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The Real Interest Rate Channel *is* Structural in Contemporary New-Keynesian Models

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Abstract

The monetary transmission mechanism in a New-Keynesian model with contemporary features is put to scrutiny. In contrast to Rupert and Šustek (2019), we find that the real interest rate channel *is* structural when the model contains empirically realistic frictions on the flow of investment. A monetary contraction (expansion) is always followed by an increase (decrease) in the real interest rate. The monetary transmission mechanism indeed operates through the real interest rate channel in this class of models.

Key words: New-Keynesian models, monetary transmission mechanism, real interest rate
JEL classification: E24, E32, E43

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1 Introduction

How does monetary policy affect inflation and output? According to contemporary New Keynesian (NK) models that are widely used in academia and central banks, it is via the *the real interest rate channel*. An increase in the short-term nominal interest rate increases the real interest rate in the presence of sticky prices. Households and firms then reduce consumption and investment, respectively. As demand and output contract, inflation declines. In a recent provocative paper, Rupert and Šustek (2019) challenge this widely held view. They write:

> *The main message of this paper is that the transmission mechanism of monetary policy in New-Keynesian models does not operate through the real interest rate channel. Any consistency with the real interest rate channel is purely observational, not structural, due to a specific parameterization.* Rupert and Šustek (2019), p. 54.

Based on their analysis using an NK model with capital, they conclude that from a monetary policy perspective either current NK models present a misleading description of the monetary transmission mechanism or policy makers need to rethink the monetary transmission channel altogether.

In this note, we show that the properties highlighted in Rupert and Šustek (2019) rely on two specific features both of which are absent in contemporary NK models that are used for monetary policy analysis by academics (for example, the literature following Christiano, Eichenbaum and Evans (2005) and Smets and Wouters (2007)), and in Central Banks (for example, Brave et al. (2012), Del Negro et al. (2013), among many others). First is that in the frictionless setting, with smooth consumption, the analysis relies on an unrealistic response of investment to a monetary policy shock —investment deviates upwards of 50% from steady state after a 1% shock to the policy rate. Depending on the persistence of the monetary

shock, the (ex-ante) real interest rate can increase, decrease, or remain unchanged, and in this sense the response is not structural. None of the contemporary NK models, however, have this feature. Second is that with capital adjustment costs, Rupert and Šustek (2019) show that the real interest rate channel arises only for sufficiently high capital adjustment costs. For low costs, the real interest rate moves in the opposite direction from the monetary shock. The real interest rate response is again not structural. None of the contemporary NK models, however, consider capital adjustment costs. Instead, these models have adjustment costs on the flow of investment, or Investment Adjustment Costs (IAC) as introduced by Christiano, Eichenbaum and Evans (2005) to match the empirical response of investment to a monetary policy shock.[1]

We illustrate that in an NK model with IAC the real interest rate channel *is* structural, contrary to the conclusions of Rupert and Šustek (2019). The real interest rate always has the same sign as that of the monetary shock. A monetary contraction raises the real rate whereas an expansion lowers it. This response of the real interest rate does not depend on the size of IAC or the degree of persistence in the monetary shock process. In this sense, the real interest rate channel in NK models *is* structural. Both consumption and investment adjust to the real interest rate. Hence, the monetary transmission mechanism indeed operates through the real interest rate channel in contemporary NK models.

In Section 2 we lay out an NK model with endogenous capital and IAC and illustrate the real interest rate channel. We consider different parameterizations of IAC and the persistence of monetary shock to support our main point. In Section 3 we conclude.

[1]The use of IAC in contemporary NK-DSGE models is ubiquitous.

2 New-Keynesian model with capital

In this section we assess the New-Keynesian model with capital and investment adjustment costs. Since the model is nearly identical to the NK model in Rupert and Šustek (2019), we simply highlight the one modified equation. The law of motion for capital with IAC takes the following form,

$$k_{t+1} = (1-\delta)k_t + i_t\left[1 - \frac{\Omega}{2}\left(\frac{i_t}{i_{t-1}} - 1\right)^2\right], \tag{1}$$

where k_t is the current capital stock, i_t is current gross investment, δ is the depreciation rate, and Ω is the IAC parameter. Under this adjustment costs specification, the more the gross investment rate differs from one the less new capital is produced from a unit of investment. In this setup Ω governs the magnitude of the costs associated with IAC. We describe the equilibrium conditions of the model and the log-linearized equations in the Appendix.

