Competition and Offshoring

José Antonio Rodríguez-López*
Department of Economics
University of California, Irvine

April 2011

Abstract

I present a model of offshoring decisions in a framework with heterogeneous firms, random adjustment costs of offshoring, and endogenous markups. The model implies an inverted-U relationship between firm-level productivity and offshoring likelihood, so that a very productive firm is not necessarily more likely to offshore than a less productive one. After a change in the competitive environment, the offshoring probability increases for some firms but declines for others. If productivity follows a power-law distribution, the distribution of offshoring firms is positively skewed—that is, most offshoring firms have productivity levels below the mean.

Keywords: competition, offshoring, heterogeneous firms, endogenous markups, random adjustment costs.
JEL codes: F12, F23.

*I thank Dan Bogart, Priya Ranjan, Guillaume Rocheteau and seminar participants at UC Irvine for comments and suggestions. E-mail: jantonio@uci.edu.
1 Introduction

Changes in the competitive environment affect firms’ offshoring decisions. Consider the trend in recent years in the U.S. computer manufacturing industry. According to the IBIS World Industry Report (2011), in 2003 this industry had 600 establishments owned by 518 firms, and 85,977 employees. By 2006, the numbers had changed to 479 establishments, 419 firms, and 31,017 employees.\(^1\) During that period, the industry started a period of strong price competition—that continues in the present day—that forced many of its participants to relocate parts of their production processes to low-wage countries, specially to China. In this example, a tougher competitive environment increased the offshoring likelihood of some firms and wiped out some other firms. In this paper, I present a model that relates a firm’s offshoring probability with its productivity. I obtain an inverted-U relationship between firm-level productivity and offshoring likelihood, so that it is not necessarily the case that a very productive firm is more likely to offshore than a less productive firm. Moreover, under plausible assumptions with respect to the distribution of productivity, I find that most offshoring firms cluster in the left (low-productivity) side of the distribution.

The objective of offshoring is to improve a firm’s cost structure by moving a part of its production process to another country in order to take advantage of differences in factor prices. In general, offshoring involves a rich source country and a developing destination country, with the difference in labor prices as the main force driving offshoring decisions. There are two organizational types of offshoring: foreign direct investment—when the offshoring firm owns a subsidiary in a foreign country—and outsourcing—when the offshoring firm subcontracts a part of its production process with a foreign firm.\(^2\) In this paper, I do not distinguish between the two types of organizational structures.

In the computer industry example above, we have a situation in which an increase in competition brings firms’ markups down, causes the exit of some firms, and drives some other firms to offshore in order to remain competitive. But, if offshoring implies lower marginal costs, why weren’t these firms offshoreing in the first place? To explain the previous story, my model needs three ingredients: firm heterogeneity in productivity, endogenous markups, and random adjustment costs of offshoring.

\(^1\)Since 2006, employment remained about constant and there were further reductions in the number of establishments and firms. By 2010, there were 435 establishments and 412 firms.

\(^2\)See Antrás and Rossi-Hansberg (2009) for a review of the literature on production organization and trade.
With firm heterogeneity in productivity, we can explain why some firms will exit after a shock that creates a tougher competitive environment: profit opportunities decline after the shock and the least productive firms prefer to exit instead of suffering a loss. I model firm heterogeneity à la Melitz (2003). That is, a firm must pay a sunk cost at the time of entry, and only then it will realize its productivity, which is drawn from a continuous distribution. As a Melitz-type model, I solve for cutoff productivity levels that determine the tradability of each good. There is one cutoff level for non-offshoring firms, and one for offshoring firms. If a firm's productivity is no less than the cutoff, the firm participates in the market. The shocks we consider affect the cutoff productivity levels and hence the tradability of some goods.

An increase in competition drives firms' markups down. This cannot be achieved with constant elasticity of substitution (CES) preferences, as they imply exogenous markups. Instead, my model includes endogenous markups and firms adjust them in response to changes in the competitive environment. For this purpose, I use the continuum-of-goods version of the symmetric translog preferences introduced by Bergin and Feenstra (2000). In Rodríguez-López (2011), I show—in a heterogeneous-firm framework—that the demand system implied by the translog preferences is as tractable as the CES demand system. Among its properties, the translog demand system generates markups that are increasing in productivity.

With respect to the question above, firms were not offshoring in the first place because the offshoring decision is costly. It involves non-negligible relocation and reorganization costs. Therefore, just as with investment, the offshoring decision is lumpy. Hence, inspired by the model of Caballero and Engel (1999) on lumpy investment decisions in a generalized (S,s) setting, I introduce random adjustment costs of offshoring. Every period, each non-offshoring firm draws an offshoring adjustment cost from a probability distribution. If the adjustment cost draw is below an endogenously determined cutoff level, the firm adjusts its production process and begins offshoring. The sunk adjustment cost has two components, one related to the firm's productivity, and one independent of it. The first component is positively related to the firm's size, as larger firms face larger reorganization costs. The second component is relatively more important for low productivity firms, as they are small in size. Under this structure, I obtain a precise offshoring probability for each non-offshoring firm. As mentioned above, the relationship between productivity and offshoring likelihood has an inverted-U shape.
After a shock that increases competition and drives up the cutoff productivity levels for both offshoring and non-offshoring firms, we obtain two opposing effects in the offshoring likelihood function for non-offshoring firms. The negative effect is driven by the decline in non-offshoring profits implied by the increase in the non-offshoring cutoff productivity level. On the other hand, the positive effect is driven by the change in the (offshoring-costs adjusted) difference between offshoring and non-offshoring profits: although both decline with a tougher competitive environment, their difference might increase, making the offshoring status more attractive relative to the non-offshoring status. Given the similarities of these effects with the effects described by Aghion et al. (2005) in their competition and innovation analysis, I use their terminology and refer to the negative force as the Schumpeterian effect, and to the positive force as the escape-competition effect. The Schumpeterian effect dominates for low-productivity firms, and the escape-competition effect dominates for high-productivity firms.

From the offshoring likelihood function, the model can solve for the proportion of offshoring firms for each level of productivity. The function for the proportion of offshoring firms then combines with the productivity distribution to derive the probability distributions for offshoring and non-offshoring firms. Using a Pareto distribution for the productivity of firms, the numerical examples show that the distribution of offshoring firms is positively skewed. That is, most offshoring firms have productivities below the mean. This result is the opposite to what we would obtain in a similar model with fixed costs of offshoring (instead of random adjustment costs).

The paper is organized as follows. Section 2 presents the model, with special emphasis in section 2.2, which describes the offshoring decision problem. In Section 2.3.2, I discuss the Schumpeterian and escape-competition effects, present theoretical results on the response of a firm’s offshoring likelihood to different types of shocks, and present some numerical examples. Finally, section 4 concludes.

2 A Model with Random Adjustment Costs of Offshoring

I present a model with three main features: firm heterogeneity in productivity, endogenous markups, and random adjustment costs of offshoring. As mentioned before, I model firm
heterogeneity and endogenous markups in a monopolistic competition setting with translog
preferences as in the model of Rodríguez-López (2011), and random adjustment costs as in
the model of Caballero and Engel (1999).

The country is inhabited by a continuum of households in the unit interval. Each house-
hold provides a unit of labor at a fixed wage level. Wages levels are, however, different be-
tween this country and the rest of the world. In particular, the wage level abroad is below the
domestic wage level. This fundamental difference is the driver of offshoring decisions.

In this section, I start by specifying preferences, demand functions, and pricing and pro-
duction decisions. Then I describe the offshoring decision and present several results on
average prices, market shares, and the composition of sellers. The section concludes with the
specification of the free-entry condition that closes the model.

2.1 Setup

2.1.1 Preferences and Demand

The preferences of the representative household are given by

\[ U = x + \eta \ln Q, \]

where \( x \) is a homogeneous good, \( Q \) is an aggregator of differentiated goods, and \( \eta \) is a parameter that indicates the degree of preference for differentiated goods. Following Feenstra (2003) and Rodríguez-López (2011), I assume that \( Q \) satisfies the symmetric translog expenditure function

\[ \ln E = \ln Q + \frac{1}{2\gamma N} \left( 1 \right) \int_{i \in \Delta} \ln p_i di + \frac{\gamma}{2N} \int_{i \in \Delta} \int_{j \in \Delta} \ln p_i (\ln p_j - \ln p_i) dj di, \]

where \( E \) is the minimum expenditure required to obtain \( Q \), \( \Delta \) denotes the set of differentiated goods available for purchase, \( N \) is the measure of \( \Delta \), \( p_i \) denotes the price of differentiated good \( i \), and \( \gamma \) indicates the level of substitutability between the varieties (a higher \( \gamma \) implies a higher degree of substitution).

The production of each unit of the homogeneous good, \( x \), requires one unit of labor. This
good is sold in a perfectly competitive market at a price of 1 and hence, the domestic wage is 1.
Given the quasilinear utility function in equation (1) and the equivalence of the wage and the homogeneous-good price, the total expenditure in differentiated goods of the representative household, $I$, is simply a fraction $\eta$ of its wage. As the wage is equal to 1, it is the case that $I = \eta$, where we must satisfy $\eta < 1$.

The demand of the representative household for differentiated good $i$ is given by $q_i = \sigma_i \frac{I}{p_i}$, where $\sigma_i$ is the share of variety $i$ in the total expenditure on differentiated goods. By Shephard’s lemma—the derivative of equation (2) with respect to $\ln p_i$—we obtain that $\sigma_i = \gamma \ln \left( \frac{\hat{p}}{p_i} \right)$, where

$$\hat{p} = \exp \left( \frac{1}{\gamma N} + \ln p \right)$$

(3)
denotes the maximum price that firms can set in the differentiated-good sector, and $\ln p = \frac{1}{N} \int_{j \in \Delta} \ln p_j dj$.

