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A Dynamic Model of COVID-19: Contagion and Implications of Isolation Enforcement

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Abstract

We present a dynamic model that produces day-to-day changes in key variables due to the COVID-19 contagion: currently infected people, accumulated infected people, recovered people, deaths, and infected people who require hospitalization. The model is calibrated to the COVID-19 outbreak in Spain so that it replicates the death toll and the phases on the daily deaths curve. Then, we study the effects of the isolation enforcement following the declaration of the State of Alarm (March 14th, 2020). The simulations indicate that both the timing and the intensity of the isolation enforcement are crucial for the COVID-19 spread. Since the infection curve was already very steep at the time of the State of Alarm declaration, a 4-day earlier intervention for social distancing would have reduced the number of COVID-19 infected people by 67%. The model also informs that the isolation enforcement does not delay the peak day of the epidemic but slows down its end. Finally, we find that when social distancing relaxes the evolution of the COVID-19 in Spain will be very sensitive to both the contagion probability (which it is expected to go down due to preventive actions) and the number of interpersonal encounters (which it is expected to go up due to the reopening of economic and social activities). We report a threshold level for the contagion pace to avoid a second COVID-19 outbreak in Spain.

Key words: COVID-19 pandemic, calibrated model simulations, isolation enforcement, policy intervention design

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1 Introduction

On March 11th 2020, the World Health Organization declared the Coronavirus Disease 2019 (COVID-19) outbreak a pandemic—a worldwide spread of the disease. As of May 2nd, there are 238,431 confirmed deaths due to COVID-19 worldwide, with the US, Italy, UK, Spain, and France being the worst hit countries, and the total number of confirmed cases has reached 3.31 million.

Unfortunately, the pandemic is still in progress unleashing a global health crisis and putting enormous pressure on health care systems. The travel related source of virus spread was quickly followed by 'community spread' where the initial source of the infection remains unidentified. Governments and public authorities have implemented mandatory actions to contain the virus spread such as travel restrictions, lockdowns, closures of public spaces, institutions, and businesses, social (and physical) distancing, and self-isolation.

Drawing on the epidemiological Susceptible-Infected-Recovered (SIR) methodology, pioneered by Kermack and McKendrick (1927), we present a discrete-time dynamic model to predict the COVID-19 contagion. Even though the model is simple, it captures the main characteristics of the contagion process and provides insights valuable for policy orientation. We calibrate the model parameters to aggregate Spanish data and present simulations to show the dramatic implications of enforcing mobility constraints over the COVID-19 spread in Spain.[1] We also present three short-term scenarios that may occur when these constraints are eased.

2 Model description

For any given day t, we have the decomposition

$$N = x_t + z_t$$

[1]Our paper can be connected to several recent contributions. Prem et al. (2020) conduct a similar exercise to ours for the city of Wuhan in China with some differences in both the calibration and the model predictions. Sebastiani et al. (2020) analyze the role of government measures in slowing and reducing COVID-19 growth in different regions in Italy. Wang et al. (2020) and Wu, Leung, Bushman, Kishore, Niehus, de Salazar, Cowling, Lipsitch and Leung (2020) estimate the evolution of the COVID-19 cases in Wuhan, while Atkeson (2020) investigates the impact of social distancing for the virus spread in the US, and Ferguson et al. (2020) analyze the impact of non-pharmaceutical interventions to contain the virus expansion in the Great Britain.

where N is the total population on the arrival day of the first person infected by COVID-19, x_t is the accumulated number of people infected by COVID-19 on day t and z_t is the accumulated number of people never infected on day t.[2] On day 1, $x_1 = 1$ and $z_1 = N - 1$. For any future day t, the law of motion for x_t is

$$x_t = x_{t-1} + \alpha y \frac{\widetilde{x}_{t-1}}{N - k_{t-1}} z_{t-1} \tag{1}$$

that adds up to its value on the previous day, x_{t-1}, the number of newly infected people $\alpha y \frac{\widetilde{x}_{t-1}}{N-k_{t-1}} z_{t-1}$. In the latter term, $0 < \alpha < 1$ is the contagion probability on each encounter between one non-infected person and one infected person, $y > 0$ is the number of people each person meets per day, $\widetilde{x}_{t-1}$ is the number of people currently infected as of day $t-1$, and k_{t-1} is the accumulated number of deaths caused by COVID-19 as of day $t-1$.[3]

The ratio $\frac{\widetilde{x}_{t-1}}{N-k_{t-1}}$ provides the share of currently infected people with respect to the surviving population at the end of day $t-1$, which determines the probability of meeting someone infected. Thus, the product of the number of encounters by the rate of infected people, $y\frac{\widetilde{x}_{t-1}}{N-k_{t-1}}$, is the number of infected people every person meets on day t. Once we multiply it by the contagion probability on each encounter, we have $\alpha y \frac{\widetilde{x}_{t-1}}{N-k_{t-1}}$ as the effective daily contagion rate per person. The number of people who have *never* been infected at the end of day $t-1$ is z_{t-1}, and they are the potential newly infected people (susceptible people in the SIR methodology). Therefore, the second term on the right side of (1), $\alpha y \frac{\widetilde{x}_{t-1}}{N-k_{t-1}} z_{t-1}$, is the number of newly infected people on day t. It explains how the number of new cases depends on both the contagion probability α, and on the *total* number of encounters between infected and non-infected individuals, $y\frac{\widetilde{x}_{t-1}}{N-k_{t-1}} z_{t-1}$.

The difference between the accumulated number of people infected, x_t, and the number of people (still) currently infected, $\widetilde{x}_t$, comes from the fact that the COVID-19 disease is neither chronic nor necessarily lethal. Let us assume, for simplicity, that the outcome of the disease is realized within an interval of days after the incubation period (outcome interval). Thus, if

[2] We assume that the initial population N remains constant in this decomposition to consider that all deaths caused by the virus infection will determine the lethality rate of the virus. The same result would be obtained if there where no migration flows and the daily natality rate would be the same as the mortality rate not-related to COVID-19. Given the short time horizon of the analysis and the focus of the paper, we have decided to fix N.

