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Abstract

We use aggregate banking data to uncover a new fact: U.S. banks counter-cyclically vary the proportion of defaulted loans that they charge-off. The variance of this "charge-offs to defaults" ratio is roughly 15 times larger than that of GDP. Canonical financial accelerator models cannot explain this variance. We show that introducing stochastic default costs into the model helps to resolve the discrepancy with the data. Estimating the augmented model on typical macroeconomic data using Bayesian techniques reveals that the estimated default cost shocks not only help account for the variance of the banking data but also help account for a significant fraction of the U.S. business cycle between 1984 and 2015.

Keywords: Charge-offs and defaults, default cost shocks, financial accelerator models, business cycles.
JEL Classification: E3, E44

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1 Introduction

Banks realize that an unpleasant but expected outcome of making loans to a large number of borrowers is that a fraction of these borrowers will default on their loans. Since banks largely make loans with borrowed funds, regulations require them to charge-off the expected value of funds that are lost due to defaults.[1] These losses stem from at least two sources. First, the value of the repossessed collateral and any seized returns may not cover the principal and interest due to the bank. Second, there may be costs related to the default (and associated bankruptcy proceedings) which need to be accounted for.

It is well known that default rates tend to increase during recessions and fall during booms. Figure 1 displays this counter-cyclical pattern with defaults shown as diamonds and real GDP as a solid line. Figure 1 also plots the behavior of charge-offs (stars) for the U.S. banking system and not surprisingly, charge-offs appear to be positively correlated with defaults and negatively correlated with GDP.[2] What is surprising is that charge-offs do not simply follow the path of defaults. While defaults are about 15 times more volatile than output, charge-offs vary much more – about 22 times more volatile than output. Moreover, a second glance at Figure 1 makes apparent that the co-movement of charge-offs and defaults is highly imperfect (correlation coefficient of 0.7).

To further investigate the joint behavior of defaults and charge-offs, we express them as a ratio and find that the ratio of charge-offs to defaults (COD henceforth) is indeed highly variable, as suggested by Figure 1, and negatively correlated with GDP. In U.S. data, the standard deviation of COD relative to the standard deviation of GDP is 15.6 and the correlation between these two variables is -0.2. More empirical moments about COD are presented in section 2.

[1] Regulators expect banks to set aside resources called loan-loss provisions when making loans. As loan losses are actually realized, the bank takes charge-offs, equaling the value of loans removed from banks books. These are then deducted from the provisions that had been made for loan losses.

[2] Using quarterly FDIC data on insured commercial banks and savings institutions covering 1984Q1 to 2016Q1, we measure charge-offs as total charge-offs while defaults are measured as loans and leases 90 days or more past due. Both series are deflated using the GDP deflator and population. All series are detrended using the HP filter with smoothing parameter $\lambda = 1600$.

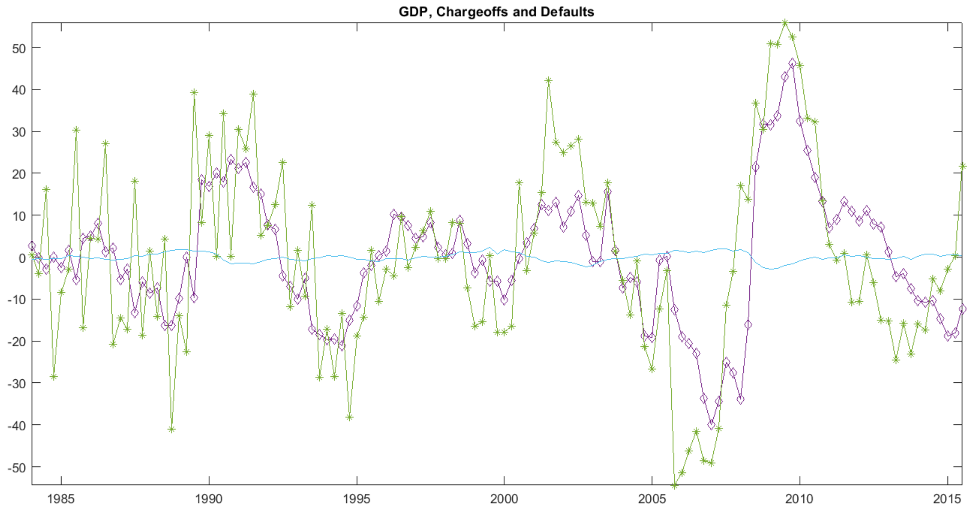

Figure 1: U.S. GDP (line); Defaults (diamonds); Charge-offs (stars)

These new stylized facts raise a number of questions that we wish to explore in this paper. First, why should we expect banks to vary the ratio of charge-offs to defaults over time? Second, why is this ratio counter-cyclical? Third, is the joint behavior of charge-offs and defaults consistent with the predictions of business cycle models? Intuitively, and somewhat mechanically, the answer to the first question is obvious. If banks expect to pay higher default costs than normal or if the return on the repossessed assets is smaller than normal then banks need to charge-off more than "usual" and this results in a rise in the ratio of charge-offs to defaults. In order to shed light on the question of why the COD ratio varies negatively with the business cycle, we turn to the canonical business cycle framework that links the banking sector with real economic activity. This framework, which encompasses the financial accelerator models of Bernanke et al. (1999) and Carlstrom and Fuerst (1997) (and models that build on it) is not only popular but particularly relevant for our question because it implies a loan contract between banks and borrowers in which the default rate is endogenously determined and responds to macroeconomic shocks. Moreover, a key aspect of the framework, consistent with reality, is a representative bank budget constraint which allows losses associated with defaults to emerge from both sources outlined above. This allows the model to make predictions about the dynamics of aggregate defaults, and implicitly, also about aggregate charge-offs. While it is well known that financial accelerator models can generate counter-cyclical defaults, little is known about their implications for charge-offs or for the COD ratio.

We take an off-the-shelf financial accelerator model augmented with New-Keynesian features in order to explore the endogenous response of COD to a large number of shocks that generate business cycles (henceforth referred to as the baseline model). We find that none of the shocks, (together or separately) are able to capture the joint behavior of charge-offs and defaults summarized above. This baseline financial accelerator model implies a near perfect correlation between the two variables and so is unable to generate much variance in the COD ratio. We discuss the reasons behind this failure in more detail in section 3.5 but it emerges from the value of repossessed assets from defaulted loans not moving enough to capture the variance in the COD ratio.

Next, we show that a simple modification of the baseline model can break the tight link between charge-offs and defaults. This modification boils down to introducing exogenous variation in default costs of the representative bank as suggested by Gunn and Johri (2013). Having established that default cost shocks can increase the variance of COD in principle, we proceed to estimate an augmented version of the baseline model (called the full model) which includes our default cost process and all of the shocks already included in the baseline model. This procedure allows aggregate U.S. data to discipline the default cost shocks. We find that the full model not only captures a significant amount of variation in aggregate banking data including COD and credit spreads, but also in the standard macroeconomic series studied in the business cycle literature. We note that COD is not used as an observable in the estimation of the full model so that it can be used to test the external validity of the baseline and full models.

To summarize, the cyclical movements of the COD ratio present a challenge to conventional financial accelerator models. Our interpretation of the data and model simulations is that variation in default costs are a disturbance to the banking system that is large enough to cause aggregate fluctuations consistent with both macroeconomic data and the COD ratio. The reason COD is counter-cyclical is because a rise in default costs causes banks to increase charge-offs more than the rise in the fraction of loans that are in default. Defaults rise because banks offer worse terms to borrowers by increasing the spread between the lending rate and the deposit rate for any given amount of leverage. This leads to reduced borrowing in equilibrium and a recession with lower investment, output and hours induced by shocks in the banking sector.

1.1 Literature review

We view our study as part of a growing literature that argues that variation in intermediation costs are a feature of U.S. data and that this variation has important implications for business cycles. Ajello (2016) explores stochastic variation in intermediation costs to generate movements in credit spreads and aggregate time series. Our focus on the joint behaviour of charge-offs over defaults allows us to distinguish between two distinct types of intermediation costs: those that are specifically related to defaulted funds and those that are not. While all intermediation costs create a wedge between the lending rate and the deposit rate (credit spread), only default costs create a wedge between charge-offs and defaults. We use our COD data to provide external validity to our estimated DSGE model with stochastic default cost shocks. In their presence, the model can generate a highly volatile COD series while in their absence, the combined effect of all macroeconomic shocks is unable to generate this volatility. Since Ajello (2016) builds on the Kiyotaki and Moore (2012) framework, there are no equilibrium defaults. As a result that model does not speak to our COD data. Despite these and other differences, we note some important implications that are common. Like us, Ajello (2016) finds that intermediation cost shocks are important drivers of aggregate output and investment. This suggests that the widely noted relationship between credit spreads and macroeconomic activity may emerge from a number of distinct sources within the financial sector.[3]

Exogenous variation in credit spreads driven by some form of intermediation cost can be found in a number of other studies. Curdia and Woodford (2009) study monetary policy in a New Keynesian model where intermediaries face stochastic variation in these costs but they do not take the model to the data. Similarly, impulse responses to shocks to a more fully specified loan production technology that takes as inputs labor and collateral can be found in Goodfriend and McCallum (2007). News about intermediation cost shocks can be found in Gunn and Johri (2011). See also earlier work in Cooper and Ejarque (2000). Our model, building on Gunn and Johri (2013), differs from all these studies in that stochastic variation in the cost of banks is embedded into a model where loan contracts with entrepreneurs are endogenously determined in equilibrium. The equilibrium combination of leverage and external finance premium (or credit spread) chosen by agents in equilibrium depend on bank costs so that neither is entirely exogenously driven but can respond to shocks within and outside the financial sector.[4] In our model loan contracts also respond to news about future variation in default costs. Indeed the estimation assigns a significant role to news shocks.

Microeconomic evidence on time varying default costs can be found in the work of Levin et al. (2004) who construct a panel of 900 U.S. firms from 1997Q1 to 2003Q3. Using balance sheet information, expected default probabilities and credit spreads, the authors estimate the parameters that govern the financial contract implied by the Bernanke et al. (1999) model, including the costs associated with defaulted loans. Levin et al. (2004) find that these costs vary systematically over the business cycle, rising during recessions and falling below mean levels during booms. While our structural DSGE

[3]The negative relationship between credit spreads and aggregate economic activity has been well known for some time but has been highlighted in Gilchrist and Zakrajek (2012) who construct an index of credit spreads and show that increases in this spread index are highly predictive of falls in future economic activity, even more so than the traditional spread between risky corporate bonds and government bonds.

[4]Aysun and Honig (2011) study the impact of different levels of intermediation costs on an economy facing sudden stops. Unlike us, they do not model default costs as a stochastic process.

model also incorporates the same equations that characterize the loan contract between intermediaries and firms, the informational content of our estimation of default costs differs in that we do not use balance sheet data to tie down our shocks. Moreover, we use aggregate data to estimate the aggregate default costs as opposed to firm level data. In section 3.5, we show the relationship between COD, our exogenous default cost shocks and an endogenous expression that captures the general equilibrium effects of all other possible shocks. This relationship potentially defines a wedge between the implications of existing financial accelerator models and our COD data which we interpret in terms of stochastic variation in default costs and show that these shocks are empirically relevant for understanding U.S. business cycles.

We view the micro evidence in Levin et al. (2004) as corroborating our work based on aggregate time series. Gunn and Johri (2013) incorporated surprise and anticipated shocks to default costs in a real version of a financial accelerator model in order to explain the boom-bust cycle associated with the Great Recession. While that paper was solely concerned with the financial crisis and associated recession, our work here shows that both surprise shocks and news shocks to the default cost process are of great importance in accounting for the joint dynamics of charge-offs and defaults, credit spreads, as well as other macroeconomic variables.

The rest of the paper is organized as follows. Section 2 provides evidence on the ratio of charge-offs to defaults. Section 3 presents a summary of the model with a focus on the financial intermediary while other model elements which are common to a host of New Keynesian and Financial Accelerator models are relegated to the Appendix. We report results in section 4 and section 5 concludes.

2 Empirical evidence on the charge-offs to defaults ratio

This section presents descriptive statistics on the COD ratio and documents its correlation to key macro and financial variables. We use quarterly FDIC data on insured commercial banks and savings institutions covering 1984Q1 to 2015Q4. We measure charge-offs as total charge-offs while defaults are measured as loans and leases 90 days or more past due. In terms of descriptive statistics, the COD ratio (no filtering or detrending) has a mean of 56% and a standard deviation of 22% in our sample. Clearly, the amount charged-off by banks is not a constant fraction of the amount defaulted on loans.

