Asymmetric Shocks in a Currency Union: Perspectives from the West African Monetary Union with a New Keynesian Model

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Abstract

How well-off or worse off a country can be by joining a currency union in the presence of structural heterogeneity and asymmetric shocks? In light of the proposed creation of a currency union for the Economic Community of the West African States (ECOWAS), we develop a three-region DSGE model to explore the question. We divide the ECOWAS into three regions, Nigeria, the existing WAEMU (West-African Economic Monetary Union), and the Rest. Considering two monetary regimes (monetary union and monetary independence), we assess the heterogeneity in the responses to asymmetric productivity and terms-of-trade shocks in these two regimes, as well as the costs related to the loss of monetary independence. It follows from the results of the different simulations that overall, a variety of conclusions can be drawn from these studies. On a priori grounds, it is not direct to say whether macroeconomic stability would increase or decrease if the ECOWAS countries were to form their monetary union. However, the results show to some extent that the ECOWAS countries perform better in the presence of asymmetric shock when they have their independent monetary regime compared to joining a monetary union. Also, shocks hitting Nigeria’s economy tend to have a more destabilizing effect on the other regions, especially when they are inside the union. Our results also suggest that the proposed monetary union for the ECOWAS region can potentially lead to welfare improvement for all the members. However, but the magnitude of the welfare gain is relatively small.

1 Introduction

The creation of the European Monetary Union in 1999, has fueled debates over the opportunity to create currency unions in several regions around the world. A currency union is an agreement between two or more economies to share a common currency or peg their cross borders exchange rates. As broadly discussed in the literature (Zika and UK [2005], Madhur [2004], Emerson [1992], etc.), there are several benefits for a country to join a currency union. The adoption of a single currency eliminates or reduces all the costs associated with the exchange rates (transaction costs, exchange-rate volatility, etc.), and as a consequence, it facilitates prices comparison and harmonization Rogoff [1996]. While the potential benefits can make a currency union attractive, several costs are also associated with joining a monetary union. The main ones are the loss of national monetary sovereignty, the loss of control over the monetary policy, and the loss of the central bank as a lender of last resort, etc. (Dornbusch [2001]). These losses can have several macroeconomic implications, especially when countries face asymmetric shocks that can destabilize their national economies.
The costs are low if the shocks are symmetric and the adjustment mechanisms are quick while they are high if the shocks are asymmetric, and the adjustment mechanisms are slow.

Given the relative success of the European Monetary Union, there has been a renewed and growing interest in creating a monetary union in other regions of the world, including the Economic Community of West African States (ECOWAS). The ECOWAS was established in 1975 to promote cooperation and economic integration in fifteen West African countries. These countries have already formed a customs union and are planning to move their economic integration a step forward by adopting a single currency. They have indicated their intention to speed up the monetary component of the integration by adopting a single currency.

The idea of forming a single currency for the whole region has raised legitimate questions about the ability of these countries to create a currency union, and in particular, their ability to deal with asymmetric shocks. Indeed, several papers have assessed the characteristics of the shocks that affect the different economies within the region. For example, using different econometric models, several papers like (Fielding and Shields, Masson and Pattillo, Bénassy-Quéré and Coupet, Buigut and Valev, van den Boogaerde and Tsangarides, Qureshi and Tsangarides) have analyzed the shocks that affected several African countries. Most of these studies concluded that African economies, and especially the ECOWAS members were, not only highly heterogeneous in their production structure, but they also faced asymmetric shocks. The latter could exacerbate the challenges of forming a monetary union. Besides, Houssa examined output growth shocks in West Africa using a dynamic factor model. He concluded that most of the countries in the region are mainly affected by negatively correlated supply shocks that will make adjustment difficult if they were to form a monetary union.

In reality, none of the studies on the viability of the formation of a monetary union in West Africa has examined the propagation mechanisms of those asymmetric shocks within the region. They have mostly focused on analyzing the characteristics of the shocks faced by the different economies within the region. They have not assessed the extent to which these shocks are transmitted within the ECOWAS region and the reactions to them, both in the originating countries and in the rest of the region. From a policy perspective, the final outcomes of these shocks are more important than their characteristics alone.

The present study plans to fill this gap by analyzing how asymmetric shocks are propagated within these ECOWAS economies, and how well-off or worse-off are these economies in the

1These countries are Benin, Burkina, Cap-Verde, Ivory Coast, Gambia, Ghana, Guinea, Guinea-Bissau, Liberia, Mali, Niger, Nigeria, Senegal, Sierra Leone, and Togo
face of asymmetric shocks when they are outside or inside a currency union. In addition, we plan to assess the presence of any asymmetry in the responses to those shocks. A structural micro-founded model is required to achieve that objective. A multi-region dynamic stochastic general equilibrium (DSGE) model that considers the optimal reactions of economic agents in the presence of changes in relative prices is needed to carry out the analysis. This type of model is now the workhorse for analyzing adjustments to shocks in currency unions. The type of shocks we consider in this analysis is country-specific, which are asymmetric by nature. In fact, asymmetric shocks are perturbations that affect a set of regions in a different way and they can be grouped into two categories. The country-specific shocks i.e. shocks that affect just one country and hence are asymmetric in both origin and impact and the shocks that are common to several regions but with different impact i.e. symmetric in origin but asymmetric in impact (see Bajo-Rubio and Díaz-Roldán [2003] and Emerson [1992]).

We build upon the recent literature on multi-country DSGE models to develop a multi-region and multisector DSGE model for the area. In contrast to most multi-country DSGE models found in the literature that consists of only two regions, (Home and Foreign), the present model features the disaggregation of the ECOWAS into three regions. The latter, share not only economic links among themselves, but also have individual relations with the rest of the world. Hence, instead of assuming in our model that the world consists only of the three economies of the region, we do account for the reality that suggests that these economies have individual links with the rest of the world. With that respect, the model can account for asymmetric terms-of-trade shocks affecting one economy of the proposed union. Yet, a previous study by Mendoza [1995] has shown that international price shocks account for nearly half of the volatility of production in African countries. Therefore, the ability to distinguish the relationships of each economy of the region with the rest of the world will be critical for the understanding of the analysis of the shocks affecting the region.

The three economies considered in the region are Nigeria, the WAEMU\(^2\) countries, and the Rest of ECOWAS countries (henceforth RECOWAS\(^3\)). In addition to differentiating the economies, we also consider two productive sectors, which are the tradable and nontradable sectors. Due to data limitation, the informal sector is not explicitly taken into consideration in this analysis. Our model builds upon the literature on new open economy models pioneered by Obstfeld and Rogoff [2000] and recent contributions in the literature on New

\(^2\)WAEMU is a monetary union formed by eight french speaking countries in West Africa that are member of the CFA zone: Benin, Burkina-Faso, Cote d’Ivoire, Guinea-Bissau, Mali, Niger, Senegal, Togo

\(^3\)We call RECOWAS the group of countries represented by Ghana, Liberia, Guinea, Sierra Leone, Gambia
Keynesian DSGE models, like Smets and Wouters [2003] and Blanchard and Galí [2007]. We consider both nominal and real frictions in the model, by introducing nominal price rigidity à-la Calvo [1983] and capital installation costs. The model is used to examine two types of asymmetric shocks affecting each of the three economies of the region: terms-of-trade shock and total factor productivity shock to the tradable sector. We analyze the propagation mechanisms and adjustments to the shocks in two regimes: monetary independence and monetary union.

The following insights emerge from the results of our simulations:

(i) In both monetary union and monetary independence, the adverse productivity shock causes a decline in real variables such as output, consumption, investment in the domestic economies, and cause an increase in the price level. With monetary independence, the central banks respond to the inflationary pressure by tightening the nominal interest rate. Also, the magnitude of the effect of the shock on output in each region is higher in monetary union compared to monetary independence.

(ii) Except for Nigeria, which in monetary independence, tends to benefit from negative productivity shock in the other regions, regardless of its source, productivity shock has a negative spillover effect on the other regions in both regimes. Terms-of-trade shocks are negatively transmitted to the three regions when they share the same currency, while in monetary independence, the shock is negatively (positively) transmitted when it originates from Nigeria (WAEMU and RECWAS).

(iii) In the monetary union regime, the shocks originating from WAEMU and RECWAS produce almost zero response in the union-wide interest rate while the interest rate reacts to shock originating from Nigeria.

(iv) Shocks originating from Nigeria tend to have a more significant effect on WAEMU and RECWAS, while the effects are relatively mitigated for shocks coming from WAEMU and RECWAS.

(vi) Both productivity and terms of trade shocks are more persistent in monetary union compared to monetary independence. Even though it is not straightforward to conclude about the degree of macroeconomic stability when the countries are outside compared to inside the ECOWAS monetary union, we observe that a monetary union that excludes Nigeria will likely promote macroeconomic stability in the other regions.

(vii) The ECOWAS currency union will lead to welfare improvement for all the three regions, but the magnitude of the welfare gain is relatively small. WAEMU has the
highest welfare gain, while Nigeria encounters the lowest welfare gain.

We organized the remainder of the paper as follows. In the next section, we review the related literature. The third section presents some relevant stylized facts about the ECOWAS countries. Section 4 describes the basic structure of the model. In section 5, we first discuss the data along with the model calibration, and we analyze the simulation results in the sixth section. The last section concludes.

2 Related Literature

Following the seminal paper of Mundell [1961] on the theory of optimal currency area (OCA), a vast and expanding literature on the feasibility of a monetary union in the presence of asymmetric shock has emerged. They mostly used the European monetary union or the United States as reference cases (Bayoumi, Eichengreen and Eichengreen [1994], Eichengreen et al. [1996], Krugman [1993] P.Krugman and D. Guillermo [1993]and Feldstein [1997]). Still, another strand of the literature analyzed the feasibility of monetary unions in other parts of the world, and mostly in Africa. Asongu, Nwachukwu and Tchamyou [2017] provide an excellent review of the studies on various proposed monetary unions in Africa, including the ECOWAS monetary union project.

