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Business Cycle**

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Abstract

We develop and test a novel prediction of the theory of collusion over the business cycle. Building on [Haltiwanger and Harrington \(1991\)](#), we present a model of collusive behaviour in the presence of persistent demand and an Antitrust Authority (AA) in a Cournot framework. The level of collusion is higher during a boom relative to a recession as collusion occurs more frequently when demand is increasing (entering into a collusive arrangement is more profitable and deviating from an existing cartel is less profitable). The model predicts that the number of discovered cartels and hence antitrust filings should be procyclical because the level of collusion is procyclical. Using a unique data set of United States Antitrust filings, we present robust evidence consistent with the model's prediction. We find that antitrust filings are procyclical even after controlling for AA's monitoring intensity. The evidence suggests that procyclical competition policies may be a cost minimizing solution to asymmetries in collusive behaviour over the business cycle.

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1. Introduction

The incentives for firms to collude in imperfectly competitive markets—coordinating their behaviour to attain profits higher than the uncoordinated level—depend on the state of the business cycle. This insight underlies the theory of collusion over the business cycle and was first studied by [Rotemberg and Saloner \(1986\)](#). Their framework showed that collusion among firms is more difficult to sustain during a boom when demand is high because of a heightened desire to cheat from the mutual understanding. [Haltiwanger and Harrington \(1991\)](#) (HH), however, showed that when demand is persistent, as observed over the business cycle, the losses from the forgone collusive profits in the immediate future rise enough to sustain a higher level of collusion during an expansion.¹ Whether collusion is more pervasive during expansions or recessions is an empirical question and has implications for competition policy.

In this paper, we develop and test a novel prediction of the theory of collusion over the business cycle.² We first present a model that introduces two modifications to HH. First, we consider Cournot competition to allow the possibility of cyclical variations in the Nash equilibrium payoffs. These cyclical variations can play a potentially important role in the gains from colluding and losses from defection over the business cycle.³ Second, we consider an Antitrust Authority (AA) that conducts monitoring and implements a randomly generated amercement (fine). This simple and tractable way of introducing competition policy allows us to model the probability that a new cartel forms and the probability that an existing cartel dissolves. We can, then, infer how the expected number of cartels varies over the business cycle and formulate our empirical hypothesis: *the frequency of discovered cartels is procyclical*.

Our model demonstrates that, in the presence of an AA, collusion occurs more frequently

¹[Kandori \(1991\)](#) shows that, under special conditions, the [Rotemberg and Saloner \(1986\)](#) finding continues to hold even when demand is serially correlated. [Bagwell and Staiger \(1997\)](#) consider a Markov demand structure in a similar framework. [Fabra \(2006\)](#) allows for the possibility of capacity constraints. [Knittel and Lepore \(2010\)](#) also consider a model with capacity constraints. Focusing on prices, they find that two types of price wars exist; mixed strategy price wars, and counter cyclical collusive pricing.

²See [Joseph E. Harrington \(2017\)](#) for an excellent review of the theory of collusion and competition policy.

³[Rotemberg and Saloner \(1986\)](#) show that Cournot competition can overturn their main result based on Bertrand competition even for i.i.d. demand, when firms face non-linear demand and/or increasing marginal costs.

in an expansion relative to a contraction. We show that there are two forces driving the level of collusion. First, entering into a collusive arrangement is more profitable in an expansion. Second, deviating from an existing cartel is less profitable in an expansion. Both of these forces work in tandem, thus, raising the overall level of collusion in an expansion. Since collusion is procyclical, it follows that the number of discovered cartels should also be procyclical, after controlling for the AA's monitoring intensity.

It is, in general, difficult to detect collusive behaviour. The previous empirical literature has examined evidence of price wars during recessions, price cycles and countercyclical ratio of prices-to-measure marginal cost to assess the presence of collusion, among other strategies (see, for example, [Aguirregabiria and Slade \(2017\)](#), for a detailed overview of the empirical literature). A strand of empirical literature employs duration models to understand how business cycle fluctuations impact cartel duration. Most closely related to our study is the work of [García Galindo et al. \(2017\)](#). Using a dataset of EU cartel formations and breakups, the authors find that both cartel formation and dissolution are procyclical. This would suggest that cartel turnover is greater when the economy is expanding. Using a hidden Markov model, research by [Hyytinen et al. \(2018\)](#) show that during the period 1951-1990, the probability of forming a cartel and continuing a cartel in the Finnish manufacturing industry is positively correlated with positive GDP shocks. Other related work includes; [Suslow \(2005\)](#) which show that cartel duration is negatively correlated with uncertainty, [Levenstein and Suslow \(2011\)](#) find that firm-specific discount rates are connected with cartel duration, and [Levenstein and Suslow \(2016\)](#) show that the duration of US cartels responds negatively to real interest rates.

Our research departs from the previous empirical literature in that we provide evidence based on the aggregate US Department of Justice AA filings reflecting discovered cartels between 1970 and 2016. To the best of our knowledge, so far this data has not been brought to bear in an empirical analysis of the cyclicity of collusion. Our main focus is to understand how the cyclical variation in AA filings relates to the business cycle. While there are many available measures of the cycle, our baseline measure is the cyclicity in aggregate US corporate profits to GDP ratio as a proxy for demand facing the imperfectly competitive sector of the economy. Once we account for the variation in filings that are a result of actions by the AA,

for example, monitoring intensity and the revised corporate leniency program, the remaining variation within AA filings is attributed to the presence of collusion in the economy.

Our empirical analysis shows evidence that there are more AA filings during an expansion. Using the negative binomial count regression model, we find that a 1 percent increase in corporate profits over GDP will lead to a 6 percent increase in AA filings. This accounts for the AA’s monitoring intensity and the US corporate leniency program. This empirical finding is robust when we consider NBER cycle dates as our measure of the business cycle. This finding has important policy implications for the AA. Specifically, procyclical monitoring policies may be a cost minimizing strategy for the AA.

The paper proceeds as follows: Section 2 presents the theoretical model, Section 3 tests our hypothesis empirically, and Section 4 concludes the paper. Proofs of the theoretical results are provided in Appendix A. In Appendix B, we present a detailed description of our data collection methodology. In Appendix C, we provide much of the derivations that appear within the body of the text. Appendix D contains additional regression tables. Appendix E contains additional tables and figures. Finally, Appendix F contains the details of our simulation analysis.

2. Theoretical Model

Our model is a variant of [Haltiwanger and Harrington \(1991\)](#) with two modifications. First, we consider Cournot competition among firms instead of Bertrand. Second, we introduce AA in the model. There are N firms producing a homogenous product that compete by simultaneously setting their level of output. To simplify the analysis, we assume that market demand is linear and takes the following form:

$$Q(P; t) = a(t) - bP, \text{ where: } a(t) > 0 \forall t, b > 0 \tag{1}$$

where $a(t)$ is a deterministic and periodic demand shifter pushing demand outward when the economy is expanding and inwards when the economy is contracting.⁴ We set the expansion–contraction cycle to a length of \bar{t} (the *period* of $a(t)$). Demand is realized in discrete time

⁴Going forward we assume that $b = 1$.

and we express the demand function as:

$$Q(P; t) = \begin{cases} a(1) - P & \text{if } t \in \{1, \bar{t} + 1, 2\bar{t} + 1, \dots\} \\ a(2) - P & \text{if } t \in \{2, \bar{t} + 2, 2\bar{t} + 2, \dots\} \\ \vdots & \\ a(\bar{t}) - P & \text{if } t \in \{\bar{t}, 2\bar{t}, 3\bar{t}, \dots\} \end{cases} \quad (2)$$

Assumption 1. *There exists a \hat{t} such that $a(\hat{t}) > a(t) \forall t \in \{1, \dots, \bar{t}\} \setminus \{\hat{t}\}$.*

Assumption 2. $a(1) < a(2) < \dots < a(\hat{t}) > a(\hat{t} + 1) > \dots > a(\bar{t}) > a(1)$. Note that $a(1) = a(i\bar{t} + 1) \forall i = 0, 1, 2, \dots$

Assumption 1 guarantees the existence of a unique maximum of $a(t)$ when $t = \hat{t}$. Assumption 2 formally characterizes the shape of the demand shifter. During each cycle, the demand shifter $a(t)$ is increasing for all $t \in (i\bar{t} + 1, (i + 1)\hat{t}) \forall i = 0, 1, 2, \dots$, and decreasing for all $t \in ((i + 1)\hat{t}, (i + 1)\bar{t} + 1) \forall i = 0, 1, 2, \dots$. Applying Assumption 2 to the definition of the demand function, we arrive at Lemma 1.

Lemma 1. $Q(P; 1) < Q(P; 2) < \dots < Q(P; \hat{t}) > Q(P; \hat{t} + 1) > \dots > Q(P; \bar{t}) > Q(P; 1)$

Proof. See Appendix.

Lemma 1 characterizes the business cycle. At any point in time when demand is increasing, we refer to as an expansion and when demand is decreasing, we refer to as a contraction. With the demand structure defined, we proceed to show that collusion occurs more frequently in an expansion rather than a contraction, all else being equal.

2.1. Cournot Nash Equilibrium

We assume all firms are symmetric and have the same linear cost function $C(q_i) = cq_i$. The marginal cost c is set such that $a(t) \geq c \forall t$. Firms maximize profits by independently choosing their level of output q_i . The total level of output is $Q = \sum_{i=1}^N q_i$. The profit maximization problem for each firm is characterized as follows:

$$\pi_i^N(t) = \max_{q_i} P(Q; t)q_i - C(q_i) = \frac{(a(t) - c)^2}{(N + 1)^2} \quad (3)$$

Equation (3) is the standard Cournot equilibrium with N firms. Since $a(t) \geq c \forall t$, the equilibrium level of profits follow the same cyclical pattern as $a(t)$. Thus, when the economy is expanding, competitive profits are increasing. In contrast, competitive profits are decreasing when the economy is contracting. Since firms make decisions based on all future profit streams, let $V_i^N(t)$ represent the present value of all future profit streams when firms commit to competition in period t . If firms discount future periods by factor δ , the value from competing in all future periods is expressed as follows:

$$V_i^N(t) = \sum_{j=0}^{\infty} \delta^j \pi_i^N(t+j) = \pi_i^N(t) + \frac{1}{1-\delta} \left[\delta \pi_i^N(t+1) + \dots + \delta^{\bar{t}} \pi_i^N(t) \right] \quad (4)$$

From (4), we observe that value depends on profits in period t and the discounted sum of all future cyclical profit streams.

2.2. Collusive Equilibrium

In addition to the competitive equilibrium, firms may decide to coordinate output and form a cartel. To coordinate collusion in equilibrium, all firms within the market must agree to join the cartel. In our model, we assume that there is an AA that investigates and prosecutes cartels. We assume that within any period, the AA can open an investigation into a specific market with probability α . Once an investigation has been opened, if the AA collects incriminating evidence, they can proceed to trial. The strength of the evidence is exogenous with a probability of successful conviction equal to ρ . The level of α and ρ are fixed over the business cycle. If a successful conviction occurs, each firm in the cartel must pay a fine F . This fine is exogenously determined at the beginning of the period and depends on various factors, for example; legal precedent and political influence. From the perspective of the firm, the fine appears random and we assume it follows an unspecified probability distribution over the support $[0, \bar{F}]$.

The cartel acts as a single firm and maximizes total profits by choice of market output Q . Total profits are divided among the N firms equally. Each firm's profit function, $\pi_i^M(t)$, (where M denotes the collusive monopoly case) is as follows:

$$\pi_i^M(t) = \frac{1}{N} \left(\max_Q P(Q; t)Q - C(Q) \right) = \frac{(a(t) - c)^2}{4N} \quad (5)$$

Equation (5) is the standard monopoly result with linear demand. Since firms have formed a cartel, there is a probability that they will be detected and successfully convicted by the AA. The expected fine faced by each firm in the cartel in period t is $\alpha\rho F$. As such, we can characterize the present value of collusion as follows:

$$\begin{aligned} V_i^M(t) &= \pi_i^M(t) + \alpha\rho(\delta V^M(t+1) - F) + (1 - \alpha\rho)\delta V_i^M(t+1) \\ &= \pi_i^M(t) + \frac{1}{1 - \delta^{\bar{t}}} \left[\delta\pi_i^M(t+1) + \dots + \delta^{\bar{t}}\pi_i^M(t) \right] - \frac{\alpha\rho F}{1 - \delta} \end{aligned} \quad (6)$$

To keep the model tractable, we assume that the AA does not monitor cartels post prosecution. The implication of this assumption is that once detected, firms will continue to collude in the future. The cartel views the fine as a cost of doing business. Thus, the present value expected losses from joining the cartel is the same irrespective of t . This is reflected in Equation (6).