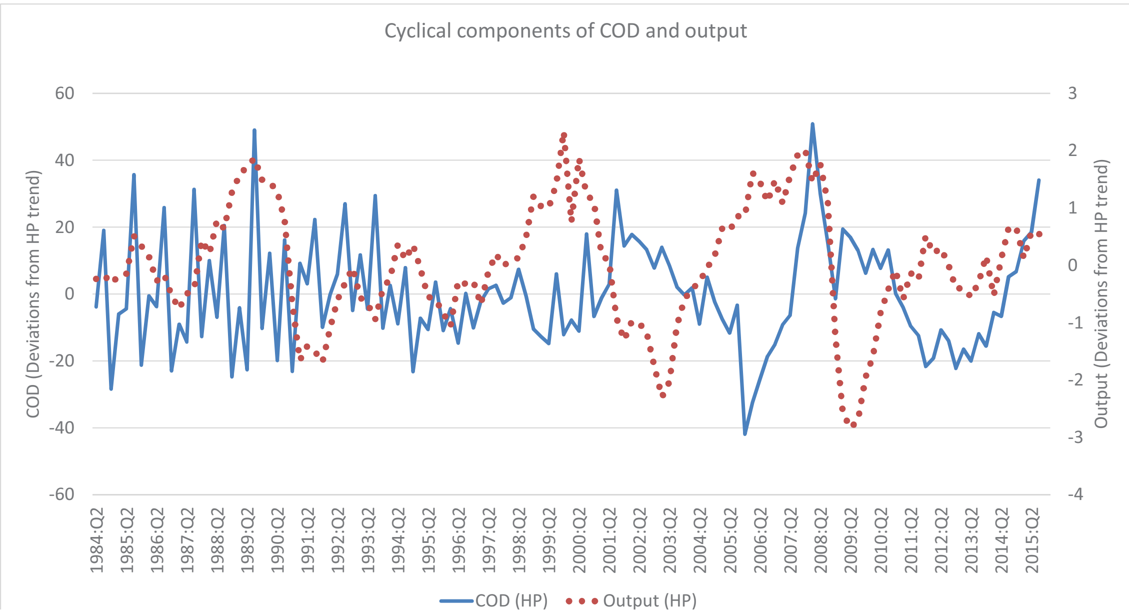

Figure 2: Cyclical component of COD and Y in U.S. data

To document the relationship of the COD ratio with other variables over the business cycle we detrend all the time series using the HP filter with a smoothing parameter equal to 1600.[5] We first compare the cyclical components of COD and GDP in Figure 2.

[5]More details on our data can be found in the Appendix.

Movements in the two series appear to be negatively correlated. This pattern is especially apparent over the last two cycles. Table 1 presents the correlation of COD with some key variables. COD is negatively correlated with GDP and aggregate investment and positively correlated with credit spread. The table also shows that credit spread is more volatile than the COD ratio (18 and 16.4 respectively) while investment (4.5) and GDP (1.1) are much less volatile.

We now turn to the description of our model.

Table 1: Statistics

Panel 1: U.S. data 1984Q1-2015Q4

	Y	I	cdt sprd	COD
correl w/ COD	-0.2	-0.3	0.4	1.0
Std Dev.	1.1	4.5	18.0	16.4

3 Model

The model economy is a relatively standard New Keynesian framework with the addition of a financial accelerator mechanism from Bernanke, Gertler and Gilchrist (1999). The model builds on Gunn (2018), which is a New Keynesian interpretation of the real model of Gunn and Johri (2013) that features a stochastic default cost process. The economy consists of a large number of identical households, a single competitive final goods firm, a continuum of monopolistically competitive intermediate goods firms, one each of a competitive capital-producer and financial intermediary, a continuum of risk-neutral entrepreneurs indexed by $i \in [0, 1]$, a continuum of monopolistically competition labour unions, a competitive employment agency and a monetary policy authority. The nominal frictions include Calvo-style wage and price stickiness with partial indexation. We follow the decentralization of Schmitt-Grohe et al. (2007) and Smets and Wouters (2007) whereby a monopolistic union buys homogeneous labour from households, transforms it into a differentiated labour inputs, and sells it to the employment agency who aggregates the differentiated labour into a composite which it then sells to intermediate goods producers. Since this particular decentralization of wage stickiness implies that consumption and hours are identical across households, for simplicity we will refer to a stand-in representative household. The monetary policy authority sets the nominal interest rate using a rule which is a function of the inflation rate, the output growth rate and the past nominal interest rate.

There are seven stochastic processes in the model: J_t (preference), ν_{p_t} (price markup), ν_{wt} (wage markup), z_t (neutral technology), θ_t (default cost), η_t (monetary policy) and m_t (marginal efficiency of investment)[6]. The processes for technology shocks and default cost shocks both include a four-period ahead anticipated component. We refer to the version of the model with all seven stochastic processes as the *full model*. In addition, throughout our analysis, we also consider a version of the model where we shut-down the default cost process θ_t such that θ is constant, only including the six remaining stochastic processes commonly found in the literature. We refer to this as the *baseline* model.

Our description of the model in the main text focuses on the financial sector portion while other model elements which are common to a host of New Keynesian and Financial Accelerator models are relegated to Appendix A.

3.1 Financial Intermediary

At the end of each period t the financial intermediary makes a portfolio of loans to the measure of entrepreneurs, with B_{it+1} denoting the loan to the i^{th} entrepreneur, funding this portfolio of loans by issuing securities, A_{t+1}, to the household that promise a risk-free gross return, R^a_{t+1}. The financial intermediary has no other sources of funds, and thus it must generate a total return on its loan portfolio in each aggregate contingency to just cover its opportunity cost of funds on the household securities. As in Bernanke et al. (1999), each risk-neutral entrepreneur bears all the aggregate risk on its loan and

[6] For more details, see the description of stochastic processes in section A.8.

thus makes state-contingent loan payments that ensure that in each aggregate state of the world the financial intermediary achieves an expected return equal to its opportunity cost of funds. This leaves the intermediary with only the idiosyncratic risk associated with individual loans, which it can diversify away by virtue of holding a large loan portfolio.

3.2 Entrepreneurs

Risk-neutral entrepreneurs accumulate physical capital and make the capacity utilization decisions for their capital. The timing of the decisions of the i^{th} entrepreneur is as follows. The entrepreneur enters into period t with predetermined capital stock K_{it}, purchased at the end of the previous period from capital producers for price q_{t-1}, as well as debt obligations B_{it}. After observing the aggregate state in period t, the entrepreneur chooses the capital utilization rate u_{it} and then rents capital services $\tilde{K}_{it} = u_{it}K_{it}$ at rental rate r_t to intermediate goods firms. The entrepreneur then sells its entire capital stock to capital-goods producers for price $\bar{q}_t$, realizing its ex-post return to that capital, R^k_{it}, given by

$$R^k_{it} = \omega_{it} \left[\frac{u_{it}r_t - a(u_{it}) + \bar{q}_t}{q_{t-1}} \right]. \tag{1}$$

In the above expression, ω_i is a random variable providing an idiosyncratic component to entrepreneur i's return, such that the ex-ante return to capital is subject to both idiosyncratic and aggregate risk. The random variable ω is i.i.d across firms and time, has cumulative distribution function $F(\omega)$, and is normalized so that $E\omega = 1$. Note that the entrepreneur observes this idiosyncratic component when realizing its return, but after making its capacity utilization decision. As in Christiano et al. (2003), entrepreneurs incur a cost $a(u_{it})$ per unit of capital in terms of goods for utilization rate u_{it}, where $a'(\cdot), a''(\cdot) > 0$, such that changing utilization influences the enterpreneur's return to capital as in (1). The parameter controling the curvature of the cost function is denoted ϵ_u.

After realizing its return, the entrepreneur makes any necessary payments to the financial intermediary to fulfill the terms of its contract determined the previous period. Finally at the end of the period, the entrepreneur chooses its desired level of capital, K_{it+1}, to hold into the following period, buying it from the capital producer for price q_t. Entrepreneurs finance these capital purchases with their own end-of-period net-worth, X_{it+1}, and new loans from the financial intermediary B_{it+1}, such that their financing satisfies

$$q_t K_{it+1} = X_{it+1} + B_{it+1}. \tag{2}$$

Entrepreneurs face a constant probability, γ, of surviving into the next period. When entrepreneurs die they consume their entrepreneurial equity, c^e_{it}. Finally, entrepreneurs supply a unit time endowment inelastically to the good-producers at wage-rate w^e_t.

3.3 Agency problem and debt-contract

The financial intermediary can observe the average return to capital R^k_t but not an entrepreneur's idiosyncratic component ω_{it}, unless it pays a monitoring cost. As a consequence the parties can adopt a financial contract that minimizes the expected agency costs, in the form of risky-debt where the monitoring costs are incurred only in states where an entrepreneur fails to make promised debt payments. In the model we pool this monitoring cost along with all other costs related to the default process and refer to them as "default costs." As in Bernanke et al. (1999) these default costs take the form of a fraction, θ_t, of the entrepreneur's gross payout, $\omega_{it} R^k_t q_{t-1} K_{it}$, however, unlike Bernanke et al. (1999), here θ_t is time-varying and follows an exogenous stationary stochastic process around its steady state. We refer to it as a *default cost (DC) shock*, common to all entrepreneurs, and observable by all agents in the economy.

At the end of period t, the entrepreneur chooses its capital expenditures, $q_t K_{it+1}$ and associated level of borrowing, B_{it+1}, with knowledge of neither the aggregate state in period $t+1$ nor the idiosyncratic realization of ω in period $t+1$, ω_{it+1}. Conditional on these choices, the terms of the contract between the financial intermediary and the entrepreneur specify a contractual non-default state-contingent gross interest rate, R^l_{it+1} that ensures that in each aggregate state of the world, the financial intermediary achieves an expected return, net of costs, equal to the its opportunity cost of funds. In the event that the entrepreneur's idiosyncratic returns are insufficient to cover its contracted debt payments, the entrepreneur defaults and goes bankrupt, handing over all remaining gross returns to the financial intermediary, leaving the gross returns less default costs to the financial intermediary. Note that given the state-contingent contract structure, the loan rate R^l_{it} will adjust in period t to reflect the ex-post realization of the aggregate state in t. We show in the appendix that such a contract results in the condition

$$[\Gamma(\bar{\omega}_{it+1}) - \theta_{t+1}G(\bar{\omega}_{it+1})]\, R^k_{t+1}q_tK_{it+1} = R^a_{t+1}\left(q_tK_{it+1} - X_{it+1}\right), \tag{3}$$

where $\bar{\omega}_{it+1}$ is a "cut-off" level of ω_{it}, defined by $\bar{\omega}_{it+1}R^k_{t+1}q_tK_{it+1} = R^l_{it+1}B_{it+1}$, $\Gamma(\bar{\omega})$ is the financial intermediary's expected share of gross returns, given by $\Gamma(\bar{\omega}_{it}) = [1 - F(\bar{\omega}_{it})]\bar{\omega}_{it} + \int_0^{\bar{\omega}_{it}} \omega dF(\omega)$, and where $G(\bar{\omega})$ is given by $G(\bar{\omega}_{it}) = \int_0^{\bar{\omega}_{it}} \omega dF(\omega)$. Equation (3) defines a menu of contracts for a given level of net-worth X_{it+1} relating the entrepreneur's choice of K_{it+1} to the cut-off level of $\bar{\omega}_{it+1}$.

3.4 Entrepreneur's problem

The i^{th} entrepreneur's gross return in period t, after realization of the aggregate state but before the resolution of idiosyncratic risk, is given by

$$V^k_{it} = \int_{\bar{\omega}_{it}}^{\infty} \omega R^k_t q_{t-1}K_{it}dF(\omega) - R^l_{it}B_{it}. \tag{4}$$

Given the definition of R^k_{it} in (1), all entrepreneurial-indexed variables are predetermined at the timing of the utilization decision, and thus we can simply represent the entrepreneur as choosing capacity utilization u_{it} to maximize $u_{it}r_t - a(u_{it})$, yielding the first-order condition

$$r_t = a'(u_{it}). \tag{5}$$

For a given level of net-worth X_{it+1}, the entrepreneur then chooses K_{it+1} capital and the loan cut-off ω_{it+1} to maximze $E_t\{V^k_{it+1}\}$ subject to the condition that the financial intermediary's expected return on the contract equal its opportunity cost of its borrowing, equation (3). Letting λ_{it+1} be the ex-post value of the Lagrange multiplier conditional on realization of the aggregate state, and writing the period $t+1$ ex-post gross returns as $V^k_{it+1} = [1 - \Gamma(\bar{\omega}_{it+1}]\, R^k_{t+1} q_t K_{it+1}$, where $1 - \Gamma(\bar{\omega}_{it+1})$ is the entrepreneur's expected share of gross returns, the first-order conditions are then

$$\Gamma'(\bar{\omega}_{it+1}) - \lambda_{t+1}\left[\Gamma'(\bar{\omega}_{it+1}) - \theta_{t+1} G'(\bar{\omega}_{it+1})\right] = 0 \tag{6}$$

$$E_t\left\{\left[1 - \Gamma(\bar{\omega}_{it+1})\right] \frac{R^k_{t+1}}{R^a_{t+1}} + \lambda_{t+1}\left(\left[\Gamma(\bar{\omega}_{it+1}) - \theta_{t+1} G(\bar{\omega}_{it+1})\right] \frac{R^k_{t+1}}{R^a_{t+1}} - 1\right)\right\} = 0 \tag{7}$$

$$\left[\Gamma(\bar{\omega}_{it+1}) - \theta_{t+1} G(\bar{\omega}_{it+1})\right] R^k_{it+1} q_t K_{it+1} - R^a_{t+1}\left(q_t K_{it+1} - X_{it+1}\right) = 0, \tag{8}$$

where (6) and (8) hold in each contingency, but (7) holds only in expectation.