2.1.2 Pricing and Production of Differentiated Goods

The market demand for differentiated good $i$ is identical to the demand of the representative household because households are located in the unit interval. Assuming that the producer of good $i$ has a constant marginal cost, $c_i$, we obtain that its profit maximizing price is

$$p_i = (1 + \mu_i)c_i,$$

(4)

where $\mu_i$ is producer’s $i$ proportional markup over the marginal cost, which is given by

$$\mu_i = \Omega \left( \frac{\hat{p}}{c_i} e \right) - 1.$$

(5)

The function $\Omega(\cdot)$ denotes the Lambert $W$ function and has very convenient properties. In particular, if $z \geq 0$ we have that $\Omega'(z) > 0$, $\Omega''(z) < 0$, $\Omega(0) = 0$, and $\Omega(e) = 1$.\(^3\) Note that $\mu_i$ is zero if $c_i = \hat{p}$ (so that the price of good $i$ equals its marginal cost), and is greater than zero if $c_i < \hat{p}$. If $c_i > \hat{p}$, producer $i$ will not sell in the domestic market.

Another useful result arising from the properties of the Lambert $W$ function is that $\ln p_i = \ln \hat{p} - \mu_i$.\(^4\) Using this result in the expression for $\sigma_i$ in section 2.1.1, we get that $\sigma_i = \gamma \mu_i$. That\(^3\)See Corless et al. (1996) for an overview of the Lambert $W$ function. Other of its properties include $\Omega'(z) = \frac{\Omega(z)}{z(1 + \Omega(z))}$ for $z \neq 0$, and $\ln[\Omega(z)] = \ln z - \Omega(z)$ when $z > 0$.

\(^4\)To obtain this result, first we rewrite the price of good $i$ as $p_i = \Omega \left( \frac{\hat{p}}{c_i} e \right) c_i$, we then take the natural log of that

5
is, the market share density of producer $i$ is directly proportional to its markup.

I assume Melitz-type firm heterogeneity in productivity. After paying a sunk entry cost of $f_E$, a firm realizes its productivity $\varphi$. Knowing its productivity, the firm can decide between using only domestic labor ($L$) or use also foreign labor ($L^*$). The foreign wage, $W^*$, is less than the domestic wage of 1. I assume that an offshoring firm splits its production process in two complementary parts, one of which stays at home while the other is moved abroad. Let $s \in \{n, o\}$ denote a firm’s offshoring status, with $n$ meaning “not offshoring” and $o$ meaning “offshoring”. Then, the production function of a producer with productivity $\varphi$ and offshoring status $s$ is given by $y_s(\varphi) = \varphi L_s$, where

$$
L_s = \begin{cases} 
L & \text{if } s = n \\
\min \left\{ \frac{L}{1-\alpha}, \frac{L^*}{\alpha \tau_o} \right\} & \text{if } s = o.
\end{cases}
$$

In $L_s$, $\alpha \in (0, 1)$ represents the fraction of the production process being offshored, while $\tau_o \geq 1$ accounts for a variable cost of making foreign labor compatible with the domestic production process.\(^5\) Denoting the price of $L_s$ with $W_s$, we obtain that $W_n = 1$ and $W_o = 1 - \alpha + \alpha \tau_o W^*$. Hence, the marginal cost of a firm with productivity $\varphi$ and offshoring status $s$ is $\frac{W_s}{\varphi}$. Throughout the paper, I assume that $\tau_o$ is small enough so that $W_o < W_n$—so that the marginal cost for a firm is always lower when offshoring.

Following equations (4) and (5), we can write the price set by a firm with productivity $\varphi$ and offshoring status $s$ as

$$p_s(\varphi) = (1 + \mu_s(\varphi)) \frac{W_s}{\varphi},$$

for $s \in \{n, o\}$, and

$$\mu_s(\varphi) = \Omega \left( \frac{\hat{\varphi}}{\frac{W_s}{\varphi}} \right) - 1.$$

Then, this firm’s equilibrium quantity and profit function are respectively given by

$$y_s(\varphi) = \left( \frac{\mu_s(\varphi)}{1 + \mu_s(\varphi)} \right) \frac{\gamma I}{W_s} \frac{\varphi}{\varphi} \text{ and } \pi_s(\varphi) = \frac{\mu_s(\varphi)^2}{1 + \mu_s(\varphi)} \gamma I,$$

where $I$ is the total expenditure in differentiated goods.

---

\(^5\) We can also think of $\tau_o$ as an iceberg offshoring cost: a producer must hire $\tau_o \geq 1$ units of foreign labor to produce the same amount of output of a unit of home labor.
2.1.3 Cutoff Productivity Levels

In this model, as in any other Melitz-type model, a cutoff productivity level indicates the tradability of each good in a market: a firm sells its differentiated good if and only if its productivity is no less than the cutoff productivity level. The existence of the upper bound for the price that firms can set, $\hat{p}$, allows us to obtain the cutoff productivity levels without the need to assume fixed costs of production (which are necessary in the Melitz (2003) model with CES preferences). Using the markup function in the previous section, we define the cutoff productivity level for firms with offshoring status $s$ as

$$\varphi_s = \inf\{\varphi : \mu_s(\varphi) \geq 0\} = \frac{W_s}{\hat{p}},$$

for $s \in \{n, o\}$. Thus, this model contains two cutoff productivity levels: $\varphi_n$ and $\varphi_o$.

Note that we can use the zero-cutoff-markup condition in (6) to replace $\hat{p}$ in the markup equation from the previous section. Therefore, we can rewrite the markup equation for a firm with productivity $\varphi$ as

$$\mu_s(\varphi) = \Omega\left(\frac{\varphi}{\varphi_s}\right) - 1$$

for $\varphi \geq \varphi_s$, and $s \in \{n, o\}$.

Moreover, combining the two expressions that stem from (6), we obtain one of the two equations we need to solve the model:

$$\varphi_o = W_o\varphi_n,$$

where we use that $W_n = 1$. As $W_o < 1$, it is always the case that $\varphi_o < \varphi_n$. Hence, a firm whose productivity is in the interval $[\varphi_o, \varphi_n)$ will only produce if it offshores.

2.2 The Offshoring Decision

Following the model of Caballero and Engel (1999) on lumpy investment decisions in a generalized $(S, s)$ framework, I model the offshoring decision on the basis of random adjustment costs. When a firm decides to offshore, it incurs in adjustment—or reorganization—costs. These costs, however, can vary over time and are not necessarily the same for firms with the same level of productivity.
From the previous sections, we know that the total profit that a firm with productivity \( \varphi \) obtains every period under state \( s \), where \( s \in \{ n, o \} \), is given by

\[
\pi_s(\varphi) = \begin{cases} 
0 & \text{if } \varphi < \varphi_s \\
\frac{\mu_s(\varphi)^2}{1+\mu_s(\varphi)} \gamma I & \text{if } \varphi \geq \varphi_s,
\end{cases}
\]

for \( \mu_s(\varphi) \) given by equation (7). Since the marginal cost is lower when a firm is offshoring, it is always the case that \( \pi_o(\varphi) \geq \pi_n(\varphi) \), with strict inequality if \( \varphi > \varphi_o \). This implies that the offshoring decision is irreversible.

At the beginning of each period, every firm that is not offshoring finds out its offshoring adjustment cost, which is proportional to the non-offshoring profits plus a cost component unrelated to its productivity.\(^6\) The firm then decides whether or not to offshore. If the firm decides to offshore, it will remain offshoring until it is hit by an exogenous death shock. If the firm does not offshore, it can die at the end of the period (after an exogenous death shock), or survive and receive a new offshoring adjustment cost at the beginning of the following period. Then, let the offshoring adjustment cost for a firm with productivity \( \varphi \) in a certain period be given by

\[
\psi(\pi_n(\varphi) + f_o),
\]

where \( \psi \) is random, non-negative, and with cumulative distribution function \( F(\psi) \). The term \( \psi \pi_n(\varphi) \) accounts for adjustment costs related to the firm’s productivity (for a given \( \psi \), these costs are increasing in productivity), while \( \psi f_o \) accounts for adjustment costs that are independent of \( \varphi \).

As in Melitz (2003), let \( \delta \) be the probability of an exogenous death shock every period. In steady state, the profit of an offshoring firm with productivity \( \varphi \) in every period, \( \pi_o(\varphi) \), is constant. Then, at the beginning of each period, the Bellman equation for the value of a firm that is not offshoring is

\[
V(\varphi, \psi) = \max \left\{ \frac{\pi_o(\varphi)}{\delta} - \psi(\pi_n(\varphi) + f_o), \pi_n(\varphi) + (1 - \delta)E \left[ V(\varphi, \psi') \right] \right\}.
\]  

\(^6\)Caballero and Engel (1999) interpret adjustment costs that are proportional to the before-change profits as the amount of profits that a firm stops receiving during the adjustment. The larger the firm, the larger these costs are.
Let $\Psi(\varphi)$ be the value for $\psi$ that makes a non-offshoring firm with productivity $\varphi$ indifferent between offshoring or not. The following proposition describes the solution for $\Psi(\varphi)$.