Regarding the ECOWAS region, the key insights from their review are the mixed conclusions reached by the various studies on the viability of a monetary union in the region. Some authors offered a very pessimistic view about the project, while others provided a more nuanced assessment of the creation of this monetary union. The pessimists raised concerns over the viability of the formation of a monetary union among the West African countries in light of the heterogeneity of the shocks in the absence of the exchange rate as a policy instrument. In what follows, we provide a brief review of the main findings in key papers related to the topic.

Batté et al. [2009] used a two-region DSGE model to discuss the type of monetary regime to be adopted by the ECOWAS monetary union. They found that in the presence of commodity price shocks, the most stabilizing regime for Nigeria is fixed money supply, while WAEMU will be better off with a fixed exchange rate regime. Coulibaly and Gnimassoun [2013] used panel cointegration techniques to analyze the optimality of the ECOWAS monetary union. They focus on the similarities and dissimilarities of ECOWAS member states in terms of economic competitiveness. Their results suggest that the WAEMU area is the most homogeneous group of countries in the Central and Western parts of Africa. They concluded that Ghana, Gambia, and, to a lesser extent by Sierra Leone could join the WAEMU countries.
to form a larger union. Debrun, Masson and Pattillo [2005] analyzed the incentives to join a monetary union on West Africa and found that asymmetric shocks are less important than government spending propensities in the evaluation of the net gains and losses from potential monetary unions. They developed a theoretical model with monetary and fiscal policy interactions to analyze the net gains or losses of the proposed West African monetary union. They found that the union is not compatible for most WAEMU countries because of high fiscal distortions for Nigeria and its terms of trade, which is negatively correlated with those of the other ECOWAS countries. However, according to their numerical results, the ECOWAS monetary union would be profitable for all members if Nigeria’s financing needs were equal to the average of the other countries. Moreover, if the union was created gradually, i.e., the non-WAEMU members were added individually to the existing monetary union (WAEMU), there would also be a welfare improvement for all member countries.

Tsangarides and Qureshi [2008] used a cluster analysis to assess the suitability and the sustainability of the monetary unification project in West Africa. Their results suggested that the ECOWAS monetary union was not relevant because of the dissimilarities in the economic characteristics of the candidate countries. Masson and Pattillo [2002] assessed the viability of a monetary union in West Africa and discussed the various institutional options for implementing monetary cooperation. They concluded that instead of trying to meet a very short deadline for the creation of a monetary union, the countries of the region should invest their energies into reinforcing convergence on low inflation, sustainable fiscal policies and structural policies necessary for strong growth.

Fielding and Shields [2001] developed an econometric VAR model to assess the impact of output and inflation shocks on the members of two existing monetary unions of the CFA Franc Zone. They found a significant and positive correlation of inflation shocks in all countries of the Franc CFA zone and concluded that as long as inflation mattered for policy and the policy response to inflation shocks was immediate, appurtenance to the zone was not a cost. In contrast, they found that output shocks were not correlated among all members of the area. They partitioned the zone into two subsets of high and low correlations of output shocks. The two subsets did not coincide with the existing boundaries of the two monetary unions of the Franc CFA Zone. Accordingly, they suggested a redefinition of the boundaries of the two monetary unions.
3 Stylized facts about ECOWAS countries

The reduction of divergence in economic indicators such as GDP growth, inflation, or public debt is the key factor for success in a currency union since countries can, in that case, react similarly to shocks, and the common central bank can react more effectively to those shocks. We explore in this section the characteristic of the economies under study in order to have a broad view of their structure. Tables 1-4 present some stylized business cycles facts for the three blocks of regions. All the variables are per capita and cover the period 1981-2012. It follows from the table1 that the ECOWAS countries do not converge in terms of growth. Nigeria, Côte d’Ivoire, Ghana, and Senegal are the dominant countries of the region with the largest GDP per capita.

In order to deepen the analysis, we compute three measures of business cycles in order to see how they are synchronized between the regions. These measures are the fluctuation in the annual growth rate of real GDP, five years rolling correlation coefficient for the GDP growth rate (Figure 1) and the correlation coefficient of the cyclical component of GDP 4 (Table 2). It appears from the panel (a) and (b) of Figure 1 that significant business cycle divergences emerge between Nigeria, WAEMU, and RECWAS. We can also observe in the panel (b) that on average, Nigeria is more correlated with WAEMU than with RECWAS. A more detailed analysis (Table 2) shows that the growth cycles of the ECOWAS countries are not significantly synchronized. We can distinguish on one side a group of countries formed by Mali, Niger, Senegal, Côte d’Ivoire, and Senegal, which have a positive and significant correlation. The synchronization of those countries can be explained by the fact that they are members of a currency union (WAEMU). On the other side, Nigeria is not synchronized with any other countries in the area. While this result is not significant, it shows the particularity of Nigeria, which is an oil exporter while all the other countries are mainly specialized in the agricultural sector. Moreover, in the RECWAS region, we have some countries such as Guinea Liberia, and Ghana that have positive and non-significant growth cycle while the economic cycles are negative and non-significant for Gambia and Ghana.

Overall, the analysis of the synchronization of the cyclical components countries shows that, except the WAEMU block, which displays a positive correlation, the ECOWAS countries are not economically synchronized. Moreover, in term of production structure, the export diversification index of the ECOWAS countries (second column of Table 1) are very low showing that those economies are not diversified. In fact, some countries in the region depend mainly on the raw agricultural product (cocoa, coffee, cotton,) and some on mineral (oil, cobalt,etc.) and the exchange

4We extract the cyclical components using the Hodrick-Prescott (HP) filter
of those commodities is concentrated on a small number of countries namely Nigeria, Cote d’Ivoire, Ghana, and Senegal. The export diversification of the ECOWAS countries has implications for the formation of their monetary union because, as highlighted by Kenen [2000], a high degree of product diversification is favorable for smoothing the functioning of a monetary union. Finally, the intra-regional trade is very low in West African countries (see table 3 and 4) with about 7.5% (3.5%) of Nigeria’s total exports (imports) that go in the ECOWAS region while the intra-ECOWAS WAEMU and RECOWAS exports share are respectively 16.3% and 10.8%. Some literature on currency union shows that belonging to a currency union leads to increase intra-trade, therefore theoretically, the ECOWAS monetary union should improve the performance of member countries regarding regional trade.

4 Structure of the model

The structure of the model inherits many characteristics of a large class of new Keynesian small open economy Dynamic Stochastic General Equilibrium model like the one developed by Smets and Wouters [2003], Christiano, Eichenbaum and Evans [2005], Gali and Monacelli [2005]. The main difference is that it is a multi-region, multi-sector model. In our model, the world is composed of three regions, namely Nigeria (N), WAEMU (W), the rest of ECOWAS countries (RE), and the rest of the world. We distinguish two sectors of production: the tradable and the nontradable sector. In each of these regions, there are four types of agents (households, firms, the government, and the monetary authority). Firms produce goods that are exported or used domestically in the local market. The government manages the fiscal policy, and the monetary authority conducts the monetary policy by setting the nominal interest rate in response to changes in inflation, output, and the exchange rate. We consider two monetary regimes, a monetary independence regime in which the independent monetary authority in each block determines its monetary policy, and a monetary union regime in which a common central bank manages the monetary policy for the whole union. We assume a monopolistic competition in the intermediate goods markets, while perfect competition is considered in the final good’s markets. Furthermore, we introduce nominal and real rigidity in the model, with the sticky prices à-la Calvo for the prices of the intermediate goods.

4.1 Households

4.1.1 Intertemporal optimization

The representative infinitely living household has preference over consumption \( C_{i,t} \), leisure \( 1 - N_{i,t} \) and has access to the financial market where he can buy and sell domestic and
foreign bonds. His lifetime utility function, which is positively related to the current level of consumption, and negatively related to labor supply is given by:

\[ U(C_{i,t}, N_{i,t}) = E_0 \sum_{t=0}^{\infty} \beta^t_i \left( \frac{C_{i,t}^{1-\sigma} - \psi N_{i,t}^{1+\phi_i}}{1 - \sigma_i} \right) \]  

Where \( i = \{ N, W, RE \} \) is the index used for regions, \( \beta \in (0, 1) \) is the discount factor, \( \sigma \) is the risk aversion parameter, \( N_{i,t} \) is the total number of hours worked, \( \phi \) is the inverse of elasticity of labor supply, \( \psi \) is the marginal dis-utility of participating in the labor market, and \( E \) is the expectation operator.

The household supplies labor services \( N_{i,t} \) to the firms with the nominal wage \( W_{i,t} \) as compensation and rents capital \( K_{i,t} \), and receives the rental rate \( R_{k_{i,t}} \). He also receives the profits, \( \Pi_{i,t} \) distributed by the monopolistic firms, and pays a lump-sum tax \( T_{i,t} \) to the government. We assume that the foreign bonds are denominated in the foreign currency so that \( e_{i,t} \) denotes the bilateral exchange rate between the country \( i \) and the rest of the world. The budget constraint is then given by:

\[ PC_{i,t}C_{i,t} + B_{i,t+1} + e_{i,t}B^*_{i,t+1} + P_{i,t}I_{i,t} = W_{i,t}N_{i,t} + P^k_{i,t}K_{i,t} + B_{i,t}(1 + r_{i,t}) + e_{i,t}B^*_{i,t}(1 + r^*_{i,t}) + \Pi_{i,t} - T_{i,t} \]  

where \( PC_{i,t} \) and \( P_{i,t} \) are respectively the aggregate prices index for country \( i \)'s private consumption and investment. \( B_{i,t} \) and \( B^*_{i,t} \) are respectively the households holding of domestic and foreign bonds that provide, respectively, the real returns \( r_{i,t} \) and \( r^*_{i,t} \).

