Denying Leniency to Cartel Instigators: Costs and Benefits

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Abstract

A large number of countries have introduced successful leniency programs into their competition law enforcement to encourage colluding firms to come forward with evidence that will help detect cartels and punish price-fixers. This paper studies a feature of some of these programs that has received relatively little attention in the literature: the inclusion of “No Immunity for Instigators Clauses” (NIICs). These provisions deny leniency benefits to parties that instigate cartel behavior or function as cartel ringleaders. Our results show that NIICs can lead to increased or decreased levels of cartel conduct. By removing the instigator’s benefit from cooperating with the authorities, a NIIC undoes some of the destabilizing benefit the leniency program was intended to generate and thereby furthers cartel stability. On the other hand, the instigator faces an asymmetrically severe punishment under a NIIC and this can reduce the incentive to instigate in the first place.

Keywords: antitrust, collusion, leniency programs, instigators

JEL Codes: L41, L12, K21
I. Introduction

Price-fixing, the term used here to represent a larger set of collusive agreements among competitors to reduce competition between themselves, has long been the most universally condemned of the antitrust offences. This is explained, at least in part, by the fact that naked price-fixing has very little or no efficiency benefits to weigh against the obvious harms caused by higher prices and associated effects. The strong antipathy toward collusion has resulted in many national competition laws that treat certain classic collusive practices as *per se* illegal, and also in this behavior being treated as criminal conduct in many jurisdictions.¹

Passing laws against collusion is one thing, but detecting and convicting participants for secretive price-fixing is quite another. As the real evidence of price-fixing resides with the participants, a number of national competition regimes have for many years provided incentives for participants to come forward with information by promising amnesty or leniency. Since its reform in 1993, the Corporate Leniency Policy introduced by the United States Department of Justice (DOJ) has been widely regarded as the most successful policy in history in detecting cartels affecting American interests.² Following on this success many other countries introduced or revised their own leniency programs (“LPs”), for example there are such programs now in Australia, Brazil, Canada, China, the European Union and India.³

The implementation of these programs and their apparent success has, not surprisingly, attracted the attention of researchers trying to understand the full implications of these programs

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¹ There are criminal prohibitions for price-fixing in, among other countries: Australia, Canada, the United Kingdom and the United States. See, e.g. Stephan (2014).
² Scott Hammond, Deputy Assistant Attorney General reported in 2005: “The Antitrust Division’s Corporate Leniency Program has been the Division’s most effective investigative tool. Cooperation from leniency applicants has cracked more cartels than all other tools at our disposal combined.” Hammond (2005).
³ Of the 56 countries with antitrust laws in place in the study by Borell et al. (2012), only 10 are reported as not having leniency policies. These include Argentina, Jordan, Malaysia, Peru, Thailand and Venezuela.
and how to optimize them for maximum social benefit.\textsuperscript{4} However, one important feature of some LPs has not been well studied. In several countries with programs, such as the United States, Australia and Brazil, leniency is not available to parties viewed as instigators or ringleaders of the cartels. We refer here to such excluding provisions as “no immunity to instigator clauses” or “NIICs”.\textsuperscript{5} Interestingly, as well, some jurisdictions have implemented NIICs only to subsequently remove them.\textsuperscript{6}

Our purpose here is to explore the implications of adding NIICs to LPs for both the establishment of collusive agreements and the detection of such agreements when they are put into effect. We find that NIICs can have ambiguous effects on the suppression of cartels. It is easy to understand why this might be the case. On the one hand, by removing the availability of leniency to instigators the NIIC undoes some of the supposed benefit of the LP itself – making more credible the instigator’s commitment to its cartel partners and thereby serving cartel stability. On the other hand, a potential instigator in a jurisdiction with a NIIC faces asymmetric, and harsher, punishments relative to its cartel partners who continue to enjoy the option of leniency applications. This can reduce any party’s incentive to instigate a cartel and it will

\textsuperscript{4} Research continues to examine the extent to which the apparent success of leniency programs – as suggested by their very wide-spread adoption around the world – is real. Miller (2009) studies the U.S. experience and finds evidence supporting the effectiveness of the American leniency program at enhancing cartel detection and deterrence capabilities. De (2010) finds evidence that leniency policies have made cartels more fragile in Europe. Brenner (2009) gets somewhat more mixed results studying the European experience: while investigations become more efficient with leniency, there is less clear evidence that cartel stability has been reduced. Borrell et al. (2012) study the effect of leniency policy on business executives’ perceptions of antitrust effectiveness as revealed in regular surveys conducted by the International Institute for Management Development. They find that leniency programs are associated with perceptions of enhanced antitrust effectiveness, particularly for countries with lower effectiveness ratings.

\textsuperscript{5} For example, in the US, in order to apply for amnesty or leniency, the leniency policy requires that “the corporation did not coerce another party to participate in the illegal activity and clearly was not the leader in, or originator of, the activity”, U.S. Department of Justice, Antitrust Division, Corporate Leniency Policy at A.6 (\url{http://www.justice.gov/atr/public/guidelines/0091.htm}).

\textsuperscript{6} For example, the EU (removed in 2006) and Canada (removed in 2010). In a unique situation, China’s State Administration for Industry and Commerce (SAIC) has a leniency policy with a NIIC (applying to “organizers of monopoly agreements”) for non-price monopoly agreements while the National Development and Reform Commission (NDRC) has no NIIC in its leniency policy related to price-monopoly agreements. See Ye (2014).
reduce incentives for the instigator (and in some cases others as well) to cooperate with the authorities once investigations are underway.

We believe that our results suggest that caution be exercised before a competition authority includes a NIIC as part of its leniency program. In addition to recognized challenges associated with identifying which participant is the actual “instigator” for the purposes of applying a NIIC, we see here that the clause may actually stabilize collusion by giving participants more confidence that others will not provide incriminating evidence to the authorities. Therefore, while a NIIC may reflect a jurisdiction’s laying greater blame for cartel behavior on instigators, it could be poor antitrust policy.

The next section of the paper briefly reviews much of the economics literature on LPs, including the few papers that touch on issues closely related to those explored here. It also provides an overview of the model used here. Section III then presents the full model. Sections IV and V present our results, respectively, for the case of an LP without and then with a NIIC. In Section VI we explore a special case of our model -- simplified in some dimensions -- that allows us to explore the implications of adding firm asymmetry. Section VII then provides our conclusions and suggestions for further research.

II. Literature and Model Overview

An earlier and seminal contribution on the economic theory of LPs was that by Motta and Polo (2003), which was followed by notable contributions from Spagnolo (2004), Aubert, Rey and Kovacic (2006), Feess and Walzl (2004), Motchenkova (2004), Chen and Harrington (2007) and Harrington (2008), among others. A detailed review of the literature on LPs is provided in Spagnolo (2008). While there is a huge variation in these models, a general conclusion was that
an LP does generally make collusion among firms more difficult, though the literature does point to some notable exceptions.

In fact, Motta and Polo (2003) themselves pointed out that, while LPs can indeed destabilize cartels, they can also have collusive effects. In particular firms may choose to collude but then report (“reveal” in their terminology) to the authorities when the probability of conviction rises, in which case the LP reduces their expected fines from collusion. They also demonstrated that if leniency is made available to firms even after an investigation has been opened, the program would be more effective – indeed, in their model, an LP is not effective if it is available only before the investigation. Rey (2003) and Spagnolo (2004) however provided models in which pre-investigation leniency is also effective since it increases the gains from deviation. This is because defecting cartel members can now reveal and evade paying potential fines.

Harrington (2008) considers the effectiveness of LPs in a novel framework, when the probability of detection and successful prosecution changes over time. The Harrington model also considers whether more than the first party to cooperate should be offered leniency. Harrington shows that the optimal leniency program should provide amnesty to only the “first-party in”, similar to the US LP, and unlike the EU and Canadian LPs.

As noted earlier, relatively little formal attention has been paid in this literature to the possible effects of the asymmetric treatment of instigators or ringleaders – in particular the inclusion of NIICs into the LP.\(^7\) That said, several authors have conjectured as to how such agreements might affect collusion and detection – in some cases suggesting possible effects

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\(^7\) As with Bos and Wandschneider (2012), we do not draw any distinction between instigator and ringleader and model them as the same kind of actor. In an interesting recent paper, Davies and De (2013) examine the record of 89 European cartels between 1990 and 2008 to determine the frequency with which ringleaders appear and the kinds of roles such firms play in their cartels.
modeled here. Two other recent papers that do formally consider asymmetric treatment are Herre et al. (2012) and Bos and Wandschneider (2012). These papers offer complementary treatments to that provided here, presenting very different models (differences highlighted here as we proceed), though both share our interest in understanding the complicated relationship between NIICs and the incidence and detection of collusion. In a model in which no one firm has enough evidence to generate a conviction and side-payments between cartelists are permitted, Herre et al. (2012) show that adding a NIIC will have little effect when the instigator (“ringleader” in their terms) has a large amount of evidence to provide authorities, particularly if the base probability of authority investigation is low. Bos and Wandschneider (2012) study the effect on the highest sustainable cartel price of introducing a NIIC. They find that excluding ringleaders will generally (but not always) lead to lower cartel prices.

Interestingly, there has been some experimental work on leniency that has considered the implications of asymmetric treatment for ringleaders. Bigoni et al. (2012) have recently provided results questioning the value of NIICs at enhancing cartel deterrence or moderating prices in active cartels.

In its structure, the model here is closest to that of Motta and Polo (2003), with the important addition of the instigation stage and special policy treatment of instigators. To facilitate exposition and comparison with this earlier important work, we employ similar notation and terminology. We model a market in which two firms, initially symmetric, compete in an infinitely-repeated game. One firm may elect to suggest a collusive agreement – this is the act of

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8 Aubert et al. (2006) and Spagnolo (2008) are notable examples.
9 Their cartel model is based on that in Bos and Harrington (2010).
10 In Bigoni et al. (2012), subjects play a differentiated Bertrand price game in an infinitely repeated game framework. While LPs deter a larger fraction of cartels from forming, they also lead to higher prices in those cartels that are not reported. If there are positive rewards for whistle-blowing, however, complete deterrence can be achieved. When the ringleader is excluded from the leniency program, they find that fewer cartels are deterred, while the prices in remaining cartels are higher than otherwise.
instigation – and if the other agrees the agreement is confirmed and a violation of the 
competition law committed. The firms realize that such an agreement could be detected and 
punished by the Antitrust Authority (AA). If convicted firms face fines of $F$ unless granted 
leniency. A conviction requires the realization of two separate events – first the AA must 
commence an investigation, second the investigation must result in a successful prosecution. 
These probabilities are taken as exogenous parameters here, determined by public policy 
decisions outside this model. The probability the AA opens an investigation is given as $\alpha$; and 
the probability of conviction, conditional on firms coming to an agreement and the AA launching 
an investigation is given as $\rho$.

After reaching an agreement, each firm independently elects whether or not to honor the 
at agreement. Importantly, we assume that defecting does not remove antitrust liability – the 
offense is committed by simply achieving agreement.\footnote{This assumption is partly based on the idea that even a defecting firm may have prices well in excess of competitive prices, such that its customers are still hurt by its entry into the agreement. Moreover, a firm may be held liable for its previous acts of collusion if it chooses to defect in a later period.} Subsequent to the realization of payoffs from colluding or defecting, the AA may (randomly) open an investigation. At this point, the parties may take advantage of the LP and choose to cooperate with the AA (“reveal”). If any firm reveals, conviction of all parties is assured and punishments handed out. Firms eligible under the LP that choose to reveal will pay a reduced fine, $F_{LP}$, here we assume $F_{LP} = 0$.\footnote{Only one firm is entitled to leniency, so if both firms reveal the “first one in” is determined randomly here. Of course, if a NIIC is in place, the instigator is not eligible for leniency.} If neither firm reveals, the continuing probability of conviction remains at $\rho$.

Any disruption to the collusive equilibrium is assumed to end the cartel forever. That is, 
should either or both firms defect from the collusive agreement or should collusion be punished
by the AA, the industry will revert to static non-cooperative Nash equilibrium behavior forever.\textsuperscript{13}

With this basic structure we examine the scope for collusion with and without a NIIC as part of the LP. To do this we focus on regions in the space defined by different values of two key enforcement parameters, $\alpha$ and $F$, over which various equilibria obtain. For some values, for example, very high values of $F$, it will always be the case that collusion cannot be supported in equilibrium. This would also be the case for high values of the probability of investigation, $\alpha$, if the probability of conviction given investigation ($\rho$) was also very high. For other values it is possible that there will be equilibria in which firms collude but reveal when an investigation starts, while there will generally be regions in which firms collude but do not reveal when an investigation is launched. We show that the introduction of the NIIC has two key effects on the parameter spaces over which collusion can arise and, when it does arise, be detected and punished.

Not surprisingly, the inclusion of a NIIC removes the incentive of the instigator to ever reveal the presence of collusion and this will reduce the possibility that the non-instigator will want to reveal to protect itself. This has the effect of narrowing the range of parameter values over which both parties choose to reveal, with the result that collusion is less likely to be detected after it has begun. The NIIC does, however, also reduce the incentive of firms to take the first step toward collusion – to be the instigator – and this can reduce the incidence of collusion. Interestingly, the first effect (reduced incentive to reveal) can lead, under some parameter values, to the support of collusion where it would otherwise not previously have been possible because of the firms’ expectations that their rivals would reveal.

\textsuperscript{13} These assumptions are similar to those in Herre et al. (2012) and Bos and Wandschneider (2012) but different from the approach in Motta and Polo (2003). In the latter case, the authors allow the parties to return to collusion after detection, under some circumstances even if the firms have chosen to reveal.
In a later section of the paper we explore a special case of the main model into which it is possible to introduce some asymmetry between firms that serves to illustrate some additional conditions under which adding a NIIC to a LP may actually support collusion. The key insight derived here is that the application of a NIIC when the instigator would otherwise be the “weakest link” (i.e. the most likely to defect and apply for immunity) will serve to make the instigator’s promise to adhere to cartel agreements more credible. This will enhance, rather than diminish, cartel stability.

III. The Model and Timing

Two symmetric firms compete in a market for an infinite number of periods. If both firms play static Nash equilibrium strategies per-period profits will be \( \pi^N \) for each. If they collude on the joint monopoly price, they will achieve per-period profits of \( \pi^M \) each.\(^{14}\) Should one firm defect on a collusive agreement while the other honors that agreement, the defector gets profits that period of \( \pi^D \) while its rival gets the payoff of \( \pi^S \). As in the standard prisoner’s dilemma, we assume that \( \pi^D > \pi^M > \pi^N > \pi^S \).

In the absence of any antitrust liability, and on the assumption that firms adopt the grim strategy of playing competitively forever should either of them defect, it is well-known that collusion can be supported if the firms put sufficient value on future profits, that is if their (common) discount factor, \( \delta \), is greater than the critical level \( \delta_0 \), given by:

\[
\delta_0 = \frac{\pi^D - \pi^M}{\pi^D - \pi^N}.
\]

\(^{14}\) We think of this collusive agreement is simply agreeing on a common (monopoly) price and do not permit colluding firms to make side-payments to each other. Given the symmetry here with the simple leniency program this is not a restrictive assumption, however under a NIIC the instigator might demand some compensation for the extra risks it is assuming. The possibility of side-payments is considered in Herre et al. (2012).
To make the following analysis meaningful, we focus on situations in which a cartel agreement would have been sustainable in the absence of antitrust enforcement; that is we assume that \( \delta > \delta_0 \).