[3] For simplicity, the contagion probability α is both constant over time and identical for all meetings, which ignores heterogeneity in the meeting duration, the degree of physical contact, the viral load of the transmitter, etc. Hence, α is considered to represent contagion probability under average circumstances. We also assume the number of daily social contacts, y, is constant and exogenous, which must be interpreted as the behavior of the representative individual.

the incubation period of the virus is T_i days, where i denotes 'incubation', the lower bound of the outcome interval is the next day after the end of the incubation period, i.e. $T_i + 1$ days. The upper bound is set to have an outcome interval with the same number of days above and below the average duration of the disease, T. Subsequently, the upper bound is $T + (T - (T_i + 1)) = 2T - (T_i + 1)$ days after the virus contagion.

The realization of the disease outcome is uniformly distributed along the days of the outcome interval.[4] Since the number of days with possible realizations of the disease is $2T - (T_i + 1) - (T_i + 1) + 1 = 2\left(T - (T_i + 1)\right) + 1$, there is a constant fraction for each daily cohort of infected people, $\frac{1}{2(T-(T_i+1))+1}$, who perceives the outcome of the disease on a given day. Therefore, the law of motion for the number of currently infected people by COVID-19 is

$$\widetilde{x}_t = \widetilde{x}_{t-1} + \alpha y \frac{\widetilde{x}_{t-1}}{N - k_{t-1}} z_{t-1} - \left(\tfrac{1}{2(T-(T_i+1))+1}\right) \sum_{j=(T_i+1)}^{2T-(T_i+1)} \left(x_{t-j} - x_{t-j-1}\right) \tag{2}$$

The individuals of each cohort can either recover (with an associated survival probability $0 < 1 - \lambda < 1$) or die (with an associated fatality probability $0 < \lambda < 1$). The evolution of accumulated deaths, k_t, is as follows

$$k_t = k_{t-1} + \lambda \left(\frac{1}{2\left(T - (T_i + 1)\right) + 1}\right) \sum_{j=(T_i+1)}^{2T-(T_i+1)} \left(x_{t-j} - x_{t-j-1}\right)$$

Naturally, the accumulated number of recovered people, h_t, is

$$h_t = h_{t-1} + (1 - \lambda) \left(\frac{1}{2\left(T - (T_i + 1)\right) + 1}\right) \sum_{j=T_i+1}^{2T-(T_i+1)} \left(x_{t-j} - x_{t-j-1}\right)$$

Since $N = x_t + z_t$, we can split up the total infected people in three possible states, $x_t = h_t + k_t + \widetilde{x}_t$, to get

$$N = h_t + k_t + \widetilde{x}_t + z_t \tag{3}$$

which means that total population, N, comprise the people who have already healed, h_t, the people who have already died, k_t, the people who are infected with their outcome not yet known, $\widetilde{x}_t$, and the people who have never been infected, z_t.

[4] This is assumed to avoid excessive complexity and due to the uncertainty on the real distribution. Furthermore, there is a large case variability on COVID-19 infections due to person-specific characteristics, for example, age, immune system capacity, early diagnosis and treatment, which makes plausible the assumption of a uniform distribution of the outcome realizations along the days of the outcome interval.

COVID-19 is an infectious virus that typically causes mild symptoms similar to the common flu, and only a minor fraction of sick people who test positive need hospitalization.[5] Nevertheless, the contagion rate of COVID-19 is very high and the capacity of hospitals to give treatment to sick people is severely constrained. In the model, we assume that a fraction θ of the infected people who have passed the incubation period, T_i, suffer from severe complications (typically, respiratory difficulties and pneumonia) and need hospitalization. Thus, the number of hospital beds, b_t, required to treat COVID-19 positive people on day t is

$$b_t = \theta \sum_{j=T_i+1}^{2T-(T_i+1)} \left(x_{t-j} - x_{t-j-1}\right)$$

where $\sum_{j=T_i+1}^{2T-(T_i+1)} \left(x_{t-j} - x_{t-j-1}\right)$ is the total number of infected people who have passed the incubation period, T_i, on day t.

To summarize, we have a dynamic system of 6 equations as follows:

$$\begin{aligned}
x_t &= x_{t-1} + \alpha y \frac{\widetilde{x}_{t-1}}{N - k_{t-1}} z_{t-1} \\
\widetilde{x}_t &= \widetilde{x}_{t-1} + \alpha y \frac{\widetilde{x}_{t-1}}{N - k_{t-1}} z_{t-1} - \left(\frac{1}{2\left(T - (T_i + 1)\right) + 1}\right) \sum_{j=(T_i+1)}^{2T-(T_i+1)} \left(x_{t-j} - x_{t-j-1}\right) \\
N &= x_t + z_t \\
k_t &= k_{t-1} + \lambda \left(\frac{1}{2\left(T - (T_i + 1)\right) + 1}\right) \sum_{j=(T_i+1)}^{2T-(T_i+1)} \left(x_{t-j} - x_{t-j-1}\right) \\
h_t &= h_{t-1} + (1 - \lambda) \left(\frac{1}{2\left(T - (T_i + 1)\right) + 1}\right) \sum_{j=(T_i+1)}^{2T-(T_i+1)} \left(x_{t-j} - x_{t-j-1}\right) \\
b_t &= \theta \sum_{j=T_i+1}^{2T-(T_i+1)} \left(x_{t-j} - x_{t-j-1}\right)
\end{aligned}$$

which determine the evolution of the 6 endogenous variables $\{x_t, \widetilde{x}_t, z_t, k_t, h_t, b_t\}$, given initial values.

[5] In fact, some of the people infected with COVID-19 are asymptomatic, which makes the spreading out of the epidemic more difficult to prevent and control by the health authorities. Anderson et al. (2020) say that "*Estimates suggest that about 80% of people with COVID-19 have mild or asymptomatic disease...*".

3 Model calibration for Spain

The baseline calibration is aimed at representing the outbreak of COVID-19 in a medium-size country. We take the case of Spain because the virus spread has been distributed quite evenly within the territory, with a similar evolution on the daily growth of confirmed cases and the reproduction number observed across the Spanish administrative provinces (ISCIII (2020)). As the variables of our model do not incorporate spatial differentiation, we find it suitable for studying the impact of the virus in territories with homogeneous contagion patterns such as Spain (and not other countries that have the pandemic concentrated close to its epicentre, such as China, Italy or the US).