To minimize differences between the responses reported here and in Rupert and Šustek (2019), we use the same calibration of parameter values: $\beta = 0.99$, $\eta = 1$, $\theta = 0.7$, $\nu = 1.5$, $\delta = 0.025$, $\alpha = 0.3$, and $\varepsilon = 0.83$.[2] We calibrate the adjustment costs parameter to 5.48 which corresponds to the estimated value in Smets and Wouters (2007). However, industry level IAC estimates tend to be smaller (Groth and Khan 2010). We show that the conclusions regarding the real interest rate channel hold under smaller adjustment costs parameter specifications. The real interest rate is reported as percentage point deviation from steady state (i.e., $R_t - \bar{R}$) while consumption, investment, and output are reported in percentage deviation from steady state (i.e., $\frac{x_t - \bar{x}}{\bar{x}}$). Figure 1 displays the response of consumption, investment, the real interest rate, and output to a 1 percentage point shock to

[2] We explored the importance of consumption habits for the real interest rate channel. Since they did not impact the conclusions drawn, we set $\varepsilon_C = 0$, as in Rupert and Šustek (2019).

the monetary policy rule.

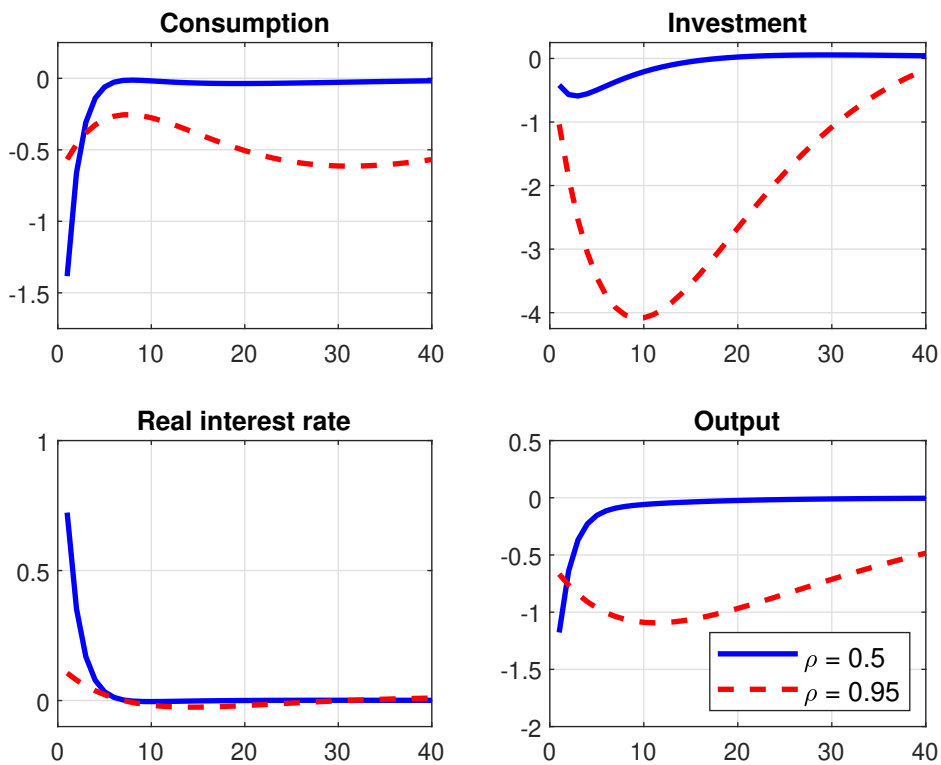

Figure 1: Impulse response functions to a monetary policy shock with investment adjustment costs

Notes: Investment adjustment costs parameter, Ω, is calibrated to 5.48. ρ is the persistence of the monetary policy shock.