**Proposition 1 (The cutoff adjustment factor)**

Given the Bellman equation (9) and a continuous $F(\psi)$, the cutoff adjustment factor for a non-offshoring firm with productivity $\varphi$, $\Psi(\varphi)$, is the unique solution to the equation

$$
\Psi(\varphi) = \frac{z(\varphi)}{\delta} - \frac{1}{\delta} \int_0^{\Psi(\varphi)} F(\psi) d\psi,
$$

where $z(\varphi) = \frac{\pi_o(\varphi) - \pi_n(\varphi)}{\pi_n(\varphi) + f_o} \geq 0$ is an offshoring-costs-adjusted measure of the distance between offshoring and non-offshoring profits.

Therefore, at the beginning of each period, for the set of non-offshoring firms with productivity $\varphi$, those drawing an adjustment factor below $\Psi(\varphi)$ become offshoring firms. We can be more precise and pin down the probability that a non-offshoring firm with productivity $\varphi$ begins to offshore in a particular period. Denoting this probability with $\Lambda(\varphi)$, it follows that $\Lambda(\varphi) = F(\Psi(\varphi))$. The following proposition describes the behavior of $\Lambda(\varphi)$.

**Proposition 2 (The probability of offshoring)**

1. $\Lambda(\varphi) = 0$ for $\varphi \leq \varphi_o$, and $\Lambda(\varphi) \to 0$ if $\varphi \to \infty$;

2. If $f_o > 0$, there is a unique maximum for $\Lambda(\varphi)$ in the interval $(\varphi_o, \infty)$. Given $\varphi_n$ and $\varphi_o$, the level of productivity that maximizes $\Lambda(\varphi)$ approaches $\varphi_n$ from the right as $f_o$ declines;

3. If $f_o = 0$, $\Lambda(\varphi) = 1$ for $\varphi \in (\varphi_o, \varphi_n]$, and is strictly decreasing for $\varphi > \varphi_n$.

Figure 1 presents a graphical description of Proposition 2. The probability of offshoring is zero for a firm with productivity at or below $\varphi_o$, as this firm cannot make positive profits even if it offshores. For firms with productivities above $\varphi_o$, it is useful to refer to our offshoring-costs-adjusted measure of the incremental profits from offshoring, $z(\varphi)$, which is the most important determinant in the shape of $\Lambda(\varphi)$. The larger $z(\varphi)$, the higher the adjustment factor that a non-offshoring firm is willing to accept, and hence the higher the probability of offshoring. Non-offshoring firms with productivities between $\varphi_o$ and $\varphi_n$ do not produce—have zero profits—and thus, their offshoring decision only depends on the comparison of the offshoring profits and the fixed component (unrelated to productivity) of the offshoring
adjustment costs, $\psi f_o$. If $f_o$ equals zero, non-offshoring firms in this range face no offshoring adjustment costs, and hence all of them become offshoring firms; if $f_o > 0$, these firms’ prospects of offshoring increase with productivity, and hence $\Lambda(\varphi)$ is increasing in this range.

For non-offshoring firms with productivities above $\varphi_n$ (so that they produce and have positive profits), their offshoring decision also considers the offshoring adjustment costs associated with their size, $\psi \pi_n(\varphi)$. For those firms close to $\varphi_n$ (from the right), they are small enough so that the most important adjustment cost they face is $\psi f_o$. Thus, if $f_o > 0$, there exists a range of firms—starting at $\varphi_n$—for which the probability of offshoring increases with productivity. As the offshoring adjustment cost related to the firm’s size becomes more important, there will be a point at which the offshoring probability starts to decline.

There are two key differences of this model with respect to heterogeneous-firm models that only consider fixed costs of offshoring (e.g. Antràs and Helpman, 2004). In the models that only consider fixed costs, every firm with a productivity no less than a cutoff productivity level will offshore: denoting that cutoff level by $\bar{\varphi}$, these models imply that $\Lambda(\varphi) = 0$ if $\varphi < \bar{\varphi}$, and $\Lambda(\varphi) = 1$ if $\varphi \geq \bar{\varphi}$. On the other hand, in this model (i) there is no cutoff level that separates offshoring and non-offshoring firms, and (ii) the most productive firms can have offshoring probabilities that are below the offshoring probabilities of much less productive firms.
2.3 Distribution and Composition of Firms

2.3.1 Productivity Distributions for Offshoring and Non-Offshoring Firms

After entry, a firm draws its productivity from the interval \([\varphi_{\min}, \infty)\) according to the cumulative distribution function \(G(\varphi)\), with probability density function denoted by \(g(\varphi)\). Hence, every period \(t\) there is a pool of firms with measure \(N_{P,t}\) that contains all the existing firms in the interval \([\varphi_{\min}, \infty)\). Given the exogenous death probability \(\delta\), \(N_{P,t+1} = (1 - \delta)N_{P,t} + N_{E,t+1}\), where \(N_{E,t+1}\) denotes the mass of entrants in \(t + 1\). In steady state, the measure of the pool of firms is constant at \(N_P\), so that \(N_E = \delta N_P\). The composition of \(N_P\) between firms that offshore and firms that do not offshore is determined by \(\delta\) and \(\Lambda(\varphi)\). For each level of productivity \(\varphi\), the steady-state proportion of offshoring firms is given by

\[
\Gamma(\varphi) = \frac{\Lambda(\varphi)}{1 - (1 - \delta)(1 - \Lambda(\varphi))}. \tag{11}
\]

The proportion of firms with productivity \(\varphi\) that do not offshore is then given by \(1 - \Gamma(\varphi)\).\(^7\)

Let \(h_o(\varphi)\) and \(H_o(\varphi)\) denote, respectively, the probability density function and the cumulative distribution function for the productivity of offshoring firms. Using \(\Gamma(\varphi)\) and \(g(\varphi)\), it is the case that

\[
h_o(\varphi) = \frac{\Gamma(\varphi)g(\varphi)}{\bar{\Gamma}}, \tag{12}
\]

where \(\bar{\Gamma} = \int_{\varphi_{\min}}^{\infty} \Gamma(\varphi)g(\varphi)d\varphi\) is the proportion of firms that offshore in steady state—i.e. the measure of offshoring firms is \(\bar{\Gamma}N_P\). Note that given that \(\Gamma(\varphi) = 0\) if \(\varphi \leq \varphi_o\), then \(\bar{\Gamma} = \int_{\varphi_o}^{\infty} \Gamma(\varphi)g(\varphi)d\varphi\).

Analogously, let \(h_n(\varphi)\) and \(H_n(\varphi)\) denote the probability density function and the cumulative distribution function for the productivity of non-offshoring firms. We then have that

\[
h_n(\varphi) = \frac{g(\varphi)[1 - \Gamma(\varphi)]}{1 - \bar{\Gamma}}. \tag{13}
\]

The measure of non-offshoring firms is \((1 - \bar{\Gamma})N_P\).

\(^7\)Note that if the death probability, \(\delta\), is equal to zero, all the firms with productivity higher than \(\varphi_o\) will offshore in the steady state.
2.3.2 Composition of Firms, Averages, and Market Shares

As mentioned in section 2.1.1, $N$ denotes the measure of the set of goods that are available for purchase. As each firm produces a single good, the set of actual producers also has measure $N$ and is a subset of the pool of firms (which has measure $N_P$). The set of actual producers is composed by non-offshoring firms, with measure $N_n$, and by offshoring firms, with measure $N_o$. That is, $N = N_n + N_o$.

From the previous sections, we know that a non-offshoring firm produces if its productivity is no less than $\varphi_n$. Given the distribution of non-offshoring firms, we have that a fraction $1 - H_n(\varphi_n)$ of the pool of non-offshoring firms—$(1 - \bar{\Gamma})N_P$—satisfies that requirement. Therefore, $N_n = (1 - H_n(\varphi_n))(1 - \bar{\Gamma})N_P$. On the other hand, every offshoring firm has a productivity level that is no less than $\varphi_o$ (i.e. $H_o(\varphi_o) = 0$ and every offshoring firm produces). Thus, $N_o = \bar{\Gamma}N_P$. Adding the expressions for $N_n$ and $N_o$, we get

$$N = [1 - (1 - \bar{\Gamma})H_n(\varphi_n)] \cdot N_P.$$ 

The following proposition presents the solution for $N_P$ as a function of the cutoff productivity levels and exogenous parameters.

**Proposition 3** (*The measure of the pool of firms*)

*Given the productivity distributions of offshoring and non-offshoring firms, the measure of the pool of firms is given by*

$$N_P = \frac{1}{\gamma \left[ (1 - \bar{\Gamma})(1 - H_n(\varphi_n))\bar{\mu}_n + \bar{\Gamma}\bar{\mu}_o \right]},$$ (14)

*where $\bar{\mu}_s = \int_{\varphi_s}^{\infty} \mu_s(\varphi)h_s(\varphi \mid \varphi \geq \varphi_s)d\varphi$ is the average markup of producers with offshoring status $s$, for $s \in \{n, o\}$.*

Once we obtain the cutoff productivity levels, we solve for $N_P$ using equation (14), and plug in the result in the expressions for $N_n$, $N_o$, and $N$.

We can also write expressions for average productivities and average prices. The average productivity and average price of producing firms—those with productivities that are no less
than the cutoff level—with offshoring status $s$, for $s \in \{n, o\}$, are respectively given by

$$
\bar{\varphi}_s = \int_{\varphi_s}^{\infty} \varphi h_s(\varphi | \varphi \geq \varphi_s) d\varphi \quad \text{and} \quad \bar{p}_s = \int_{\varphi_s}^{\infty} p_s(\varphi) h_s(\varphi | \varphi \geq \varphi_s) d\varphi.
$$

The overall average price can then be written as $\bar{p} = \frac{N_n}{N} \bar{p}_n + \frac{N_o}{N} \bar{p}_o$.

