

CHANGES MAY BE MADE TO THIS COURSE OUTLINE UP TO SEPTEMBER 17 2019

**CARLETON UNIVERSITY
DEPARTMENT OF ECONOMICS**

**ECON 4001 A
MATHEMATICAL ANALYSIS IN ECONOMICS**

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Office Hours: Monday 2:45pm-4:45 pm, Wednesday 1:15-2:15 pm, and by appointment

Fall 2019

Lectures: Mondays and Wednesdays, 11:35-12:55 PM

Place: TB 202.

Tutorial: Mondays, 1:05 PM to 2:25 PM

Place: TB 202.

Course Objectives:

A presentation of various mathematical tools. An analysis of static optimization: including the Weierstrass, Lagrange, Kuhn-Tucker and the envelope theorems. A presentation of dynamic economic analysis including dynamic optimization based on the Hamiltonian and the Maximum Principle. Various applications of these mathematical techniques to problems in economic theory.

Prerequisites:

ECON 3001 with a grade of C+ or higher. Prerequisites will not be waived, and no registration will be permitted without the required prerequisites. Students who believe they have taken a similar background course or courses from another university must provide appropriate documentation to the Department of Economics Undergraduate Administrator, Amanda Wright.

Required textbooks:

Michael Hoy, John Livernois, Chris McKenna and Ray Rees, *Mathematics for Economics* (third edition), MIT Press, 2011. ISBN: 9780262516228. Each chapter has many problems with solutions provided at the end of the book for the odd numbered problems. This book is referred to as Hoy.

Knut Sydsaeter, Peter Hammond, Atle Seierstad, Arne Strom, *Further Mathematics for Economic Analysis* (2nd edition), Prentice Hall, 2008. This book is referred to as Sydsaeter.

Supplementary reference books:

Alpha C. Chiang and Kevin Wainwright. *Fundamental Methods of Mathematical Economics*, 4th ed., McGraw-Hill, 2013. This book is referred to as Chiang.

William E. Boyce and Richard C. Di Prima. *Elementary Differential Equations*, 10th ed., John Wiley and Sons, Inc, 2012.

COURSE OUTLINE AND READING LIST

The required readings are from the Hoy and Sydsaeter textbooks. Additional readings and topics may be assigned as the instructor sees fit. It is assumed that you know the material covered in ECON 3001 and ECON 4001 will make use of what you have learned in that course. Sections 1 and 2 of the course outline will review briefly some of these concepts that you have studied in ECON 3001. However, these sections will especially emphasize new concepts. If you are rusty, you are encouraged to review rules of differentiation (see any textbook on calculus and Hoy or Chiang for a very brief review) and of integration (see any textbook in calculus and Hoy, ch 16, especially sec 16.1 and 16.5 for a very brief review), matrix algebra (see Hoy, ch 8, 9 and 10 section 10.1), complex numbers and trigonometry (see for example, Hoy, appendix, and any textbook on algebra and trigonometry). Reviewing integration techniques, series and sequences is especially important as these will be needed to solve differential and difference equations and perform dynamic analysis.

Useful references to review calculus are: James Stewart, *Single Variable Calculus*, 8th ed, 2015 and James Stewart, *Multivariable Calculus*, 8th ed, 2015. A useful reference to review trigonometry is: James Stewart, Lothar Redlin and Saleem Watson. *Algebra and Trigonometry*, 4th ed, 2015.

The emphasis in this course will be on solving problems.

1. Topics in linear algebra

- 1.1. Basic rules for matrices and determinants
- 1.2. Linear independence, rank of a matrix
- 1.3. Eigenvalues and eigenvectors
- 1.4. Diagonalization
- 1.5. Quadratic forms

Lecture Notes

Sydsaeter, ch 1 sections 1.1-1.8

Hoy, ch 10 sections 10.2-10.3

2. Topics in multivariable calculus

- 2.1. Gradients and directional derivatives
- 2.2. Convex sets
- 2.3. Concave and convex functions
- 2.4. Quasiconcave and quasiconvex functions
- 2.5. Taylor's formula

Lecture Notes

Sydsaeter, ch 2 sections 2.1-2.6

Hoy, ch 5, ch 11 sections 11.4 and 11.5

Chiang, ch 11 sections 11.5 and ch 12 section 12.4

3. Static optimization theory: review and new concepts

- 3.1. Unconstrained optimization
 - 3.1.1. First order conditions
 - 3.1.2. Second order conditions
- 3.2. Constrained optimization
 - 3.2.1. First order conditions
 - 3.2.2. Second order conditions
- 3.3. Nonlinear programming
 - 3.3.1. Kuhn-Tucker concave programming
 - 3.3.2. Quasi-concave programming