3.5 Charge-offs and defaults

In sections 1 and 2 we reported that defaults and charge-offs are not perfectly correlated in U.S. banking data and that their ratio, COD, is highly volatile. Here we show why the baseline model has a hard time replicating these facts and how the introduction of default cost shocks in the full model can, in theory, help the model match those facts.

We calculate the value of defaults in the model as

$$Def_t = F(\bar{\omega}_t) R^\ell_t B_t = F(\bar{\omega}_t) \bar{\omega}_t R^k_t q_t K_t. \tag{9}$$

Charge-offs are the difference between the value of defaulted loans and the resources obtained by financial intermediaries as part of the default process after incurring all associated expenses. These include monitoring entrepreneurs who defaulted and the resources needed to appropriate the returns of these projects. They are calculated as follows

$$Chof_t = F(\bar{\omega}_t)R_t^{\ell}B_t - (1-\theta_t)G(\bar{\omega}_t)R_t^k q_t K_t = [F(\bar{\omega}_t)\bar{\omega}_t - (1-\theta_t)G(\bar{\omega}_t)]R_t^k q_t K_t. \quad (10)$$

We can combine the two equations above to write

$$COD_t = \frac{Chof_t}{Def_t} = \left[1 - \frac{(1-\theta_t)G(\bar{\omega}_t)}{F(\bar{\omega}_t)\bar{\omega}_t}\right] \quad (11)$$

which makes it clear that the wedge between charge-offs and defaults depends on the ratio appearing in square brackets. If θ were a fixed number, as is the case in the baseline model, only general equilibrium changes in $\bar{\omega}_t$ would produce variation in this wedge. As explained in section 3.3, $\bar{\omega}_t$ is the cutoff value of ω_t such that the entrepreneur's gross payout exactly equals the contracted amount. In other words, $\bar{\omega}_t$ is defined by $\bar{\omega}_t R_t^k q_{t-1} K_t = R_t^{\ell} B_t$ and it adjusts in response to period t shocks. For example, in response to a positive productivity shock, the average return to capital R_t^k increases while the loan rate R^{ℓ} decreases forcing $\bar{\omega}_t$ down. This implies that fewer entrepreneurs (with worse shocks) default on their loans. At the same time, the average return on the defaulted projects is also lower. Recall that $G(\bar{\omega}_t)$ calculates the average productivity of entrepreneurs who default while $F(\bar{\omega})$ is a CDF which implies that both of these quantities are increasing in $\bar{\omega}$. The simultaneous impact of a change in $\bar{\omega}_t$ on both the numerator and denominator of COD should imply relatively small changes in COD in the baseline model compared to the data. If our conjecture is correct, then the baseline model will display (i) a high correlation between charge-offs and defaults, and (ii) a low variance in the COD ratio. Of course, very large shocks of just the right magnitude could in principle generate the required volatility but they would run afoul of other macroeconomic data such as real GDP, aggregate investment etc. Our solution is to discipline shocks in the baseline model using standard observables used in the New Keynesian DSGE literature and then look at the implied movement in COD from the estimated baseline model in the next section. The results in Section 4.1 show that our conjecture is correct.

Now suppose that θ_t is an exogenous random variable. As can be seen from (11), shocks to θ have a direct positive effect on COD, over and above the general equilibrium effect coming through the response of $\bar{\omega}$ discussed above. As a result, default cost shocks break the tight link that exists between charge-offs and defaults in the baseline model and potentially provide a source for increasing the variance of COD relative to the baseline model. If, however, changes in $\bar{\omega}_t$ undo the impact of changes in θ_t, then we may not get the desired results. The overall impact of both endogenous and exogenous sources of movement in COD can only be ascertained by simulating the full model. We provide these results in section 4.2.

Finally, recall that the remaining parts of the model are described in Appendix A. That appendix also contains additional details about the agency problem and debt-contracting problem as well as a definition of equilibrium.

4 Results

4.1 Baseline model: no default cost shocks

Our derivation of the COD ratio implied by the baseline model (see section 3.5) led to the conjecture that the model with a constant θ would generate a very low variance in that ratio and nearly perfectly correlated charge-offs and defaults. We test this conjecture by studying the implications of an estimated version of the baseline model, holding θ constant and only including the six stochastic processes commonly found in the literature. Measurement error is introduced on consumption, hours, real wage and investment. The observables used in the model estimation are commonplace – nominal interest rate, the inflation rate, total hours, and the growth rates of GDP, aggregate consumption, aggregate investment, and real wage. In addition to these, financial accelerator models also include the credit spread and we follow suit. Details on data construction and sources can be found in Appendix B.

4.1.1 Parameterization

In this section we explain how we attribute values to parameters. We group model parameters in two sets. The first one contains parameters that are not estimated while the second one contains those estimated using Bayesian methods as in An and Schorfheide (2007). Since the main goal of estimating the baseline model is to obtain the overall movement in $\bar{\omega}_t$ due to the combined general equilibrium effect of all the shocks in the baseline model, we choose to calibrate all non-shock related parameters to the values estimated in the literature. Later, when we estimate the full model with default cost shocks added to the current set of shocks, this approach will have the advantage that all changes will be the result of new estimates of the stochastic elements of the model and none will be due to changes in parameters. For completeness, we note that our results are not sensitive to actually estimating the typical parameters around our calibrated values.

For parameters in the non-estimated set, we use typical values established in the literature or we choose the parameters to match relevant steady state quantities in the model economy with analogous quantities in the data. The values of parameters not estimated are shown in Table 2. Beginning with the parameters common to standard real-business cycle models, we set the share of labor in production, α to 0.67, and the depreciation of physical capital, δ to 0.025. On the preference side, we set the household's subjective discount factor β to 0.99, implying a net annualized risk-free interest rate of 4.1%, and implying a quarterly gross return on household financial assets $R^d = (1 + r^f)^{0.25} = 1.0101$. Three other parameters appear in the utility function in equation (A.16). They are the curvature parameter on the disutility of labour (σ_L), the weight on the disutility of labour in the utility function (Ψ_L), and the consumption habit persistence parameters (b). We follow Christiano et al (2014a) and set $\sigma_L = 1$. We set Ψ_L so that the household time allocated to market work hours is normalized to 0.3. We come back to the habit parameter below.

Table 2: Parameters not estimated

Description	Parameter	Value
Household subjective discount factor	β	0.99
Curvature on disutility of labour	σ_L	1
Steady state hours-worked	N_{ss}	0.3
Habit persistence parameter	b	0.8
Capital depreciation rate	δ	0.025
Curvature, investment adjustment cost	s''	6
Curvature, utilization cost	ϵ_u	2
Labour share in production	α	0.67
Household share of total labour in production	τ	0.99
Monetary policy smoothing parameter	ρ_{rn}	0.8
Monetary policy weight on inflation	ϕ_{pi}	1.75
Monetary policy weight on output growth	ϕ_y	0.2
Calvo price stickiness	ζ_p	0.7
Calvo wage stickiness	ζ_w	0.8
Price indexing weight on inflation	ι_p	0.2
Wage indexing weight on inflation	ι_w	0.5
Steady state price markup	$\lambda_p \equiv 1/\nu_p$	1.1
Steady state wage markup	$\lambda_w \equiv 1/\nu_w$	1.1
Steady state government spending-GDP ratio	$\frac{G}{Y}$	0.18
Steady state default rate	$F_{\bar{\omega}}$	0.0076
Steady state external finance premium	EFP	0.005
Steady state fractional monitoring cost	θ	0.12

For the parameters associated with the financial contract and the entrepreneur, we follow Bernanke et al. (1999) in setting these parameters. In steady state, the external finance spread, $R^k - R^d$, equals 0.005 quarterly, leverage, K/X, is approximately 2, and the fraction of entrepreneurs defaulting each quarter is 0.0076. We set the quarterly survival rate of entrepreneurs to 0.9799, the variance of $\log \bar{\omega}$ to 0.0908, and steady-state fraction of gross returns lost in default, θ, to 0.12.

We now discuss the parameters in the New Keynesian block of the baseline model. Values are taken to be round numbers in the range between well known studies such as Smets and Wouters (2007) and Ajello (2016).

The habit persistence parameter (b) is set to .8 which is in the range of .7 to .85 found in the literature. We set price and wage stickiness parameters ($\zeta_p = .7$ and $\zeta_w = .8$ respectively) to be within the tight range found in the literature, noting that the latter tends to be estimated higher in many studies. Similarly, the price and wage indexation parameters ($\iota_p = .2$ and $\iota_w = .5$), lie between the values estimated in Ajello and Smets and Wouters (2007). The nominal interest rate smoothing parameter (ρ_{rn}) is usually found to be very persistent. We use a value of 0.8. For the monetary policy weight on inflation (ϕ_π) we use 1.75 and for the monetary policy weight on output (ϕ_y) we use 0.2. The cost functions curvature parameters associated with utilization and adjustment costs (ϵ_u and s'' respectively) are set to 2 and 6, respectively.

We now turn to the autocorrelation and variance of our shocks. These are all estimated using Bayesian methods using the prior distributions reported in Table 3. First, all of the shocks autocorrelation parameters share a beta distribution with mean 0.5 and standard deviation 0.2. Second, the standard deviations of all anticipated and unanticipated innovations share an inverse gamma distribution with mean 1 and a standard deviation of 10. The upper bound of the uniform distribution of the standard deviation of the measurement error on any series (consumption, hours, investment and wages) is 10% of the standard deviation of that series. The lower bound of the uniform prior distribution is 0.001 for all four observables.

Table 3: Baseline Model: priors and posteriors

Description	**Parameter**	**Prior mean**	**Posterior mean**	**90% HPD interval**		**Prior distrib.**	**Prior std dev.**
	autocorrelation of shocks						
Technology process	ρ_z	0.5	0.359	0.293	0.421	beta	0.2
MEI process	ρ_{mei}	0.5	0.974	0.970	0.978	beta	0.2
Preference process	ρ_J	0.5	0.498	0.400	0.558	beta	0.2
Monetary policy process	ρ_η	0.5	.0035	.0004	.0067	beta	0.2
Price markup process	ρ_{ν_p}	0.5	0.918	0.905	0.932	beta	0.2
Wage markup process	ρ_{ν_w}	0.5	0.233	0.206	0.257	beta	0.2
	standard deviation of shocks						
Technology, unanticipated	ϵ_z	1	9.545	7.393	11.133	inverse gamma	10
Technology, anticipated	ϵ_z^4	1	16.614	14.378	19.450	inverse gamma	10
MEI, unanticipated	ϵ_m	1	7.259	6.863	7.801	inverse gamma	10
Preferences, unanticipated	ϵ_J	1	25.678	24.127	27.815	inverse gamma	10
Monetary policy, unanticipated	ϵ_η	1	0.378	0.139	0.979	inverse gamma	10
Price markup, unanticipated	ϵ_{ν_p}	1	0.3667	0.319	0.413	inverse gamma	10
Wage markup, unanticipated	ϵ_{ν_w}	1	0.448	0.387	0.506	inverse gamma	10
	standard deviation of measurement errors						
Hours		0.203	0.404	0.402	0.405	uniform	0.117
Consumption growth		0.031	0.061	0.061	0.061	uniform	0.017
Investment growth		0.094	0.160	0.159	0.162	uniform	0.053
wage growth		0.044	0.086	0.086	0.086	uniform	0.025

Finally, we solve the model by using standard methods to linearize the non-linear system about its steady state.

4.1.2 Estimation results

Table 3 also reports the posterior distributions of estimated parameters. The top panel illustrates the degree of persistence of shocks. Price markup shocks and marginal efficiency of investment (MEI) shocks are the most persistent with a first-order autocorrelation coefficient above 0.9. Technology shocks, preference shocks and wage markup shocks are moderately persistent (autocorrelation in the 0.3-0.5 range). Monetary policy shocks essentially exhibit no serial correlation.

