Optimal Monetary Policy With Large Shocks*

Martín Harding† Jesper Lindé‡ David Lopez-Salido§ Mathias Trabandt¶

February 4, 2026

Abstract

This paper explores how the interaction of large supply shocks with state-dependent pricing constrains optimal monetary policy in new ways. Strategic complementarities arising from kinked-demand curves affect firms' incentive to set prices depending on the size and the sign of the cost-push shock. Aggressive policy responses—to curb inflation driven by large adverse cost-push shocks—contrast with optimal attenuation in the policy response to more favorable shocks. The size and sign of the shocks thus uncovers new state-dependent tradeoffs for central banks. Identifying these new policy tradeoffs, arising from commitment and discretionary policies, requires moving away from linear-quadratic frameworks.

JEL Classification: E1, E3, E5.

Keywords: Inflation dynamics, New Keynesian model, inflation risk, monetary policy, linearized model, nonlinear model, real rigidities, optimal monetary policy.

---

*Any views in this paper are solely the responsibility of the authors and do not necessarily agree with the Bank of Canada, the IMF, the Board of Governors of the Federal Reserve System, or those of any other person associated with these institutions. We are grateful for comments and suggestions by Borağan Aruoba and Stefano Gnocchi, as well as seminar participants at the Bank of Canada and the 2024 Bank of Finland-CEPR annual conference. Mathias Trabandt is grateful for hospitality of the Monetary Modeling Unit at the Monetary and Capital Markets Department of the International Monetary Fund. Replication codes are available at www.mathiastrabandt.com.

†Bank of Canada, 234 Wellington Street, Ottawa, ON K1A 0G9, Canada, E-mail: mharding@bankofcanada.ca.

‡IMF (Monetary and Capital Markets Department) and CEPR, E-mail: jlinde@imf.org.

§Board of Governors of the Federal Reserve System (20th Street and Constitution Avenue N.W., Washington, D.C. 20551, USA) and CEPR, Email: david.lopez-salido@frb.gov.

¶Goethe University Frankfurt, Theodor-W.-Adorno-Platz 3, 60323 Frankfurt am Main, Germany and Halle Institute for Economic Research (IWH) and CEPR, E-mail: mathias.trabandt@gmail.com.

1 Introduction

The recent global surge in inflation, with a prominent role for adverse supply shocks, serves as a stark reminder of the profound impact that large shocks can have on economic dynamics. At an aggregate level, firms' pricing behavior appears to imply that prices rise swiftly in response to mounting marginal costs yet exhibit a reluctance to fall with the same vigor when costs decrease. This asymmetric behavior of firms underscores the intricate nature of price-setting mechanisms and their implications for inflationary pressures.[1]

The interaction of cost-push shocks and kinked demand can indeed amplify the effects of these shocks. Cost-push shocks, such as increases in energy or raw material prices, raise marginal costs for firms. Given the kinked demand curve, firms are hesitant to adjust their prices upward in response to higher costs, fearing a significant loss of market share if competitors do not follow suit. However, this price rigidity can lead to reduced profit margins, prompting firms to cut back on production or investment. Additionally, if firms collectively decide to pass on the increased costs to consumers, the inelastic nature of the demand below the kink means that prices may rise more sharply, leading to higher overall inflation. Consequently, the combination of cost-push shocks and the kinked demand curve can therefore exacerbate economic volatility by amplifying inflationary pressures and reducing output.

In this paper we formalize these ideas drawing from the work by Kimball (1995) and Dotsey and King (2005), Levin et al. (2008) and Harding, Lindé and Trabandt (2022, 2023). This research has developed relatively simple models with kinked demand consistent with the asymmetric aggregate pricing dynamics discussed earlier. Recent work by Aruoba et al. (2024) shows a detailed discussion on how a Kimball demand system helps macro models match micro pricing moments. In addition, Alexander et al. (2024) estimate a significant degree of strategic complementarity using Canadian microdata and show that a Kimball model can be calibrated to obtain the degree of strategic complementarity generated by a multi-sector oligopoly model.

The Kimball demand schedule generates a nonlinear Phillips curve, which illuminates that propagation of supply shocks can be state-dependent, particularly when the economy strays far from its steady state. This nonlinear framework is consistent with the observation that the empirical distribution of inflation is positively skewed and reveals that central banks face state-dependent policy trade-offs. Hence, the prescriptions of the standard linear quadratic approach to optimal policy may be misguided when inflation is driven well above the central banks target.

[1] Recent work by Gagliardone et al.(2024) argues that state-dependent pricing behavior of firms have become increasingly pivotal to understand post-covid inflation price dynamics.

Our paper addresses the question: What are the optimal monetary policy prescriptions in response to significant adverse supply shocks that have state-dependent propagation? We explore optimal monetary policy within a New Keynesian model that incorporates important nonlinear pricing dynamics caused by strategic complementarities among price setters arising from kinked demand functions.

We use our nonlinear model to examine the impact of large shocks on optimal monetary policy. We characterize Ramsey optimal policy within both fully nonlinear and linear-quadratic (LQ) frameworks, allowing us to identify the effects of nonlinearities in price setting on optimal policy (well-studied in the literature). Our analysis focuses on the role of strategic complementarities in price setting due to kinked demand curves and how optimal policy differs between small and large shocks to firms' marginal costs. We study the differences between commitment versus discretionary policy—and we compare outcomes using a simple quadratic loss function against those derived from households' welfare.

Our findings highlight the significant impact of strategic complementarities on optimal monetary policy responses to cost-push shocks. We observe that within a nonlinear framework, optimal policy takes a more aggressive approach to curbing inflationary pressures from larger shocks. Moreover, the optimal policy response is more gradual and persistent with strategic complementarities arising from kinked demand, as they introduce real rigidities that amplify and increase the persistence in monetary policy transmission. Furthermore, our analysis reveals the limitations of the linear-quadratic approach in capturing the nuanced dynamics of state-dependent inflation when the economy is exposed to large shocks. In doing so, throughout the paper, we compare optimal policy prescriptions with a standard Taylor rule benchmark, demonstrating that such a rule is severely suboptimal. A central bank under optimal commitment opts for a higher output cost to stabilize inflation, resulting in much lower policy rate volatility.

We emphasize the benefits of commitment in shaping optimal policy outcomes with strategic complementarities. Favorable economic outcomes under a gradual but persistent optimal policy response to adverse supply shocks hinge on the ability of the central bank to commit to the policy plan. We compare this optimal commitment benchmark with a discretionary policy where the central bank does not take agents' expectations about the future into account. Under discretionary policy, the persistent optimal policy response often dissipates, highlighting the importance of commitment in navigating tumultuous economic landscapes. The sharp but short-lived jump in the policy rate under discretion is similar to when policy is following a standard Taylor rule. Despite the front-loaded response, the overall policy plan is looser under discretion resulting in higher inflation and larger distortionary effects of adverse cost-push shocks.

A recent paper by Karadi et al. (2024) finds that optimal policy in response to large shocks leverages the lower sacrifice ratio to reduce inflation and stabilize the frequency of price adjustments in a menu cost model with state dependent pricing following Golosov and Lucas (2007). A key difference between our model and theirs is that our model features significant asymmetries in the optimal policy to large negative and positive cost push shocks, while their model implies approximately symmetric responses to large shocks regardless if they are positive or negative. In our model with a banana-shaped Phillips curve as in Phillips (1958), the central bank implements a tighter monetary policy stance when inflationary pressures are rising above target by more that it eases the policy stance to fight same-sized deflationary pressures absent any effective lower bound considerations on policy rates.

The remainder of the paper is organized as follows. Section 2 presents the model, while Section 3 discusses how we solve and parameterize it. Section 4 discusses our results. Finally, Section 5 provides concluding remarks.

2 Model

This section briefly summarizes a simplified version of Calvo (1983) pricing model. In the model, we abstract from capital accumulation and monetary distortions (i.e., associated with the money holdings of households.). We also assume that the market for assets is complete, while households supply their labor services to firms in a perfectly competitive labor market where wages are fully flexible.

2.1 Households

Households seek to maximize $E_0 \sum_{t=0}^{\infty} \beta^t \varsigma_t \, U(C_t, N_t)$, where $\beta \in (0,1)$ is the discount factor and C_t and N_t are consumption and hours worked, respectively. The variable ς_t is an exogenous shock to the discount factor. We assume that $\nu_{t+1} = \frac{\varsigma_{t+1}}{\varsigma_t}$ is exogenous with $\nu = 1$ in steady state. We focus in the following period utility

$$u(C_t, N_t) = \frac{C_t^{1-\sigma} - 1}{1-\sigma} - \frac{{N_t}^{1+\chi}}{1+\chi}, \tag{1}$$

where the parameters $\sigma > 0$ and $\chi \geq 0$ represent the relativer risk aversion and the inverse of the Frisch labor supply elasticity, respectively.

We assume that financial markets are complete and that households supply their labor services to firms in a perfectly competitive labor market. The representative household then chooses decision rules for consumption C_t, labor N_t, and a nominal bonds portfolio B_{t+1} to maximize its utility subject a period budget constraint:

$$C_t + E_t[Q_{t,t+1}B_{t+1}/P_{t+1}] \leq \frac{B_t}{P_t} + (1+\tau_t)\frac{W_t}{P_t}N_t + \Phi_t - T_t \tag{2}$$

where $Q_{t,t+1}$ is the stochastic discount factor for computing the real value at period t of one unit of consumption goods at period $t+1$, W_t is the nominal wage rate, and T_t is the real lump-sum tax, Φ_t is the real dividend income. The parameter τ_t denotes a proportional (to labor income) subsidy given to the households–funded by a non-distortionary lump-sum tax, T_t. The employment subsidy might be chosen to offset the distortion associated with the presence of imperfect competition in goods market. The first order conditions for the household's optimization are given by

$$C_t^{\sigma} N_t{}^{\chi} = (1+\tau_t)\frac{W_t}{P_t} \tag{3}$$

$$Q_{t,t+1} = \beta\nu_{t+1}\left(\frac{C_t}{C_{t+1}}\right)^{\sigma} \tag{4}$$

In addition, if i_t denotes the risk-free gross nominal interest rate at period t, the first-order condition for bonds can be written as:

$$1 = \beta E_t \nu_{t+1}\left(\frac{C_t}{C_{t+1}}\right)^{\sigma}\frac{i_t}{\Pi_{t+1}} \tag{5}$$

where the gross inflation rate is defined as $\Pi_{t+1} = P_{t+1}/P_t$.