Table 1: Calibration of model parameters for Spain

Population	$N = 47 \times 10^6$
Fatality rate	$\lambda = 0.0085$
Disease average duration (days)	$T = 16$
Incubation period (days)	$T_i = 5$
Hospitalization rate	$\theta = 0.0528$
Daily encounters per person in normal times	$y = 25$
Contagion probability	$\alpha = 0.01615$

The total population is $N = 47$ million people to coincide approximately with the population of Spain in 2020. For the fatality rate, λ, we follow Anderson et al. (2020) who provide an estimated range between 0.3% and 1% with reference on the data released by the World Health Organization.[6] Typically, the Infection Fatality Rate (IFR), defined as $\frac{\text{confirmed deaths}}{\text{confirmed+unconfirmed cases}}$, is lower than the Case Fatality Rate (CFR), measured as $\frac{\text{confirmed deaths}}{\text{confirmed cases}}$. Since some of the COVID-19 cases are not reported because they are either asymptomatic or the tests have not been taken these two indicators tend to be quite different, with a higher value of the CFR over the IFR. Our model produces the IFR. Spain may experience a relatively high IFR due to the population aging (in 2019 people over 75 years old represented 9.54% of the total population) and the much stronger severity of COVID-19 on the elderly.[7] As for capacity, Spain

[6] Recently, Wu, Leung, Bushman, Kishore, Niehus, de Salazar, Cowling, Lipsitch and Leung (2020) have lowered the estimate of the case fatality risk (measured as the the probability of dying after developing symptoms) of COVID-19 in Wuhan to 1.4%.

[7] For comparative purpose, the percentage of population over 75 years old in the UK was 8.29% in 2018.

has approximately 300 hospital beds per 100,000 people (below the EU average of about 372 beds), and health coverage is guaranteed by the government with a well-developed public provision of hospitals and treatments. Balancing out these arguments, we set $\lambda = 0.0085$ (0.85%), above the median value of the range suggested by Anderson et al. (2020).

The incubation period for COVID-19 is about 5 or 6 days and there is an average period of 10 days or more (longer than a common flu) of confrontation between the immune system and the virus (Anderson et al. (2020)). Therefore, we set an average disease duration at $T = 15$ days and the incubation period last for 5 days, $T_i = 5$.[8] Thus, the calibrated outcome interval runs from $(T_i + 1) = 6$ days after the contagion to $2T - (T_i + 1) = 26$ days after the contagion.

Ferguson et al. (2020) estimate the COVID-19 hospitalization rate for the population of the Great Britain using a subset of cases obtained from China. Their estimate is 4.4%. For Spain, as we assume that in its population there is a higher fraction of elderly people than in either Great Britain or China, we set the hospitalization rate at $\theta = 0.0528$ (5.28%), which implies a 20% higher value than the one reported in Ferguson et al. (2020).

The daily number of two-people encounters per day is subject to heterogeneity because it clearly depends on the specific social and economic characteristics of the individuals (the type of job, social/leisure activities, age, etc.), as well as on the social norms and habits of a country or territory. People gatherings for social and economic activities are quite common in Spain. Thus, we set $y = 25$ meetings to represent an average behavior of Spanish citizens in normal times, though recognizing the uncertainty and variance that affect this model parameter.[9] On March 14th, 2020, the Spanish government declared a state of emergency, the "State of Alarm" (SoA) in response to the COVID-19 outbreak in Spain. The decree contemplated mobility restrictions, school and socioeconomic activity suspensions, and home confinement for the population. Fifteen days after the SoA declaration (March 29th, 2020), the government passed further actions and enforced home confinement to every person whose job is not related to either health care or basic needs. On April 13th, the government gave legal permission to resume the production activity on the manufacturing and construction sectors conditioned to the compliance with protective actions to prevent the virus contagion at the workplace (wearing protection gear, keeping interpersonal distance, reorganizing shifts to minimize workers concentration, etc.). The calibrated model can represent the SoA as a pol-

[8] Prem et al. (2020) assumed a similar incubation period (6.4 days) and a shorter average disease duration (between 9.4 and 13.4 days) in their case study for Wuhan.

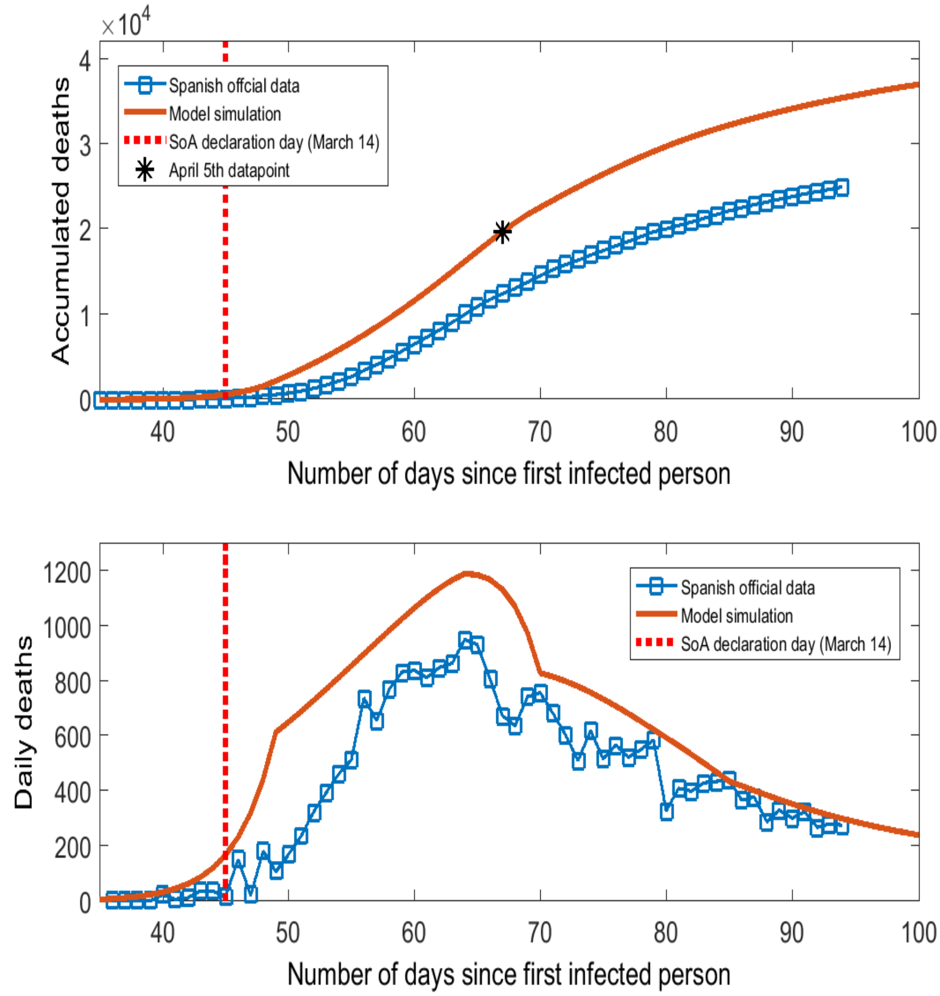
[9] Prem et al. (2020) assume much lower values of daily contacts for Wuhan.