As we now introduce antitrust enforcement and a leniency programs, the timing of the game, within each period becomes:

**Stage 0:** The AA moves first, setting the probability of investigation, \( \alpha \) and probability of successful prosecution (given investigation has started), \( \rho \). We take these decisions as exogenously given in our analysis.\(^{15}\)

**Stage 1:** The firms choose whether or not to instigate. If neither instigates, then the firms compete in the product market in non-cooperative Nash fashion, get the corresponding payoffs and the game ends for that period. We move to period 2, where the game again starts from stage 1. If both firms instigate, one is assigned the role of instigator randomly.

**Stage 2:** If there has been instigation by one firm, the other firm either agrees to form a cartel or refuses to do so. In case of the latter, the game ends for this period, and the firms move on to the next period with one-period non-cooperative Nash payoffs.

**Stage 3:** If firms agree to collude at stage 2, the firms set the prices (or quantities) in the product market, either colluding or defecting from the collusion. They obtain their one period collusive payoffs, or their payoffs from defecting (or being cheated on).

**Stage 4:** With probability \( \alpha \), the Antitrust Authority (AA) begins an investigation of the industry.

**Stage 5:** Either (or both) firm(s) may apply for leniency by revealing information to the AA.

**Stage 6:** If either or both firms have revealed, the cartel is convicted with certainty and members are punished as provided for in the cartel law and the LP. If neither firm reveals, conviction is

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\(^{15}\) Motta and Polo’s (2003) treatment includes an analysis of optimal enforcement policies.
obtained with probability $\rho$. A firm eligible for the LP that reveals pays a fine of zero, other firms pays $F$. If both firms are eligible for leniency and reveal, they each secure leniency with probability of $\frac{1}{2}$.

In the event of any defection or successful cartel prosecution firms revert to static Nash non-cooperative play forever.

Note that with the timing as outlined in stages 3 to 5, we are ruling out the possibility that firms will reveal as they defect, before any investigation has been launched. While our approach follows the timing in Motta and Polo (2003), other papers have employed this alternative timing (e.g. Harrington (2008) and Chen and Rey (2013)). In Section VII below – and in more detail in Appendix 3 – we modify our model to allow for revealing with defection and derive results qualitatively similar to those from our main model.

### IV. Analysis of the Leniency Program

At the outset we should note that firms face a stationary environment over time in the sense that the probabilities of investigation and conviction, $\alpha$ and $\rho$, are the same in every period. Accordingly, in every period $t > 1$, the firms will want to continue the collusive agreement as long as they choose to enter into the agreement in period 1 and the cartel has not broken down (due to defection or conviction) prior to $t$. Thus, we can analyze a firm’s incentives to collude, defect, or reveal in the same way for every period.

The equilibrium concept we use is subgame perfect equilibrium. Moreover, we assume that a firm would choose a (weakly or strongly) dominant strategy in any subgame in which such a strategy exists. This helps reduce the number of equilibria we have to consider.

#### IV.1 The Revelation Game at Stage 5
To study the subgame perfect equilibrium, we start with firms’ choices at stage 5 in any period $t$. Assuming that a collusive agreement has been reached ($t = 1$) or continued ($t > 1$) at the beginning of this period, we have three possible scenarios at stage 5: (1) neither firm has defected from the collusive agreement at stage 3 in this period; (2) one of the two firms has defected from the agreement in the period, or (3) both firms have defected from the agreement in the period.

Let $V^C$ denote a firm’s expected payoff from entering into a collusive agreement. To simplify presentation, we define a new variable, $\tilde{\rho}$, which takes on the value $\rho$ if AA launches an investigation, and 0 otherwise. Using this notation, we can write the firms’ payoffs associated with different strategies as given in Table 1 for the subgame associated with the scenario in which neither firm has defected at stage 3.

### Table 1: The Revelation Game at Stage 5 in the case where neither firm has defected

<table>
<thead>
<tr>
<th>Firm 2</th>
<th>Reveal</th>
<th>Not reveal</th>
</tr>
</thead>
<tbody>
<tr>
<td>Firm 1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Reveal</td>
<td>$\pi^M + \frac{\delta \pi^N}{1-\delta} - \frac{F}{2}$, $\pi^M + \frac{\delta \pi^N}{1-\delta} - \frac{F}{2}$</td>
<td>$\pi^M + \frac{\delta \pi^N}{1-\delta}$, $\pi^M + \frac{\delta \pi^N}{1-\delta} - F$</td>
</tr>
<tr>
<td>Not reveal</td>
<td>$\pi^M + \frac{\delta \pi^N}{1-\delta} - F$, $\pi^M + \frac{\delta \pi^N}{1-\delta}$</td>
<td>$\pi^M + \tilde{\rho} \left[ \frac{\delta \pi^N}{1-\delta} - F \right] + (1 - \tilde{\rho}) \delta V^C$, $\pi^M + \tilde{\rho} \left[ \frac{\delta \pi^N}{1-\delta} - F \right] + (1 - \tilde{\rho}) \delta V^C$</td>
</tr>
</tbody>
</table>

It is easy to see from Table 1 that (Not Reveal, Not Reveal) would be a Nash equilibrium in this subgame if

$$\pi^M + \tilde{\rho} \left[ \frac{\delta \pi^N}{1-\delta} - F \right] + (1 - \tilde{\rho}) \delta V^C \geq \pi^M + \frac{\delta \pi^N}{1-\delta}.$$  

(1)
From (1) we can solve the critical value of $\tilde{\rho}$ below which (Not Reveal, Not Reveal) would be a Nash equilibrium in the subgame represented by Table 1, defined by

$$\rho^* = \frac{\delta[(1-\delta)V^C - \pi^N]}{\delta[(1-\delta)V^C - \pi^N] + (1-\delta)F}. \quad (2)$$

Note from (2) that $\rho^* > 0$ if and only if $V^C > \pi^N / (1-\delta)$, i.e. the payoff from collusion has to be higher than that from competition. The latter is, of course, a condition for firms to enter into a collusive agreement in the first place. Therefore, $\rho^* > 0$ holds along any equilibrium path following a collusive agreement.

Note that (Reveal, Reveal) is always a Nash equilibrium in the subgame represented in Table 1. If this equilibrium always prevails for both values of $\tilde{\rho}$, collusion collapses at the end of the first period, with each firm receiving an expected fine of $F/2$. Anticipating this, each firm would choose Defect at stage 3. This, in turn, implies that the two firms would not enter into a collusive agreement in the first place. To keep the analysis interesting, we assume that in the event that AA does not launch an investigation (i.e., if $\tilde{\rho} = 0$), each firm would choose Not Reveal in the subgame represented in Table 1.

Moreover, we also need to specify what criterion we use to select an equilibrium in cases where $\tilde{\rho} = \rho$ and $\rho < \rho^*$. Among the many refinements of Nash equilibriums in the literature, we choose one that can best reflect the ideas we want to capture in this model. One of these ideas is related to the fact that the NIIC eliminates the incentives of the instigator to reveal. Accordingly, the non-instigator does not have to be concerned about the possibility of revelation by the instigator when the NIIC is in place. In contrast, without the NIIC the non-instigator

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$^{16}$ Alternatively, if we assume that (Reveal, Reveal) is a Nash equilibrium in Table 1 for both values of $\tilde{\rho}$, the two firms would never collude because of the deterrence effect of the leniency program. The NIIC, on the other hand, causes collusion to arise under some circumstances (as shown in Appendix 1). In other words, the NIIC will expand the set of parameter values over which collusion occurs. Therefore, adopting this alternative assumption would strengthen our conclusion that the NIIC can expand the set of parameter values over which collusion occurs.
always has to contend with this possibility. This, we believe, should affect the firms’ behavior in the equilibria with and without the NIIC.

To capture and examine the above idea in our model, we use “strategic riskiness” as a selection criterion to select the equilibrium in cases where \( \tilde{\rho} = \rho \) and \( \rho < \rho^* \).\(^{17}\) In a symmetric game such as the one represented in Table 1, an equilibrium is less strategically risky (and is therefore selected) if each firm’s equilibrium strategy is the best reply to the other firm’s strategy of randomizing with equal probability between Reveal and Not Reveal. Hence, strategic riskiness captures the idea that, without the NIIC, each firm is not certain that the other firm will choose Not Reveal and its decision takes into account this uncertainty.

To find the conditions under which firms enter into and honor a collusive agreement, we must consider what happens if one of them defects. Table 2 illustrates the subgame that they face at stage 5 if firm 1 has defected from the collusive agreement at stage 3. We can see from Table 2 that Reveal is a strictly dominant strategy if \( \tilde{\rho} = \rho \), in which case the unique Nash equilibrium involves both firms choosing Reveal. If \( \tilde{\rho} = 0 \), (Not Reveal, Not Reveal) is also a Nash equilibrium in this stage game. However, Reveal is a weakly dominant strategy. As indicated earlier, in cases where there is a dominant strategy, we assume that the firms will choose that strategy in the equilibrium; that is, they choose Reveal in Table 2.

We can construct another table like Table 2 for the case where firm 2 has defected from the collusive agreement at stage 3. Since it will be symmetric to Table 2, we omit it here for brevity. In addition, we also omit the analysis for the case where both firms have defected from the collusive agreement. Given that our interest is in finding the conditions under which collusion

\(^{17}\) Strategic riskiness as a selection criterion can be thought of as an extension of Harsanyi and Selten’s (1988) concept of risk dominance to infinitely repeated games. In an earlier paper on leniency, Spagnolo (2004) uses this concept, which is developed theoretically in Blonski et al (2013) and Blonski and Spagnolo (2014). We are grateful to an anonymous referee for drawing this literature to our attention.
occurs, the latter is not needed for the analysis of a firm’s incentives to enter into and honor a collusive agreement.

### Table 2: The Revelation Game at Stage 5 in the case where firm 1 has defected

<table>
<thead>
<tr>
<th>Firm 1</th>
<th>Reveal</th>
<th>Not reveal</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reveal</td>
<td>[\pi^D + \frac{\delta \pi^N}{1-\delta} - \frac{F}{2}, \pi^S + \frac{\delta \pi^N}{1-\delta} - \frac{F}{2}]</td>
<td>[\pi^D + \frac{\delta \pi^N}{1-\delta}, \pi^S + \frac{\delta \pi^N}{1-\delta} - F]</td>
</tr>
</tbody>
</table>
| Not reveal | \[\pi^D + \frac{\delta \pi^N}{1-\delta} - F, \pi^S + \frac{\delta \pi^N}{1-\delta}\] | \[\pi^D + \frac{\delta \pi^N}{1-\delta} - \hat{p}F,\]
| | | \[\pi^S + \frac{\delta \pi^N}{1-\delta} - \hat{p}F\] |

### IV.2 The Boundary between C/R and C/NR Equilibria

Given the above model specifications, we can make a distinction between two types of equilibria associated with Table 1. In the first one, both firms would choose Reveal if there is an investigation, but Not Reveal if there is no investigation in period \(t\). We name this type of equilibria C/R equilibrium, in which firms collude and then reveal (if there is an investigation). In the second type of equilibria, both firms choose Not Reveal independent of whether there is an investigation. We refer to this as the C/NR ("collude and not reveal") equilibrium.

Before we investigate the characteristics of C/R and C/NR equilibria in detail, we first derive the condition that defines the boundary between C/R equilibria and C/NR equilibria. As indicated earlier, a C/R equilibrium exists whenever there is a C/NR equilibrium under the leniency program, and, in such situations of multiple equilibria, we use the principle of strategic riskiness to select the relevant equilibrium. Accordingly, our goal here is to derive a boundary
condition that separates the situations where a C/R equilibrium is risk-dominant from those
where a C/NR equilibrium is risk-dominant.

Let \( V^{CR} \) denote the sum of discounted profit stream in a C/R equilibrium, and \( V^{CNR} \) the sum of
discounted profit stream in a C/NR equilibrium. Using Table 1, we can express the former as

\[
V^{CR} = \pi^M + (1 - \alpha)\delta V^{CR} + \alpha \left[ \frac{\delta \pi^N}{1 - \delta} - \frac{F}{2} \right].
\]  

(3)

Solving (3), we obtain:

\[
V^{CR} = \frac{\pi^M + \alpha \delta \pi^N / (1 - \delta) - \alpha F / 2}{1 - (1 - \alpha)\delta}.
\]  

(4)

Similarly, we can express \( V^{CNR} \) as:

\[
V^{CNR} = \pi^M + \alpha \rho (\frac{\delta \pi^N}{1 - \delta} - F) + (1 - \alpha \rho)\delta V^{CNR}.
\]  

(5)

Solving the above to obtain:

\[
V^{CNR} = \frac{\pi^M + \alpha \rho \left[ \frac{\delta \pi^N}{1 - \delta} - F \right]}{1 - \delta (1 - \alpha \rho)}.
\]  

(6)

Since the game in Table 1 is symmetric, a risk-dominant equilibrium can be found by
identifying each firm’s best reply to the other firm’s mixed strategy with equal probabilities.
This implies that (Reveal, Reveal) is a risk-dominant equilibrium if each firm’s expected payoff
from choosing Reveal,

\[
\frac{1}{2} \left( \pi^M + \frac{\delta \pi^N}{1 - \delta} - \frac{F}{2} \right) + \frac{1}{2} \left( \pi^M + \frac{\delta \pi^N}{1 - \delta} \right),
\]  

(7)

is at least as high as that from choosing Not Reveal.\(^{18}\)

\(^{18}\) In (8), we assume that each firm knows that the other firm will choose either Reveal in every period or Not Reveal
in every period whenever they are in this subgame. If it observes Not Reveal in the present period, it can infer that
the other firm will choose Not Reveal in all future periods. Thus, in (8) we use \( V^{CNR} \) for \( V^C \).
Using (6), we can rewrite this condition as:

\[ F \geq \Phi(\alpha) = \frac{2\delta(1-\rho)(\pi^M - \pi^N)}{(1-\delta)(1+2\rho) + 3\alpha\delta\rho}. \]  

In other words, \( F = \Phi(\alpha) \) defines the boundary that separates the C/R equilibria from the C/NR equilibria. It can be shown that \( \Phi'(\alpha) < 0 \), which means that the curve \( F = \Phi(\alpha) \) is downward-sloping (see, for example, Figure 1).

**IV.3 The C/R Equilibria**

In order for firms to reach and honor a collusive agreement in a C/R equilibrium, the following two incentive compatibility constraints have to be satisfied:

\[(IC1) \quad V^{CR} \geq V^D, \text{ and} \]

\[(IC2) \quad V^{CR} \geq V^N, \]

where \( V^D \) is the expected payoff associated with defecting from the agreement and \( V^N \) is the payoff from playing the non-cooperative equilibrium forever.\(^{19}\) Using Table 2, we can express \( V^D \) as

\[ V^D = \pi^D + \frac{\delta\pi^N}{1-\delta} - \frac{F}{2}. \]  

The payoff from playing the non-cooperative equilibrium is standard:

\[ V^N = \frac{\pi^N}{(1-\delta)}. \]  

\(^{19}\) Though we refer to IC2 as an incentive compatibility constraint, it might also be described as a participation constraint.
Depending on the value of $F$, one of IC1 and IC2 will be redundant. To be more specific, it can be shown that $V^D > V^N$ if and only if $F < 2(\pi^D - \pi^N)$. Therefore, IC1 is tighter if $F < 2(\pi^D - \pi^N)$, and IC2 is tighter if $F > 2(\pi^D - \pi^N)$.