2.2 Firms

2.2.1 Market Structure

We assume the existence of a continuum of monopolistically competitive firms producing differentiated intermediate goods that are used as inputs by a (perfectly competitive) firm producing a single final good. Following Kimball (1995) we assume that firms face demand schedules with demand elasticity that is increasing in their relative price due to strategic complementarities in price setting. This implies that firms are less aggressive in their pricing behavior. Thus, the final good is produced by a representative, perfectly competitive firm with the following general technology

$$\int_0^1 G(y_t(j))\, dj = 1, \tag{6}$$

where $y_t(j) = \frac{Y_t(j)}{Y_t}$, and $Y_t(j)$ is the quantity of intermediate good j used as an input. The function G satisfies that $G' > 0$, $G'' < 0$, and $G(1) = 1$. The final good firm chooses input demands $Y_t(j)$ to maximize profits, subject the previous technological constraint (6). In general, the solution to this problem does not produce closed form solutions. Instead, in this paper we follow Dotsey and King (2005), so the specific aggregator G takes the following form:

$$G(y) = \frac{\epsilon}{\epsilon - 1}\frac{1}{1+\eta}\left[(1+\eta)y - \eta\right]^{\frac{\epsilon-1}{\epsilon}} - \left[\frac{\epsilon}{\epsilon - 1}\frac{1}{1+\eta} - 1\right]$$

where $\epsilon = \tilde{\epsilon}(1+\eta)$ and $\tilde{\epsilon} > 1$ is the standard price elasticity of demand in Dixit-Stiglitz. The parameter η (henceforth Kimball parameter) controls the degree of complementarity ($\eta < 0$) or substitutability ($\eta > 0$) of firms' pricing decisions. The Dixit-Stiglitz emerges for $\eta = 0$. It can be shown that the solution of the firm problem yields the set of demand schedules given by

$$y_t(j) = \frac{1}{1+\eta}\left[p_t(j)^{-\epsilon}\lambda_t^{\epsilon} + \eta\right] \tag{7}$$

where $p_t(j) = \frac{P_t(j)}{P_t}$, P_t is the aggregate price level and $P_t(j)$ corresponds to the intermediate goods price; and λ_t is the Lagrange multiplier of the restriction (6). The multiplier takes the form

$$\lambda_t = \left(\int_0^1 p_t(j)^{1-\epsilon}\, dj\right)^{\frac{1}{1-\epsilon}}. \tag{8}$$

Finally, the zero profit condition implies that the aggregate price index is given by the following expression, $P_t = \frac{1}{1+\eta}\left(\int_0^1 P_t(j)^{1-\epsilon}\, dj\right)^{\frac{1}{1-\epsilon}} + \frac{\eta}{1+\eta}\int_0^1 P_t(j)\, dj$, which can be written as

$$1 = \frac{1}{1+\eta}\left(\int_0^1 p_t(j)^{1-\epsilon}\, dj\right)^{\frac{1}{1-\epsilon}} + \frac{\eta}{1+\eta}\int_0^1 p_t(j)\, dj \tag{9}$$

Notice that the Dixit-Stiglitz aggregator is a special case when $\eta = 0$.

2.2.2 Technology

The production function for a typical intermediate goods firm (say, the one producing good j) is given by:

$$Y_t(j) = A_t N_t(j) \tag{10}$$

where A_t represents total factor productivity and $N_t(j)$ represents the labor services hired by firm j. We assume that A_t will follow a first order autoregressive process with constant mean, A, and ρ_A as the first order autocorrelation coefficient.

2.2.3 Nominal Rigidities

Because of the presence of market power, intermediate firms are assumed to set nominal prices in a staggered fashion, according to the stochastic time dependent rule proposed by Calvo (1983). Each firm resets its price with probability $1-\theta$ each period, independently of the time elapsed since the last adjustment.

Profit maximization of firm i is give by:

$$\max_{P^*_{t,i}} \; E_t \sum_{j=0}^{\infty} (\beta\theta)^j \, \varsigma_{t+j}\Omega_{t+j} \left[(1+\tau_{p,t+j}) \, P^*_{t,i} - MC_{t+j} \right] Y_{t+j,i}$$

subject to

$$Y_{t+j,i} = \frac{1}{1+\eta} \left[\left(\frac{P^*_{t,i}}{P_t} \right)^{-\epsilon} \lambda_t^{\epsilon} + \eta \right] Y_{t+j} \tag{11}$$

where Ω_{t+j} denotes the Lagrange multiplier on the household budget constraint, $\tau_{p,t}$ is a per-unit of output sold subsidy paid to firms. MC_t denotes nominal marginal cost given by

$$MC_t = \phi_t \frac{W_t}{A_t}, \tag{12}$$

where it follows that real marginal cost is given by $mc_t = \phi_t \frac{W_t}{A_t P_t}$. The variable ϕ_t in eq. (12) denotes an exogenous cost-push shock, i.e. an exogenous variation in firm's marginal cost. As this shock is inefficient, we will make alternative assumptions about the extent to which it affects potential output.

The first order condition associated with profit maximization for the intermediate inputs firms that set prices at time t can be written as follows:

$$p^*_t = \frac{1}{1+\tau_{p,t}} \frac{z_{2,t}}{z_{1,t}} + \frac{1}{\epsilon - 1} \left(p^*_t\right)^{1+\epsilon} \frac{z_{3,t}}{z_{1,t}} \tag{13}$$

where $p^*_t = \frac{P^*_t}{P_t}$ denotes the relative price of the profit maximizing price at time t and the variables $z_{1,t}$, $z_{2,t}$, and $z_{3,t}$ satisfy the following set of equations:

$$\begin{aligned}
z_{1,t} &= \beta\theta E_t\{\nu_{t+1}\Pi_{t+1}^{\epsilon-1} z_{1,t+1}\} + U_{C_t} Y_t \Delta_{2,t}^{\frac{\epsilon}{1-\epsilon}} \\
z_{2,t} &= \beta\theta E_t\{\nu_{t+1}\Pi_{t+1}^{\epsilon} z_{2,t+1}\} + \frac{\epsilon}{\epsilon-1} U_{C_t} mc_t Y_t \Delta_{2,t}^{\frac{\epsilon}{1-\epsilon}} \\
z_{3,t} &= \beta\theta E_t\{\nu_{t+1}\Pi_{t+1}^{-1} z_{3,t+1}\} + \eta \, U_{C_t} Y_t
\end{aligned}$$

where Π_{t+1} and mc_t represent gross inflation rate between t and $t+1$ and the real marginal cost at time t, respectively. Finally, we have made use of the following relationship, $\lambda_t = \Delta_{2,t}^{\frac{1}{1-\epsilon}}$, where the variable $\Delta_{2,t}$ is defined below.

From the definition of aggregate prices (9) it follows

$$1 = \frac{1}{1+\eta} \Delta_{2,t}^{\frac{1}{1-\epsilon}} + \frac{\eta}{1+\eta} \Delta_{3,t} \tag{14}$$

where $\Delta_{2,t} \equiv \int_0^1 p_t(j)^{1-\epsilon}dj$ and $\Delta_{3,t} \equiv \int_0^1 p_t(j)dj$ follow the processes

$$\Delta_{2,t} = (1-\theta)\ \left(p_t^*\right)^{1-\epsilon} + \theta {\pi_t}^{\epsilon-1}\Delta_{2,t-1} \tag{15}$$

$$\Delta_{3,t} = (1-\theta)\ p_t^* + \theta {\pi_t}^{-1}\Delta_{3,t-1} \tag{16}$$

Notice that the Dixit-Stiglitz model ($\eta = 0$) implies that $\Delta_{2,t} = 1$, and $z_{3,t} = 0$, so equation (13) can be explicitly solved for optimal relative price as follows

$$p_t^* = \frac{z_{2,t}}{z_{1,t}}. \tag{17}$$

2.3 Social Resource Constraint

Given that relative prices are not equal across firms, there is a wedge between aggregate output measure in terms of production inputs and the aggregate demand measured in terms of the composite good. Hence, the sum of all firms outputs measured in terms of factor inputs can be obtained as follows

$$Y_t^* = \int_0^1 Y_t(j)dj = \int_0^1 A_t\ N_t(j)dj = A_t\ N_t \tag{18}$$

where $N_t = \int_0^1 N_t(j)dj$. Goods market equilibrium implies that the previous definition of aggregate output is not the relevant one since it sums across differentiated goods, i.e.

$$Y_t^* = \int_0^1 Y_t(j)dj = \left[\frac{1}{1+\eta}\lambda_t^\epsilon \int_0^1 p_t(j)^{-\epsilon}\ dj + \frac{\eta}{1+\eta}\right] Y_t = \Delta_t Y_t \tag{19}$$

where the wedge $\Delta_t \equiv \left[\frac{1}{1+\eta}\lambda_t^\epsilon \int_0^1 p_t(j)^{-\epsilon}\ dj + \frac{\eta}{1+\eta}\right]$. Using expressions (18) and (19) yields the following social resource constraint

$$C_t + G_t = Y_t(1+g_t) = \frac{A_t}{\Delta_t} N_t \tag{20}$$

where g_t represents the exogenous fraction of government consumption on aggregate demand.

From the definition of the wedge Δ_t, it is clear that it can be written as a function of the relative price dispersion as follows:

$$\Delta_t \equiv \frac{1}{1+\eta}\lambda_t^\epsilon \Delta_{1,t} + \frac{\eta}{1+\eta} \tag{21}$$

where $\Delta_{1,t} \equiv \int_0^1 p_t(j)^{-\epsilon}\ dj$ is a measure of relative price distortions. Given that the aggregate composite good is a function of the multiplier, we can use expression (8) and the previous definition of $\Delta_{2,t}$ to see that $\lambda_t = \Delta_{2,t}^{\frac{1}{1-\epsilon}}$, which implies that the wedge Δ_t can be written as follows

$$\Delta_t \equiv \frac{1}{1+\eta}\Delta_{2,t}^{\frac{\epsilon}{1-\epsilon}}\Delta_{1,t} + \frac{\eta}{1+\eta} \tag{22}$$

Finally, following Yun (1996), the Calvo-type of staggered price setting allows one to write the two measures of relative price distortion as follows

$$\Delta_{1,t} = (1-\theta)\ \left(p_t^*\right)^{-\epsilon} + \theta {\pi_t}^{\epsilon} \Delta_{1,t-1} \tag{23}$$

As noticed before, under $\eta = 0$, then $\Delta_{2,t} = \lambda_t = 1$, $\Delta_t = \Delta_{1,t}$, and expression (15) will correspond to the standard equation linking aggregate inflation and the relative price of the newly set prices.