The second panel of Table 3 reveals a great deal of variation in the estimated standard deviations of the different shocks. They range from less than .5 for monetary policy shocks, price and wage markup shocks all the way to 25.7 for preference shocks. Unanticipated and anticipated shocks to technology, and MEI range from 7 for MEI to 16.6 for anticipated technology shocks. The last panel reports the mean of the estimated posterior of the standard deviation of measurement error which tends to lie below .5. All posterior means are slightly higher than the prior means.

4.1.3 Implications of estimated baseline model

We now compare the baseline model's predictions to the statistics reported in the first panel of Table 4 based on U.S. data. The second panel of that table reports two sets of statistics. The first row reports the theoretical moments of HP filtered variables implied by the estimated shock parameters. The second set uses the smoothed time series implied by the parameter estimates *and the estimated shocks* (we HP filter the smoothed series just like we HP filter U.S. time series). As conjectured, the standard deviation of the COD ratio is much lower than its counterpart in U.S. data (less than 1 *versus* 15.6) and the correlation of defaults and charge-offs is nearly perfect.

Table 4: Baseline Model

	$\frac{SD(chof)}{SD(y)}$	$\frac{SD(def)}{SD(y)}$	$\frac{SD(COD)}{SD(y)}$	$cor(chof, def)$
Panel 1: U.S. data 1984Q1-2015Q4				
	21.8	14.7	15.6	0.7
Panel 2: Estimated Baseline Model (no default cost shocks)				
Theoretical (HP 1600)	6.6	6.4	0.3	1
Simulated (HP 1600)	18.4	17.8	0.6	1
Panel 3: Higher Variance (Theoretical; HP 1600)				
double std deviation of ϵ_z	7.4	7.1	0.3	1
double std deviation of ϵ_z^4	6.3	6.0	0.2	1
double std deviation of ϵ_m	6.8	6.6	0.3	1
double std deviation of ϵ_J	5.3	5.1	0.2	1
double std deviation of ϵ_η	7.4	7.1	0.3	1
double std deviation of ϵ_{ν_p}	7.4	7.1	0.3	1
double std deviation of ϵ_{ν_w}	6.7	6.4	0.3	1

As additional evidence that the shocks' general equilibrium effect on $\bar{\omega}$ is too small to produce significant variation in COD in the baseline model, we conduct the following exercise: (i) keeping all parameters at the values shown in Table 3; (ii) we double the standard deviation of one of the shocks: (iii) then we calculate theoretical moments and report them in the third panel of Table 4. Clearly, doubling the standard deviations of shocks has very little impact on the relative standard deviation of the COD ratio and no effect at all on the correlation of defaults and charge-offs. We conclude that the baseline model cannot produce the joint behavior of defaults and charge-offs.

4.2 The full model: adding stochastic default cost shocks

4.2.1 Default costs shocks — charge-offs and defaults statistics

We argued above using equation (11) that variation in θ has the potential to lower the correlation between charge-offs and defaults as well as raise the variance of COD. We now add these default cost shocks to the baseline model (adopting the parameters estimates reported in Table 3) and use model simulations to document how the relevant statistics change with the standard deviation of anticipated and unanticipated θ shocks (denoted σ_θ^4 and σ_θ^0 respectively) as well as with the autocorrelation parameter ρ_θ. The second panel of Table 5 is divided into three parts. In the first part, we increase the standard deviation of the surprise component from 1 to 10 keeping $\rho_\theta = 0.9$ (no anticipated shocks are included). In the second part we increase the standard deviation of the anticipated component from 1 to 10 keeping $\rho_\theta = 0.9$ (no surprise shocks are included). In the third part, we increase ρ_θ from .5 to .99 (only surprise shocks are included).

Table 5: Specifications with Default Cost Shocks

			$\frac{SD(chof)}{SD(y)}$	$\frac{SD(def)}{SD(y)}$	$\frac{SD(COD)}{SD(y)}$	$cor(chof, def)$
Panel 1: U.S. data 1984Q1-2015Q4						
			21.8	14.7	15.6	0.7
Panel 2: Default costs shocks added to Baseline Model						
			(Theoretical; HP 1600)			
σ_θ^0	σ_θ^4	ρ_θ				
0	0	0	6.6	6.4	0.3	1
1	0	0.9	6.7	6.4	0.8	0.99
10	0	0.9	12.4	7.3	7.1	0.86
0	1	0.9	6.7	6.4	0.7	0.99
0	10	0.9	10.5	7.6	7.0	0.75
10	0	0.5	9	6.4	5.7	0.78
10	0	0.99	15.3	9.6	7.2	0.93
Panel 3: Estimated Full Model						
Simulated (HP 1600)			17.9	24.4	25.9	0.28

Looking at the last two columns of Table 5, it is clear that increasing the variance of the anticipated and unanticipated intermediation costs shocks have large (and very similar) impacts on these statistics. The standard deviation of COD relative to that of GDP rises from below 1 to 7 while the correlation between charge-offs and defaults falls from .99 to .86 with unanticipated shocks and .99 to .75 with anticipated shocks. Increasing ρ_θ causes an increase in the correlation of charge-offs and defaults and a small positive impact on the relative standard deviation of COD. Additional analysis, not displayed here, reveals that the larger the variance of the default cost shocks, the lower the correlation and the larger is the relative standard deviation of COD. The first two columns of the table reveal that increasing the volatility of default costs has a larger influence on the volatility of charge-offs than on defaults, and this helps to explain why COD becomes more variable.

4.2.2 Estimation of the full model with default cost shocks

The results in section 4.2.1 reveal that the endogenous movements in $\bar{\omega}$ in response to θ_t shocks do not reverse the primary effects on COD. The next step is to discipline the size of these shocks to U.S. data and use the full model, estimated without the COD series, to make predictions about the moments of COD. These can then be compared to the U.S. data moments discussed earlier. To do this, we use the same parameter values and macroeconomic data as used in the estimation of the baseline model. In addition, all stochastic series included in the baseline model are also included here with the same priors. The only difference is the addition of default cost shocks. As before, for the priors, all shocks are treated exactly the same. Table 6 displays the prior and posterior distributions from this exercise.[7]

There is some variation across shocks in their degree of persistence and volatility. Default cost shocks, technology shocks and price markup shocks are the most persistent (autocorrelation coefficient=0.96) while wage markup shocks exhibits very little serial correlation (coefficient=0.04). The persistence of other shocks lie between these two extremes. The default cost shocks have the highest standard deviations (10 for the unanticipated component and 12 for the anticipated one) followed by mei shocks (6) and preference shocks (3.5). All other shocks have standard deviations less than one. The standard deviation of measurement errors is also quite small and close to the priors.

[7] Estimated shocks are in Figure 6 while historical and simulated/smoothed variables are displayed in Figure 7. These figures are in Appendix C.

Table 6: Full Model: priors and posteriors

Description	**Parameter**	**Prior mean**	**Posterior mean**	**90% HPD interval**		**Prior distrib.**	**Prior std dev.**
	autocorrelation of shocks						
Technology process	ρ_z	0.5	0.9607	0.9319	0.9907	beta	0.2
MEI process	ρ_{mei}	0.5	0.8961	0.847	0.947	beta	0.2
Default cost process	ρ_θ	0.5	0.9627	0.9471	0.9787	beta	0.2
Preference process	ρ_J	0.5	0.8028	0.7405	0.8684	beta	0.2
Monetary policy process	ρ_η	0.5	0.498	0.404	0.5914	beta	0.2
Price markup process	ρ_{ν_p}	0.5	0.9576	.9069	0.9977	beta	0.2
Wage markup process	ρ_{ν_w}	0.5	0.0354	0.0044	0.0652	beta	0.2
	standard deviation of shocks						
Technology, unanticipated	ϵ_z	1	0.286	0.217	0.355	invg	10
Technology, anticipated	ϵ_z^4	1	0.322	0.233	0.413	invg	10
MEI, unanticipated	ϵ_m	1	6.162	4.252	8.060	invg	10
Default cost, unanticipated	ϵ_θ	1	10.141	8.747	11.472	invg	10
Default cost, anticipated	ϵ_θ^4	1	12.078	10.541	13.594	invg	10
Preferences, unanticipated	ϵ_J	1	3.501	3.073	3.934	invg	10
Monetary policy, unanticipated	ϵ_η	1	0.133	0.119	0.144	invg	10
Price markup, unanticipated	ϵ_{ν_p}	1	0.130	0.118	0.141	invg	10
Wage markup, unanticipated	ϵ_{ν_w}	1	0.436	0.388	0.482	invg	10
	standard deviation of measurement errors						
Hours		0.203	0.275	0.174	0.382	uniform	0.117
Consumption growth		0.031	0.058	0.054	0.061	uniform	0.017
Investment growth		0.094	0.175	0.161	0.186	uniform	0.053
wage growth		0.044	0.041	0.003	0.074	uniform	0.025

Comparing the estimates of the shock processes to those of the baseline model, we note a few changes. In the full model, the posterior mean of the standard deviation of technology shocks falls a lot for both anticipated and unanticipated shocks. Preference shocks are also estimated to be much less volatile. In terms of persistence, monetary policy shocks display an increased autocorrelation in the full model.

Panel 3 of Table 5 displays the relative standard deviation of the COD ratio for the smoothed COD series based on the actual estimated shocks in the full model. It is clear that the full model is successful at producing volatility in COD with a relative standard deviation of 25.9 *versus* 0.6 in the baseline model. It is also successful at lowering the correlation of charge-offs and defaults (0.28 *versus* 1 in the baseline model). Furthermore, the third panel of Table 7 shows that in the full model COD is negatively correlated with output and investment just like in U.S. data (see panel 1). Remarkably, the full model matches exactly the correlation of COD and credit spreads. This correlation is 0.4 while the baseline model predicts a correlation of 1 (see panel 2). We reiterate that the shocks in the full model were estimated without using COD as an observable.

Table 7: Statistics

Panel 1: U.S. data 1984Q1-2015Q4

	Y	I	cdt sprd	COD
correl w/ COD	-0.2	-0.3	0.4	1.0
Std Dev.	1.1	4.5	18.0	16.4

Panel 2: Baseline Model

	Y	I	cdt sprd	COD
correl w/ COD	-0.5	-0.4	1.0	1.0
Std Dev.	1.1	4.0	18.0	0.6

Panel 3: Full Model

	Y	I	cdt sprd	COD
correl w/ COD	-0.5	-0.8	0.4	1.0
Std Dev.	1.1	4.5	18.0	27.3

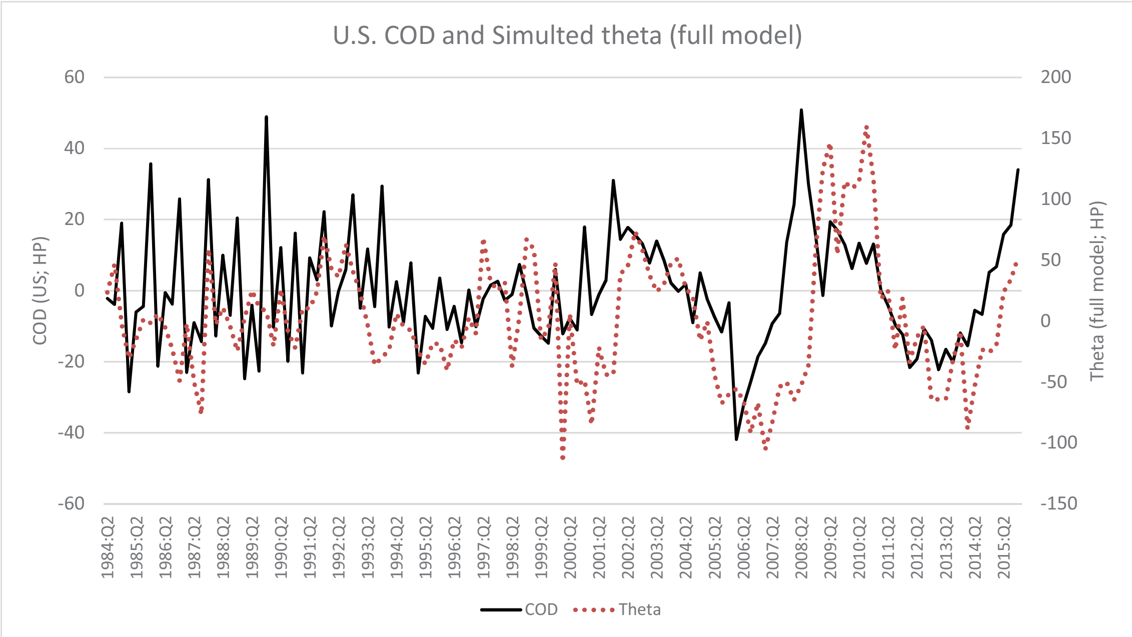

Figure 3: COD (U.S. data) and θ (full model)

Gunn and Johri (2013) argue that default cost shocks help understand the boom-bust cycle associated with the "great recession". A glance at Figure 3 reveals that COD was below trend in the quarters preceding the financial crisis while output boomed. This was followed by a sharp rise in COD above trend during the crisis while output plummeted below trend. The path of the default cost shock θ based on the estimated model is displayed in Figure 3. It provides a visualization of the ideas discussed in Gunn and Johri (2013). In addition, Figure 3 suggests that variation in default costs may have played an important role in other boom-bust episodes as well.