The impulse response functions show that consumption, investment, and output fall in response to a positive monetary policy shock. In contrast to the ambiguity in Rupert and Šustek (2019) regarding the response of the real interest rate, when the model has frictions on the flow of investment, the real interest rate always rises in response to a positive monetary policy shock. This result holds under both shock persistence specifications.[3] Experimenting with the model, we find that even with highly persistence shocks ($\rho = 0.999$) the real interest

[3]We do not report impulse responses for the case where $\rho = 0$, but the real interest rate also rises in this case.

rate rises when IAC is present in the model.[4]

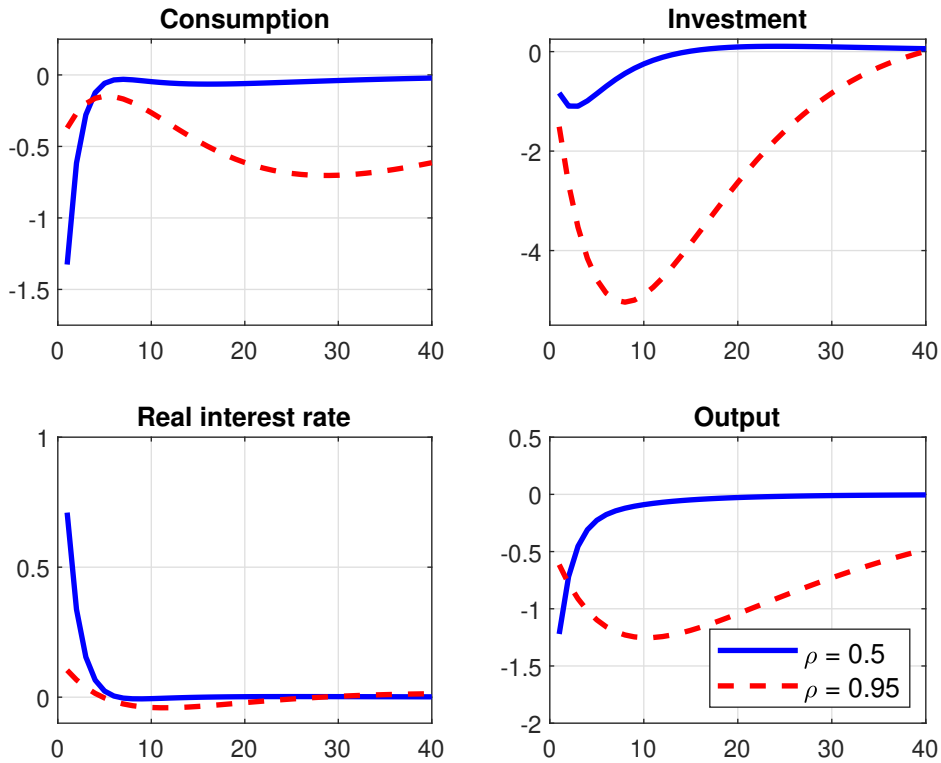

Figure 2: Impulse response functions to a monetary policy shock with investment adjustment costs

Notes: Investment adjustment costs parameter, Ω, is calibrated to 2.5. ρ is the persistence of the monetary policy shock.

The IAC parameter determines the strength of the de-linkage between the real interest rate and the marginal product of capital. We document that the real interest rate channel is robust to lower IAC parameters. Figure 2 displays the response of consumption, investment, the real interest rate, and output to a monetary policy shock when $\Omega = 2.5$ — a much smaller IAC parameter than typically estimated in the DSGE literature.[5] Naturally as the adjustment costs associated with the flow of investment fall, the response of investment to

[4]In an NK model without capital, the real interest rate always rises after a positive monetary shock (see, for example, Galí (2015), and also noted in Rupert and Šustek (2019)).

[5]For reference, Christiano et al. (2014) find an estimate of $\Omega = 10.78$.

a monetary policy shock becomes larger. However, the real interest rate always rises.