The appendix contains the set of nonlinear dynamic and steady state equations for the economy with sticky prices as well as for the economy with flexible prices (potential/efficient economy).

2.4 Monetary Policy

We consider two cases. First, the central bank maximizes social welfare,

$$E_0 \sum_{t=0}^{\infty} \beta^t \varsigma_t U(C_t, N_t)$$

subject to the set of nonlinear equilibrium equations provided in the appendix and the zero lower bound constraint for the gross nominal interest rate $i_t \geq 1$. We consider optimal monetary policy under commitment from a timeless perspective. This corresponds to the Ramsey optimal policy starting from the steady state, where all the Lagrange multipliers associated to the planner's problem are initialized at their steady state values.

Second, we consider the case when the central bank follows a Taylor rule (subject to the zero lower bound constraint on the nominal interest rate)

$$i_t/i = \max\left[1/i, \left\{\frac{\widetilde{rr}_t}{\widetilde{rr}}\right\}^{r_{\widetilde{rr}}} \left\{\frac{\Pi_t}{\overline{\Pi}}\right\}^{r_\pi} \left\{\frac{Y_t}{Y} / \frac{\tilde{Y}_t}{\tilde{Y}}\right\}^{r_x}\right]$$

where $\tilde{Y}_t$ denotes flexible price output:

$$\tilde{Y}_t = \left[\frac{\tilde{\epsilon}-1}{\tilde{\epsilon}} \frac{(1+\tau_t)\,(1+\tau_{p,t})\, A_t^{1+\chi}}{\phi_t^{\iota}(1-g_t)^{\sigma}}\right]^{\frac{1}{\sigma+\chi}}$$

where $\iota \in \{0,1\}$ is a switch that allows to specify whether or not cost-push shocks affect potential output.

The flexible price real interest rate $\widetilde{rr}_t$ is determined by:

$$1 = \beta E_t \nu_{t+1} \left(\frac{1-g_t}{1-g_{t+1}}\right)^{\sigma} \left(\frac{\tilde{Y}_t}{\tilde{Y}_{t+1}}\right)^{\sigma} \widetilde{rr}_t$$

See the appendix for more details about the flexible price economy.

We also consider the implications of assuming discretionary optimal monetary policy (i.e. the policy maker takes private sector expectations as given) and compare the implications to our baseline case of optimal monetary policy under commitment. We assume that under discretion the policy maker does not take the future behavior of agents into account. In practice, this means that we do not take derivatives with respect to forward-looking terms when solving the planner's optimization problem.

Finally, we also consider the case when the central bank minimizes a standard quadratic loss function (with commitment or discretion):

$$\min E_0 \frac{1}{2} \sum_{t=0}^{\infty} \beta^t \left[\left\{ \ln\left(\frac{\Pi_t}{\bar{\Pi}}\right) \right\}^2 + \lambda \left\{ \ln\left(\frac{Y_t}{Y}\right) - \ln\left(\frac{\tilde{Y}_t}{\tilde{Y}}\right) \right\}^2 \right]$$

where $\lambda \geq 0$.

2.5 Log-linear Model

Log-linearizing the model around a zero inflation steady state yields the following set of equilibrium equations (see the appendix for details):

$$\hat{\Pi}_t = \beta E_t \hat{\Pi}_{t+1} + \kappa_p \gamma (\sigma + \chi) \left(\hat{Y}_t - \hat{\tilde{Y}}_t \right) + \kappa_p \gamma (1 - \iota) \hat{\phi}_t - \left(\kappa_{\breve{\tau}_p} - \kappa_p \right) \gamma \breve{\tau}_{p,t}$$

$$\left(\hat{Y}_t - \hat{\tilde{Y}}_t \right) = E_t \left(\hat{Y}_{t+1} - \hat{\tilde{Y}}_{t+1} \right) - \frac{1}{\sigma} E_t \left[\hat{i}_t - \hat{\Pi}_{t+1} - \hat{\tilde{r}}r_t \right]$$

$$\hat{\tilde{r}}r_t = \beta E_t \left[\sigma \left(\hat{\tilde{Y}}_{t+1} - \hat{\tilde{Y}}_t \right) - \frac{\sigma}{1-g} \Delta \breve{g}_{t+1} - \hat{\nu}_{t+1} \right]$$

$$(\sigma + \chi) \hat{\tilde{Y}}_t = (1 + \chi) \hat{A}_t + \frac{\sigma}{1-g} \breve{g}_t - \iota \hat{\phi}_t + \frac{1}{1+\tau} \breve{\tau}_t + \frac{1}{1+\tau_p} \breve{\tau}_{p,t}$$

where

$$\kappa_p = \frac{(1-\theta)(1-\beta\theta)}{\theta}, \quad \kappa_{\breve{\tau}_p} = \frac{(1-\theta)\left(1-\beta\theta\rho_{\tau_p}\right)}{\theta} \frac{1}{1+\tau_p}$$

$$\gamma = \frac{1}{1-\mu\eta}, \quad \mu = \frac{\tilde{\epsilon}}{\tilde{\epsilon}-1}$$

Following e.g. Levin, Lopez-Salido and Yun (2007), we assume that the central bank solves a linear-quadratic (LQ) problem by minimizing the following loss function (see Levin, Lopez-Salido and Yun (2007)

$$\min E_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{\theta \tilde{\epsilon}}{2(1-\theta)(1-\beta\theta)} \hat{\Pi}_t^2 + \frac{\sigma + \chi}{2} \left(\hat{Y}_t - \hat{\tilde{Y}}_t \right)^2 \right]$$

subject to the Phillips curve, new-IS curve, and the zero lower bound constraint for the gross nominal interest rate $\widehat{i}_t \geq -\ln(i)$. Note that η affects the planner's constraint in the linear system above but not the objective function. This implies that the Kimball demand system can have important implications for optimal policy even in the LQ setup. The planner takes potential output and the potential real rate as given. Note that the loss function derivations assume that the flexible price economy is efficient, i.e. a sales subsidy to firms removes the monopoly distortion. See the parameterization section for details.

Alternatively, we consider the case when the central bank follows a Taylor rule (subject to the zero lower bound constraint on the nominal interest rate):

$$\widehat{i}_t = \max \left[-\ln(i), r_{\tilde{r}r} \widehat{\tilde{r}r}_t + r_\pi \hat{\Pi}_t + r_x \left(\hat{Y}_t - \hat{\tilde{Y}}_t \right) \right].$$

We also consider the implications of assuming discretionary optimal monetary policy (i.e. the policy maker takes private sector expectations as given) and compare the implications to our baseline case of optimal monetary policy under commitment. In addition, we also consider the case when the central bank minimizes the following loss function (with commitment or discretion):

$$\min E_0 \frac{1}{2} \sum_{t=0}^{\infty} \beta^t \left[\hat{\Pi}_t^2 + \lambda \left(\hat{Y}_t - \hat{\tilde{Y}}_t \right)^2 \right]$$

where $\lambda \geq 0$.

3 Equilibrium, Solution and Parameterization

The appendix provides full sets of equilibrium equations for the nonlinear and linear model. We solve the model with the nonlinear solver ('simul') embedded in Dynare. Technically, the solution method is the one developed by Fair and Taylor (1983). Stochastic simulation of the linearized and nonlinear model are carried out under certainty equivalence, i.e. a sequence of unexpected so-called MIT shocks.

We obtain the Ramsey planner optimality conditions used in the model by updating and applying the routines developed in Levin and Lopez-Salido (2004) and Levin, Onatski, Williams, and Williams (2005). The appendix also provides the steady state as well as details about how we solve the model.

Table 1 contains the parameter values that we use in the analysis. These parameter values are either taken from Harding, Lindé and Trabandt (2022, 2023), or are standard in the literature.

Table 1: Model parameter values

Parameters		
β	0.995	Discount factor
σ	2	Inverse EIS
χ	2.5	Inverse Frisch elasticity
θ	0.66	Calvo price rigidity
$\tilde{\epsilon}$	11	Substitution elasticity
η	-8	Kimball
r_π	1.5	Taylor rule: inflation
r_x	0.125	Taylor rule: output gap
$r_{\widetilde{rr}}$	1	Taylor rule: real potential rate
ι	0	Cost-push does not affect $\widehat{\widetilde{Y}}_t$
Steady State of Endogenous and Exogenous Variables		
Π	1	Gross inflation
g	0	Government Consumption
A	1	Production Technology
ϕ	1	Production Cost-push
τ	0	Labor Subsidy
τ_p	$\frac{\tilde{\epsilon}}{\tilde{\epsilon}-1}-1$	Sales Subsidy
ν	1	Discount factor

We assume $\Pi = 1$ in the non-stochastic steady state. We set the sales subsidy to $\tau_p = \frac{\tilde{\epsilon}}{\tilde{\epsilon}-1} - 1$. With this, we obtain $mc = 1$ in the non-stochastic steady state. This way, the non-stochastic steady state and the flexible price economy are efficient. Note that this assumption is important as it underlies the second order approximation of the welfare function in the log-linear model.

Finally, unless noted otherwise, we assume that all exogenous variables follow AR(1) processes with unit variance and AR(1) coefficients of 0.25.

In the stochastic simulations, we simulate the model for 15,000 periods.

4 Results

This section conveys our key results. We start by documenting the propagation of small and large adverse cost-push shocks under alternative assumptions about policy shocks. Adverse cost-push shocks put upward pressure on inflation, and we compare their transmission under the Ramsey optimal policy in our nonlinear model, the linear-quadratic optimal response, and the effects under a standard simple interest rate rule. Next, we show the distributional implications of alternative policies when we simulate a long stochastic sample with the model. All these experiments assume commitment, either in the formulation of the optimal policies or to the simple policy rule, but in the last subsection we compare the effects of adverse cost-push shocks under commmitment with their transmission under discretion (i.e. the policy maker does not internalize expectations about agents' forward-looking behavior in its policy conduct). In the final subsection, we analyze the transmission of equally-sized large favorable cost-push shocks that lead to lower inflation and stimulate output. A key finding is that the optimal response is asymmetric to large adverse and favorable cost push shocks.

4.1 Impulses to Adverse Cost-push Shocks

In this section, we study the transmission of small and large positive cost-push shocks (ϕ_t in equation 12). We size small shocks to imply a movement in annualized inflation with 0.5 percentage points on impact under the standard Taylor rule in the linearized formulation of the model. In terms of the workhorse estimated Smets and Wouters (2007) model, this represents the equivalent of a 1/2 standard deviation shock, as annualized inflation in the SW model rises with 1 percentage points following a 1 standard deviation price markup shock. To approximate a large shock, we consider a two standard deviation shock that drives up annualized inflation with 2 percent on impact.