icy intervention that significantly reduces the number of physical contacts among citizens. Hence, we will capture the effects of the SoA intervention by reducing, on the SoA declaration day, the number of interpersonal daily encounters from $y = 25$ to $y = 4$. The tighter lockdown actions, that came into force 15 days past the SoA declaration, are represented as an additional 35% cut in the number of personal contacts to $y = 2.6$ encounters per day.[10] Once the tightening is partially relaxed, 30 days past the SoA declaration, the number of daily contacts returns to $y = 4$ and we also cut the contagion probability by 25% to capture the preventive effect of the new working conditions.

The choice of the day in which the isolation is enforced can be crucial for the posterior extension of the disease (as we will document below). Thus, we paid special attention to selecting the day of our model series when the policy intervention took place in Spain. The first confirmed infected person in Spain was a German tourist who tested positive of COVID-19 in La Gomera (Canary Islands) on January 29, 2020. In turn, we consider January 29 as day 1, and the SoA declaration day (March 14) is day 45. The tightening of the SoA, which reduced work permissions only to jobs related to essential needs, took place on March 29, which is identified as day 60 of the series. The conditioned return to some of the economic activities (April 13) corresponds to day 75.

[10] The Spanish Minister of internal affairs commented on a press conference on the first day after the suspension of all non-basic economic activities that traffic in public transportation fell 34% compared to the previous working day.

Figure 1: **Deaths caused by COVID-19 in Spain.**



The contagion probability α measures the speed at which the virus spreads. In Spain, the COVID-19 showed exponential growing patterns in the early stages with doubling times for confirmed cases and deaths between 2 and 4 days and a reproduction number, R_0, between 4.0 and 7.0 (ISCIII (2020)).[11] As the true number of infected people cannot be observed in the data, we have calibrated the value of α to match the series of deaths caused by COVID-19 in Spain (which are comparable between the model simulations and the data).[12] Official

[11] Ferguson et al. (2020) assume a doubling time of the confirmed cases of COVID-19 at 5 days. As for the reporduction number, they say "*Based on fits to the early growth-rate of the epidemic in Wuhan, we make a baseline assumption that* $R_0 = 2.4$ *but examine values between 2.4 and 2.6.*".

[12] Moreover, a direct observation of the contagion probability α is not possible because the incubation period

data frequently underestimate the number of COVID-19 deaths because many casualties take place outside hospitals (home, elderly residences). Wu, McCann, Katz and Peltier (2020) find these missing deaths to be a very large number for the outbreak in Spain by comparing the excess over the historical average of mortality registration with the official number reported by the government. Specifically, Wu, McCann, Katz and Peltier (2020) calculate that by April 5, 2020, the number of accumulated deaths caused by COVID-19 in Spain should be 19,700 instead of the officially reported value of 12,400 (7,300 missing deaths). We have chosen the value of the primary contagion probability, α, to match the datapoint of 19,700 deaths on April 5 in the series of accumulated deaths generated by the model. This criterion determined setting $\alpha = 0.01615$. Figure 1 shows the official data (up to the latest available observation from May 2nd) and model simulations of accumulated and daily deaths caused by COVID-19 in Spain.[13] Both the phases and peak day of the curve of daily deaths is well replicated by the model, with the gap due to the missing deaths reported by Wu, McCann, Katz and Peltier (2020). Such difference tends to shrink over the downward phase, which is consistent with a larger number of tests taken and the mitigation of the problems for the diagnosis provision that have characterized the peak days of the COVID-19 epidemic in Spain.

4 Simulation results

We have programmed the simulations in Matlab.[14] For initial values, we consider that on day 1, $t = 1$, there is one imported contagion and one person gets infected while the rest of the population had no virus, i.e. $x_1 = \widetilde{x}_1 = 1$. Then, we run the calibrated six-equation model forward over the next 365 days to analyze the effects of the SoA declaration for the COVID-19 spread in Spain. In addition, we simulate the model under alternative decisions on the timing and intensity of the policy intervention. The variables to be discussed here are the number of infected people, accumulated deaths and the number of hospital beds required to treat COVID-19 (infected people who need hospitalization). The benchmark case is the "no intervention" scenario, keeping $y = 25$ as calibrated for normal times in Spain. If

of COVID19 is typically long (5 or 6 days) and many infected people are not tested. These difficulties justify the criterion chosen to calibrate α based on the matching between *ex post* observations of model simulations and the data.

[13] The model prediction for the COVID-19 death toll on April 25 is 33,048 deaths. The official number released on that day was 22,902 which would raise the nuber of missing deaths from 7,300 on April 5 to 10,146 on April 25.

[14] The Matlab code written to carry out the model simulations is available upon request.

there would have been no intervention, the model prediction is that almost all the Spanish people would have been infected (46.95 million people) and, applying the fatality rate, 0.85% of them (nearly 400 thousand people) would have died.

The estimated effects of the SoA intervention are displayed as red lines in Figure 2.[15] In comparison to the no intervention scenario (black lines), the curves of infected people and hospitalized people shift down and widen up as a clear example of the 'flattening of the curve' pattern. If we compare the values of the columns of "No intervention" and "Day 45 (SoA)" reported from Table 2, we find impressive effects. Thus, the SoA declaration is estimated to reduce the accumulated number of infected people from 46.95 million to 5 million, the maximum number of people who need hospitalization from nearly 2.4 million people to 155 thousand people (a 93.5% cut), and in accumulated deaths from almost 400 thousand to 42.5 thousand (a 89% cut).