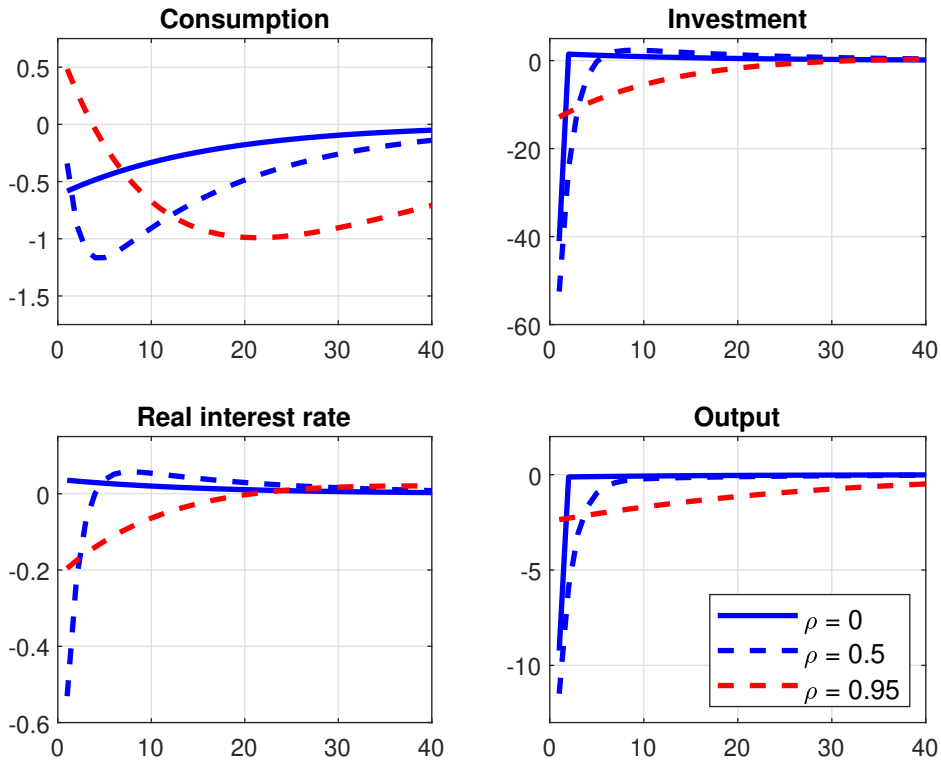

Figure 3: Impulse response functions to a monetary policy shock in Rupert and Šustek (2019)

Notes: Investment adjustment costs parameter, Ω, is calibrated to 0. ρ is the persistence of the monetary policy shock.

In the absence of costs associated with the flow of investment, capital is extremely sensitive to changes in the real interest rate which lead to large changes in investment. To illustrate this point, Figure 3 displays the response of consumption, investment, the real interest rate, and output to a 1 percentage point increase in ξ_t when investment is frictionless ($\Omega = 0$). The impulse response functions reported here are identical to those in Figures 1, 3 and 4 in Rupert and Šustek (2019). Two points are worth emphasizing: First, the response of investment to a 1 percentage point shock to the monetary policy rate is unrealistic. Invest-

ment deviates from steady state by 13-53% depending upon the persistence in the shock.[6] Second, the response of the real interest rate is ambiguous. When shock persistence is low, the real interest rate rises. But even moderate amounts of persistence lead to a fall in the real interest rate. While the emphasis in their exercise is on the qualitative features of the model, our point is that drawing implications based on these model properties, as they do, is problematic.

3 Conclusion

We highlight that the real interest rate channel is central to the monetary transmission mechanism in contemporary NK models. A monetary contraction (expansion) is followed by an increase (decrease) in the real interest rate. The presence of investment adjustment costs make the real interest rate channel a structural feature in this class of models.

[6]The response of investment to a monetary shock is not shown in Rupert and Šustek (2019).