Figure 1 compares impulse responses to a small adverse cost-push shock under alternative policy assumptions. When policy follows the Taylor rule the central bank swiftly tightens the policy rate to contain the rise in inflation, followed by the familiar drop in GDP. The tightening is short-lived and after four quarters the policy stance has all but normalized. Note also that for this relatively small shock nonlinearities play a minor role and there are no big differences between the nonlinear (blue solid line) and linearized (blue dashed line) model under the Taylor rule.

The optimal policy response to the same shock looks very different. The increase in the policy rate is now much more gradual and persistent. Inflation increases by less on impact and, importantly, goes below zero before returning to its equilibrium. That is, the central bank implements a deflationary stabilization policy which is associated with a much bigger cost in terms of output. The gradual and persistent policy response reflects the important role of the real rigity implied by the strategic complementarities in price setting for monetary policy transmission. Note also that there are now some noticeable differences between the nonlinear and linearized (LQ) model. The nonlinear model features a stronger and more persistent tightening to contain inflation because of

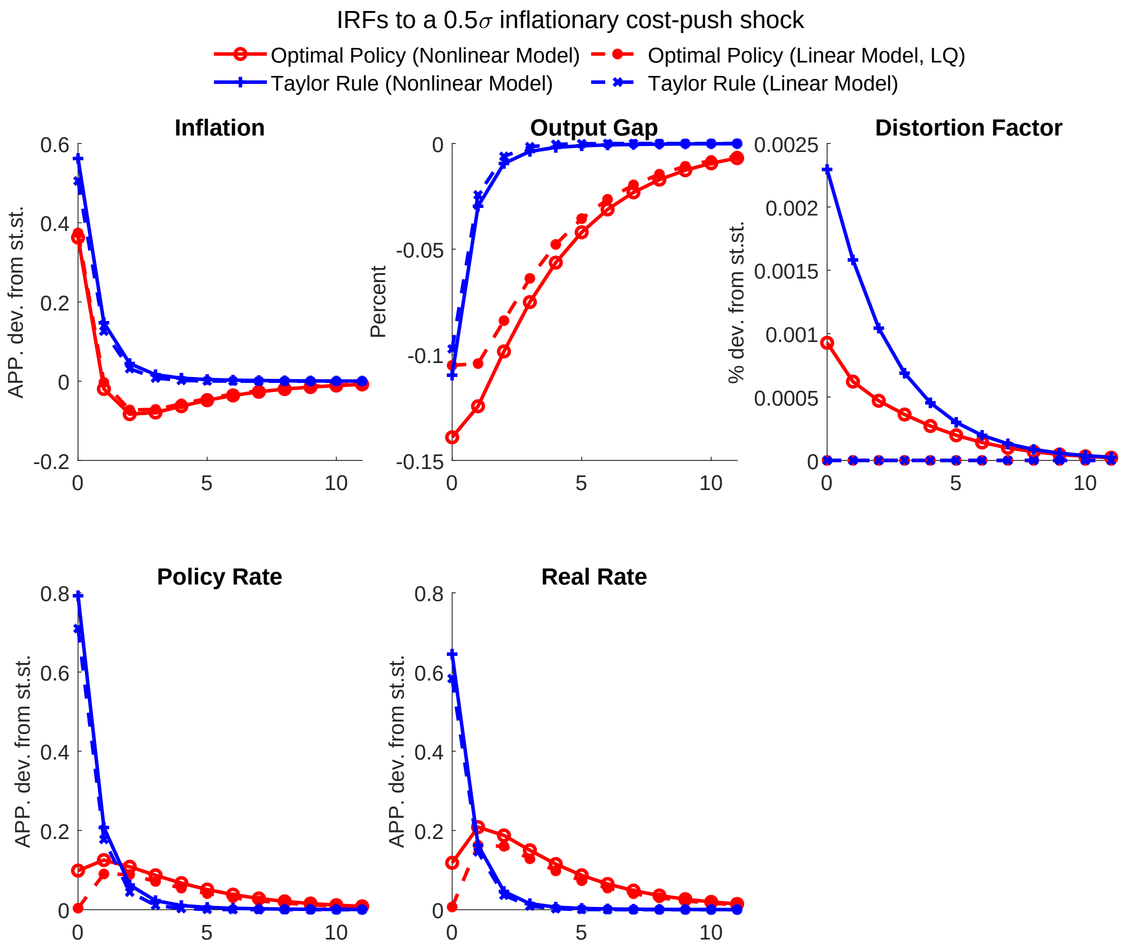

Figure 1: Impulse responses to a small adverse cost-push shock (0.5 sd.).

the asymmetric pricing behavior of firms, which results in a larger cost in terms of GDP. Finally, the more aggressive tightening prescribed by the optimal policy results in a smaller efficiency cost measured by the distortion factor Δ_t.

Figure 2 computes the same impulse responses as in Figure 1 but to a large adverse cost-push shock. Again starting with the Taylor rule setup, inflation in the linearized model now goes up by 2pp. reflecting the larger shocks size. Nonlinearities play a much bigger role and inflation jumps to 3.2pp. in the nonlinear model. This triggers a stronger—albeit still relatively short-lived—tightening with the corresponding larger drop in output.

The optimal policy becomes much more aggressive as the shock becomes larger. In fact, inflation now increases by less in the nonlinear model than in the LQ model under the optimal policy and there is a more protracted deflationary episode following the shock. This is due to a much more aggressive tightening in the nonlinear model in response to the shock. The policy rate increases

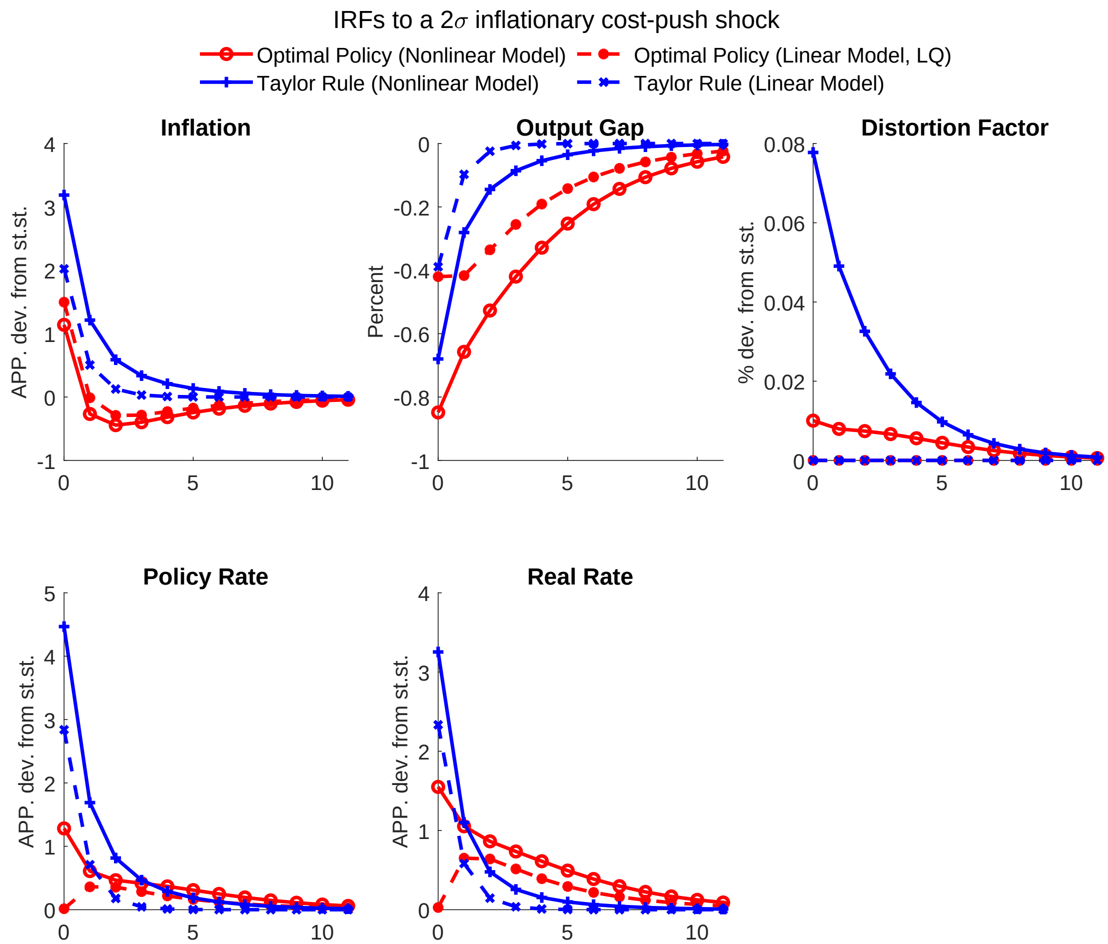

Figure 2: Impulse responses to a large adverse cost-push shock (2 sd.).

by 1.3pp. on impact compared to 0.1pp. in Figure 1 and is still very persistent. The differences between the nonlinear and LQ setups become larger as the shock size increases because the central bank takes the asymmetric price setting behavior of firms into account and is willing to pay a higher cost in terms of output to avoid large inflation surges. The reason is that, as inflation increases, cost-push shocks propagate more and more strongly and the policy trade-off for the central bank to stabilize inflation worsens.