Once a cartel has been formed, firms can deviate from the production plan. By doing so, the defecting firm will achieve a competitive advantage leading to a higher level of profits. However, once defection has occurred, firms will revert to producing at the Nash level of quantity in all future periods as a punishment strategy. If a firm decides to defect from a collusive arrangement, it solves the following maximization problem to realize profits $\pi_i^D(t)$ (where the D denotes defection):

$$\pi_i^D(t) = \max_{q_i} P(q^i + (N-1)q^M; t)q_i - C(q_i) = \frac{(a(t) - c)^2(N+1)^2}{16N^2} \quad (7)$$

The present value of defecting from the collusive arrangement in period t is characterized as follows:

$$\begin{aligned} V^D(t) &= \pi_i^D(t) - \alpha\rho F + \delta \sum_{i=0}^{\infty} \delta^i \pi_i^N(t+1+i) \\ &= \pi_i^D(t) - \alpha\rho F + \frac{1}{1 - \delta^{\bar{t}}} \left[\delta\pi_i^N(t+1) + \dots + \delta^{\bar{t}}\pi_i^N(t) \right] \end{aligned} \quad (8)$$

We can combine the results of Equations (3), (5), and (7) into the following Lemmas:

Lemma 2. $\pi_i^D(t) > \pi_i^M(t) > \pi_i^N(t)$, $\forall t \in \{1, \dots, \bar{t}\}$

Proof. See Appendix.

Lemma 3. $\pi_i^j(1) < \pi_i^j(2) < \dots < \pi_i^j(\hat{t}) > \pi_i^j(\hat{t} + 1) > \dots > \pi_i^j(\bar{t}) > \pi_i^j(1) \forall j \in \{M, N, D\}$

Proof. See Appendix.

We have shown that profits are cyclical, profits from defection are greater than monopoly profits, and collusive profits are greater than competitive profits. We now characterize how profits from defection, collusion, and competition change relative to one another over the business cycle.

2.3. Collusion Over the Business Cycle

We start by defining the function $\Delta_i(t)$ as the one-shot gain from collusion in period t and $G_i(t)$ to be the one-shot gain from defection in period t :

$$\Delta_i(t) = \pi_i^M(t) - \pi_i^N(t) = \frac{(a(t) - c)^2(N - 1)^2}{4N(N + 1)^2} > 0, \forall N > 1 \quad (9)$$

$$G_i(t) = \pi_i^D(t) - \pi_i^M(t) = \frac{(a(t) - c)^2(N - 1)^2}{16N^2} > 0, \forall N > 1 \quad (10)$$

The one-shot gain from defection and the one-shot gain from collusion track the business cycle. Formally:

Lemma 4. $\Delta_i(1) < \Delta_i(2) < \dots < \Delta_i(\hat{t}) > \Delta_i(\hat{t} + 1) > \dots > \Delta_i(\bar{t}) > \Delta_i(1)$ and $G_i(1) < G_i(2) < \dots < G_i(\hat{t}) > G_i(\hat{t} + 1) > \dots > G_i(\bar{t}) > G_i(1)$.

Proof. See Appendix. □

2.3.1. Markets with No Collusion

Suppose there are \mathcal{M} identical markets where collusion is not present.⁵ Let t' and t'' be such that $t' < \hat{t} < t''$ and $Q(P; t') = Q(P; t'')$. Since t' is an expansionary period, and t'' is a contractionary period, the value gained from colluding is greater in an expansion relative to a contraction.

Theorem 1. $V_i^M(t') - V_i^N(t') > V_i^M(t'') - V_i^N(t'') \forall t' < \hat{t} < t''$ such that $Q(P; t') = Q(P; t'')$.

⁵Identical markets are markets that share the same demand function and have the same marginal cost.

Proof. See Appendix.

Theorem 1 says that when the economy is expanding, all else equal, the gain from forming a cartel is greater in an expansion when compared to a contraction. It is, however, possible that the fine is set in a way such that $V_i^M(t') - V_i^N(t') < 0$ at some t' . In this case, although the value from colluding in an expansion is still greater, firms will not collude because doing so leads to a loss. The implication of Theorem 1 is that when $V_i^M(t') - V_i^N(t') > 0$ we expect more markets to form cartels in an expansion. Although the single period gain from collusion is the same in both the expansion and the contraction, firms that begin colluding when the economy is doing well take advantage of growth in the gains from collusion. There are increasing gains from collusion in an expansion and decreasing gains from collusion in a contraction.

Since monitoring is not available to the AA, the present value of the expected fine from colluding is the same in both an expansion and a contraction. If monitoring was available, the firm would also face foregone collusive profits from being detected. In other words, the expected cost of being detected in an expansion exceeds the expected cost of being detected in a contraction. This result complicates the analysis and severely limits tractability.

We now frame our analysis in terms of the critical fine, \hat{F} , which can prompt the formation of a cartel. For any $F < \hat{F}$ cartel formation is profitable. To determine \hat{F} , we solve the following equation:

$$V_i^M(t) - V_i^N(t) = 0 \tag{11}$$

to get:

$$\hat{F}(\alpha, \rho, \delta, t) = \frac{1 - \delta}{\alpha\rho(1 - \delta^t)} [\Delta_i(t) + \dots + \delta^{t-1} \Delta_i(t - 1)] \tag{12}$$

Lemma 5. $\hat{F}(\alpha, \rho, \delta, t') > \hat{F}(\alpha, \rho, \delta, t'') \forall t' < \hat{t} < t''$ such that $Q(P; t') = Q(P; t'')$

Proof. See Appendix.

Lemma 5 says that the critical value of the fine is greater in an expansion relative to a contraction. Since F is random, the probability that a cartel forms in an expansion is

greater than in a contraction. We assume that the fine imposed by the AA is described by a cumulative density function (CDF) $H(\cdot)$ over the support $[0, \bar{F}]$, where \bar{F} is the maximum fine allowed by law. The probability that a market forms a cartel in period t is $H(\hat{F}(\alpha, \rho, \delta, t)) = \text{P}(F \leq \hat{F}(\alpha, \rho, \delta, t))$.

Theorem 2. $H(\hat{F}(\alpha, \rho, \delta, t')) > H(\hat{F}(\alpha, \rho, \delta, t'')) \forall t' < \hat{t} < t''$ such that $Q(P; t') = Q(P; t'')$.

Proof. See Appendix.

Theorem (2) says that the probability of a market forming a cartel is greater in an expansion relative to a contraction. If we assume there are \mathcal{M} identical markets without collusion, the expected number of markets that begin colluding at time t is:

$$\mathcal{M} \cdot H(\hat{F}(\alpha, \rho, \delta, t)) \quad (13)$$

Lemma 6. $\mathcal{M} \cdot H(\hat{F}(\alpha, \rho, \delta, t')) > \mathcal{M} \cdot H(\hat{F}(\alpha, \rho, \delta, t''))$.

Lemma 6 says that the expected number of markets that begin colluding is greater in an expansion since the value gained from forming a cartel is greater in an expansion relative to a contraction. We would, therefore, expect more markets to begin colluding when the economy is expanding.

2.3.2. Industries with Collusion

We define the present value of all future losses from defecting in period t as:

$$L(t, \delta) = \sum_{s=0}^{\infty} \delta^{s\bar{t}} [\delta \Delta_i(t+1) + \dots + \delta^{\bar{t}} \Delta_i(t)] = \frac{1}{1 - \delta^{\bar{t}}} [\delta \Delta_i(t+1) + \dots + \delta^{\bar{t}} \Delta_i(t)] \quad (14)$$

We can therefore represent the present value gained from defecting in period t as $V_i^D(t) - V_i^M(t) = G(t) - \alpha\rho F - L(t, \delta)$. We now show that the gain from defecting is lower in an expansion versus a contraction.

Theorem 3. $V_i^D(t'') - V_i^M(t'') > V_i^D(t') - V_i^M(t') \forall t' < \hat{t} < t''$ such that $Q(P; t') = Q(P; t'')$.

See Theorem 4 and 7 in [Haltiwanger and Harrington \(1991\)](#).

Proof. See Appendix.

Theorem 3 states that, all else being equal, the gain from deviation in an expansion is smaller relative to a contraction because firms forgo high collusive profits in the immediate future. This makes deviations less profitable for firms. Since fewer firms deviate in an expansion there is a higher degree of collusion. Another way to frame this analysis is to determine the critical fine, \tilde{F} , such that any fine above this critical value will prompt firms to defect from the cartel. We solve the following equation for \tilde{F} :

$$V_i^D(t) - V_i^M(t) = 0 \quad (15)$$

to get:

$$\tilde{F}(\alpha, \rho, \delta, t) = \frac{1 - \delta}{\alpha \rho \delta} [G(t) - L(t)] \quad (16)$$

Lemma 7. $\tilde{F}(\alpha, \rho, \delta, t') > \tilde{F}(\alpha, \rho, \delta, t'') \forall t' < \hat{t} < t''$ such that $Q(P; t') = Q(P; t'')$.

Proof. See Appendix.

Lemma 7 says that the maximum fine required to sustain a cartel is greater in an expansion. The probability that $F > \tilde{F}$ is larger in a contraction. We would, therefore, expect to see more defection in a contraction. The probability that a collusive market will continue to collude at period t is $H(\tilde{F}(\alpha, \rho, \delta, t)) = P(F \leq \tilde{F}(\alpha, \rho, \delta, t))$. The probability that a firm defects within a given market is $1 - H(\tilde{F}(\alpha, \rho, \delta, t))$.

Theorem 4. $1 - H(\tilde{F}(\alpha, \rho, \delta, t')) < 1 - H(\tilde{F}(\alpha, \rho, \delta, t'')) \forall t' < \hat{t} < t''$ such that $Q(P; t') = Q(P; t'')$

Proof. See Appendix.

From Theorem 4 we conclude that the probability of defection for a firm is lower in an expansion relative to a contraction. If we assume that an economy has K identical collusive markets, the expected number of collusive markets that collapse is:

$$K \cdot [1 - H(F(\alpha, \rho, \delta, t))] \quad (17)$$

As a result of Theorem 4, we have the lemma:

Lemma 8. $K \cdot [1 - H(F(\alpha, \rho, \delta, t'))] < K \cdot [1 - H(F(\alpha, \rho, \delta, t''))]$.

Lemma 4 says that the expected number of collusive markets that collapse in an expansion is less relative to a contraction.

Theorem (2) and (4) imply that when the economy is expanding, we should expect more cartels to form and fewer to defect, all else being equal. The level of collusive behavior is procyclical. Suppose that at a specific point in time there are K collusive markets and M competitive markets. We assume that M and K are fixed. We consider what happens to the stock of collusive industries given periods t' and t'' such that $Q(P; t') = Q(P; t'')$. We assume that t' represents a period in an expansion while t'' represents a period in a contraction. We let $S(t)$ represent the expected stock of cartels in period t . The expected stock of cartels is:

$$S(t) = K + M \cdot H(\hat{F}(\alpha, \rho, \delta, t)) - K \cdot [1 - H(\tilde{F}(\alpha, \rho, \delta, t))] \quad (18)$$

From Lemma (6) and (8) we see that:

$$\begin{aligned} M \cdot H(\hat{F}(\alpha, \rho, \delta, t')) &> M \cdot H(\hat{F}(\alpha, \rho, \delta, t'')) \\ -K \cdot [1 - H(\tilde{F}(\alpha, \rho, \delta, t'))] &> -K \cdot [1 - H(\tilde{F}(\alpha, \rho, \delta, t''))] \end{aligned} \quad (19)$$

If we add both sides of the inequality in (19) and add K to both sides we get:

$$\begin{aligned} K + M \cdot H(\hat{F}(\alpha, \rho, \delta, t')) - K \cdot [1 - H(\tilde{F}(\alpha, \rho, \delta, t'))] &> \\ K + M \cdot H(\hat{F}(\alpha, \rho, \delta, t'')) - K \cdot [1 - H(\tilde{F}(\alpha, \rho, \delta, t''))] & \end{aligned} \quad (20)$$

Using the definition of the expected stock of cartels:

$$S(t') > S(t'') \quad (21)$$

Thus, the expected stock of cartels is greater in the expansion relative to the contraction given a contraction and an expansion with identical demand. Since the AA will open an investigation with probability α , the expected number of investigations is:

$$\alpha S(t') > \alpha S(t'') \quad (22)$$

From Equation (22) we conclude that the expected number of antitrust filings will be greater in an expansion versus a contraction. The difference in filings between the two periods is driven by the overall stock of collusive behavior in each period. This result provides the basis for testing how corporate profits impacts the number of AA filings over the expansion and contraction phases of the business cycle.