For the market shares, we know from section 2.1.1 that the market share density of producer $i$ is given by $\sigma_i = \gamma \mu_i$. For a firm with productivity $\varphi$ and offshoring status $s$, we write the market share density as $\sigma_s(\varphi) = \gamma \mu_s(\varphi)$: given the offshoring status $s$, more productive firms charge higher markups, and have larger market shares. Integrating the previous expression over all firms with the same status, we obtain that the total market share of firms with offshoring status $s$ is given by $\sigma_s = \gamma N_s \bar{\mu}_s$, for $s \in \{n, o\}$. It must be the case that $\sigma_n + \sigma_o = 1$.

### 2.4 Free-Entry Condition

As in Melitz (2003), firms enter as long as the expected value of entry is no less than the sunk entry cost, $f_E$.

A potential entrant knows that the expected profit for a firm with productivity $\varphi$ for its first period of existence is

$$
\bar{\pi}(\varphi) = (1 - \Lambda(\varphi)) \pi_n(\varphi) + \Lambda(\varphi) \{ \pi_o(\varphi) - E(\psi | \psi \leq \Psi(\varphi)) (\pi_n(\varphi) + f_o) \},
$$

which is a weighted average between the non-offshoring profits and the offshoring profits minus the expected adjustment cost, with the weights determined by the offshoring probability, $\Lambda(\varphi)$. Taking into account the exogenous death shock at the end of every period, the potential entrant also knows that the expected profit of a firm with productivity $\varphi$ for its $t$th period of existence is given by

$$
\bar{\pi}^t(\varphi) = (1 - \delta)^{t-1} \left\{ (1 - \Lambda(\varphi))^{t-1} \bar{\pi}(\varphi) + \left[ 1 - (1 - \Lambda(\varphi))^{t-1} \right] \pi_o(\varphi) \right\},
$$

where the first term inside the brackets accounts for the expected profit at time $t$ if the firm is not yet offshoring by $t - 1$, while the second term accounts for the profit the firm receives at $t$ if it offshores by $t - 1$. Of course, $\bar{\pi}^1(\varphi) = \bar{\pi}(\varphi)$. Hence, given the productivity distribution of new entrants (with probability density function $g(\varphi)$), a potential entrant’s expected value of entry is given by $\bar{\pi}_E = \int_{\varphi_o}^{\infty} \left\{ \sum_{t=1}^{\infty} \bar{\pi}^t(\varphi) \right\} g(\varphi) d\varphi$. Substituting the expressions for $\bar{\pi}^t(\varphi)$ and
\( \bar{\pi}(\varphi), \bar{\pi}_E \) can be written as

\[
\bar{\pi}_E = \int_{\varphi_o}^{\infty} \frac{1}{\delta + (1 - \delta)\Lambda(\varphi)} \left\{ (1 - \Lambda(\varphi))\pi_n(\varphi) + \Lambda(\varphi) \left[ \frac{\pi_o(\varphi)}{\delta} - E(\psi | \psi \leq \Psi(\varphi)) (\pi_n(\varphi) + f_o) \right] \right\} g(\varphi) d\varphi.
\]

(15)

Using equations (11), (12), and (13), I can rewrite equation (15) in terms of \( h_o(\varphi) \) and \( h_n(\varphi) \). Hence, the free-entry condition, \( \bar{\pi}_E = f_E \), is given by

\[
(1 - \bar{\Gamma}) \int_{\varphi_o}^{\infty} \frac{\pi_n(\varphi)}{\delta} h_n(\varphi) d\varphi + \bar{\Gamma} \int_{\varphi_o}^{\infty} \left[ \frac{\pi_o(\varphi)}{\delta} - E(\psi | \psi \leq \Psi(\varphi)) (\pi_n(\varphi) + f_o) \right] h_o(\varphi) d\varphi = f_E.
\]

(16)

Note that the expression for \( \bar{\pi}_E \) in the free-entry condition presents the value of entry as a weighted average between a potential entrant’s lifetime expected profits if it never offshores, and the expected lifetime offshoring profits minus the one-time adjustment cost, with the weights determined by the steady-state proportion of offshoring firms, \( \bar{\Gamma} \).

This concludes the model. After obtaining \( \Psi(\varphi) \) and \( \Lambda(\varphi) \), we use equations (8) and (16) to solve for \( \varphi_o \) and \( \varphi_n \). Once we obtain the cutoff productivity levels, we can solve for the rest of the variables: average markups, average prices, average productivities, market shares, \( N_P \), \( N_n \), \( N_o \), and \( N \).

3 Competitive Environment and Offshoring Likelihood

In this section, I look into the model’s implications for the effect of changes in the competitive environment on firms’ offshoring decisions. I analyze changes in the competitive environment by altering the values of three parameters: the degree of product substitutability (\( \gamma \)), the degree of preference for differentiated goods (\( \eta \)), and the fixed component driver of offshoring adjustment costs (\( f_o \)). I also look at the model’s responses for changes in the cost of making foreign labor compatible with the domestic firms’ production process (\( \tau_o \)).

3.1 The Schumpeterian and Escape-Competition Effects

The net effect of a change in the competitive environment on a firm’s offshoring likelihood is the result of two opposing forces. Using the terminology of Aghion et al. (2005), I refer to the negative effect of competition in the offshoring probability as the Schumpeterian effect, and to its positive effect as the escape-competition effect. The magnitude of these effects varies ac-
cording to each firm’s productivity, but it is possible to separate out the set of firms for which the Schumpeterian effect dominates, from the set of firms for which the escape-competition effect dominates.

Intuitively, the Schumpeterian effect refers to the cleansing effect of competition, which limits the ability of the least productive firms to improve their cost structure by offshoring. This effect exists if and only if a more competitive environment increases the cutoff level $\varphi_o$, as this implies a negative effect on expected offshoring profits. A dominant Schumpeterian effect drives the offshoring probability to zero for some of the firms (those with productivities between the old and new $\varphi_o$), and a decline in this probability for others.

On the other hand—and along the same lines of the competition and innovation story of Aghion et al. (2005)—the escape-competition effect refers to the impact of increased competition on the incremental profits from offshoring. That is, although a tougher competitive environment might decrease both non-offshoring and offshoring profits, the gap between them might increase for some firms, causing an increase in these firms’ offshoring likelihood. Intuitively, for these firms offshoring becomes more attractive as a mean to “escape” competition.

I characterize a tougher competitive environment for differentiated-good firms as a scenario where any of the following happens: (i) an increase in the parameter of substitutability for differentiated goods, $\gamma$; (ii) a increase in the preference parameter for differentiated goods (versus the homogeneous good), $\eta$; and (iii) a decrease in the fixed component (unrelated to productivity) driver of offshoring adjustment costs, $f_o$. The second case is equivalent to an increase in total expenditure in differentiated goods, $I$, as we know from section 2.1.1 that $\eta = I$.

Given that all the model’s information is contained in the cutoff productivity levels, it is useful to define a term that captures how the cutoff levels respond to each of the parameters in the previous four cases. Let $\zeta_{\varphi,s, b}$ denote the elasticity of $\varphi_s$ with respect to parameter $b$, for $s \in \{n, o\}$. Taking the natural logarithm of equation (8), we have that $\ln \varphi_o = \ln W_o + \ln \varphi_n$, where from section 2.1.2 we know that $W_o = 1 - \alpha + \alpha \tau_o W^*$. As $W_o$ is independent of $\gamma$, $\eta$, and $f_o$, if follows that the elasticity of $\varphi_o$ with respect to each of these parameters is identical to the elasticity of $\varphi_n$; that is, $\zeta_{\varphi,o, \ell} = \zeta_{\varphi,n, \ell}$ if $\ell \in \{\gamma, \eta, f_o\}$. We can simplify the notation and refer to $\zeta_{\ell}$ as the elasticity of any of the cutoff levels with respect to $\ell$, for $\ell \in \{\gamma, \eta, f_o\}$. The
following proposition describes the model’s implications for the effects of competition on a firm’s offshoring likelihood.

**Proposition 4 (Changes in the competitive environment and offshoring likelihood)**

1. If $f_0 > 0$ and $\zeta_\omega > 0$, for $\omega \in \{\gamma, \eta\}$, we have that for an increase in competition driven by an increase in $\omega$:

   (i) There exists a unique productivity level, $\varphi_\omega^*$, that separates non-offshoring firms according to the dominant competition effect.

   (ii) For a non-offshoring firm with productivity $\varphi$, the Schumpeterian effect dominates if $\varphi < \varphi_\omega^*$, while the escape-competition effect dominates if $\varphi > \varphi_\omega^*$.

2. If $f_0 > 0$ and $\zeta_{f_0} < 0$, and after replacing $\omega$ by $f_0$, statements (i) and (ii) from part 1 hold for an increase in competition driven by a decrease in $f_0$.

These results differ drastically from the implications of a model with fixed costs of offshoring in which only the most productive firms offshore. In such a model, a tougher competitive environment would shift up the offshoring cutoff productivity level and we would only obtain a crude version of the Schumpeterian effect: the probability of offshoring drops from 1 to 0 for the firms between the old and new cutoff level.

