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WHEN SALES MAXIMIZATION IS PROFIT-MAXIMIZING: A TWO-STAGE GAME

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Very Preliminary.

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Abstract

This paper attempts to revive interest in the sales maximization hypothesis by proving that there are conditions under which such an objective can be profitable. Competition is modelled as a two-stage game. In the first stage owners write contracts with managers instructing them on whether to maximize sales, profits or some combination. In the second stage, the managers play a Cournot quantity game selecting outputs to maximize the chosen objective function. In such a game, sales maximization represents a more aggressive strategy that can enlarge a firm's market share and may or may not increase its profits. When the objective is to maximize a weighted sum of profits and sales, pure profit maximization (i.e. putting all the weight on profits) never proves optimal.

I. Introduction

In a now-famous book published in 1959, William Baumol suggested that profit maximization may not be the objective of all firms. He went on to study the implications of an assumption of sales maximization for these firms. Baumol's work is probably the most famous of the large literature that offered a variety of alternative objective functions, including Williamson's general managerial preference function and Simon's notion of "satisficing". (See Clarkson and Miller [1982], pp. 28-38 for a short discussion of this literature.)

We might attribute the fact that so much of this work has faded from memory to two causes. First, though these theories were often more difficult to use, it is not clear that they enriched empirical work, as compared to the simple profit maximization hypothesis. Second, many economists have enough faith in the competitive processes (in both output and corporate control markets) and its Darwinian effects, that they believe that only profit-maximizing firms could exist in competitive markets. Combine this with a belief that most of the economy, particularly the market for corporate control, is very competitive (or contestable), and a sales-maximizing firm must indeed be a rare thing.

My purpose here is to resurrect the sales maximization hypothesis by proving that it may be profitable, thereby addressing this second problem. The results come from modelling competition between firms as a two-stage game. In the first stage, owners of firms write contracts with managers telling them what to maximize. The second stage is a fairly standard Cournot-type quantity game with managers selecting output to maximize their directed objective, assuming the other firms' outputs are fixed.

The results indicate that, under certain conditions, it may indeed be optimal for owners to tell managers to maximize the dollar value of sales. This translates into a more aggressive strategy in the following quantity game that will lead to a higher market share and possibly higher profits. There may be a prisoners' dilemma equilibrium here, in which all firms try to maximize sales when they would all be better off should they only maximize profits.

The next section lays out the simple duopoly model and presents the basic results. Some extensions are contained in Section III, while the fourth section offers the conclusions.

II. Duopoly

Since we will need the ability to derive explicit solutions defining equilibria under different regimes, it is particularly helpful to start with a very simply model. Here we begin by studying a duopoly, with linear demand and identical constant marginal costs.

For convenience we will assume that each firm has one owner, who does want to maximize profits, and one manager who does what he is told by the owner. While we assume away agency problems here, it would be interesting to incorporate them in future work. Agency is, in a sense, the modern theory that explains behaviour that is not profit-maximizing.

We model competition in this market as a two-stage game. In the first stage, each owner writes a contract with her manager telling him what his objective should be when he selects an output. This contract must take the form of a credible commitment. The candidate objective functions we consider here involve profit maximization or sales (dollar value, not physical units) maximization. In the next section, a hybrid is studied in which the objective is to maximize a weighted sum of sales and profits.

In the second stage of the game the managers play a quantity setting game with Nash conjectures. That is, each selects an output that maximizes his objective function, on the assumption that his rival's output is fixed.

As indicated, demand is linear and we express it as:

(1) $$P = a - bX$$

where $X = x_1 + x_2$, the sum of the two firms' outputs, and P is price. Marginal costs, c, are assumed to be constant.