Using (4) and (10), we rewrite the IC1 constraint, $V^{CR} \geq V^D$, as:

$$F \geq \Gamma(\alpha) \equiv \frac{2(\pi^D - \pi^M) - 2(1-\alpha)\delta(\pi^D - \pi^N)}{(1-\alpha)(1-\delta)}. \quad (12)$$

Define $\alpha_i \equiv 1 - \delta_0 / \delta$. Using (12) we can easily show that $\Gamma(\alpha) > 0$ for $\alpha > \alpha_i$, and that $\Gamma'(\alpha) > 0$. Thus, $\Gamma(\alpha)$ is an upward-sloping curve with a horizontal intercept $\alpha_i$, as can be seen in Figure 1.

The positive slope of $\Gamma(\alpha)$ has the interesting implication that, starting from a point just below this curve, a larger fine ($F$) would actually induce firms to enter into a collusive agreement. The reason for this counter-intuitive observation is that $F$ enters both (4) and (5) with a negative sign. In other words, a larger fine reduces the payoff from collusion and the payoff from defection. From (4) and (10) we find that

$$\frac{\partial V^D}{\partial F} < \frac{\partial V^{CR}}{\partial F} < 0. \quad (13)$$

Thus, a larger fine reduces the payoff from defection by more than it reduces the payoff from collusion, increasing the range of parameter values under which collusion will be sustainable.

Using (4) and (11), we rewrite the IC2 constraint, $V^{CR} \geq V^N$, as:

$$F \leq \Psi(\alpha) \equiv \frac{2(\pi^M - \pi^N)}{\alpha}. \quad (14)$$

It is easy to see from (14) that $\Psi(\alpha)$ decreases in $\alpha$. As shown in Figure 1, $\Gamma(\alpha)$ and $\Psi(\alpha)$ intersect at $\alpha_3 \equiv 1 - \delta_0$ and $F = 2(\pi^D - \pi^N)$.
In addition to IC1 and IC2, we also need to consider condition (9), which determines the boundary between the C/R equilibria and C/NR equilibria. To determine the position of the \( \Phi(\alpha) \) curve relative to the \( \Psi(\alpha) \) curve, we compare (9) and (14) for \( \alpha \) in the range \([0, \alpha_3]\). Figure 1 illustrates a situation where the \( \Phi(\alpha) \) curve lies below the \( \Psi(\alpha) \) curve for all \( \alpha \in [0, \alpha_3] \). As shown in Appendix 2, a sufficient condition for this situation, i.e., \( \Phi(\alpha) < \Psi(\alpha) \) for all \( \alpha \in (0, \alpha_3) \), is \( \delta \leq 1/(2 - \delta_0) \). On the other hand, if \( \delta \) exceeds this threshold and \( \rho \) is sufficiently small, a portion of \( \Phi(\alpha) \) curve lies above the \( \Psi(\alpha) \) curve for \( \alpha \) close to \( \alpha_3 \).

In the remainder of section IV (only), we will present the analysis under the assumption that \( \delta \leq 1/(2 - \delta_0) \). We do so in order to keep at a manageable level the number of cases we have to discuss. Interested readers are referred to Appendix A2.2 for an analysis of the cases associated with \( \delta > 1/(2 - \delta_0) \).

To complete the derivation of a subgame perfect equilibrium in this case, we consider the firms’ decisions at stages 1 and 2 in the first period. For a set of parameters such that (9), (12) and (14) hold, entering into a collusive agreement yields a higher payoff than competition. Hence, each firm has an incentive to be the instigator and to propose a collusive agreement at stage 1, and the other firm will have an incentive to agree to the agreement at stage 2. Accordingly, there are two symmetric subgame perfect equilibria associated in this case, one with firm 1 being the instigator and the other one with firm 2 being the instigator.\(^{20}\)

Note that (9) defines the boundary for the region over which a C/R equilibrium could occur. Below the \( \Phi(\alpha) \) curve, firms would not reveal even if there is an investigation. Thus, this is the region for potential C/NR equilibria. It can be shown that \( \partial \Phi / \partial \rho < 0 \). The latter implies that as

\(^{20}\) The same reasoning applies to the analysis of C/NR equilibria below. Hence, it will not be repeated.
\( \rho \) falls, the \( \Phi(\alpha) \) curve in Figure 1 shifts upward, shrinking the region for potential C/R equilibria while enlarging the region for potential C/NR equilibria.

On the other hand, note from (12) and (14) that \( \Gamma(\alpha) \) and \( \Psi(\alpha) \) are independent of \( \rho \). Thus, these two curves do not shift as the value of \( \rho \) changes.

Therefore, the region for C/R equilibria is the shaded area bounded by the curves \( \Psi(\alpha) \), \( \Gamma(\alpha) \) and \( \Phi(\alpha) \) in Figure 1. Define \( \alpha_2 \) as the solution to \( \Gamma(\alpha) = \Phi(\alpha) \). Then we have the following observations about the C/R equilibria.

**Proposition 1**\(^{21} \): A C/R equilibrium is less strategically risky only if \( F \geq \Phi(\alpha) \). Moreover,

(i) if \( \alpha < \alpha_2 \), a C/R equilibrium occurs for \( F \in [\Phi(\alpha), \Psi(\alpha)] \), but there is no collusion for \( F > \Psi(\alpha) \).

(ii) if \( \alpha \in (\alpha_2, \alpha_3) \), a C/R equilibrium occurs for \( F \in [\Phi(\alpha), \Gamma(\alpha)] \), but there is no collusion for \( F \in (\Phi(\alpha), \Gamma(\alpha)) \) and \( F > \Psi(\alpha) \).

(iii) if \( \alpha > \alpha_3 \), there is no collusion for any \( F > \Phi(\alpha) \).

Proposition 1 suggests that a C/R equilibrium is not possible if the probability of investigation (\( \alpha \)) is sufficiently high. Note also that the effect of a larger fine (\( F \)) on collusion is not monotonic for \( \alpha \) in the intermediate range between \( \alpha_2 \) and \( \alpha_3 \). Here, there is no collusion if the fine is slightly less than \( \Gamma(\alpha) \), but an increase in the fine to between \( \Gamma(\alpha) \) and \( \Psi(\alpha) \) will actually lead to collusion. On the other hand, a further increase in the fine above \( \Psi(\alpha) \) eliminates collusion.

\(^{21}\) The proofs of all propositions and lemmas are presented in Appendix 2.
Now we consider the case $F < \Phi(\alpha)$, i.e., in the region under the $\Phi(\alpha)$ curve in Figure 1. In this case, Not Reveal is a less strategically risky strategy if there is an investigation by AA. For a C/NR equilibrium to occur, the following to incentive compatibility conditions must be satisfied:

**(IC3)** $V_{V}^{\text{CNR}} \geq V_{D}$, and

**(IC4)** $V_{V}^{\text{CNR}} \geq V_{N}^{\text{CNR}}$.

We will first consider IC4. Using (6) and (11) we can show that it is satisfied if and only if

$$F \leq \frac{\pi^{M} - \pi^{N}}{\alpha \rho}.$$  

Moreover, using (9) we can find that $F < \Phi(\alpha)$ implies (15). In other words, $V_{V}^{\text{CNR}} \geq V_{N}^{\text{CNR}}$ is not a binding constraint given that $F < \Phi(\alpha)$.

Turning to IC3, we use (6) and (10) to find that it holds if and only if

$$[\alpha \rho (1 - \delta/2) - (1 - \delta)/2] F \leq \delta(1 - \alpha \rho)(\pi^{D} - \pi^{N}) - (\pi^{D} - \pi^{M}).$$  

Note that the left-hand side of (16) can be positive or negative depending on the magnitudes of $\alpha$, $\delta$ and $\rho$. If it is positive (respectively, negative), (16) implies $F < $ (respectively, $>\) \Omega(\alpha)$, where

$$\Omega(\alpha) = \frac{\delta(1 - \alpha \rho)(\pi^{D} - \pi^{N}) - (\pi^{D} - \pi^{M})}{\alpha \rho (1 - \delta/2) - (1 - \delta)/2}.$$  

Note that the left-hand side of (16), and hence the denominator of (17), is negative for all $\alpha \in (0,1)$ if $\rho < \frac{1 - \delta}{2 - \delta}$. On the other hand, the right-hand side of (16), and equivalently the
numerator of (17), is positive for all \( \alpha \) if \( \rho < \alpha_1 \). Hence, \( \alpha_1 \) and \( \frac{1 - \delta}{2 - \delta} \) are the two critical values of \( \rho \) that affect the sign of \( \Omega(\alpha) \).

The relative magnitudes of \( \alpha_1 \) and \( \frac{1 - \delta}{2 - \delta} \) depend on the value of \( \delta \). It can be shown that
\[
\alpha_1 < \frac{1 - \delta}{2 - \delta} \quad \text{if and only if} \quad \delta < \frac{2 \delta_0}{1 + \delta_0}.
\]
Accordingly, the characteristics of C/NR equilibria depend on the magnitudes of \( \delta \) and \( \rho \).

Let \( \alpha_4 \) denote the positive root to the quadratic equation in \( \alpha \) implied by \( \Omega(\alpha) = \Phi(\alpha) \). In other words, the \( \Omega(\alpha) \) curve and \( \Phi(\alpha) \) curve intercept at \( \alpha = \alpha_4 \). Depending on the values of other parameters, \( \alpha_4 \) may be less than or greater than 1. Figure 2 illustrates a situation in which \( \alpha_4 < 1 \).

**Proposition 2:** A C/NR equilibrium is less strategically risky only if \( F < \Phi(\alpha) \). Moreover,

(i) in the case where \( \rho < \min\{\alpha_1, \frac{1 - \delta}{2 - \delta}\} \), a C/NR equilibrium prevails for any value of \( \alpha \in (0,1) \) as long as \( F < \Phi(\alpha) \).

(ii) in the case where \( \delta < \frac{2 \delta_0}{1 + \delta_0} \) and \( \rho > \alpha_1 \), a C/NR equilibrium prevails if the amount of fine satisfies \( \Phi(\alpha) > F > \max\{0, \Omega(\alpha)\} \) for \( \alpha < \min\{\alpha_4, 1\} \). There is no collusion if \( F < \Omega(\alpha) \).

(iii) in the case where \( \delta > \frac{2 \delta_0}{1 + \delta_0} \) and \( \rho > \frac{1 - \delta}{2 - \delta} \), a C/NR equilibrium prevails if the amount of fine satisfies \( F < \min\{\Phi(\alpha), \Omega(\alpha)\} \).

Proposition 2 is better understood with the aid of Figures 1, 2 and 3. Part (i) of the proposition says that if the probability of conviction (\( \rho \)) is low, a C/NR equilibrium prevails in
any point under the \( \Phi(\alpha) \) curve in Figure 1. In this case, the IC3 constraint is not binding given that \( F < \Phi(\alpha) \). Part (ii) of the proposition corresponds to the lower portion of Figure 2, where the \( \Omega(\alpha) \) curve is upward-sloping. Figure 2 is drawn for a situation where only a portion of the \( \Omega(\alpha) \) curve lies below the \( \Phi(\alpha) \) curve. However, for some smaller \( \rho \) (but still larger than \( \alpha_1 \)), the \( \Omega(\alpha) \) curve is everywhere below the \( \Phi(\alpha) \) curve for all \( \alpha \in (0, 1) \). Note that for \( \alpha \) between \( \alpha_1 / \rho \) and \( \alpha_4 \) in Figure 2, there is no collusion if \( F \) is small, but collusion occurs for a larger \( F \).

Looking at this from a slightly different perspective, Figure 2 shows that collusion can be deterred even if \( F \) is small, provided that both \( \alpha \) and \( \rho \) are sufficiently large.

Finally, part (iii) of the proposition is illustrated in Figure 3, where the \( \Omega(\alpha) \) curve is downward-sloping. Here, the \( \Omega(\alpha) \) curve (representing IC3) becomes a binding constraint for collusion if \( \alpha \) is large enough. Figure 3 is drawn for a situation where the \( \Omega(\alpha) \) curve hits the horizontal axis at \( \alpha < 1 \). However, for some smaller \( \rho \) (but still larger than \( \frac{1 - \delta}{2 - \delta} \)), the \( \Omega(\alpha) \) curve lies above the horizontal axis for all \( \alpha \in (0, 1) \).

**IV.5 Policy Implications**

A close examination of Propositions 1 and 2, along with Figures 1 – 3, reveals the impact of the three policy parameters, \( \alpha \), \( F \) and \( \rho \), on firms’ incentives to collude and, in the event of investigation, to reveal. First, a large probability of investigation (\( \alpha \)) is quite effective in deterring collusion in the region where a C/R equilibrium may arise. But it is not as effective in the region where a C/NR equilibrium may occur. If the probability of conviction is low, a large probability of investigation by itself may have no effect on collusion (see figure 1). To be effective, a large probability of investigation needs to be coupled with a large probability of conviction.
Second, a larger fine does not always reduce collusion. As has been noted above, if the probability of investigation falls in the interval \((\alpha_2, \alpha_3)\) or \((\alpha_1 / \rho, \alpha_4)\) in Figure 2, an increase in the fine can move the equilibrium from a region of no collusion into one of collusion. Intuitively, this possibility exists because the fine affects both the payoff from collusion and the payoff from defection. It occurs when an increase in the amount of fine reduces the former by less than it reduces the latter, thus relaxing the relevant incentive compatibility constraint for collusion. On the other hand, if we restrict ourselves to the regions where collusion occurs, revealing occurs only if the fine is sufficiently large, i.e., if \(F \geq \Phi(\alpha)\). Therefore, if the goal of a leniency program is to encourage cartel members to come forward, a sufficiently large fine is needed to achieve this.

Third and finally, a smaller probability of conviction reduces the occurrences of colluding and revealing (if investigated), but it increases the incidence of colluding and not revealing. Therefore, in a jurisdiction in which the bar for cartel conviction is very high, the leniency program may be less successful in inducing more revealing by firms.

V. Analysis of the Leniency Program with a NIIC

Next we assume that a NIIC is attached to the leniency program. Without loss of generality, much of our analysis will be conducted on the premise that firm 1 is the instigator of the cartel and as such is not eligible for leniency. The analysis is symmetric for the case where firm 2 is the instigator of the cartel. It can be shown that in situations where collusion occurs, there are two subgame perfect equilibria, one with firm 1 as the instigator and the other with firm 2 as the instigator.

We will use “\(^\wedge\)" to indicate the variables and parameters associated with the NIIC regime. Because they are treated differently under the NIIC regime, the payoffs from collusion are
different for the two firms. Accordingly, we define \( \hat{\mathbf{V}}^c_i \) as firm \( i \)'s expected payoff in a collusive equilibrium, and \( \hat{\mathbf{V}}^d_i \) as firm \( i \)'s expected payoff from defection. The payoff from competition, however, remains the same as given by (6).

The procedure we use to determine the equilibria in this case is the same as in section IV. To speed the exposition here we relegate the detailed analysis of this case to Appendix 1. We present below a summary of this analysis.

1. Since the firms are asymmetric in this case, we must consider a pair of incentive compatibility constraints for each of IC1, IC2, and IC3. In each case, only one of each pair of constraints will be binding.

2. The first incentive compatibility constraint, which we called IC1 and which related to the choice between C/R and defecting, is tighter for firm 2 (the non-instigator) than for firm 1. We name this constraint of firm 2 as IC1′ and label its curve \( \hat{\Gamma} \).

3. The second incentive compatibility constraint, IC2, which related to the choice between C/R and playing non-cooperatively (i.e. Nash), is tighter for firm 1 (the instigator) than for firm 2. This constraint of firm 1 (now denoted by IC2′) defines the curve \( \hat{\Psi} \).

4. The third incentive constraint, IC3, related to the choice between C/NR and defecting, is tighter for firm 2 than for firm 1. This constraint of firm 2 is now referred to as IC3′ and its curve as \( \hat{\Omega} \).