The prospect that the agent plays a Ponzi game and accumulate an infinite amount of assets is ruled out by imposing the following transversality conditions:

\[ \lim_{t \to \infty} E_0 \left( \frac{B_{i,t}}{1 + r_{i,t}} \right) = 0 \]  

\[ \lim_{t \to \infty} E_0 \left( \frac{B^*_{i,t}}{1 + r^*_{i,t}} \right) = 0 \]  

The physical capital stock which is owned by the household evolves according to the following motion law:

\[ K_{i,t+1} = (1 - \delta_i)K_{i,t} + I_{i,t} \left[ 1 - S \left( \frac{I_{i,t}}{I_{i,t-1}} \right) \right] \]  

where \( \delta_i \) denotes the physical rate of depreciation and \( S \left( \frac{I_{i,t}}{I_{i,t-1}} \right) = \frac{\kappa_i}{2} \left( \frac{I_{i,t}}{I_{i,t-1}} - 1 \right)^2 \) is the investment adjustment costs.
The Lagrangian of the household is given by:

$$\max_{X_t} L_0 = E_0 \sum_{t=0}^{\infty} \beta^t \left\{ U(C_{i,t}, N_{i,t}) + \lambda_{i,t}(W_{i,t}N_{i,t+k} + R_{i,t}^kK_{i,t} + B_{i,t}(1 + r_{i,t}) + e_{i,t}B_{i,t}^*(1 + r_{i,t}^*) + \Pi_{i,t} - P_{C,i,t}C_{i,t} - B_{i,t+1} - e_{i,t}B_{i,t+1}^* - P_{I,i,t}I_{i,t}) + \mu_{i,t} \left\{(1 - \delta_{i})K_{i,t} + I_{i,t} \left[1 - S \left(\frac{I_{i,t}}{I_{i,t-1}}\right)\right] - K_{i,t+1}\right\}\right\}$$

We derive the optimality conditions for the representative household with respect to consumption, labor supply, capital, domestic and foreign bond holdings:

$$C_{i,t-\sigma_i} = \lambda_{i,t}P_{C,i,t}$$

$$\psi_{i,t} N_{i,t}^{\phi_i} = \lambda_{i,t} W_{i,t}$$

$$\lambda_{i,t} = \beta_i E_t \lambda_{i,t+1}(1 + r_{i,t+1})$$

$$\lambda_{i,t} = \beta_i E_t \lambda_{i,t+1} \frac{e_{i,t+1}}{e_{i,t}} (1 + r_{i,t+1})$$

$$Q_{i,t} = \frac{\mu_{i,t}}{\lambda_{i,t}}$$

$$Q_{i,t} = \beta_i E_t \left[\left(\frac{\lambda_{i,t+1}}{\lambda_{i,t}}\right)(1 - \delta_i)Q_{i,t+1} + R_{i,t+1}^k\right]$$

$$P_{i,t} = Q_{i,t} - Q_{i,t}S \left(\frac{I_{i,t}}{I_{i,t-1}}\right) - Q_{i,t}S' \left(\frac{I_{i,t}}{I_{i,t-1}}\right) + \beta E_t \lambda_{i,t+1} Q_{i,t+1}S' \left(\frac{I_{i,t+1}}{I_{i,t}}\right) \left(\frac{I_{i,t}}{I_{i,t-1}}\right)^2$$

$$\lambda_{i,t}$$ and $$\mu_{i,t}$$ are the Lagrange multipliers related to, respectively, the budget constraint and the capital accumulation equation.

Equations (7)-(12) summarize the inter-temporal decisions of households in goods and asset markets. $$\lambda_{i,t}$$ represents the shadow price i.e. the maximum price that the consumer is willing to pay to have an extra unit of consumption in the future. This shadow price is equal to the marginal utility of aggregate consumption in equation (7). $$\mu_{i,t}$$ is the marginal benefit in terms of the utility of one unit of investment. Equation (8) describes the labor supply resulting from the optimal choice of the agent between consumption and leisure. Equations (9) and (10) suggest that the marginal utility of consuming one more unit today is equal to the expected present value of the additional future consumption obtained by investing in domestic and foreign bonds. Equation (11) represents the Tobin’s Q and states that the marginal cost of investment is equal to the marginal benefit and finally (12) indicate that the value of currently installed capital depends on its future expected value, taking into account
the depreciation rate and the expected rate of return.

4.1.2 Intratemporal optimization

Consumption $C_{i,t}$ is a composite good consisting of tradable $C^T_{i,t}$ and nontradable goods $C^N_{i,t}$ that can be given by the following CES aggregator.

$$C_{i,t} = \left((1 - \omega_i)C^T_{i,t} - \frac{\eta - 1}{\eta} + \omega_i C^N_{i,t} - \frac{\eta - 1}{\eta}\right)^{-\frac{\eta}{\eta - 1}} (14)$$

where the parameter $\eta$ is the intra-temporal elasticity of substitution of consumption between tradable and nontradable goods. The higher is this latter, and the more substitutable the goods are. The parameter $\omega$ is the weight of the nontradable bundle in aggregate consumption. The objective of the representative household is to minimize the expenditures on the tradable and nontradable goods while maintaining a certain target level of the consumption good. This optimization problem yields the following demand functions for the tradable and nontradable goods:

$$C^T_{i,t} = (1 - \omega_i) \left(\frac{P^T_{C_{i,t}}}{P_{C_{i,t}}}\right)^{-\frac{\eta - 1}{\eta}} C_{i,t} \quad (15)$$

$$C^N_{i,t} = \omega_i \left(\frac{P^N_{i,t}}{P_{C_{i,t}}}\right)^{-\frac{\eta - 1}{\eta}} C_{i,t} \quad (16)$$

where $P_{C_{i,t}}, P^T_{C_{i,t}}, P^N_{i,t}$ are respectively the consumer price index and the prices index on tradable and nontradable goods. The overall consumer price index is given by:

$$P_{C_{i,t}} = \left((1 - \omega_i)P^T_{C_{i,t}} + \omega_i P^N_{i,t}\right)^{-\frac{1}{\eta}} \quad (17)$$

The tradable goods are either domestically produced or imported from abroad. The consumption of tradable is a combination of domestically produced tradable good, $(C^T_{i,t}^{TH})$, and of the aggregate of imported goods $(C^T_{i,t}^{TF})$.

$$C^T_{i,t} = \left((1 - \alpha_i)C^T_{i,t}^{TH} - \frac{\epsilon - 1}{\epsilon} + \alpha_i C^T_{i,t}^{TF} - \frac{\epsilon - 1}{\epsilon}\right)^{-\frac{\epsilon - 1}{\epsilon - 1}} \quad (18)$$

$\alpha_i$ is the trade share parameter and $\epsilon_i$ the elasticity of substitution between domestically produced goods and the composite of foreign goods.

Like in the previous case for total consumption, the intratemporal optimization problem
leads to the following demand functions for locally produced and the composite of imported tradable goods:

\[ C_{TH_{i,t}}^{T} = (1 - \alpha_i) \left( \frac{P_{TH_{i,t}}^{T}}{P_{C_{i,t}}^{T}} \right)^{-\epsilon_i} C_{C_{i,t}}^{T} \]  \hspace{1cm} (19)

\[ C_{TF_{i,t}}^{T} = \alpha_i \left( \frac{P_{TF_{i,t}}^{T}}{P_{C_{i,t}}^{T}} \right)^{-\epsilon_i} C_{C_{i,t}}^{T} \]  \hspace{1cm} (20)

where \( P_{TH_{i,t}}^{T} \) and \( P_{TF_{i,t}}^{T} \) are, respectively, the prices of domestically produced tradable and imported goods. The tradable goods price index is given by:

\[ P_{C_{i,t}}^{T} = \left( (1 - \alpha_i) P_{TH_{i,t}}^{T_1} + \alpha_i P_{TF_{i,t}}^{T_1} \right)^{\frac{1}{1-\epsilon_i}} \]  \hspace{1cm} (21)

The composite of imported tradable goods from abroad is made of goods produced in the union or imported from the rest of the world. It is a CES aggregate of imported good from the rest of the world \( C_{TROW_{i,t}}^{T} \), and the index of consumption goods produced by the other members of the union:

\[ C_{TF_{i,t}}^{T} = \left( (1 - \tau_i) C_{TROW_{i,t}}^{T_1} + \tau_i C_{TU_{i,t}}^{T_1} \right)^{\frac{\epsilon_i}{\nu_i - \epsilon_i}} \]  \hspace{1cm} (22)

The parameter \( \nu_i \) is the elasticity of substitution between goods produced in the union, and the rest of the world and \( \tau_i \) measures the weight of the tradable bundle produced by the other members of the union.