4.2 Distributional Implications

In this section, we simulate the model for a long sample of 15,000 observations for cost-push shocks for the 4 alternative formulations of the policies discussed above. We then use the simulated data and study densities and various moments for the simulated series. Figure 3 provides densities of endogenous variables for optimal monetary policy in the nonlinear model and the LQ setup in red

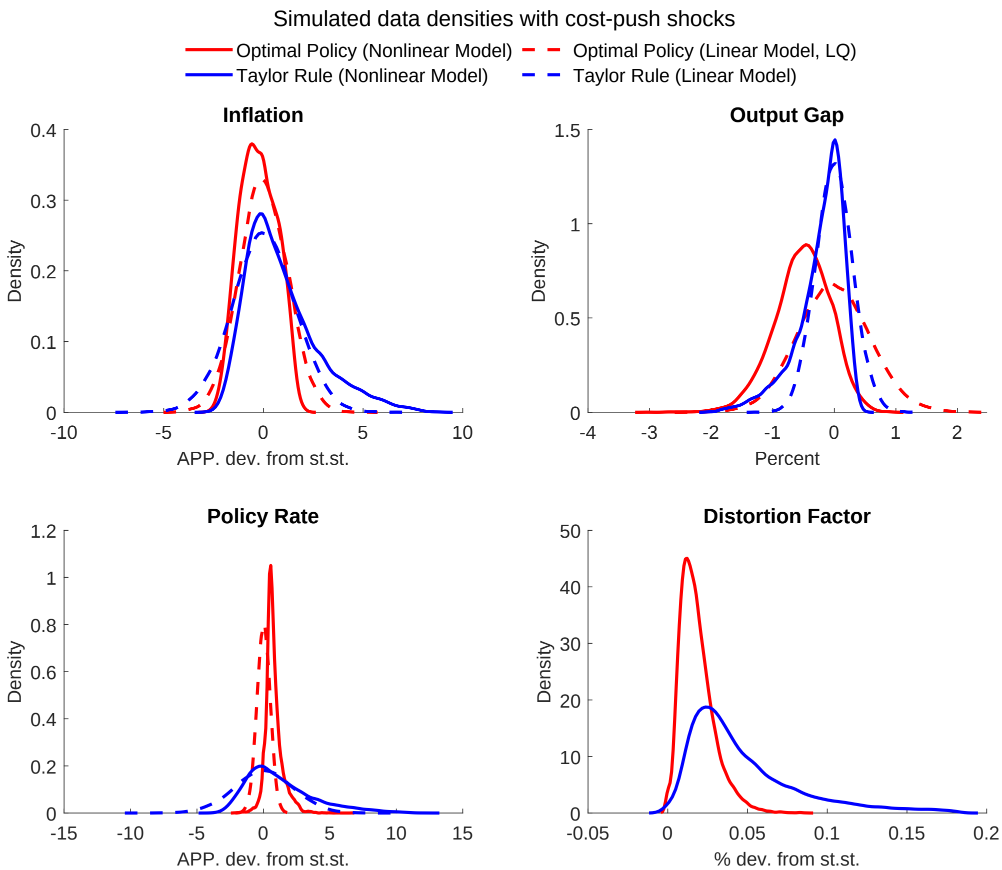

Figure 3: No ZLB: optimal policy in nonlinear model and LQ setup. Densities after long simulation sample with cost-push shocks.

after a long simulation with cost-push shocks without imposing the zero lower bound (ZLB).[2] For comparison, the blue lines show the simulations with the same shocks under the Taylor rule. Table **??** contains the associated first, second, and third moments of the simulated data for the 4 setups.

The top-left panel shows the dramatic effects of optimal policy for inflation dynamics. The blue solid line shows that under the Taylor rule inflation has a distribution with a high positive skewness of 1.1 in the nonlinear model. The optimal policy in solid red completely eliminates this skewness, bringing inflation much more tightly centered around the target and cutting the standard deviation in half. Inflation becomes slightly negative on average because of the central bank's intention to avoid large inflation surges. The top-right panel shows that this can only be achieved with a substantial cost in terms of output. The output gap distribution shifts to the left

[2]We show simulations without imposing the ZLB to tease out the nonlinear effects coming from asymmetries in price setting alone. We show results for these simulation imposing the ZLB in Figure C.1 in the appendix.

and has a much larger negative tail compared to the Taylor rule case. The bottom-left panel shows the policy rate dynamics. The policy rate becomes notably less volatile under the optimal policy, with a standard deviation 4 times smaller than under the Taylor rule in the nonlinear model. Also, the optimal policy is able to stabilize inflation with a policy rate that is on average lower than under the Taylor rule.

Comparing the nonlinear model (solid red) with the LQ setup (dashed red) illustrates the crucial role of nonlinearities in price setting for optimal policy design. The top-left panel shows that the LQ setup does not feature a deflationary bias. This is because the exacerbated upside inflation risk of the nonlinear model disappears in the LQ model, which features symmetric price setting behavior. This is clearly echoed in the top-right and bottom-left panels, which show that the output gap distribution is shifted to the left and the policy rate distribution is shifted to the right. In sum, optimal policy is more hawkish in the nonlinear model because the central bank takes into account that it becomes harder to stabilize inflation once it starts increasing far above the target. By the same token, the LQ approach misses the important role of state-dependent inflation dynamics for the design of optimal monetary policy. To verify that the differences between the nonlinear and LQ setup are due to the strategic complementarities in price setting we repeat this exercise in the Dixit-Stiglitz model (i.e., the model without strategic complementarities). Figure C.2 in the appendix shows that these differences disappear once strategic complementarities are turned off.

Table 2: Simulated data moments (no ZLB).

	Optimal policy					
	Nonlinear model			Linear (LQ) model		
	mean	std	skewness	mean	std	skewness
Inflation	-0.2	0.9	0	0	1.2	0
Output	-0.5	0.5	-0.3	0	0.6	0
Policy rate	0.8	0.6	1.1	0	0.5	0
Distortion	0	0	1.2	0	0	–
	Taylor rule					
	Nonlinear model			Linear model		
	mean	std	skewness	mean	std	skewness
Inflation	0.9	1.8	1.1	0	1.6	0
Output	-0.2	0.4	-1.3	0	0.3	0
Policy rate	1.3	2.6	1.0	0	2.2	0
Distortion	0	0	1.5	0	0	–

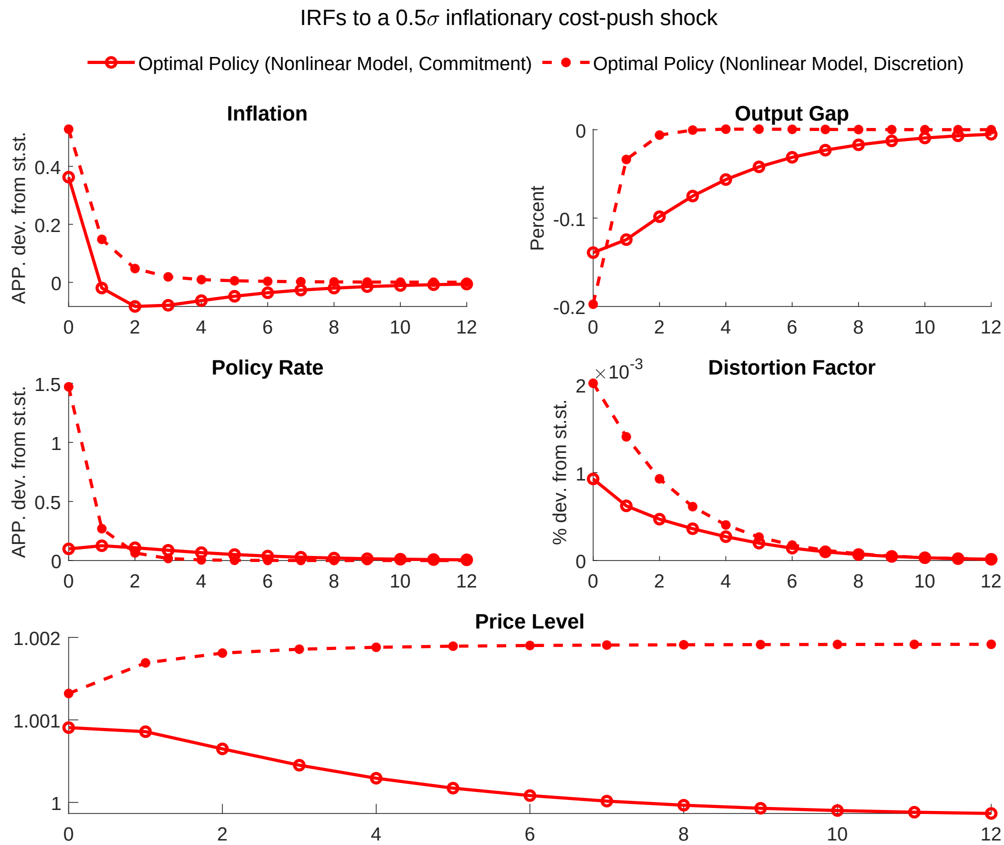

Figure 4: Discretion vs. Commitment: Impulse responses to a small adverse cost-push shock (0.5 sd.).

4.3 Commitment vs. Discretion

Figure 4 compares the cases of commitment and discretion in the nonlinear model. It provides impulse responses after an adverse small cost-push shock that drives up inflation with 0.5 percent when monetary policy is given by the simple instrument rule in Figure 1. Hence, the red solid line with dots is the same in Figure 4 as in Figure 1. Under discretion, the central bank takes as given market expectations about the future when choosing optimal monetary policy. Persistently tight optimal policy prescriptions under commitment are replaced with a transitory (stronger) tightening under discretion which yields an inflationary bias. This is further illustrated in the bottom panel showing the dynamics of the price level. Under commitment, the price level jumps on impact and the gradually converges to a point of slight optimal deflation. By contrast, the inflationary bias under discretion implies that the price level increases steadily and ends up well above its initial level once the shock fully dissipates.

Next Figure 5 shows the impact of a large 2 standard deviation adverse shock cost-push shock. Large shock in this case is four times the size of the "small" 0.5 standard deviation shock reported in Figure 4. The message is the same as in Figure 4 and results are only more extreme. Optimal policy under commitment implies a small optimal deflation, while discretion carries a large inflationary bias.