4.1 The timing of social distancing

As shown in the bottom right-hand cell of Figure 2, the number of people who need to be hospitalized show no apparent variation in the first days after the SoA declaration in comparison to the no intervention case. The reason for this lack of effects is that the reduction in the hospitalized people will not be realized before the end of the 5-day incubation period. Precisely, it is day 51 (6 days after the SoA day) when the slope of the red line flattens as there are fewer infected people who develop symptoms and need to be hospitalized. We represent the Spanish hospital bed capacity as the horizontal dash line in the diagram of the bottom right-hand side of Figures 2 and 3.[16] The model estimates that between days 50 and 78 (nearly one month) the demand for hospital beds exceeds capacity.[17] The downward phase is fast for some days after the peak day but it turns slower on day 70 onwards (coinciding with the end of the outcome interval assumed in the calibration).

[15] It should be noticed that the axes of Figures 2 and 3 have been truncated from above due to the huge value of the variables in the no intervention case.

[16] Spain has an overall amount of around 141,000 hospital beds. Let us suppose that in normal times the capacity utilization rate is 60%. Thus, the hospital beds capacity to cope with the COVID-19 spread in Spain is assumed to be 60% of 141,000 which is 84,600 units.

[17] This result of the model can be corroborated by numerous examples of rush instalments of field hospitals and temporary capacity extensions (hotels, extra space in hospital buildings) in Spain during the worst days of the COVID-19 pandemic. The Spanish health authorities have announced that the largest field hospital put up to treat COVID-19 patients (IFEMA fair and exhibitions facility in Madrid) will be closing on May 1st when it started receiving COVID-19 patients on March 25th.

Figure 2: **Effects of alternative timings for the isolation policy in Spain following the COVID-19 outbreak**

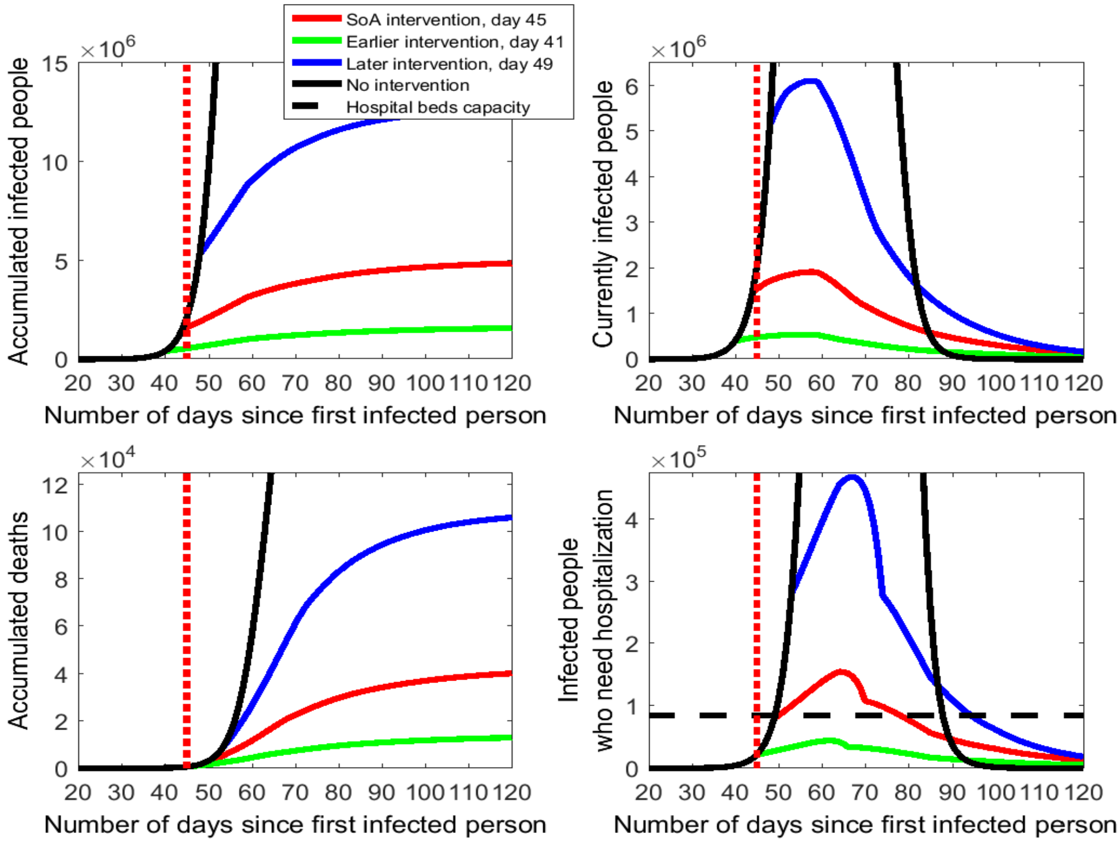


Table 2: **Simulation results of the timing of social distancing in Spain**

	No intervention	Day 41	Day 45 (SoA)	Day 49
• Accumulated infected people, millions	46.95	1.65	5.00	12.74
• Accumulated deaths, thousands	399.1	14.0	42.5	108.3
• Daily peak of hospitalized people, thousands	2383	44.3	155.1	468.7
Peak day	68	62	64	67

A 4-day earlier intervention (day 41) would have been prevented many infections and reduced the number of deaths and the hospitalization needs (see the flattening and pushing down of the green lines in Figure 2 relative to the red lines). Numbers reported in Table 2 support an important point on undertaking early action. In a scenario with social distancing enforced 4 days earlier, the model estimates a reduction by 67% in the accumulated numbers of infected people (from 5 million to 1.65 million) and deaths (28.5 thousand lives are estimated that would have been saved). Moreover, the number of required hospitalizations drops by 71% on peak day, from 155,100 to 44,300, which could have been totally covered by the Spanish health care system (see Figure 2).

The 4-day postponement of the intervention to day 49 would have increased infected people, hospitalization needs, and deaths by a factor close to 2.5 (see the blue lines in Figure 2 and the numbers reported in Table 2). The situation would have been catastrophic for the health assistance of more than 330 thousand people who need medical treatment on the peak day, when this number is more than 6 times the Spanish hospitalization capacity.