2.4. Model Simulation

To help illustrate our findings, we assume a specific functional form of the demand shifter. We consider a parameterized version of the simple sine function to capture the cyclical nature of the business cycle as follows.

$$a(t) = d \left(\frac{v}{d} + \sin \left(\frac{(2\pi)t}{\bar{t}} - \frac{\pi(\bar{t} + 4)}{2\bar{t}} \right) \right) \quad (23)$$

The shape of the function is defined by three parameters; d , v , and \bar{t} . They determine the the peak level of demand, length of the cycle, and the trough level of demand, respectively. The maximum value of the function is equal to $v + d$, while the minimum value is $v - d$. We assume that $v \geq d$. The length of the cycle is defined by \bar{t} . Note that the second term in the sine function, $\frac{\pi(\bar{t}+4)}{2\bar{t}}$, is an offset so the cycle starts at the minimum level of demand. We can simplify Equation (23) as follows;

$$a(t) = v - d \cos \left(\frac{2\pi(t - 1)}{\bar{t}} \right) \quad (24)$$

Since the magnitude of $a(t)$ is unimportant, we assume that $v = d = 1$ to get:

$$a(t) = 1 - \cos \left(\frac{2\pi(t - 1)}{\bar{t}} \right) \quad (25)$$

We know that $a(t) \geq c$, to simplify our analysis we let $c = 0$. Thus, the demand function becomes:

$$Q(P; t) = \left(1 - \cos \left(\frac{2\pi(t - 1)}{\bar{t}} \right) \right) - P \quad (26)$$

We calibrate the parameters based on aggregate US data. We assume that each period t represents a quarter. Based on the NBER data, the average length of the business cycle, trough to trough, is 19 quarters, thus we set $\bar{t} = 19$. To impute the discount factor, we use a real interest rate of $r = 0.023$, corresponding to the 2016 value for the US. The implied

discount factor is $\delta = 0.98$. We also assume that the number of firms in a market is set exogenously, we arbitrarily use $N = 8$. Using 2017 antitrust data, we calculate the percent of 4-digit NAICS codes under investigation. This number is approximately 2% and we set $\alpha = 0.02$. Finally, according to [Warin et al. \(2006\)](#), between 1996 and 2005, the antitrust conviction rates hovered around 50%. Thus, we set $\rho = 0.50$.

Table 1: Parameter Assumptions

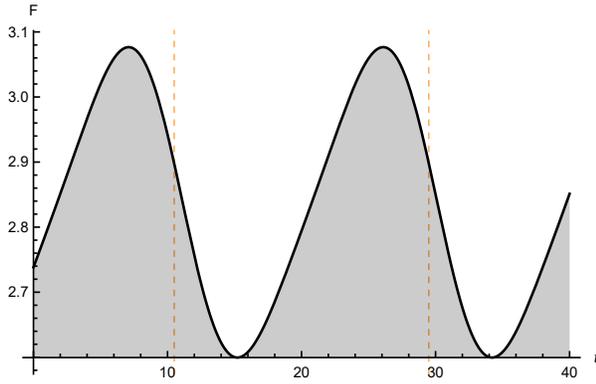
Parameter	Interpretation	Value	Source
\bar{t}	Business Cycle Length (Quarters)	19	NBER Cycles
δ	Discount Factor (Real Interest Rate)	0.98	World Bank
α	Rate of Investigations	0.02	US DOJ Antitrust
ρ	Probability of Prosecution	0.50	American Bar Association
N	Market Size	8	Arbitrary

Figure 1a shows the values of F such that $V_i^M(t) \geq V_i^N(t)$. The solid black line represents \hat{F} . The area below the curve represents the values of \hat{F} such that firms will form a cartel. Figure 1b shows the values of \hat{F} for both a contraction and an expansion such that demand is equal. The upper axis shows the contractionary periods that map to the lower axis which are expansionary periods. The mapping is based on periods with equal demand. The probability that F is below \hat{F} is larger in an expansion compared to a contraction. This shows that it is easier to form a cartel in an expansion. Figure 1c and Figure 1d are for existing cartels where the solid black line represents \tilde{F} . From Figure 1d, we see that the probability that F is below \tilde{F} is larger in an expansion compared to a contraction. In other words, the probability that a cartel collapses is larger in a contraction versus an expansion. We conclude that in an expansion more firms join cartels and fewer cartels collapse.

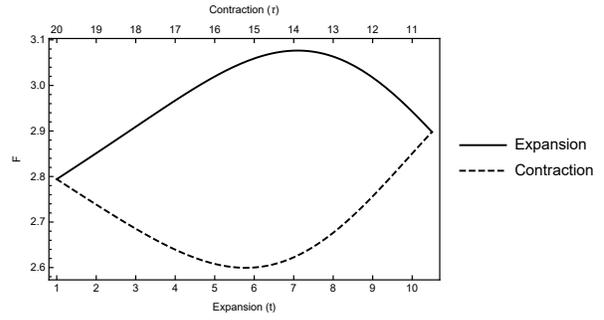
2.5. The Optimality of Procyclical Antitrust Policy

According to the model, there is a greater level of collusive behavior when the economy is expanding versus when the economy is contracting. What implication does this have for the AA? We assume that the goal of the AA is to maximize the overall expected number of convictions. Consider a simple two-period model and assume that the AA has a finite budget

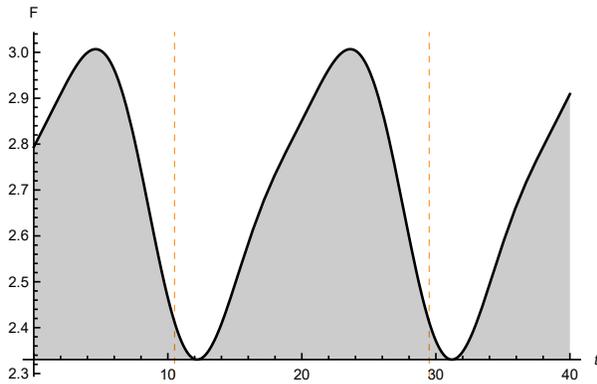
Figure 1: Various Levels of F over the Cycle that Trigger Collusion and Defection



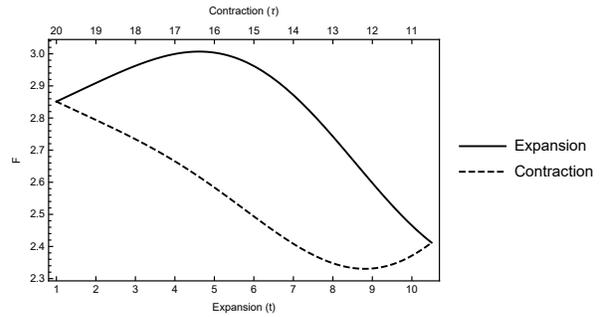
(a) F such that $V_i^M(t) = V_i^N(t)$



(b) F to Trigger Collusion



(c) F such that $V_i^M(t) = V_i^D(t)$



(d) F to Trigger Defection

to be allocated across two periods. The size of the budget in each period influences the AA's capacity to take on investigations (i.e., the probability of opening an investigation). Let W_t represent the budget in period t and \bar{W} represent the total budget to be allocated over two periods. The probability of an investigation takes the following functional form capturing the dependency on the budget size:

$$0 \leq \alpha(W_t) \leq 1 \text{ and } \alpha(0) = 0, \alpha'(W_t) > 0, \alpha''(W_t) < 0 \quad (27)$$

The property $\alpha'(\cdot) > 0$ captures the notion that an increase in the budget increases the capacity to take on new investigations and hence the number of investigations increases. The property $\alpha''(\cdot) < 0$ captures the notion that each additional dollar allocated towards an investigation has diminishing returns. As officers divide their time across an increasing number of investigations, their marginal productivity decreases. We make the following assumption on the form of (27);

$$\alpha(W_t) = 1 - \exp(-aW_t) \quad (28)$$

This convenient function satisfies all the properties. We include the parameter $a > 0$ which measures the rate at which the function approaches the horizontal asymptote of 1. The budget constraint of the AA is:

$$W_1 + W_2 \leq \bar{W} \quad (29)$$

We let M_t represent the number of cartels in period t . We assume that $M_1 > M_2$, which according to our model means that period 1 is an expansion and period 2 is a contraction. As the parameter a increases, the marginal benefit of allocating additional dollars to investigations increases. The objective of the AA is to maximize the expected number of prosecuted cartels as follows:

$$\max_{W_1, W_2} \alpha(W_1) \rho M_1 + \alpha(W_2) \rho M_2 \text{ s.t } W_1 + W_2 \leq \bar{W} \quad (30)$$

The constraint is binding since $\alpha'(\cdot) > 0$. The first order necessary conditions (FONCs) are:

$$\frac{d\alpha(W_1^*)}{dW_1} M_1 + \frac{d\alpha(W_2(W_1^*))}{dW_1} \frac{dW_2}{dW_1} M_2 = 0 \quad (31)$$

Using the functional form for $\alpha(\cdot)$ we get:

$$\exp(-aW_1^*)M_1 - \exp(-a(\bar{W} - W_1^*))M_2 = 0 \quad (32)$$

Solving Equation (32);

$$W_i^* = \frac{\bar{W}}{2} - \frac{1}{2a} \log\left(\frac{M_j}{M_i}\right) \quad (33)$$

Taking into consideration the boundary conditions, if $M_1 > M_2$:

$$W_1^* = \min\left(\bar{W}, \frac{\bar{W}}{2} + \frac{1}{2a} \log\left(\frac{M_1}{M_2}\right)\right), \quad W_2^* = \bar{W} - W_1^* \quad (34)$$

From Equation (34) we see that if the level of collusion is the same in both periods, the AA will split the budget equally across the two periods. However, based on the level of collusion in each period, the AA will adjust their expenditures toward the period with more cartels. As we can see, this adjustment is dampened by the parameter $a > 0$. In terms of the theoretical model, the AA should impose a procyclical monitoring policy that would allocate more resources to an expansion and less resources to a contraction.

3. Empirical Analysis

In this section, we test a key prediction of the model that collusion is more pervasive during expansions relative to contractions. Specifically, based on the model, our hypothesis is as follows:

Hypothesis: *AA filings should be positively related with the business cycle after controlling for AA's monitoring intensity.*

We divide our analysis into a brief discussion on the collection and compilation of the dataset in Section 3.1, a discussion of our empirical methodology in Section 3.2, and a presentation of our findings in Section 3.3.