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A The model

A.1 Households

The household problem is given by,

$$\text{Max} \sum_{t=0}^{\infty} \beta^t \left\{ \log\left(C_t - \varepsilon_C C_{t-1}\right) - \frac{L_t^{1+\eta}}{1+\eta} \right\},$$

subject to the following budget constraint and law of motion for capital which includes investment adjustment costs,

$$W_t L_t + R_t^k K_t + \left(\frac{1+i_{t-1}}{1+\pi_t}\right) B_{t-1} = C_t + I_t + B_t,$$
$$K_{t+1} = (1-\delta)K_t + I_t\left[1 - \frac{\Omega}{2}(\frac{I_t}{I_{t-1}} - 1)^2\right],$$

yielding the following equilibrium conditions,

$$\lambda_t = \frac{1}{C_t - \varepsilon_C C_{t-1}} - E_t\left\{\frac{\beta\varepsilon_C}{C_{t+1} - \varepsilon_C C_t}\right\}, \quad (2)$$
$$L_t^{\eta} = \lambda_t W_t, \quad (3)$$
$$1 = \beta E_t\left\{\left(\frac{\lambda_{t+1}}{\lambda_t}\right)\left(\frac{1+i_t}{1+\pi_{t+1}}\right)\right\}, \quad (4)$$
$$q_t = \beta E_t\left\{\left(\frac{\lambda_{t+1}}{\lambda_t}\right)\left(R_{t+1}^k + (1-\delta)q_{t+1}\right)\right\}, \quad (5)$$
$$1 = q_t\left[1 - \frac{\Omega}{2}\left(\frac{I_t}{I_{t-1}} - 1\right)^2 - \Omega\left(\frac{I_t}{I_{t-1}} - 1\right)\left(\frac{I_t}{I_{t-1}}\right)\right] + \beta\Omega E_t\left\{q_{t+1}\left(\frac{\lambda_{t+1}}{\lambda_t}\right)\left(\frac{I_{t+1}}{I_t} - 1\right)\left(\frac{I_{t+1}}{I_t}\right)^2\right\}, \quad (6)$$

where λ is the Lagrange multiplier on the budget constraint and q is the ratio of the Lagrange multipliers on the law of motion for capital and the budget constraint.

A.2 Intermediate firms

Intermediate firms use a constant returns to scale technology and minimize costs subject to meeting demand. Wages and rental rates are common to all firms,

$$\text{Min } R_t^k K_t + W_t L_t \qquad \text{s.t.}$$
$$K_t(i)^{\alpha} L_t(i)^{1-\alpha} \geq \left(\frac{P_t(i)}{P_t}\right)^{\frac{1}{\varepsilon-1}} Y_t,$$

which yields the optimal mix of capital and labour in production and marginal cost,

$$\frac{W_t}{R_t^k} = \left(\frac{1-\alpha}{\alpha}\right)\left(\frac{K_t}{L_t}\right), \tag{7}$$

$$\chi_t = \left(\frac{R_t^k}{\alpha}\right)^{\alpha}\left(\frac{W_t}{1-\alpha}\right)^{1-\alpha}. \tag{8}$$

Letting θ denote the probability that a firm cannot adjust its prices, the firm chooses $P_t(j)$ taking into account it may not be able to change its price for a very long time and maximizes real profit,

$$\text{Max } E_t \sum_{s=0}^{\infty} (\beta\theta)^s \left(\frac{U'(C_{t+s})}{U'(C_t)}\right)\left(\frac{P_t(j)}{P_{t+s}}\left(\frac{P_t(j)}{P_{t+s}}\right)^{\frac{1}{\varepsilon-1}} Y_{t+s} - \frac{\chi_{t+s}}{P_{t+s}}\left(\frac{P_t(j)}{P_{t+s}}\right)^{\frac{1}{\varepsilon-1}} Y_{t+s}\right),$$

which yields the standard New-Keynesian Phillips Curve,

$$\pi_t = \beta E_t \pi_{t+1} + \Psi \hat{\chi}_t \tag{9}$$

where $\Psi = \frac{(1-\beta\theta)(1-\theta)}{\theta}\frac{1-\alpha}{1-\alpha+\frac{\alpha}{1-\varepsilon}}$.