We can use variance decomposition analysis based on the full model to understand the contribution of default cost shocks to U.S. business cycles at various horizons. The top panel of Table 8 reports the unconditional variance decomposition results. Since default cost shocks were shown in Gunn and Johri (2013) to cause large movements in credit spreads, it is not surprising that they account for 98% of the variance in credit spreads. Interestingly, they also account for a noticeable fraction (at least 30%) of the variance of all observables except real wage growth. Price markup shocks also have an important role. They account for more than 15% of the variance of hours worked, output growth, inflation and wage rate growth. Preference shocks explain half of the variance in consumption growth while wage markup shocks explain 70% of the variance of wage growth. Finally, MEI shocks explain nearly 20% of the variance in investment growth.

Table 8: Model with default cost shocks: variance decomposition

Observable	z	z^4	m	η	ν_p	ν_w	J	θ	θ^4
Unconditional variance decomposition									
Hours worked	1.01	1.33	12.19	3.17	18.44	7.06	4.75	17.54	34.51
Consumption growth	1.45	1.55	2.33	1.52	7.49	3.15	50.02	10.97	21.52
Output growth	1.77	1.52	7.72	4.95	16.97	4.85	6.5	14.87	40.86
Investment growth	0.45	0.41	19.25	0.93	5.66	1.36	0.64	26.97	44.33
Nominal int. rate	1.01	0.61	10.9	5.13	9.09	4.35	3.39	22.68	42.83
Inflation	2.33	1.12	7.96	2.03	21.21	9.44	3.29	17.84	34.79
Wage growth rate	1.78	0.85	0.89	0.14	19.05	70.65	0.09	2.12	4.44
Credit spread	0.04	0.03	0.75	0.58	0.7	0.15	0.09	56.68	40.98
1-Period ahead Conditional variance decomposition									
Hours worked	2.83	0.86	3.91	5.84	12.13	6.84	5.52	18.46	43.61
Consumption growth	1.54	1.45	1.12	1.88	7.83	3.15	60.03	7.66	15.34
Output growth	1.5	0.95	3.87	6.01	15.55	3.98	5.68	18.55	43.92
Investment growth	0.38	0.24	20.91	1.08	4.97	1.1	0.44	29.01	41.87
Nominal int. rate	1.53	0.07	5.35	21.46	13.33	6.51	3.31	15.63	32.81
Inflation	3.23	0.01	5.64	1.77	28.85	12.81	2.81	14.79	30.08
Wage growth	1.96	0.08	0.53	0.13	20.74	71.64	0.11	1.52	3.3
Credit spread	0.05	0.01	1.62	1.1	1.02	0.19	0.03	87.41	8.57

Conditional variance decompositions at horizons one, three, eight and twelve periods ahead share the patterns documented above for the unconditional decomposition. One difference between the latter results and the one-period ahead variance decomposition (see second panel of Table 8) is the role of monetary policy shocks that account for 21% of the variance in the nominal interest rate.[8] The one-period ahead decomposition gives us a sense of the magnitude of the pure news effect of an anticipated shock (i.e. the effects on

[8]Three, eight and twelve-period ahead variance decompositions are in Appendix C.

endogenous variables after the news is received but before the shock is actually realized).[9] The second panel of Table 8 shows that the pure news effect of the anticipated default cost shock is very significant for several of the observables. A pattern appearing in all variance decomposition results is that, except for credit spreads, the anticipated component of default cost shocks always explains more than the unanticipated component.

Figure 4 shows the impulse responses to a surprise positive shock to θ. A rise in θ_t means that all else equal, the financial intermediary loses a larger share of the value of defaulting loans in that period. In order to cover its lower net of costs return on defaulted loans and satisfy its zero profit condition, the financial intermediary then raises the loan rate r_t^l, driving up the credit spread. This increases the proportion of loans that default as entrepreneurs are forced to pay higher borrowing costs. Moreover, since the financial intermediary loses a larger fraction of each unit of defaulted loans due to the rise in θ_t, charge-offs have to increase more than defaults, and the COD ratio rises immediately. Additionally, since θ_t is persistent, the rise in θ_t implies a rise in θ_{t+1}, altering the terms for new loan contracts established in period t. In particular, the rise in θ_{t+1} shifts the menu of contracts describing the combinations of cut-off productivity and leverage consistent with the terms of the contract, such that the rise in θ_{t+1} reduces the level of leverage consistent with the contract. All else equal, this leads to a fall in demand for new physical capital by entrepreneurs leading into period $t+1$. This kicks off a chain of general equilibrium effects that leads to an overall fall in aggregate activity. See Gunn and Johri (2013) for a detailed discussion.

Figure 5 shows the impulse responses to a four period out anticipated positive shock to θ that is eventually realized four periods out. The impact of the anticipated shock in Figure 5 is similar to that of the case of the persistent surprise shock in Figure 4,

[9]Note that Sims (2016) argues that unconditional variance decompositions and conditional variance decompositions at horizons greater than the news shock anticipation horizon (defined as the time gap between the period when the news is received and the period where the exogenous variable actually changes) do not provide an accurate calculation of the pure news effect of an anticipated shock because they combine together the pure news effect of the anticipated shock on observables and the effects triggered by the actual realization of the shock.

except that with no actual change in θ in the initial period, the initial impact on the financial intermediary's zero-profit condition is absent. Instead, the sole initial impact effect is through the anticipated shift in contractual terms leading into the expected change in θ in four periods, resulting in a direct drop in the demand for new capital and economic activity in that period. This expected impact on economic activity in four periods then triggers a drop in activity in the preceding periods through various inter-temporal channels described in detail in Gunn and Johri (2013) and Gunn (2018). Importantly - and in contrast to the surprise shock case in Figure 4 - all of the initial response of credit spreads, charge-offs and CODs in the three periods preceding the rise in θ are due to general equilibrium effects only[10]. Thus while credit spreads and charge-offs rise on impact, they only reach their peak later when default costs θ actually rise. Furthermore, COD barely moves on impact, movements in $\bar{\omega}$ being its only driving force absent any variation in θ.

The larger contribution of the anticipated component of default cost shocks shown in the variance decompositions earlier can also be seen in Figures 4 and 5. To make the comparison of these two sets of responses more meaningful, we set the standard deviations of the surprise and news components to the same value. A comparison of Figures 4 and 5 reveals that the initial responses of output growth, inflation, hours, wage growth, consumption growth and nominal interest rate are similar in both figures but slightly larger in Figure 5 (anticipated shock). The slightly larger responses to an anticipated shock, combined with the larger estimated standard deviation of anticipated shocks (20% larger than surprise shocks) explain the larger shares of variance of observables explained by the anticipated component of default cost shocks versus their unanticipated component.

To understand why the surprise component of default cost shocks is more important than the anticipated component for the variance of the credit spread (especially at short

[10]Note however that if the news shock is realized four periods out, there will be a rise in θ four periods out that will impact the financial intermediary's budget constraint in that future period through the realized change in θ in the future, thereby also impacting COD significantly in that period. In contrast, a pure unrealized news shock to θ can only impact COD through general equilibrium effects at all horizons.