The solution to the problem of finding the optimal allocation of expenditures on goods imported from the rest of the world and the index of goods produced in by the other members of the union gives the following demand functions:

\[ C_{TROW_{i,t}} = (1 - \tau_i) \left( \frac{P_{M_{ROW}^{T}}^{ROW}}{P_{TF_{i,t}}^{T}} \right)^{-\nu_i} C_{C_{i,t}}^{T} \]  \hspace{1cm} (23)

\[ C_{TU_{i,t}}^{T} = \tau_i \left( \frac{P_{TU_{i,t}}^{T}}{P_{TF_{i,t}}^{T}} \right)^{-\nu_i} C_{C_{i,t}}^{T} \]  \hspace{1cm} (24)

where \( P_{C_{i,t}}^{T} \) and \( P_{TF_{i,t}}^{T} \) are the price indices of tradable from the union, the rest of the world and the overall foreign tradable price. The index price of the foreign tradable
goods is given by:

\[ P_{C_{i,t}}^{TF} = \left( \tau_i P_{C_{i,t}}^{TU_{1-\nu_i}} + (1 - \tau_i) P_{M_{i,t}}^{ROW_{1-\nu_i}} \right)^{\frac{1}{1-\nu_i}} \]  

(25)

Finally, the index of goods produced by the other members of the union \((C_{i,t}^{TU})\) is a CES composite of goods imported from other countries \(l \ (l \neq i)\).

\[ C_{i,t}^{TU} = \left[ \sum_l (\Omega_{il}) C_{i,t}^{TL_{1-\Gamma_i}} \right]^{\frac{\Gamma_i}{\Gamma_i - 1}} \]  

(26)

\(\Gamma_i\) is the elasticity of substitution between goods produced in the foreign countries \(l\) and \(\Omega_{il}\) measures the share of tradable imported bundle produced in the country \(l\). The household allocates expenditure according to the demand functions for goods that are imported from each country \(l\):

\[ C_{i,t}^{TL} = \Omega_{il} \left( \frac{P_{C_{i,t}}^{TL}}{P_{C_{i,t}}^{TU}} \right)^{-\Gamma_i} C_{i,t}^{TU} \]  

(27)

The price index for the goods produced in the union is given by:

\[ P_{C_{i,t}}^{TU} = \left[ \sum_l \Omega_{il} e_{i,t} (P_{C_{i,t}}^{TL})^{1-\Gamma_i} \right]^{\frac{1}{1-\Gamma_i}} \]  

(28)

We use the same procedure for the investment good. We assume that the demand for investment \(I_{i,t}\) is a CES of a basket of nontradable and tradable investment goods. This later is a composite of home and foreign tradable investment goods which in turn is a composite of imported investment goods from the union and from the rest of the world:
The optimal demand function of investment goods and the associated prices indexes are defined by:
\[ I_{i,t}^T = (1 - \iota_i) \left( \frac{P_{I_{i,t}}^T}{P_{I_{i,t}}} \right)^{-\zeta_i} I_{it} \]  
(33)

\[ I_{i,t}^N = \iota_i \left( \frac{P_{I_{i,t}}^N}{P_{I_{i,t}}} \right)^{-\zeta_i} I_{it} \]  
(34)

\[ I_{i,t}^{TF} = (1 - \nu_i) \left( \frac{P_{I_{i,t}}^{TF}}{P_{I_{i,t}}} \right)^{-\chi_i} I_{i,t}^T \]  
(35)

\[ I_{i,t}^{TH} = \nu_i \left( \frac{P_{I_{i,t}}^{TH}}{P_{I_{i,t}}} \right)^{-\chi_i} I_{i,t}^T \]  
(36)

\[ I_{i,t}^{TU} = (1 - \psi_i) \left( \frac{P_{I_{i,t}}^{TU}}{P_{I_{i,t}}^{TF}} \right)^{-\mu_i} I_{i,t}^{TF} \]  
(37)

\[ I_{i,t}^{TR} = \psi_i \left( \frac{e_{i,t} M_{ROW_{i,t}}^{TF}}{P_{I_{i,t}}^{TF}} \right)^{-\mu_i} I_{i,t}^{TF} \]  
(38)

\[ I_{i,t}^{Tl} = \kappa_i \left( \frac{P_{I_{i,t}}^{Tl}}{P_{I_{i,t}}^{TU}} \right)^{-\xi_i} I_{i,t}^{TU} \]  
(39)

\[ P_{i,t} = \left( (1 - \iota_i) P_{I_{i,t}}^{T1-\zeta_i} + \iota_i P_{i,t}^N \right)^{\frac{1}{1-\iota_i}} \]  
(40)

\[ P_{i,t}^T = \left( (1 - \nu_i) P_{i,t}^{TF1-\chi_i} + \nu_i P_{i,t}^{TH1-\chi_i} \right)^{\frac{1}{1-\nu_i}} \]  
(41)

\[ P_{i,t}^{TF} = \left( (1 - \psi_i) P_{i,t}^{TU1-\mu_i} + \psi_i e_{i,t} P_{i,t}^{ROW1-\mu_i} \right)^{\frac{1}{1-\psi_i}} \]  
(42)

\[ P_{i,t}^{TU} = \left[ \sum_l \kappa_i \iota_{i,t} (P_{I_{i,t}}^{Tl})^{1-\xi_i} \right]^{\frac{1}{1-\xi_i}} \]  
(43)

### 4.2 Firms

On the production side, we assume that there are two types of firms so that the production takes place in two steps. In the first steps, a continuum of intermediate goods producers (wholesale firm) indexed by \( j \in [0, 1] \) produces a differentiated good by combining capital and labor taking the rental rate of capital and the wage rate as given. They operate in a monopolistically competitive environment where they set prices in a staggered manner following Calvo [1983]. Secondly, the final goods producers (retailers) combine all the varieties of the intermediate firms to produce a homogeneous final good and sell it to buyers in a perfectly competitive market. For simplicity, we assume that capital and labor are homogeneous, and there is free mobility of both inputs in the economy. This implies that we have the same
wage and rental rate of capital in both sectors.

4.2.1 Domestic intermediate goods producers

In each economy $i \in (N, W, RE)$, there is a continuum of intermediate goods-producing firms $j$, which produces a differentiated good $Y_i(j)$. They use the following Cobb-Douglas production function that exhibits a constant-returns-to-scale technology in both input:

$$Y^s_{i,t}(j) = A^s_{i,t} K^s_{i,t}(j)^{\alpha^s_i} N^s_{i,t}(j)^{1-\alpha^s_i}$$  \hspace{1cm} (44)

With $s = \{\text{Domestic Tradable (TH)}, \text{Nontradable (N)}\}$, $A^s_{i,t}$ is a country-specific factor productivity shock that follow an exogenous AR(1) process.

$$\log(A^s_{i,t}) = \rho^s_i \log(A^s_{i,t-1}) + \epsilon^s_{a,i,t}$$  \hspace{1cm} (45)

where $\epsilon^s_{a,i,t}$ is a serially uncorrelated shock with zero mean and standard deviation $\sigma_{a,i}$. The intermediate goods firms take the price level $W_{i,t}$ and $R^k_{i,t}$ as given and minimize their production costs under the technology constraint so as to find the optimal demand for factor uses. The cost minimization problem is then giving by:

$$\text{Min}_{K^s_{i,t}, N^s_{i,t}} W_{i,t} N^s_{i,t} + R^k_{i,t}$$  \hspace{1cm} (46)

subject to

$$Y_{i,t} = A^s_{i,t} K^s_{i,t} N^{s1-\alpha^s_i}_{i,t}$$  \hspace{1cm} (47)

From the first-order conditions, we get the expression for the marginal cost of firms producing domestic tradable and nontradable goods:

$$mc^s_{i,t} = \frac{W_{i,t} 1-\alpha^s_i R^k_{i,t} \alpha^s_i 1}{1-\alpha^s_i}$$  \hspace{1cm} (48)

The equation (48) states that at the optimum, the marginal cost depends negatively on technology and positively on nominal wage and rental rate.

4.2.2 Final good producers

The final goods producers use all varieties $j \in (0,1)$ of intermediate goods as input to produce the final good $Y_{i,t}$ by using a Dixit-Stiglitz production function (Dixit and Stiglitz...
They operate in a perfectly competitive market and face the following problem:

Max \[ y^s_{i,t}(j) \]  
subject to \[ y^s_{i,t} = \left[ \int_0^1 y^s_{i,t}(j) \frac{\epsilon^p_{i-1}}{\epsilon^p_i} dj \right] \frac{\epsilon^p_i}{\epsilon^p_{i-1}} \]  

The input demand function associated to this problem can be written as follow:

\[ y^s_{i,t}(j) = \left( \frac{P^s_{i,t}(j)}{P^s_{i,t}} \right)^{-\epsilon^p_i} Y^s_{i,t} \]  

The price of the final good is:

\[ P^s_{i,t} = \left( \int_0^1 P^s_{i,t}^{1-\epsilon^p_i}(j) dj \right)^{\frac{1}{1-\epsilon^p_i}} \]  

4.3 Price setting

Final goods prices are perfectly flexible, whereas those of the intermediate goods are sticky. Following Gali and Monacelli [2005], we assume that the prices of the differentiated goods are set in a staggered style à la Calvo [1983]. This suggests the presence of nominal rigidity in the model. According to Calvo, firms receive in each period a signal indicating whether they can review their prices. Hence, at a given time, a random fraction of firms that are called the forward-looking firms can adjust their prices optimally while the remaining backward-looking firms set their prices based on rules of thumb using historical information about the price level.

Let’s suppose that in every period t, firms producing tradable and nontradable goods are allowed to reset their price with probability \( 1 - \theta^s_i \) in a forward-looking manner. The proportion of firms that do not receive any price updating signal index their price to the past observed inflation rate while the remaining proportion can adjust their price level and choose the optimal prices \( p^s_{i,t} \). The average price level in the economy is then giving by:

\[ P^s_{i,t} = \left[ (1 - \theta^s_i)P^s_{i,t}^{(1-\epsilon^p_i)} + \theta^s_iP^s_{i,t-1}^{1-\epsilon^p_i} \right]^{\frac{1}{1-\epsilon^p_i}} \]  

Firms that are allowed to re-optimize their price in period t choose the optimal price by maximizing the present discounted value of their future profits subject to the demand
function.

\[
\max_{p^*_i(t)} \quad \Pi = E_t \sum_{k=0}^{\infty} \theta^k \Delta_{k,t+k} \left[ \frac{p^s_{i,t+k}(j)}{p^s_{i,t+k}} y_{i,t+k}(j) - \frac{mc^s_{i,t+k}}{P^s_{i,t+k}} y_{i,t+k}(j) \right] \quad (54)
\]

subject to

\[
Y_{i,t+k}^s(j) = \left[ \frac{p^s_{i,t+k}(j)}{P^s_{i,t+k}} \right]^{-\epsilon} y_{i,t+k} \quad (55)
\]

Where \( \Delta_{k,t+k} \) is the stochastic discount factor

From the first-order condition of the profit maximization problem and by assuming that all firms choose the same price level, we obtain the following expression for the optimal price level:

\[
P^s_{i,t} = e^{p_i} \frac{\sum_{k=0}^{\infty} \theta^k \Delta_{k,t+s} \frac{mc^s_{i,t+k}}{p^s_{i,t+k}} y_{i,t+k}}{\sum_{k=0}^{\infty} \theta^k \Delta_{k,t+s} \frac{y_{i,t+k}}{p^s_{i,t+k}}} \quad (56)
\]

The equation (56) shows that the firms maximize their expected future profits by choosing an optimal price that is equal to a constant markup over a discounted stream of expected future nominal marginal cost. Finally, after some manipulation the expression of the New-Keynesian Phillips curve can be derived:

\[
\pi^s_{i,t} = \beta_i E_{i,t+1} \pi^s_{i,t+1} + \lambda^s_i mc^s_{i,t} \quad (57)
\]

with \( \lambda^s_i = \frac{(1 - \beta_i \theta^s_i)(1 - \theta^s_i)}{\theta^s_i} \) \quad (58)

The equation (57) suggests that current inflation rates for the domestic tradable and non-tradable goods are individually determined by that the expected value of future inflation rates and their marginal cost.