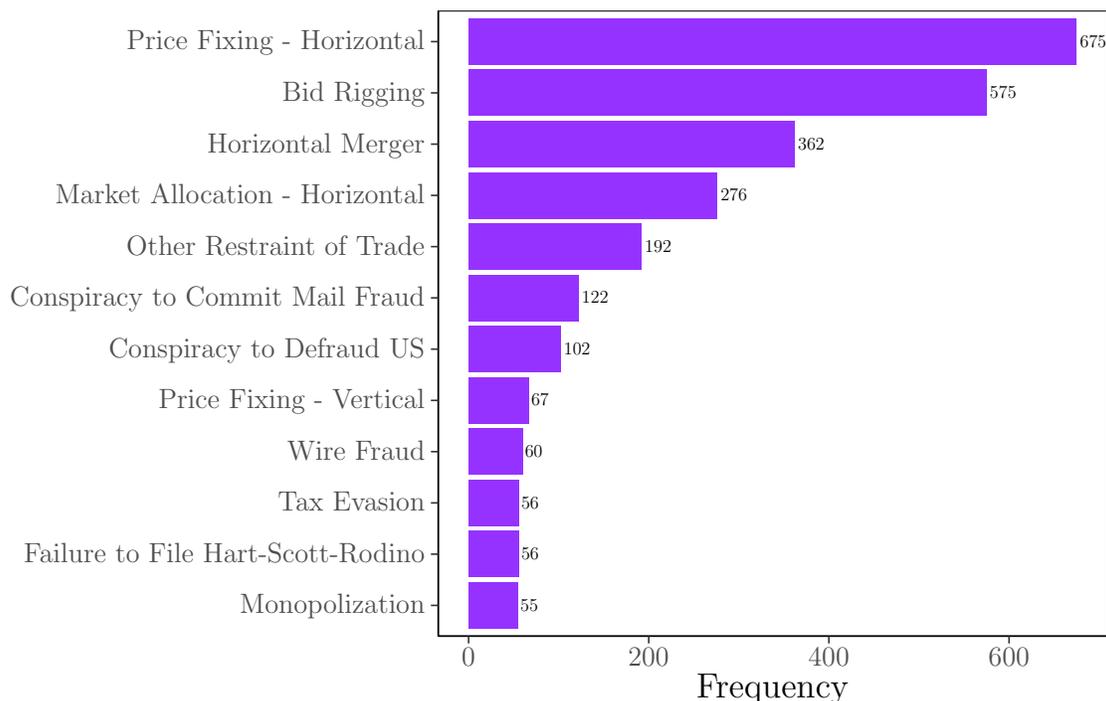
3.1. Data Compilation and Description

We compile the data from four major sources, the largest of which spans the years 1903 to 2017. The four major components of our data are: the frequency of cases brought fourth by the US Department of Justice (DOJ) Antitrust Division, corporate profits from the Federal

Economic Reserve Data (FRED) database, the Annual Appropriations of the DOJ Antitrust Division, and the NBER Business Cycle Dates.

We compile the data on case frequency from publicly available information on the DOJ's online repository of antitrust cases dating back to 1941.⁶ The main page of the repository contains a hyperlink for each case that has been filed. We developed a web scraper that accesses each link and collects the following data: case open date, case name, case type, case violation, market, industry code, and component. In total, 2079 cases were extracted.⁷ We aggregate the filing frequencies every quarter. In Figure 2 we graph the top 12 antitrust violations between 1941 and 2018.⁸ Note that these violations are not mutually exclusive as filings may have multiple violations associated with them. A good example is U.S. V. James P. Heffernan (Steel Drums). The defendant was charged with horizontal price fixing and mail fraud. The mail fraud charge comes as an indirect consequence of price fixing.

Figure 2: Frequency of Antitrust Filings by Type 1941-2018



⁶See <https://www.justice.gov/atr/antitrust-case-filings-alpha>

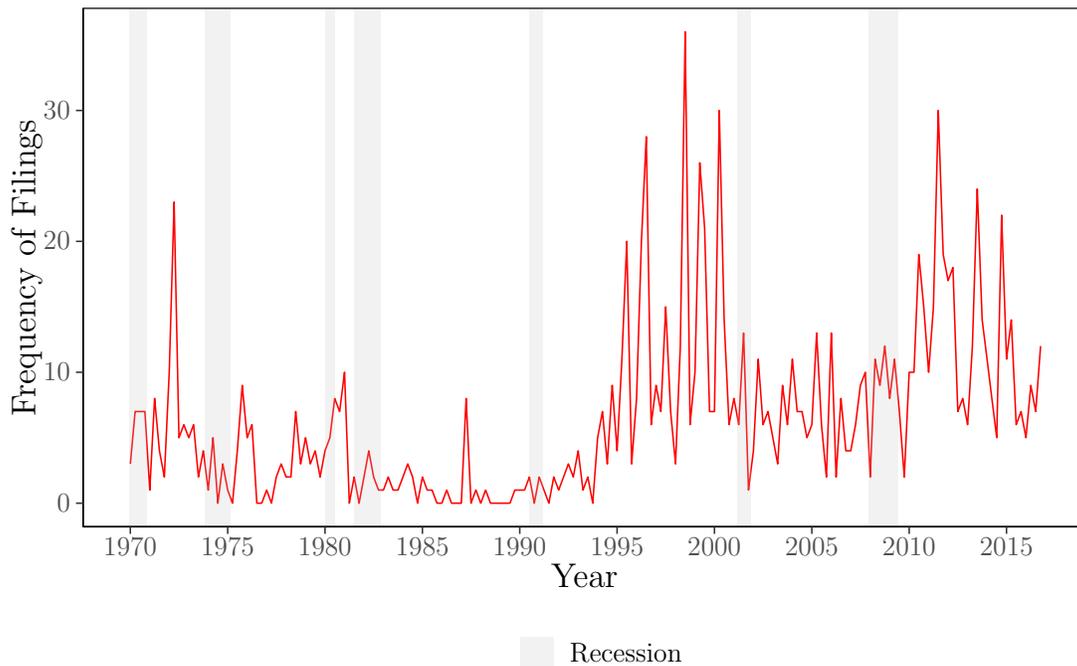
⁷As of August 4th, 2018

⁸We have excluded cases that had an empty space as a violation. This could be an unintended effect of the scrapping process. These accounted for 159 cases.

We observe that horizontal price fixing, bid rigging, horizontal market allocation, and other restraints of trade constitute a large share of the antitrust workload. This is followed by merger related filings (horizontal mergers and Hart-Scott-Rodino filings). In terms of other violations, we observe that conspiracy to commit mail fraud, conspiracy to defraud US, tax evasion, and wire fraud typical co-occur with other legal challenges like price fixing and bid rigging.

In the pre-1970 sample, there are only 20 cases over a 29 year period, on average, less than one case per quarter. This may signal one of two things; first, the competition landscape may have been fundamentally different. For example, enforcement of price fixing may not have been a priority. Second, there may be cases missing from the DOJ website. We, therefore, truncate our sample at 1970. Figure 3 shows the frequency of collusion related filings with the associated frequencies in Table 7 (Appendix E).

Figure 3: Frequency of Antitrust Filings (Collusion Related) 1970-2016 (Annual)



There are two important observations to note. First, the average level of filings jumps in 1993. This corresponds to the exact year that the US DOJ implemented its revised leniency

program.⁹ As expected, there is an increase in the average number of filings post revised leniency program. Second, in most cases, there appears to be a decline in filings either during or after a recession. This may indicate that the level of collusion during a recession is relatively low, a pattern consistent with the prediction of the model. In addition to the filing frequency over time, it is helpful to look at the distribution of filing frequency. Figures 4 and 5 show the kernel density plots for all filings and collusion related filings.

Figure 4: Kernel Density - All

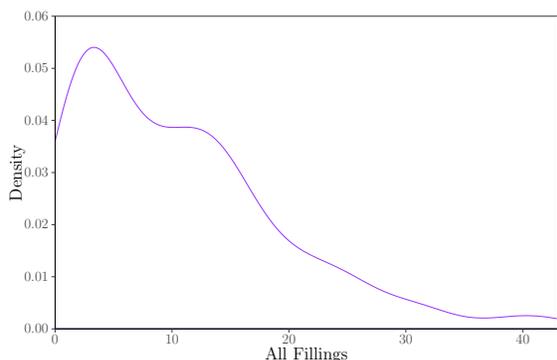
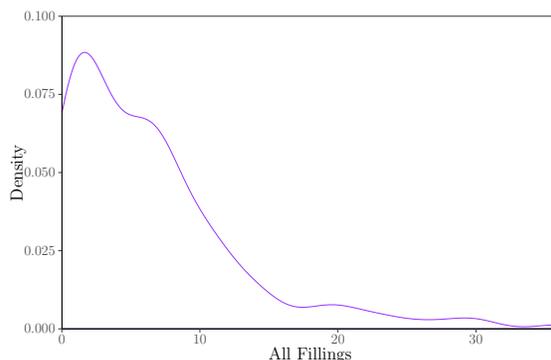


Figure 5: Kernel Density - Collusion



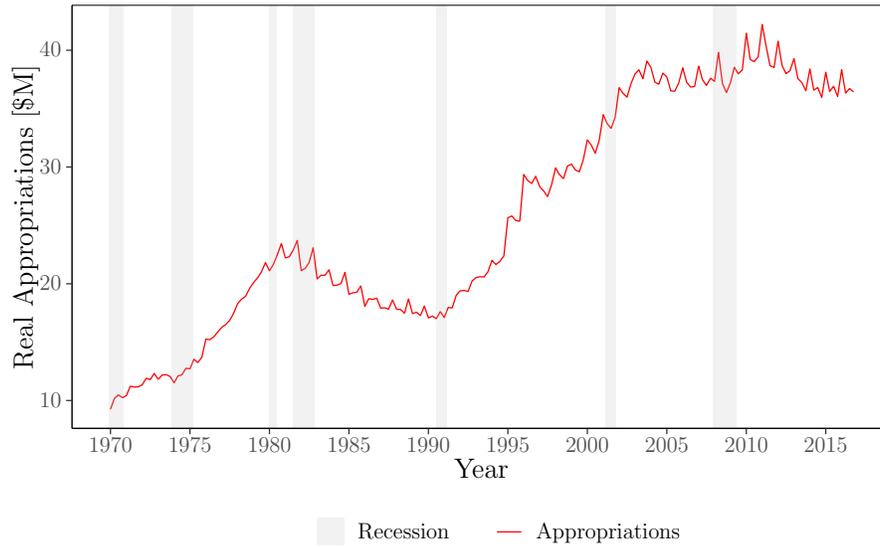
We observe that filing frequency is skewed to the right. Most of the time, the number of filings per quarter is localized around zero. Given that our dependent variable is a count and that the data is right skewed, the application of a count regression model (Poisson or negative binomial) will allow us to deal with the non-normality of the OLS errors and force predicted values to be non-negative.

We collect the Appropriations figures for the DOJ Antitrust Division from the “*Antitrust Division Appropriation Figures Since Fiscal Year 1903*” data set on the Data.gov repository.¹⁰ Appropriations have been recorded on an annual basis from 1903 to 2016. To account for inflationary pressures on appropriations, we adjust using the GDP deflator. Since the appropriations data is recorded annually, we disaggregate by weighting the annual total across each quarter using total government expenditures from FRED. For each year from 1960 to 2016 we calculate the % of total expenditures that were incurred each quarter and apply them to the annual appropriations. Figure 6 shows the annual appropriations data.

⁹See <https://www.justice.gov/atr/file/810281/download>

¹⁰See <https://www.justice.gov/atr/appropriation-figures-antitrust-division>

Figure 6: Quarterly Real Budget Appropriations 1970-2016 (\$M - 2010)



We use the after tax corporate profits-to-GDP ratio (excluding inventory valuation adjustment and capital consumption adjustment), as a proxy for the business cycle. Since the theory suggests that firm profits drive the decision to collude, looking at aggregate corporate profits provides us with a measure of the business cycle that links directly to our model. As a robustness check, we also estimate a model using the NBER cycle dates. Figure 7 shows corporate profits along with the NBER cycle dates in gray.

Figure 7: Corporate Profits 1970-2016

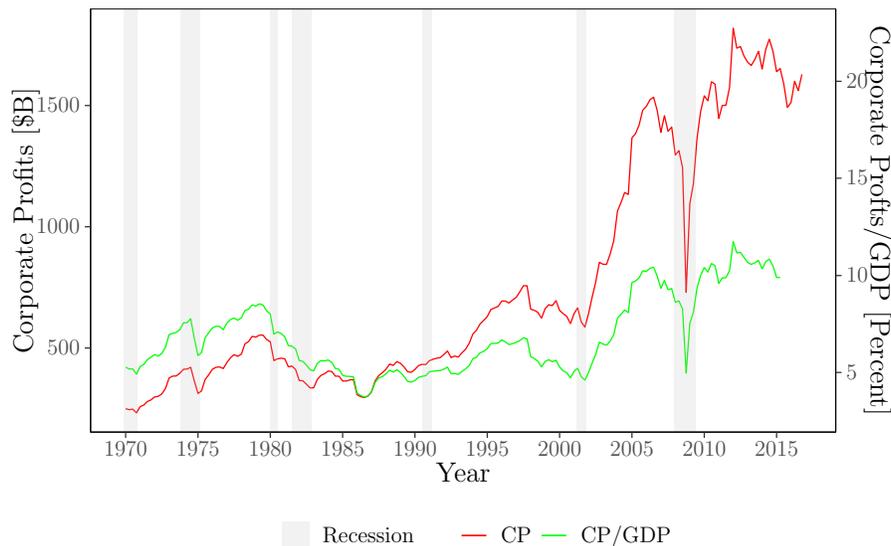


Table 2 presents the descriptive statistics of the data. Our findings show that there has been on average 11 antitrust cases per quarter since 1970. Once we include only price fixing and collusion filings, we find there were, on average, 6 filings per quarter. The average quarterly real budget for the antitrust division is approximately \$26M. Looking at real corporate profits, the average level across the sample is 761B, or 7% of GDP.

