Rules-Based Monetary Policy and the Threat of Indeterminacy When Trend Inflation Is Low

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Abstract

Low inflation is not perceived as a potential threat to determinacy and macroeconomic stability. Should the Fed return to a rules-based monetary policy, the prospect of indeterminacy would be particularly acute if the Fed adopted a mixed policy rule with the nominal interest rate responding to the output gap and output growth. This is true for a rate of inflation as low as that observed on average since the early 1990s. This finding contrasts sharply with the existing literature where the threat of indeterminacy was high before 1983 and almost nonexistent afterwards. Key to our result is a strong interaction between low trend inflation, sticky wages and technological trend growth. Accounting for a cost channel of monetary policy and a roundabout production process increases the threat of indeterminacy under low inflation. When removing the output gap or output growth from the mixed rule, we find that a rule responding to output growth sharply widens the scope for stability. By stark contrast, the results obtained under a rule reacting to the output gap only essentially mimic those with the mixed rule.


Keywords: Low trend inflation; Taylor rule; Output gap; Output growth; Indeterminacy; Sticky wages; Trend growth; Working capital; Roundabout production.

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1 Introduction

“In many conversations with central bankers I hear nostalgia for what they call normal policy times, and I have urged policy makers to renormalize rather than a new-normalize policy—to return to a rules-based monetary strategy as soon as possible.”


Faced with a near decade of unconventional monetary policy stuck at the Zero Lower Bound (ZLB) on nominal interest rates, a number of economists including Volcker (2014), Calomiris, Ireland, and Levy (2015), Ireland and Levy (2017), and Taylor (2015) have recommended that the Fed should return to more conventional rules-based monetary policy many believe has contributed to greater macroeconomic stability from 1983 to the onset of the Great Recession. The premise behind this proposal is that implementing a rule-based policy will signal the Fed’s intention to go back to a comprehensible strategy of communicating to the public the rationale behind its policy actions and explaining the relationship between these actions and its main objectives. These two elements presumably played a key role in achieving macroeconomic stability during the so-called Great Moderation.

The merits of a rule-based monetary policy is also indirectly supported by the recent works of Wu and Xia (2016), Wu and Zhang (2017) and Debortoli, Galí, and Gambetti (2018), who offer evidence in favor of the hypothesis of “perfect substitutability” between conventional and unconventional monetary policies. This hypothesis holds that unconventional policy when the ZLB binds produced outcomes that resemble rules-based policy in the pre-ZLB period. Therefore, unconventional monetary policy at the ZLB would more or less mimic conventional monetary policy with the Fed following a Taylor rule without the ZLB.

To ease the return to rules-based policy, Blanchard, Dell’Ariccia, and Mauro (2010), Ball (2013) and Krugman (2014) have advocated for a moderate increase in the inflation target between 3% and 4%. Likewise, implementing this proposal would raise inflation and interest rates on average. According to conventional wisdom, a level of trend inflation of 3-4% should not represent a threat to determinacy and macroeconomic stability more generally.

In the standard textbook New Keynesian (NK) model, a rule-based monetary policy set in compliance with the Taylor principle guarantees a unique rational expectations equilibrium (REE). The Taylor principle works well in such a context because higher interest rates lower inflation by

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1We do not have in normal times the counterfactual that the Fed followed unconventional policies with an outcome similar to the Great Moderation.

2By standard or textbook NK model, we mean one with sticky prices, no capital accumulation and zero steady-state inflation.
curtailing aggregate spending. Yet, Christiano, Trabandt, and Walentin (2011) uncover an intriguing result: when the standard NK model is modified to account for two theoretical refinements, namely the potential significance of a cost channel for monetary policy and a roundabout production structure (Basu, 1995), complying with the Taylor principle no more guarantees a unique REE. The reason for this is that working capital implies a direct impact of the nominal interest rate on real marginal costs, while roundabout production exacerbates this effect.

Contrasting with conventional wisdom, we show that a monetary policy set in accordance with a Taylor rule widely used in the recent literature can pose a threat to determinacy even at a level of trend inflation close to the average U.S. rate of inflation since the early 1990s. To make this key point, we use an expanded version of the medium-scale New Keynesian (MSNK) model proposed by Christiano, Eichenbaum, and Evans (2005). Therefore, our framework includes Calvo-style nominal wage and price rigidities, and real adjustment frictions like consumer habit formation, investment adjustment costs and variable capital utilization.

To this relatively standard MSNK framework, we add trend growth in neutral and investment-specific technical progress (e.g. see Smets and Wouters, 2007; Justiniano and Primiceri, 2008; Justiniano, Primiceri, and Tambalotti, 2010, 2011), a cost channel, roundabout production, positive trend inflation and an inertial Taylor rule. Furthermore, to the difference of Christiano and Eichenbaum (1992), Christiano, Eichenbaum, and Evans (1997, 2005) and Ravenna and Walsh (2006), who assume that working capital is needed to finance the wage bill, firms in our model use intra-period loans to finance their outlays of intermediate inputs, labor and capital services. Using this framework, we identify conditions leading to (in)determinacy at low levels of trend inflation between 0 and 3% for different specifications of rules-based monetary policy. We show that the prospect of (in)determinacy depends critically on the choice of theoretical ingredients and policy rules assumed in the simulations.

There is no consensus about the exact specification of a policy rule followed by the Fed during the postwar period. The policy rule in the textbook NK model says that interest rates adjust to inflation and the level of the output gap, with the output gap defined as the difference between actual output and the level of output under flexible nominal wages and prices (Galí, 2008, Ch. 3). Still, another rule widely used in the estimation of MSNK models after Smets and Wouters (2007) says that interest rates react to deviations of inflation from a steady-state target, to the output

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3 Other examples of models where working capital serves to finance more factor payments than simply the wage bill include Chowdhury, Hoffmann, and Schabert (2006), Christiano, Trabandt, and Walentin (2011), Ascari, Phaneuf, and Sims (2018) and Phaneuf, Sims, and Victor (2018)
gap and to output growth. The central bank also smooths movements in interest rates. We call this particular policy rule the mixed rule.\footnote{See also Justiniano, Primiceri, and Tambalotti (2010, 2011) and Khan and Tsoukalas (2011, 2012). The mixed rule is also used by Coibion and Gorodnichenko (2011) to identify sources of (in)determinacy during the postwar U.S. period, and by Coibion, Gorodnichenko, and Wieland (2012) to study optimal inflation rate in the NK model.}

In Smets and Wouters (2007), the mixed rule ensures determinacy over the entire postwar period and two subsamples which are 1966:1-1979:II and 1984:I-2004:IV. This is in contrast to Clarida, Galí, and Gertler (2000) and Coibion and Gorodnichenko (2011), who show that the U.S. economy was in a state of indeterminacy prior to 1980, and in a determinate state after 1982. Two reasons possibly explain these differences. First, Smets and Wouters report estimates showing that the Fed’s response to inflation was active (coefficient on inflation greater than 1) in the two subperiods. By contrast, Clarida, Galí, and Gertler (2000) and Coibion and Gorodnichenko (2011) report that the response to inflation was passive (coefficient on inflation between 0 and 1) prior to 1980 and active after 1982. Second, concomitantly to using the mixed rule, Smets and Wouters assume automatic indexation of non-reoptimized wages and prices to the last quarter’s rate of inflation and steady-state inflation, making trend inflation and trend growth irrelevant for equilibrium dynamics to a first-order approximation and thus neutralizing the impact of positive trend inflation.

However, the indexation assumption has been the object of criticisms (Woodford, 2007; Cogley and Sbordone, 2008; Chari, Kehoe, and McGrattan, 2009; Christiano, Eichenbaum, and Trabandt, 2016) One is that it lacks microeconomic foundations. Another is that it counterfactually implies that all wages and prices in the economy change every three months, something which is inconsistent with micro studies on wage and price adjustments (Bils and Klenow, 2004; Nakamura and Steinsson, 2008; Eichenbaum, Jaimovich, and Rebelo, 2011; Barattieri, Basu, and Gottschalk, 2014). Moreover, Phaneuf, Sims, and Victor (2018) show that a model without indexation better accounts for VAR evidence about the inflation responses to monetary and non-monetary shocks than one with indexation. Also, Ascari, Phaneuf, and Sims (2018) provide a comprehensive survey of the evidence from micro data on wage indexation in the U.S. and European countries. They find that indexation is essentially absent from the data, a conclusion which also reached by Barattieri, Basu, and Gottschalk (2014) based on U.S. micro data. For these reasons, we omit indexation from our model.

We explore the conditions leading to determinacy under low trend inflation assuming four different specifications of the Taylor rule. The first specification is the mixed Taylor rule described above. The second one is a Taylor rule responding to the level of the output gap but not to output growth. The third is a rule reacting to output growth but not to the output gap. Finally, the
fourth is the mixed rule with an interest rate smoothing of order two estimated by Coibion and Gorodnichenko (2011).