5. In the revelation subgame at stage 5, Not Reveal becomes a dominant (at least weakly) strategy for the instigator (since it gains nothing from revealing). Given our assumption that the firms will play (weakly) dominant strategies, we obtain a unique equilibrium in this subgame. This means we do not need to use the strategic riskiness selection criterion

\[\text{As before, incentive compatibility constraint IC4 is redundant.}\]
to identify an equilibrium outcome. Because of this change, the boundary between C/R and C/NR equilibria regions (previously $\Phi$) will move upward to become $\hat{\Phi}$ (as will be elaborated below in Lemma 2).

To determine the effects of the NIIC on equilibria, we compare the equilibrium conditions with and without the clause. We will proceed by first examining how the NIIC affects the incentive compatibility conditions for collusion, represented by functions $\Psi',\Gamma,\Omega,\hat{\Psi},\hat{\Gamma}$ and $\hat{\Omega}$. Then we consider how the NIIC changes the boundary conditions that separate the C/R equilibria from the C/NR equilibria, represented by functions $\Phi$ and $\hat{\Phi}$. Finally, we combine the two to show that the NIIC can expand the set of parameter values over which collusion can arise (which we will refer to for simplicity as “increasing collusion”) under some circumstances.

The incentive compatibility conditions are based on comparisons of the firms’ payoffs under collusion, defection, and competition. Note that the NIIC has no impact on the payoff from competition ($V^N$) and the payoff in a C/NR equilibrium ($V^{CNR}$). Moreover, it can be shown that $\hat{V}_1^{CR} < V^{CR} < \hat{V}_2^{CR}$ and $\hat{V}_1^D < V^D < \hat{V}_2^D$. Thus, for firm 1 (the instigator) the NIIC reduces both the payoff from collusion (in a C/R equilibrium) and the payoff from defection. In other words, while the NIIC reduces the instigator’s incentive to enter into a collusive agreement, it also decreases the firm’s incentive to defect in the event of collusion. On the other hand, the NIIC increases the payoff from collusion (in a C/R equilibrium) for firm 2, but it also enhances its incentives to defect and reveal. On the surface, it is not obvious whether the NIIC tightens or relaxes these incentive compatibility constraints.

A more careful comparison of the incentive compatibility conditions under the two regimes reveals that NIIC tightens these constraints. To present this formally, define a pair of sets,
\[ S \equiv \{ (\alpha, F) \in (0,1) \times R_+ \mid \Psi(\alpha) \geq F \geq \Gamma(\alpha) \} \quad \text{and} \quad \hat{S} \equiv \{ (\alpha, F) \in (0,1) \times R_+ \mid F \leq \hat{\Psi}(\alpha), \alpha \leq \hat{\Gamma} \} , \]

for potential C/R equilibria. The former is the set of \((\alpha, F)\) that satisfies the incentive compatibility constraint associated with a C/R equilibrium, IC1 and IC2, while the latter is the set of \((\alpha, F)\) that satisfies the counterparts under the NIIC regime, IC1' and IC2'. Similarly, we can define another pair of sets for potential C/NR equilibria,

\[ T \equiv \{ (\alpha, F) \in (0,1) \times R_+ \mid kF \leq k\Omega(\alpha) \} \]

where

\[ k = \alpha \rho (1 - \delta / 2) - (1 - \delta) / 2 , \]

and

\[ \hat{T} \equiv \{ (\alpha, F) \in (0,1) \times R_+ \mid F \leq \hat{\Omega}(\alpha) \} . \]

Set \( T \) is the collection of \((\alpha, F)\) that satisfies the incentive compatibility constraint associated with a C/NR equilibrium, IC3, and \( \hat{T} \) is the set of \((\alpha, F)\) that satisfies the counterpart under the NIIC regime, IC3'.

**Lemma 1**: \( \hat{S} \) is a strict subset of \( S \), and \( \hat{T} \) is a strict subset of \( T \).

Lemma 1 states that the set of \((\alpha, F)\) that satisfies the incentive compatibility constraints for collusion under the NIIC regime is smaller than that under the leniency program without a NIIC. This seems to suggest that the NIIC should indeed achieve its intended effect of decreasing collusion. Given that the NIIC tightens the incentive compatibility constraints for collusion in both the C/R and C/NR equilibria, one might expect the NIIC should reduce the occurrence of collusion.

However, an analysis of the boundary conditions \((\Phi \text{ and } \hat{\Phi})\) indicates that the above intuition is incomplete. By denying leniency for the cartel instigator, the NIIC removes its incentives to reveal. This, in turn, makes the cartel more stable and hence more valuable to the instigator and the non-instigator, reducing the latter’s incentives to reveal. This shifts the boundary between C/R and C/NR equilibria. Indeed, using (9) and (28), we can show:

\[ \text{We use this } k \text{ term to control for the switching of signs that determines whether } \Omega \text{ is a positively or negatively sloped function of } \alpha. \text{ See equations (15) and (16) and accompanying discussion.} \]

28
Lemma 2: $\Phi(\alpha) > \Phi(\alpha)$ for all $\alpha \in [0,1]$.

Lemma 2 suggests that, given that collusion has occurred, the NIIC enlarges the set of $(\alpha, F)$ over which the C/NR equilibrium prevails. In other words, while cartels may be less likely to occur under the NIIC, firms are less likely to reveal when a cartel does happen.

A less obvious implication of the boundary shift is that it can enlarge the set of $(\alpha, F)$ for which collusion occurs. Indeed, with the aid of Lemmas 1 and 2, we can establish the following:

Proposition 3: The NIIC reduces collusion if $F \geq \hat{\Phi}(\alpha)$. It does not increase collusion (i.e. increase the range over which collusion is sustainable) if $F \leq \Phi(\alpha)$. However, if $F$ is in the intermediate range, then the NIIC may expand the set of parameter values over which collusion is sustainable and, in situations where collusion prevails with and without the NIIC, it reduces revelation.

Figures 4 and 5 are examples that illustrate Proposition 3. Figure 4 is drawn under the conditions that

$$\rho < \frac{\delta - \delta_0}{2(1 - \delta_0) + \delta} \text{ and } \delta \leq \frac{1}{2}. \quad (18)$$

It can be viewed as a combination of Figure 1 and the figure that would arise from the same parameter conditions under the NICC, for example this considers a case of a sufficiently small $\rho$. In the diagram, area A represents those combinations of $(\alpha, F)$ with which collusion does not occur under the leniency program without a NIIC but does arise under the NIIC regime. Note that this area lies between the curves $\Phi(\alpha)$ and $\hat{\Phi}(\alpha)$. Also lying between these two curves is area B, in which the NIIC turns C/R equilibria into C/NR equilibria. Here, the NIIC does not induce more collusion but it reduces the occurrence of revealing. Located above the $\hat{\Phi}(\alpha)$ curve
is area C, which represents the reduction in collusion as a result of the NIIC. Finally, the NIIC does not increase or decrease collusion in the area under the $\Phi(\alpha)$ curve in Figure 4.

This last observation, however, is tied to the conditions in (18). Figure 5 illustrates an example that can arise under a different set of conditions. Below the $\Phi(\alpha)$ curve in this diagram, area D represents the reduction in collusion as a result of the NIIC. On the other hand, areas A, B, and C represent the same effects of the NIIC as their counterparts in Figure 4.

More generally, we can derive the following condition for the NIIC to increase collusion.

**Proposition 4:** The NIIC increase collusion for $F \in (\Phi(\alpha), \hat{\Phi}(\alpha))$ if

$$\rho < \frac{\delta - \delta_0}{2 + \delta - 3\delta_0}. \quad (19)$$

It is important to note that condition (19) permits a fairly wide range of $\rho$. The critical value of $\rho$ given by the right-hand side of (19) can, for $\delta$ and $\delta_0$ in the appropriate ranges, be greater than the critical values of $\rho$ contained in Proposition 2, namely $\alpha_{1}$ and $(1 - \delta)/(2 - \delta)$. In other words, Propositions 4 is relevant for all three cases in Proposition 2. Figures 6 and 7 are just two examples of the cases that can arise when condition (19) is satisfied.

Finally, it is also worth noting that condition (19) is sufficient, but not necessary, for the NIIC to increase collusion for $F$ in the region $(\Phi(\alpha), \hat{\Phi}(\alpha))$. In other words, the NIIC may increase collusion even if $\rho$ exceeds the threshold given in (19).

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24 To be precise, the conditions for Figure 5 to arise are $\delta_0 > (7 - \sqrt{17})/4$, $\delta \in (2(1 - \delta_0)/\delta_0, 1/(2 - \delta_0))$, and $\rho \in (\alpha_{1}, (\delta - \delta_0)/(2 + \delta)(1 - \delta_0))$. See Appendix 2 for the derivation of these conditions and (18).

25 Note also that Proposition 4 is not subject to the assumption $\delta \leq 1/(2 - \delta_0)$ in section IV.
VI. Asymmetric Firms

In this section, we extend our model to allow firms to be asymmetric in their own right. Specifically, we suppose that, because of differences on the cost and/or demand side, the one-period profit from collusion, defection, or competition is different for different firms.

Accordingly, the one-period profit from each of these actions is denoted by \( \pi_i^N \), \( \pi_i^D \), and \( \pi_i^N \), where the subscript denotes firm \( i (= 1, 2) \). Define \( \delta_{i} = (\pi_i^D - \pi_i^M)/(\pi_i^D - \pi_i^N) \), and assume that the firms’ common discount factor \( \delta \) satisfies \( \delta > \max\{\delta_{i1}, \delta_{i2}\} \), so that collusion can be supported in the absence of antitrust enforcement. Following the same procedure as in previous sections, we can use the firms’ incentive compatibility constraints and the boundary conditions to derive \( \Gamma_i(\alpha), \Psi_i(\alpha), \Omega_i(\alpha), \Phi_i(\alpha), \hat{\Gamma}_i, \hat{\Psi}_i(\alpha), \hat{\Omega}_i(\alpha), \) and \( \hat{\Phi}_i(\alpha) \) for each firm \( i \).

The main point we want to make with this extended model is that when firms are asymmetric, the NIIC can have an additional adverse effect on competition. We will make this point by focusing on the case of C/NR equilibria only. This will allow us to make our point clearly and effectively without being distracted by the discussions of the myriad of scenarios that arise in this extended model. Since, from (9) and Lemma 2, we know that \( \Phi_i'(\alpha) < 0 \) and \( \hat{\Phi}_i(\alpha) > \Phi_i(\alpha) \) for \( \alpha \in [0, 1] \), we accomplish this by assuming that \( F < \min\{\Phi_1(1), \Phi_2(1)\} \). To reduce further the number of scenarios we must discuss, we assume that \( F < \min\{\pi_1^D - \pi_1^N, \pi_2^D - \pi_2^N\} \). This assumption ensures that, under the NIIC regime, the instigator’s payoff from defection is higher than that from competition. Combining these assumptions, we will focus on the equilibria for \( F \) fixed at a value in the range \( F < \min\{\Phi_1(1), \Phi_2(1), \pi_1^D - \pi_1^N, \pi_2^D - \pi_2^N\} \).
We start with the leniency program without a NIIC. With asymmetric firms, the incentive compatibility constraint, IC3, is different for different firms. Using firm $i$’s counterpart to (16), we find that IC3 for firm $i$ is equivalent to

$$\alpha \leq \alpha_{Ci} \equiv \frac{\delta (\pi_i^D - \pi_i^N) - (\pi_i^D - \pi_i^M) + (1 - \delta)F / 2}{\rho [\delta (\pi_i^D - \pi_i^N) + (1 - \delta / 2)F]}.$$

Similarly, we can derive the IC3’ for the non-instigator (indicated by superscript NI) under the NIIC as:

$$\alpha \leq \hat{\alpha}_{CNi} \equiv \frac{\delta (\pi_i^D - \pi_i^N) - (\pi_i^D - \pi_i^M)}{\rho [\delta (\pi_i^D - \pi_i^N) + F]}.$$

With asymmetric firms, it is no longer the case that the incentive compatibility constraint of the non-instigator necessarily implies that of the instigator. Thus, we use firm $i$’s counterpart to (5) to derive its incentive compatibility constraint for collusion as an instigator (indicated by superscript I):

$$\alpha \leq \hat{\alpha}_{CI} \equiv \frac{\delta (\pi_i^D - \pi_i^N) - (\pi_i^D - \pi_i^M) + (1 - \delta)F}{\rho [\delta (\pi_i^D - \pi_i^N) + (1 - \delta)F]}.$$

Notice that the critical values given in (20) – (22) can all exceed 1 for a sufficiently small $\rho$. In that case, collusion would occur for all values of $\alpha \in (0,1)$, and the presence or absence of the NIIC would not make any difference.\(^{26}\) Given that our interest here is to study the effect of the NIIC, we rule out this scenario by assuming that $\rho > 1 - \delta_{oi} / \delta$ for $i = 1$ and 2. This assumption ensures that each of (20) and (21) becomes binding for at least some $\alpha \in (0,1)$.

The relative magnitudes of these critical values of $\alpha$ can be ranked as follows.

**Lemma 3:** \(\hat{\alpha}_{CNI} > \alpha_{CI} > \hat{\alpha}_{CNI}^{NI}\).\(^{26}\)

\(^{26}\) We need to keep in mind that our analysis here is confined to the region of potential C/NR equilibria. The NIIC would make a difference if we broaden the range of $F$ to bring in C/R equilibria.
Lemma 3 suggests that for a given firm, the NIIC tightens the incentive compatibility constraint if it is a non-instigator but relaxes it if it is an instigator. This by itself, however, is not sufficient to ensure that the NIIC would reduce collusion. To the contrary, the NIIC may increase collusion, as we will show below.

To simplify the discussion of the equilibria in this extended model, we assume, without loss of generality, that \( \alpha_{c1} < \alpha_{c2} \). We will first present the conditions under which a C/NR equilibrium occurs with and without the NIIC. This lays the foundation for deriving the condition under which the NIIC increases collusion.

**Proposition 5:** Given the assumptions in this section,

(i) A C/NR equilibrium prevails under the leniency program without NIIC if \( \alpha \leq \alpha_{c1} \).

(ii) A C/NR equilibrium prevails under the NIIC regime if \( \alpha \leq \max \{ \min \{ \alpha_{c1}^I, \alpha_{c2}^{NI} \}, \alpha_{c1}^{NI} \} \).

Part (i) of Proposition 5 says that firm 1 is the marginal firm that determines the critical value of \( \alpha \) for collusion in the absence of the NIIC. Part (ii) of the proposition, however, implies that firm 1 is not necessarily the marginal firm under the NIIC regime. Recall that in our model the identity of the instigator is determined endogenously at stages 1 and 2 in the first period of the game. If a collusive agreement instigated by firm \( i \) yields higher payoffs than competition for both firms, firm \( i \) will have an incentive to propose such an agreement at stage 1 and the other firm will accept it. Note that firm \( i \)’s incentive to be an instigator is independent of whether the other firm 2 wants to instigate a cartel. Therefore, the critical value of \( \alpha \) for collusion is the larger of the two critical values associated with each of the two firms being the instigator. The critical value of \( \alpha \) for collusion is \( \alpha < \min \{ \alpha_{c1}^I, \alpha_{c2}^{NI} \} \) if firm 1 is the instigator. On the other
hand, if firm 2 is the instigator, the critical value is \( \hat{\alpha}_{c_1}^N \), (which is smaller than \( \hat{\alpha}_{c_2}^I \) by Lemma 3).