Table 2: Descriptive Statistics (1970-2016)

	N	Mean	Median	Sd
Filing Frequency	188	10.64	9.00	9.00
Filing Frequency (Collusion)	188	6.44	5.00	6.53
Filing Frequency (Mergers)	188	1.91	1.50	1.83
Real Budget Appropriations (M - 2010)	188	25.84	22.35	9.70
Real Corporate Profits (B - 2010)	188	761.46	550.53	478.74
Corporate Profits/GDP	182	6.86	6.41	2.01

Since the descriptive statistics can provide insights into the relationships between filing frequency and various covariates, we also look at a variety of conditional statistics. Table 8 (Appendix E) presents the conditional descriptive statistics based on the recession indicator. Most notably, on average, there is a higher level of total filings and collusion based filings when the economy is in an expansion. Additionally, we see that mergers between the two groups are about the same on average. We also observe that real corporate profits are positively correlated with the business cycle.

We can gain some insight into how the introduction of the revised leniency program has impacted filings by looking at how the average level of filings differs before and after its introduction. Table 9 (Appendix E) provides these conditional statistics. There is a sharp difference in the average level of filing before and after 1993. The total number of filings and collusion based filing increased by a factor of 4 or greater. Mergers have not increased as dramatically, which suggests that the increase in collusion related filing can be attributed to the revised leniency program and not another institutional factor.

Finally, we look at the average level of filings over various levels of corporate profit rates.

Table 3 provides these conditional statistics. We find that the overall level of filings and collusion related filings are increasing in profitability relative to the state of the economy. This evidence corroborates the hypothesis that there are more filings when the economy is expanding.

Table 3: Conditional Descriptive Statistics (1970-2016)

	N	Mean	Median	Sd
Low Corporate Profits [3%,6%)				
Filing Frequency	83	7.78	4.00	8.61
Filing Frequency (Collusion)	83	4.73	2.00	6.82
Filing Frequency (Mergers)	83	1.83	1.00	1.78
Mid Corporate Profits [6%,9%)				
Filing Frequency	63	9.73	8.00	8.14
Filing Frequency (Collusion)	63	5.94	5.00	5.13
Filing Frequency (Mergers)	63	1.68	1.00	1.75
High Corporate Profits (9%,12%]				
Filing Frequency	36	18.00	16.50	7.75
Filing Frequency (Collusion)	36	11.03	10.00	6.47
Filing Frequency (Mergers)	36	2.08	2.00	1.98

* During 1970-2015, the lowest CP/GDP was 3%, while the highest was 12%.

In the next section, we provide a brief introduction to the econometric methodology and a detailed analysis of our results.

3.2. Methodology

The frequency of AA filings is a count variable. Because of this standard linear regression models may not provide the best econometric fit. This is especially true if the data is right-skewed. Looking at Figure 5, the frequency of AA filings is skewed to the right when we consider both the collusion subset and the all filings case. We decide to estimate our model using the Negative Binomial approach as per [Cameron and Trivedi \(2010\)](#). There are two

main reasons for favoring this methodology. First, as we show in the results section¹¹, our data exhibits over-dispersion which cannot be controlled for by the standard Poisson model. Second, we don't believe that zero counts are generated by a distinct process, thus ruling out zero-inflated models. We briefly cover the estimation procedure below.

Suppose the probability that a specific count occurs follows a Poisson distribution:

$$y_t \sim \text{Poisson}(v_t \mu_t) \quad (35)$$

We describe μ_t as a function of regressors while v_t is unobserved heterogeneity. Based on the distribution of y_t :

$$\mathbb{E}[y_t | \mathbf{X}_t, v_t] = v_t \exp(\mathbf{X}_t' \boldsymbol{\beta}) \quad (36)$$

y_t has the following conditional PDF:

$$f(y_t | \mathbf{X}_t, v_t) = \frac{\exp(-v_t \mu_t) (v_t \mu_t)^{y_t}}{y_t!} \quad (37)$$

Next suppose that $v_t > 0$, i.i.d, and follows a Gamma distribution:

$$v_t \sim \text{Gamma}(1/\alpha, \alpha) \quad (38)$$

v_t is characterized by the following PDF, where α is the scale parameter and $1/\alpha$ is the shape parameter:

$$g(v_t) = \frac{v_t^{(1-\alpha)/\alpha} \exp(-v_t/\alpha)}{\Gamma(1/\alpha) \alpha^{1/\alpha}} \quad (39)$$

We note that the negative binomial regression arises as a Gamma-Poisson mixture

$$f(y_t | \mathbf{X}_t) = \int_0^\infty f(y_t | \mathbf{X}_t, v_t) g(v_t) dv_t = \frac{\Gamma(1/\alpha + y_t)}{\Gamma(y_t + 1) \Gamma(1/\alpha)} \left(\frac{1}{1 + \alpha \mu_t} \right)^{1/\alpha} \left(\frac{\alpha \mu_t}{1 + \alpha \mu_t} \right)^{y_t} \quad (40)$$

Based on this derivation the unconditional mean of the count variable is as follow:

$$\mathbb{E}[y_t | \mathbf{X}_t] = \exp(\mathbf{X}_t' \boldsymbol{\beta}) \quad (41)$$

In our case the observed heterogeneity takes the following form:

$$\mathbb{E}[y_t | \mathbf{X}_t] = \exp(\alpha + \beta_1 \text{CYCLE}_t + \beta_2 \text{LENIENCY}_t + \beta_3 \text{BUDGET}_t) \quad (42)$$

¹¹Evidenced by the significance of the α coefficient.

$CYCLE_t$ represents corporate profits per GDP after tax (without IVA and CCAdj), $LENIENCY_t$ represents the leniency indicator ($t \geq 1993$), and $BUDGET_t$ is detrended real annual budget appropriations. Going forward, all coefficients should be interpreted as either a unit change in the log expected counts or partial elasticities (a unit change in the regressor leads to a percent change in the count).

3.3. Results

In this section we present the results of our empirical estimation. We consider two specifications. In the first case we assume that the AA has a constant monitoring intensity over the business cycle. In the second case we control for both the monitoring intensity and the revised leniency program. These controls allow us to isolate the variation in investigations due to factors not within the AAs control. According to our model, the source of this variation is the cyclical variation in the degree of overall collusion in the economy. Thus positive correlation between corporate profits and AA filings would be evidence in favour procyclical collusion.

Table 4: Negative Binomial Regression Results - Collusion Filings (1970-2016)

	<i>Dependent variable:</i>			
	Frequency of Filings (Collusion)			
	(1)	(2)	(3)	(4)
<i>CYCLE</i>	0.174*** (0.035)	0.160*** (0.035)	0.074** (0.032)	0.061* (0.033)
Detrended Budget (M - 2010)		0.057** (0.022)		0.043** (0.020)
Leniency			1.155*** (0.161)	1.149*** (0.160)
Constant	0.602** (0.300)	0.677** (0.293)	0.576** (0.245)	0.655*** (0.247)
Observations	182	182	182	182
α (LR Test $\alpha = 0$)	0.790***	0.772***	0.490***	0.484***
Wald χ^2	24.69***	33.55***	86.32***	90.56***
Pseudo R ²	0.021	0.025	0.078	0.082

Note:

*p<0.1; **p<0.05; ***p<0.01; Robust standard errors reported in parenthesis.

Using the negative binomial methodology, we employ maximum likelihood estimation to estimate the model coefficients. This model implicitly assumes that the AA has a constant monitoring intensity across the business cycle. The coefficient on *CYCLE* is 0.174 and significant at the 1% level. This result corroborates our theory that the AA tends to have more filings in an expansion rather than a contraction. Looking at all filings in Table 5 (Appendix D), we see that the sign and level of significance are similar.

We now control for time varying monitoring intensity. We use detrended real appropriations as a proxy for monitoring intensity. As one would expect, detrended budget positively impacts the number of AA filings. In the collusion filings case, an unanticipated real appropriations increase, on average, leads to an increase in AA filings. This result is significant at the 5% level. The coefficient associated with *CYCLE* is 0.160, and is significant at the 1% level. Even after controlling for monitoring intensity over the cycle, we still find investigations are positively correlated with expansions. This evidence supports the theory that the increase in investigations during an expansion is associated with a higher degree of collusive behaviour. The result in the all filings regression in Table 5 (Appendix D) is similar.

To account for the structural change in the data in 1993, we include an indicator for the introduction of the revised leniency program. We find that the leniency dummy explains a large portion of the variation in filings. The coefficient is positive and statistically significant at the 1% level in all regressions. This is consistent with the US experience in that the introduction of the revised leniency program lead to a proliferation in self-reporting. We see that the coefficient associated with *CYCLE* is 0.061 and is statistically significant at the 10% level. The introduction of the corporate leniency program as a covariate has the effect of reducing the explanatory power of *CYCLE*. Again, these results are both consistent with the theory.

Finally, as a robustness check, we replace corporate profits with the NBER cycle dates in Table 6 (Appendix D). We use a dummy to indicate an expansion. We find that the coefficient associated with the expansion dummy is positive, however not statistically significant. The sign supports our theory and the reduction in significance is likely due to a reduction in the variation of the expansion dummy. When we collate our results, we find that the data is in

favor of the argument that collusion is procyclical. That is to say, we should expect to find more collusion when the economy is expanding.

4. Conclusion

We explored the impact that the business cycle has on collusive behavior in a Cournot framework with an Antitrust Authority. We found that collusion is more prevalent when the economy is expanding. There are two major explanations for this. First, we find that in markets where there is no collusion, the per period gain from collusion is high in the immediate future when compared to a similar period in a contraction. Second, in markets where collusion is present, as in [Haltiwanger and Harrington \(1991\)](#), we find that the loss from defection is high in the immediate future, this suggests that defection in an expansion is costly. As a result, more cartels will form in an expansion relative to a contraction. In addition, when the economy is expanding, fewer cartels will dissolve. This implies that the overall level of collusion in the economy is procyclical. This leads to our hypothesis that AA filings are procyclical.

We tested the prediction of the theory using a unique dataset of US antitrust filings. We find that AA filings are procyclical. A 1 percent increase in corporate profits over GDP leads to a 6 percent increase in AA filings. This finding controls for the AA's monitoring intensity and the US corporate leniency program and is robust to different measures of the business cycle. Our empirical findings suggest that procyclical monitoring policies may be the optimal cost minimizing route for the allocation of the DOJ resources.