Lecture Notes

Sydsaeter, ch 3 sections

Hoy, ch 6, 12 and 15

Chiang, ch 9, 11, 12 and 13

4. Topics in integration and series

- 4.1. Review of rules of integration
- 4.2. Integration by parts, substitution or partial fractions
- 4.3. Leibniz's formula
- 4.4. Multiple integrals
- 4.5. Sequences and series

Lecture Notes

Sydsaeter, ch 4 sections

Hoy, ch

Chiang, ch

5. Differential equations and dynamic analysis

- 5.1. Linear first order differential equations (FODE)
- 5.2. Nonlinear FODE
- 5.3. Second and higher order differential equations
- 5.4. System of simultaneous differential equations

Lecture Notes

Sydsaeter, ch 5-7

Hoy, ch 17, 21-24

Boyce and Di Prima ch 2, 3, 5, 7

Chiang, ch 15-16 and 19

6. Difference equations and dynamic analysis*

- 6.1. Linear first order difference equations
- 6.2. Nonlinear first order difference equations
- 6.3. Second and higher order difference equations
- 6.4. System of simultaneous difference equations

Lecture Notes
Sydsaeter, ch 11
Hoy, ch 18-20 and 24
Chiang, ch 17-19

7. Dynamic optimization and optimal control

- 7.1. The maximum principle and the Hamiltonian
- 7.2. Optimization and discounting
- 7.3. Alternative boundary conditions
- 7.4. Infinite time horizon problems
- 7.5. Constraints on the control variables
- 7.6. Free terminal time problems
- 7.7. Multiple control variables

Lecture Notes
Sydsaeter, ch 9 and 10
Hoy, ch 25
Chiang, ch 20

8. Dynamic programming and discrete time optimization*

Lecture Notes
Sydsaeter, ch 12

*The extent of coverage will depend on time availability.

Course evaluation:

Mid-term exam: 30% Monday November 4, 2019.

Assignments: 20%

Comprehensive final exam (covers the whole material unless specified otherwise) which is formally scheduled during the examination period: 50%.

All exams will be based on solving problems, proving theorems or results and may possibly require you to provide short answers. Assignments will consist of solving problems or proving theorems and results.

Students who miss a midterm exam for a valid reason (documented appropriately) will have the associated weight added to that of the final exam. Should the final exam be missed for any reason, a grade of F will

be assigned, and the student must arrange for the possibility of a deferred examination through the Registrar's Office within 3 working days of the exam. If granted by the Registrar's Office, this exam will take place during the time designated in the Calendar. Special final exams (or mid-terms) will not be given. Standing in a course is determined by the course instructor subject to the approval of the Faculty Dean. This means that grades submitted by the instructor may be subject to revision. No grades are final until they have been approved by the Dean.

Assignments:

Some assignments will be given during the term. Details and due dates will be provided in class. Late assignments will not be accepted. The answers will be given by the T.A. during the tutorial. Although you may discuss the material with your classmates, the assignments need to be done on your own. See statement on plagiarism below.

Re-marking:

Any request for remarking must be submitted in writing within two weeks of that piece of work first being returned to the class. The request should contain a detailed explanation of why you feel you should receive a higher mark. Please note that remarking will apply to the entire work not just the contentious issue. As a result, the revised mark may be higher than, lower than or the same as the original mark.

Course Requirements:

Students who miss a midterm for a valid reason (for example, documented with a doctor's note) will have its weight transferred to the final exam. Otherwise, a grade of 0 will be assigned. Students who do not hand in an assignment but have a valid reason (for example, documented with a doctor's note) will have its weight transferred to the final exam. Otherwise, a grade of 0 for that assignment will be given.

Students must fulfill all course requirements in order to achieve a passing grade (D- or higher). Students who miss the final exam must apply for a deferred examination at the registrar's office. If granted, the deferred exam will take place during the time designated in the calendar. A student who misses the midterms as well as the final exam will be considered to have inadequate term work and may not be eligible for a deferred exam.

CuLearn and e-mail:

This course will make use of CuLearn.

Lecture notes, review questions, problems, references to additional readings and any other material will be posted on CuLearn. Class announcements will be sent by e-mail using CuLearn.

It is each student's responsibility to check this website for posted material as well as the Carleton University email system for correspondence. If you do not have a CuLearn account, please visit: www.carleton.ca/culearn.