If the owners in the first stage had told their managers to maximize profits, we would observe the familiar Cournot-Nash equilibrium. Each manager would choose output x_i, to maximize profits:

$$(2) \qquad \Pi_i = (P - c)x_i \qquad \text{for } i = 1,2 .$$

Differentiating (2) with respect to x_i, holding x_j (the other firm's output) constant, yields the first-order condition

$$(3) \qquad P - c - bx_i = 0$$

which can be solved for x_i, giving the reaction functions:

$$(4) \qquad x_i = \frac{a - c - bx_j}{2b} \qquad \text{for } i, j = 1,2, i \neq j.$$

Solving the two equations in (4), we have

$$(5) \qquad x_1 = x_2 = \frac{a - c}{3b}$$

so that

(6) $$X = x_1 + x_2 = \frac{2(a - c)}{3b}$$

and profits for each firm can be easily shown to be

(7) $$\Pi_i = \frac{(a - c)^2}{9b} .$$

This is all very familiar. But now consider the incentive of the owner of firm one to change the contract with her manager, requiring him instead to maximize sales. This will have the effect of pushing her firm's reaction function outward, capturing a larger market share. Credibility could be a problem here. The manager must believe that his rewards will be based upon how well he maximizes sales rather than profits, even though he must know that it is profits his owner really cares about. At any rate, we assume he faithfully follows orders.

The manager of firm one will then choose x_1 to maximize sales

(8) $$S_1 = Px_1,$$

yielding a first-order condition replacing (3) of

(9) $$P - bx_1 = 0 .$$

The corresponding reaction function for firm one is now

(10) $$x_1 = \frac{a - bx_2}{2b} \; .$$

Combining (10) with the reaction function (4) for firm two gives us our new equilibrium quantities

(11) $$x_1 = \frac{a + c}{3b}$$

(12) $$x_2 = \frac{a - 2c}{3b}$$

and

(13) $$X = \frac{2a - c}{3b} \; .$$

Simple calculations reveal that firm one's profits will now be

(14) $$\Pi_1 = \frac{(a - 2c)(a + c)}{9b} \; .$$

The profitability of this new contract is easily determined by comparing the profits in (14) to those in (7). Profits will be higher with sales maximization if

$$(a - 2c)(a + c) > (a - c)^2$$

which can be simplified to (assuming $c > 0$)

$$(15) \qquad a > 3c \quad .$$

Therefore, we have our first result:

Proposition 1: If $a > 3c$, $c > 0$ and one firm's manager is maximizing profits, it will be profitable for the other firm's owner to instruct her manager to maximize sales.

The simplicity of this condition may seem a little surprising, particularly the irrelevance of b, the slope of the demand curve. We shall return to this question shortly.

First, we must complete the characterization of the two-stage equilibrium. If $a > 3c$, we know that each owner will want to write a sales maximization contract with her manager. But suppose firm one does this first, will firm two necessarily follow? This will expand output and lower price still further, leaving both firms worse off than when the managers were maximizing profit.

If firm one maximizes sales while firm two maximizes profits, firm one's profits are given by (14) and firm two's by (16):

$$(16) \qquad \Pi_2 = \frac{(a - 2c)^2}{9b} \quad .$$

Notice that this will be positive if $a > 3c$, but it will represent lower profits than firm two had earned when firm one's manager also maximized profits.

If both owners encourage managers to maximize sales, both reaction functions will be of the form given in (10) and the equilibrium outputs will be

$$(17) \qquad x_i = \frac{a}{3b} \qquad \text{for } i = 1,2$$

$$(18) \qquad X = \frac{2a}{3b} .$$

Profits for each firm are then

$$(19) \qquad \Pi_i = \frac{a(a - 3c)}{9b}$$

which is easily seen to be less than profits when both firms maximized profits. The question now, however, is whether this is better for firm two than being the only firm maximizing profits. Switching to a sales maximization approach will be profitable for firm two if

$$a(a - 3c) > (a - 2c)^2$$

which can be reduced to (again assuming $c > 0$)

(20) $a > 4c$,

giving our second result.

Proposition 2: There are three possible equilibria to this two-stage duopoly game. If $a < 3c$, both owners will order managers to maximize profits, and outputs and profits will be as in (5) and (7). If $a > 4c$, both managers will be instructed to maximize sales, yielding the outputs and profits given by (17) and (19). Finally, if $3c < a < 4c$, one firm will choose to maximize profits while the other maximizes sales. The sales maximizing firm will have an output and profits as given in (11) and (14) while the other has an output and profits given by (12) and (16).