In short, the simulation results indicate that the choice of the day for setting the enforcement of social distancing has critical consequences on the evolution of the virus spread.

4.2 The intensity of social distancing

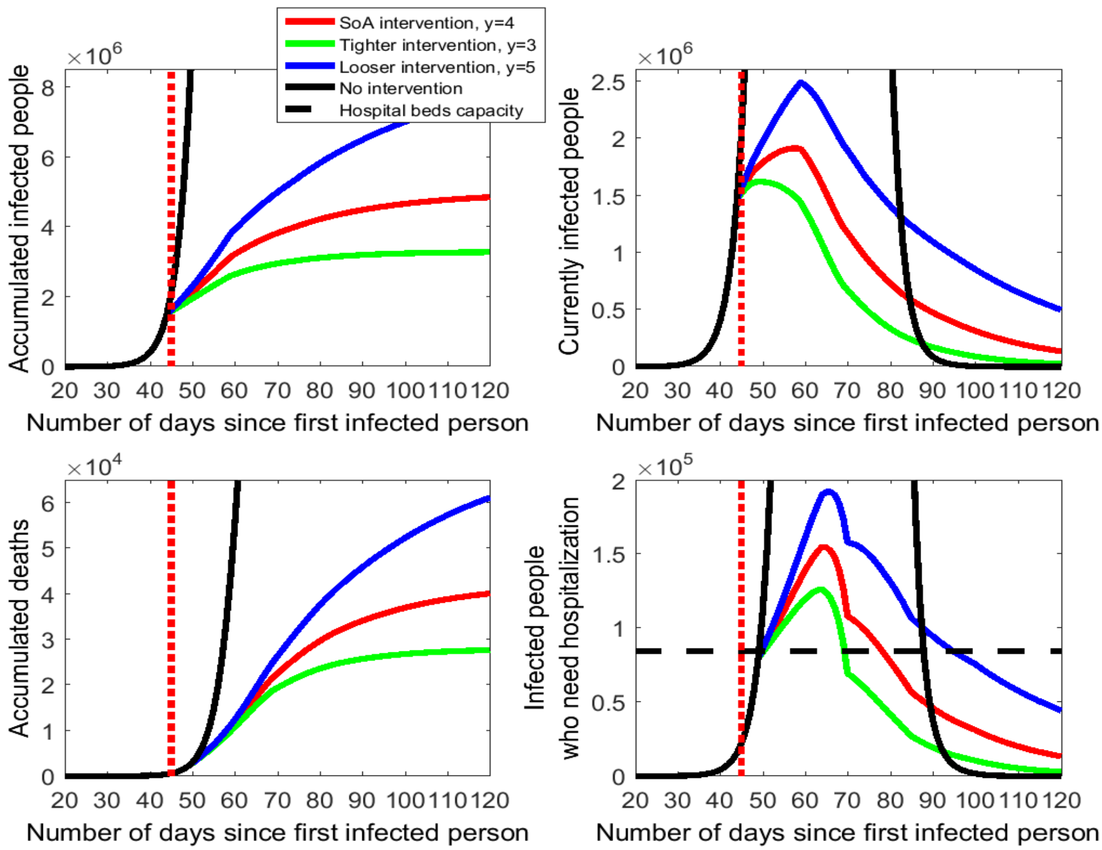
The effects of different degrees of intensity of the social distancing action taken by the Spanish government are documented in Figure 3 and Table 3. Thus, we compare the cases of $y = 3$ (more intensity on isolation) and $y = 5$ (less intensity on isolation) to the calibrated setting of $y = 4$ for the SoA procurement. Once again, the quantitative effects are very large (although somehow not as large as they were for the timing of the intervention).

Table 3: **Simulation results of the intensity of social distancing in Spain**

	No intervention	$y = 3$	$y = 4$ (SoA)	$y = 5$
• Accumulated infected people, millions	46.95	3.30	5.00	8.57
• Accumulated deaths, thousands	399.1	28.0	42.5	72.9
• Daily peak of hospitalized people, thousands	2383	126.3	155.1	192.4
Peak day	68	63	64	66

Both the numbers reported in Table 3 and the green lines on Figure 3 indicate that only reducing the SoA enforcement in one more interpersonal meeting would produce an estimated decrease in the number of accumulated deaths by 34% (from 42.5 thousand to 28 thousand) and in the peak number of people who need hospitalization by 19% (from 155 thousand to 126 thousand). By contrast, a looser implementation of the SoA with $y = 5$ daily encounters per person would have an important cost in human lives (the accumulated number of deaths would rise by 30 thousand) and on the number of people who need hospitalization (on peak day 37 thousand more). Actually, the health care system would be on the verge of collapsing because for 45 consecutive days (between day 50 and day 94, both included) more hospital beds would be required than the installed capacity.

Figure 3: **Effects of alternative intensities for the isolation policy in Spain following the COVID-19 outbreak**



4.3 The effects of isolation enforcement on the epidemic duration

Next, we analyze the duration of the epidemic under alternative scenarios. Usually, social distancing and isolation policies are considered to cause the flattening of the curve on the epidemic characterized by both lower peak values (shift down and widening of the curve) and a later observation of these peak values (shift to the right of the curve). The delay on the observed peak is sometimes used by commentators and policy makers as a justification for not implementing a severe isolation enforcement due to a longer epidemic duration. Figure

4 displays the plots of the model-generated daily series of currently infected people in Spain under alternative scenarios of isolation: no intervention ($y = 25$), SoA ($y = 4$ from day 45), earlier SoA ($y = 4$ from day 41), and tighter SoA ($y = 3$ from day 45). The full-sized vision of Figure 4 clearly illustrates the dramatic effects of isolation to produce the flattening of the curve. Figure 4 also shows that the isolation policies do not involve any shifting of the curve to the right because the peak day is not delayed following any isolation intervention.

Table 4 reports earlier peak days of currently infected people upon any isolation enforcement compared to no intervention (Tables 2 and 3 also documented earlier peak days on hospitalization needs).

The duration of the epidemic is apparently similar in all cases displayed in Figure 4, as by day 100 numbers converge towards the zero line on the number of currently infected people. From the SoA declaration, day 45, to approximately day 85 the number of currently infected people without intervention is dramatically higher than any case of isolation enforcement. After day 85 or so, Figure 4 seems to indicate that the black line (no intervention) falls below the other lines (isolation enforcement).