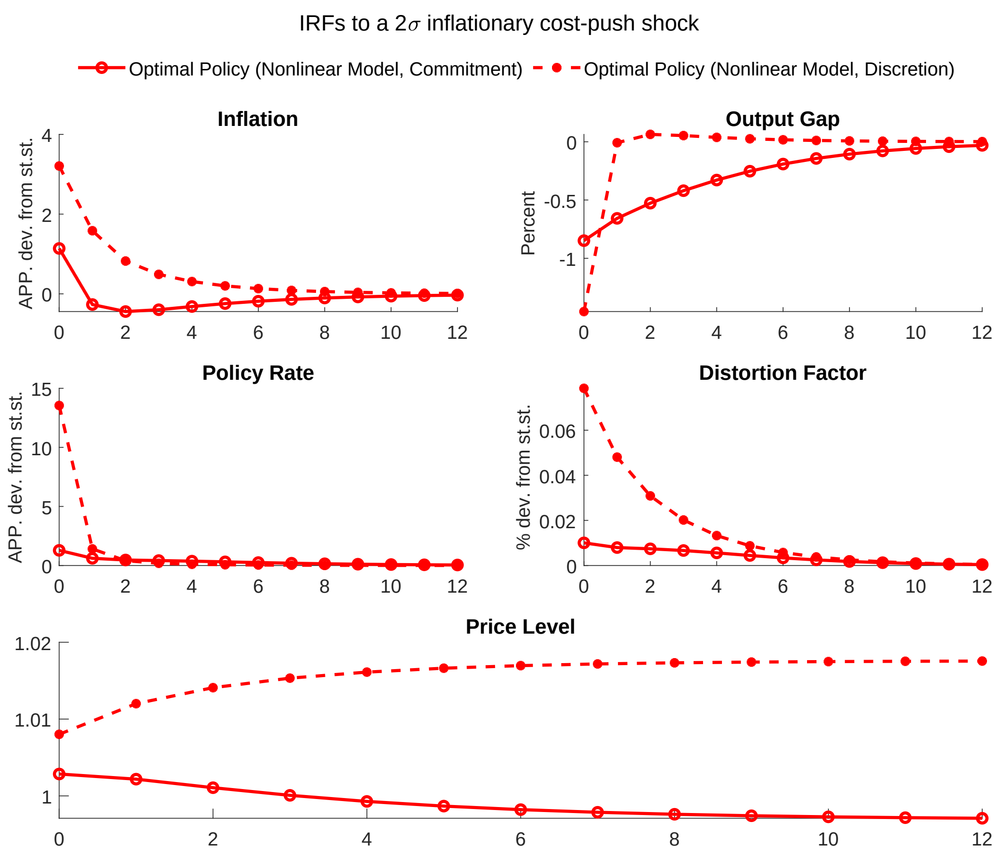

Figure 5: Discretion vs. Commitment: Impulse responses to a large adverse cost-push shock (2 sd.).

4.4 Impulses to Favorable Cost-push Shocks

In this section, we study the transmission of large negative cost-push shocks (i.e. large negative realizations of ϕ_t in equation 12). We do not study small shocks as we did in Figure 1 since the transmission of small positive and negative cost-push are approximatively equal. Following the experiment in Figure 2, we consider a two standard deviation shock that drives down annualized inflation by 2 percent on impact in the linearized model with a Taylor rule.

Figure 6 compares the impulse responses to a a large favorable cost-push shock under alternative policy assumptions. The experiment is isomorphic to that in Figure 2, but with a flipped sign. Hence, when policy follows the Taylor rule in the linearized model, the responses in Figure 6 are identical to those in Figure 2 (but with the sign flipped) as we do not impose an ELB on policy rates in our simulations. However, for the nonlinear version of the model, we see strong nonlinearities under the Taylor rule. By comparing the impulses in Figures 6 and 2 in this case (blue solid line with crosses), we see that the propagation of the shock is strongly attenuated for a favorable shock relative the case with a large adverse shock, reflecting the "banana-shaped" Phillips curve in our model with kinked demand. So, absent effective lower bound considerations, the transmission of large negative cost-push shocks to both output and inflation are notably more modest than large positive cost-push shock under a simple interest rate rule. The inflation part stems from the fact that a large negative shock drives the economy towards the flat portion of the Phillips curve, which implies that the central bank does not have to raise real and nominal rates as much to combat inflation in this case. As a consequence, the transmission to output is attenuated as well. That is, output expands less for a large favorable cost-push than it falls for large adverse cost push shock when the central bank follows a Taylor rule in our nonlinear model, because it needs to engineer a smaller cut in the real rate path to fight deflationary pressures in the former case.

As expected, the optimal policy response in the LQ model is also symmetric under the large positive and negative shocks.[3] However, the optimal policy responses in the nonlinear formulation of the model are highly asymmetric for the equally-sized negative and positive shocks, which can seen by comparing the impulses in this case (red solid lines with circles in Figures 6 and 2). Output expands notably less for the deflationary cost-push shock than it falls for the adverse shock, reflecting that the planner needs to raise the real rate path to lean against the adverse inflationary cost-push more than is cuts the real rate path to lean against the favorable deflationary cost-push shock. This policy behavior allows inflation to fall more for a favorable cost-push shock than what inflation is allowed to rise for an adverse cost-push shock. In fact, the planner leans against the expansion in output by initially raising both the nominal and real policy rate for the deflationary cost-push shock. Hence, nominal policy rates initially move in the same direction for both inflationary and deflationary cost-push shocks, albeit notably less in the case with deflationary impulses. The less aggressive response prescribed by the optimal policy for deflationary cost-push

[3]Notice that in this case our assumption of a non-binding ELB is irrelevant without shock uncertainty as the policy rate moves so little that the ELB would not bind anyway given our calibration of the steady state real interest rate.

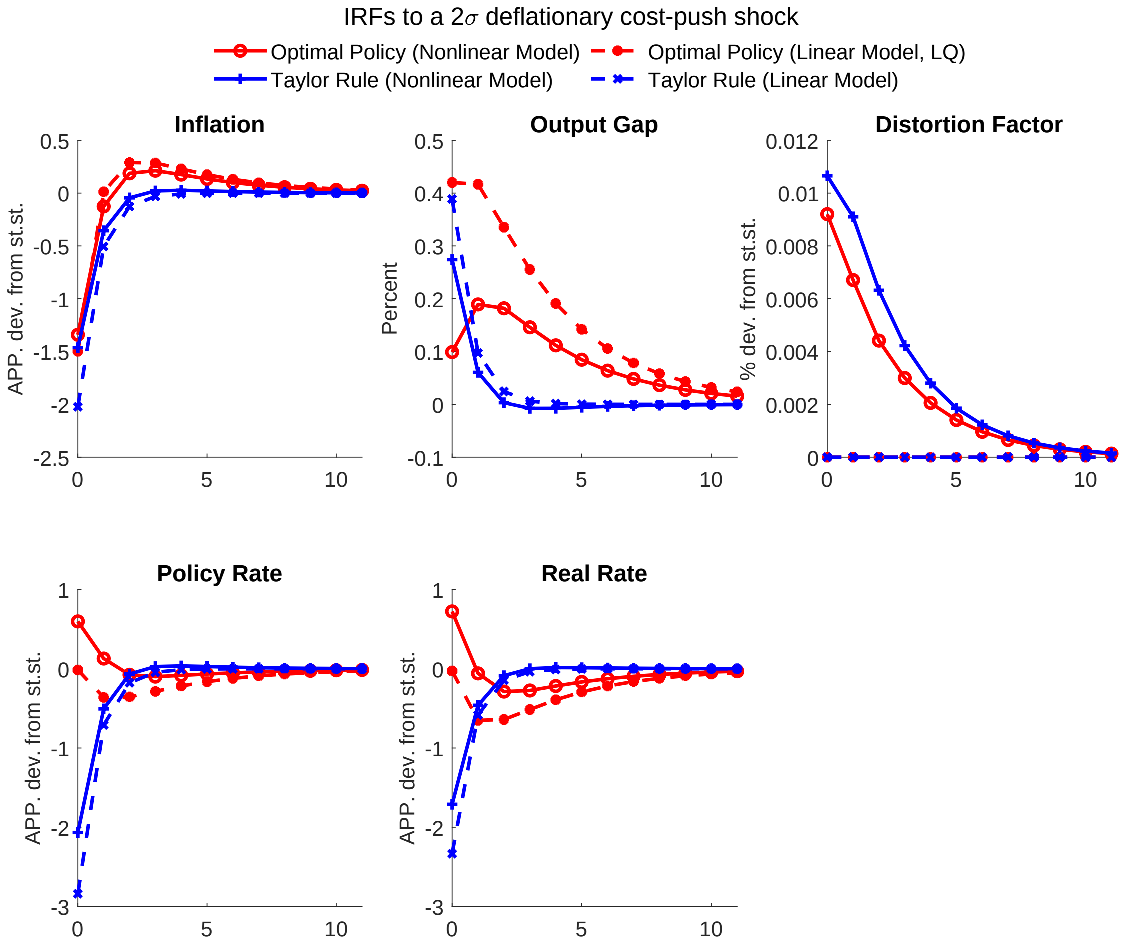

Figure 6: Impulse responses to a large favorable cost-push shock (2 sd.).

shocks is associated with a smaller efficiency cost measured by the distortion factor Δ_t. In sum, our model with a "banana-shaped" Phillips curve implies an optimal asymmetric response to large deflationary and inflationary cost-push shocks, in contrast to the optimal policy response in the Karadi et al. (2024) menu cost model which features an "s-shaped" Phillips curve.

5 Conclusion

This paper explores the implications of large adverse supply shocks and state-dependent pricing on optimal monetary policy within a New Keynesian framework. Building upon recent research, we explore nonlinear dynamics of inflation propagation, emphasizing the role of strategic complementarities in pricing. By contrasting responses to small and large shocks, as well as commitment versus discretion, we find time-varying policy trade-offs confronting central banks. An additional important insight from our nonlinear Phillips curve model—in the spirit of Phillips (1958)—is that the impulses to large deflationary and inflation cost-push shocks are asymmetric.

Our results highlight the necessity for aggressive policy measures in curbing inflationary pressures exacerbated by large adverse shocks. Moreover, we underscore the limitations of linear-quadratic models in capturing state-dependent inflation dynamics, emphasizing the significance of commitment in navigating turbulent economic landscapes.

In future research, we intend to explore various avenues. First, we will examine optimal policy under varying inflationary conditions when central banks adhere to a simple loss function, and analyze optimal policy strategies when central banks are constrained by an effective lower bound and subject to uncertainty about future shocks. Moreover, we intend to study the robustness of our findings under short-term planning horizons and explore the potential for simplified policy rules to approximate optimal outcomes within a nonlinear framework.

References

Adam, Klaus, and Roberto M. Billi (2006), "Optimal Monetary Policy under Commitment with a Zero Bound on Nominal Interest Rates", *Journal of Money, Credit, and Banking* 38(7), 1877-1905.

Adam, Klaus, and Roberto M. Billi (2007), "Discretionary Monetary Policy and the Zero Lower Bound on Nominal Interest Rates", *Journal of Monetary Economics* 54(3), 728-752.

Adjemian, Stéphane, Matthieu D. Paries, and Stéphane Moyene (2008), "Towards a monetary policy evaluation framework", European Central Bank Working Paper Series No. 942.

Adrian, T., N. Boyarchenko, and D. Giannone (2019), "Vulnerable Growth," *American Economic Review* 109(4), 1263-89.

Alexander, Patrick, Lu Han, Oleksiy Kryvtsov, and Ben Tomlin, 2024. "Markups and Inflation in Oligopolistic Markets: Evidence from Wholesale Price Data," Staff Working Papers 24-20, Bank of Canada.

Alves, S. A. L. (2014). "Lack of Divine Coincidence in New-Keynesian Models," *Journal of Monetary Economics* 67, 33–46.