So we see that both symmetric and nonsymmetric equilibria are possible in this model. There are, of course, multiple (i.e. two) equilibria when $3c < a < 4c$, one in which the first firm maximizes profits and the other sales, and another in which these objectives are reversed.

Before moving on to generalize the model a little, we should consider what lies behind the condition (15) that makes it optimal for at least one firm to maximize sales. Specifically, we may wonder why the slope parameter does not enter the condition - is elasticity irrelevant? The answer is no. The absence of b is due to the particular properties of the simple model we have here. With linear demand and constant marginal costs, it is easy to show that the market elasticity of demand at the equilibrium point will be independent of b.

If both firms maximize profits, total output is given by (6) and it is a simple matter to prove that the (absolute value of the) elasticity at this point will be

$$(21) \qquad \eta_{\Pi} = \frac{-dX}{dP} \cdot \frac{P}{X} = \frac{a + 2c}{2(a - c)} = \frac{r + 2}{2(r - 1)} \qquad (\text{where } r \equiv a/c)$$

which is independent of b, the slope, and depends only on r, the ratio of a and c, not their levels.

Notice also that $d\eta_{\Pi}/dr < 0$. Interestingly, this means that the condition that makes a switch to sales maximization more profitable will also make demand less elastic. This may seem counter-intuitive, since output expansions with inelastic demands will have larger (negative) price effects. What happens is that with higher values of the ratio r, the percentage difference between the outputs given by (6) and (13) shrinks. As a result, price changes by a smaller total percentage as r rises, even though it is more sensitive to marginal changes in quantity.

The slope parameter does, of course, effect the magnitude of the difference between the levels of profits given in (7) and (14), though not its sign. This means that if we introduced some differential cost in writing or enforcing the two types of contracts it would also play a role in determining the choice of contract. For example, suppose profits contracts were more costly to monitor because of possible haggling over how to define profits. Assume the cost difference can be represented by a fixed cost z subtracted from the profits given by (7). Now firm one will switch to sales maximization if

$$\frac{(a - 2c)(a + c)}{9b} > \frac{(a - c)^2}{9b} - z$$

or

$$a > 3c - \frac{9bz}{c}. \tag{22}$$

III. Some Generalizations

We begin by taking up the question of how the profitability of sales maximization is affected by the number of oligopolists. Is there something special about duopoly that makes Proposition 1 possible? It turns out that adding firms actually weakens the condition necessary for sales maximization to be profitable.

The regular Cournot equilibrium with n firms is very easy to calculate. Each of the n firms has a first-order condition:

$$a - bX - c - bx_i = 0 \qquad \text{for } i = 1,2\ldots n, \tag{23}$$

where now $X = \sum_{j=1}^{n} x_j$. In equilibrium, with symmetry, $X = nx_i$ and each firm's output will be

$$x_i = \frac{a - c}{b(n + 1)} \tag{24}$$

so that

$$X = \left(\frac{n}{n + 1}\right)\left(\frac{a - c}{b}\right). \tag{25}$$

Profits for each firm will be

$$\Pi_i = \frac{1}{b}\left(\frac{a - c}{n + 1}\right)^2. \tag{26}$$

Should firm one elect to maximize sales, its first-order condition becomes

(27) $$a - bX - bx_1 = 0 .$$

One firm solving condition (27) together with n - 1 firms solving (23) yields

(28) $$x_1 = \frac{a + c(n - 1)}{b(n + 1)}$$

(29) $$x_i = \frac{a - 2c}{b(n + 1)} \qquad \text{for } i = 2,3\ldots,n$$

and

(30) $$X = \frac{an - c(n - 1)}{b(n + 1)} .$$

Profits for firm one will now be:

(31) $$\Pi_1 = \frac{(a - 2c)[a + c(n - 1)]}{b(n + 1)^2} .$$

If the profits in (31) exceed those in (26) the switch to sales maximization will have been profitable. This requires

$$(a - 2c)[a + c(n - 1)] > (a - c)^2$$

which reduces to

$$\text{(32)} \qquad a > c\left(\frac{2n - 1}{n - 1}\right) .$$

Let the term $\left(\frac{2n - 1}{n - 1}\right)$ be called $K(n)$ and notice that $K(2) = 3$ as before. The properties we care about here are the sign of dK/dn and the limit (if any) K approaches as n gets large. These are easy enough to see.

$$\frac{dK}{dn} = \frac{-1}{(n - 1)^2} < 0$$

and $$\lim_{n \to \infty} K(n) = 2 .$$

Thus, condition (32) is actually less demanding when there are more than two firms.

Proposition 3: Other things equal, the more firms there are in the market the weaker the condition necessary for sales-maximization to dominate profit maximization. There is a limit, however. If $a < 2c$ no number of firms will make sales maximization optimal.

Finally, let us consider a more complicated contract. Suppose that owners could instruct managers to maximize a weighted sum of profits and sales, and call this W_i:

$$W_i = \alpha(P - c)x_i + (1 - \alpha)Px_i$$

or

$$W_i = (P - \alpha c)x_i \ .$$

Now we can ask about the conditions under which $\alpha = 1$ (profit maximization), $\alpha = 0$ (sales maximization) or $0 < \alpha < 1$.

We return to the duopoly model and assume that firm two is maximizing profits. What value for α will maximize firm one's profits? The reaction functions become

$$x_1 = \frac{a - \alpha c - bx_2}{2b}$$

$$x_2 = \frac{a - c - bx_1}{2b}$$

yielding equilibrium outputs

$$x_1 = \frac{a - (2\alpha - 1)c}{3b}$$

$$x_2 = \frac{a - (2 - \alpha)c}{3b}$$

and

$$X = \frac{2a - (\alpha + 1)c}{3b} .$$

Profits for firm one will be

$$(33) \qquad \Pi_1 = \frac{[a - (2 - \alpha)c][a - (2\alpha - 1)c]}{9b} .$$

The owner of firm one will then pick α to maximize Π_1 as given in (33). The first-order condition

$$\frac{d\Pi_1}{d\alpha} = \frac{1}{9b} \{[a - (2 - \alpha)c](-2c) + [a - (2\alpha - 1)c]c\} = 0$$

reduces to

(34) $$\alpha^* = \frac{5c - a}{4c}.$$

Notice that as long as $a > c$, α^* must be strictly less than one. Thus we have the result:

Proposition 4: Suppose contracts in which owners instruct managers to maximize W_i (a weighted average of profits and sales) are feasible. Then it will never be optimal for an owner to order full profit maximization (i.e. α^* will never equal one).

Thus, profit maximization is never going to be profit-maximizing. The optimal contract, α^*, will also depend only upon the ratio a/c. If we restrict α to lie in the range [0,1] we can see from (34) that full-blown sales maximization ($\alpha^* = 0$) becomes optimal when $a \geqslant 5c$.

The variability of α simply allows firm one to select equilibria from among the set of points along firm two's reaction function. Hence, the value given in (34), since it is profit-maximizing for firm one, must reproduce the Stackelberg equilibrium we would observe if firm one was the leader and firm two the follower. This is straightforward to verify.

Solving for the full symmetric equilibrium is somewhat more complicated. Let α_1 and α_2 represent the two firms' weights on profits. Their reaction functions will be

$$x_1 = \frac{a - \alpha_1 c}{2b} - \frac{x_2}{2}$$

$$x_2 = \frac{a - \alpha_2 c}{2b} - \frac{x_1}{2}$$

yielding equilibria outputs for any given pair (α_1, α_2) of

$$(35) \qquad x_1 = \frac{a - (2\alpha_1 - \alpha_2)c}{3b}$$

$$(36) \qquad x_2 = \frac{a - (2\alpha_2 - \alpha_1)c}{3b}$$

and then

$$(37) \qquad X = \frac{2a - (\alpha_1 + \alpha_2)c}{3b} .$$

We must now find the value for (for example) α_1 that maximizes firm one's profits, (i.e. $d\Pi_1/d\alpha_1 = 0$) at a point where $\alpha_1 = \alpha_2$ for symmetry. The first-order condition is

$$(38) \qquad \frac{d\Pi}{d\alpha_1} = (a - bX - c)\frac{dx_1}{d\alpha_1} - bx_1 \frac{dX}{d\alpha_1} = 0 .$$