Our strategy is to search for the minimum response of interest rates to deviations of inflation from target, denoted as $\alpha_{\pi}$, consistent with a unique Rational Expectations Equilibrium (REE). Our baseline calibration assumes parameter values of the policy rule consistent with estimates generally found in the empirical literature, with the exception of course of $\alpha_{\pi}$ which we search for. The average waiting time between wage and price adjustments is set at 9 months, although we also assess the sensitivity of our results to varying the degrees of nominal rigidities. The share of intermediate inputs into gross output is 0.5, while the fraction of factor payments financed by working capital ranges from 0 to 1.

We first examine the conditions leading to (in)determinacy when using the mixed rule. We find that without working capital, the smallest value of $\alpha_{\pi}$ consistent with determinacy is 1.3 for an inflation trend of 0, 1.9 for a trend inflation of 2%, and 2.5 for a trend of 3%. With low trend inflation, these represent relatively large deviations from the Taylor principle. With a fraction of factor payments financed by working capital equal to 1/2, the smallest $\alpha_{\pi}$ consistent with a unique REE increases to 1.6 with zero trend inflation, to 2.3 with 2% trend inflation and 2.9 with a 3% trend. With all factor payments financed by working capital, we find no response to inflation consistent with determinacy.

These findings raise the following question: What are the key features of our model driving these findings? We show that with a fraction of factor payments financed by working capital of 0 and 1/2, the main factors driving our indeterminacy results are non-zero trend inflation, sticky wages and trend growth. With flexible nominal wages and no economic growth, we find that indeterminacy can be prevented for $\alpha_{\pi}$ values lower than the estimates generally found in the literature for a trend inflation of 2% and 3%. If we further remove working capital and roundabout production from the model, we then find that determinacy can be achieved with a weakly active (near one) response of interest rates to inflation. Therefore, the more distant a particular model is from our baseline framework, the closer to the Taylor principle are the conditions leading to determinacy.

The interaction between trend inflation and sticky wages is critical for our findings because households with positive long-run inflation would like to reset their wages each period, but only a fraction can. This leads to significant steady-state wage dispersion, driving a wedge between aggregate labor supply and demand. It also leads to higher wage markups on average, as updating households choose higher wages than they otherwise would to protect their future real wages from
inflation. This higher average wage markup moves the economy further from the first best allocation, increasing the threat of indeterminacy. Trend growth simply increases the severity of these monopolistic distortions.

These results lead to another question: Is there an alternative to the mixed rule that more safely achieves determinacy? To answer this question, we first remove output growth from the policy rule with interest rate responding to the output gap. We find that the conditions leading to (in)determinacy closely mimic those under the mixed rule.

Things are sharply different when we shut down the reaction to the output gap, leaving only that to output growth. We find that the policy rule more safely guarantees a unique REE than the mixed rule does, and it does so by a significant margin. That is, a policy rule reacting to output growth ensures determinacy for a much larger set of policy responses to inflation. Whether the model accounts or not for working capital, the minimum response of interest rates to inflation consistent with determinacy is 1 whether trend inflation is 0, 2% or 3%. The value of $\alpha_\pi$ consistent with determinacy is therefore significantly smaller under the growth rule.

Why reacting to output growth rather than to output gap helps achieving a state of determinacy under low trend inflation? In the textbook NK sticky-price model, and hence according to conventional stabilization, lowering interest rates when output is below potential is what the monetary authority is expected to do. Our different results obtained under a policy rule reacting to output growth follows from the natural rate property of the model. That is, when the level of output is below potential, output growth tends to be high, calling for higher, not lower, interest rates which better ensures a unique REE. Thus, the larger the misallocation in the model that caused by the deviation from the flexible price output, the greater the tension in the two opposing forces in the policy rule: output gap vs output growth.

Relative to the standard NK model, our MSNK model implies stronger monopolistic distortions in the steady state, the most important arising from the interaction between trend inflation, sticky wages and trend growth. The stronger distortions take the economy farther away in the steady state from the efficient level of output entering the output gap. This in turn calls for a more aggressive reaction to inflation as suggested by our findings. Of the two opposing influences on interest rates, that of the output gap or output growth, we find that the impact of output gap is disproportionately large relative to that of output growth. This explains why under the mixed rule or a rule reacting only to output gap, determinacy requires a much stronger response to inflation. By the same token, this is also why a rule incorporating output growth only widens the range of inflation responses consistent with determinacy.
We show that our main results are robust to varying the degrees of nominal rigidities in some acceptable range. Lowering the Calvo probability of non-reoptimized prices from $2/3$ to $0.55$ as assumed by Coibion and Gorodnichenko (2011) does not affect our findings obtained under the mixed rule. Increasing the Calvo probability of non-reoptimized wages from $2/3$ to $3/4$ to be more consistent with the evidence in Barattieri, Basu, and Gottschalk (2014) has a negligible impact on findings with a policy rule reacting to output growth only.

A final question is how our results are affected if we use the post-1982 estimates of the mixed rule of Coibion and Gorodnichenko (2011)? Based on a sticky-price model without capital accumulation including firm-specific labor and non-stationary neutral technological progress, these authors offer evidence suggesting that determinacy was achieved after 1982 through a combination of a lower trend inflation (3%) and a “hawkish” policy stance. This turns out to not be the case in our baseline model. Remarkably, with their post-1982 estimates of the mixed rule, our baseline model implies an indeterminate state. The main reason for this apparently surprising result is that, although the coefficient on output growth in the Coibion-Gorodnichenko rule is much larger at $2.21$ than the coefficient on the output gap at $0.44$ after 1982, the impact of output gap on the prospect of indeterminacy is disproportionately large relative to that of output growth as we have stressed above.

The rest of the paper is organized as follows. Section 2 describes our model. Section 3 provides a discussion of our calibration. Section 4 presents and discusses our results under the mixed Taylor rule. Section 5 presents our findings with a policy rule responding to output growth only. Section 6 looks at the implications of our model for determinacy in the post-1982 era under the mixed Taylor rule of Coibion and Gorodnichenko (2011). Section 7 discusses related literature. Section 8 contains concluding remarks.

## 2 The Baseline Model

Our baseline DSGE model employs the Calvo specification of staggered wage and price adjustment based on the optimizing behavior of monopolistically competitive households and firms. It also includes other frictions such as consumer habit formation, investment adjustment costs and variable capital utilization. Real per capita output growth stems from deterministic trend growth in neutral and investment-specific technological progress.

We add to this relatively standard medium-size New Keynesian model, a cost channel, roundabout production and non-zero steady-state inflation. To close the model, we assume that monetary policy first obeys the mixed Taylor rule. Next, we use a Taylor rule with a reaction to short-run
deviations of inflation from target and output growth from steady state. The inflation target is exogenously fixed.

Note that since our only focus is on (in)determinacy issues under the mixed Taylor rule and alternative where interest rates react only to output growth for measure of economic activity, we present a baseline model in its deterministic version.

2.1 Gross Output

Gross output, \( X_t \), is produced by a perfectly competitive firm using a continuum of intermediate goods, \( X_{jt}, j \in (0, 1) \) and the following CES production technology:

\[
X_t = \left( \int_0^1 X_{jt}^{(1+\lambda_p)} \frac{1}{\lambda_p} \, dj \right)^{1+\lambda_p},
\]

where \( \lambda_p \) is the desired (or steady-state) markup of price over marginal cost for intermediate firms.

Profit maximization and a zero-profit condition for gross output leads to the following downward sloping demand curve for the \( j^{th} \) intermediate good:

\[
X_{jt} = \left( \frac{P_{jt}}{P_t} \right)^{-\frac{(1+\lambda_p)}{\lambda_p}} X_t,
\]

and \( P_{jt} \) is the price of good \( j \), while \( P_t \) is the aggregate price index:

\[
P_t = \left( \int_0^1 P_{jt}^{\frac{1}{\lambda_p}} \, dj \right)^{-\lambda_p}.
\]

2.2 Intermediate Goods Producers and Price Setting

A monopolist produces intermediate good \( j \) according to the following production function:

\[
X_{jt} = \max \left\{ g_A \Gamma_{jt} \left( K_{jt}^\alpha L_{jt}^{1-\alpha} \right)^{1-\phi}, \Upsilon_t F, 0 \right\},
\]

where \( \Gamma_{jt} \) denotes the intermediate inputs, \( \hat{K}_{jt} \) is capital services, \( L_{jt} \) the labor input used by the \( j^{th} \) producer and \( g_A \) is the gross growth rate of neutral technology. \( \Upsilon_t \) represents a growth factor composed of trend growth in neutral and investment-specific technologies. \( F \) is a fixed cost implying that profits are zero in the steady state and ensuring the existence of balanced growth path.

The growth factor is given by the composite technological process:

\[
\Upsilon_t = \left( g_A \right)^{\frac{1}{(1-\phi)(1-\alpha)}} \left( g_{\varepsilon_t} \right)^{\frac{\alpha}{1-\alpha}},
\]
where \( g_t \) is the gross growth rate of investment specific technology.