Proposition 5 implies that the NIIC will increase collusion if
\[
\max \left\{ \min \{\hat{\alpha}_{c_1}^I, \hat{\alpha}_{c_2}^N\}, \hat{\alpha}_{c_1}^N \right\} > \alpha_{c_1}. \text{ This can indeed arise uncertain circumstances.}
\]

**Proposition 6:** Given the assumptions in this section, the NIIC increases collusion if and only if
\[
\hat{\alpha}_{c_2}^N > \alpha_{c_1}. \quad (27)
\]

Figures 6 and 7 illustrate the two situations implied by Proposition 6. Under the NIIC regime, either firm can be an instigator in equilibrium if \( \alpha < \hat{\alpha}_{c_1}^N \), but collusion is possible only with firm 1 as the instigator if \( \alpha > \hat{\alpha}_{c_1}^N \). In latter case, the critical value of \( \alpha \) for collusion is given by
\[
\min \{\hat{\alpha}_{c_1}^I, \hat{\alpha}_{c_2}^N\}. \quad \text{Figure 6 illustrates a situation where } \hat{\alpha}_{c_2}^N < \hat{\alpha}_{c_1}^I, \text{ in which case firm 2 is the marginal firm that determines the critical value of } \alpha \text{ under the NIIC regime. Figure 7, on the other hand, demonstrates a situation where } \hat{\alpha}_{c_2}^N > \hat{\alpha}_{c_1}^I, \text{ in which case firm 1 is the marginal firm under the NIIC regime.}
\]

**VII. Discussion and Conclusions**

From earlier research, notably that of Motto-Polo (2003) we already knew that leniency programs can have unintended effects, in that they can actually enhance cartel stability under certain conditions. This paper extends this earlier work, most significantly by considering the implications of layering a policy of denying leniency to instigators or whistleblowers on top of a more standard leniency program.

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27 From (20)-(21) it is not difficult to see that parameter values exist such that this condition can hold.
The results here suggest first that, the NIIC does have the effect of tightening incentive compatibility constraints around decisions to collude or defect – an effect that would, by itself tend to be pro-competitive. However, the policy also reduced the incentive parties have to reveal collusive activity to the antitrust authority, with two important implications: (i) collusion may be supportable under parameter conditions that would not have supported collusion without the NIIC; and (ii) regions in the parameter space in which collusion would have been revealed after the start of an investigation will, with a NIIC, become regions with collusion but no revealing. It is also true that there can be regions in which the NIIC can successfully deter instigation in the first place, so that collusion is not supportable that would have arisen absent the NIIC.

In our model, firms’ opportunity to reveal comes after an investigation has been launched. As noted in section III, some models in the leniency literature have featured different timing in which revealing can happen as a firm defects from the cartel agreement. In Appendix 3 we modify our model’s timing in this way and show that – even though we will no longer observe revealing in equilibrium – our other key result carries over. Specifically, we show that the NIIC can increase collusion for some parameter values.

Clearly, the wide variety of potential results makes drawing firm policy conclusions on the wisdom of including such clauses difficult. Perhaps the best that can be said is that it is very hard to make a strong policy case for including NIICs based on the deterrence or detection of cartels at this point. It also suggests that further work would be very valuable. We could, for example, incorporate asymmetry into the larger model studied here, though we do not expect that to change our qualitative conclusions much. More promisingly, we could explore the implication of less complete leniency policies and partial NIICs and we could optimize public

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28 It may be the case that some NIICs were motivated by a moral distaste for granting leniency to the most culpable of cartel members, rather than a careful consideration of the efficiency effects of the policy.
policy with respect to these instruments. It would also be very useful, as an important robustness check, to consider more than two firms in the cartel: with only two firms and a NIIC in place, the non-instigator need not fear a “rush to the courthouse” and so might behave in a qualitatively different way than it would were it to be one of several non-instigators. Finally, another modeling assumption that could be usefully explored was that once ended collusion can never be restarted. As with some other papers in the leniency literature we might consider the implications of letting the cartel restart (after detection and punishment, but not after defection) after some number of periods.
\[ \Psi: V^{\text{CR}} = V^{\text{N}} \]

\[ \Gamma: V^{\text{CR}} = V^{\text{D}} \]

\[ \Omega: V^{\text{CNR}} = V^{\text{D}} \]

\[ \Phi: C/R \text{ vs. } C/NR \]
$\Psi: V_{CR} = V^N$

$\Gamma: V_{CR} = V^D$

$\Omega: V_{CNR} = V^D$

$\Phi: C/R \text{ vs. } C/NR$

Figure 2
\[ \Psi: V^{CR} = V^N \]
\[ \Gamma: V^{CR} = V^D \]
\[ \Omega: V^{CNR} = V^D \]
\[ \Phi: C/R \text{ vs. } C/NR \]
\[ \Psi: V_{CR} = V^N \]
\[ \Gamma: V_{CR} = V^D \]
\[ \Omega: V_{CNR} = V^D \]
\[ \tilde{\Phi}: V_{CNR} = V_{CR} \]
\[\Psi: V^{CR} = V^N\]
\[\Gamma: V^{CR} = V^D\]
\[\Omega: V^{CNR} = V^D\]
\[\Phi: V^{CNR} = V^{CR}\]
Figure 6

Collusion without NIIC

Collusion with NIIC

Figure 7

Collusion without NIIC

Collusion with NIIC

\[ 0 \quad \hat{\alpha}_{c1}^{NI} \quad \alpha_{c1} \quad \hat{\alpha}_{c1}^I \quad \hat{\alpha}_{c2}^N \quad 1 \]


Connor, John M. Cartel Amnesties Granted: Worldwide Whistleblowers, Mimeo, Purdue University, 2008.


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Appendix 1: Detailed Analysis of the NIIC Case

In this appendix, we present the complete analysis of the equilibrium under the assumption that a NIIC is attached to the leniency program. We will proceed in the same order as in section IV.

A1.1 The Revelation Game at Stage 5

As in section IV, we start with an examination of each firm’s incentives to reveal at stage 5 in period \( t \). If neither firm defects at stage 3, the firms face the situation at stage 5 as represented by Table A1-1.

Table A1-1: The Revelation Game at Stage 5 in the case where neither firm has defected

<table>
<thead>
<tr>
<th>Firm 2</th>
<th>Firm 1</th>
<th>Reveal</th>
<th>Not reveal</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reveal</td>
<td>( \pi^M + \frac{\delta \pi^N}{1-\delta} - F, \pi^M + \frac{\delta \pi^N}{1-\delta} )</td>
<td>( \pi^M + \frac{\delta \pi^N}{1-\delta} - F, \pi^M + \frac{\delta \pi^N}{1-\delta} - F )</td>
<td>( \pi^M + \frac{\delta \pi^N}{1-\delta} - F )</td>
</tr>
<tr>
<td>Not reveal</td>
<td>( \pi^M + \frac{\delta \pi^N}{1-\delta} - F, \pi^M + \frac{\delta \pi^N}{1-\delta} )</td>
<td>( \pi^M + \frac{\delta \pi^N}{1-\delta} - F )</td>
<td>( \pi^M + \frac{\delta \pi^N}{1-\delta} - F )</td>
</tr>
</tbody>
</table>

In the subgame represented by Table A1-1, Not Reveal is a weakly dominant strategy for firm 1 (the instigator) since the NIIC removes the incentives for the firm to reveal. Then (Not Reveal, Not Reveal) will be a Nash equilibrium in this subgame if firm 2 does not have an incentive to reveal, i.e., if

\[
\pi^M + \frac{\delta \pi^N}{1-\delta} \geq \pi^M + \frac{\delta \pi^N}{1-\delta} - F + (1 - \bar{\rho})\delta \hat{V}^C_2, \quad (A1)
\]
Note that (A1) is the same as (1) except that $V^C$ is now replaced by $\hat{V}_2^C$. Accordingly, we can substitute $\hat{V}_2^C$ for $V^C$ in (2) to obtain the critical value of $\hat{\rho}$ that determines firm 2’s choice between Reveal and Not Reveal, denoted by $\hat{\rho}^*$. 

Note an important difference that NIIC makes to the equilibrium outcome in this subgame. Given that firm 1 never chooses Reveal (because it is weakly dominated), the strategy profile (Reveal, Reveal) is not an equilibrium outcome if $\rho < \hat{\rho}^*$. Accordingly, we no longer need to use strategic riskiness to select an equilibrium outcome.

From (2) it is easy to see that $\hat{\rho}^* > 0$ as long as $\hat{V}_2^C \geq \pi^N/(1 - \delta)$ (collusion yields a higher payoff than competition). This implies that along any equilibrium path, neither firm will reveal in the event that AA does not launch an investigation ($\hat{\rho} = 0$) in period $t$. On the other hand, in the event that there is an investigation ($\hat{\rho} = \rho$), firm 2 will reveal if and only if $\rho > \hat{\rho}^*$. Therefore, a C/NR equilibrium could occur if $\rho < \hat{\rho}^*$, and a C/R equilibrium could prevail if $\rho > \hat{\rho}^*$.

**Table A1-2: The Revelation Game at Stage 5 in the case where firm 1 has defected**

<table>
<thead>
<tr>
<th>Firm 1</th>
<th>Reveal</th>
<th>Not reveal</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reveal</td>
<td>$\pi^D + \delta\pi^N 1-\delta - F$, $\pi^S + \delta\pi^N 1-\delta$</td>
<td>$\pi^D + \delta\pi^N 1-\delta - F$, $\pi^S + \delta\pi^N 1-\delta - F$</td>
</tr>
<tr>
<td>Not reveal</td>
<td>$\pi^D + \delta\pi^N 1-\delta - F$, $\pi^S + \delta\pi^N 1-\delta - F$, $\pi^D + \delta\pi^N 1-\delta - F$</td>
<td>$\pi^D + \delta\pi^N 1-\delta - \rho F$, $\pi^S + \delta\pi^N 1-\delta - \rho F$</td>
</tr>
</tbody>
</table>

To find a firm’s payoff from defection, consider the situation in Table A1-2, which represents the subgame after firm 1 has defected. We can see from Table A1-2 that Reveal is a strictly dominant strategy for firm 2 if $\hat{\rho} = \rho$. If $\hat{\rho} = 0$, on the other hand, Reveal is a weakly...
dominant strategy for firm 2. Under our assumption that a firm chooses a dominant strategy whenever there is one, firm 2 chooses Reveal in the subgame represented by Table A1-2. Given that firm 2 chooses Reveal, firm 1 is indifferent between Reveal and Not Reveal. In fact, the payoffs for the two firms are the same between (Reveal, Reveal) and (Not Reveal, Reveal). Hence, we can select either of these as the equilibrium outcome in this subgame.

We can construct another table like Table A1-2 for the case where firm 2 has defected at stage 3. Applying the same logic as in the preceding paragraph, the relevant equilibria in this stage game are (Reveal, Reveal) and (Not Reveal, Reveal), both yield the same payoffs to the two firms.

A1.2 The C/R Equilibria under the NIIC Regime

We first consider the C/R equilibria, which could arise if \( \hat{\rho} > \hat{\rho}^* \). In this case, the equilibrium strategy profile played in the subgame game represented by Table A1-1 is (Not Reveal, Reveal) if there is an investigation and (Not Reveal, Not Reveal) if there is no investigation. Then at stage 3, the cartel instigator’s expected payoff, if it chooses to honor the collusive agreement, is given by

\[
\hat{\nu}_1^{CR} = \pi^M + (1-\alpha)\delta\hat{\nu}_1^{CR} + \alpha\left[\frac{\delta\pi^N}{1-\delta} - F \right]. \quad (A2)
\]

Solving (A2) to obtain:

\[
\hat{\nu}_1^{CR} = \frac{\pi^M + \alpha\delta\pi^N/(1-\delta) - \alpha F}{1 - (1-\alpha)\delta}. \quad (A3)
\]

Similarly, firm 2’s expected payoff, if it chooses to stick with the collusive agreement, is given by
\[ \hat{V}_2^{CR} = \pi^M + (1 - \alpha)\delta\hat{V}_2^{CR} + \alpha \left[ \frac{\delta\pi^N}{1 - \delta} \right]. \] \hfill (A4)

Solving (A4) to obtain:

\[ \hat{V}_2^{CR} = \frac{\pi^M + \alpha\delta\pi^N / (1 - \delta)}{1 - (1 - \alpha)\delta}. \] \hfill (A5)

If firm \( i \) chooses to defect from the collusive agreement, its expected payoff is

\[ \hat{V}_1^D = \pi^D + \frac{\delta\pi^N}{1 - \delta} - F \] \hfill (A6)

for the cartel instigator, and

\[ \hat{V}_2^D = \pi^D + \frac{\delta\pi^N}{1 - \delta} \] \hfill (A7)

for the other firm.

It is easy to see from (4), (10), (A3), and (A5) – (A7) that \( \hat{V}_1^{CR} < \hat{V}_1 < \hat{V}_2^{CR} \) and \( \hat{V}_1^D < V^D < \hat{V}_2^D \). In other words, the NIIC reduces firm 1’s payoff and raises firm 2’s payoff in both the case where a firm chooses to honor the collusive agreement and the case where it defects from the agreement.

Since the payoffs of these two firms are no longer symmetric under the NIIC regime, we must consider two pairs of the incentive compatibility constraints for collusion, one pair for each firm. They are: \( \hat{V}_1^{CR} \geq \hat{V}_1^D \) and \( \hat{V}_1^{CR} \geq V^N \) for firm 1, and \( \hat{V}_2^{CR} \geq \hat{V}_2^D \) and \( \hat{V}_2^{CR} \geq V^N \) for firm 2.

It is easier to consider first the incentive compatibility constraints of firm 2. Since \( \pi^D > \pi^N \), it is clear from (11) and (A7) that \( \hat{V}_2^D > V^N \). Thus, the binding incentive compatibility constraint for firm 2 is \( \hat{V}_2^{CR} \geq \hat{V}_2^D \), which, for ease of comparison with section IV, will be named IC1'.

Using (A5) and (A7) we rewrite the condition \( \hat{V}_2^{CR} \geq \hat{V}_2^D \) as:
\[ \alpha \leq 1 - \frac{\delta \rho}{\delta} \equiv \alpha_1. \quad (A8) \]

In the \( \alpha - F \) space, (A8) represents all the points to the left of the vertical line at \( \alpha = \alpha_1 \), which is named as the \( \Gamma \) curve in Figure A1-1.

For firm 1, on the other hand, \( \hat{V}_1^D > V^N \) if and only if \( F < \pi^D - \pi^N \). Therefore, the binding incentive compatibility condition for firm 1 to enter into a collusive agreement is \( \hat{V}_1^{CR} \geq \hat{V}_1^D \) for \( F \leq \pi^D - \pi^N \), and \( \hat{V}_1^{CR} \geq \hat{V}_1^N \) for \( F > \pi^D - \pi^N \). It can be shown, using (A3), (A5) – (A7), that \( \hat{V}_2^{CR} \geq \hat{V}_2^D \) implies \( \hat{V}_1^{CR} \geq \hat{V}_1^D \). Thus, the condition \( V_2^{CR} \geq V_2^D \) determines the boundary for collusion in the case \( F \leq \pi^D - \pi^N \).

Now consider the case \( F > (\pi^D - \pi^N) \). Here firm 1’s incentive compatibility constraint is \( \hat{V}_1^{CR} \geq V^N \), which is the counterpart to IC2 in section IV and will be named as IC2′. Using (6) and (A3) we can rewrite it as:

\[ F \leq \hat{\Psi}(\alpha) \equiv \frac{\pi^M - \pi^N}{\alpha}. \quad (A9) \]

It is easy to see from (A9) that \( \hat{\Psi}(\alpha) \) decreases in \( \alpha \).