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A. Appendix

Proof of Lemma 1. Start by subtracting P from each side of the inequality in Assumption 2.

$$\begin{aligned} a(1) - P < a(2) - P < \dots < a(\hat{t}) - P > a(\hat{t} + 1) - P > \dots \\ &> a(\bar{t}) - P > a(1) - P \end{aligned} \quad (\text{A-1})$$

Apply the definition of $Q(P; t) = a(t) - P$;

$$\begin{aligned} Q(P; 1) < Q(P; 2) < \dots < Q(P; \hat{t}) > Q(P; \hat{t} + 1) > \dots \\ &> Q(P; \bar{t}) > Q(P; 1) \end{aligned} \quad (\text{A-2})$$

□

Proof of Lemma 2.

$$\pi_i^M(t) - \pi_i^N(t) = \frac{(a(t) - c)^2(N - 1)^2}{4N(N + 1)^2} > 0, \quad \forall N > 1 \quad (\text{A-3})$$

$$\pi_i^D(t) - \pi_i^M(t) = \frac{(a(t) - c)^2(N - 1)^2}{16N^2} > 0, \quad \forall N > 1 \quad (\text{A-4})$$

Using the two results above, we can easily see that $\pi_i^M(t) > \pi_i^N(t)$ and $\pi_i^D(t) > \pi_i^M(t)$. Thus, by transitivity, $\pi_i^D(t) > \pi_i^M(t) > \pi_i^N(t)$. □

Proof of Lemma 3. for $j \in \{N, M, D\}$, we know that $\pi_i^j(t) = f_j(N)(a(t) - c)^2$ where $f_j(N) > 0 \forall N > 1$. As such, $\pi_i^j(t) \propto (a(t) - c)^2$. Since $(a(t) - c) \geq 0$, and using the fact that $a(1) < a(2) < \dots < a(\hat{t}) > a(\hat{t} + 1) > \dots > a(\bar{t}) > a(1)$, it must be the case that $(a(1) - c)^2 < (a(2) - c)^2 < \dots < (a(\hat{t}) - c)^2 > (a(\hat{t} + 1) - c)^2 > \dots > (a(\bar{t}) - c)^2 > (a(1) - c)^2$. Using the proportionality relationship, it must then be the case that $\pi_i^j(1) < \pi_i^j(2) < \dots < \pi_i^j(\hat{t}) > \pi_i^j(\hat{t} + 1) > \dots > \pi_i^j(\bar{t}) > \pi_i^j(1) \forall j \in \{M, N, D\}$ □

Proof of Lemma 4. We know that $\Delta_i(t) = f_\Delta(N)(a(t) - c)^2$ and $G_i(t) = f_G(N)(a(t) - c)^2$ where $f_\Delta(N) > 0$ and $f_G(N) > 0 \forall N > 1$. Therefore, we now that the following holds; $\Delta_i(t) \propto (a(t) - c)^2$ and $G_i(t) \propto (a(t) - c)^2$. The remainder of the proof follows the same logic as the proof for Lemma 3. For brevity, we omit this. □

Proof of Theorem 1. See Theorem 4 and 7 in [Haltiwanger and Harrington \(1991\)](#). We would like to show that $V_i^M(t') - V_i^N(t') > V_i^M(t'') - V_i^N(t'')$.

$$\begin{aligned} & [V_i^M(t') - V_i^N(t')] - [V_i^M(t'') - V_i^N(t'')] = \\ & \frac{[\Delta_i(t') + \dots + \delta^{\bar{t}-1}\Delta_i(t' - 1)]}{1 - \delta^{\bar{t}}} - \frac{[\Delta_i(t'') + \dots + \delta^{\bar{t}-1}\Delta_i(t'' - 1)]}{1 - \delta^{\bar{t}}} \end{aligned} \quad (\text{A-5})$$

Let's assume that the first term is greater than the second term;

$$\frac{[\Delta_i(t') + \dots + \delta^{\bar{t}-1}\Delta_i(t' - 1)]}{1 - \delta^{\bar{t}}} > \frac{[\Delta_i(t'') + \dots + \delta^{\bar{t}-1}\Delta_i(t'' - 1)]}{1 - \delta^{\bar{t}}} \quad (\text{A-6})$$

$$\Delta_i(t') + \dots + \delta^{\bar{t}-1}\Delta_i(t' - 1) > \Delta_i(t'') + \dots + \delta^{\bar{t}-1}\Delta_i(t'' - 1) \quad (\text{A-7})$$

Let's define the following ;

$$A = \Delta_i(t') + \dots + \delta^{t''-t'-1}\Delta_i(t'' - 1) \quad (\text{A-8})$$

$$B = \Delta_i(t'') + \dots + \delta^{\bar{t}+t'-t''-1}\Delta_i(t' - 1)$$

We can express (A-7) as follows;

$$A + \delta^{t''-t'}B > B + \delta^{\bar{t}+t'-t''}A \quad (\text{A-9})$$

$$\frac{A}{1 - \delta^{t''-t'}} > \frac{B}{1 - \delta^{\bar{t}+t'-t''}} \quad (\text{A-10})$$

The left hand side is just the present value of the stream $(\Delta_i(t') + \dots + \Delta_i(t'' - 1))$ repeated every $t'' - t'$ periods and the right hand side is just the stream $(\Delta_i(t'') + \dots + \Delta_i(t' - 1))$ repeated every $\bar{t} + t' - t''$ periods. It must be the case that;

$$\Delta_i(\tau) \geq \Delta_i(\tau'), \forall \tau \in \{t', t' + 1, \dots, t'' - 1\}, \forall \tau' \in \{t'', t'' + 1, \dots, t' - 1\} \quad (\text{A-11})$$

To think about [A-10](#) intuitively, the set $\{t', t' + 1, \dots, t'' - 1\}$ are all of the time periods in the peak of a cycle during some period length \bar{t} , while $\{t'', t'' + 1, \dots, t' - 1\}$ are all of the time periods during the trough. Thus, it must be the case that any period from $\{t', t' + 1, \dots, t'' - 1\}$ yields a higher gain than EVERY period from $\{t'', t'' + 1, \dots, t' - 1\}$. Therefore;

$$\frac{[\Delta_i(t') + \dots + \delta^{\bar{t}-1}\Delta_i(t' - 1)]}{1 - \delta^{\bar{t}}} > \frac{[\Delta_i(t'') + \dots + \delta^{\bar{t}-1}\Delta_i(t'' - 1)]}{1 - \delta^{\bar{t}}} \quad (\text{A-12})$$

Therefore, $V_i^M(t') - V_i^N(t') > V_i^M(t'') - V_i^N(t'')$. \square

Proof of Lemma 5. We need to show that $\hat{F}(\alpha, \rho, \delta, t') > \hat{F}(\alpha, \rho, \delta, t'')$. From Theorem 1, we showed that;

$$\Delta_i(t') + \dots + \delta^{\bar{t}-1} \Delta_i(t' - 1) > \Delta_i(t'') + \dots + \delta^{\bar{t}-1} \Delta_i(t'' - 1) \quad (\text{A-13})$$

We also know that;

$$\hat{F}(\alpha, \rho, \delta, t) = \frac{1 - \delta}{\alpha \rho (1 - \delta^{\bar{t}})} [\Delta_i(t) + \dots + \delta^{\bar{t}-1} \Delta_i(t - 1)] \quad (\text{A-14})$$

From (A-13) we can arrive at;

$$\begin{aligned} \frac{1 - \delta}{\alpha \rho (1 - \delta^{\bar{t}})} [\Delta_i(t') + \dots + \delta^{\bar{t}-1} \Delta_i(t' - 1)] \\ > \frac{1 - \delta}{\alpha \rho (1 - \delta^{\bar{t}})} [\Delta_i(t'') + \dots + \delta^{\bar{t}-1} \Delta_i(t'' - 1)] \end{aligned} \quad (\text{A-15})$$

And using the definition in (A-14);

$$\hat{F}(\alpha, \rho, \delta, t') > \hat{F}(\alpha, \rho, \delta, t'') \quad (\text{A-16})$$

□

Proof of Theorem 3. We know that $G(t'') = G(t')$. We want to show that $L(t', \delta) - G(t') > L(t'', \delta) - G(t'')$. Since $G(t'') = G(t')$, it remains to show that $L(t', \delta) > L(t'', \delta)$. Using the definition of $L(t, \delta)$;

$$\frac{1}{1 - \delta^{\bar{t}}} [\delta \Delta_i(t' + 1) + \dots + \delta^{\bar{t}} \Delta_i(t')] > \frac{1}{1 - \delta^{\bar{t}}} [\delta \Delta_i(t'' + 1) + \dots + \delta^{\bar{t}} \Delta_i(t'')] \quad (\text{A-17})$$

Let's define the following;

$$A = \delta \Delta_i(t' + 1) + \dots + \delta^{t''-t'} \Delta_i(t'') \quad (\text{A-18})$$

$$B = \delta \Delta_i(t'' + 1) + \dots + \delta^{\bar{t}-t''+t'} \Delta_i(t') \quad (\text{A-19})$$

Using the definition of A and B;

$$A + \delta^{t''-t'} B > B + \delta^{\bar{t}-t''+t'} A \quad (\text{A-20})$$

$$\frac{A}{1 - \delta^{t''-t'}} > \frac{B}{1 - \delta^{\bar{t}-t''+t'}} \quad (\text{A-21})$$

Note that the left hand side is just the present value of $(\Delta_i(t' + 1), \dots, \Delta_i(t''))$ over the interval $t'' - t'$. The right hand side is the present value of $(\Delta_i(t'' + 1), \dots, \Delta_i(t'))$ over

the interval $\bar{t} - t'' + t'$. Each element in A dominates each element in B, that is to say $\Delta_i(\tau') \geq \Delta_i(\tau'') \forall \tau' \in \{t' + 1, \dots, t''\}, \forall \tau'' \in \{t'' + 1, \dots, t'\}$. As is such, the left hand side is greater than the right hand side which completes the proof, showing that;

$$L(t', \delta) - G(t') > L(t'', \delta) - G(t'') \quad (\text{A-22})$$

□

Proof of Lemma 7. From Theorem 3 we see that $L(t') - G(t') > L(t'') - G(t'')$. It is also true that;

$$\frac{\delta\alpha\rho}{1-\delta} [L(t') - G(t')] > \frac{\delta\alpha\rho}{1-\delta} [L(t'') - G(t'')] \quad (\text{A-23})$$

Now applying the definition of \tilde{F} ;

$$\tilde{F}(\alpha, \rho, \delta, t') > \tilde{F}(\alpha, \rho, \delta, t'') \quad (\text{A-24})$$

□

B. Data Collection

The US DOJ Antitrust Division lists all Antitrust case filings in alphabetical order on their website. Cases appear as early as the 1940's and as recent as 2017. Each case has a hyperlink that directs the end user to general information about the case. Each case has information on, among others: filing date, type, and type of violations. In addition to basic descriptive information, the DOJ includes documents relevant to the case. Since there are approximately 1900 cases, we needed to develop an efficient method of extracting the basic descriptive information for each case. We proceeded by developing a web scrapper in R.

The alphabetical list on the DOJ website has a hyperlink associated with each filing. Upon inspecting the web page, we found that the web address associated with each case is embedded in the HTML code. We first used the “XML” package in R to parse the HTML document associated with the alphabetical listing of Antitrust filings. We used “htmlTreeParse” to generate an R data structure that represents the HTML document. Once we created this data structure, we searched this data structure for all “`a`” tags and extracted the corresponding “`href`” elements. Since the resulting list contains all links within the HTML document, we needed to differentiate links that correspond to a case and links that point elsewhere on the DOJ website. We noticed that links associated with a particular case had the following string within the link name “`/atr/case`”. Thus, we deleted all additional links that did not contain “`/atr/case`”, which left us with only those links that map to specific cases. We were left with a list of full links that corresponded to all Antitrust filings as listed on the alphabetical list on the DOJ website.

Once we had the list of web links that corresponded to all antitrust cases listed on the DOJ website, we used a loop to parse the HTML code on each web link. Using the web link and identifying certain patterns within the HTML documents, we were able to extract the following information: Case Open Date, Case Name, Case Type, Case Violation, Market, Industry Code, and Component. After extracting a list of cases and their corresponding dates, we counted the number of filings that occurred in a given quarter from 1970 to 2016.