### 4.4 Monetary and fiscal policy

We consider two types of monetary regimes in the model: monetary union and monetary independence. By following existing work in the DSGE literature (Justiniano and Preston [2010]; Smets and Wouters [2007]), we approximate the monetary policy using the Taylor-type rule.
4.4.1 Policy under monetary independence and monetary union

To reflect the monetary policy framework under an independent monetary regime, we assume that the central Banks in each block of country conduct a monetary policy by managing the short-term nominal interest rate \( R_{i,t} \) using the Taylor rule. They adjust their interest rate in response to the deviation of inflation, output, and exchange rate from their respective targets.

\[
\frac{R_{i,t}}{R_{ss,i}} = \left( \frac{R_{i,t-1}}{R_{ss,i}} \right)^{\gamma_r^i} \left[ \left( \frac{Y_{i,t}}{Y_{ss,i}} \right)^{\gamma_y^i} \left( \frac{\Pi_{i,t}}{\Pi_{ss,i}} \right)^{\gamma_{\pi}^i} \left( \frac{e_{i,t}}{e_{ss,i}} \right)^{\gamma_e^i} \right]^{1-\gamma_r^i} \epsilon_{i,t}^m
\]

(59)

where \( \gamma_r^i \) is the degree of interest rate smoothing, \( \gamma_{\pi}^i \), \( \gamma_y^i \) and \( \gamma_e^i \) are respectively the relative weights on inflation, output and exchange rate. The error term \( \epsilon_{i,t}^m \) is a monetary policy shock. \( R_{ss,i} \) is the steady-state nominal interest rate. The total output \( Y_{i,t} \) in the country \( i \) is defined by:

\[
P_i Y_{i,t} = P_i^{TH} Y_{i,t}^{TH} + P_i^{N} Y_{i,t}^{N}
\]

(60)

\[
P_{i,t} = P_{i,t}^{TH} w_i P_{i,t}^{N1-w_i}
\]

(61)

where \( w_i \) the relative weight of the tradable sector in the aggregate output of country \( i \).

Likewise, for the monetary union regime, we postulate an interest rate rule where the common central bank responds to deviations of output, inflation, and changes in the real exchange rate for the whole union:

\[
\frac{P_{U,t}}{R_{U,ss}} = \left( \frac{P_{U,t-1}}{R_{U,ss}} \right)^{\gamma_r^U} \left[ \left( \frac{Y_{U,t}}{Y_{U,ss}} \right)^{\gamma_y^U} \left( \frac{\Pi_{U,t}}{\Pi_{U,ss}} \right)^{\gamma_{\pi}^U} \left( \frac{e_{U,t}}{e_{U,ss}} \right)^{\gamma_e^U} \right]^{1-\gamma_r^U} \epsilon_{U,t}^m
\]

(62)

\[
P_{U,t} Y_{U,t} = \sum_i P_{i,t} Y_{i,t}
\]

(63)

\[
P_{U,t} = \prod_i P_{i,t}^{w_{i,U}}
\]

(64)

\[
\Pi_{U,t} = \prod_i \Pi_{i,t}^{w_{i,U}}
\]

(65)

where \( \Pi_{U,t} \) is the union-wide inflation level and is represented by the geometric mean of the country level inflation. \( Y_{U,t} \) is the aggregate GDP of the union and is the sum of the goods produced in each region. \( P_{U,t} \) is the aggregate GDP deflator in the union and the relative weight \( w_{i,U} \) and \( w_{i,U} \in [0,1] \) are respectively calculated on the basis of the participation of the
country $i$ in the aggregate output and consumption of the union:

$$w^p_i = \frac{Y_{i,t}}{\sum_i Y_{i,t}} \quad (66)$$

$$w^\pi_i = \frac{c_{i,t}}{\sum_i c_{i,t}} \quad (67)$$

### 4.4.2 Government budget constraint and current account balance

Regarding the fiscal policy, the government consumes and finance any shortfall in his revenue or repay any surplus by levying a lump-sum tax or subsidy. It faces a period-by-period budget constraint with taxes and newly issued government bonds on the income side, and public spending and maturing bonds on the expenditure side. We assume that government spending is exclusively on nontradable goods and governed by an exogenous stochastic process. The government budget constraint is defined by:

$$P_{N,t} G_{i,t} + R_{i,t} B_{i,t} = T_{i,t} + B_{i,t+1} \quad (68)$$

Government expenditures, $G_{i,t}$ follow a stationary AR(1) process:

$$G_{i,t} = \rho^g G_{i,t-1} + \epsilon^g_{i,t} \quad (69)$$

where $\epsilon^g_{i,t}$ is a government spending shock.

Finally, by combining the household’s budget constraint, government budget, and the profit function of the firms, we obtain the following current account balance that describes the evolution of the economy’s net foreign assets:

$$e_t B_{i,t+1}^* = e_{i,t} B_{i,t}^* R_{i,t}^* + P_{i,t}^{TH} E X_{i,t} - P_{i,t}^{TF} C_{i,t}^{TF} \quad (70)$$

### 4.5 International price

The ECOWAS countries are small compared to the global economies, and therefore they consider the world interest rate as given. Hence, we assume $R_{i,t}^*$ follow an AR(1) process:

$$R_{i,t}^* = \rho^r R_{i,t-1}^* + \epsilon^r_{i,t} \quad (71)$$
The bilateral terms of trade between the domestic economy, and the rest of the world is defined by:

$$TOT_{i,t} = \frac{PX_{i,t}^{ROW}}{PM_{i,t}^{ROW}}$$  \hspace{1cm} (72)

Each region is a price-taker vis-a-vis the rest of the world markets and hence, the export and import prices from the rest of the world are exogenous.

$$PX_{i,t}^{ROW} = \rho_{i}^{px} PX_{i,t-1}^{ROW} + \epsilon_{i,t}^{px}$$  \hspace{1cm} (73)

$$PM_{i,t}^{ROW} = \rho_{i}^{pm} PM_{i,t-1}^{ROW} + \epsilon_{i,t}^{pm}$$  \hspace{1cm} (74)

### 4.6 Market clearing conditions and model dynamics

We end the model description with the specification of the market-clearing conditions. In equilibrium, all markets must clear in each country. The goods market equilibrium conditions require production to be equal to aggregate demand in each country. In the domestic tradable sector, goods produced are consumed by households, invested domestically or sold abroad:

$$Y_{i,t}^{TH} = C_{i,t}^{TH} + I_{i,t}^{TH} + EX_{i,t}^{ROW} + \sum_{j} EX_{i,t}^{j}$$  \hspace{1cm} (75)

where $EX_{i,t}^{ROW}$ and $EX_{i,t}^{j}$ represent respectively the exports of the country $i$ to the rest of the world and to the other countries of the model.

$$EX_{i,t}^{ROW} = \tau \left( \frac{PX_{i,t}^{ROW}}{P_{i,t}^{TH}} \right)^{-\nu} Y_{i,t}^{ROW}$$  \hspace{1cm} (76)

$$EX_{i,t}^{j} = C_{j,t}^{Ti} + I_{j,t}^{Ti}$$  \hspace{1cm} (77)

where the output level for the rest of the world follows the following exogenous process:

$$y_{i,t}^{ROW} = \rho_{i}^{y^{ROW}} y_{i,t}^{ROW} + \epsilon_{i,t}^{ROW}$$  \hspace{1cm} (78)

Similarly, the market-clearing condition in the nontradable sector imply that, in each country, the nontradable good is either consumed, invested, or purchased by the government:

$$Y_{i,t}^{N} = C_{i,t}^{N} + I_{i,t}^{N} + G_{i,t}$$  \hspace{1cm} (79)
In the factor markets (labor and capital), the wage rate and the rental rate of capital must respectively adjust to clear the markets.

\[ K_{i,t} = \int_0^1 K_{i,t}^{TH} + \int_0^1 K_{i,t}^N \]  \hspace{1cm} (80)

\[ N_{i,t} = \int_0^1 N_{i,t}^{TH} + \int_0^1 N_{i,t}^N \]  \hspace{1cm} (81)

The dynamics of the model is represented by the intertemporal decision in the goods and asset market (7)-(12), the intratemporal consumption decision (15)-(23) and investment decision (33)-(38), the sectoral production function (44), the real marginal cost (48), the sectoral Phillips curves (57), the equations for the monetary policy (59)-(65), the government budget constraint and the current account balance (68) and (70), and the equilibrium conditions (75)-(80).

5 Data, calibration, and simulation

This section presents the data used in the model, the calibration of the parameters as well as the simulation method. As in many studies, the calibration exercise consists to either borrow the parameters from the literature on the economies of similar structure, or estimates them from time-series data for the economies studied, or a mix of both. In this study, we calibrate the parameters using both the existing literature and time series estimation. We choose the parameters to be consistent with the assumptions of previous work on similar countries. For simplicity, all behavioral parameters between the three regions are assumed to be identical. This assumption allows us to consider that the only differences in the model structure are the policy setting and the long-run structural differences in country size and trade patterns. The dynamic of the model is analyzed using variance decomposition and impulse responses. We will focus on the effect of asymmetric disturbance on the real side of the three regions.