Intellectual Property Rights Regarding Course Material:

Please note: "Classroom teaching and learning activities, including lectures, discussions, presentations, etc., by both instructors and students, are copy protected and remain the intellectual property of their respective author(s). All course materials, including PowerPoint presentations, outlines, and other materials, are also

protected by copyright and remain the intellectual property of their respective author(s). Students registered in the course may take notes and make copies of course materials for their own educational use only. Students are not permitted to reproduce or distribute lecture notes and course materials publicly for commercial or non-commercial purposes without express written consent from the copyright holder(s)." (See <https://carleton.ca/provost/2015/calendar-dates-and-course-materials-sharing-website/>)

Tutorials:

Attendance is compulsory. The tutorial period may be used for lectures, for exams or for tutoring by the TA, for problem sessions, review sessions and going over answers to assignments and exams. Each week, exercises will be assigned in class. You are expected to do them since these problems will help you prepare for the mid-term and final exams.

Plagiarism:

Please be aware that plagiarism is a serious offence at Carleton and should be recognized and avoided. For further information on how to do so, please see "Pammett on Plagiarism and Paraphrasing" at www.carleton.ca/economics/courses/writing-preliminaries.

The University Senate defines plagiarism as "*presenting, whether intentionally or not, the ideas, expression of ideas or work of others as one's own.*" This can include:

- reproducing or paraphrasing portions of someone else's published or unpublished material, regardless of the source, and presenting these as one's own without proper citation or reference to the original source;
- submitting a take-home examination, essay, laboratory report or other assignment written, in whole or in part, by someone else;
- using ideas or direct, verbatim quotations, or paraphrased material, concepts, or ideas without appropriate acknowledgment in any academic assignment;
- using another's data or research findings;
- failing to acknowledge sources through the use of proper citations when using another's works and/or failing to use quotation marks;
- handing in "substantially the same piece of work for academic credit more than once without prior written permission of the course instructor in which the submission occurs."

Plagiarism is a serious offence that cannot be resolved directly by the course's instructor. The Associate Dean of the Faculty conducts a rigorous investigation, including an interview with the student, when an instructor suspects a piece of work has been plagiarized. Penalties are not trivial. They can include a final grade of "F" for the course.

Requests for Academic Accommodation:

You may need special arrangements to meet your academic obligations during the term. For an accommodation request, the processes are as follows:

Pregnancy obligation:

Please contact your instructor with any requests for academic accommodation during the first two weeks of class, or as soon as possible after the need for accommodation is known to exist. For more details, visit the Equity Services website: carleton.ca/equity/wp-content/uploads/Student-Guide-to-Academic-Accommodation.pdf

Religious obligation:

Please contact your instructor with any requests for academic accommodation during the first two weeks of class, or as soon as possible after the need for accommodation is known to exist. For more details, visit the Equity Services website: carleton.ca/equity/wp-content/uploads/Student-Guide-to-Academic-Accommodation.pdf

Academic Accommodations for Students with Disabilities:

The Paul Menton Centre for Students with Disabilities (PMC) provides services to students with Learning Disabilities (LD), psychiatric/mental health disabilities, Attention Deficit Hyperactivity Disorder (ADHD), Autism Spectrum Disorders (ASD), chronic medical conditions, and impairments in mobility, hearing, and vision.

If you have a documented disability requiring academic accommodations in this course, please contact the **Paul Menton Centre for Students with Disabilities (PMC)** at 613-520-6608 or pmc@carleton.ca for a formal evaluation. If you are already registered with the PMC, contact your PMC coordinator to send me your Letter of Accommodation at the beginning of the term, and no later than two weeks before the first in-class scheduled test or exam requiring accommodation (if applicable). You must also contact the PMC no later than two weeks before the first in-class scheduled test or exam requiring accommodation (if applicable). After requesting accommodation from PMC, meet with your instructor as soon as possible to ensure accommodation arrangements are made. carleton.ca/pmc

Survivors of Sexual Violence:

As a community, Carleton University is committed to maintaining a positive learning, working and living environment where sexual violence will not be tolerated, and its survivors are supported through academic accommodations as per Carleton's Sexual Violence Policy. For more information about the services available at the university and to obtain information about sexual violence and/or support, visit: carleton.ca/sexual-violence-support

Accommodation for Student Activities:

Carleton University recognizes the substantial benefits, both to the individual student and for the university, that result from a student participating in activities beyond the classroom experience. Reasonable accommodation must be provided to students who compete or perform at the national or international level. Please contact your instructor with any requests for academic accommodation during the first two weeks of class, or as soon as possible after the need for accommodation is known to exist. <https://carleton.ca/senate/wp-content/uploads/Accommodation-for-Student-Activities-1.pdf>

For more information on academic accommodation, please contact the departmental administrator or visit: students.carleton.ca/course-outline .