There are three conditions in Proposition 4 with respect to the elasticities of the cutoff productivity levels: $\zeta_\gamma > 0$, $\zeta_\eta > 0$, and $\zeta_{f_0} < 0$. Given that I am not assuming any particular distributions for $\varphi$ and $\psi$ (the random adjustment factor), it is not straightforward to derive precise requirements to satisfy these conditions. In my simulations in the following sections, I prove numerically that this is indeed the case for a Pareto distribution of productivity—widely used in heterogeneous-firm models—and a uniform distribution for the random adjustment factor.

Before introducing the numerical examples, we can also look at the response of the model to changes in the cost of making foreign labor compatible with the domestic production process, $\tau_o$. In this case we have that $W_o$ depends on $\tau_o$. In particular, a decline in $\tau_o$ decreases $W_o$, so that the gap between the non-offshoring and offshoring marginal costs widens. Given this relationship, the elasticities of the cutoff rules ($\varphi_o$ and $\varphi_n$) with respect to $\tau_o$ are not equivalent. Specifically, we get $\zeta_{\varphi_o, \tau_o} = \frac{\alpha\tau_o W_o^*}{W_o} + \zeta_{\varphi_n, \tau_o}$. The following proposition describes the relationship between $\tau_o$ and a firm’s offshoring likelihood.
Proposition 5 (Changes in $\tau_o$ and offshoring likelihood)

If $f_o > 0$ and $\zeta_{\varphi_o, \tau_o} \in \left(0, \frac{\alpha \tau_o W^*}{W_o}\right)$, after a decrease in $\tau_o$, there is an increase in the probability of offshoring for every non-offshoring firm above the new $\varphi_o$.

If $\tau_o$ declines and $\zeta_{\varphi_o, \tau_o} \in \left(0, \frac{\alpha \tau_o W^*}{W_o}\right)$, $\varphi_o$ declines and $\varphi_n$ increases. As offshoring becomes more attractive, some low productive non-offshoring firms whose offshoring likelihood was zero before the change, now start considering offshoring. But also, competition from new offshoring firms causes some non-offshoring firms to stop producing (those between the old and new $\varphi_n$). These firms, however, are also more likely to offshore after the change, as they try to escape competition. As with Proposition 4, the condition $\zeta_{\varphi_o, \tau_o} \in \left(0, \frac{\alpha \tau_o W^*}{W_o}\right)$ is satisfied in my numerical simulations in the following section.

3.2 The Benchmark Case

For the benchmark steady state, I set parameter values that ensure an interior solution that gives enough freedom of movement to our parameters of interest. Although this is not a calibration exercise, for most of the model’s parameters I use values that are common in calibrated models.

I assume that firm productivity is Pareto distributed in the interval $[\varphi_{\text{min}}, \infty)$ with shape parameter $k > 1$ (a high $k$ implies low heterogeneity, with firms clustering near the lower bound, $\varphi_{\text{min}}$). Following Ghironi and Melitz (2005), I set $\varphi_{\text{min}}$ to 1, and $k$ to 3.5 (close to their 3.4 value). I set the parameter of substitutability among varieties, $\gamma$, at 2, which is the value used by Bergin and Feenstra (2000, 2001). As mentioned in section 2.1.1, the domestic wage is 1. The parameter of preference for differentiated goods, $\eta$, is set at 0.75. As $\eta$ is equivalent to the total expenditure in differentiated goods, $I$, the assumed value implies that the representative household spends 75% of its income on differentiated goods. I assume $\alpha = 0.5$, which means that every offshoring firm ships abroad half of its production process. The wage in the foreign country, $W^*$, is set at 0.5. I set $\tau_o$ at 1.4, so that an offshoring firms must hire 1.4 units of foreign labor to produce the same amount as one unit of domestic labor. The values for $\alpha$, $W^*$, and $\tau_o$ imply that $W_o = 0.85$—the marginal cost for each firm is 15% lower if it offshores. Regarding the offshoring adjustment costs, I assume that the adjustment factor, $\psi$, follows an uniform distribution in the interval $[0, \psi_{\text{max}}]$. The upper bound, $\psi_{\text{max}}$, must be large enough so that—combined with $f_o$—offshoring is not overly easy. Thus, I set $\psi_{\text{max}} = 30$ and
\( f_o = 0.05 \). Without any loss of generality, the value of the sunk entry cost, \( f_E \), is set at a level that allows for an equilibrium offshoring cutoff level, \( \varphi_o \), well above \( \varphi_{\text{min}} \). I choose \( f_E = 0.25 \).

Finally, consistent with U.S. evidence on annual job destruction (see Davis, Faberman and Haltiwanger, 2006), I set the death rate, \( \delta \), at 0.1. Table 1 presents the solution.

**Table 1: Solution for Benchmark Steady State**

<table>
<thead>
<tr>
<th>Productivity levels</th>
<th>Prices</th>
<th>Composition of Firms</th>
<th>Markups and Shares</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \varphi_o )</td>
<td>1.122</td>
<td>( \hat{p} ) 0.757</td>
<td>( N ) 2.723</td>
</tr>
<tr>
<td>( \varphi_n )</td>
<td>1.320</td>
<td>( \bar{p} ) 0.638</td>
<td>( N_o ) 1.474</td>
</tr>
<tr>
<td>( \bar{\varphi}_o )</td>
<td>1.734</td>
<td>( \bar{\bar{p}}_o ) 0.621</td>
<td>( N_n ) 1.249</td>
</tr>
<tr>
<td>( \bar{\varphi}_n )</td>
<td>1.869</td>
<td>( \bar{\bar{p}}_n ) 0.658</td>
<td></td>
</tr>
</tbody>
</table>

From Table 1 we see that the average productivity levels follow the same order as the cutoff levels: \( \varphi_o < \varphi_n \) and \( \bar{\varphi}_o < \bar{\varphi}_n \). Hence, contrary to a heterogeneous-firm model with fixed costs of offshoring, in this model it is possible for the average productivity of offshoring firms to be lower than the average productivity of non-offshoring firms. This result can be reversed (i.e. obtain \( \varphi_o < \varphi_n \) and \( \bar{\varphi}_o > \bar{\varphi}_n \)) for larger levels of \( f_o \), as the offshoring adjustment cost unrelated to the firm’s productivity level, \( \psi f_o \), is the main driver in the offshoring decision of low productivity firms.

Even though the average productivity of offshoring firms is lower in the benchmark steady state, their average price is lower than the non-offshoring firms’ average price. Still, even with lower prices, offshoring firms manage to obtain higher average markups than non-offshoring firms. Finally, note that although offshoring firms \( (N_o) \) represent about 54.1% of the total mass of producing firms \( (N) \), their market share is 61.6%. That is, offshoring firms capture a larger part of the market through their low prices, limiting the number of competitors.

Propositions 4 and 5 are written in terms of the response of the probability of offshoring for non-offshoring firms, \( \Lambda(\varphi) \). Figure 2 shows \( \Lambda(\varphi) \) in the benchmark steady state, along with the proportion of offshoring firms for each level of productivity \( \varphi \), \( \Gamma(\varphi) \). Note that \( \Gamma(\varphi) \) looks like a scaled function of \( \Lambda(\varphi) \). Indeed, deriving the equation for \( \Gamma(\varphi) \) in (11), we get that \( \text{sgn}(\Gamma'(\varphi)) = \text{sgn}(\Lambda'(\varphi)) \). Non-offshoring firms with productivity close to 1.74 have the greatest offshoring probability each period (about 11%), and among all the firms with that level of productivity, 55.5% are offshoring.

Although from Figure 2 we know the proportion of offshoring firms for each level of pro-
ductivity, it does not show the exact distribution of offshoring firms. In equation (12) in section 2.1.2, I obtained the probability density function of offshoring firms, \( h_o(\phi) \), as a function of not only \( \Gamma(\phi) \), but also \( g(\phi) \). If \( g(\phi) \) is the density function of a power-law distribution, like the Pareto distribution in this numerical example, \( \Gamma(\phi) \) and \( h_o(\phi) \) can be substantially different.\(^8\) Figure 3 presents \( h_o(\phi) \), along with \( H_o(\phi) \) (the cumulative distribution function), and shows that this is indeed the case.

Figure 3a shows that, under a Pareto distribution of productivity, the implications of this model are very far from a story in which only the most productive firms offshore. The peak of \( h_o(\phi) \) is very close to the cutoff rule for non-offshoring firms. The distribution is positively skewed: most offshoring firms are clustered towards the left of the distribution. In Figure 3b we can see, for example, that (i) about 17.5% of the total mass of offshoring firms would not be producing if they were not offshoring, and (ii) 50% of offshoring firms have productivities below 1.55—well below the average productivity levels in Table 1.

In the following sections we analyze deviations from the benchmark steady state.

---

\(^8\)The Pareto distribution is very popular in heterogeneous-firm models, as it fits the distribution of firms in the U.S. and Europe (see Chaney, 2008).
3.3 Simulations

Following Propositions 4 and 5, I separately consider the model’s responses to an increase in the parameter of substitutability, $\gamma$, and to a decline in the foreign-labor-compatibility cost, $\tau_o$. I leave the analysis of the model’s responses to changes in $\eta$ and $f_o$ for Appendix B (online), as the results are very similar to the effects of $\gamma$.

I consider an increase in $\gamma$ from 2 to 2.5, and a decline in $\tau_o$ from 1.4 to 1. For the second case, it implies that $W_o$ declines from 0.85 to 0.75—so that the difference in marginal costs between non-offshoring and offshoring firms rises from 15% to 25%. Table 2 presents the solution for each case, along with the benchmark solution for comparison purposes.