Without roundabout production, \( \phi = 0 \) and \( \Upsilon^t \) reverts to the conventional deterministic growth factor with growth in neutral and investment-specific productivity. From (5), one sees that as \( \phi \) gets larger, it amplifies the effects of stochastic growth in neutral productivity on output and its components. Therefore, for a given level of stochastic growth in neutral productivity, the economy will grow faster the larger is the share of intermediate inputs in production.

The cost-minimization problem of a typical \( j \) firm is:

\[
\min_{\Gamma_t, \hat{K}_t, L_t} \left( 1 - \psi + \psi R_t \right) \left( P_t \Gamma_{jt} + R^k_t \hat{K}_{jt} + W_t L_{jt} \right),
\]

subject to:

\[
g^t_{A} \Gamma^\phi_{jt} \left( \hat{K}^\alpha_{jt} L^{1-\alpha}_{jt} \right)^{1-\phi} - \Upsilon^t F \geq \left( \frac{P_{jt}}{P_t} \right)^{-\theta} X_t.
\]

This formulation allows firms to have access to funds borrowed from a financial intermediary at the beginning of period \( t \) to finance a fraction \( \psi \) (\( 0 \leq \psi \leq 1 \)) of their factor payments, which they then reimburse at the end of period \( t \) at the gross nominal interest rate \( R_t \). \( R^k_t \) is the nominal rental price of capital services \( \hat{K}_{jt} \) (the product of utilization, \( u_t \), and physical capital, \( K_t \)), and \( W_t \) is the nominal wage index.

Defining \( \Psi_t \equiv (1 - \psi + \psi R_t) \), and then solving the cost-minimization problem yields the real marginal cost,

\[
mc_t = \bar{\phi} \varphi^t_{A} (1-\alpha)(\phi-1) \Psi_t \left[ \left( \frac{r^k_t}{w_t} \right)^{\alpha} \left( \frac{1-\alpha}{w_t} \right)^{(1-\alpha)} \right]^{1-\phi},
\]

and the demand functions for the intermediate inputs and primary factor inputs,

\[
\Gamma_{jt} = \phi \frac{mc_t}{\Psi_t} \left( X_{jt} + \Upsilon^t F \right),
\]

\[
K_{jt} = \alpha (1-\phi) \frac{mc_t}{\Psi_t r^k_t} \left( X_{jt} + \Upsilon^t F \right),
\]

\[
L_{jt} = (1-\alpha)(1-\phi) \frac{mc_t}{\Psi_t w_t} \left( X_{jt} + \Upsilon^t F \right),
\]

where \( \bar{\phi} \equiv \phi^{-\phi} (1-\phi)^{\phi-1} \left( \alpha^{-\alpha} (1-\alpha)^{\alpha-1} \right)^{1-\phi} \), \( r^k_t \) is the real rental price on capital services and \( w_t \) is the real wage.
Intermediate firms allowed to reoptimize their price choose a price \( P^*_t \), and those not allowed to reoptimize keep their price unchanged. The price-setting rule is hence given by

\[
P_{jt} = \begin{cases} 
P^*_t & \text{with probability } 1 - \xi_p \\ 
P_{j,t-1} & \text{with probability } \xi_p 
\end{cases}. \tag{11}
\]

When reoptimizing its price, a firm \( j \) chooses a price that maximizes the present discounted value of future profits, subject to (2) and to cost minimization:

\[
\max_{P_{jt}} \mathbb{E}_t \sum_{t=0}^{\infty} \xi^s_p \beta^s \Lambda_{t+s} \left[ P_{jt} X_{j,t+s} - MC_{t+s} X_{j,t+s} \right], \tag{12}
\]

where \( \beta \) is the discount factor, \( \Lambda_t \) is the marginal utility of nominal income to the representative household owning the firm, \( \xi^s_p \) is the probability that a wage chosen in period \( t \) will still be in effect in period \( t + s \), and \( MC_{t+s} \) is the nominal marginal cost.

Solving the problem yields the following optimal price:

\[
E_0 \sum_{s=0}^{\infty} \xi^s_p \beta^s \Lambda_{t+s} \frac{1}{\lambda_p} \left( \frac{p^*_t}{\pi_{t+1,t+s}} - (1 + \lambda_p) mc_{t+s} \right) = 0, \tag{13}
\]

where \( \lambda^r_t \) is the marginal utility of an additional unit of real income received by the household, \( p^*_t = \frac{P^*_t}{P_t} \) is the real optimal price and \( \pi_{t+1,t+s} = \frac{P_{t+s}}{P_t} \) is the cumulative inflation rate between \( t+1 \) and \( t+s \).

### 2.3 Households and Wage Setting

There is a continuum of households, indexed by \( i \in [0, 1] \), who are monopoly suppliers of labor. They face a downward-sloping demand curve for their particular type of labor given in (18). Each period, households face a fixed probability, \( (1 - \xi_w) \), that they can reoptimize their nominal wage. As in Erceg, Henderson, and Levin (2000), utility is separable in consumption and labor. State-contingent securities insure households against idiosyncratic wage risk arising from staggered wage-setting. Households are then identical along all dimensions other than labor supply and wages.

The problem of a typical household, omitting dependence on \( i \) except for these two dimensions, is:

\[
\max_{C_t, L_t, K^1, B^1, Z_t} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left( \ln (C_t - hC_{t-1}) - \eta \frac{L^1_{it}}{1+\chi} \right), \tag{14}
\]
subject to the following budget constraint,

\[ P_t \left( C_t + \frac{I_t}{g_{z,t}^t} + \frac{a(u_t)K_{t-1}}{g_{z,t}^t} \right) + \frac{B_{t+1}}{R_t} \leq W_{it}L_{it} + R_t^k u_t K_{t-1} + B_t + \Pi_t + T_t, \]  

(15)

and the physical capital accumulation process,

\[ K_{t+1} = g_{z,t}^t \left( 1 - S \left( \frac{I_t}{I_{t-1}} \right) \right) I_t + (1 - \delta)K_t. \]  

(16)

\( C_t \) is real consumption and \( h \) is a parameter determining internal habit. \( L_{it} \) denotes hours and \( \chi \) is the inverse Frisch labor supply elasticity. \( I_t \) is investment, and \( a(u_t) \) is a resource cost of utilization, satisfying \( a(1) = 0, a'(1) = 0, \) and \( a''(1) > 0. \) This resource cost is measured in units of physical capital. \( W_{it} \) is the nominal wage paid to labor of type \( i \), \( B_t \) is the stock of nominal bonds that the household enters the period with. \( \Pi_t \) denotes the distributed dividends from firms. \( T_t \) is a lump-sum transfer from the government. \( S \left( \frac{I_t}{I_{t-1}} \right) \) is an investment adjustment cost, satisfying \( S(1) = 0, S'(1) = 0, \) and \( S''(1) > 0, \) \( \delta \) is the depreciation rate of physical capital.

2.4 Employment Agencies

A large number of competitive employment agencies combine differentiated labor skills into a homogeneous labor input which is sold to intermediate firms, according to:

\[ L_t = \left( \int_0^1 \frac{1}{W_{it}^{1+\lambda_w} di} \right)^{1+\lambda_w}, \]  

(17)

where \( \lambda_w \) is the desired (or steady-state) markup of wage over the household’s marginal rate of substitution.

Profit maximization by the perfectly competitive employment agencies implies the following labor demand function:

\[ L_{it} = \left( \frac{W_{it}}{W_t} \right)^{\frac{1+\lambda_w}{-\lambda_w}} L_t, \]  

(18)

where \( W_{it} \) is the wage paid to labor of type \( i \) and \( W_t \) is the aggregate wage index:

\[ W_t = \left( \int_0^1 W_{it}^{\frac{1}{\lambda_w}} di \right)^{-\lambda_w}. \]  

(19)

2.5 Wage setting

The wage-setting rule is given by:
\[ W_{it} = \begin{cases} W_{it}^* & \text{with probability } 1 - \xi_w \\ W_{i,t-1} & \text{with probability } \xi_w, \end{cases} \tag{20} \]

where \( W_{it}^* \) is the reset wage. When allowed to reoptimize its wage, the household chooses the nominal wage that maximizes the present discounted value of flow utility flow (14) subject to demand schedule (18). From the first-order condition, we the have the following optimal wage rule:

\[ E_t \sum_{s=0}^{\infty} (\beta \xi_w)^s \frac{\lambda_{t+s}^r L_{it+s}}{\lambda_w^r} \left[ \frac{w_t^*}{\pi_{t+1,t+s}} - (1 + \lambda_w) \frac{\eta L^X_{it+s}}{\lambda_{t+s}^r} \right] = 0, \tag{21} \]

where \( \xi_w^s \) is the probability that a wage chosen in period \( t \) will still be in effect in period \( t + s \), and \( w_t^* \) is the reset wage denoted in real terms.