A.3 Final goods firms

The final goods sector is perfectly competitive and aggregates intermediate inputs to produce final goods. The final goods problem is given by,

$$\text{Max } P_t Y_t - \int_0^1 P_t(i) Y_t(i),$$

where $Y_t = \left(\int_0^1 Y_t(i)^{\varepsilon}\right)^{\frac{1}{\varepsilon}}$, which yields the standard downward sloping demand function for intermediate firm i's input,

$$Y_t(i) = \left(\frac{P_t(i)}{P_t}\right)^{\frac{1}{\varepsilon-1}} Y_t,$$

that states that the demand for input i is a function of its relative price and price elasticity of demand.

A.4 Monetary policy rule and market clearing conditions

Following Rupert and Šustek (2019), we set the weight on the output gap to 0 in the Taylor rule,

$$i_t = i + \nu\pi_t + \xi_t, \tag{10}$$

where ξ_t is an AR(1) process with varying degrees of persistence given by,

$$\log \xi_t = \rho_m \log \xi_{t-1} + \epsilon_t, \qquad \rho_m \in (0,1), \epsilon \sim N(0, \sigma_m^2). \tag{11}$$

Lastly, the aggregate resource constraint states that all output is either invested or consumed,

$$Y_t = C_t + I_t \tag{12}$$

B Solving for the steady state

There are 12 endogenous variables (C_t, L_t, W_t, R_t^k, K_t, I_t, λ_t, q_t, χ_t, i_t, ξ_t, and π_t) described by equations (1)-(11) and the law of motion for capital. Equation (5) implies that in the steady state $q = 1$. Using this, equation (4) can be written as,

$$1 = \beta(R^k + 1 - \delta),$$

substituting in the steady state real rental rate $R^k = \alpha K^{\alpha-1} L^{1-\alpha}$ and rearranging,

$$\frac{K}{L} = \left(\frac{\alpha}{\frac{1}{\beta} - 1 + \delta}\right)^{\frac{1}{1-\alpha}}.$$

The production function can be written in the following form, $Y = (\frac{K}{L})^\alpha L$, and the steady state level of investment is $I = \delta K$. Then the aggregate resource constraint can be rewritten as follows (dividing both sides by L),

$$\left(\frac{K}{L}\right)^\alpha = \frac{C}{L} + \delta\frac{K}{L},$$

substituting in the steady state value for λ and the wage rate W, the intratemporal condition (2) can be written as,

$$\frac{C}{L} = \left(\frac{1-\beta\varepsilon_C}{1-\varepsilon_C}\right)(1-\alpha)(\frac{K}{L})^{\alpha}L^{-(\eta+1)}.$$

Finally, plugging equations for C/L and K/L into the aggregate resource constraint and rearranging yields,

$$L^{\star} = \left[\left(\frac{1-\varepsilon_C}{1-\beta\varepsilon_C}\right)\left(\frac{1}{1-\alpha}\right)\left(\frac{\delta\alpha}{\frac{1}{\beta}-1+\delta}\right)\right]^{\frac{1}{-(\eta+1)}}.$$

By simple substitution one can solve for the remaining steady state values.

C Log-linearizing the model

Log-linearizing equations (1)-(11) and the capital law of motion yields the following system of equations[7] ,

$$(1-\varepsilon_C)(1-\beta\varepsilon_C)\hat{\lambda}_t = \theta\hat{C_{t-1}} - (1+\beta\varepsilon_C^2)\hat{C}_t + \beta\varepsilon_C\hat{C_{t+1}} \tag{13}$$

$$\eta\hat{L}_t = \hat{W}_t + \hat{\lambda}_t \tag{14}$$

$$\hat{\lambda}_t = \hat{\lambda_{t+1}} + \hat{i}_t - E_t\pi_{t+1} \tag{15}$$

$$\hat{q}_t = \hat{\lambda_{t+1}} - \hat{\lambda}_t + \beta\left(\hat{R^k_{t+1}} + (1-\delta)\hat{q_{t+1}}\right) \tag{16}$$

$$\hat{q}_t = \Omega(1+\beta)\hat{I}_t - \Omega\hat{I_{t-1}} - \beta\Omega\hat{I_{t+1}} \tag{17}$$