C. Supplementary Material

C.1. Nash Equilibrium

$$\pi_i^N(t) = \max_{q_i} P(Q; t)q_i - C(q_i) = \max_{q_i} \left(a(t) - \sum_{i=1}^N q_i \right) q_i - cq_i \quad (\text{C-25})$$

The First Order Necessary Condition;

$$a(t) - \sum_{i=1}^N q_i - q_i - c = 0 \quad (\text{C-26})$$

We assume symmetric firms ($q = q_i$);

$$q = \frac{a(t) - c}{N + 1} \quad (\text{C-27})$$

The absolute margins is expressed as follows;

$$P - c = a(t) - Nq - c = a(t) - \frac{N}{N + 1} (a(t) - c) - c = \frac{a(t) - c}{N + 1} \quad (\text{C-28})$$

As such, the Nash profits are;

$$\pi_i^N(t) = \frac{(a(t) - c)^2}{(N + 1)^2} \quad (\text{C-29})$$

C.2. Collusive Equilibrium

$$\pi_i^M(t) = \frac{1}{N} \max_Q P(Q; t)q_i - C(q_i) = \max_Q (a(t) - Q) Q - cQ \quad (\text{C-30})$$

The First Order Necessary Condition;

$$a(t) - Q - Q - c = 0 \quad (\text{C-31})$$

Therefore;

$$Q = \frac{a(t) - c}{2} \text{ and } q^M = \frac{a(t) - c}{2N} \quad (\text{C-32})$$

The absolute margins for the industry is;

$$P - c = a(t) - Q - c = a(t) - c - \frac{a(t) - c}{2} = \frac{a(t) - c}{2} \quad (\text{C-33})$$

As such, the collusive profits for each firms are;

$$\pi_i^M(t) = \frac{(a(t) - c)^2}{4N} \quad (\text{C-34})$$

C.3. Defection Profits

$$\begin{aligned}\pi_i^D(t) &= \max_{q_i} P(q^i + (N-1)q^M; t) q_i - C(q_i) \\ &= \max_{q_i} (a(t) - q_i - (N-1)q^M) q_i - cq_i\end{aligned}\tag{C-35}$$

The First Order Necessary Condition;

$$a(t) - q_i - (N-1)q^M - q_i - c = 0\tag{C-36}$$

Substituting in the value of q^M ;

$$a(t) - q_i - (N-1)\frac{a(t) - c}{2N} - q_i - c = 0\tag{C-37}$$

Solving the First Order Condition;

$$q_i = \frac{(a(t) - c)(N+1)}{4N}\tag{C-38}$$

Solving for total output;

$$Q = q_i + (N-1)q^M = \frac{(a(t) - c)(N+1)}{4N} + (N-1)\frac{a(t) - c}{2N} = \frac{(a(t) - c)(3N-1)}{4N}\tag{C-39}$$

Solving for absolute margins;

$$P - c = a(t) - c - Q = a(t) - c - \frac{(a(t) - c)(3N-1)}{4N} = \frac{(a(t) - c)(N+1)}{4N}\tag{C-40}$$

Thus,

$$\pi_i^D(t) = (P - c)q_i = \frac{(a(t) - c)^2 (N+1)^2}{16N^2}\tag{C-41}$$

C.4. Geometric Series with Repeating Elements

Let's suppose that within a geometric series there are elements that repeat themselves every \bar{t} periods. We can view each repeated iteration as a unique element repeated every \bar{t} periods. Let's consider the example of repeated Nash profits every $\bar{t} = 3$ periods. The value of perpetual competition can be expressed as follows;

$$\begin{aligned}V_i^N(t) &= \sum_{j=0}^{\infty} \delta^j \pi_i^N(t+j) = \pi_i^N(t) + \delta \pi_i^N(t+1) + \delta^2 \pi_i^N(t+2) + \delta^3 \pi_i^N(t) \\ &\quad + \delta^4 \pi_i^N(t+1) + \delta^5 \pi_i^N(t+2) + \dots \\ &= [\pi_i^N(t) + \delta \pi_i^N(t+1) + \delta^2 \pi_i^N(t+2)] \\ &\quad + \delta^3 [\pi_i^N(t) + \delta \pi_i^N(t+1) + \delta^2 \pi_i^N(t+2)] + \dots\end{aligned}\tag{C-42}$$

If we were to generalize this result, we can quickly see that ;

$$V_i^N(t) = \sum_{j=0}^{\infty} \delta^j \pi_i^N(t+i) = \sum_{i=0}^{\infty} \delta^{\bar{t}j} \left[\pi_i^N(t) + \dots + \delta^{\bar{t}-1} \pi_i^N(t + \bar{t} - 1) \right] \quad (\text{C-43})$$

Notice that the term in brackets is invariant over the subscript j ;

$$V_i^N(t) = \left[\pi_i^N(t) + \dots + \delta^{\bar{t}-1} \pi_i^N(t + \bar{t} - 1) \right] \sum_{j=0}^{\infty} \delta^{\bar{t}j} \quad (\text{C-44})$$

The summation collapses to an infinite geometric series of the form;

$$V_i^N(t) = \frac{\left[\pi_i^N(t) + \dots + \delta^{\bar{t}-1} \pi_i^N(t + \bar{t} - 1) \right]}{1 - \delta^{\bar{t}}} \quad (\text{C-45})$$

We point out that the above result can be represented as follows;

$$V_i^N(t) = \frac{\left[\pi_i^N(t) + \dots + \delta^{\bar{t}-t} \pi_i^N(\bar{t}) + \delta^{\bar{t}-t+1} \pi_i^N(1) + \dots + \delta^{\bar{t}-1} \pi_i^N(t-1) \right]}{1 - \delta^{\bar{t}}} \quad (\text{C-46})$$

Or more compactly;

$$V_i^N(t) = \frac{\left[\pi_i^N(t) + \dots + \delta^{\bar{t}-1} \pi_i^N(t-1) \right]}{1 - \delta^{\bar{t}}} \quad (\text{C-47})$$

C.5. Difference In Profits

First we consider $\pi_i^M(t) - \pi_i^N(t)$;

$$\begin{aligned} \pi_i^M(t) - \pi_i^N(t) &= \frac{(a(t) - c)^2}{4N} - \frac{(a(t) - c)^2}{(N+1)^2} \\ &= (a(t) - c)^2 \frac{(N+1)^2 - 4N}{4N(N+1)^2} \\ &= \frac{(a(t) - c)^2 (N-1)^2}{4N(N+1)^2} \end{aligned} \quad (\text{C-48})$$

Next we consider $\pi_i^D(t) - \pi_i^M(t)$;

$$\begin{aligned} \pi_i^D(t) - \pi_i^M(t) &= \frac{(a(t) - c)^2 (N+1)^2}{16N^2} - \frac{(a(t) - c)^2}{4N} \\ &= (a(t) - c)^2 \frac{4N(N+1)^2 - 16N^2}{4N16N^2} \\ &= \frac{(a(t) - c)^2 4N(N-1)^2}{4N16N^2} \\ &= \frac{(a(t) - c)^2 (N-1)^2}{16N^2} \end{aligned} \quad (\text{C-49})$$

D. Regression Results

Table 5: Negative Binomial Regression Results - All Filings (1970-2016)

	<i>Dependent variable:</i>			
	Frequency of Filings (All)			
	(1)	(2)	(3)	(4)
<i>CYCLE</i>	0.169*** (0.027)	0.149*** (0.027)	0.054** (0.024)	0.042* (0.024)
Detrended Budget (M - 2010)		0.057*** (0.017)		0.032** (0.016)
Leniency			1.208*** (0.130)	1.189*** (0.130)
Constant	1.13*** (0.230)	1.249*** (0.224)	1.177*** (0.176)	1.256*** (0.179)
Observations	182	182	182	182
α (LR Test $\alpha = 0$)	0.584***	0.561***	0.284***	0.276***
Wald χ^2	39.52***	60.25***	147.26***	156.96***
Pseudo R ²	0.024	0.030	0.103	0.107

Note: *p<0.1; **p<0.05; ***p<0.01; Robust standard errors reported in parenthesis.

Table 6: Negative Binomial Regression Results - AA Filings - NBER (1970-2016)

	<i>Dependent variable:</i>			
	Frequency of Filings			
	All	All	Collusion	Collusion
	(1)	(2)	(3)	(4)
Expansion (=1)	0.316*	0.060	0.378**	0.075
	(0.165)	(0.115)	(0.165)	(0.151)
Detrended Budget (M - 2010)	0.069***	0.036**	0.070***	0.051***
	(0.018)	(0.015)	(0.023)	(0.019)
Leniency		1.268***		1.239***
		(0.118)		(0.145)
Constant	2.069***	1.462***	1.512***	0.967***
	(0.152)	(0.108)	(0.146)	(0.143)
Observations	188	188	182	188
α (LR Test $\alpha = 0$)	0.635***	0.268***	0.839***	0.470***
Wald χ^2	17.35***	161.24***	13.07***	90.25***
Pseudo R ²	0.009	0.106	0.008	0.079

Note:

*p<0.1; **p<0.05; ***p<0.01; Robust standard errors reported in parenthesis.

E. Other Figures

Table 7: Frequency of Filings by Type Across Years (1970-2016)

	All Filings	Collusion Related	Merger Related
1970-1974	143.00	109.00	33.00
1975-1979	83.00	59.00	18.00
1980-1984	90.00	57.00	31.00
1985-1989	50.00	16.00	22.00
1990-1994	95.00	48.00	24.00
1995-1999	385.00	259.00	70.00
2000-2004	297.00	166.00	43.00
2005-2009	310.00	145.00	41.00
2010-2014	424.00	280.00	50.00
2015-2016*	124.00	71.00	27.00

* Only a two year period

Table 8: Conditional Descriptive Statistics (1970-2016)

	N	Mean	Median	Sd
Expansion				
Filing Frequency	162	10.96	9.00	9.25
Filing Frequency (Collusion)	162	6.67	5.00	6.84
Filing Frequency (Mergers)	162	1.94	2.00	1.83
Real Budget Appropriations (M - 2010)	162	26.25	22.78	9.52
Corporate Profits (B - 2010)	162	787.91	579.53	490.96
Corporate Profits/GDP	156	6.93	6.44	2.09
Contraction				
Filing Frequency	26	8.69	6.50	7.17
Filing Frequency (Collusion)	26	5.00	4.00	3.82
Filing Frequency (Mergers)	26	1.73	1.00	1.87
Real Budget Appropriations (M - 2010)	26	23.30	21.49	10.61
Real Corporate Profits (B - 2010)	26	596.62	428.54	359.45
Corporate Profits/GDP	26	6.48	6.27	1.43

Table 9: Conditional Descriptive Statistics (1970-2016)

	N	Mean	Median	Sd
Pre-Revised Leniency Program (1993)				
Filing Frequency	92	4.49	3.00	4.39
Filing Frequency (Collusion)	92	2.80	2.00	3.37
Filing Frequency (Mergers)	92	1.29	1.00	1.47
Post-Revised Leniency Program (1993)				
Filing Frequency	96	16.54	15.00	8.33
Filing Frequency (Collusion)	96	9.92	8.00	6.93
Filing Frequency (Mergers)	96	2.50	2.00	1.95

F. Simulation Details

F.1. Competition

As calculated in Section 2.1, the profits from competition are as follows;

$$\pi_i^N(t) = \frac{\left(\cos\left(\frac{2\pi(t-1)}{\bar{t}}\right) - 1\right)^2}{(N+1)^2} \quad (\text{F-50})$$

Using the estimated parameters we plot competitive profits in Figure 8. As we can see, competitive profits track the periodic pattern embedded in the demand shifter $a(t)$. Profits peak when $t = 10.5$ and are at their lowest when $t = 1$. These extrema repeat every 19 periods. We plot the value of the competitive strategy in Figure 9. We find that competitive value is periodic, however, it does not track the same pattern embedded in $a(t)$. In fact, the maximum value occurs at approximately $t = 7.1$ (and every 19 periods after). The maximum value does not occur at $t = 10.5$ since future profits are discounted by δ . The discount factor causes value to peak before demand. This offset is increasing in δ . As $\delta \rightarrow 0$, the offset disappears.

Figure 8: Competitive Profits Over the Cycle ($\bar{t} = 19$)

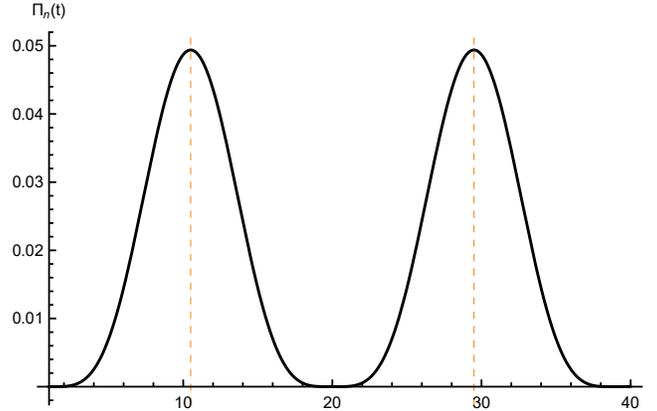
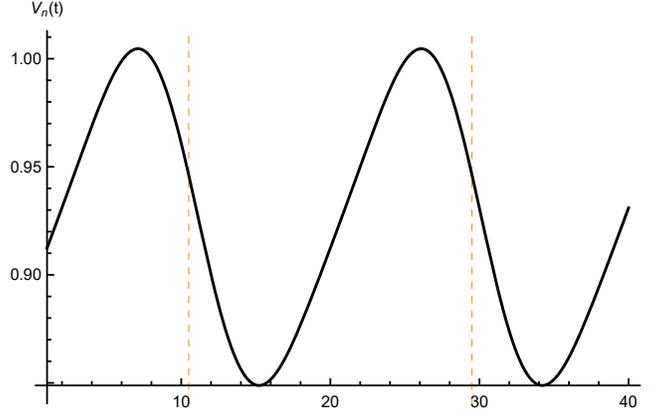


Figure 9: Value from Competition Over the Cycle ($\bar{t} = 19$)



F.2. Cartel Profits

As derived in Section 2.2, the profits from forming a cartel are as follows;

$$\pi_i^M(t) = \frac{\left(\cos\left(\frac{2\pi(t-1)}{t}\right) - 1\right)^2}{4N} \quad (\text{F-51})$$

We plot both the profit from collusion and the profit from competition in Figure 10. Collusive profits track the periodic trend in the demand shifter $a(t)$. In addition, profits peak when $t = 10.5$ and are at their lowest when $t = 1$, repeating this pattern every 19 periods. We observe that the difference between collusive profits and competitive profits is increasing during an expansion and decreasing during a contraction. Assuming that there is no AA, we plot the value from forming a cartel and the value from competition in Figure 11.