### 2.6 Monetary Policy

We will consider two different monetary policy rules. The first one is the mixed output gap-output growth rule:

\[ \frac{R_t}{R} = \left( \frac{R_{t-1}}{R} \right)^{\rho_R} \left[ \left( \frac{\pi_t}{\pi} \right)^{\alpha_\pi} \left( \frac{Y_t}{Y_t^*} \right)^{\alpha_y} \left( \frac{Y_t}{Y_{t-1}^*} g_{Y_{t-1}}^{-1} \right)^{\alpha_{dy}} \right]^{1-\rho_R} \xi_t^r, \tag{22} \]

where \( R \) is the steady-state nominal interest rate, \( \pi_t \) is the rate of inflation in period \( t \), \( \pi \) is the fixed inflation target, \( Y_t^* \) is the level of output at flexible wages and prices, \( g_{Y} \) is steady-state output growth, \( \rho_R \) is a smoothing parameter, and \( \alpha_\pi, \alpha_y \) and \( \alpha_{dy} \) are control parameters.

The mixed rule implies that the nominal interest rate adjusts in response to deviations of inflation from target, as well as to the level of the output gap and to deviations of the growth rate of output in period \( t \) from steady-state output growth.\(^5\) The output gap is defined as the actual level of output relative to the level of output under flexible wages and prices. This type of rule has been widely used in the estimation of medium-scale DSGE models.\(^6\)

### 2.7 Market-Clearing and Equilibrium

Market-clearing for capital services, labor, and intermediate inputs requires that

\[ \int_0^1 \hat{K}_{jt} dj = \hat{K}_t, \]

\[ \int_0^1 L_{jt} dj = L_t, \] and \[ \int_0^1 \Gamma_{jt} dj = \Gamma_t. \]

\(^5\)Assuming that output growth depends instead on the rate of change of the output gap has little effect on our main findings.

\(^6\)See, for example, Smets and Wouters (2007); Justiniano, Primiceri, and Tambalotti (2010); Justiniano, Primiceri, and Tambalotti (2011); Coibion and Gorodnichenko (2011); Coibion, Gorodnichenko, and Wieland (2012); Justiniano, Primiceri, and Tambalotti (2010); Justiniano, Primiceri, and Tambalotti (2011); Khan and Tsoukalas (2011); and Khan and Tsoukalas (2012), among others. The output growth rule sets \( \alpha_y = 0 \), so the central bank does not respond to the output gap.
Gross output can be written as:

\[ X_t = g_t^\phi (K_t^{\alpha} L_t^{1-\alpha})^{-\phi} - \Upsilon F. \]  

(23)

Value added, \( Y_t \), is related to gross output, \( X_t \), by

\[ Y_t = X_t - \Gamma_t, \]  

(24)

where \( \Gamma_t \) denotes total intermediates.

The resource constraint of the economy is:

\[ Y_t = C_t + \frac{I_t}{g_{\ell t}} + \frac{a(u_t)K_t}{g_{\ell t}}. \]  

(25)

3 Calibration

The calibration for our simulations is explained as follows. Some parameters are calibrated to conventional long-run targets in the data, while others are based on the previous literature. The calibration is summarized in Table 1, with the unit of time being a quarter. Some parameter values like \( \beta = 0.99 \), \( b = 0.8 \), \( \eta = 6 \), \( \chi = 1 \), \( \delta = 0.025 \) and \( \alpha = 0.33 \) are standard in the literature and require no explanation.

Other parameters deserve some explanations. The parameter governing the size of investment adjustment costs is \( \kappa = 3 \), which is slightly higher than the estimate in Christiano, Eichenbaum, and Evans (2005), but slightly lower than the one in Justiniano, Primiceri, and Tambalotti (2010, 2011). The parameter on the squared term in the utilization adjustment cost is set at \( \gamma_2 = 0.025 \), which is somewhat higher than the value chosen by Christiano, Eichenbaum, and Evans (2005), but somewhat lower than the estimate reported by Justiniano, Primiceri, and Tambalotti (2010, 2011).

The values assigned to \( \lambda_p \), denoting the steady-state price markup and to \( \lambda_w \), denoting the steady-state wage markup, are such that the desired price and wage markups are 20 percent if trend inflation is zero, which is consistent with Rotemberg and Woodford (1997) and Huang and Liu (2002).

Both the Calvo probabilities of wage non-re-optimization, \( \xi_w \), and price non-re-optimization, \( \xi_p \), are set at \( 2/3 \); this implies an average waiting time between wage and price changes of 9 months. These values are not far from the estimates reported by Christiano, Eichenbaum, and Evans (2005) for their one-shock model and by Smets and Wouters (2007) for their multi-shock model. Evidence
on the frequency of price adjustments using disaggregated consumer price data by Bils and Klenow (2004) suggests a median (as opposed to average) waiting time of price adjustments of about 5.1 months. The evidence in Nakamura and Steinsson (2008) is more consistent with our calibration of $\xi_p$ since they report a range for the average frequency of price changes between 7 and 11 months depending on the price categories on which their evidence is based. Evidence from disaggregated wage data presented in Barattieri, Basu, and Gottschalk (2014) implies a higher degree of wage stickiness than assumed here, between 12 and 15 months. Thus, we view as conservative our baseline values of $\xi_p$ and $\xi_w$.

Given that the next section will search for values of $\alpha_\pi$ consistent with a unique REE, the only parameters that need to be calibrated in the mixed Taylor rule are $\rho_r$, the interest rate smoothing parameter which is set at 0.8, $\alpha_y$, the coefficient on the output gap set at 0.2, and $\alpha_{dy}$, the coefficient on output growth also set at 0.2. These are relatively standard values in the literature when using in the estimation U.S. postwar data dating back to the 1950s and 1960s. When the central bank responds to output growth but not to the output gap, $\alpha_y = 0$. When simulating the model, we assume that the steady-state gross inflation target is fixed and set at an annualized average rate of inflation of 0, 2% or 3%.

Our model also accounts for real per capita output growth. Mapping the model to the data, the trend growth rate of the IST term, $g_{\varepsilon I}$, equals the negative of the growth rate of the relative price of investment goods. To measure this in the data, we define investment as expenditures on new durables plus private fixed investment, and consumption as consumer expenditures of nondurables and services. These series are from the BEA and cover the period 1960:I-2007:III, to leave out the financial crisis. The relative price of investment is the ratio of the implied price index for investment goods to the price index for consumption goods. The average growth rate of the relative price from the period 1960:I-2007:III is -0.00472, so that $g_{\varepsilon I} = 1.00472$. Real per capita GDP is computed by subtracting the log civilian non-institutionalized population from the log-level of real GDP. The average growth rate of the resulting output per capita series over the period is 0.005712, so that $g_Y = 1.005712$ or 2.28 percent a year. Given the calibrated growth of IST from the relative price investment data ($g_{\varepsilon I} = 1.0047$), we then pick $g_A^{1-\phi}$ to generate the appropriate average growth rate of output. This implies $g_A^{1-\phi} = 1.0022$ or a measured growth rate of TFP of about 1 percent per year.

The parameter $\psi$ measures the fraction of factor payments financed by working capital. Although some evidence confirms the existence of a cost channel (Ravenna and Walsh, 2006; Christiano, Eichenbaum, and Evans, 2005; Chowdhury, Hoffmann, and Schabert, 2006; Tillman, 2009),

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7See Ascari, Phaneuf, and Sims (2018) for a detailed description of how these data are constructed.
we do not dispose of evidence on the fraction of factor payments financed by working capital. For this reason, in our model simulations, we alternatively set \( \psi = 0 \) (no working capital), \( 1/2 \) or 1. The parameter \( \phi \), measuring the share of payments to intermediate inputs in total production, is set at \( \phi = 0.5 \) following Basu (1995), Dotsey and King (2006) and Christiano, Trabandt, and Walentin (2011).

4 Rule-Based Monetary Policy and the Threat of Indeterminacy

Using the baseline model described in Section 2, we identify in this section the conditions leading to (in)determinacy with the central bank first setting interest rates conditioned on the mixed Taylor rule. Next, we look at a Taylor rule reacting only to output growth and no response to the output gap. Specifically, we search for values of \( \alpha_\pi \) consistent with determinacy in our baseline framework and some alternative models with fewer theoretical ingredients.

4.1 The Mixed Taylor Rule and the Prospect of Indeterminacy

This subsection addresses the following question: What are the values of \( \alpha_\pi \) consistent with determinacy if monetary policy is set in accordance with the mixed Taylor rule? To answer this question, we use the baseline model proposed in Section 2. While doing so, parameters other than \( \alpha_\pi \) keep the values we have assigned them in our calibration. Table 2 displays the values of \( \alpha_\pi \) consistent with determinacy for an inflation trend of 0, 2% and 3% (annualized), and a fraction of factor payments financed by working capital of 0, \( 1/2 \) and 1.

A first thing to note is that with a trend inflation of 0 and no working capital (\( \psi = 0 \)), achieving a unique REE requires that \( \alpha_\pi \geq 1.3 \). That is, strict application of the Taylor Principle (\( \alpha_\pi > 1 \)) no more guarantees determinacy. With 0 trend inflation and a fraction of factor payments financed via intra-period loans set at \( 1/2 \), determinacy is achieved only if \( \alpha_\pi \) belongs to the interval \([1.6, 4.2]\). In the extreme case where \( \psi = 1 \), we find no value of \( \alpha_\pi \) consistent with determinacy whether trend inflation is 0, 2% or 3%.