5.1 Data and calibration

We obtain national accounts data on GDP, consumption, investment, exports, and imports in constant prices from the World Bank. Output in the tradable sector is measured by the total real production in manufacturing sectors and primary industries (agriculture, fishing, forestry, and mining). The output of nontradable includes construction, transportation, storage, service public, finance, and real estate. The tradable sectors account for respectively
61%, 51%, and 55% of GDP in Nigeria, WAEMU, and the rest of ECOWAS. This is consistent with the standard value used in the literature for low-income countries (Lombardo and Ravenna [2012] and Kose and Riezman [2001], etc.). The share of consumption in output is respectively 68%, 73%, and 77% for Nigeria, WAEMU and RECOWAS, while the share of investment in output is 13%, 16%, and 21% for the three regions respectively. The steady-state value of the world interest rate is 0.04, which is consistent with the rate faced by African countries on international markets. The discount factor $\beta_i$, is set at 0.98, as is commonly used in the literature. Following Ostry and Reinhart [1992], we set the risk aversion parameter $\sigma_i$ to 2.61. This value is computed using a panel of developing countries. We assume that the elasticity of substitution between tradable and nontradable consumption and investment are 0.9 while the elasticity of substitution of tradable goods between regions is 1.5. It is the values used by Dagher, Gottschalk and Portillo [2012] in a study for Ghana. Regarding the capital depreciation rate, $\delta$, we assigned a value of 0.05, which is commonly used in the literature. The Calvo parameter $\theta$, is equal to 0.8. The coefficient of output and inflation in the Taylor rule are respectively set to 0.8, 1.5, which means that the central bank gives higher weight to inflation targeting compared to output targeting. Regarding the exchange rate in the Taylor rule, it is set to 0.8 for Nigeria and RECOWAS as they have a flexible exchange rate. This value has to be large in the context of fixed exchange rate, so we consider the value 1.7 for WAEMU. Finally, we set the persistence and standard deviation of the region-specific technology and terms-of-trade shocks to 0.75 and 0.01 respectively. All the calibrated parameters are in table 6

5.2 Variance decomposition

We evaluate the dynamic properties of the model by discussing the variance decomposition i.e. the relative importance of the different shocks in explaining the major macroeconomic variables. Tables 7 and 8 present the contribution of each shock, namely the tradable and the terms of trade shock to the variability of the selected endogenous variables for each block of countries. We can observe that in the monetary independence regime, long run variations in the main variables in a given region are driven almost entirely by idiosyncratic shocks hitting the same region, while in the monetary union, we can observe a cross-country contribution of the shocks to the variability of the variables. Furthermore, in monetary union productivity shock originating from Nigeria is the main factor behind variations in output, consumption, investment, inflation and labor. Another fact that is important to highlight is that beside the Nigeria productivity shock that generates a high cross-country variation, terms of trade shock is domestically the second drivers of variation in each region. Overall, we can conclude that
asymmetric supply disturbances play a major role in the variability of the main variables in our model. More the ECOWAS region is more susceptible to productivity shocks, especially the one coming from NIGERIA. Finally, when they have a common central bank, the three blocks of region are significantly affected by all shocks regardless of the origin of this latter, and the terms of trades shock represent the second important source of fluctuations in the ECOWAS region.

5.3 Impulse responses analysis

In this section, we analyze the impact of productivity and terms of trade shocks on the three regions in order to highlight the insight that can be derived from the model. All the shocks are implemented as a 1% standard deviation of the variable of interest. We explore the transmission mechanism of the different shocks across the region by assuming that there is no international correlation of the shocks so that they are asymmetric. In other words, the shocks are applied to the model asymmetrically (i.e. to one region at the time). Our focus on the international implications of the shocks provides information on the heterogeneous effects they have on each region. Throughout the analysis, we will call the home economy the region where the shock originates and the foreign economies the two other regions in the model. For example, if the source of the shock is Nigeria (home economy), we will consider WAEMU and RECOWAS as the foreign regions. For each shock, we report the impulse-response functions in both monetary union (red line) and monetary independence (blue dashed line) regimes, and they are displayed in Figures 2-7. All the IRFs represent an absolute deviation from the steady-state because we simulate the model in level.

5.3.1 Tradable productivity shock

Shock transmission in monetary independence

We start by considering a temporary negative tradable productivity shock in Nigeria of one standard deviation. Figure 2 displays the local effect on the Nigerian’s economy and the international impact of the shock on selected variables in the two other regions (that is WAEMU and RECOWAS).

We observe that in the presence of negative tradable productivity shock, Nigeria’s output decreases in the regime of monetary independence. As a result, the income and wealth of households decrease, and this drop in their wealth leads them to push their demand for domestic goods downward, so aggregate consumption also falls. Moreover, the negative income effect combined with the decline in the marginal productivity of capital decreases the incen-
tive of households to accumulate more capital and therefore, there is a drop in investment. Note that due to the substitution effect between the tradable and the nontradable goods, the shock affects output, consumption, and investment in the nontradable sector positively as they all increase following the productivity decrease in the tradable sector. The shock also induces an immediate decline in labor supply, which can be explained by the presence of nominal rigidity in our model (Gali [1999]). As labor starts recovering, the marginal return of capital increases as well, so investment increases gradually till reaching its initial level. The decline in productivity also leads to an increase in inflation and the appreciation of the nominal exchange rate. Consequently, there is a drop in the competitiveness of the Nigerian economy that leads to a decline in their total exports. Moreover, the initial response of the central bank to the inflationary pressure is to hike the nominal interest rate, but the inflation starts to decrease after few periods and the interest rate decreases accordingly in order to contain the deflationary pressure.

The movements in the exchange rate mainly explain the dynamics in foreign economies. Recall that the technology shock affects both the demand and the supply side of the economy and will be internationally transmitted through different trade channels depending on the magnitude of the trade links between the regions. Hence, the negative productivity shock in Nigeria has different effects on WAEMU and RECOWAS because of their trade pattern with Nigeria. On the one hand, we observe that the negative productivity shock in Nigeria is transmitted to the foreign economies by decreasing their real output and leads the exchange rate to appreciate in WAEMU and depreciate in RECOWAS. The depreciation of the exchange rate decreases the relative price of RECOWAS’ goods, shifting demand away from imported Nigeria’s goods and toward domestic goods. The results show also that WAEMU is more sensitive to productivity shock originating from Nigeria, mainly in terms of change in output, interest rate, and employment. The effects go in the opposite direction for the WAEMU economy.

Consider now a tradable productivity shock originating from WAEMU. The mechanism on the home economy is quite similar to the case of Nigeria, i.e., marginal costs increase, and households decrease their demand for consumption, investment, and are less willing to work. The shock increases inflation, and through the Taylor rule, the monetary authority responds by increasing the nominal interest rate. The cross-country effect of the shock shows that it has a negligible effect on Nigeria’s main variables, i.e., output, consumption, investment. However, due to the substitution effect generated by a high price level, WAEMU’s consumers switch their demand away from domestic goods toward imported goods, which causes an increase in Nigeria’s exports. On the other hand, the shock hurts RECOWAS’ output because it has a relatively large elasticity to import from WAEMU. This shock leads in fact to an
improvement of WAEMU terms of trade, and consequently an increase in RECO WAS import bills.

Finally, when the source of the shock is the RECO WAS region, the decline in productivity increases firms’ marginal costs and thereby increasing the price level. The hike in prices makes the monetary policy to start an upward trend of adjustments on the interest rate. The shock has a positive impact on Nigeria’s variables and is negatively transmitted to WAEMU, and the channel is indeed through international trade. RECO WAS nominal exchange rate depreciates following the shock, and this makes its goods more attractive on the WAEMU market and hence reduces WAEMU’s trade balance.

**Shock transmission in monetary union**

We consider now the effect of the productivity shock in the regime of monetary union. The creation of a monetary union removes the exchange rate fluctuations between the three regions and consequently, the bilateral nominal exchange rate no longer plays a primary role of shocks absorber. The response of the shock in the domestic economy is very similar to the case of monetary independence, i.e., output, consumption, and investment decrease in response to the shock. The decline in productivity also leads to an increase in the price level. However, in all the regions, output fall by more under monetary union than under monetary independence. The main differences observed in the cross-country transmission of the shock can be summarized as follow: Figures 2-4 show that the productivity shocks originating from WAEMU and RECO WAS produce almost zero response in the union-wide interest rate while the interest reacts to shock originating from Nigeria. This destabilizing effect of Nigeria on the union-wide interest rate is due to its high weight on the union-wide inflation and output. Moreover, regardless the source of the productivity shock, output (inflation) is more (less) volatile in Nigeria when it joins the monetary union compared to the case of independent monetary policy, except when the shock originates from WAEMU where we have no significant difference in the variability of this two variables for Nigeria. For WAEMU and RECO WAS, both Output and inflation are less volatile in monetary union compared to monetary independence except when the shock originates from Nigeria where we have the opposite effect. These results show that it is not straightforward to conclude about the degree of macroeconomic volatility in Nigeria when it is outside compared to inside the ECOWAS monetary union. The results also suggest that a monetary union that excludes Nigeria is more likely to promote macroeconomic stability for WAEMU and RECO WAS, while with Nigeria in the union, they are more likely lead to have high macroeconomic volatility. In the regime where we have an independent monetary policy, productivity reduction in one
region is not always negatively transmitted to the other regions, while in the case of a monetary union, regardless of the source of the shock, it is negatively transmitted to the foreign countries. The productivity drop in Nigeria for example leads to an increase in the union-wide interest rate, which reduces wealth in the entire union. The tightening of the monetary policy results in a higher cost of borrowing, and consequently, households decrease their consumption, and firms decrease their production and labor demand. Moreover, the exchange rate of common currency depreciates and causes a switch in expenditure toward goods produced in the union. Finally, the effects of the shocks are more persistent in the regime of a monetary union compared to the case of monetary independence.