Aruoba, S. Borağan, Pablo Cuba-Borda, and Frank Schorfheide (2018), "Macroeconomic Dynamics Near the ZLB: A Tale of Two Countries", *Review of Economic Studies* 85(1), 87–118.

Aruoba, S. Borağan, Eugene Oue, Felipe Saffie, and Jonathan L. Willis, (2024), "Non-Constant Demand Elasticities, Firm Dynamics and Monetary Non-Neutrality: Role of Demand Shocks", NBER Working Paper No. 32518.

Ball, Laurence, Daniel Leigh and Prachi Mishra (2022), "Understanding U.S. Inflation During the COVID Era" NBER Working Paper No. 30613.

Beaudry, Paul, Thomas J. Carter and Amartya Lahiri (2022). "Looking Through Supply Shocks versus Controlling Inflation Expectations: Understanding the Central Bank Dilemma,"Staff Working Papers 22-41, Bank of Canada.

Blanchard, O. and Galí, J. (2007). "Real Wage Rigidities and the New Keynesian Model." *Journal of Money, Credit, and Banking* 39(1): 35–65.

Boehm, Christoph E., and Nitya Pandalai-Nayar, (2022), "Convex Supply Curves", *American Economic Review*, 112 (12): 3941-69.

Calvo, Guillermo, "Staggered Prices in a Utility-Maximizing Framework,' Journal of Monetary Economics, 1983, 12(3), pp. 383-98.

Canova, F. (2007). Methods for Applied Macroeconomic Research. Princeton University Press.

Christiano, Lawrence J., Martin Eichenbaum and Charles Evans (2005), "Nominal Rigidities and the Dynamic Effects of a Shock to Monetary Policy", *Journal of Political Economy* 113(1), 1-45.

Costain, James, Anton Nakov, Borja Petit, (2022), "Flattening of the Phillips Curve with State-Dependent Prices and Wages", *The Economic Journal* 132, 546–581.

Debortoli, Davide, Jinill Kim, Jesper Lindé and Ricardo Nunes (2019), "Designing a Simple Loss Function for Central Banks: Does a Dual Mandate Make Sense?," *The Economic Journal* 129, 2010-2038.

Del Negro, Marco, Frank Schorfheide, Frank Smets, and Rafael Wouters, 2007, "On the Fit of New Keynesian Models." *Journal of Business & Economic Statistics* 25 (2): 123–43.

Dixit, Avinash K, and Joseph E. Stiglitz (1977), "Monopolistic Competition and Optimum Product Diversity", *American Economic Review* 67(3), 297-308.

Dotsey, Michael and King, Robert G., (2005), "Implications of state-dependent pricing for dynamic macroeconomic models," *Journal of Monetary Economics* 52(1), 213-242.

Dupraz, Stéphane, (2017), "A Kinked-Demand Theory of Price Rigidity", Manuscript.

Edge, R. M., Laubach, T., & Williams, J. C. (2007). Learning and shifts in long-run productivity growth. Journal of Monetary Economics, 54(8), 2421-2438.

Engle, Robert F. (1982), "Autoregressive Conditional Heteroscedasticity with Estimates of the Variance of United Kingdom Inflation," *Econometrica* 50(4), 987-1007.

Erceg, C. J., Henderson, D. W., & Levin, A. T. (2000). Optimal monetary policy with staggered wage and price contracts. Journal of Monetary Economics, 46(2), 281-313.

Erceg, Christopher J. & Levin, Andrew T., 2003. "Imperfect credibility and inflation persistence," Journal of Monetary Economics, Elsevier, vol. 50(4), pages 915-944, May.

Fair, Ray C. and John B. Taylor (1983), "Solution and Maximum Likelihood Estimation of Dynamic Rational Expectations Models", *Econometrica* 51(4), 1169-1185.

Fernández-Villaverde, Jesús, Gordon, Grey, Guerrón-Quintana, Pablo and Juan F. Rubio-Ramírez (2015), "Nonlinear Adventures at the Zero Lower Bound", *Journal of Economic Dynamics and Control* 57, 182–204.

Forbes, Kristin J. and Gagnon, Joseph and Collins, Christopher G., 2021a, "Low Inflation Bends the Phillips Curve Around the World: Extended Results", *Peterson Institute for International Economics Working Paper No. 21-15.*

Forbes, Kristin J. and Gagnon, Joseph and Collins, Christopher G., 2021b, "Pandemic inflation and nonlinear, global Phillips curves", *Peterson Institute for International Economics Working Paper No. 21-15*, VoxEU.org, https://voxeu.org/article/pandemic-inflation-and-nonlinear-global-phillips-curves.

Gagliardone, Luca, Mark Gertler, Simone Lenzu, and Joris Tielens (2024), "State Dependence and Non-linear Inflation Dynamics", presentation at the International Research Forum on Monetary Policy, Federal Reserve Board.

Gelos, Gaston and Yulia Ustyugova, 2017, "Inflation responses to commodity price shocks – How and why do countries differ?", *Journal of International Money and Finance*, 72, 28-47.

Golosov, Mikhail and Robert E. Lucas (2007) "Menu Costs and Phillips Curves," *Journal of Political Economy* 115, 171–199.

Goodfriend, Marvin (1993), "Interest Rate Policy and the Inflation Scare Problem: 1979-1992," *FRB Richmond Economic Quarterly* 79(1), 1-23.

Gopinath, Gita (2022), "How will the Pandemic and War Shape Future Monetary Policy?," Speech delivered at the Jackson Hole Symposium, August 26.

Guerrieri, Luca and Matteo Iacoviello (2017), "Collateral constraints and macroeconomic asymmetries," *Journal of Monetary Economics* 90(C), 28-49.

Gust, Christopher J., Edward P. Herbst, J. David López-Salido, and Matthew E. Smith (2017). "The Empirical Implications of the Interest-Rate Lower Bound," *American Economic Review*

107(7), 1971-2006.

Hamilton, James D. Time Series Analysis. Princeton University Press, 1994.

Harding, Martín, Jesper Lindé, and Mathias Trabandt (2022), "Resolving the missing deflation puzzle," *Journal of Monetary Economics* 126(C), 15-34.

Harding, Martín, Jesper Lindé, and Mathias Trabandt (2023), "Understanding Post-Covid Inflation Dynamics," *Journal of Monetary Economics, forthcoming.*

Hazell, J., J. Herreno, E. Nakamura, and J. Steinsson (2022), "The Slope of the Phillips Curve: Evidence from U.S. States," *The Quarterly Journal of Economics*, 137, 1299– 1344.

Ilut, Cosmin, Rosen Valchev, and Nicolas Vincent (2020), "Paralyzed by fear: Rigid and discrete pricing under demand uncertainty," *Econometrica* 88(5), 1899–1938.

Juillard, Michel (1996), "Dynare : A Program for the Resolution and Simulation of Dynamic Models with Forward Variables Through the Use of a Relaxation Algorithm," CEPREMAP Working Paper 9602.

Kimball, M. S. (1995), "The Quantitative Analytics of the Basic Neomonetarist Model," *Journal of Money, Credit, and Banking* 27(4), 1241–1277.

Karadi, Peter, Anton Nakov, Galo Nuno, Ernesto Pasten, and Dominik Thaler (2024), "Strike While the Iron is Hot - Optimal Monetary Policy with a Nonlinear Phillips Curve", manuscript.

Levin, Andrew, J. David López-Salido (2004). "Optimal Monetary Policy with Endogenous Capital Accumulation", manuscript, Federal Reserve Board.

Levin, Andrew T., J. David López-Salido, Edward Nelson, and Tack Yun (2008). "Macroeconometric equivalence, microeconomic dissonance, and the design of monetary policy," *Journal of Monetary Economics* 55, 48-62.

Levin, A., Onatski, A., Williams, J., Williams, N., 2005. "Monetary Policy under Uncertainty in Microfounded Macroeconometric Models." In: NBER Macroeconomics Annual 2005, Gertler, M., Rogoff, K., eds. Cambridge, MA: MIT Press.

Levin, Andrew, David López-Salido, and Tack Yun, (2007), "Strategic Complementarities and Optimal Monetary Policy", CEPR Press Discussion Paper No. 6423.

Levin, A., J. David López-Salido, Edward Nelson, Tack Yun, (2008), "Macroeconometric equivalence, microeconomic dissonance, and the design of monetary policy", *Journal of Monetary Economics*, Volume 55, 2008, Pages S48-S62.

L'Huillier, Jean-Paul and Gregory Phelan, (2023), "Can Supply Shocks Be Inflationary with a Flat Phillips Curve?", *Manuscript.*

Lindé, Jesper and Mathias Trabandt (2018), "Should We Use Linearized Models to Calculate Fiscal Mulipliers?", *Journal of Applied Econometrics* 33(7), 937–965.

Ljungqvist, Lars, and Thomas J. Sargent. Recursive Macroeconomic Theory. The MIT Press, 2018.

López-Salido, J. David and Francesca Loria, 2020. "Inflation at Risk," Finance and Economics Discussion Series 2020-013, Board of Governors of the Federal Reserve System.

Nakata, Taisuke (2017), "Uncertainty at the Zero Lower Bound," *American Economic Journal: Macroeconomics* 9(3), 186-221.

Phillips, A. W. (1958), "The Relationship between Unemployment and the Rate of Change of Money Wages in the United Kingdom 1861-1957," *Economica* 25, 283–299.

Rotemberg, J.J. and Woodford, M. (1998). "An Optimisation-Based Econometric Framework for the Evaluation of Monetary Policy: Expanded Version." NBER Technical Working Paper 0233.

Smets, Frank and Raf Wouters (2007), "Shocks and Frictions in US Business Cycles: A Bayesian DSGE Approach", *American Economic Review* 97(3), 586-606.

Yun, Tack, (1996), "Nominal price rigidity, money supply endogeneity, and business cycles," *Journal of Monetary Economics*, 37, issue 2-3, p. 345-370.

Yun (2005), Optimal Monetary Policy with Relative Price Distortions, American Economic Review, Vol. 95 (1), March 2005, 89-109.

Woodford, M. (2003). *Interest and Prices,* Princeton: Princeton University Press.