An increase in $\gamma$—due, for example, to a change in tastes that causes consumers to perceive less product differentiation among the varieties—makes it harder for low productivity firms (who also charge higher prices) to survive. The tougher competitive environment is reflected in higher cutoff (and average) productivity levels. Every surviving cuts its price and then, the average prices of both offshoring and non-offshoring firms declines. The market becomes more concentrated ($N$ declines), and the percentage of offshoring firms increases (along with their market share, $\sigma_o$). The average markups of producing firms remains about
Table 2: Model’s Responses to Changes in $\gamma$ and $\tau_o$

<table>
<thead>
<tr>
<th></th>
<th>Benchmark</th>
<th>Increase in $\gamma$ (from 2 to 2.5)</th>
<th>Decline in $\tau_o$ (from 1.4 to 1)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Productivity</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\phi_o$</td>
<td>1.122</td>
<td>1.236</td>
<td>1.039</td>
</tr>
<tr>
<td>$\phi_n$</td>
<td>1.320</td>
<td>1.454</td>
<td>1.386</td>
</tr>
<tr>
<td>$\Phi_o$</td>
<td>1.734</td>
<td>1.888</td>
<td>1.652</td>
</tr>
<tr>
<td>$\Phi_n$</td>
<td>1.869</td>
<td>2.078</td>
<td>2.008</td>
</tr>
<tr>
<td><strong>Prices</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\hat{p}$</td>
<td>0.757</td>
<td>0.688</td>
<td>0.722</td>
</tr>
<tr>
<td>$\bar{p}$</td>
<td>0.638</td>
<td>0.579</td>
<td>0.593</td>
</tr>
<tr>
<td>$\bar{p}_o$</td>
<td>0.621</td>
<td>0.567</td>
<td>0.584</td>
</tr>
<tr>
<td>$\bar{p}_n$</td>
<td>0.658</td>
<td>0.595</td>
<td>0.622</td>
</tr>
<tr>
<td><strong>Composition of firms</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\bar{N}$</td>
<td>2.723</td>
<td>2.172</td>
<td>2.398</td>
</tr>
<tr>
<td>$N_o$</td>
<td>1.474</td>
<td>1.253</td>
<td>1.811</td>
</tr>
<tr>
<td>$N_n$</td>
<td>1.249</td>
<td>0.919</td>
<td>0.587</td>
</tr>
<tr>
<td>$\bar{N}/\bar{N}_o$</td>
<td>0.541</td>
<td>0.577</td>
<td>0.755</td>
</tr>
<tr>
<td><strong>Markups and shares</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\bar{\mu}_o$</td>
<td>0.209</td>
<td>0.203</td>
<td>0.223</td>
</tr>
<tr>
<td>$\bar{\mu}_n$</td>
<td>0.154</td>
<td>0.158</td>
<td>0.164</td>
</tr>
<tr>
<td>$\sigma_o$</td>
<td>0.616</td>
<td>0.637</td>
<td>0.808</td>
</tr>
<tr>
<td>$\sigma_n$</td>
<td>0.384</td>
<td>0.363</td>
<td>0.192</td>
</tr>
</tbody>
</table>

A decline in $\tau_o$—due, for example, to improvements in training and/or education of foreign workers—increases the relative cost of domestic labor, and hence the gap between $\phi_o$ and $\phi_n$. As it is easier to offshore, $\phi_o$ declines. To satisfy the free entry condition, the increase in the expected profits from offshore must be canceled out by a decrease in the expected non-offshore profits; thus, $\phi_n$ increases. Prices respond in the same direction as with an increase in $\gamma$. There is also a decrease in the mass of producing firms, $N$, but—given that the mass of offshore firm increases—this is driven by the disappearance of non-offshore firms. As expected, the market share and average markup of offshore firms increase. As a result of change in the distribution of non-offshore firms, their average markup also increases.

Propositions 4 and 5 are in terms of the probability of offshore, $\Lambda(\phi)$. Figure 4 shows the changes in the probability of offshore for our two cases, with $\Lambda(\phi)$ denoting the benchmark steady state, and $\Lambda(\phi)'$ denoting the new steady state. Figure 4a provides a graphical descrip-
tion of Proposition 4. After the change in $\gamma$, the productivity level $\varphi_\gamma^*$ separates the firms for which the Schumpeterian effect dominates, from those for which the escape-competition effect dominates. On the other hand, Figure 4b is a graphical description of Proposition 5. After a decline in $\tau_o$, the offshoring probability increases for every non-offshoring firm with productivity above the new (and lower) offshoring cutoff level, $\varphi'_o$.

Finally, Figure 5 shows the changes in the distribution of offshoring firms. I denote the new steady state probability density and cumulation distribution functions by $h_o(\varphi)'$ and $H_o(\varphi)'$, respectively. As expected, the distribution of offshoring firms shifts to the right after an increase in $\gamma$, and to the left after a decline in $\tau_o$.

To summarize, in both cases the market concentration increases, average prices decline, and the market share of offshoring firms increases. In the first case ($\gamma$ increases), the mass of offshoring firms declines and their average productivity rises. On the other hand, in the second case ($\tau_o$ declines), the mass of offshoring firms increases and their average productivity declines, as most of the new offshoring firms have low productivity levels.

4 Conclusion

This paper shows that more productive firms are not necessarily more likely to offshore than less productive firms. Moreover, if productivity follows a power-law distribution, most offshoring firms have productivity levels below the mean: it will not be true that only the most productive firms offshore.

Although this model refers to the firm's offshoring decision, it can be applied to different settings. In particular, the model is also relevant for the recent literature on innovation with heterogeneous firms. Indeed, the model in the previous pages can be rewritten in terms of innovation just by changing the production function of a firm with productivity $\varphi$ to $y_s(\varphi) = Z_s\varphi L$, where $Z_s$ represents the aggregate productivity factor under technology status $s$, for $s\{n,o\}$. We can relabel $n$ as “normal” technology and $o$ as “outstanding” technology. Replacing the word “offshoring” by “innovation” and some other minor changes, the rest of the model and its implications would be identical.

The model can be easily extended to a multi-country framework to answer questions

11See, for example, Atkeson and Burstein (2010) and Costantini and Melitz (2008).
(a) After an increase in $\gamma$

(b) After a decline in $\tau_o$

Figure 4: Changes in the probability of offshoring
Figure 5: Changes in the distribution of offshoring firms
about the effects of trade liberalization on offshoring or innovation decisions. We could also impose more sophisticated offshoring structures as in Grossman and Rossi-Hansberg (2008), with an endogenous determination of the fraction of the production process being offshored.
A Appendix: Proofs

Proof of Proposition 1. The variable $\Psi(\varphi)$ denotes the value for $\psi$ that makes a firm indifferent between offshoring or not. In the Bellman equation (9), this implies that

$$\frac{\pi_o(\varphi)}{\delta} - \Psi(\varphi) (\pi_n(\varphi) + f_o) = \pi_n(\varphi) + (1 - \delta)E[V(\varphi, \psi')] .$$

Solving for $\Psi(\varphi)$ we obtain

$$\Psi(\varphi) = \frac{1}{\pi_n(\varphi) + f_o} \left( \frac{\pi_o(\varphi)}{\delta} - \pi_n(\varphi) \right) - \frac{1 - \delta}{\pi_n(\varphi) + f_o} E[V(\varphi, \psi')] . \tag{A-1}$$

Given $\Psi(\varphi)$, we can rewrite the value function as

$$V(\varphi, \psi) = \begin{cases} \frac{\pi_o(\varphi)}{\delta} - \psi (\pi_n(\varphi) + f_o) & \text{if } \psi \leq \Psi(\varphi) \\ \frac{\pi_o(\varphi)}{\delta} - \Psi(\varphi) (\pi_n(\varphi) + f_o) & \text{if } \psi > \Psi(\varphi) . \end{cases}$$

From this expression, we can then get that

$$E[V(\varphi, \psi')] = \frac{\pi_o(\varphi)}{\delta} - E[\min \{ \psi', \Psi(\varphi) \}] (\pi_n(\varphi) + f_o) . \tag{A-2}$$

Plugging in equation (A-2) into equation (A-1), we find that

$$\Psi(\varphi) = z(\varphi) + (1 - \delta)E[\min \{ \psi', \Psi(\varphi) \}] , \tag{A-3}$$

where $z(\varphi) = \frac{\pi_o(\varphi) - \pi_n(\varphi)}{\pi_n(\varphi) + f_o}$. Note that $z(\varphi) \geq 0$, as $\pi_o(\varphi) \geq \pi_n(\varphi)$ for every $\varphi$.