5.3.2 Terms of trade shock

Shock transmission in monetary independence

We assess now the macroeconomic responses of West African economies to external price shocks, i.e., the impact of negative country-specific terms of trade shocks, which can be understood as terms-of-trade deterioration in the home economy in the context of independent monetary policy. Figures 5-7 display the response of the main variables. Regardless of the source of the shock, a worsening terms-of-trade leads to an economic slowdown in the home economy because of the substitution effect, which leads the households to reduce their consumption for tradable goods and cause a negative effect on aggregate production. Also, inflation increases and the monetary authority responds by increasing the policy rate. The international transmission of the shock is such that the response of the main variables is similar in both WAEMU and RECOWAS. Terms-of-trade deterioration in Nigeria generates a negative co-movement in output, consumption, investment in WAEMU, and RECOWAS. This is due to the negative correlation of Nigeria’s terms of trade to the other regions. Recall that Nigeria is an oil exporter while WAEMU and RECOWAS are mainly specialized in the agricultural sector.

When the source of the shock is WAEMU or RECOWAS, the shock is positively transmitted to the other regions because an increase in their world import price, lead them to shift their demand toward the other ECOWAS region with an increase in the exports of their respective trading partners and hence an improvement in their trade balance. However, Nigeria’s response to terms of trade shocks coming from WAEMU and RECOWAS remains insignificant. Overall the effect of terms-of-trade shocks in the domestic economies is consistent with the literature, i.e., a worsening terms-of-trade cause a drop in the relative price of tradable goods. The shock causes an increase in domestic inflation and a depreciation of the exchange rate. The monetary authority raises the interest rate in response to this inflationary pressure.
The foreign countries’ response to terms-of-trade is not trivial and depends on the bilateral trade integration in the region. Terms-of-trade shocks coming from Nigeria tend to have a negative spillover effect on the two other regions while the shock is positively transmitted when it originates from WAEMU and RECOWAS.

**Shock transmission monetary union**

Regarding the negative terms of trade shock in the monetary union setting, it causes a decline in output consumption and investment in all three regions and a hike in domestic inflation. By comparison to the other regime, both WAEMU and RECOWAS loose from the terms of trade deterioration in Nigeria in the case of monetary union. Recall that Nigeria produces a large share of the union’s output, and hence, when the shock originates in Nigeria, the negative effect on its economy spillover the whole region. Likewise, the terms of trade deterioration in RECOWAS (WAEMU) are also negatively transmitted to WAEMU(RECOWAS). Therefore, in the context of monetary union, the negative terms-of-trade shock in one region of the union leads the households of the other country to be worse off.

Overall, it follows from the analysis of the impulse response functions that the cross country effect of both productivity and terms of trade shocks depend on various characteristic including the trade linkage, the movement of the exchange rate and the monetary policy rule. We find that the transmission of shocks originating in Nigeria is stronger under both monetary regimes showing that ECOWAS countries are more vulnerable to shocks affecting Nigeria.

## 6 Welfare analysis

We evaluate in this section the benefits of joining the ECOWAS currency union through a welfare analysis. Following Schmitt-Grohé and Uribe [2004] and Kim et al. [2008]. We compute a utility base welfare by using the second-order approximation of the utility function. The second-order Taylor approximation of the utility around the steady-state is:

\[
EU(C_{i,t}, N_{i,t}) = U(C_i, N_i) + C_i^{1-\sigma_i}E(\tilde{C}_{i,t}) + \psi_i N_i^{1+\phi_i}E(\tilde{N}_{i,t}) - \frac{1}{2} \sigma_i C_i^{1-\sigma_i} var(\tilde{C}_{i,t}) - \frac{1}{2} \psi_i \phi N_i^{1+\phi_i} var(\tilde{N}_{i,t})
\]

(82)

Where \(\tilde{X}_{i,t} = (\tilde{C}_{i,t}, \tilde{N}_{i,t})\) is the deviation of \(X_{i,t}\) from its steady-state value. and \(var(\tilde{X}_{i,t})\) is the variance of \(X_{i,t}\).

The unconditional welfare gain or loss \(\varpi_{total}\) from changing regime is represented by the
permanent change in the steady-state consumption required to achieve the same expected utility function.

\[ EU^{MI}((1 + \varpi^{total}_i)C_{i,t}, N_{i,t}) = EU^{MU}(C_{i,t}, N_{i,t}) \]  

(83)

If \( \varpi^{total} > 0 \), then the households are better off under monetary union compared to monetary independence. The unconditional welfare can be decomposed into two components: the change in consumption due to the effect of the shock on the mean of the variables \( \varpi^{mean} \) and the effect of the shock on the variance of the variables \( \varpi^{var} \) (see Kollmann [2002] and Bergin, Shin and Tchakarov [2007]). Hence the total welfare cost is given by:

\[ \varpi^{total}_i = \varpi^{mean}_i + \varpi^{var}_i \]  

(84)

Table 9 shows the welfare loss or gain when the three regions move from the state of monetary independence regime to monetary union. It reports the change in consumption relative to the steady-state level for the three regions that would be needed to have the same utility under both regimes. The result shows that the ECOWAS currency union will lead to welfare improvement for all the regions, but the magnitude of the welfare gain is relatively small. Note that the gain in welfare is mostly driven by the increase in consumption mean. WAEMU will have the highest welfare gain with a 1.44% increase in consumption while Nigeria will encounter the lowest welfare gain with 0.88% positive change in consumption.
7 Conclusion

This paper contributes to the literature on currency union by examining the transmission mechanism of idiosyncratic shocks on the economies of countries sharing the same currency. More specifically, we investigate the mechanism of propagation of asymmetric shocks in the projected monetary union for the Economic Community of West African States (ECOWAS) and their welfare implications. For that purpose, we divided the ECOWAS area into three regions, namely Nigeria, the existing West African Economic Monetary Union (WAEMU) and the rest (RECOWAS), and developed a multi-region New Keynesian model with nominal and real rigidity. We assess the dynamics of the model through countries-specific productivity and terms of trade shocks and proceed to a comparative analysis of two monetary regimes: monetary independence and monetary union.

Our main findings can be summarized as follow: in both monetary union and monetary independence, the adverse productivity shock causes a decline in real variables such as output, consumption, investment in the domestic economies, and cause an increase in the price level. With an independent monetary policy, the central banks respond to the inflationary pressure by tightening the nominal interest rate. Secondly, irrespective of the source of the productivity shocks, they have an adverse spillover effect on the other regions when they have an independent monetary policy- however, Nigeria’s output increases in response to negative productivity shock in the other regions. In monetary union, we observe a positive propagation of the productivity shock across the regions. Terms-of-trade shocks are negatively transmitted to the three regions when they share the same currency, while in monetary independence, the shock is negatively (positively) transmitted when it originates from Nigeria (WAEMU and RECOWAS). Moreover, the shocks originating from WAEMU and RECOWAS produce almost zero response in the union-wide interest rate, while the interest rate reacts to shock originating from Nigeria. Shocks originating from Nigeria tend to have a more significant effect on WAEMU and RECOWAS while the effects are relatively mitigated for shocks coming from WAEMU and RECOWAS. Also, both productivity and terms of trade shocks are more persistent in monetary union compared to monetary independence.

Even though it is not straightforward from our results to conclude about the degree of macroeconomic stability in the ECOWAS area when they have an independent monetary regime versus a monetary union, we observe that a monetary union that excludes Nigeria will likely reduce macroeconomic fluctuations in the other regions.

Regarding the welfare implication of the change in monetary regime, our results show that the ECOWAS currency union will lead to welfare improvement for all the three regions, but
the magnitude of the welfare gain is relatively small. WAEMU has the highest welfare gain while Nigeria encounters the lowest welfare gain. The results of this study suggest that additional adjustment and stabilization mechanisms in the presence of asymmetric shocks are required to fully compensate for the absence of an independent monetary policy if the ECOWAS countries were to form their monetary union.
References


Bayoumi, Tamim A, Barry J Eichengreen and Barry J Eichengreen. 1994. “One money or many?: Analyzing the prospects for monetary unification in various parts of the world.”


Qureshi, Mahvash Saeed and Mr Charalambos G Tsangarides. 2006. What is fuzzy about clustering in West Africa? Number 6-90 International Monetary Fund.


van den Boogaerde, Mr Pierre and Mr Charalambos G Tsangarides. 2005. *Ten years after the CFA franc devaluation: progress toward regional integration in the WAEMU*. Number 5-145 International Monetary Fund.