Non-zero trend inflation has two main effects. First, it increases the minimum value of \( \alpha_\pi \) consistent with determinacy. With \( \psi = 0 \), the value of \( \alpha_\pi \) consistent with determinacy rises to 1.9 with 2% trend inflation and to 2.5 with 3% trend. Second, when combined with \( \psi = 1/2 \), a 2% inflation trend results in a smaller interval consistent with determinacy, that is \( \alpha_\pi \in [2.3, 4.2] \). With 3% trend inflation, the interval narrows to \([2.9, 4.1]\). Therefore, the values of \( \alpha_\pi \) consistent with a unique REE significantly deviate from the Taylor principle as trend inflation is only mildly positive.
These findings are of interest in light of estimates of $\alpha_\pi$ found in the broader literature using U.S. data dating back to the 1950s and 1960s (Smets and Wouters, 2007; Justiniano, Primiceri, and Tambalotti, 2010, 2011; Khan and Tsoukalas, 2011, 2012). These MSNK models incorporate the mixed Taylor rule and indexation, but abstract from working capital and roundabout production. With indexation, nominal wages and prices are indexed to past and steady-state inflation in a way that neutralizes the impact of non-zero trend inflation on (in)determinacy. These estimates range from 1.7 to 2.1. Based on the simulation results we have presented so far, these estimates would be consistent with determinacy if: i) $\psi = 0$ and trend inflation was less than 2% or ii) $\psi = 1/2$ and trend inflation was 0.

### 4.2 Factors Driving Indeterminacy Under the Mixed Rule

At this stage, a question that arises naturally is the following: What are the key factors driving our indeterminacy results at fairly low rates of trend inflation under the mixed Taylor rule? To answer this question, we consider different model ingredients and how they impact our results. The results of these exercises are reported in Table 3, which is similar to Table 2, but shuts off different model features to isolate their roles. We focus on five scenarios.

Panel (i) considers the case where nominal wages are flexible by setting $\xi_w = 0$. With 0 trend inflation and no working capital ($\psi = 0$), $\alpha_\pi \geq 1$ is sufficient to ensure determinacy relative to $\alpha_\pi \geq 1.3$ with sticky wages. With a trend inflation of 2%, $\alpha_\pi \geq 1.1$ will be consistent with determinacy compared to $\alpha_\pi \geq 1.9$ with sticky wages, while with 3% trend inflation, $\alpha_\pi \geq 1.3$ is needed relative to $\alpha_\pi \geq 2.5$ with sticky wages. With $\psi = 1/2$ and 0 trend inflation, it must be that $\alpha_\pi \in [1.3, 4.2]$ for determinacy. With a 2% or 3% trend inflation, $\alpha_\pi$ must be in the interval $[1.4, 4.2]$. This means that with 2% trend inflation, the lower bound on $\alpha_\pi$ consistent with determinacy is 1.64 times higher with sticky wages than with flexible wages, whereas with a 3% trend, it is 2.1 times higher. In the extreme case where all input costs are fully financed by working capital, there is no single value of $\alpha_\pi$ consistent with a unique REE.

Panel (ii) looks at the case where economic growth is shut off from our baseline model. For this, we set the trend growth rates of IST and neutral productivity to zero. The impact of economic growth on our results is significant. With $\psi = 0$ and a trend inflation of 0, $\alpha_\pi \geq 1$ is sufficient to achieve determinacy in the no growth scenario, while with 2% and 3% trend inflation, $\alpha_\pi \geq 1.4$ and $\alpha_\pi \geq 1.6$ are required for determinacy. With $\psi = 1/2$ and a trend inflation of 0, $\alpha_\pi$ must be in the interval $[1.2, 4.4]$. With 2% trend inflation, $\alpha_\pi$ must be found in the interval $[1.6, 4.3]$, meaning that the lower bound consistent with determinacy is 1.44 times higher with economic growth. With 3% trend inflation, $\alpha_\pi \in [2.0, 4.3]$, which implies that the lower bound consistent with determinacy
is 1.45 times higher with trend growth. With $\psi = 1$ and 0 trend inflation, determinacy is obtained only if $\alpha_\pi \in [1.5, 1.8]$. With a trend inflation of 2% or 3%, there is no value of $\alpha_\pi$ achieving determinacy.

Panel (iii) shows the results assuming flexible nominal wages and no trend growth. With $\psi = 0$, determinacy is achieved for $\alpha_\pi \geq 1$ with 0 trend inflation, $\alpha_\pi \geq 1.1$ with 2% trend inflation and $\alpha_\pi \geq 1.3$ with 3% trend inflation. With $\psi = 1/2$, $\alpha_\pi$ must be in the interval $[1.3, 1.8]$ with a trend inflation of 0, $\alpha_\pi \in [1.4, 4.3]$ with 2% trend and $\alpha_\pi \in [1.4, 4.2]$ with 3% trend. With $\psi = 1$, the intervals consistent with determinacy are $[1.5, 4.4]$ and $[1.6, 1.8]$ with 0 and 2% trend inflation, respectively. With $\psi = 1$ and a 3% inflation trend, there is no value of $\alpha_\pi$ consistent with determinacy. Therefore, for values of $\alpha_\pi$ like those typically found in the literature, determinacy will normally be achieved if nominal wages are flexible and there is no economic growth. These results hence confirm that sticky wages and trend growth are the two key factors driving the prospect of (in)determinacy in our baseline model.

Panel (iv) reports the results of a case where working capital and roundabout production are both omitted from the model ($\psi = 0, \phi = 0$). With 0 trend inflation, $\alpha_\pi \geq 1.3$ is sufficient to ensure determinacy as in our baseline model. With 2% trend inflation, determinacy is achieved if $\alpha_\pi \geq 1.8$ compared to 1.9 in our baseline. With 3% trend inflation, $\alpha_\pi \geq 2.3$ will be sufficient for determinacy relative to 2.5 in our baseline. These results can also be compared with those from our baseline model where $\psi = 1/2$ and 1. Recall that with $\psi = 1/2$, $\alpha_\pi$ must be in the intervals $[1.6, 4.2], [2.3, 4.2]$ and $[2.9, 4.1]$ for a trend inflation of 0, 2% and 3%, respectively. With $\psi = 1$, we found no single value of $\alpha_\pi$ generating a unique REE in our baseline model. Therefore, working capital and roundabout production, two features emphasized by Christiano et al. (2011), contribute non-trivially to the threat of indeterminacy under the mixed Taylor rule although they are not the key factors driving our results.

Panel (v) considers a fifth case in which nominal wages are flexible, there is no economic growth, no working capital and no roundabout production. The resulting model is a NK price-setting framework augmented with capital accumulation and real frictions. With $\psi = 0$ and a trend inflation of 0, the minimum value needed for determinacy is $\alpha_\pi \geq 1$. With 2% and 3% trend inflation, the necessary condition for determinacy is $\alpha_\pi \geq 1.1$. With $\psi = 1/2$, the corresponding figures are 1.1 and 1.2. With $\psi = 1$, $\alpha_\pi \in [1.3, 6]$ will be consistent with determinacy, and this whether trend inflation is 0, 2% or 3%. Therefore, based on this augmented version of the standard sticky-price model, determinacy should be safely achieved for values equal to postwar estimates of the mixed Taylor rule generally found in the literature.
To summarize the results presented in this section, we have shown that under the mixed Taylor rule, the threat of indeterminacy can be real at low rates of trend inflation of 2% and 3%. This is true when assuming that a moderate fraction of factor payments is financed by intra-period loans or that all input costs are fully financed by working capital ($\psi = 1$). Then, the minimum value of $\alpha_\pi$ consistent with determinacy is significantly higher than those typically found in estimated MSNK models for the U.S. postwar period.

Next, we have identified the key factors driving our new results. We have shown that sticky wages and trend growth in neutral and investment-specific technological progress are the main factors generating our results if the extent of working capital is moderate. When we abstract from sticky wages and trend growth, there is a serious threat to determinacy only if trend inflation reaches 3% and factor payments are fully financed by working capital. Finally, in a sticky-price model without working capital, roundabout production and economic growth, the prospect of indeterminacy at low rates of trend inflation vanishes in light of postwar estimates of the mixed policy rule generally reported in the literature.

5 Policy Rule Responding to Output Growth

The mixed Taylor rule involving responses of the nominal interest rate both to the level of output gap and to output growth, we now ask which of responding to output gap or output growth matters most for our findings.

5.1 Output Gap vs Output Growth in the Taylor Rule

We ask whether responding to the level of output gap or to output growth has the strongest impact on the minimum values of $\alpha_\pi$ consistent with determinacy. Table 4 compares $\alpha_\pi$'s consistent with determinacy in three versions of our baseline model that differ only from the specification of the Taylor rule assumed in the simulations. Panel (i) reports the results assuming the mixed Taylor rule with a reaction to output gap and output growth. Panel (ii) presents the results with the policy rule responding to output gap only. Panel (iii) displays the results with the policy rule responding to output growth only.