Although peak days are anticipated due to isolation, Table 4 reports that the downsizing of the epidemic is faster under the no intervention than with any case of isolation enforcement. If we look at the forecast estimates on day 105 (60 days after the SoA declaration), the no intervention scenario would have 1,000 infected people (0.003% of the value on peak day) while the number under the SoA enforcement would still be 254 thousand (13% of the value on peak day). Either earlier or stricter isolation actions reduce the number of infected people to 92 thousand and 65 thousand, respectively. If we look ahead at 90 days after the SoA declaration, all scenarios would lead to small numbers of remaining infected people (virtually 0 for the no intervention case and between 9 thousand and 71 thousand with isolation enforcement).[18] These numbers call for a cautious design of the calendar for the isolation downsizing that restores gradually the economic and social activities when the number of active cases is sufficiently low.[19]

[18]Even though the experiment of a tighter isolation enforcement is estimated to save less lives than the earlier enforcement case (compare the numbers provided, respectively, in Tables 2 and 3), the model simulations indicate that a tighter enforcement would bring down the number of people infected faster than the earlier enforcement.

[19]While our focus is on studying the effects of immediate mobility controls in dealing with the ongoing health crisis, their unavoidable drastic effects on economic activity are underway. An early example is Eichenbaum et al. (2020) who embed an epidemiological model in a macroeconomic general equilibrium model to study the tradeoff between the severity of decline in output and lives saved.

Figure 4: **Estimated duration of COVID-19 epidemic in Spain on alternative isolation enforcement scenarios**

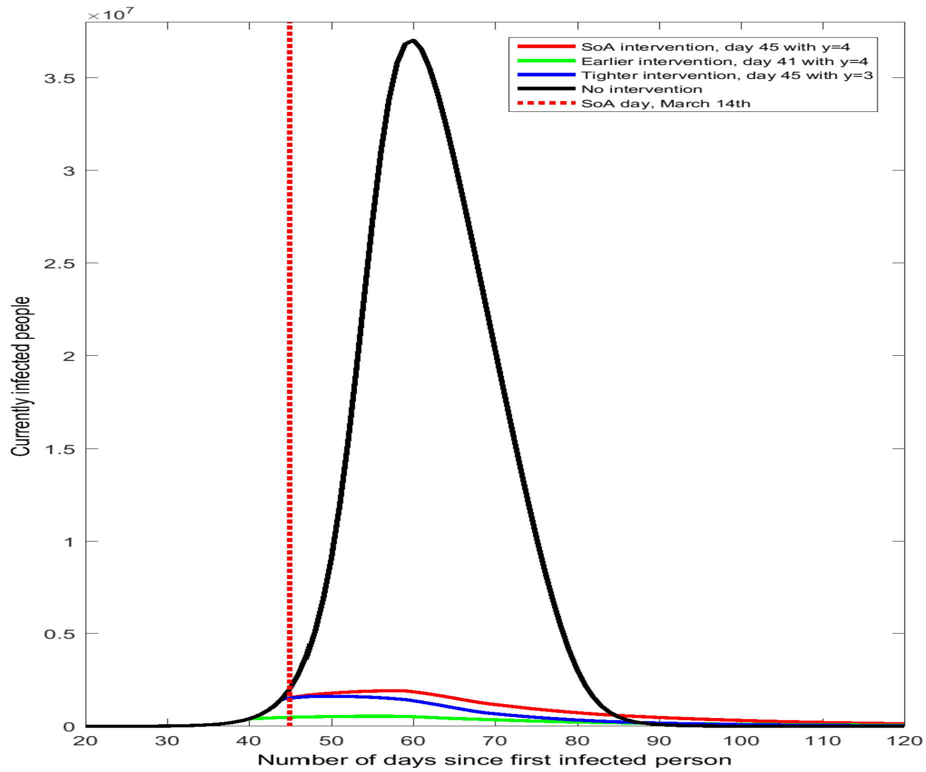


Table 4: **Simulation results of the duration of the COVID-19 epidemic in Spain**

	No intervention	SoA	Earlier SoA	Tighter SoA
Peak day for currently infected people	60	58	55	49
Number of currently infected people, thousands				
• on day 45 (SoA declaration)	2020	1538	482	1515
• on peak day	37029	1914	542	1620
• on day 75 (30 days after SoA declaration)	10273	928	279	481
• on day 105 (60 days after SoA declaration)	1	254	92	65
• on day 135 (90 days after SoA declaration)	0	71	32	9

4.4 A second peak?

The State of Alarm in Spain contemplates a gradual return to normality from May 11 (day 102), when the general quarantine period will be over and many of the isolation enforcement actions (home lockdowns and mobility restrictions) will cease. Family meetings will be permitted with some limitations of the duration and the number of relatives involved. In addition, most shops, bars, restaurants and hotels will be reopened subject to controls on interpersonal distance and continuous disinfection. In turn, the number of daily encounters between Spanish citizens will rise. For the virus spread containment, health authorities are announcing that some activities will still remain suspended (schools, music concerts, people-attending sports competitions,...), preventive actions will be required for both working and using public transportation (wearing masks and gloves, regular disinfections and hand washing, keeping a safe interpersonal distance), and a wide public provision of tests for a rapid identification and self-isolation of positive cases will be available. These mitigation actions will likely cut the contagion probability.[20] The after-lockdown stage of the epidemic would be therefore characterized by a higher value in the number of daily meetings per person, y, and a lower value in the primary contagion probability, α. These two changes have opposing effects on the contagion pace, that have not be considered so far and will be discussed next. Since αy is the product of the primary contagion probability, α, times the number of daily encounters, y, we can refer to it as the maximum contagion probability (i.e., the one associated to the case of meeting infected people in all the daily encounters). The calibrated value of αy for the SoA stage prior to the end of lockdown is $\alpha y = (0.75)(0.01615)(25 - 21) = (0.0121)(4) = 0.0484$ (4.84%). Taking $\alpha y = 4.84\%$ as the benchmark value, we will examine the evolution of the COVID-19 curve under 3 possible scenarios for αy after May 11:

- A high value of maximum contagion probability: $\alpha y = (0.01)(10) = 0.10$ (or 10%)
- A moderate value of maximum contagion probability: $\alpha y = (0.01)(8) = 0.08$ (or 8%)
- A low value of maximum contagion probability: $\alpha y = (0.01)(6) = 0.06$ (or 6%)

These scenarios combine a lower primary contagion probability (α falls from 0.0121 to 0.01) with a higher number of daily encounters per person (y rises from 4 to 10, 8, or 6). Figure 5 and Table 5 show the results.