Let us now show that

$$E[\min \{ \psi', \Psi(\varphi) \}] = \Psi(\varphi) - \int_0^{\Psi(\varphi)} F(\psi) d\psi;$$

$$= \Psi(\varphi) - \int_0^{\Psi(\varphi)} \left[ \Psi(\varphi) - E[\psi'|\psi' \leq \Psi(\varphi)] \right] F(\psi) d\psi$$

$$= \Psi(\varphi) - \int_0^{\Psi(\varphi)} \left[ \Psi(\varphi) - \int_0^{\Psi(\varphi)} (\Psi(\varphi) - \psi) F(\psi) d\psi \right] F(\psi) d\psi$$

$$= \Psi(\varphi) - \int_0^{\Psi(\varphi)} F(\psi) d\psi .$$

Substituting the previous expression into equation (A-3) we obtain that the cutoff adjustment factor solves the equation

$$\Psi(\varphi) = \frac{z(\varphi)}{\delta} - \frac{1 - \delta}{\delta} \int_0^{\Psi(\varphi)} F(\psi) d\psi . \tag{A-4}$$

To show that the solution is unique, let

$$G(\Psi(\varphi)) = \Psi(\varphi) + \frac{1 - \delta}{\delta} \int_0^{\Psi(\varphi)} F(\psi) d\psi - \frac{z(\varphi)}{\delta} , \tag{A-5}$$

so that $G(\Psi(\varphi)) = 0$ is equivalent to equation (A-4). We know that $\Psi(\varphi) \in [0, \infty)$ and from equation (A-5) we obtain that $G(0) = -\frac{z(\varphi)}{\delta} \leq 0$. Note also that $G(\Psi(\varphi)) \to \infty$ as $\Psi(\varphi) \to \infty$. Therefore, given that $G(\Psi(\varphi))$ is continuous, there is at least one solution for $G(\Psi(\varphi)) = 0$ in the interval $[0, \infty)$. Using Leibniz’s rule, we get $G'(\Psi(\varphi)) = 1 + \frac{1 - \delta}{\delta} F(\Psi(\varphi)) > 0$ for every $\Psi(\varphi)$.
Hence, as \( G(\Psi(\varphi)) \) is strictly increasing, the solution must be unique. ■

**Proof of Proposition 2.** For part 1, note first that \( \pi_n(\varphi) = \pi_o(\varphi) = 0 \) for \( \varphi \leq \varphi_o \). Then, \( z(\varphi) = 0 \) if \( \varphi \leq \varphi_o \). From equation (A-4), note that if \( z(\varphi) = 0 \), the equilibrium \( \Psi(\varphi) \) solves the equation

\[
\Psi(\varphi) = -\frac{1 - \delta}{\delta} \int_0^{\Psi(\varphi)} F(\psi)d\psi.
\]

As \( \Psi(\varphi) \geq 0 \) and \(-\frac{1 - \delta}{\delta} \int_0^{\Psi(\varphi)} F(\psi)d\psi \leq 0\), it follows that the solution is \( \Psi(\varphi) = 0 \). As \( \psi \) is a continuous random variable in the interval \([0, \infty)\), it must be the case that \( F(0) = 0 \). Therefore, \( \Lambda(\varphi) = F(\Psi(\varphi)) = 0 \) if \( \varphi \leq \varphi_o \).

As \( \Lambda(\varphi) = 0 \) when \( z(\varphi) = 0 \), to prove that \( \Lambda(\varphi) \to 0 \) as \( \varphi \to \infty \), it is enough to show that \( z(\varphi) \to 0 \) as \( \varphi \to \infty \). Note that we can rewrite \( z(\varphi) \) as

\[
z(\varphi) = \frac{\pi_o(\varphi) - 1}{1 + \pi_o(\varphi)}.
\]

The limit of \( \pi_s(\varphi) \), for \( s \in \{n, o\} \), as \( \varphi \to \infty \) is given by

\[
\lim_{\varphi \to \infty} \pi_s(\varphi) = \lim_{\varphi \to \infty} \left[ \Omega \left( \frac{\varphi}{\varphi_s} e \right) - 2 + \frac{1}{\Omega \left( \frac{\varphi}{\varphi_s} e \right)} \right] \gamma I = \infty.
\]

Hence, using L’Hôpital’s rule we can write the limit of \( z(\varphi) \) as

\[
\lim_{\varphi \to \infty} z(\varphi) = \lim_{\varphi \to \infty} \frac{\pi_o(\varphi)}{\pi_n(\varphi)} - 1.
\]

We then get

\[
\lim_{\varphi \to \infty} \frac{\pi_o(\varphi)}{\pi_n(\varphi)} = \lim_{\varphi \to \infty} \frac{1 - \frac{1}{\Omega \left( \frac{\varphi}{\varphi_o} e \right)}}{1 - \frac{1}{\Omega \left( \frac{\varphi}{\varphi_o} e \right)}} = 1,
\]

so that \( \lim_{\varphi \to \infty} z(\varphi) = 0 \).

For part 2, note that \( \Lambda'(\varphi) = f(\Psi(\varphi))\Psi'(\varphi) \), where \( f(\cdot) \) is the probability density function for \( \psi \). For \( \Psi'(\varphi) \), we derive equation (10) with respect to \( \varphi \) and use Leibniz’s rule to get \( \Psi'(\varphi) = z'(\varphi) \delta + (1 - \delta) F(\Psi(\varphi)) \). Hence,

\[
\Lambda'(\varphi) = \frac{f(\Psi(\varphi))z'(\varphi)}{\delta + (1 - \delta) \Lambda(\varphi)}.
\]

Given that \( f(\Psi(\varphi)) \) and the denominator are both positive, it is the case that the sign of \( \Lambda'(\varphi) \) is identical to the sign of \( z'(\varphi) \). I focus then on \( z'(\varphi) \).

In the interval \((\varphi_o, \varphi_n)\), \( \pi_n(\varphi) = 0 \) so that \( z(\varphi) = \frac{\pi_o(\varphi)}{f_o} \). Thus, with \( f_o > 0 \) so that \( z(\varphi) \) is finite, we have

\[
z'(\varphi) = \frac{\pi_o'(\varphi)}{f_o} = \frac{1}{f_o \varphi} \left[ \frac{\mu_o(\varphi)}{1 + \mu_o(\varphi)} \right] \gamma I \quad \text{(A-6)}
\]

for \( \varphi \in (\varphi_o, \varphi_n) \). As \( \mu_o(\varphi) > 0 \) for \( \varphi > \varphi_o \), it follows that \( z'(\varphi) > 0 \) for \( \varphi \in (\varphi_o, \varphi_n) \). Therefore, \( \Lambda(\varphi) \) is strictly increasing in the interval \((\varphi_o, \varphi_n)\), so that a maximum for \( \Lambda(\varphi) \) cannot exist in that region. Hence, if the maximum for \( \Lambda(\varphi) \) exists, it must be in the region where \( \varphi \geq \varphi_n \). I will prove that this is the case.
If \( \varphi \geq \varphi_n \), we get

\[
z'(\varphi) = \left[ \frac{(\mu_o(\varphi) - \mu_n(\varphi)) (1 + \mu_n(\varphi)) \gamma I}{\varphi [\mu_n(\varphi)^2 \gamma I + (1 + \mu_n(\varphi)) f_o]^2 (1 + \mu_o(\varphi))} \right] [f_o - \mu_o(\varphi) \mu_n(\varphi) \gamma I]. \tag{A-7}
\]

Note that if \( \varphi = \varphi_n \) (so that \( \mu_n(\varphi) = 0 \)), equation (A-7) collapses to equation (A-6). As \( \mu_o(\varphi) > \mu_n(\varphi) \) for every \( \varphi \in [\varphi_n, \infty) \), the first term in brackets is always positive. The second term in brackets gives the sign of \( z'(\varphi) \) and in particular, it determines the value of \( \varphi \) that maximizes \( z(\varphi) \)—and hence \( \Lambda(\varphi) \). Letting \( \hat{\varphi} \) denote the argument that maximizes \( z(\varphi) \), it follows that \( \hat{\varphi} \) solves the equation

\[
f_o - \mu_o(\hat{\varphi}) \mu_n(\hat{\varphi}) \gamma I = 0. \tag{A-8}
\]

To show that this is indeed a maximum and that is unique, note that \( \mu_o(\varphi) \mu_n(\varphi) \) is strictly increasing in the interval \([\varphi_n, \infty)\) because \( \mu'_s(\varphi) > 0 \), for \( s \in \{n, o\} \). Hence, \( z'(\varphi) > 0 \) if \( \varphi \in [\varphi_n, \hat{\varphi}] \), and \( z'(\varphi) < 0 \) if \( \varphi \in (\hat{\varphi}, \infty) \). Note also that given \( \varphi_n \) and \( \varphi_o \), a lower \( f_o \) implies a lower \( \hat{\varphi} \) (so that \( \mu_o(\hat{\varphi}) \mu_n(\hat{\varphi}) \gamma I \) is smaller). As \( f_o \) approaches zero, it follows that \( \mu_n(\hat{\varphi}) \) must get closer to zero. That is, \( \hat{\varphi} \to \varphi_n \) from the right.

