1.72</td>
<td>-8.62***</td>
</tr>
<tr>
<td>LnL turnout</td>
<td>4.27</td>
<td>4.37</td>
<td>4.07</td>
<td>0.75</td>
<td>-1.64</td>
<td>-8.85***</td>
</tr>
<tr>
<td>LnWinningmargin</td>
<td>2.37</td>
<td>3.28</td>
<td>1.54</td>
<td>0.461</td>
<td>-7.53***</td>
<td></td>
</tr>
</tbody>
</table>

***(**)[*] significantly different from zero at 1% (5%) [10%] using the MacKinnon (1996) critical values of -3.51 (-2.90) [-2.59].
Table 2: The Linear Parametric Effects of Aggregate Government Size on Private Output Per Capita: Canada 1929 - 2011

<table>
<thead>
<tr>
<th>Dependent Variable Regression Type</th>
<th>(1) LNPYPC OLS</th>
<th>(2a) LNPYPC Threshold q &lt; 3.62</th>
<th>(2b) LNPYPC Threshold q &gt; 3.62</th>
<th>(3a) LNPYPC Threshold q &lt; 3.54</th>
<th>(3b) LNPYPC Threshold q &gt; 3.54</th>
</tr>
</thead>
<tbody>
<tr>
<td>LnGShare</td>
<td>-0.319***</td>
<td>-0.130**</td>
<td>-0.965***</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(9.26)</td>
<td>(2.73)</td>
<td>(17.9)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Pred_LnGShare#</td>
<td></td>
<td></td>
<td></td>
<td>0.0445 (0.796)</td>
<td>-0.458*** (3.40)</td>
</tr>
<tr>
<td>LnAgric</td>
<td>-0.114***</td>
<td>-0.045</td>
<td>-0.299***</td>
<td>-0.147**</td>
<td>-0.227***</td>
</tr>
<tr>
<td></td>
<td>(4.36)</td>
<td>(1.33)</td>
<td>(9.76)</td>
<td>(2.59)</td>
<td>(3.18)</td>
</tr>
<tr>
<td>LnImratio</td>
<td>0.018*</td>
<td>0.046***</td>
<td>-0.024</td>
<td>0.088***</td>
<td>-0.190***</td>
</tr>
<tr>
<td></td>
<td>(1.70)</td>
<td>(3.92)</td>
<td>(1.56)</td>
<td>(6.84)</td>
<td>(4.13)</td>
</tr>
<tr>
<td>LnYoung</td>
<td>-0.178***</td>
<td>-0.374***</td>
<td>-0.269***</td>
<td>-0.232***</td>
<td>0.378**</td>
</tr>
<tr>
<td></td>
<td>(3.88)</td>
<td>(10.39)</td>
<td>(4.90)</td>
<td>(4.01)</td>
<td>(1.94)</td>
</tr>
<tr>
<td>LnUSiip</td>
<td>0.463***</td>
<td>0.437***</td>
<td>0.224***</td>
<td>0.270***</td>
<td>0.697***</td>
</tr>
<tr>
<td></td>
<td>(14.62)</td>
<td>(10.23)</td>
<td>(5.55)</td>
<td>(11.78)</td>
<td>(6.11)</td>
</tr>
<tr>
<td>Constant</td>
<td>7.39***</td>
<td>7.80***</td>
<td>11.15***</td>
<td>7.56***</td>
<td>3.90***</td>
</tr>
<tr>
<td></td>
<td>(36.77)</td>
<td>(45.6)</td>
<td>(27.06)</td>
<td>(29.6)</td>
<td>(3.20)</td>
</tr>
<tr>
<td>No. of Observations R² Durbin-Watson (8,83) Engle/Granger (ADF Estimated Tipping point</td>
<td>83 .993 0.869 -4.78*</td>
<td>52 .996 31 .995 56 .984 27 .950</td>
<td>37.2% 37.2% 34.5% 34.5%</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
| absolute value of t statistic in brackets; ***(***)[*] significantly different from zero at 1% (5%)[10%]