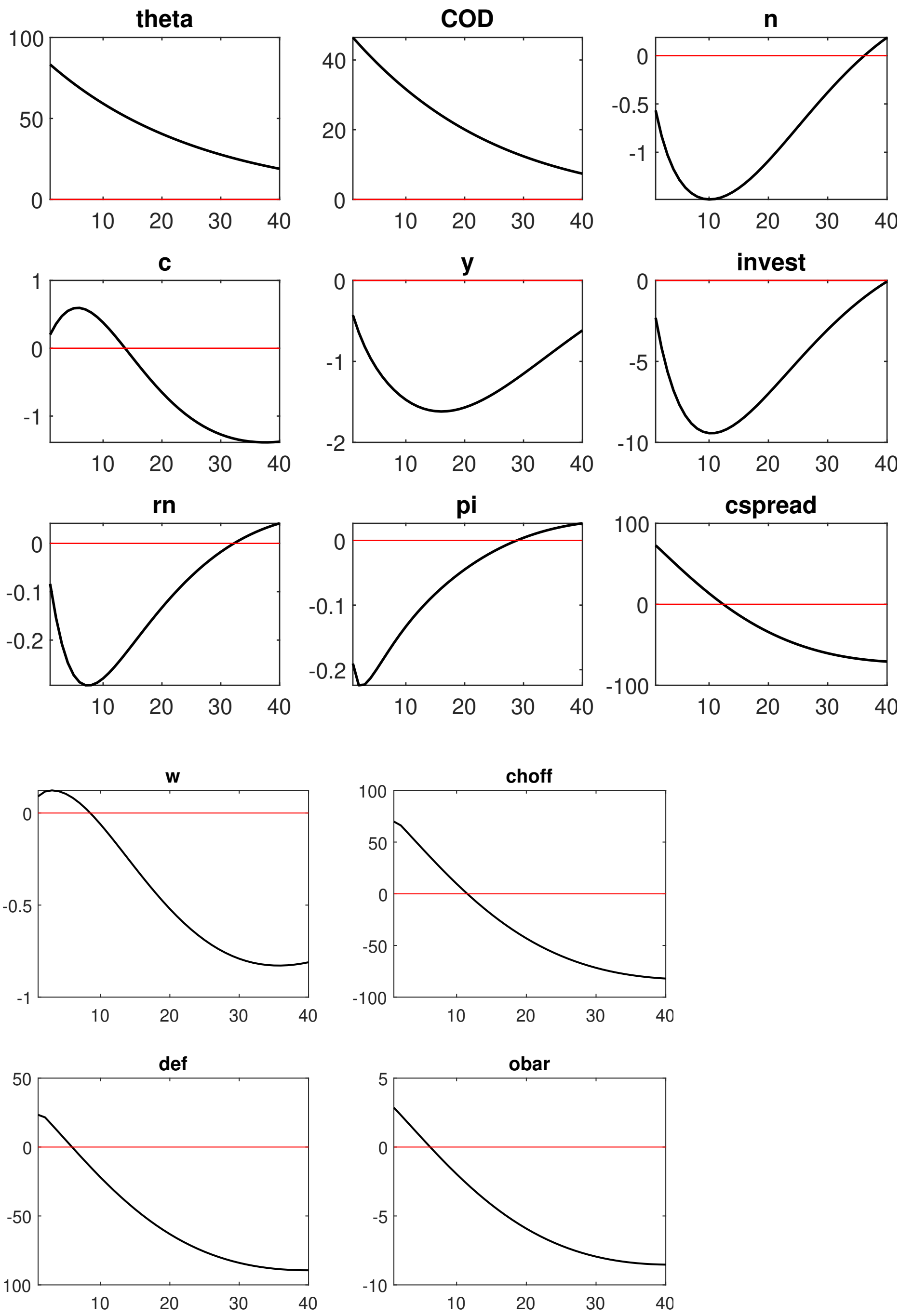

Figure 4: IRFs surprise θ shock

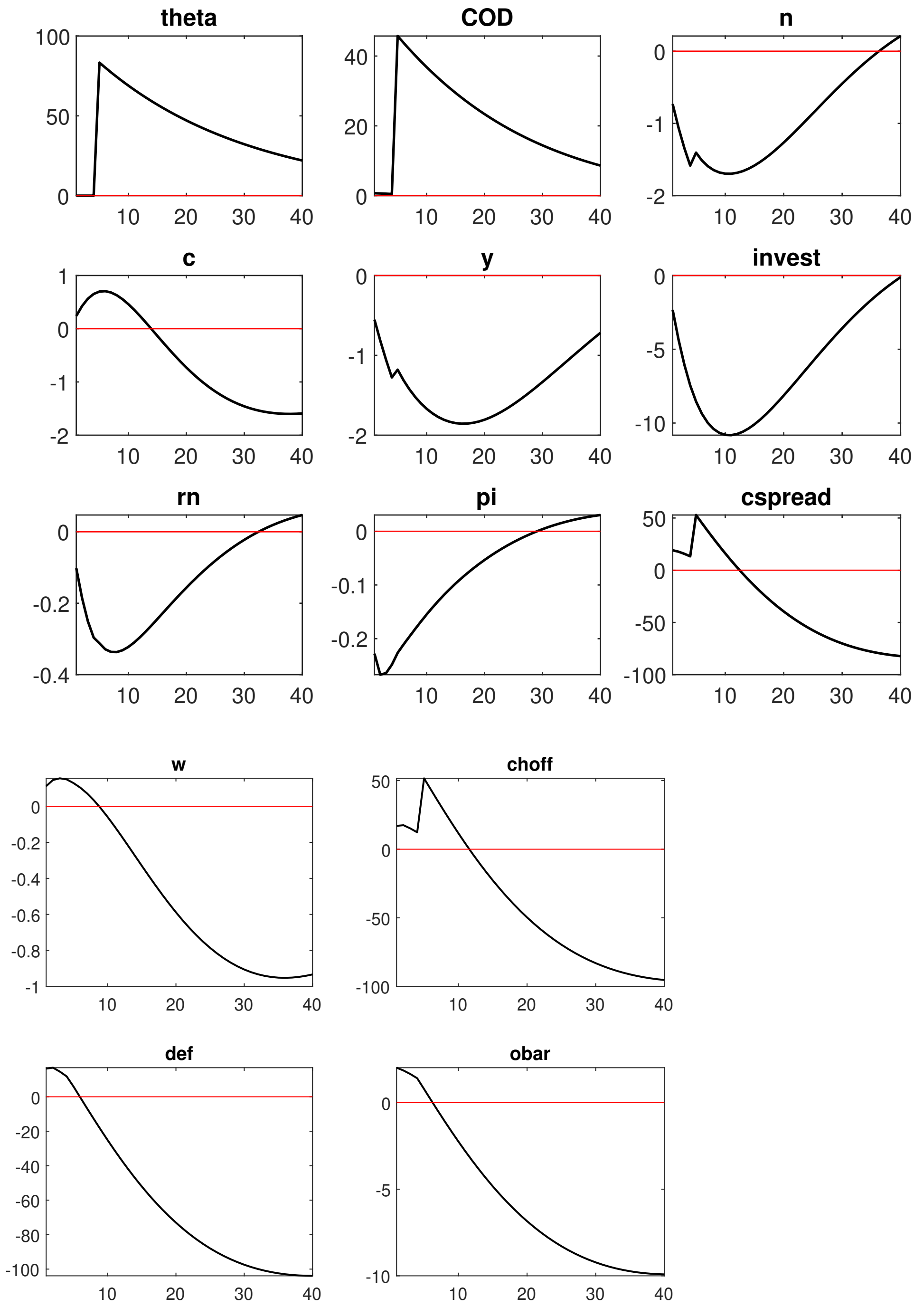

Figure 5: IRFs anticipated θ shock

horizons), it is instructive to relate this result to the impulse responses in figures 4 and 5 and to the correlation of credit spreads with output and investment in U.S. data. The 1-period ahead variance decomposition shows that unanticipated θ shocks explain almost all of the variance in credit spread in the very short run. As explained above and as seen in the figures, it is clear that unanticipated default cost shocks are more effective at producing variance in credit spread in the very short run since the initial response to a θ news shocks is much smaller (about 3 times smaller) than the response to a surprise θ shock. Also, as Figure 5 shows, the initial response of credit spread to a θ news shock is muted with most of the adjustment in credit spread happening when θ actually goes up four periods after the news arrives. This pattern of response would tend to produce a negative correlation between current output (investment) and credit spreads four period later. However, the correlation of output (investment) with credit spreads four period later is only 0.09 (0.04) in our U.S. data. Hence, to capture the co-movement of credit spread with key macro aggregates like output and investment, anticipated default cost shocks play a less prominent role than surprise shocks.

We conclude that a business cycle model augmented with default cost shocks can not only help rationalize the behavior of COD and credit spread but also play an important role in the observed variation of hours worked, investment growth, and output growth over the business cycle.

5 Conclusions

U.S. banks are required to charge-off the value of losses incurred on delinquent loans including all costs incurred as part of the default process. We show that the total amount charged-off does not mechanically follow the total value of defaulted funds as both vary over the business cycle. Moreover, the ratio of charge-offs to defaults is highly volatile and negatively correlated with GDP and positively correlated with credit spreads.

In this paper we show that the canonical business cycle model with bank lending and endogenous defaults associated with Bernanke et al. (1999) cannot explain the patterns discussed above. This occurs because default costs rise and fall in proportion to the value of defaulted loans. Next, we show that the introduction of default cost shocks in the model can reconcile the model predictions with the data. Finally we discipline the default cost shocks to U.S. macroeconomic data in a medium scale New Keynesian Financial Accelerator model with a large number of other stochastic processes in addition to default cost shocks. The model augmented with default costs fits U.S. macroeconomic data well and variance decomposition exercises reveal that default cost shocks play a significant role in explaining variance in these series. Our results suggest that between 1984 and 2015 shocks within the financial sector contributed to over half the variance in the growth rate of real GDP. We use the ratio of total charge-offs over defaults to provide external validity to the estimated model by comparing the predicted moments to actual U.S. data and find that the model does a good job in predicting moments associated with this ratio.

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A Additional Model Detail

A.1 Agency problem and debt-contract

The discussion in the main text regarding the financial intermediary implies that in each aggregate state in period t, the financial intermediary's budget constraint is

$$\xi_t = R_t^a A_t, \tag{A.1}$$

where ξ_t is the intermediary's return on its entire loan portfolio after idiosyncratic uncertainty has been realized, and where R_t^a and A_t are predetermined.

In the financial contract, the cut-off value $\bar{\omega}_{it}$ is defined as

$$\bar{\omega}_{it+1} R_{t+1}^k q_t K_{it+1} = R_{t+1}^l B_{it+1}. \tag{A.2}$$

If the entrepreneur's realization exceeds the threshold such that $\omega_{t+1}(i) \geq \bar{\omega}_{t+1}(i)$, the entrepreneur pays the financial intermediary the contracted amount $R_{it+1}^l B_{it+1}$, keeping the amount $\omega_{it+1} R_{t+1}^k q_t K_{it+1} - R_{it+1}^l B_{it+1}$. If $\omega_{it+1} < \bar{\omega}_{it+1}$, the entrepreneur defaults, receives nothing, and the financial intermediary receives $(1-\theta_t)\omega_{it+1} R_{t+1}^k q_t K_{it+1}$. As with R_{it}^l, $\bar{\omega}_{it}$ adjusts to reflect the aggregate ex-post realizations of the aggregate state in period t.

Given these contract details, we can write the financial intermediary's expected return on a given loan contract in a given aggregate contingency in period $t+1$ as

$$\xi_{it+1} = [1 - F(\bar{\omega}_{it+1})] R_{it+1}^l B_{it+1} + (1-\theta_{t+1}) \int_0^{\bar{\omega}_{it+1}} \omega R_{t+1}^k q_t K_{it+1} dF(\omega) \tag{A.3}$$

Substituting in (A.2), we can write (A.3) in terms of the cut-off $\bar{\omega}$ as

$$\xi(\bar{\omega}_{it+1}, \theta_{t+1}) = \left[[1 - F(\bar{\omega}_{it+1})]\bar{\omega}_{it+1} + (1-\theta_{t+1}) \int_0^{\bar{\omega}_{it+1}} \omega dF(\omega) \right] R_{t+1}^k q_t K_{it+1}. \tag{A.4}$$

Defining the financial intermediary's expected share of gross returns $\Gamma(\bar{\omega})$ as

$$\Gamma(\bar{\omega}_{it}) = [1 - F(\bar{\omega}_{it})]\bar{\omega}_{it} + \int_0^{\bar{\omega}_{it}} \omega dF(\omega), \tag{A.5}$$

and defining $G(\bar{\omega})$ as

$$G(\bar{\omega}_{it}) = \int_0^{\bar{\omega}_{it}} \omega dF(\omega), \tag{A.6}$$

we can re-write the financial intermediary's expected return on a given loan contract in a given aggregate contingency as

$$\xi_{t+1}(\bar{\omega}_{it+1}, \theta_{t+1}) = [\Gamma(\bar{\omega}_{it+1}) - \theta_{t+1} G(\bar{\omega}_{it+1})] R_{t+1}^k q_t K_{it+1}, \tag{A.7}$$

where the terms in square brackets represent the financial intermediary's share of profits net of default costs. The requirement that the financial intermediary earn an expected return in every aggregate contingency equal to its opportunity cost of funds,

$$\xi_{t+1}(\bar{\omega}_{it+1}, \theta_{t+1}) = R_{t+1} B_{it+1} \tag{A.8}$$

then serves as a restriction to define a menu of contracts over loan quantity and cut-off value for the entrepreneur. Substituting in $q_t K_{it+1} = X_{it+1} + B_{it+1}$ and (A.7) we can then write this as

$$\left[\Gamma(\bar{\omega}_{it+1}) - \theta_{t+1} G(\bar{\omega}_{it+1})\right] R^k_{t+1} q_t K_{t+1}(i) = R^a_{t+1} \left(q^n_t K_{it+1} - X_{it+1}\right) \tag{A.9}$$

which for a given level of net-worth X_{it+1} defines a menu of contracts relating the entrepreneur's choice of K_{it+1} to the cut-off $\bar{\omega}_{it+1}$.

A.2 Entrepreneur's contract problem

The entrepreneur's expected gross return, conditional on the ex-post realization of the aggregate state but before the resolution of idiosyncratic risk, is given by

$$V^k_{it+1} = \int_{\bar{\omega}_{it+1}}^{\infty} \omega R^k_{t+1} q_t K_{it+1} dF(\omega) - R^l_{it+1} B_{it+1}. \tag{A.10}$$

Substituting in the definitions above yields

$$V^k_{it+1} = \left[1 - \Gamma(\bar{\omega}_{it+1})\right] R^k_{t+1} q_t K_{it+1}, \tag{A.11}$$

where $1 - \Gamma(\bar{\omega}_{it+1})$ is the entrepreneur's expected share of gross returns.

For a given level of net-worth X_{it+1}, the entrepreneur's optimal contacting problem is then

$$max_{K_{it+1}, \bar{\omega}_{it+1}} E_t\{V^k_{it+1}\} \tag{A.12}$$

subject to the condition that the financial intermediary's expected return on the contract equal its opportunity cost of its borrowing, equation (3). Letting λ_{it+1} be the ex-post value of the Lagrange multiplier conditional on realization of the aggregate state, the first-order conditions are then

$$\Gamma'(\bar{\omega}_{it+1}) - \lambda_{t+1}\left[\Gamma'(\bar{\omega}_{it+1}) - \theta_{t+1} G'(\bar{\omega}_{it+1})\right] = 0 \tag{A.13}$$

$$E_t\left\{\left[1 - \Gamma(\bar{\omega}_{it+1})\right] \frac{R^k_{t+1}}{R^a_{t+1}} + \lambda_{t+1}\left(\left[\Gamma(\bar{\omega}_{it+1}) - \theta_{t+1} G(\bar{\omega}_{it+1})\right] \frac{R^k_{t+1}}{R^a_{t+1}} - 1\right)\right\} = 0 \tag{A.14}$$

$$\left[\Gamma(\bar{\omega}_{it+1}) - \theta_{t+1} G(\bar{\omega}_{it+1})\right] R^k_{it+1} q_t K_{it+1} - R^a_{t+1}\left(q^n_t K_{it+1} - X_{it+1}\right) = 0 \tag{A.15}$$

where (A.13) and (A.15) hold in each contingency, but (A.14) holds only in expectation.

A.3 Household

The stand-in household's lifetime utility is given by

$$E_0 \sum_{t=0}^{\infty} \beta^t J_t \left[\log(C_t - bC_{t-1}) - \Psi_L \frac{{N_t^h}^{1+\sigma_L}}{1+\sigma_L} \right] \tag{A.16}$$

where C_t is consumption, N_t is hours-worked, β is the subjective discount factor and J_t follows an exogenous stochastic preference process which we refer to as a *preference shock*.

The household enters into each period with real financial securities A_t which serve as deposits with the financial intermediary, and nominal bonds B_t^n, earning risk-free gross real rate of return R_t^a and risk-free gross nominal rate of return R_t^n respectively, receiving nominal wage W_t^h for supplying hours N_t^h to the labour union, and receiving a share of real profits from the capital-producers, goods-producers, financial intermediary, labour union and employment agency, denoted collectively as F_t. At the end of the period, the household chooses its consumption C_t, its holdings of financial securities A_{t+1} and nominal nominal bonds B_{t+1}^n. The household's period t budget constraint is given by

$$C_t + A_{t+1} + \frac{B_{t+1}^n}{P_t} = R_t^a A_t + R_t^n \frac{B_t^n}{P_t} + \frac{W_t^h}{P_t} N_t^h + F_t, \tag{A.17}$$

where P_t is the price of the final good in terms of the nominal unit under the control of the central bank. The household's problem is to choose sequences of C_t, N_t^h, A_{t+1} and B_{t+1}^n to maximize (A.16) subject to (A.17).