Small changes in the value of the maximum contagion probability αy result in quite dif-

[20] Additionally, as the Summer season approaches the average temperature in Spain turns substantially warmer which may reduce the transmission power of COVID-19. This effect would imply a lower contagion probability, α.

ferent trajectories for the COVID-19 spread in Spain. When αy rises from 4.84% to 10%, the curve of currently infected people quickly bends uphill with a 100-day long period of a continuous increasing (see the blue line in Figure 5). Hence, the resulting second wave would be even worse than the one suffered in March: it would last longer and the peak number of currently infected people would be observed with 2.85 million. The effects in the accumulated number of infected people and deaths would be dramatic (see Table 5).

A more moderate increase in the maximum contagion probability from 4.84% to 8% would still produce a second peak of the virus spread but with a smaller prevalence than the first peak. As the red line of Figure 5 shows, the number of infected people would feature a low positive slope from May to September. On the second peak day (around mid-September), the number of currently infected people is 462 thousand, approximately 1/4 of the value observed in late March. The death toll and the accumulated number of infections would be more than doubling the numbers obtained with no mitigation of social distancing (reported in Table 2) because the COVID-19 epidemic would be present in Spain for the whole year.

Figure 5: **COVID-19 contagion spread in Spain after social distancing mitigation**

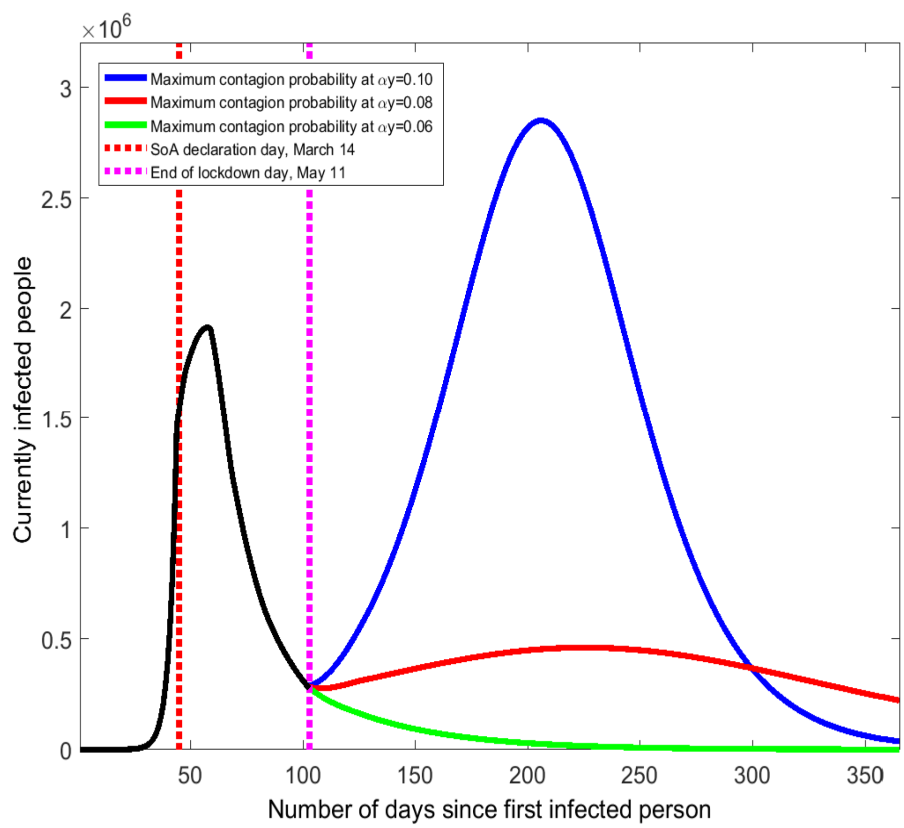


Table 5: **Simulation results of the COVID-19 spread in Spain after social distance mitigation**

	$\alpha y = 0.10$	$\alpha y = 0.08$	$\alpha y = 0.06$
• Accumulated infected people, millions	25.12	11.24	5.35
• Accumulated deaths, thousands	213.2	93.6	45.5
• Second peak of currently infected people, (Yes/No)	Yes	Yes	No
Infected people on second peak day, thousands	2851	462	-
Second peak day	207	226	-

If the increase of the maximum contagion probability after May 11 were small (from 4.84% to 6%), there would be no second wave of the COVID-19 outbreak and the curve would keep moving downhill (see green line of Figure 5). The accumulated numbers of infected people and death would barely increase. This result shows that a second wave can be avoided if the change in the maximum contagion probability, αy, is sufficiently low. We have searched for the threshold of αy that determines whether the curve turns upward or continues downward. Such critical level is found at $\alpha y = 0.0761$ (7.61%) that delivers a flat line of active cases after the vertical dotted May 11 line in Figure 5. Thus, the Spanish health authorities should monitor that the maximum contagion probability stays below 7.61% to prevent a second COVID-19 peak.

5 Conclusions

We presented a dynamic discrete-time model of the COVID-19 spread that provides information on six variables relevant for the quantitative analysis of many ongoing containment efforts.

The model has been calibrated to Spanish data to quantify the impact of alternative isolation enforcements in the face of the COVID-19 pandemic. Compared to the no intervention scenario, the State of Alarm declaration is estimated to cut the number of accumulated deaths by 89% and the number of hospital beds needed by 93.5%. Both an earlier and a more intense intervention could have been crucial for further reductions in infected people, deaths and hospitalizations. The isolation enforcement does not delay the peak day of the epidemic but slows down its end.

The model estimates that the day of the State of Alarm declaration (March 14) Spain had

1.5 million actively infected people, on peak day (March 27) it reached 1.9 million and on the last day of forced home confinement (May 10) it will have dropped to 300 thousand. The mitigation of isolation enforcement could bring a second wave of the COVID-19 outbreak in Spain if the maximum contagion probability rises from 4.84% to beyond 7.61%.

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