Next, we consider the boundary condition that separates the region of potential C/R equilibria and that of the potential C/NR equilibria. Substituting (A5) into for \( V^C \) in (2), we derive

\[ \hat{\rho}^* = \frac{\delta(\pi^M - \pi^N)}{\delta(\pi^M - \pi^N) + (1 - \delta + \alpha \delta)F}. \quad (A10) \]

It is clear from (A10) that \( \hat{\rho}^* > 0 \). Moreover, it can be shown \( \rho \geq \hat{\rho}^* \) if and only if

\[ F \geq \hat{\Phi}(\alpha) \equiv \frac{\delta(1 - \rho)(\pi^M - \pi^N)}{(1 - \delta + \alpha \delta)\rho}. \quad (A11) \]
It can be verified from (A11) that $\hat{\Phi}(\alpha)$ decreases in $\alpha$.

In Figures A1-1 and A1-2, $\hat{\Phi}(\alpha)$ defines the boundary between the C/R region and C/NR region. It can be shown that $\partial \hat{\Phi} / \partial \rho < 0$. It implies that as $\rho$ falls, the $\hat{\Phi}(\alpha)$ curve in these two diagrams shifts upward, shrinking the region of C/R equilibria while enlarging the region of potential C/NR equilibria. Figure A1-1 is drawn under the assumption that $\rho > \frac{\delta - \delta_0}{1 + \delta - 2\delta_0}$, in which case the $\hat{\Phi}(\alpha)$ curve lies below the $\hat{\Psi}(\alpha)$ curve for all $\alpha \leq \alpha_1$. Figure A1-2, on the other hand, is drawn for $\rho < \frac{\delta - \delta_0}{1 + \delta - 2\delta_0}$, in which case the $\hat{\Phi}(\alpha)$ curve crosses the $\hat{\Psi}(\alpha)$ curve at an $\alpha < \alpha_1$. In both diagrams, the shaded area above the $\hat{\Phi}(\alpha)$ curve is the region over which a C/R equilibrium occurs.

A1.3 The C/NR Equilibria under the NIIC Regime

Now consider the case $\rho < \hat{\rho}^*$, or equivalently $F < \hat{\Phi}(\alpha)$. In this case, the equilibrium strategy profile played in the subgame represented by Table A1-1 is (Not Reveal, Not Reveal) for both values of $\tilde{\rho}$. Since neither firm ever chooses to reveal, the NIIC has no effect on each firm’s payoff in this situation. Hence, each firm’s payoff remains the same as $V_{CNR}$, given by (6).

In this case, there are three incentive compatibility constraints, namely, $V_{CNR} \geq \hat{V}_1^D$, $V_{CNR} \geq \hat{V}_2^D$ and $V_{CNR} \geq V^N$. However, two of these constraints are not binding. To be more specific, $V_{CNR} \geq \hat{V}_1^D$ is implied by $V_{CNR} \geq \hat{V}_2^D$ because $\hat{V}_1^D < \hat{V}_2^D$. Moreover, since $\hat{V}_2^D > V^N$,

\[\text{Note from (A8) and (A9) that } \hat{\Gamma} \text{ and } \hat{\Psi}(\alpha) \text{ are independent of } \rho. \text{ Thus, in Figures A1-1 and A1-2 these two curves do not shift as the value of } \rho \text{ changes.} \]
also implies that $V^{CNR} > V^N$. Therefore, the only binding constraint we need to consider is $V^{CNR} \geq \hat{V}^D_2$, which we will call IC3'.

Using (6) and (A7), we rewrite IC3' as

$$F \leq \hat{\Omega}(\alpha) = \frac{(1-\alpha \rho)\delta(\pi^D - \pi^N) - (\pi^D - \pi^M)}{\alpha \rho}. \quad (A12)$$

It can be shown that $\hat{\Omega}$ decreases in $\alpha$ and $\rho$. Accordingly, $\hat{\Omega}(\alpha)$ is a downward-sloping curve in Figures A1-1 and A1-2. A fall in $\rho$ would shift the curve upward. Moreover, it can be verified that $\hat{\Omega}(\alpha) = \Phi(\alpha)$ at $\alpha = \alpha_1$, and $\hat{\Omega}(\alpha) > \Phi(\alpha)$ if and only if $\alpha < \alpha_1$. In the latter case, (A12) is not binding. Therefore, the $\hat{\Omega}(\alpha)$ curve determines the boundary of collusion only for $\alpha > \alpha_1$, as shown in Figures A1-1 and A1-2.

**A1.4 Properties of the Equilibria**

To complete the analysis of the equilibria under the NIIC regime, we consider the firms’ choices at stages 1 and 2 in the first period of the game. If conditions IC1' and IC2' are satisfied in the case $F \geq \hat{\Phi}(\alpha)$, or if IC3' is satisfied in the case $F < \hat{\Phi}(\alpha)$, entering into a collusive agreement yields a higher payoff to each firm than competition. Even though the instigator earns a lower payoff than the non-instigator, it is still the best response of a firm to propose a collusive agreement at stage 1 if it expects that the other firm will not propose. Therefore, there are two subgame perfect equilibria, one with firm 1 being the instigator and the other with firm 2 being the instigator. (In the following discussion, however, we will continue to focus on the case where firm 1 is the instigator).

From the proceeding analysis, we can summarize the conditions for the equilibria under the NIIC Regime as follows.
**Proposition A1:**  (i) A C/R equilibrium occurs if \( \hat{\Psi}(\alpha) \geq F \geq \hat{\Phi}(\alpha) \). The latter can be satisfied only if \( \alpha \leq \alpha_1 \).

(ii) A C/NR equilibrium occurs if \( F < \hat{\Phi}(\alpha) \) for \( \alpha \leq \alpha_1 \), or if \( F < \hat{\Omega}(\alpha) \) for \( \alpha > \alpha_1 \).

(iii) No collusion occurs in equilibrium if \( F > \max\{\hat{\Psi}(\alpha), \hat{\Phi}(\alpha)\} \) for \( \alpha \leq \alpha_1 \), or if \( F > \hat{\Omega}(\alpha) \) for \( \alpha > \alpha_1 \).

Recall that Figures A1-1 and A1-2 are drawn for different ranges of \( \rho \). If \( \rho > (\delta - \delta_0)/(1 + \delta - 2\delta_0) \), the \( \hat{\Gamma} \) curve, which comes from firm 2’s incentive compatibility condition in a C/R equilibrium, determines part of the boundary between collusion and no collusion (see Figure A1-1). If \( \rho < (\delta - \delta_0)/(1 + \delta - 2\delta_0) \), on the other hand, this segment of the \( \hat{\Gamma} \) curve is submerged in the region of C/NR equilibria, and as such is no longer relevant. Instead, the \( \hat{\Phi}(\alpha) \) curve determines part of the boundary between collusion and no collusion (see Figure A1-2).
\[ \Psi: V^{CR} = V^N \]
\[ \Gamma: V^{CR} = V^D \]
\[ \Omega: V^{CNR} = V^D \]
\[ \Phi: V^{CNR} = V^{CR} \]
\[ \hat{\Psi}: V^{CR} = V^{N} \]
\[ \hat{\Gamma}: V^{CR} = V^{D} \]
\[ \hat{\Omega}: V^{CNR} = V^{D} \]
\[ \hat{\Phi}: V^{CNR} = V^{CR} \]
Appendix 2: Proofs

A2.1. Proof of the Sufficient Condition for $\Phi(\alpha) < \Psi(\alpha)$ for all $\alpha \in (0, \alpha_3]$

Using (9) and (14), we find that $\Phi(\alpha) < \Psi(\alpha)$ if and only if

$$H(\alpha) \equiv \alpha \delta (4 \rho - 1) + (2 \rho + 1)(1 - \delta) > 0. \quad (A13)$$

It is easy to show that $H'(\alpha) \geq 0$ if and only if $\rho \geq 1/4$. Since $H(0) = (2 \rho + 1)(1 - \delta) > 0$, (A13) holds for any value of $\alpha$ and $\delta$ between 0 and 1 if $\rho \geq 1/4$. For the case $\rho < 1/4$, we use (A13) to show that $H(\alpha_3) > 0$ at $\rho = 0$ if $\delta \leq 1/(2 - \delta_0)$. Since $\partial H(\alpha)/\partial \rho > 0$, we have $H(\alpha_3) > 0$ for any $\rho > 0$ under the same condition. Given that $H'(\alpha) < 0$ for $\rho < 1/4$, the condition $\delta \leq 1/(2 - \delta_0)$ is sufficient to ensure that $H(\alpha) > 0$ and hence $\Phi(\alpha) < \Psi(\alpha)$ for $\alpha \in (0, \alpha_3]$.

A2.2. Analysis of C/R Equilibrium under the Assumption $\delta > 1/(2 - \delta_0)$

The preceding discussion implies that $\delta \leq 1/(2 - \delta_0)$ is a sufficient, but not a necessary, condition for $\Phi(\alpha) < \Psi(\alpha)$ to hold in the range $\alpha \in (0, \alpha_3]$. Using (A13) we find that

$$H(\alpha_3) > 0 \text{ if and only if } 
\rho > \frac{\delta(2 - \delta_0) - 1}{2 + 2\delta - 4\delta \delta_0}. \quad (A14)$$

Note that the right-hand side of (A14) is positive if and only if $\delta > 1/(2 - \delta_0)$.

Suppose $\delta > 1/(2 - \delta_0)$. We still have $\Phi(\alpha) < \Psi(\alpha)$ as long as the value of $\rho$ satisfies (A14). In other words, all of the results in section IV are still valid as long as (A14) holds. On the other hand, in the case where (A14) is violated, we have $\Phi(\alpha) > \Psi(\alpha)$ for $\alpha$ close to $\alpha_3$.

Then in Figures 1 – 3, the $\Phi(\alpha)$ curve lies above the $\Gamma(\alpha)$ curve for $\alpha \leq \alpha_3$, which implies that
the $\Gamma(\alpha)$ curve no longer determines the boundary between C/R equilibria and no collusion.

Define $\bar{\alpha}_2$ as the solution to $\Phi(\alpha) = \Psi(\alpha)$. Part (i) of Proposition 1 is applicable for $\alpha < \bar{\alpha}_2$, and part (iii) is applicable for $\alpha > \bar{\alpha}_2$. Part (ii) of Proposition 1 is no longer relevant.

### A2.3. Conditions for Figures 4 and 5 to Arise

In Figure 4, we have $\hat{\Omega}(\alpha) > \Psi(\alpha)$ and $\Psi(\alpha) > \Phi(\alpha)$ for $\alpha \in [\alpha_1, 1]$. Using (14) and (A12), we find that $\hat{\Omega}(\alpha) > \Psi(\alpha)$ holds for $\alpha \leq 1$ if

$$\rho < \frac{\delta - \delta_0}{2(1 - \delta_0) + \delta}. \quad (A15)$$

Using (A13) we show that a sufficient condition for $\Psi(\alpha) > \Phi(\alpha)$ to hold for $\alpha \leq 1$ is $\delta \leq 1/2$.

Moreover, we need to ensure that $\rho$ satisfies the condition in part (i) of Proposition 2, which is associated with the equilibrium scenarios illustrated in Figure 4. In this regard, observe that

$$\min\left\{\frac{1 - \delta}{2 - \delta}, \frac{\delta - \delta_0}{2(1 - \delta_0) + \delta}\right\} = \frac{\delta - \delta_0}{2(1 - \delta_0) + \delta} \quad (A16)$$

for $\delta \leq 1/2$. Hence, (18) is sufficient for the situation in Figure 4 to arise.

In Figure 5, $\Phi(\alpha) < \Psi(\alpha)$ for $\alpha \in (0, \alpha_1]$ and $\hat{\Omega}(\alpha) > \Psi(\alpha)$ for $\alpha \in (\alpha_1, \alpha_3)$. As shown above in section A2.1, the former holds if $\delta < 1/(2 - \delta_0)$. Using (14) and (A12), we find that the latter is satisfied as long as

$$\rho < \frac{\delta - \delta_0}{(2 + \delta)(1 - \delta_0)}. \quad (A17)$$

Note that in Figure 5, the equilibrium scenarios without the NIIC are associated with part (ii) of Proposition 2, which arises if $\rho > \alpha_1$ and $\delta < 2\delta_0/(1 + \delta_0)$. It can be shown that

$$\alpha_1 < (\delta - \delta_0)/(2 + \delta)(1 - \delta_0) \quad \text{if and only if} \quad \delta > 2(1 - \delta_0)/\delta_0. \quad \text{Moreover, we need}$$
 proved that $\delta_0 > (7 - \sqrt{17})/4$ to ensure that $2(1 - \delta_0)/\delta_0 < 1/(2 - \delta_0)$. This restriction on $\delta_0$ also implies that $1/(2 - \delta_0) < 2\delta_0/(1 + \delta_0)$. Therefore, $\delta_0 > (7 - \sqrt{17})/4$, $\delta \in (2(1 - \delta_0)/\delta_0, 1/(2 - \delta_0))$ and $\rho \in (\alpha_1, (\delta - \delta_0)/(2 + \delta)(1 - \delta_0))$ are sufficient for the situation in Figure 5 to arise.

A2.4. Proof of Propositions and Lemmas

Proof of Proposition 1: From (7) – (9) we know that a C/R equilibrium is less risky if $F \geq \Phi(\alpha)$. Under the assumption that $\delta \leq 1/(2 - \delta_0)$, the $\Phi(\alpha)$ curve lies below the $\Psi(\alpha)$ curve for all $\alpha \in (0, \alpha_3]$. Then the $\Psi(\alpha)$ curve determines the boundary between C/R equilibria and no collusion for $F \geq \Psi(\alpha_3) = \Gamma(\alpha_3)$, and the $\Gamma(\alpha)$ curve determines the same boundary for $F \in [\Gamma(\alpha_2), \Gamma(\alpha_3)]$. Since $\Psi'(\alpha) < 0$, $\Gamma'(\alpha) > 0$ and $\Phi'(\alpha) < 0$, we have the results in parts (i), (ii) and (iii). QED

Proof of Proposition 2: From (7) – (9) we know that a C/NR equilibrium is less risky if $F < \Phi(\alpha)$. Let $A$ and $B$ denote the numerator and dominator of $\Omega(\alpha)$ in (17), respectively. Then $\Omega > 0$ as long as $A$ and $B$ have the same sign. Otherwise, $\Omega < 0$. Note that $A > 0$ for all $\alpha$ if $\rho < \alpha_1$, and $B < 0$ for all $\alpha$ if $\rho < (1 - \delta)/(2 - \delta)$. Moreover, $\Omega(\alpha)$ approaches either $+\infty$ or $-\infty$ as $\alpha \rightarrow (1 - \delta)/[\rho(2 - \delta)]$. Using (17), we find

$$\Omega'(\alpha) = -B\delta\rho(\pi^D - \pi^N) - A\rho(1 - \delta/2)$$

which is positive if both $A$ and $B$ are negative, but negative if both $A$ and $B$ are positive.

Setting $\Omega(\alpha) = \Phi(\alpha)$, we obtain a quadratic equation of the form $\alpha^2 + Q\alpha - R = 0$ with $Q > 0$ and $R > 0$. It is easy to see that this equation has a positive root and a negative root. Let $\alpha_4$ denote the positive root.
Part (i) of Proposition 2 follows from that, in the case where \( \rho < \min \{ \alpha_i, (1 - \delta)/(2 - \delta) \} \), we have \( B < 0 \) and \( \Omega(\alpha) = A / B < 0 \), and hence (16) is satisfied for all \( \alpha \in (0, 1) \).