From (35) and (37) we see that

$$\frac{dx_1}{d\alpha_1} = \frac{-2c}{3b} \quad \text{and} \quad \frac{dX}{d\alpha_1} = \frac{-c}{3b} .$$

Solving (38) setting $\alpha_2 = \alpha_1$ to satisfy the symmetry condition, the equilibrium contract, $\tilde{\alpha}_1$, is seen to be:

$$(39) \qquad \tilde{\alpha}_1 = \frac{6c - a}{5c} .$$

Restricting $\tilde{\alpha}_i$ to lie between 0 and 1 gives us:

Proposition 5: In a game in which both owners instruct their managers to maximize a weighted average of sales and profits, and α represents the weight on profits, the following are the resulting symmetric equilibria:

(i) For $a \geqslant 6c$, $\tilde{\alpha} = 0$ (i.e. both firms put full weight on sales)

(ii) For $c < a < 6c$, $\tilde{\alpha} = \frac{6c - a}{5c}$, so that $0 < \tilde{\alpha} < 1$.

Notice that as long as $a > c$, $\tilde{\alpha} < 1$, so that pure profit maximization is still never optimal. Also note that as long as $\tilde{\alpha}$ is still less than one, increasing a (i.e. reducing the elasticity of demand) will reduce the optimal weight on profits. Hence, we can say that the more profitable the market, the less weight managers will put on profits in their maximand.

IV. Conclusions

This paper is part of a growing literature that analyses competition by viewing the process as a multi-stage game. Such models have become popular in industrial organization, where they have been used to study entry deterrence (see, for example Dixit [1980] and Ware [1984]) and the effects of Cournot selections of capacity on subsequent Bertrand price competition equilibria (Kreps and Scheinkman [1983]). They have also been used in finance, to explore the effect of debt/equity levels on Cournot competition (Brander and Lewis [1986]) and in international trade to study the games played by governments attempting to set trade policies to advantage domestic suppliers competing with foreign firms (Brander and Spencer [1984] and [1985] and Cooper and Riezman [1986]).

Here we have seen that conditions may exist under which it is optimal for owners, who care about profits, to instruct managers to maximize sales rather than profits. If the early motivation for the sales maximization hypothesis comes from an apparent managerial emphasis on sales, these results may explain this emphasis. It may not be irrational, or caused by any agency problem. Indeed, managers may be following orders perfectly.

It was important to the analysis that once the owner's contract with the manager is set, the owner leaves the firm's operation up to the manager. Otherwise a credibility problem could arise since at the sales maximization equilibrium it would in fact be profitable for each firm to cut back output somewhat. It does not seem unreasonable, however, to view these management contracts as credible commitments to play a more aggressive strategy in the quantity game. Here, the separation of ownership from control may have its advantages.

Several extensions of this work suggest themselves. First, of course, the generality of the results could be tested by exploring the effects of non-linear demand and non-constant marginal costs. And differences between firms (in costs, for example) could be studied. Probably more interesting, would be an investigation of what other elements might be included in a credible management contract to encourage more aggressive play. Bonuses and rewards for growth in market share might be desirable, for example. Or it may prove optimal to introduce nonlinearities into the objective function. The manager's compensation could be a convex function of sales, for example, with an increasing marginal reward for expanding sales. Finally, it would be useful to try to introduce agency considerations into this model. If sales are easier to predict and monitor than profits, sales maximization contracts may have advantages over and above those described here.

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