Looking at this table, what is striking is the high similarity of results obtained under the mixed Taylor rule and a policy rule reacting only to output gap. To a first-order approximation, the results with the output gap closely mimic those obtained under the mixed Taylor rule. Next, we ask how our findings are affected when interest rates react only to output growth. We find that the conditions leading to determinacy are markedly different from those obtained under the mixed
policy rule. Without working capital ($\psi = 0$), $\alpha_\pi \geq 1$ guarantees a unique REE whether trend inflation is 0, 2% or 3%. With a fraction of factor payments financed by working capital set at $\psi = 1/2$, $\alpha_\pi \in [1, 4.2]$ safely guarantees determinacy. Finally, in the extreme case where $\psi = 1$, determinacy is achieved if $\alpha_\pi \in [1, 1.8]$ under 0 trend inflation. With an inflation trend of 2% and 3%, the interval consistent with determinacy is $[1, 1.7]$.

What do we learn about rules-based monetary policy from these findings? First, the mixed Taylor rule widely used recently in the estimation of MSNK models represents a serious threat to determinacy at a low level of trend inflation seen through the lens of our baseline model. Second, when looking at whether reacting to the output gap or to output growth drives our indeterminacy results under the mixed rule, we have found that responding to output gap has a disproportionally large impact on our determinacy results relative to output growth. Third, our findings suggest that a policy rule reacting only to output growth considerably widens the range of $\alpha_\pi$-values consistent with determinacy.

5.2 Varying the Degrees of Nominal Rigidities

Our baseline calibration has set the Calvo probabilities of wage and price non-reoptimization at $\xi_p = \xi_w = 2/3$. However, micro level evidence on the frequency of price adjustments by Bils and Klenow (2004) suggests that the probability of price non-reoptimization is somewhat lower. Coibion and Gorodnichenko (2011) and Coibion, Gorodnichenko, and Wieland (2012) set $\xi_p = 0.55$. On the other hand, micro level evidence on the frequency of wage adjustments by Barattieri, Basu, and Gottschalk (2014) implies a somewhat higher probability of wage non-reoptimization between 0.75 and 0.8.

Table 5 presents the results of two experiments. The first experiment (Panel i) looks at the prospect of indeterminacy under the mixed Taylor rule for $\xi_p = 0.55$. The second experiment (Panel ii) revisits the conditions leading to determinacy under a policy rule reacting to output growth with $\xi_w = 3/4$. Lowering $\xi_p$ from $2/3$ to 0.55 essentially has no impact on the conditions ensuring determinacy in our baseline model under the mixed rule. The same is true when increasing $\xi_w$ from $2/3$ to $3/4$ and assuming that the Taylor rule reacts to output growth only in our baseline model.

6 The Mixed Rule and Indeterminacy After 1982

This section asks the following question: What if we take into account post-1982 estimates of the mixed rule? In the literature, evidence covering subsample periods has been mixed. For instance,
using a Bayesian procedure, Smets and Wouters (2007) find no evidence of significant variations in the estimates of the mixed Taylor rule from 1966:II-1979:II to 1984:I-2004:IV. As a matter of fact, their subsample estimates are relatively similar to parameter values of the mixed rule assumed by our baseline calibration. Therefore, using their subsample estimates have no significant impact on the results we have reported.


Unlike the mixed Taylor rule in our baseline model, Coibion, Gorodnichenko, and Wieland (2012) estimate a rule allowing for interest smoothing of order two of the form:

\[
\frac{R_t}{R} = \left( \frac{R_{t-1}}{R} \right)^{\rho_1} \left( \frac{R_{t-2}}{R} \right)^{\rho_2} \left[ \left( \frac{\pi_t}{\pi} \right)^{\alpha_\pi} \left( \frac{Y_t}{Y_t^*} \right)^{\alpha_y} \left( \frac{Y_t - g_{t-1} Y_{t-1}}{Y_t^*} \right)^{\alpha_{dy}} \right]^{1-\rho_1-\rho_2} \varepsilon_t.
\]

They report significant changes in the estimates of the mixed Taylor rule from 1969-1978 to 1983-2002. They report an estimate of \( \alpha_\pi \) of 0.79 prior to 1979 and 1.58 after 1982. The estimate of \( \alpha_y \) is 0.48 and 0.44, respectively, while that of \( \alpha_{dy} \) is 0.04 and 2.21. Therefore, in the years following 1982, the coefficient on inflation has been two times its pre-1979 value. Meanwhile, the coefficient on output growth has surged from 0.04 prior to 1979 to 2.21 after 1982. Therefore, it has been much bigger than the coefficient on the output gap. Meanwhile, the smoothing parameters prior to 1979 are \( \rho_1 = 1.39 \) and \( \rho_2 = -0.49 \), and those after 1982, \( \rho_1 = 1.12 \) and \( \rho_2 = -0.18 \). Based on their estimates of the mixed rule, in particular on the much stronger responses of interest rates to inflation and output growth, CG have referred to pre-1979 monetary policy as “dovish” and to post-1982 policy as “hawkish”.

Table 6 reports the results of an experiment where we use in our baseline model the post-1982 CG estimates of the mixed policy rule, except the coefficient on inflation we are searching for. The rest of the parameters keep the values assigned by our calibration. What is striking about these results is that determinacy seems more difficult to achieve under the post-1982 CG rule. Under 0 trend inflation and no working capital (\( \psi = 0 \)), \( \alpha_\pi \geq 1.7 \) is consistent with determinacy compared to \( \alpha_\pi \geq 1.3 \) conditioned on our baseline calibration. With positive trend inflation, the minimum requirement on \( \alpha_\pi \) to achieve determinacy increases. With 2% trend inflation it rises to \( \alpha_\pi \geq 3.1 \), and with 3% trend inflation it increases to \( \alpha_\pi \geq 3.9 \). CG report an estimate of \( \alpha_\pi \) which is 1.58 assuming a level of trend inflation of 3%.
Now, if $\psi = 1/2$, $\alpha_\pi$ must belong to the intervals $[2.3, 18.1]$, $[3.6, 18]$ and $[4.5, 17.9]$ for an inflation of 0, 2% and 3%, respectively. In the case where the cost of all inputs is financed via intra-period loans ($\psi = 1$), these intervals become $[2.9, 7.6]$, $[4.2, 7.6]$ and $[5.3, 7.6]$, respectively. What emerges for these findings, is that the minimum requirement on $\alpha_\pi$ consistent with determinacy is significantly higher when we use the estimates of CG.

Recall we have shown conditioned on our baseline calibration that the conditions leading to determinacy with a policy rule reacting to the output gap closely mimic those under the mixed policy rule. CG obtain an estimate of the response to the output gap which is significantly stronger than our baseline (0.44 vs 0.2). As a result, the minimum requirement on $\alpha_\pi$ to achieve determinacy is significantly higher since the impact of the output gap on the prospect of determinacy is disproportionally large relative to that of output growth as we have seen previously, and this despite the fact that the response to inflation was much stronger after 1982.

7 Related Literature

Our findings have so far suggested that a policy rule reacting only to output growth leaves the monetary authority with significantly more room to achieve a unique REE than a policy rule responding to the level of the output gap and to output growth or to the output gap only. Some reasons have been advanced in the literature as to why targeting the growth rate of output might be preferred to targeting the level of the output gap. Walsh (2003), for one, argues it is unclear that stabilizing inflation and the level of the output gap are objectives that central banks actually or should pursue. He provides some evidence, based on the standard NKPC model, that a policy aimed at stabilizing inflation and the rate of change of output imparts inertia that possibly improves stabilization relative to pure discretion or inflation targeting.

Ascari and Ropele (2009) highlight the potentially destabilizing role of a policy rule reacting to the level of the output gap under positive trend inflation in a standard sticky-price model without capital accumulation. Here, using a sticky-price model with capital accumulation and real frictions that abstracts from other theoretical refinements, we have presented evidence that a policy rule reacting both to the level of the output gap and to output growth does not represent a serious threat to determinacy for a level of trend inflation of 3% or less. We have shown that such a model calls for a reaction of interest rates to inflation that represents only a small departure from the Taylor principle.
Coibion, Gorodnichenko, and Wieland (2012) show that a policy rule responding to the growth rate of output as opposed to the level of the output gap, can help restore determinacy for plausible inflation responses. They reach this conclusion using a sticky-price model without capital accumulation that includes firm-specific labor and non-stationary neutral technological progress. Next, they combine the level of the output gap and output growth into a Taylor rule estimated from a subsample of data from 1983 to 2002. Given their estimates of the mixed Taylor rule and a percentage of trend inflation of 3% or 6%, they draw 10,000 times from the distribution of the estimated parameters and assess the fraction of draws that yield a determinate REE for a given level of trend inflation. They find that conditioned on their mixed Taylor rule, more than 99 percent of the empirical distribution of parameters yields determinacy at a level of trend inflation of 3% in the post-1982 period.\(^8\) Contrasting sharply with their findings, we find according to our baseline model that their estimates of the mixed Taylor rule are not consistent with determinacy for a level of trend inflation of 3%.