$$\delta\hat{I}_t = \hat{K_{t+1}} - (1-\delta)\hat{K}_t \tag{18}$$

$$\hat{R^k_t} = R^k(\hat{L}_t - \hat{K}_t + \hat{W}_t) \tag{19}$$

$$\hat{\chi}_t = \frac{\alpha}{R^k}\hat{R^k_t} + (1-\alpha)\hat{W}_t \tag{20}$$

$$\pi_t = \Psi\hat{\chi}_t + \beta E_t\pi_{t+1} \tag{21}$$

$$\hat{Y}_t = \alpha\hat{K}_t + (1-\alpha)\hat{L}_t \tag{22}$$

$$\hat{i^n_t} = \upsilon\pi_t + \xi_t \tag{23}$$

$$\hat{Y}_t = \frac{C}{Y}\hat{C}_t + \frac{I}{Y}\hat{I}_t \tag{24}$$

$$\tag{25}$$

[7]Recall that $\hat{x_t} = \frac{x_t - x}{x}$ for all variables excluding the nominal interest rate and return on capital, which are expressed in percentage point deviations (i.e., $x_t = x_t - x$).

D Dynare codes

The figures can be recreated using the following Dynare code. It is also possible download the files (which includes a Matlab file to recreate the exact figures in the paper) from www.joshuabrault.com/research or https://carleton.ca/khan/research/.[8]

```
// Dynare codes for Brault and Khan, 2019

var Y C L W RK K I LAMBDA Q MC MT PI i R;
varexo eps_m;
parameters BETA HABIT ETA DELTA OMEGA ALPHA NU PSI THETA EPS RHOM;

BETA = 0.99;
HABIT = 0;
ETA = 1;
DELTA = 0.025;
OMEGA = 5.48;
ALPHA = 0.3;
NU = 1.5;
THETA = 0.7;
RHOM = 0.5;
EPS = 0.83;
PSI = ((1-BETA*THETA)*(1-THETA)/THETA)*((1-ALPHA)/(1-ALPHA+(ALPHA/(1-EPS))));

model(linear);
// Some local variables
#LBAR =  ((1-((DELTA*ALPHA)/((1/BETA)-1+DELTA)))*(1/(1-ALPHA))*((1-HABIT)/(1-BETA*HABIT)))^(1/(-ETA-1));
#KBAR = ((ALPHA)/((1/BETA)-1+DELTA))^(1/(1-ALPHA))*LBAR;
#IBAR = DELTA*KBAR;
#YBAR = KBAR^(ALPHA)*LBAR^(1-ALPHA);
#CBAR = YBAR-IBAR;
#RKBAR = (1/BETA)- 1 + DELTA;

// model equations, see section C in appendix for details
(1-HABIT)*(1-BETA*HABIT)*LAMBDA = HABIT*C(-1) - (1+BETA*HABIT^2)*C + BETA*HABIT*C(+1);
ETA*L = LAMBDA + W;
LAMBDA = LAMBDA(+1) + i - PI(+1);
Q = LAMBDA(+1) - LAMBDA + BETA*(RK(+1)+(1-DELTA)*Q(+1));
Q = OMEGA*(1+BETA)*I - OMEGA*I(-1) - BETA*OMEGA*I(+1);
DELTA*I = K - (1-DELTA)*K(-1);
RK = RKBAR*(W-K(-1)+L);
Y = ALPHA*K(-1)+(1-ALPHA)*L;
MC = (ALPHA/RKBAR)*RK + (1-ALPHA)*W;
PI = PSI*MC + BETA*PI(+1);
i = NU*PI + MT;
Y = (CBAR/YBAR)*C + (IBAR/YBAR)*I;
MT = RHOM*MT(-1) + eps_m;

// Auxillary equation to define the ex-post real interest rate
R = i - PI(+1);
end;

steady;
shocks;
var eps_m; stderr 0.01;
end;

stoch_simul(order=1, irf=40, nomoments) K Y C PI MC i R I MT;
```

[8] All computations were done using Matlab 2018b and Dynare 4.5.7 (Adjemian et al. (2011)). In Dynare, capital is a predetermined variable which implies it must show up as dated $t-1$. As is customary, we lead capital by 1 period.