### Table 1: Stylized facts

<table>
<thead>
<tr>
<th></th>
<th>Export diversification index</th>
<th>GDP per capita (constant USD)</th>
<th>GDP growth (%)</th>
<th>ISM to GDP ratio</th>
<th>Inflation (%)</th>
<th>Export (% GDP)</th>
<th>Import (% GDP)</th>
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<tr>
<td>Nigeria</td>
<td>6.02 5.83</td>
<td>1353.77 1774.33</td>
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<td>9.00 3.41</td>
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<td>Burkina-Faso</td>
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<td>347.40 503.89</td>
<td>-0.09 5.92</td>
<td>34.84 90.91</td>
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<td>26.72 26.61</td>
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<tr>
<td>Cote-d'Ivoire</td>
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<td>Guinea-Bissau</td>
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<td>3.11 4.89</td>
<td>99.81 50.45</td>
<td>4.89 2.35</td>
<td>16.33 24.17</td>
<td>31.16 32.16</td>
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<td>2.21 4.17</td>
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<td>6.31 1.89</td>
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<td>36.85 41.01</td>
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<tr>
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<td>1.52 2.03</td>
<td>95.15 84.95</td>
<td>6.03 2.78</td>
<td>39.04 36.69</td>
<td>40.71 34.11</td>
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### Table 2: Correlation of cyclical components

<table>
<thead>
<tr>
<th>Countries</th>
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<th>RECWAS</th>
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<td></td>
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<td>BEN BFA</td>
<td>CPV GMB GHN GIN LIB SLE</td>
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<td>1</td>
<td></td>
</tr>
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<td>Burkina-Faso</td>
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<td>0.07 1</td>
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</tr>
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<td>Cote-d'Ivoire</td>
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<td>0.18 0.46***</td>
<td>-0.02 -0.07</td>
</tr>
<tr>
<td>Guinea-Bissau</td>
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<td>0.32* 0.49***</td>
<td>0.20 0.00</td>
</tr>
<tr>
<td>Mali</td>
<td>0.03</td>
<td>0.11 0.49***</td>
<td>-0.03 0.52***</td>
</tr>
<tr>
<td>Niger</td>
<td>-0.05</td>
<td>0.16 0.33*</td>
<td>0.40 0.00</td>
</tr>
<tr>
<td>Senegal</td>
<td>-0.05</td>
<td>0.16 0.33*</td>
<td>0.40 0.00</td>
</tr>
<tr>
<td>Togo</td>
<td>0.11</td>
<td>0.04 0.11</td>
<td>0.27 0.33*</td>
</tr>
<tr>
<td>Cabo-Verde</td>
<td>0.01</td>
<td>-0.05 0.16</td>
<td>0.09 0.03</td>
</tr>
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<td>Gambia</td>
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<td>-0.21 0.26</td>
<td>0.08 0.13</td>
</tr>
<tr>
<td>Guinea</td>
<td>0.00</td>
<td>0.34* 0.01</td>
<td>-0.11 0.16</td>
</tr>
<tr>
<td>Liberia</td>
<td>-0.15</td>
<td>-0.01 0.18</td>
<td>0.21 0.01</td>
</tr>
<tr>
<td>Sierra-Leone</td>
<td>-0.07</td>
<td>0.03 -0.07</td>
<td>0.05 -0.05</td>
</tr>
</tbody>
</table>

Source: Author calculation using World bank data

Significance at 1% (***) , 5% (**), 10% (*)

36
Figure 1: Comparison of business cycle

![Comparison of business cycle](image)

Table 3: Export share

<table>
<thead>
<tr>
<th></th>
<th>NIGERIA</th>
<th>WAEMU</th>
<th>RECOWAS</th>
</tr>
</thead>
<tbody>
<tr>
<td>NIGERIA</td>
<td>3.92</td>
<td>3.58</td>
<td></td>
</tr>
<tr>
<td>WAEMU</td>
<td>9.79</td>
<td></td>
<td>6.58</td>
</tr>
<tr>
<td>RECOWAS</td>
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<td>6.27</td>
<td></td>
</tr>
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Source: Author calculation using UN COMTRADE data

Table 4: Import share

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<th>RECOWAS</th>
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<td></td>
</tr>
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<td>WAEMU</td>
<td>8.30</td>
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<td>RECOWAS</td>
<td>4.98</td>
<td>4.26</td>
<td></td>
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Source: Author calculation using UN COMTRADE data
Table 5: Weight in regional aggregate data

<table>
<thead>
<tr>
<th>Variables</th>
<th>NIGERIA</th>
<th>WAEMU</th>
<th>RE CowAS</th>
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<tr>
<td>Output 69.93</td>
<td>20.33</td>
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<td>Consumption 74.79</td>
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Source: Author calculation using World bank data

Table 6: Calibration

<table>
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<th>Parameter</th>
<th>Description</th>
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<th>WAEMU</th>
<th>RE CowAS</th>
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<tbody>
<tr>
<td>σ</td>
<td>Risk aversion parameter</td>
<td>2.61</td>
<td>2.61</td>
<td>2.61</td>
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<tr>
<td>φ</td>
<td>Inverse labor supply elasticity</td>
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<td>1</td>
<td>1</td>
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<tr>
<td>β</td>
<td>Discount factor</td>
<td>$\frac{1}{1+r}$</td>
<td></td>
<td></td>
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<tr>
<td>δ</td>
<td>Depreciation rate of capital</td>
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<td>0.05</td>
<td>0.05</td>
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<tr>
<td>$\eta$, $\zeta$</td>
<td>Elast. of subst. between T and NT consumption and investment goods</td>
<td>0.9</td>
<td>0.9</td>
<td>0.9</td>
</tr>
<tr>
<td>$\epsilon$, $\chi$, $\mu$, $\Gamma$, $\varphi$</td>
<td>Elast. of subst. between varieties of Tradable consumption and investment goods</td>
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<td>1.5</td>
<td>1.5</td>
</tr>
<tr>
<td>β</td>
<td>Elast. of subst. between the varieties of export goods</td>
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<td>1.5</td>
<td>1.5</td>
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<tr>
<td>$\rho_{TH,NT}$</td>
<td>Persistence in domestic T and NT productivity</td>
<td>0.9</td>
<td>0.9</td>
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<tr>
<td>$\rho_{PM}$</td>
<td>Persistence in world import price</td>
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<tr>
<td>$\sigma_{TH,NT}$</td>
<td>Standard deviation of productivity shock</td>
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<td>0.01</td>
<td>0.01</td>
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<tr>
<td>$\sigma_{PM}$</td>
<td>Standard deviation of import price shock</td>
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<td>0.01</td>
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<td>$\theta_{TH,NT}$</td>
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<td>0.8</td>
<td>0.8</td>
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<td>$\gamma^{\pi}$</td>
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<td>1.5</td>
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<td>1.7</td>
<td>0.8</td>
</tr>
</tbody>
</table>

Steady-state value

| C/Y Ratio of private consumption to GDP | 0.68 | 0.73 | 0.77 |
| I/Y Ratio of Investment GDP            | 0.13 | 0.16 | 0.21 |
| G/Y Ratio of government spending to GDP | 0.09 | 0.15 | 0.10 |
| X/Y Ratio of export to GDP             | 0.31 | 0.24 | 0.26 |
| I/Y Ratio of Import to GDP             | 0.21 | 0.34 | 0.38 |
Table 7: Variance decomposition: Monetary independence (in percentage)

<table>
<thead>
<tr>
<th>monetary independence</th>
<th>$\epsilon_{ath}$ (NG)</th>
<th>$\epsilon_{tot}$ (NG)</th>
<th>$\epsilon_{ath}$ (W)</th>
<th>$\epsilon_{tot}$ (W)</th>
<th>$\epsilon_{ath}$ (RE)</th>
<th>$\epsilon_{tot}$ (RE)</th>
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</thead>
<tbody>
<tr>
<td><strong>NIGERIA</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
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<tr>
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<td>0</td>
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<tr>
<td>Labour</td>
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<td>70.2</td>
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<tr>
<td><strong>WAEMU</strong></td>
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<td>0.05</td>
<td>43.07</td>
<td>56.67</td>
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<td>0.18</td>
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</tr>
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<td>Output</td>
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<td>0.01</td>
<td>0.12</td>
<td>0.02</td>
<td>37.92</td>
<td>61.93</td>
</tr>
<tr>
<td>Consumption</td>
<td>0.02</td>
<td>0.06</td>
<td>0.48</td>
<td>0.06</td>
<td>34.4</td>
<td>64.98</td>
</tr>
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<td>0.08</td>
<td>0.01</td>
<td>17.64</td>
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<td>0.02</td>
<td>0.17</td>
<td>0.02</td>
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Table 8: Variance decomposition: Monetary union (in percentage)

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<th>monetary union</th>
<th>$\epsilon_{ath}$ (NG)</th>
<th>$\epsilon_{tot}$ (NG)</th>
<th>$\epsilon_{ath}$ (W)</th>
<th>$\epsilon_{tot}$ (W)</th>
<th>$\epsilon_{ath}$ (RE)</th>
<th>$\epsilon_{tot}$ (RE)</th>
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<td>1.76</td>
<td>0.37</td>
<td>19.23</td>
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<td>13.29</td>
<td>1.85</td>
<td>0.57</td>
<td>16.21</td>
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<tr>
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<td>1.96</td>
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<tr>
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<td>7.39</td>
<td>1.38</td>
<td>0.35</td>
<td>27.7</td>
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<td>12.01</td>
<td>2.62</td>
<td>0.55</td>
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<td>20.79</td>
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<td>14.28</td>
<td>11.65</td>
<td>16.22</td>
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<tr>
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<td>1.85</td>
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<tr>
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<td>0.25</td>
<td>4.28</td>
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</tr>
<tr>
<td>Output</td>
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<td>1.93</td>
<td>1.88</td>
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<tr>
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<td>0.22</td>
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<tr>
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<td>2.34</td>
<td>0.23</td>
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<td>23.78</td>
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Table 9: Welfare: % variation in consumption

<table>
<thead>
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<th></th>
<th>NIGERIA</th>
<th>WAEMU</th>
<th>RECWAS</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\omega_{\text{mean}}$</td>
<td>0.60</td>
<td>0.78</td>
<td>0.80</td>
</tr>
<tr>
<td>$\omega_{\text{var}}$</td>
<td>0.28</td>
<td>0.65</td>
<td>0.61</td>
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<tr>
<td>$\omega_{\text{total}}$</td>
<td>0.88</td>
<td>1.43</td>
<td>0.41</td>
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</table>

Source: Author calculation

Figure 2: Effect of tradable productivity Shock originating from Nigeria
Figure 3: Effect of tradable productivity shock originating from WAEMU
Figure 4: Effect of tradable productivity Shock originating from RECWAS
Figure 5: Effect of terms of trade Shock originating from Nigeria
Figure 6: Effect of terms of trade Shock originating from WAEMU
Figure 7: Effect of terms of trade shock originating from RECWAS.