In the case where \( \delta < 2\delta_0/(1 + \delta_0) \) and \( \rho > \alpha_1, \rho \) can be either greater or less than \( (1 - \delta)/(2 - \delta) \). If \( \rho \in (\alpha_1, (1 - \delta)/(2 - \delta)) \), we have \( B < 0, \Omega(\alpha) > 0 \) and \( \Omega'(\alpha) > 0 \) for \( \alpha \in (\alpha_1 / \rho, 1) \). If \( \rho \in ( (1 - \delta)/(2 - \delta), 1) \), we have \( B < 0, \Omega(\alpha) > 0 \) and \( \Omega'(\alpha) > 0 \) for \( \alpha \in (\alpha_1 / \rho, (1 - \delta)/\rho(2 - \delta)) \), and \( \Omega(\alpha) \) approaches +\( \infty \) as \( \alpha \) approaches \( (1 - \delta)/\rho(2 - \delta) \) from the left. Since \( \Phi'(\alpha) < 0 \), the \( \Phi(\alpha) \) curve and \( \Omega(\alpha) \) curve intercept at \( \alpha_4 < (1 - \delta)/\rho(2 - \delta) \). These results imply that for \( \rho \) in either of these two intervals, the \( \Omega(\alpha) \) curve is upward-sloping and (16) takes the form \( F \geq \Omega(\alpha) \) for \( \alpha \) between \( \alpha_1 / \rho \) and \( \min \{ \alpha_4, 1 \} \). For \( \alpha \leq \alpha_1 / \rho, \Omega(\alpha) < 0 \) and hence \( F \geq \Omega(\alpha) \) holds for any positive \( F \). Taking into consideration the \( \Phi(\alpha) \) curve, we conclude that a C/NR equilibrium prevails if \( \Phi(\alpha) > F > \max \{ 0, \Omega(\alpha) \} \) for \( \alpha < \min \{ \alpha_4, 1 \} \).

In the case where \( \delta > 2\delta_0/(1 + \delta_0) \) and \( \rho > (1 - \delta)/(2 - \delta) \), \( \rho \) can be either greater or less than \( \alpha_1 \). If \( \rho \in ((1 - \delta)/(2 - \delta), \alpha_1) \), we have \( B > 0, \Omega(\alpha) > 0 \) and \( \Omega'(\alpha) < 0 \) for \( \alpha \in ((1 - \delta)/\rho(2 - \delta), 1) \). If \( \rho \in (\alpha_1, 1) \), we have \( B > 0, \Omega(\alpha) > 0 \) and \( \Omega'(\alpha) < 0 \) for \( \alpha \in ((1 - \delta)/\rho(2 - \delta), \alpha_1 / \rho) \). These results imply that for \( \rho \) in either of these two intervals, the \( \Omega(\alpha) \) curve is downward-sloping and (16) takes the form of \( F \leq \Omega(\alpha) \). Therefore, we conclude that a C/NR equilibrium prevails if \( F < \min \{ \Phi(\alpha), \Omega(\alpha) \} \). QED

Proof of Proposition A1: Using (A9) and (A11), we find

\[
\dot{\Psi}(\alpha) - \dot{\Phi}(\alpha) = \frac{(\pi^M - \pi^N)\left[\rho(1 - \delta) - \alpha\delta(1 - 2\rho)\right]}{\alpha\rho(1 - \delta + \alpha\delta)}, \quad (A19)
\]
which has a positive sign if either \( \rho \geq 1/2 \) or \( \rho < 1/2 \) and \( \alpha < \rho (1-\delta)/[\delta(1-2\rho)] \). Note from (A8) that a C/R equilibrium can occur only if \( \alpha \leq \alpha_1 \). It can be shown that in the case \( \rho < 1/2 \), \( \alpha_1 < \rho (1-\delta)/[\delta(1-2\rho)] \) if and only if \( \rho > (\delta-\delta_0)/(1+\delta-2\delta_0) \). Noting that \((\delta-\delta_0)/(1+\delta-2\delta_0) < 1/2\), we conclude that \( \hat{\Psi}(\alpha) > \hat{\Phi}(\alpha) \) for all \( \alpha \leq \alpha_1 \) (see Figure A1-1) as long as \( \rho > (\delta-\delta_0)/(1+\delta-2\delta_0) \). If, on the other hand, \( \rho < (\delta-\delta_0)/(1+\delta-2\delta_0) \), we have \( \hat{\Psi}(\alpha) < \hat{\Phi}(\alpha) \) for \( \alpha \in (\rho (1-\delta)/[\delta(1-2\rho)], \alpha_1) \), which gives rise to the situation illustrated in Figure A1-2.

Using (A11) and (A12), we find that \( \hat{\Phi}(\alpha) = \hat{\Omega}(\alpha) \) at \( \alpha_1 \) and \( \hat{\Phi}(\alpha) < \hat{\Omega}(\alpha) \) for \( \alpha < \alpha_1 \). The results then follow from the definitions of \( \hat{\Gamma}(\alpha) \), \( \hat{\Phi}(\alpha) \), \( \hat{\Psi}(\alpha) \) and \( \hat{\Omega}(\alpha) \). QED

**Proof of Lemma 1:** First, we consider \( S \) and \( S^* \). Using (12) and (14), we find that \( \Gamma(\alpha_3) = \Psi(\alpha_3) = 2(\pi^D - \pi^N) \). From (14) and (A9), it is clear that \( \Psi(\alpha) > \hat{\Psi}(\alpha) \). Thus,

\[
F \leq \hat{\Psi}(\alpha) \text{ implies } F < \Psi(\alpha). \quad \text{From (12), we find that } \Gamma(\alpha_1) = 0 \text{ and } \Gamma'(\alpha) > 0 \text{ for } \alpha > \alpha_1.
\]

Then \( \Gamma^{-1}(F) > \alpha_1 \) for any \( F > 0 \). Hence, \( \alpha \leq \hat{\Gamma} \) implies \( \alpha < \Gamma^{-1}(F) \). Therefore, \( \hat{S} \) is a strict subset of \( S \).

Second, we compare \( T \) and \( \hat{T} \). We consider, separately, the three cases in Proposition 2.

(i) In the case where \( \rho < \min \{\alpha_1,(1-\delta)/(2-\delta)\} \), \( k < 0 \) and \( F \geq \Omega(\alpha) \) holds for any \( F > 0 \) and \( \alpha \in (0,1) \). On the other hand, \( \hat{T} \) admits only those \( F \leq \hat{\Omega}(\alpha) \). Thus, \( \hat{T} \) is a strict subset of \( T \) in this case.

(ii) In the case where \( \delta < 2\delta_0/(1+\delta_0) \) and \( \rho > \alpha_1 \), we have \( k < 0 \), \( \Omega'(\alpha) > 0 \) and \( \Omega(\alpha) \geq 0 \) for \( \alpha \in [\alpha_1/\rho, (1-\delta)/\rho(2-\delta)] \). On the other hand, \( \hat{\Omega}'(\alpha) < 0 \) and \( \hat{\Omega}(\alpha) \geq 0 \) for \( \alpha \leq \alpha_1/\rho < 1 \). Hence, \( \hat{\Omega}^{-1}(F) < \Omega^{-1}(F) \) for any \( F > 0 \). This implies that \( \hat{T} \) is a strict
subset of $T$ in this case. Graphically, $T$ (respectively, $\hat{T}$) contains all the points to the left of the curve represented by $F = \Omega(\alpha)$ (respectively, $F = \hat{\Omega}(\alpha)$) for $F > 0$. Since the $F = \Omega(\alpha)$ curve lies to the right of the $F = \hat{\Omega}(\alpha)$ curve for $F > 0$, $T$ contains more points than $\hat{T}$.

(iii) In the case where $\delta > 2\delta_0/(1 + \delta_0)$ and $\rho > (1 - \delta)/(2 - \delta)$, we have $k > 0$, $\Omega(\alpha) < 0$ and $\Omega(\alpha) \geq 0$ for $\alpha \in ((1 - \delta)/\rho(2 - \delta), \alpha_1/\rho]$. From (17) and (A12), we find that $\Omega(\alpha) > \hat{\Omega}(\alpha)$ for $\alpha \in ((1 - \delta)/\rho(2 - \delta), \alpha_1/\rho)$. Moreover, $\Omega(\alpha) \to \infty$ as $\alpha$ approaches $(1 - \delta)/\rho(2 - \delta)$ from the left, and $\hat{\Omega}(\alpha) \to \infty$ as $\alpha \to 0$. Hence, $F \leq \Omega(\alpha)$ implies $F \leq \Omega(\alpha)$ but not the other way around, i.e., $\hat{T}$ is a strict subset of $T$. QED

Proof of Lemma 2: Using (9) and (A11), we can show that $\hat{\Phi}(\alpha) - \Phi(\alpha) > 0$ for $\alpha \in [0, 1]$. QED

Proof of Proposition 3: Since $\hat{\Phi}(\alpha) > \Phi(\alpha)$ (by Lemma 2), $F \geq \hat{\Phi}(\alpha)$ implies $F > \Phi(\alpha)$.

Then a C/R equilibrium could prevail both with and without the NIIC if $F \geq \hat{\Phi}(\alpha)$. The NIIC reduces collusion if $F \geq \hat{\Phi}(\alpha)$ because $\hat{S}$ is a strict subset of $S$.

In the case in which $F \leq \Phi(\alpha)$, we have $F < \hat{\Phi}(\alpha)$. Then a C/NR equilibrium could prevail both with and without the NIIC. If the boundary of C/NR equilibria without the NIIC is determined by (16), then Lemma 1 (specifically, $\hat{T} \subset T$) implies that the NIIC reduces collusion. However, for $\rho < \min\{\alpha_1, (1 - \delta)/(2 - \delta)\}$ set $T$ does not impose any restrictions on the boundary of C/NR equilibria, in which case the NIIC does not necessarily reduce collusion.

In the case in which $F \in (\Phi(\alpha), \hat{\Phi}(\alpha))$, a C/R equilibrium without the NIIC and a C/NR equilibrium with the NIIC prevail if there is collusion before and after the NIIC, which means that the NIIC reduces revelation. The examples in Figures 4 and 5 prove the point that the NIIC may increase collusion. QED
Proof of Proposition 4: A sufficient condition for the NIIC to increase collusion is that
\[ \hat{Q}(\alpha) > \max\{\Psi(\alpha), \Phi(\alpha)\} \] at \( \alpha = \alpha_1 \), in which case there is collusion with the NIIC but no collusion without it for \( F \) between \( \max\{\Psi(\alpha_1), \Phi(\alpha_1)\} \) and \( \hat{Q}(\alpha_1) \). (The \( \Gamma(\alpha) \) curve does not affect the boundary of collusion at \( \alpha = \alpha_1 \) because \( \Gamma(\alpha_1) = 0 \)). Using (9) and (A12), we find that \( \hat{Q}(\alpha) > \Phi(\alpha) \) at \( \alpha = \alpha_1 \). Using (14) and (A12), we can show that \( \hat{Q}(\alpha) > \Psi(\alpha) \) at \( \alpha = \alpha_1 \) if (19) holds. In other words, (19) is a sufficient condition for the NIIC to increase collusion.

Regarding the two thresholds of \( \rho \) in Proposition 2, it is easy to show that
\[ (\delta - \delta_0)/(2 + \delta - 3\delta_0) > \alpha_1 \] if and only if \( \delta_0 > 2/3 \), and \( (\delta - \delta_0)/(2 + \delta - 3\delta_0) > (1 - \delta)/(2 - \delta) \) if and only if \( \delta > (2 - \delta)/(3 - 2\delta_0) \). Under these conditions for \( \delta_0 \) and \( \delta \), there is a range of \( \rho \) for which both (19) and \( \rho > \max\{\alpha_1,(1 - \delta)/(2 - \delta)\} \) hold. QED

Proof of Lemma 3: Follows from the comparison of (20), (21) and (22). QED

Proof of Proposition 5: Since \( \alpha_{c1} < \alpha_{c2} \), the IC3 constraint is satisfied for both firms if \( \alpha \leq \alpha_{c1} \).

The assumptions in this section ensure that the other conditions for a C/NR equilibrium without the NIIC are satisfied. Hence, a C/NR equilibrium without the NIIC prevails if \( \alpha \leq \alpha_{c1} \).

Under the NIIC regime, a C/NR equilibrium with firm \( i \) as the instigator could be sustained if \( \alpha \leq \min\{\hat{\alpha}^I_{c1}, \hat{\alpha}^NI_{c1}\} \) (\( j \neq i \)). Lemma 3 and \( \alpha_{c1} < \alpha_{c2} \) imply that \( \min\{\hat{\alpha}^I_{c2}, \hat{\alpha}^{NI}_{c1}\} = \hat{\alpha}^{NI}_{c1} \). Note that a firm has an incentive to propose and the other will accept a collusive agreement at stages 1 and 2 of the game as long as the agreement yields higher payoffs than competition for both firms. Hence, the critical value of \( \alpha \) for collusion is the larger of the two critical values associated with each of the two firms being the instigator. In other words, a C/NR equilibrium prevails if \( \alpha \leq \max\{\min\{\hat{\alpha}^I_{c1}, \hat{\alpha}^{NI}_{c1}\}, \hat{\alpha}^NI_{c1}\} \). QED
Proof of Proposition 6: Recall that without the NIIC, a C/NR equilibrium prevails if $\alpha \leq \alpha_{c1}$.

Then the NIIC increases collusion if and only if

\[ \max \left\{ \min \{ \hat{\alpha}_1, \hat{\alpha}_{C2} \}, \hat{\alpha}_{C1} \right\} > \alpha_{c1}. \]  \quad (A20)

Since $\hat{\alpha}_{C1}^N < \alpha_{c1}$ by Lemma 3, (A20) is satisfied if and only if $\min \{ \hat{\alpha}_1, \hat{\alpha}_{C2}^N \} > \alpha_{c1}$. There are two cases to consider:

(a) If $\hat{\alpha}_{C2}^N \leq \hat{\alpha}_1$, then $\min \{ \hat{\alpha}_1, \hat{\alpha}_{C2}^N \} = \hat{\alpha}_{C2}^N$, in which case $\hat{\alpha}_{C2}^N > \alpha_{c1}$ ensures that (A20) holds.

(b) If $\hat{\alpha}_{C2}^N > \hat{\alpha}_1$, then $\min \{ \hat{\alpha}_1, \hat{\alpha}_{C2}^N \} = \hat{\alpha}_1$, in which case (A20) holds by Lemma 3.

Therefore, $\hat{\alpha}_{C2}^N > \alpha_{c1}$ is sufficient for (A20) to hold. To prove that this condition is necessary, observe that if $\hat{\alpha}_{C2}^N \leq \alpha_{c1}$, (A20) does not hold in case (a) and the situation in case (b) cannot arise. QED
Appendix 3: Alternative Timing

Here we present and analyze a modified game in which each firm makes a decision on whether to report to the antitrust authority (the AA) at the same time as its decision on whether to defect. Specifically, the game proceeds as follows:

- **Stages 1**: Each firm chooses simultaneously whether to instigate.
- **Stage 2**: If there has been instigation by one firm, the other firm either agrees to form a cartel or refuses to do so.
- **Stage 3**: Each firm chooses from the following four possible actions: Collusion and Not Revealing (C/NR), Collusion and Revealing (C/R), Defection and Not Revealing (D/NR), and Defection and Revealing (D/R).
- **Stage 4**: The AA launches an investigation with probability $\alpha$. If a collusive agreement is in place and neither firm reveals, the investigation leads to a conviction with probability $\rho$.

All other aspects of this model are the same as those in the original model in the main text. We will continue to use strategic riskiness as a selection criterion in cases of multiple equilibriums in a stage game.