For part 3, note that if \( f_o = 0 \), \( z(\varphi) = \frac{\pi_n(\varphi)}{\pi_n(\varphi)} - 1 \). Then, \( z(\varphi) \to \infty \) if \( \varphi \in (\varphi_o, \varphi_n] \) (because \( \pi_n(\varphi) \) equals zero). From equation (A-4), it follows that it must be the case that \( \Psi(\varphi) \to \infty \) in this interval. Then, \( \Lambda(\varphi) = 1 \) if \( f_o = 0 \) and \( \varphi \in (\varphi_o, \varphi_n] \). On the other hand, for \( \varphi > \varphi_n \), we substitute \( f_o = 0 \) in equation (A-7) to get

\[
z'(\varphi) = -\frac{(\mu_o(\varphi) - \mu_n(\varphi)) (1 + \mu_n(\varphi)) \mu_o(\varphi)}{\varphi \mu_o(\varphi)^2 (1 + \mu_o(\varphi))}. \tag{A-9}
\]

As \( \mu_o(\varphi) > \mu_n(\varphi) \), equation (A-9) is always negative. That is, \( \Lambda(\varphi) \) is strictly decreasing if \( f_o = 0 \) and \( \varphi > \varphi_n \). \( \blacksquare \)

**Proof of Proposition 3.** In section 2.1.1, we obtained that \( \ln p_i = \ln \hat{p} - \mu_i \). Then, for a firm with productivity \( \varphi \) and offshoring status \( s \), for \( s \in \{n, o\} \), we have that \( \ln p_s(\varphi) = \ln \hat{p} - \mu_s(\varphi) \). Hence, the average log price of firms with offshoring status \( s \) is given by \( \ln \bar{p}_s = \ln \hat{p} - \bar{\mu}_s \), where

\[
\bar{\mu}_s = \int_{\varphi_s}^{\infty} \mu_s(\varphi) h_s(\varphi \mid \varphi \geq \varphi_s) d\varphi
\]

is the average markup of this group of firms. We can then use the expressions for \( \ln \bar{p}_n \) and \( \ln \bar{p}_o \) in the overall average log price, \( \ln \bar{p} = \frac{N_n}{N} \ln \bar{p}_n + \frac{N_o}{N} \ln \bar{p}_o \), to get

\[
\ln \hat{p} - \ln \bar{p} = \frac{N_n}{N} \bar{\mu}_n + \frac{N_o}{N} \bar{\mu}_o. \tag{A-10}
\]

Now, from equation (3), we can solve for \( \ln \hat{p} - \ln \bar{p} \) as \( \ln \hat{p} - \ln \bar{p} = \frac{1}{\gamma N} \). Plugging in this result in equation (A-10) we get

\[
\frac{1}{\gamma} = N_n \bar{\mu}_n + N_o \bar{\mu}_o.
\]

Finally, substituting in the previous equations our expressions for \( N_n \) and \( N_o \) from section (2.3.2), \( N_n = (1 - H_n(\varphi_n))(1 - \Gamma) N_P \) and \( N_o = \Gamma N_P \), we solve for \( N_P \) as

\[
N_P = \frac{1}{\gamma \left[ (1 - \Gamma)(1 - H_n(\varphi_n))\bar{\mu}_n + \Gamma \bar{\mu}_o \right]}
\]

\(^{12}\text{This implies that as long as } f_o > 0 \text{, } z'(\varphi) \text{ is continuous.}
Proof of Proposition 4. For part 1, it is enough to work with the response of $\Lambda(\varphi)$ to $\gamma$, as the derivatives with respect to $I(\eta)$ are similar (we only need to replace $\gamma$ with $I$). Thus, we obtain $\frac{d\Lambda(\varphi)}{d\gamma}$ and derive the conditions that determine its sign.

Similar to the proof of Proposition 2, we obtain that

$$\frac{d\Lambda(\varphi)}{d\gamma} = \left[ \frac{f(\Psi(\varphi))}{\delta + (1 - \delta)\Lambda(\varphi)} \right] \frac{dz(\varphi)}{d\gamma}.$$

As the term in brackets is positive, we only need to focus on $\frac{dz(\varphi)}{d\gamma}$, where $z(\varphi) = \frac{\pi_o(\varphi) - \pi_n(\varphi)}{\pi_n(\varphi) + f_o}$ and $\pi_s(\varphi)$ is defined as in the beginning of section 2.2. Using the formula for the derivative of the Lambert function in footnote 3, we obtain

$$\frac{dz(\varphi)}{d\gamma} = \left[ \frac{(\mu_o(\varphi) - \mu_n(\varphi))I}{(\pi_n(\varphi) + f_o)(1 + \mu_n(\varphi))(1 + \mu_n(\varphi))} \right] \times [\mu_o(\varphi)\mu_n(\varphi)\gamma I \zeta_f - f_o(\mu_o(\varphi) + \mu_n(\varphi) + \mu_o(\varphi)\mu_n(\varphi)) - f_o \zeta_f],$$

where $\mu_s(\varphi) = 0$ if $\varphi \leq \varphi_n$, and is greater than zero otherwise, for $s \in \{n, o\}$. The first term in brackets is non-negative, and strictly positive as long as $\varphi > \varphi_o$. Then, for $\varphi > \varphi_o$, the sign of $\frac{dz(\varphi)}{d\gamma}$ is determined by the sign of $\Upsilon_1(\varphi) = \mu_o(\varphi)\mu_n(\varphi)\gamma I \zeta_f + f_o(\mu_o(\varphi) + \mu_n(\varphi) + \mu_o(\varphi)\mu_n(\varphi)) - f_o \zeta_f$.

Note that if $\zeta_f > 0$, $\Upsilon_1(\varphi) \rightarrow -f_o \zeta_f < 0$ as $\varphi \rightarrow \varphi_o$ from the right. Also $\Upsilon_1(\varphi) \rightarrow \infty$ as $\varphi \rightarrow \infty$ (because $\mu_s(\varphi) \rightarrow \infty$ as $\varphi \rightarrow \infty$ for $s \in \{n, o\}$). Therefore, given that $\Upsilon_1(\varphi)$ is continuous, there is at least one solution for $\Upsilon_1(\varphi) = 0$ in the interval $(\varphi_o, \infty)$. Given that $\mu_s(\varphi) > 0$ if $\varphi \geq \varphi_s$, for $s \in \{n, o\}$, it follows that $\Upsilon_1(\varphi)$ is strictly increasing in $\varphi$. Therefore, the solution to $\Upsilon_1(\varphi) = 0$, $\varphi_s$ is unique. Note that if $\varphi \in (\varphi_o, \varphi_s)$, then $\Upsilon_1(\varphi) < 0$ and $\frac{dz(\varphi)}{d\gamma} < 0$. On the other hand, if $\varphi > \varphi_s$, then $\Upsilon_1(\varphi) > 0$ and $\frac{dz(\varphi)}{d\gamma} > 0$.

For part 2, we also get that $\text{sgn} \left( \frac{d\Lambda(\varphi)}{d\gamma} \right) = \text{sgn} \left( \frac{dz(\varphi)}{d\gamma} \right)$. We follow similar steps to part 1 to obtain

$$\frac{dz(\varphi)}{df_o} = \left[ \frac{I(\varphi) - \mu_n(\varphi))I}{(\pi_n(\varphi) + f_o)(1 + \mu_n(\varphi))(1 + \mu_n(\varphi))f_o} \right] \times [\mu_o(\varphi)\mu_n(\varphi)\gamma I \zeta_f - f_o(\mu_o(\varphi) + \mu_n(\varphi) + \mu_o(\varphi)\mu_n(\varphi)) - f_o \zeta_f].$$

Similar to part 1, the sign of $\frac{dz(\varphi)}{df_o}$ is determined by $\Upsilon_2(\varphi) = \mu_o(\varphi)\mu_n(\varphi)\gamma I \zeta_f + f_o(\mu_o(\varphi) + \mu_n(\varphi) + \mu_o(\varphi)\mu_n(\varphi)) - f_o \zeta_f$.

Note that if $\zeta_f < 0$, $\Upsilon_2(\varphi) \rightarrow -f_o \zeta_f > 0$ as $\varphi \rightarrow \varphi_o$ from the right. Also $\Upsilon_2(\varphi) \rightarrow -\infty$ as $\varphi \rightarrow \infty$. Given that $\Upsilon_2(\varphi)$ is continuous, there is at least one solution for $\Upsilon_2(\varphi) = 0$ in the interval $(\varphi_o, \infty)$. Given that $\mu_s(\varphi) > 0$ if $\varphi \geq \varphi_s$, for $s \in \{n, o\}$, it follows that $\Upsilon_2(\varphi)$ is strictly decreasing in $\varphi$. Therefore, the solution to $\Upsilon_2(\varphi) = 0$, $\varphi_s$ is unique. Note that if $\varphi \in (\varphi_o, \varphi_f)$, then $\Upsilon_2(\varphi) > 0$ and $\frac{dz(\varphi)}{df_o} > 0$. On the other hand, if $\varphi > \varphi_f$, then $\Upsilon_2(\varphi) < 0$ and $\frac{dz(\varphi)}{df_o} < 0$. ■

Proof of Proposition 5. We need to prove that $\frac{d\Lambda(\varphi)}{df_o} < 0$. As before, it is the case that $\text{sgn} \left( \frac{d\Lambda(\varphi)}{d\gamma} \right) = \text{sgn} \left( \frac{dz(\varphi)}{df_o} \right)$. Following similar steps as in the proof of Proposition 4, and using
the fact that \( \zeta_{\varphi_n,\tau_0} = \zeta_{\varphi_0,\tau_0} - \frac{\alpha W^*\tau_0}{W_0} \), we get
\[
\frac{dz(\varphi)}{d\tau_0} = \left[ \frac{\gamma I}{(\pi_n(\varphi) + f_0)^2(1 + \mu_0(\varphi))(1 + \mu_n(\varphi))\tau_0} \right] \times \left[ \mu_0(\varphi)\mu_n(\varphi)\gamma I[\mu_0(\varphi)\zeta_{\varphi_n,\tau_n} - \mu_n(\varphi)\zeta_{\varphi_0,\tau_0}] - f_0[\mu_0(\varphi) - \mu_n(\varphi)][\zeta_{\varphi_n,\tau_n} - \frac{\alpha W^*\tau_0 f_0}{W_0}\mu_n(\varphi)(1 + \mu_0(\varphi))] \right].
\]

The first term in brackets is positive as long as \( \varphi > \varphi_0 \). If \( \zeta_{\varphi_n,\tau_0} > 0 \) and \( \zeta_{\varphi_n,\tau_0} < 0 \), the second term in brackets is always negative. Hence \( \frac{dz(\varphi)}{d\tau_0} < 0 \).  \( \blacksquare \)
References


IBIS WORLD INDUSTRY REPORT (2011): Report 33411a: Computer Manufacturing in the US.