Letting λ_t be the Lagrange multiplier associated with the household's budget constraint, the first-order conditions with respect to C_t, N_t^h, A_{t+1} and B_{t+1}^n are respectively

$$\lambda_t = \frac{J_t}{C_t - bC_{t-1}} - \beta b E_t \frac{J_{t+1}}{C_{t+1} - bC_t} \tag{A.18}$$

$$\lambda_t \frac{W_t^h}{P_t} = \Psi_L J_t {N_t^h}^{\sigma_L} \tag{A.19}$$

$$\lambda_t = \beta E_t R_{t+1}^a \tag{A.20}$$

$$\lambda_t = \beta E_t R_{t+1}^n \frac{P_t}{P_{t+1}}. \tag{A.21}$$

A.4 Final goods firm and intermediate goods firms

The final goods firm produces the final good Y_t by combining differentiated intermediate goods $y_{jt}, j \in [0, 1]$, according to the technology

$$Y_t = \left[\int_0^1 y_{jt}^{\nu_{pt}} dj \right]^{\frac{1}{\nu_{pt}}}, \quad 0 < \nu_p \leq 1. \tag{A.22}$$

where ν_{p_t} follows an exogenous stochastic process which we refer to as a *price markup shock*. The producer acquires each j^{th} intermediate good at price P_{jt}, and sells the final good at price P_t where it may be used as a consumption or as an input into the production of investment goods. Each period the producer chooses intermediate goods y_{jt} $\forall j$ to maximize profits $P_t Y_t - \int_0^1 P_{jt} y_{jt} dj$, yielding a standard demand curve

$$y_{jt} = \left[\frac{P_{jt}}{P_t}\right]^{\frac{1}{\nu_p - 1}} Y_t, \tag{A.23}$$

for the j^{th} intermediate good, and nominal price index

$$P_t = \left[\int_0^1 P_{jt}^{\nu_p/(\nu_p-1)} dj\right]^{\frac{(\nu_p-1)}{\nu_p}}. \tag{A.24}$$

The j^{th} intermediate goods firms produces the differentiated good y_{jt} according to the technology

$$y_{jt} = z_t \tilde{n}_{jt}^{\alpha} \tilde{k}_{jt}^{1-\alpha}, \tag{A.25}$$

where z_t is total factor productivity that follows an exogenous stochastic process which we refer to as a *technology shock*, $\tilde{n}_{jt}$ is total hours-worked, and $\tilde{k}_{jt}$ is physical capital services. Hours-worked is a composite of both household and entrepreneurial labour, such that $\tilde{n}_{jt} = n_{jt}^{\Omega}(n_{jt}^{e})^{1-\Omega}$, where n_{jt} is worker labour, n_{jt}^{e} is entrepreneurial labour , and where Ω parameterizes the elasticity of the hours composite to household labour. Capital services is defined by $\tilde{k}_{jt} = u_{jt} k_{it}$, where k_{jt} is the stock of physical capital and u_{jt} is the utilization rate of that stock, chosen by the entrepreneurs.

The j^{th} firm hires n_{jt} and n_{jt}^{e} at wage rates W_t and w_t^{e} respectively, rents capital services $\tilde{k}_{jt}$ at rate r_t, and sells its output at price P_{jt}. Intermediate goods firms have market power, and can thus set prices subject to the demand curve (A.23). The firms face Calvo frictions in setting their prices such that each period they can re-optimize prices with probability $1 - \zeta_p$. A firm that is unable to re-optimize its price in a given period re-sets it according to the indexation rule $P_{jt} = P_{jt-1} \pi_{t-1}^{\iota_p} \pi^{1-\iota_p}, \quad 0 \leq \iota_p \leq 1$, where $\pi_t = P_t / P_{t-1}$ and π is its steady state, and where $0 \leq \iota_p \leq 1$. A firm that can re-optimize its price in period t chooses its price P_{jt}^{*} to maximize

$$E_t \sum_{s=0}^{\infty} \zeta_p^s \beta^s \frac{\lambda_{t+s} P_t}{\lambda_t P_{t+1}} \left[P_{jt}^{*} (\Pi_{k=1}^{s} \pi_{t+k-1}^{\iota_w} \pi^{1-\iota_w}) y_{jt+s} - P_{t+s} S(y_{jt+s})\right], \tag{A.26}$$

where $\beta^s \frac{\lambda_{t+s} P_t}{\lambda_t P_{t+1}}$ is the household owner's nominal discount factor, given the production technology (A.25) and the demand curve for y_{jt}, and where $S(y_{jt})$ is the firm's real cost function as a solution to its cost-minimization problem for a given level of output y_{jt}.

A.5 Employment agency and employment unions

The employment agency combines differentiated labour $n_{qt}, q \in [0,1]$, into a composite N_t according to

$$N_t = \left[\int_0^1 n_{qt}^{\nu_{wt}} dq\right]^{\frac{1}{\nu_{wt}}}, \quad 0 < \nu_w \leq 1. \tag{A.27}$$

where ν_{wt} follows an exogenous stochastic process which we refer to as a *wage markup shock*. Each period the agency acquires each q^{th} differentiated labour service at wage W_{qt} from the labour union, and sells the composite labour to the intermediate goods producers for wage W_t. The agency chooses n_{qt} $\forall q$ to maximize profits $W_t N_t - \int_0^1 W_{qt} n_{qt} dq$, yielding a demand function

$$n_{qt} = \left[\frac{W_{qt}}{W_t}\right]^{\frac{1}{\nu_w - 1}} N_t, \tag{A.28}$$

for the q^{th} labour type, and wage index

$$W_t = \left[\int_0^1 W_{qt}^{\nu_w/(\nu_w-1)} dq\right]^{\frac{(\nu_w-1)}{\nu_w}}. \tag{A.29}$$

The q^{th} labour union acquires labour N_t^h from the household at wage W_t^h, differentiates it into labour type $n_{qt}, q \in [0,1]$, and then sells it to the employment agency for wage W_{qt}. The unions have market power, and can thus choose the wage for each labour type subject to the labour demand curve (A.28). The unions face Calvo frictions in setting their wages, such that each period they can re-optimize wages with probability $1 - \zeta_w$. A union that is unable to re-optimize wages re-sets it according to the indexation rule $W_{qt} = W_{qt-1}\pi_{t-1}^{\iota_w}\pi^{1-\iota_w}, \quad 0 \leq \iota_w \leq 1$, where $\pi_t = P_t/P_{t-1}$ and π is its steady state, and where $0 \leq \iota_w \leq 1$. A union that can re-optimize its wage in period t chooses its wage W_{qt}^* to maximize

$$E_t \sum_{s=0}^{\infty} \zeta_w^s \beta^s \frac{\lambda_{t+s} P_t}{\lambda_t P_{t+1}} \left[W_{qt}^* (\Pi_{k=0}^s \pi_{t+k-1}^{\iota_w} \pi^{1-\iota_w}) - W_{t+s}^h\right] n_{qt+s}, \tag{A.30}$$

subject to the demand curve for n_{qt}.

A.6 Capital-producer

The competitive capital-goods producer operates a technology that combines existing capital with new investment goods to create new installed capital. At the end of each period it purchases existing capital K_t^k from entrepreneurs at price $\bar{q}_t$, combining it with investment I_t to yield new capital stock K_t^{nk}, which it sells back to entrepreneurs in the same period at price q_t. The capital-producer faces investment adjustment costs in the creation of new capital, and incurs depreciation in the process, so that

$$K_t^{nk} = (1-\delta)K_t^k + I_t\left[1 - S\left(\frac{m_t I_t}{I_{t-1}}\right)\right], \tag{A.31}$$

where m_t follows an exogenous stochastic process that we refer to as a *marginal efficiency of investment (MEI) shock* (see Justiniano et al. (2010)), and $S(x)$ is an investment adjustment cost function based on Christiano et al. (2005) with the properties $S(x) = 0$, $S'(x) = 0$, and $S''(x) = s''$, where s'' is a parameter. The capital producer's period t profits are given by $\Pi_t^k = q_t^n K_t^{nk} - q_t K_t^k - I_t$. Since the capital producer faces intertemporal investment adjustment costs, it faces a dynamic problem, choosing K_t^{nk}, K_t^k and I_t to maximize

$$E_0 \sum_{t=0}^{\infty} \frac{\beta^t \lambda_t}{\lambda_0} \Pi_t^k \tag{A.32}$$

subject to (A.31).

The capital producer's first-order conditions are given by

$$\bar{q}_t = q_t(1-\delta) \tag{A.33}$$

$$q_t^n - \frac{1}{\Upsilon_t} - q_t^n S\left(\frac{m_t I_t}{I_{t-1}}\right) - q_t^n S'\left(\frac{m_t I_t}{I_{t-1}}\right)\frac{m_t I_t}{I_{t-1}} + E_t\left\{\frac{\beta\lambda_{t+1}}{\lambda_t} q_{t+1}^n m_{t+1} \frac{I_{t+1}^2}{I_t^2} S'\left(\frac{m_{t+1} I_{t+1}}{I_t}\right)\right\}. \tag{A.34}$$

A.7 Monetary policy

Monetary policy takes the form of a monetary authority that sets the gross nominal interest rate R_{t+1}^n according to a rule in the form

$$\frac{R_{t+1}^n}{R^n} = \left(\frac{R_t^n}{R^n}\right)^{\rho_R} \left[\left(\frac{\Pi_t}{\Pi}\right)^{\phi_\pi} \left(\frac{Y_t}{Y_{t-1}}\right)^{\phi_y}\right]^{1-\rho_R} \eta_t, \tag{A.35}$$

where variables without subscripts are steady-state values, Π_t is the gross inflation rate, and η_t follows an exogenous stochastic process that we refer to as a *monetary policy shock*.

A.8 Stochastic processes

There are 7 stochastic processes in the model: J_t (preference), ν_{pt} (price markup), ν_{wt} (wage markup), z_t (technology), θ_t (DC), η (monetary policy) and m_t (MEI). All the stochastic processes Ξ_t, where $\Xi = J_t, \nu_{pt}, \nu_{wt}, z_t, \theta_t, \eta_t, m_t$, evolve according to the stationary process

$$\ln(\Xi_t/\Xi) = \rho_\Xi \ln(\Xi_{t-1}/\Xi) + u_{\Xi t}, \tag{A.36}$$

where $\rho_\Xi < 1$, Ξ denotes the mean of the process and $u_{\Xi t}$ is the shock innovation. We potentially allow for news shocks to the technology and default cost processes, such that for these processes, the innovation $u_{\Xi t}$ contains both an anticipated and unanticipated component, whereas for the remaining stochastic processes, the innovation $u_{\Xi t}$ contains only an unanticipated component, such that

$$u_{\Xi t} = \begin{cases} \epsilon^{p}_{\Xi t-p} + \epsilon^{0}_{\Xi t} & \Xi = \{z_t, \theta_t\} \\ \epsilon^{0}_{\Xi t} & \Xi = \{J_t, \nu_{pt}, \nu_{wt}\, m_t\, \eta_t\} \end{cases} \tag{A.37}$$

where $p > 0$, $\epsilon^{p}_{\Xi t-p}$ is a news shock that agents receive in period $t-p$ about the innovation in t, and $\epsilon^{0}_{\Xi t}$ is a surprise shock. All shocks are mean zero and uncorrelated over time and with each other. The news and surprise shocks have standard deviation σ^{p}_{Ξ} and σ^{0}_{Ξ} respectively.

A.9 Equilibrium

Equilibrium in this economy is defined by contingent sequences of C_t, $c^e_t(i)\forall i$, N_t, N_{t^h}, $n_{jt}\forall j$, $u_{jt}\forall j$, $n^e_{jt}\forall j$, $P_{jt}\forall j$, $y_{jt}\forall j$, I_t, A_{t+1}, $K_{it+1}\forall i$, $u_{it}\forall i$, $B_{it+1}\forall i$, $\bar{\omega}_{it+1}\forall i$, K^{nk}_t, K^k_t, B^n_{t+1}, W_t, W^h_t, W^e_t, W_{qt}, r_t, R^a_{t+1}, $R^l_{it+1}\forall i$, R^k_t, $\bar{q}_t$, q_t, R^n_t, P_t, that satisfy the following conditions: (i) the allocations solve the household's, final goods-producer's, intermediate goods producers', financial intermediary's, entrepreneurs', capital producer's, employment agency's and employment union's problems, taking prices as given, (ii) all markets clear, (iii) the resource constraint $C_t + C^e_t + q^n_t\Phi(\frac{I_t}{K_t}K_t) + \theta_t G(\bar{\omega}_t)q^n_{t-1}R^k_t K_t = Y_t$ holds, where $\int_0^1 K_{it+1} = K_{t+1}$, $\int_0^1 B_{it+1} = B_{t+1}$, $\int_0^1 X_{it+1} = X_{t+1}$, $\int_0^1 c^e_{it+1} = C^e_{t+1}$, $\int_0^1 N^e_i = N^e = 1$ and where all entrepreneurs choose the same cut-off such that $\bar{\omega}_{it+1} = \bar{\omega}_{t+1} \quad \forall i$, and therefore $R^l_{it+1} = R^l_{t+1} \quad \forall i$.