We focus on the case where the two firms are identical *ex ante*, and continue to use $\pi^N$, $\pi^M$, $\pi^D$ and $\pi^S$ to denote a firm’s per-period profits under static Nash equilibrium, collusion and defection. In addition, we define $\pi^B$ as a firm’s profits in a period when both firms defect at the same time. We assume that $\pi^D > \pi^M > \pi^N > \pi^B \geq \pi^S$.

Let $V^{\text{CNR}}$ denote a firm’s discounted sum of profits when both firms play Collusion and Not Revealing in every period for as long as they are not caught by the AA. The value of $V^{\text{CNR}}$ is not affected by the change of timing in this model. Then from (6) in the main text, we have
\[ V_{\text{CNR}}^{\text{NM}} = \frac{\pi^M + \alpha \rho \left[ \delta \pi^N / (1 - \delta) - F \right]}{1 - \delta (1 - \alpha \rho)} \]  

(A21)

Below we examine the equilibriums of this model for the case of a leniency program without and then with a NIIC. From this examination we will derive the conditions under which the NIIC increases the occurrence of collusion.

### A3.1. Leniency Program without the NIIC

The analysis proceeds in the same way as the one in the main text. We will start by considering the firms' decisions at stage 3 of the modified game after the two firms have reached a collusive agreement. In Table A3-1 are the payoff pairs associated with different strategy profiles of the two firms.

**Table A3-1: The Subgame at Stage 3 in the absence of the NIIC**

<table>
<thead>
<tr>
<th>Firm 2</th>
<th>Collusion and Not Revealing (C/NR)</th>
<th>Collusion and Revealing (C/R)</th>
<th>Defection and Not Revealing (D/NR)</th>
<th>Defection and Revealing (D/R)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Collusion and Not Revealing (C/NR)</td>
<td>( V_{\text{CNR}}^{\text{NM}} )</td>
<td>( \pi^M + \frac{\delta \pi^N}{1 - \delta} - F ), ( \pi^M + \frac{\delta \pi^N}{1 - \delta} )</td>
<td>( \pi^S + \frac{\delta \pi^N}{1 - \delta} - \alpha \rho F ), ( \pi^D + \frac{\delta \pi^N}{1 - \delta} )</td>
<td>( \pi^S + \frac{\delta \pi^N}{1 - \delta} - F ), ( \pi^D + \frac{\delta \pi^N}{1 - \delta} )</td>
</tr>
<tr>
<td>Collusion and Revealing (C/R)</td>
<td>( \pi^M + \frac{\delta \pi^N}{1 - \delta} )</td>
<td>( \pi^M + \frac{\delta \pi^N}{1 - \delta} - F ), ( \pi^M + \frac{\delta \pi^N}{1 - \delta} )</td>
<td>( \pi^S + \frac{\delta \pi^N}{1 - \delta} - \alpha \rho F ), ( \pi^D + \frac{\delta \pi^N}{1 - \delta} )</td>
<td>( \pi^S + \frac{\delta \pi^N}{1 - \delta} - F ), ( \pi^D + \frac{\delta \pi^N}{1 - \delta} )</td>
</tr>
<tr>
<td>Defection and Not Revealing (D/NR)</td>
<td>( \pi^D + \frac{\delta \pi^N}{1 - \delta} - \alpha \rho F ), ( \pi^S + \frac{\delta \pi^N}{1 - \delta} - \alpha \rho F )</td>
<td>( \pi^D + \frac{\delta \pi^N}{1 - \delta} - F ), ( \pi^S + \frac{\delta \pi^N}{1 - \delta} )</td>
<td>( \pi^B + \frac{\delta \pi^N}{1 - \delta} - \alpha \rho F ), ( \pi^D + \frac{\delta \pi^N}{1 - \delta} )</td>
<td>( \pi^B + \frac{\delta \pi^N}{1 - \delta} - F ), ( \pi^D + \frac{\delta \pi^N}{1 - \delta} )</td>
</tr>
<tr>
<td>Defection and Revealing (D/R)</td>
<td>( \pi^D + \frac{\delta \pi^N}{1 - \delta} )</td>
<td>( \pi^D + \frac{\delta \pi^N}{1 - \delta} - F ), ( \pi^D + \frac{\delta \pi^N}{1 - \delta} )</td>
<td>( \pi^B + \frac{\delta \pi^N}{1 - \delta} - F ), ( \pi^D + \frac{\delta \pi^N}{1 - \delta} )</td>
<td>( \pi^B + \frac{\delta \pi^N}{1 - \delta} - F ), ( \pi^D + \frac{\delta \pi^N}{1 - \delta} )</td>
</tr>
</tbody>
</table>
It is easy to see from Table A3-1 that the strategy profile (D/R, D/R) (i.e., both firms choose Defection and Revealing) is a Nash equilibrium in this stage-game. Moreover, (C/NR, C/NR) (i.e., both firms choose Collusion and Not Revealing) can be a Nash equilibrium under certain conditions. To see the latter, note that the strategies C/R and D/NR are dominated by D/R for each firm. As a result, (C/NR, C/NR) is a Nash equilibrium if

\[ V_{CNR} \geq \pi^D + \frac{\delta \pi^N}{1 - \delta}. \quad (A22) \]

In the case where (A22) is satisfied and hence there are two Nash equilibriums at stage 3, we select the equilibrium that is less risky in the sense of Blonski and Spagnolo (2014). To be more specific, the Collusion and No Revealing (CNR) equilibrium is less risky than the Defect and Reveal equilibrium if

\[ V_{CNR} + \left[ \pi^S + \frac{\delta \pi^N}{1 - \delta} - F \right] \geq \left[ \pi^D + \frac{\delta \pi^N}{1 - \delta} \right] + \left[ \pi^B + \frac{\delta \pi^N}{1 - \delta} - \frac{F}{2} \right]. \quad (A23) \]

Rearranging (A23), we obtain

\[ V_{CNR} - \left[ \pi^D + \frac{\delta \pi^N}{1 - \delta} \right] \geq \pi^B - \pi^S + \frac{F}{2} (> 0). \quad (A24) \]

It is easy to see that (A24) implies (A22). Therefore, (C/NR, C/NR) would be the equilibrium outcome at stage 3 if (A24) is satisfied.

To find out the restrictions imposed by (A24) on the parameters of the model, substituting (A21) for \( V_{CNR} \) into (A24), we obtain:

\[ F \leq \Delta \equiv \frac{\delta (1 - \alpha \rho)(\pi^D - \pi^N) - (\pi^D - \pi^M) - [1 - \delta(1 - \alpha \rho)](\pi^B - \pi^S)}{\alpha \rho + [1 - \delta(1 - \alpha \rho)]/2}. \quad (A25) \]

A necessary condition for (A25) to hold is that \( \Delta > 0 \). The latter is satisfied if \( \delta > \delta_c \) and \( \alpha \rho < 1 - \delta_c / \delta \), where
\[
\delta_c \equiv \frac{(\pi^D - \pi^M) + (\pi^B - \pi^S)}{(\pi^D - \pi^N) + (\pi^B - \pi^S)} < 1. \quad (A26)
\]

It is easy to verify that \( \delta_c > \delta_0 \).

For Collusion and No Revealing to be the equilibrium outcome of the entire game, it is necessary that each firm’s discounted sum of profits, \( V^{CNR} \), is at least as high as that of no collusion, \( \pi^N/(1 - \delta) \). This condition is satisfied by (A24) because \( \pi^D > \pi^N \).

If (A24) is not satisfied, on the other hand, Defection and Revealing becomes the equilibrium outcome at stage 3. Since \( \pi^N > \pi^B \) implies
\[
\frac{\pi^N}{1 - \delta} > \pi^B + \frac{\delta \pi^N}{1 - \delta} - \frac{F}{2}, \quad (A27)
\]
the firms will not enter into a collusive agreement at stages 1 and 2 of the game. Therefore,

**Proposition A2**: Under a leniency program without the NIIC, both firms choose Collusion and Not Revealing in equilibrium if \( \delta > \delta_c, \alpha \rho < 1 - \delta_c/\delta \), and \( F \) satisfies (A25). Otherwise, no collusion occurs in equilibrium.

**A3.2. Leniency Program with the NIIC**

Now we consider the subgame at stage 3 under the NIIC. Without loss of generality, suppose that this subgame is reached after firm 1 has instigated. As can be seen from the payoff pairs in Table A3-2, for firm 1 D/R is now weakly dominated by D/NR. Note, moreover, that in the event that firm 2 chooses C/NR, firm 1 would earn a strictly higher payoff by playing D/NR than D/R. Since (C/NR, C/NR) is a possible equilibrium in this subgame, it is reasonable to assume that firm 1 will never play D/R in equilibrium.

The strategy profile (C/NR, C/NR) would indeed be a Nash equilibrium at stage 3 if
\[
V^{CNR} \geq \pi^D + \frac{\delta \pi^N}{1 - \delta} - \alpha \rho F \quad (A28)
\]
holds for firm 1 and (A22) holds for firm 2. Note, however, that the former is implied by the latter. On the other hand, (D/NR, D/R) is a Nash equilibrium at stage 3 irrespective of whether (A22) is satisfied or not.

**Table A3-2: The Subgame at Stage 3 in the presence the NIIC (Firm 1 being the instigator)**

<table>
<thead>
<tr>
<th>Firm 2</th>
<th>Collusion and Not Revealing (C/NR)</th>
<th>Collusion and Revealing (C/R)</th>
<th>Defection and Not Revealing (D/NR)</th>
<th>Defection and Revealing (D/R)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Firm 1</td>
<td>$V_{CNR, V_{CNR}}$</td>
<td>$\pi^M + \delta \pi^N_{1-\delta} - F, \quad \pi^M + \delta \pi^N_{1-\delta}$</td>
<td>$\pi^S + \delta \pi^N_{1-\delta} - \alpha \rho F, \quad \pi^D + \delta \pi^N_{1-\delta}$</td>
<td>$\pi^S + \delta \pi^N_{1-\delta} - F, \quad \pi^D + \delta \pi^N_{1-\delta}$</td>
</tr>
<tr>
<td></td>
<td>$\pi^M + \delta \pi^N_{1-\delta} - F, \quad \pi^M + \delta \pi^N_{1-\delta}$</td>
<td>$\pi^S + \delta \pi^N_{1-\delta} - F, \quad \pi^D + \delta \pi^N_{1-\delta}$</td>
<td>$\pi^S + \delta \pi^N_{1-\delta} - F, \quad \pi^D + \delta \pi^N_{1-\delta}$</td>
<td>$\pi^S + \delta \pi^N_{1-\delta} - F, \quad \pi^D + \delta \pi^N_{1-\delta}$</td>
</tr>
<tr>
<td></td>
<td>$\pi^D + \delta \pi^N_{1-\delta} - \alpha \rho F, \quad \pi^S + \delta \pi^N_{1-\delta}$</td>
<td>$\pi^D + \delta \pi^N_{1-\delta} - F, \quad \pi^S + \delta \pi^N_{1-\delta}$</td>
<td>$\pi^B + \delta \pi^N_{1-\delta} - \alpha \rho F, \quad \pi^D + \delta \pi^N_{1-\delta}$</td>
<td>$\pi^B + \delta \pi^N_{1-\delta} - F, \quad \pi^D + \delta \pi^N_{1-\delta}$</td>
</tr>
<tr>
<td></td>
<td>$\pi^D + \delta \pi^N_{1-\delta} - F, \quad \pi^S + \delta \pi^N_{1-\delta}$</td>
<td>$\pi^D + \delta \pi^N_{1-\delta} - F, \quad \pi^S + \delta \pi^N_{1-\delta}$</td>
<td>$\pi^B + \delta \pi^N_{1-\delta} - F, \quad \pi^D + \delta \pi^N_{1-\delta}$</td>
<td>$\pi^B + \delta \pi^N_{1-\delta} - F, \quad \pi^D + \delta \pi^N_{1-\delta}$</td>
</tr>
</tbody>
</table>

The (C/NR, C/NR) equilibrium involves a smaller strategic risk than the (D/NR, D/R) equilibrium for the two firms if

$$V^{CNR}_{CNR} + \left[ \pi^S + \frac{\delta \pi^N}{1-\delta} - F \right] \geq \left[ \pi^D + \frac{\delta \pi^N}{1-\delta} - \alpha \rho F \right] + \left[ \pi^B + \frac{\delta \pi^N}{1-\delta} \right] \quad (A29)$$

holds for firm 1 and

$$V^{CNR}_{CNR} + \left[ \pi^S + \frac{\delta \pi^N}{1-\delta} - \alpha \rho F \right] \geq \left[ \pi^D + \frac{\delta \pi^N}{1-\delta} \right] + \left[ \pi^B + \frac{\delta \pi^N}{1-\delta} \right] \quad (A30)$$
holds for firm 2. It can be verified that (A30) implies (A29). In addition, (A30) also implies (A22). Therefore, (C/NR, C/NR) would prevail as the equilibrium outcome at stage 3 as long as (A30) is satisfied.

If (A30) does not hold, on the other hand, (D/NR, D/R) becomes the equilibrium outcome at stage 3. Since \[ \pi_N > \pi_B \] implies
\[ \frac{\pi^N}{1 - \delta} > \frac{\delta \pi^N}{1 - \delta} > \frac{\pi^B}{1 - \delta} > \frac{\delta \pi^N}{1 - \delta} - F, \]
the firms will not enter into a collusive agreement at stages 1 and 2 of the game.

Substituting (A21) for \( V^{CNR} \) into (A31), we obtain
\[ F \leq \Delta \equiv \frac{\delta (1 - \alpha\rho)(\pi^D - \pi^N) - (\pi^D - \pi^M) - [1 - \delta(1 - \alpha\rho)](\pi^B - \pi^S)}{\alpha\rho[2 - \delta(1 - \alpha\rho)]}. \] (A32)

Note that the numerator of \( \Delta \) is the same as that of \( \Delta \) in (A25). Hence, \( \Delta > 0 \) if \( \delta > \delta_c \) and \( \alpha\rho < 1 - \delta_c/\delta \). Then we have:

**Proposition A3:** Under a leniency program with the NIIC, both firms choose Collusion and Not Revealing in equilibrium if \( \delta > \delta_c, \alpha\rho < 1 - \delta_c/\delta \), and \( F \) satisfies (A32). Otherwise, no collusion occurs in equilibrium.

**A3.3. Effects of the NIIC**

A comparison of Propositions A2 and A3 suggests that the effect of the NIIC depends on which of (A25) and (A32) is a tighter constraint. To be more specific, the NIIC expands the set of parameters for which Collusion and No Revealing is the equilibrium outcome if (A25) is tighter than (A32).

**Proposition A4:** If \( \delta > \delta_c \) and \( \alpha\rho < \min\{1/2, 1 - \delta_c/\delta\} \), then \( \Delta > \Delta > 0 \) and the NIIC causes Collusion and No Revealing to become the equilibrium outcome for \( F \) in the range \( (\Delta, \Delta) \).
Proof: Noting that the numerators of $\Delta$ and $\hat{\Delta}$ are identical, we can show that $\hat{\Delta} > \Delta$ if $\alpha \rho < 1/2$.

The result then follows from Propositions A2 and A3. QED

Therefore, this modified model yields qualitatively the same conclusion as the original model; that is, the NIIC can increase collusion under certain conditions. Specifically, if the time discount factor is not too small, the probability of detection and conviction ($\alpha \rho$) is not too large and the fine upon conviction is in the intermediate range, there would be no collusion in the absence of the NIIC but there is collusion with the NIIC in place.

On the other hand, these two models differ in that, in the present model, no firm ever reports to the AA in equilibrium. But the richer structure of the original model generates equilibriums in which firms actually choose to reveal under some circumstances. In the latter case, the NIIC reduces the occurrence of revelation in equilibrium.