Sims (2013) explores which measure of economic activity targeted by the monetary authority would be more beneficial to welfare. He uses a standard MSNK model abstracting from working capital and roundabout production. He shows that a policy rule reacting to output growth is welfare enhancing relative to a rule responding to the level of the output gap.

Using a Bayesian procedure, Arias et al. (2017) estimate a MSNK model embedding a mixed Taylor rule as in Coibion, Gorodnichenko, and Wieland (2012). The estimation is fitted to the Great Moderation. They find that for moderate inflation targets between 2% and 4%, the probability of determinacy is near unity conditioned on the mixed rule. However, this probability somewhat drops conditional on model-free estimates of the monetary policy rule based on real-time data.

Hirose, Kurozumi, and Zandweghe (2017) estimate a sticky-price model without capital accumulation where non-zero steady-state inflation plays an active role. In their preferred model, a fraction of price-setting firms behave as "rule-thumbers" following Galí and Gertler (1999), setting prices when allowed to change to the last quarter's rate of inflation. Using the mixed Taylor rule, they report that the response of interest rates to inflation was weakly active in the pre-1979 period (coefficient of 1.03) and much more active in the post-1982 period (coefficient of 2.73). Based on their estimated model, they establish the probability of indeterminacy to be nearly one in the pre-1979 period and nearly 0 in the post-1982 period.

\(^8\)They even find that with the estimated post-1982 policy reaction function and a trend inflation of 6%, the fraction of draws consistent with determinacy in the post-1982 period is 62.2%. 

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8 Conclusion

Is the prospect of determinacy affected when trend inflation is increased from 2% to 3-4% while the Fed concomitantly returns to a rules-based monetary policy? Until now, the answer offered by the broad literature was that a trend inflation of 4% or less does not pose a real threat to determinacy. In contrast to conventional wisdom, we have shown that conditioned on a Taylor rule popularized by Smets and Wouters (2007), which has been widely used afterwards in the estimation of MSNK models, an inflation trend of only 2% represents a threat to determinacy according to the model we have proposed in this paper. The policy rule responsible for this apparently surprising result is one where the nominal interest rate reacts to inflation, to the level of the output gap and to output growth.

Given the results we have presented, three questions may come to mind. First, why is it that these results went remarkably overlooked in the literature? Second, knowing that inflation has averaged 3.52% during the postwar period and 2.2% after 1990, was the U.S. economy always in a state of indeterminacy during the postwar period with the Fed running monetary policy based on the mixed Taylor rule? Or alternatively, would it be possible that the U.S. economy was never in a state of indeterminacy during the postwar period with the Fed following a Taylor rule responding only to output growth?

Our answer to the first question is the following. There are presumably two reasons why our results went overlooked in the literature. The first reason is that MSNK models have generally assumed automatic quarterly indexation of non-reoptimized nominal wages and prices to past and long-run inflation that more or less neutralizes the impact of steady-state inflation on the prospect of indeterminacy under the mixed Taylor rule. The second reason, is that this class of models has also ignored the potential impact of working capital interacting with roundabout production as emphasized by Christiano, Trabandt, and Walentin (2011) in the context of a simpler NK price-setting model. Our own results suggest that with a trend inflation of 0, no working capital and no roundabout production, determinacy can be achieved with a response of interest rates to inflation not deviating significantly from the Taylor principle even when the central bank sets its policy in accordance with the mixed Taylor rule.

Our answers to the second and third questions are less clear-cut. For one, we hardly believe that the U.S. economy has always been in a state of indeterminacy throughout the postwar period with the Fed using the mixed Taylor rule. If this is the case, another possibility is that the economy was never in a state of indeterminacy during the postwar period with the Fed setting its policy from a rule responding only to output growth. This can be verified by bringing to the data versions
of our model with the mixed Taylor rule and a policy rule reacting only to output growth. Finding, that the economy was never in a state of indeterminacy during the postwar period would call for a reassessment of several important questions, including the identification of the sources of the Great Inflation and Great Moderation. We plan to undertake this task in the near future.
References


Table 1: Parameter Values

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<th>Parameter</th>
<th>Value</th>
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<td>$\alpha$</td>
<td>$1/3$</td>
<td>Capital share</td>
</tr>
<tr>
<td>$\psi$</td>
<td>$0, 1/2, 1$</td>
<td>Fraction of input financed</td>
</tr>
<tr>
<td>$\rho_i$</td>
<td>0.8</td>
<td>Taylor rule smoothing</td>
</tr>
<tr>
<td>$\alpha_{\pi}$</td>
<td>1.5</td>
<td>Taylor rule inflation</td>
</tr>
<tr>
<td>$\alpha_y$</td>
<td>0.2</td>
<td>Taylor rule output growth</td>
</tr>
<tr>
<td>$\lambda_y$</td>
<td>0.2</td>
<td>Taylor rule output gap</td>
</tr>
<tr>
<td>$\pi$</td>
<td>$0, 2, 3$</td>
<td>Steady state inflation (annual percentage rate)</td>
</tr>
<tr>
<td>$g_{t,i}$</td>
<td>1.0047</td>
<td>Gross growth of investment specific technology</td>
</tr>
<tr>
<td>$g_A$</td>
<td>$1.0022^{1-\phi}$</td>
<td>Gross growth of neutral productivity</td>
</tr>
</tbody>
</table>

Note: This table shows the values of the parameters used in quantitative analysis of the model. A description of each parameter is provided in the right column.
<table>
<thead>
<tr>
<th>Trend inflation ((\pi))</th>
<th>Fraction of input ((\psi))</th>
<th>0</th>
<th>1/2</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>0%</td>
<td>(\alpha_\pi \geq 1.3)</td>
<td>(\alpha_\pi \in [1.6, 4.2])</td>
<td>empty set</td>
<td></td>
</tr>
<tr>
<td>2%</td>
<td>(\alpha_\pi \geq 1.9)</td>
<td>(\alpha_\pi \in [2.3, 4.2])</td>
<td>empty set</td>
<td></td>
</tr>
<tr>
<td>3%</td>
<td>(\alpha_\pi \geq 2.5)</td>
<td>(\alpha_\pi \in [2.9, 4.1])</td>
<td>empty set</td>
<td></td>
</tr>
</tbody>
</table>

Note: This table shows the values of the minimum response of interest rates to deviations of inflation from target (\(\alpha_\pi\)), which are consistent with determinacy for an inflation trend of 0%, 2% and 3% (annualized), and a fraction of factor payments financed by working capital of 0, 1/2 and 1.
Table 3: Determinacy and mixed Taylor rule in alternative models

Panel (i): $\xi_w = 0$

<table>
<thead>
<tr>
<th>Trend inflation ($\pi$)</th>
<th>Fraction of input ($\psi$)</th>
<th>0</th>
<th>1/2</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>0%</td>
<td>$\alpha_x \geq 1.0$</td>
<td>$\alpha_x \in [1.3, 4.2]$</td>
<td>empty set</td>
<td></td>
</tr>
<tr>
<td>2%</td>
<td>$\alpha_x \geq 1.1$</td>
<td>$\alpha_x \in [1.4, 4.2]$</td>
<td>empty set</td>
<td></td>
</tr>
<tr>
<td>3%</td>
<td>$\alpha_x \geq 1.3$</td>
<td>$\alpha_x \in [1.4, 4.2]$</td>
<td>empty set</td>
<td></td>
</tr>
</tbody>
</table>

Panel (ii): $g_z = g_v = 1$

<table>
<thead>
<tr>
<th>Trend inflation ($\pi$)</th>
<th>Fraction of input ($\psi$)</th>
<th>0</th>
<th>1/2</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>0%</td>
<td>$\alpha_x \geq 1.0$</td>
<td>$\alpha_x \in [1.2, 4.4]$</td>
<td>$\alpha_x \in [1.5, 1.8]$</td>
<td></td>
</tr>
<tr>
<td>2%</td>
<td>$\alpha_x \geq 1.1$</td>
<td>$\alpha_x \in [1.4, 4.3]$</td>
<td>$\alpha_x \in [1.6, 1.8]$</td>
<td></td>
</tr>
<tr>
<td>3%</td>
<td>$\alpha_x \geq 1.6$</td>
<td>$\alpha_x \in [2.0, 4.3]$</td>
<td>empty set</td>
<td></td>
</tr>
</tbody>
</table>