# Pred_LnGShare = -9.35 + 1.59***LnRegistered + 0.075LnWinningmargin + 1.08**LnTurnout + 0.336 LnOpen with R² = .399. The equation passes the link test for model specification in the sense that the prediction value squared has no explanatory power.
Table 3: Quadratic estimates of the Effect of Aggregate Government Size on Private Output Per Capita: Canada 1929 - 2011

<table>
<thead>
<tr>
<th>Dependent Variable</th>
<th>Regression Type</th>
<th>Parametric Form</th>
<th>(1) LNPYPC</th>
<th>(2) LNPYPC</th>
<th>(3) LNPYPC</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>OLS</td>
<td>DOLS(^1)</td>
<td>2SLS; DOLS(^1)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Quadratic</td>
<td>Quadratic</td>
<td>Quadratic</td>
</tr>
<tr>
<td>LnGShare</td>
<td></td>
<td></td>
<td>0.830</td>
<td>3.09**</td>
<td>14.65***</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>(1.74)</td>
<td>(2.50)</td>
<td>(2.69)</td>
</tr>
<tr>
<td>LnGShare(_sq)</td>
<td></td>
<td></td>
<td>-0.172</td>
<td>-0.510***</td>
<td>-2.12***</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>(2.40)</td>
<td>(2.91)</td>
<td>(2.79)</td>
</tr>
<tr>
<td>LnAgric</td>
<td></td>
<td></td>
<td>-0.121</td>
<td>-0.125***</td>
<td>-0.328***</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>(4.19)</td>
<td>(2.80)</td>
<td>(3.49)</td>
</tr>
<tr>
<td>LnImratio</td>
<td></td>
<td></td>
<td>0.025</td>
<td>0.002</td>
<td>0.159**</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>(2.04)</td>
<td>(0.11)</td>
<td>(2.20)</td>
</tr>
<tr>
<td>LnYoung</td>
<td></td>
<td></td>
<td>-0.258</td>
<td>-0.197**</td>
<td>-1.18**</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>(6.73)</td>
<td>(2.02)</td>
<td>(2.56)</td>
</tr>
<tr>
<td>LnUSiip</td>
<td></td>
<td></td>
<td>0.429</td>
<td>0.442***</td>
<td>-0.360</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>(10.70)</td>
<td>(5.17)</td>
<td>(1.01)</td>
</tr>
<tr>
<td>Constant</td>
<td></td>
<td></td>
<td>5.978</td>
<td>1.90</td>
<td>-5.30***</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>(8.98)</td>
<td>(1.31)</td>
<td>(3.78)</td>
</tr>
<tr>
<td>No. of Observations</td>
<td></td>
<td></td>
<td>83</td>
<td>78</td>
<td>78</td>
</tr>
<tr>
<td>R(^2)</td>
<td></td>
<td></td>
<td>.993</td>
<td>.673</td>
<td>.994</td>
</tr>
<tr>
<td>Durbin-Watson (8,83)</td>
<td></td>
<td></td>
<td>.673</td>
<td>.673</td>
<td>.748</td>
</tr>
<tr>
<td>Engle/Granger (ADF)</td>
<td></td>
<td></td>
<td>-4.22</td>
<td>-5.78</td>
<td>0.008</td>
</tr>
<tr>
<td>Test of Endogenous regressors: P-value</td>
<td>(^)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Estimated Tipping point</td>
<td></td>
<td></td>
<td>11.5%</td>
<td>20.70%</td>
<td>31.8%</td>
</tr>
</tbody>
</table>

absolute value of t statistic in brackets; ***(***)[*] significantly different from zero at 1% (5%)[10%]
\(^1\) DOLS uses contemporaneous first difference and two leads and lags of first differences together with Newey-West adjusted standard errors to account for correlations across among independent variables within and across time.
\(^\#\) 2SLS uses as instruments of government size: lnregistered, lnturnout, lnwinningmargin and lnOpen.
Figure 3: Nonparametric estimates of the conditional effects of Government Size and Controls on LNPYPC without instruments for endogeneity
Figure 4: Nonparametric estimates of the conditional effects of Government Size and Controls on LNPYPC after instrumenting for endogeneity
Figure 5
Federal and Consolidate Non-interest Government Expenditure as a percentage of GDP: 1929 - 2011