Equilibrium in the capital goods market implies that $K^{nk}_t = K_{t+1}$ and $K^k_t = K_t$, and equilibrium in the securities market implies that $A_t = B_t$. Nominal bonds are in zero net-supply such that $B^n_t = 0$.

In equilibrium the financial intermediary's return on its entire loan portfolio just covers its opportunity cost of funds, implying that its budget constraint holds in every aggregate contingency and after idiosyncratic uncertainty is resolved as

$$\left[\Gamma(\bar{\omega}_{t+1}) - \theta_{t+1}G(\bar{\omega}_{t+1})\right] R^k_{t+1} q_t K_{t+1} = R^a_{t+1}A_{t+1}. \tag{A.38}$$

Aggregate net-worth evolves as the accumulated gross returns of surviving entrepreneurs plus their labour income. Letting V_t be aggregate gross entrepreneurial returns, we can compute it as the average gross idiosyncratic returns,

$$V_t = \left[1 - \Gamma(\bar{\omega}_t)\right] R^k_t q^n_{t-1} K_t, \tag{A.39}$$

which after making substitutions yields

$$V_t = R^k_t q^n_{t-1} K_t - \left[R^a_t B_t + \theta_t G(\bar{\omega}_t) R^k_t q^n_{t-1} K_t\right], \tag{A.40}$$

so that aggregate net-worth evolves as

$$X_{t+1} = \gamma V_t + w^e_t. \tag{A.41}$$

Finally, entrepreneurial consumption C^e_t is equal to the aggregated gross return of dying entrepreneurs,

$$C_t^e = (1-\gamma)V_t. \tag{A.42}$$

For reference later in the discussion of our results, we also define the equilibrium real risk-free net interest rate as $r_t^f = \frac{1}{E_t \beta \frac{\lambda 1t+1}{\lambda_{1t}}} - 1$, the credit spread as $R_t^l - R_t^a$, and leverage as $L_t = \frac{q_t^n K_{t+1}}{X_{t+1}}$.

B Data

- Real Gross Domestic Product, 3 Decimal, Billions of Chained 2009 Dollars, Quarterly, Seasonally Adjusted Annual Rate.
  Source: search on series code GDPC96 at https://fred.stlouisfed.org/
- Gross Domestic Product - Implicit Price Deflator - 1996=100, Seasonally Adjusted
  Source: search on series code GDPDEF at https://fred.stlouisfed.org/
- Personal Consumption Expenditures, Billions of Dollars, Seasonally Adjusted Annual Rate
  Source: search on series code PCEC at https://fred.stlouisfed.org/
- Fixed Private Investment, Billions of Dollars, Seasonally Adjusted Annual Rate
  Source: search on series code FPI at https://fred.stlouisfed.org/
- Civilian Employment: Sixteen Years and Over, Thousands, Seasonally Adjusted
  Source: search on series code CE16OV at https://fred.stlouisfed.org/
- Effective Federal Funds Rate
  Source: search on FEDFUNDS at https://fred.stlouisfed.org/
- Average Weekly Hours Duration, Nonfarm Business, All Persons, : index, 1992 = 100, Seasonally Adjusted
  Source: search on series code PRS85006023 at https://fred.stlouisfed.org/
- Hourly Compensation Duration, Nonfarm Business, All Persons, : index, 1992 = 100, Seasonally Adjusted.
  Source: Search series id PRS85006103 at U.S. Bureau of Labour Statistics,
  http://data.bls.gov/cgi-bin/srgate
- Labor Force Status : Civilian noninstitutional population - Age : 16 years and over - Seasonally Adjusted - Number in thousands.
  Source: Search series id LNS10000000 at U.S. Bureau of Labour Statistics,
  http://data.bls.gov/cgi-bin/srgate
- Credit Spread: Moody's Seasoned Baa Corporate Bond Yield Relative to Yield on 10-Year Treasury Constant Maturity, Percent, Quarterly, Not Seasonally Adjusted.
  Source: search on series code BAA10YM at https://fred.stlouisfed.org/
- Charge-offs: Total charge-offs on Total Loans and Leases, All FDIC-Insured Institutions, Millions of Dollars,
  Source: Quarterly Loan Portfolio Performance Indicators
  https://www.fdic.gov/bank/analytical/qbp/timeseries/loan-performance.xls
- Defaults: Loans 90 days or more past due, All FDIC-Insured Institutions, Millions of Dollars,
  Source: Quarterly Loan Portfolio Performance Indicators
  https://www.fdic.gov/bank/analytical/qbp/timeseries/loan-performance.xls

- QUSPAMUSDA: Total Credit to Private Non-Financial Sector, Adjusted for Breaks, for United States. Source: search on series code QUSPAMUSDA at https://fred.stlouisfed.org/

C Additional results

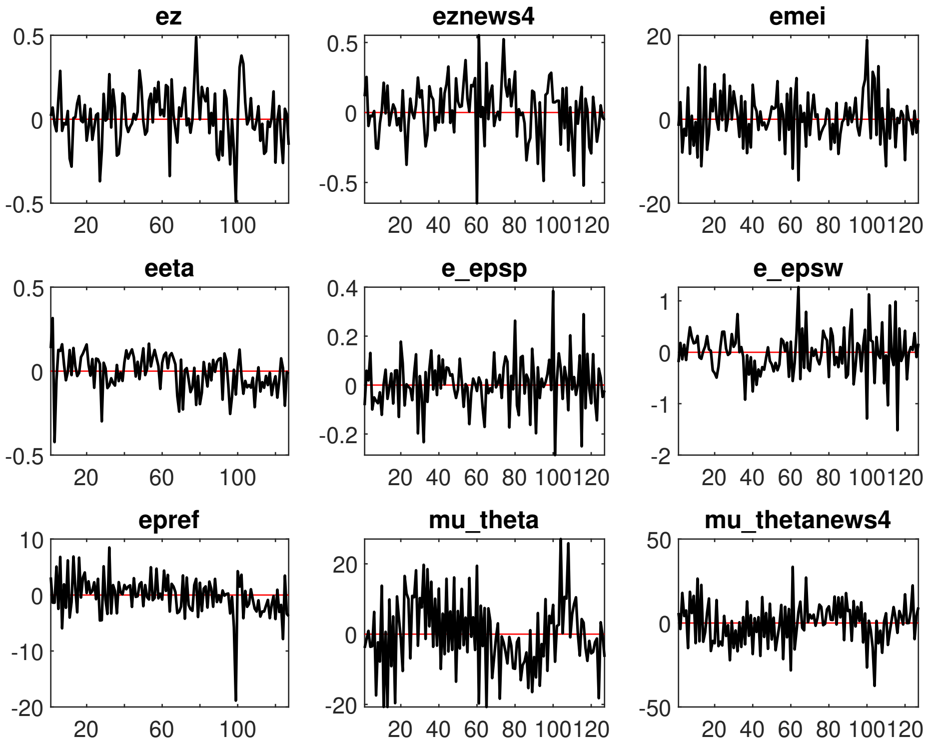

Figure 6: Shocks from full model estimation

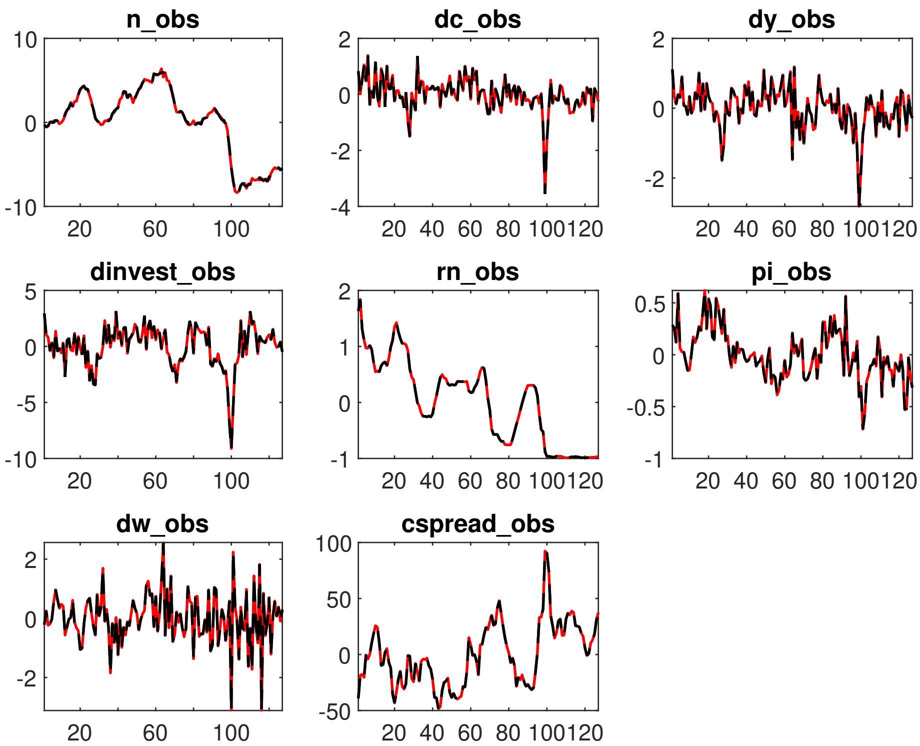

Figure 7: Historical and Smoothed Variables - full model estimation

Table 9: Conditional variance decomposition

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Observable	z	z^4	m	η	ν_p	ν_w	J	θ	θ^4
3-Period ahead									
Hours worked	0.51	1.32	8.11	4.06	15.03	6.41	4.98	17.67	41.9
Consumption growth	1.9	1.97	1.3	1.63	9.52	3.8	51.88	9.33	18.66
Output growth	1.78	1.36	8.28	4.38	16.95	4.54	4.75	17.21	40.75
Investment growth	0.41	0.32	18.35	0.83	5.26	1.17	0.51	28.09	45.06
Nominal int. rate	1.38	0.03	8.02	8.79	11.66	5.88	3.66	19.81	40.78
Inflation	2.26	0.03	7.37	1.97	19.32	8.97	3.22	18.89	37.97
Wage growth	2.21	0.37	0.52	0.12	23.79	68.13	0.12	1.49	3.26
Credit spread	0.05	0.01	1.93	1.2	1.06	0.17	0.02	86.85	8.71
8-Period ahead									
Hours worked	0.25	0.64	11.05	2.37	16.87	5.82	2.54	20.34	40.12
Consumption growth	1.99	2.45	1.55	1.57	9.66	3.81	50.32	9.58	19.08
Output growth	1.7	1.75	8.88	3.74	15.77	4.16	4.61	17.74	41.65
Investment growth	0.40	0.43	16.15	0.68	5.07	1.08	0.52	27.82	47.85
Nominal int. rate	0.72	0.27	9.74	2.39	5.4	3.01	2.93	25.87	49.67
Inflation	1.34	0.71	8.23	1.71	11.18	5.29	2.79	23.47	45.28
Wage growth	2.16	1.51	0.74	0.12	23.41	66.28	0.12	1.81	3.86
Credit spread	0.03	0.01	2.81	1.08	0.79	0.1	0.03	61.15	34.01
12-Period ahead									
Hours worked	0.25	0.55	11.11	1.73	15.57	4.91	1.63	22.03	42.23
Consumption growth	1.74	2.14	2.62	1.42	8.48	3.4	47.32	11.11	21.77
Output growth	1.67	1.72	8.67	3.72	15.52	4.19	4.86	17.84	41.81
Investment growth	0.41	0.43	16.34	0.71	5.11	1.13	0.54	27.67	47.67
Nominal int. rate	0.5	0.29	9.17	1.56	3.61	2.08	2.32	27.66	52.81
Inflation	1.16	0.63	7.8	1.51	9.76	4.58	2.48	24.62	47.46
Wage growth	2.07	1.47	1.25	0.14	22.44	63.9	0.12	2.85	5.75
Credit spread	0.03	0.02	4.89	1.26	0.76	0.1	0.12	58.08	34.73