Panel (iii): $\xi_w = 0$ and $g_z = g_v = 1$

<table>
<thead>
<tr>
<th>Trend inflation ($\pi$)</th>
<th>Fraction of input ($\psi$)</th>
<th>0</th>
<th>1/2</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>0%</td>
<td>$\alpha_x \geq 1.0$</td>
<td>$\alpha_x \in [1.3, 4.2]$</td>
<td>$\alpha_x \in [1.5, 1.8]$</td>
<td></td>
</tr>
<tr>
<td>2%</td>
<td>$\alpha_x \geq 1.1$</td>
<td>$\alpha_x \in [1.4, 4.3]$</td>
<td>$\alpha_x \in [1.6, 1.8]$</td>
<td></td>
</tr>
<tr>
<td>3%</td>
<td>$\alpha_x \geq 1.3$</td>
<td>$\alpha_x \in [1.4, 4.2]$</td>
<td>empty set</td>
<td></td>
</tr>
</tbody>
</table>

Panel (iv): $\phi = 0$

<table>
<thead>
<tr>
<th>Trend inflation ($\pi$)</th>
<th>Fraction of input ($\psi$)</th>
<th>0</th>
<th>1/2</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>0%</td>
<td>$\alpha_x \geq 1.3$</td>
<td>$\alpha_x \geq 1.5$</td>
<td>$\alpha_x \in [1.6, 5.8]$</td>
<td></td>
</tr>
<tr>
<td>2%</td>
<td>$\alpha_x \geq 1.8$</td>
<td>$\alpha_x \geq 2.0$</td>
<td>$\alpha_x \in [2.2, 5.7]$</td>
<td></td>
</tr>
<tr>
<td>3%</td>
<td>$\alpha_x \geq 2.3$</td>
<td>$\alpha_x \geq 2.5$</td>
<td>$\alpha_x \in [2.6, 5.7]$</td>
<td></td>
</tr>
</tbody>
</table>

Panel (v): $\xi_w = 0$, $g_z = g_v = 1$, $\phi = 0$

<table>
<thead>
<tr>
<th>Trend inflation ($\pi$)</th>
<th>Fraction of input ($\psi$)</th>
<th>0</th>
<th>1/2</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>0%</td>
<td>$\alpha_x \geq 1.0$</td>
<td>$\alpha_x \geq 1.1$</td>
<td>$\alpha_x \in [1.3, 6.0]$</td>
<td></td>
</tr>
<tr>
<td>2%</td>
<td>$\alpha_x \geq 1.1$</td>
<td>$\alpha_x \geq 1.2$</td>
<td>$\alpha_x \in [1.3, 6.0]$</td>
<td></td>
</tr>
<tr>
<td>3%</td>
<td>$\alpha_x \geq 1.1$</td>
<td>$\alpha_x \geq 1.2$</td>
<td>$\alpha_x \in [1.3, 6.0]$</td>
<td></td>
</tr>
</tbody>
</table>

Note: This table shows the values of the minimum response of interest rates to deviations of inflation from target ($\alpha_x$), which are consistent with determinacy for an inflation trend of 0%, 2% and 3% (annualized), and a fraction of factor payments financed by working capital of 0, 1/2 and 1 in alternative models.
### Table 4: Determinacy: output gap vs output growth

#### Panel (i): gap and growth

<table>
<thead>
<tr>
<th>Trend inflation (π)</th>
<th>Fraction of input (ψ)</th>
<th>0</th>
<th>1/2</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>0%</td>
<td>α_π ≥ 1.3</td>
<td>α_π ∈ [1.6, 4.2]</td>
<td>empty set</td>
<td></td>
</tr>
<tr>
<td>2%</td>
<td>α_π ≥ 1.9</td>
<td>α_π ∈ [2.4, 4.2]</td>
<td>empty set</td>
<td></td>
</tr>
<tr>
<td>3%</td>
<td>α_π ≥ 2.5</td>
<td>α_π ∈ [2.9, 4.1]</td>
<td>empty set</td>
<td></td>
</tr>
</tbody>
</table>

#### Panel (ii): gap only

<table>
<thead>
<tr>
<th>Trend inflation (π)</th>
<th>Fraction of input (ψ)</th>
<th>0</th>
<th>1/2</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>0%</td>
<td>α_π ≥ 1.3</td>
<td>α_π ∈ [1.6, 4.2]</td>
<td>empty set</td>
<td></td>
</tr>
<tr>
<td>2%</td>
<td>α_π ≥ 1.9</td>
<td>α_π ∈ [2.4, 4.1]</td>
<td>empty set</td>
<td></td>
</tr>
<tr>
<td>3%</td>
<td>α_π ≥ 2.6</td>
<td>α_π ∈ [3.0, 4.1]</td>
<td>empty set</td>
<td></td>
</tr>
</tbody>
</table>

#### Panel (iii): growth only

<table>
<thead>
<tr>
<th>Trend inflation (π)</th>
<th>Fraction of input (ψ)</th>
<th>0</th>
<th>1/2</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>0%</td>
<td>α_π ≥ 1.0</td>
<td>α_π ∈ [1.0, 4.2]</td>
<td>α_π ∈ [1.0, 2.3]</td>
<td></td>
</tr>
<tr>
<td>2%</td>
<td>α_π ≥ 1.0</td>
<td>α_π ∈ [1.0, 4.2]</td>
<td>α_π ∈ [1.0, 2.3]</td>
<td></td>
</tr>
<tr>
<td>3%</td>
<td>α_π ≥ 1.0</td>
<td>α_π ∈ [1.0, 4.2]</td>
<td>α_π ∈ [1.0, 2.3]</td>
<td></td>
</tr>
</tbody>
</table>

Note: This table shows the values of the minimum response of interest rates to deviations of inflation from target (α_π), which are consistent with determinacy for an inflation trend of 0%, 2% and 3% (annualized), and a fraction of factor payments financed by working capital of 0, 1/2 and 1 for different specifications of the Taylor rule.
Table 5: Determinacy for alternative degrees of nominal rigidities

Panel (i): mixed Taylor rule and $\xi_p = 0.55$

<table>
<thead>
<tr>
<th>Trend inflation ($\pi$)</th>
<th>Fraction of input ($\psi$)</th>
<th>0</th>
<th>1/2</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>0%</td>
<td>$\alpha_\pi \geq 1.3$</td>
<td>$\alpha_\pi \in [1.6, 4.2]$</td>
<td>empty set</td>
<td></td>
</tr>
<tr>
<td>2%</td>
<td>$\alpha_\pi \geq 1.9$</td>
<td>$\alpha_\pi \in [2.3, 4.2]$</td>
<td>empty set</td>
<td></td>
</tr>
<tr>
<td>3%</td>
<td>$\alpha_\pi \geq 2.4$</td>
<td>$\alpha_\pi \in [2.8, 4.2]$</td>
<td>empty set</td>
<td></td>
</tr>
</tbody>
</table>

Panel (ii): policy rule reacting to output growth and $\xi_w = 0.75$

<table>
<thead>
<tr>
<th>Trend inflation ($\pi$)</th>
<th>Fraction of input ($\psi$)</th>
<th>0</th>
<th>1/2</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>0%</td>
<td>$\alpha_\pi \geq 1.0$</td>
<td>$\alpha_\pi \in [1.0, 4.2]$</td>
<td>$\alpha_\pi \in [1.0, 1.8]$</td>
<td></td>
</tr>
<tr>
<td>2%</td>
<td>$\alpha_\pi \geq 1.1$</td>
<td>$\alpha_\pi \in [1.1, 4.2]$</td>
<td>$\alpha_\pi \in [1.0, 1.7]$</td>
<td></td>
</tr>
<tr>
<td>3%</td>
<td>$\alpha_\pi \geq 1.1$</td>
<td>$\alpha_\pi \in [1.1, 4.2]$</td>
<td>$\alpha_\pi \in [1.0, 1.7]$</td>
<td></td>
</tr>
</tbody>
</table>

Note: This table shows the values of the minimum response of interest rates to deviations of inflation from target ($\alpha_\pi$), which are consistent with determinacy for an inflation trend of 0%, 2% and 3% (annualized), and a fraction of factor payments financed by working capital of 0, 1/2 and 1 for alternative degrees of nominal rigidities.
Table 6: Determinacy under Coibion and Gorodnichenko (2011) policy rule

<table>
<thead>
<tr>
<th>Trend inflation (π)</th>
<th>Fraction of input (ψ)</th>
<th>0</th>
<th>1/2</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>0%</td>
<td>α_π ≥ 1.7</td>
<td>α_π ∈ [2.3, 18.1]</td>
<td>α_π ∈ [2.9, 7.6]</td>
<td></td>
</tr>
<tr>
<td>2%</td>
<td>α_π ≥ 3.1</td>
<td>α_π ∈ [3.6, 18.0]</td>
<td>α_π ∈ [4.2, 7.6]</td>
<td></td>
</tr>
<tr>
<td>3%</td>
<td>α_π ≥ 3.9</td>
<td>α_π ∈ [4.5, 17.9]</td>
<td>α_π ∈ [5.3, 7.6]</td>
<td></td>
</tr>
</tbody>
</table>

Note: This table shows the values of the minimum response of interest rates to deviations of inflation from target (α_π), which are consistent with determinacy for an inflation trend of 0%, 2% and 3% (annualized), and a fraction of factor payments financed by working capital of 0, 1/2 under Coibion and Gorodnichenko (2011) policy rule.