Figure 10: Collusive Profits Over the Cycle ($\bar{t} = 19$)

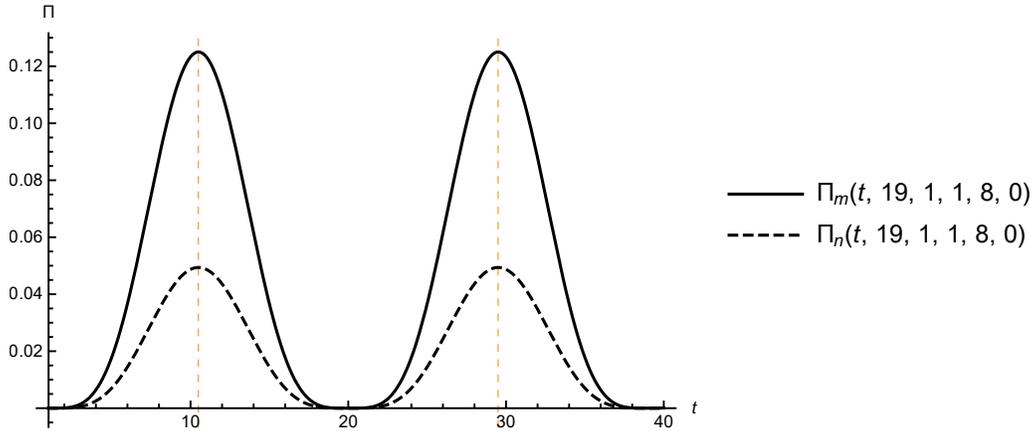
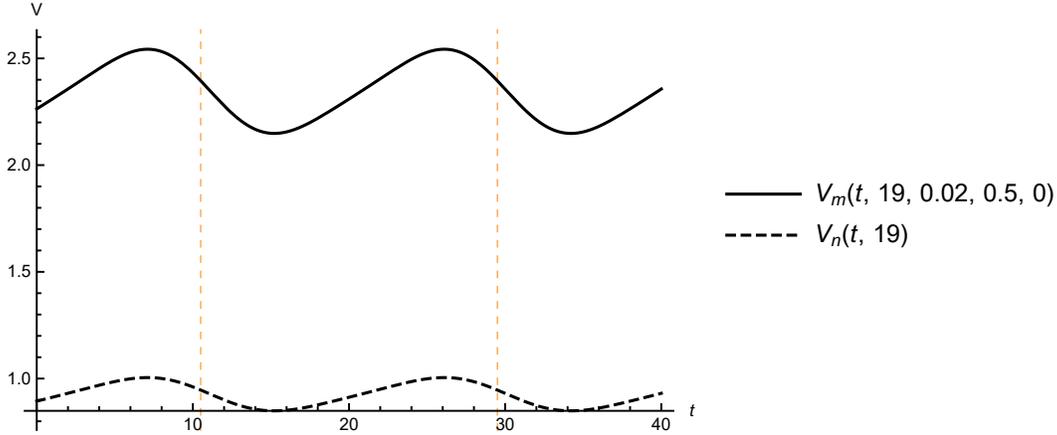


Figure 11: Collusive Value Over the Cycle ($\bar{t} = 19$)



Notice that the value from forming a cartel is large relative to the competitive strategy. This is the case because we used a high discount factor. As per the *Folk Theorem*, in infinitely repeated games we know that a strategy can be sustained as a subgame perfect equilibrium as long as firms are sufficiently patient. In our model, firms are almost perfectly patient in the sense that they are willing to trade current cash flows for future cash flows on a one-to-one basis. Since patience alone is not enough to prevent the formation of cartels, the fine imposed by the judicial authority can lower the value of collusion, thus leading to a result where the gain from collusion may be negative at various points over the business cycle.

F.3. Defection Profits

As derived in Section 2.2, the profits from defection are expressed as follows;

$$\pi_i^D(t) = \frac{\left(\cos\left(\frac{2\pi(t-1)}{T}\right) - 1\right)^2 (N+1)^2}{16N^2} \quad (\text{F-52})$$

We plot the profits from collusion versus the profits from defection in Figure 12. As can be seen, the profit from defection is at least as great as the profits from continuing to collude. In addition, profits peak when $t = 10.5$ and are at their lowest when $t = 1$. These extrema repeat every 19 periods. We observe that the difference between defection profits and collusive profits is increasing during an expansion and decreasing during a contraction. Assume that there is no AA, we plot the value from defection and collusion in Figure 13.

Figure 12: Defection Profits Over the Cycle ($\bar{t} = 19$)

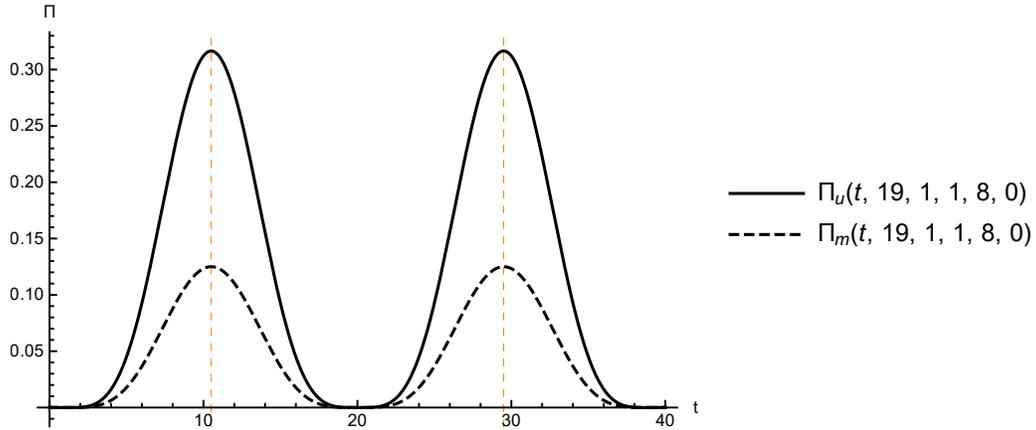
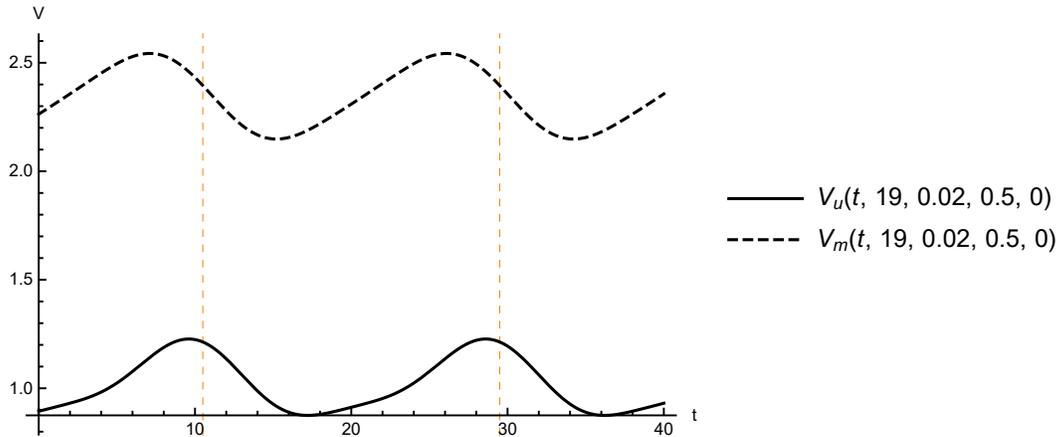


Figure 13: Value From Defection Over the Cycle ($\bar{t} = 19$)

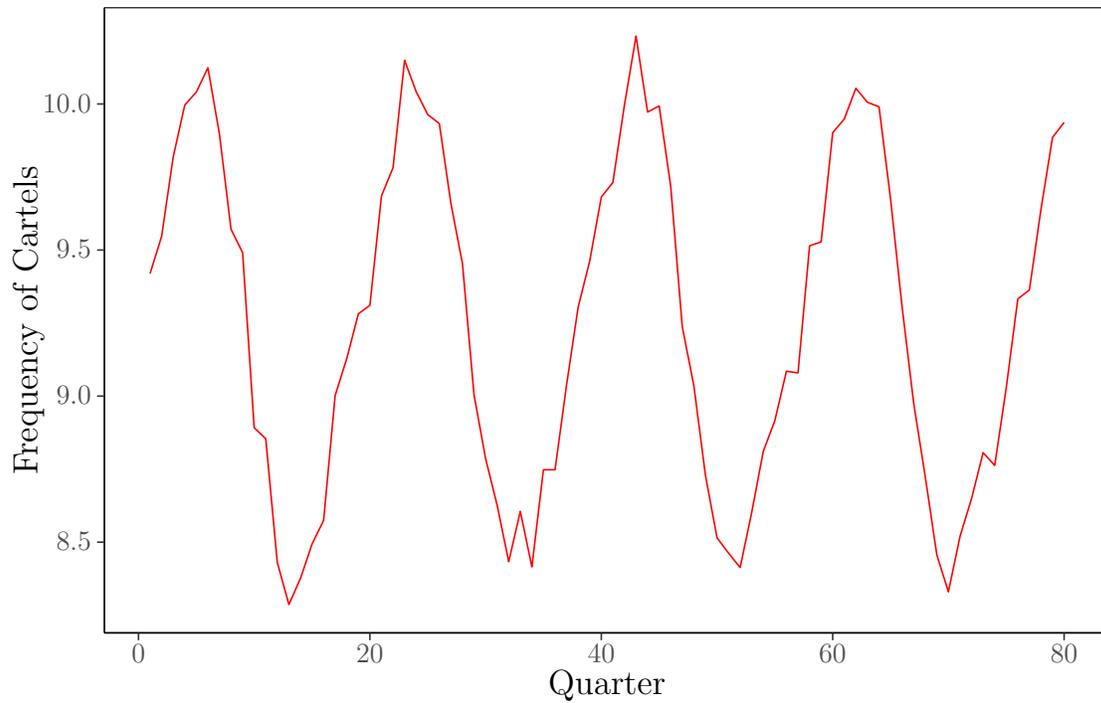


F.4. Simulated Economy

We simulate an economy with 20 identical markets over 80 quarters. We assume that marginal costs are equal to zero, the length of the business cycle is 19 quarters, the probability of an investigation is 2%, and the probability of conviction is 50%. The size of each market is drawn from a discrete uniform distribution over the support 2 to 20. Additionally, the fine is drawn from a continuous uniform distribution over the support 0 to 5. The choice of distribution and upper bounds are arbitrary and have no impact on the pattern observed in the data. At each period, we determine if a competitive market forms a cartel. If a cartel has already been formed, we determine if the cartel will collapse. We perform this simulation 10,000 times and take the average number of cartels across all simulations. Our results are

plotted in Figure 14 below.

Figure 14: Simulation Results



We find that the level of collusive behaviour tracks the business cycle. The highest level of collusive behaviour occurs during the expansion while the lowest level occurs in a contraction. We conclude that, even when the number of firms is random, we should still expect to observe a cyclical pattern in overall collusive behaviour.