Appendix A Dynamic and Steady State Equilibrium Equations

A.1 Sticky Price Economy

Nonlinear Dynamic Equilibrium Equations

$$U_t = \frac{C_t^{1-\sigma} - 1}{1-\sigma} - \frac{{N_t}^{1+\chi}}{1+\chi}$$

$$V_t = U_t + \beta E_t \nu_{t+1} V_{t+1}$$

$$U_{C_t} = C_t^{-\sigma}$$

$$C_t^{\sigma} {N_t}^{\chi} = (1+\tau_t) w_t$$

$$1 = \beta E_t \nu_{t+1} \left(\frac{C_t}{C_{t+1}} \right)^{\sigma} \frac{i_t}{\Pi_{t+1}}$$

$$p_t^* = \frac{1}{1+\tau_{p,t}} \frac{z_{2,t}}{z_{1,t}} + \frac{1}{\epsilon - 1} \left(p_t^*\right)^{1+\epsilon} \frac{z_{3,t}}{z_{1,t}}$$

$$z_{1,t} = \beta\theta E_t \{ \nu_{t+1} \Pi_{t+1}^{\epsilon-1} z_{1,t+1} \} + U_{C_t} Y_t \Delta_{2,t}^{\frac{\epsilon}{1-\epsilon}}$$

$$z_{2,t} = \beta\theta E_t \{ \nu_{t+1} \Pi_{t+1}^{\epsilon} z_{2,t+1} \} + \frac{\epsilon}{\epsilon-1} U_{C_t} mc_t Y_t \Delta_{2,t}^{\frac{\epsilon}{1-\epsilon}}$$

$$z_{3,t} = \beta\theta E_t \{ \nu_{t+1} \Pi_{t+1}^{-1} z_{3,t+1} \} + \eta U_{C_t} Y_t$$

$$mc_t = \phi_t \frac{w_t}{A_t}$$

$$1 = \frac{1}{1+\eta} \Delta_{2,t}^{\frac{1}{1-\epsilon}} + \frac{\eta}{1+\eta} \Delta_{3,t}$$

$$\Delta_{1,t} = (1-\theta) \left(p_t^*\right)^{-\epsilon} + \theta \Pi_t^{\epsilon} \Delta_{1,t-1}$$

$$\Delta_{2,t} = (1-\theta) \left(p_t^*\right)^{1-\epsilon} + \theta \Pi_t^{\epsilon-1} \Delta_{2,t-1}$$

$$\Delta_{3,t} = (1-\theta) p_t^* + \theta \Pi_t^{-1} \Delta_{3,t-1}$$

$$C_t + g_t Y_t = Y_t$$

$$Y_t = \frac{A_t}{\Delta_t} N_t$$

$$\Delta_t \equiv \frac{1}{1+\eta} \Delta_{2,t}^{\frac{\epsilon}{1-\epsilon}} \Delta_{1,t} + \frac{\eta}{1+\eta}$$

plus a specification for Monetary Policy

Number of equations: 18

Number of endogenous variables: 18 (Δ_t, $\Delta_{3,t}$, $\Delta_{2,t}$, $\Delta_{1,t}$, Y_t, N_t, $p_t^*, \Pi_t, mc_t, w_t, z_{3,t}, U_{C_t}, z_{2,t}, z_{1,t}, C_t, i_t, U_t, V_t$)

Number of exogenous variables: 6 ($g_t, A_t, \phi_t, \tau_t, \tau_{p,t}, \nu_t$)

Non-stochastic Steady State Set $\nu = 1$. Assume the central bank chooses a level of inflation in steady state, Π. Then, we can solve for the non-stochastic steady state using the following set of equations

$$p^* = \left[\frac{1}{1+\eta} \left[\frac{1-\theta}{1-\theta\Pi^{\epsilon-1}} \right]^{\frac{1}{1-\epsilon}} + \frac{\eta}{1+\eta} \frac{1-\theta}{1-\theta\Pi^{-1}} \right]^{-1}$$

$$\Delta_1 = \frac{(1-\theta)\left(p^*\right)^{-\epsilon}}{1-\theta\Pi^{\epsilon}}$$

$$\Delta_2 = \frac{(1-\theta)\left(p^*\right)^{1-\epsilon}}{1-\theta\Pi^{\epsilon-1}}$$

$$\Delta_3 = \frac{(1-\theta)p^*}{1-\theta\Pi^{-1}}$$

$$\Delta \equiv \frac{1}{1+\eta} \Delta_2^{\frac{\epsilon}{1-\epsilon}} \Delta_1 + \frac{\eta}{1+\eta}$$

$$mc = \frac{p^* - \frac{1}{\epsilon-1}\left(p^*\right)^{1+\epsilon} \eta \frac{1-\beta\theta\Pi^{\epsilon-1}}{(1-\beta\theta\Pi^{-1})\Delta_2^{\frac{\epsilon}{1-\epsilon}}}}{\frac{1}{1+\tau_p}\frac{\epsilon}{\epsilon-1}\frac{1-\beta\theta\Pi^{\epsilon-1}}{1-\beta\theta\Pi^{\epsilon}}}$$

$$w = \frac{A}{\phi} mc$$

$$Y = \left[\frac{(1+\tau)w}{(1-g)^{\sigma}\left(\Delta/A\right)^{\chi}} \right]^{\frac{1}{\sigma+\chi}}$$

$$C = (1-g)Y$$

$$N = Y\Delta/A$$
$$U = \frac{C^{1-\sigma}-1}{1-\sigma} - \frac{N^{1+\chi}}{1+\chi}$$
$$V = \frac{U}{1-\beta}$$

$$U_C = C^{-\sigma}$$

$$i = \pi/\beta$$

$$z_1 = \frac{C^{-\sigma} Y \Delta_2^{\frac{\epsilon}{1-\epsilon}}}{1-\beta\theta\Pi^{\epsilon-1}}$$
$$z_2 = \frac{C^{-\sigma} mc Y \Delta_2^{\frac{\epsilon}{1-\epsilon}}}{1-\beta\theta\Pi^{\epsilon}} \frac{\epsilon}{\epsilon-1}$$
$$z_3 = \frac{\eta C^{-\sigma} Y}{1-\beta\theta\Pi^{-1}}$$

A.1.1 Nonlinear Flexible Price Economy

Setting $\theta = 0$ and defining flexible price variables with a tilde gives:

$$\tilde{Y}_t = \left[\frac{\tilde{\epsilon}-1}{\tilde{\epsilon}} \frac{(1+\tau_t)(1+\tau_{p,t}) A_t^{1+\chi}}{\phi_t^{\iota}(1-g_t)^{\sigma}}\right]^{\frac{1}{\sigma+\chi}}$$

where $\iota \in \{0,1\}$. If $\iota = 1$ then cost push shocks affect potential output. When $\iota = 0$ then cost push shocks do not affect potential output, i.e. they are inefficient shocks as in e.g. Smets and Wouters (2007) and many others.

$$1 = \beta E_t \nu_{t+1} \left(\frac{1-g_t}{1-g_{t+1}}\right)^{\sigma} \left(\frac{\tilde{Y}_t}{\tilde{Y}_{t+1}}\right)^{\sigma} \widetilde{rr}_t$$

$$\tilde{U}_t = \frac{\left[(1-g_t)\tilde{Y}_t\right]^{1-\sigma} - 1}{1-\sigma} - \frac{\left[\tilde{Y}_t/A_t\right]^{1+\chi}}{1+\chi}$$
$$\tilde{V}_t = \tilde{U}_t + \beta E_t \nu_{t+1} \tilde{V}_{t+1}$$

Note that $p_t^* = 1$ and $\Delta_t = 1$ and in steady, we have

$$\tilde{U} = \frac{\left[(1-g)\tilde{Y}\right]^{1-\sigma} - 1}{1-\sigma} - \frac{\left[\tilde{Y}/A\right]^{1+\chi}}{1+\chi}$$

$$\tilde{V} = \frac{\tilde{U}}{1-\beta}$$

$$\tilde{Y} = \left[\frac{\tilde{\epsilon}-1}{\tilde{\epsilon}} \frac{(1+\tau)(1+\tau_p) A^{1+\chi}}{\phi^{\iota}(1-g)^{\sigma}}\right]^{\frac{1}{\sigma+\chi}}$$

$$\widetilde{rr} = 1/\beta$$

where the above equations assume $\nu_t = \nu_{t+1} = 1$.

A.2 Log-linear model

Log-linearize model around zero inflation steady state:

$$0 = E_t\left[\hat{\nu}_{t+1} + \sigma\hat{C}_t - \sigma\hat{C}_{t+1} + \hat{i}_t - \hat{\Pi}_{t+1}\right]$$

Define:

$$1 + \tau_{p,t} = \Lambda_{t,p}$$

Recall the set of nonlinear equilibrium equations

$$p_t^* = \frac{1}{\Lambda_{t,p}} \frac{z_{2,t}}{z_{1,t}} + \frac{1}{\epsilon - 1} (p_t^*)^{1+\epsilon} \frac{z_{3,t}}{z_{1,t}}$$

$$z_{1,t} = \beta\theta E_t\{\nu_{t+1}\Pi_{t+1}^{\epsilon-1} z_{1,t+1}\} + U_{C_t} Y_t \Delta_{2,t}^{\frac{\epsilon}{1-\epsilon}}$$

$$z_{2,t} = \beta\theta E_t\{\nu_{t+1}\Pi_{t+1}^{\epsilon} z_{2,t+1}\} + \frac{\epsilon}{\epsilon-1} U_{C_t} mc_t Y_t \Delta_{2,t}^{\frac{\epsilon}{1-\epsilon}}$$

$$z_{3,t} = \beta\theta E_t\{\nu_{t+1}\Pi_{t+1}^{-1} z_{3,t+1}\} + \eta U_{C_t} Y_t$$

It is useful to rewrite these equations as follows using a change of variables $\tilde{z}_{3,t} = z_{3,t}/\eta$ and $\tilde{z}_{2,t} = \frac{z_{2,t}}{\frac{\epsilon}{\epsilon-1}}$

$$p_t^* = \frac{\frac{\epsilon}{\epsilon-1}}{\Lambda_{t,p}} \frac{\tilde{z}_{2,t}}{z_{1,t}} + \frac{\eta}{\epsilon - 1} (p_t^*)^{1+\epsilon} \frac{\tilde{z}_{3,t}}{z_{1,t}}$$

$$z_{1,t} = \beta\theta E_t\{\nu_{t+1}\Pi_{t+1}^{\epsilon-1} z_{1,t+1}\} + U_{C_t} Y_t \Delta_{2,t}^{\frac{\epsilon}{1-\epsilon}}$$

$$\tilde{z}_{2,t} = \beta\theta E_t\{\nu_{t+1}\Pi_{t+1}^{\epsilon} \tilde{z}_{2,t+1}\} + U_{C_t} mc_t Y_t \Delta_{2,t}^{\frac{\epsilon}{1-\epsilon}}$$

$$\tilde{z}_{3,t} = \beta\theta E_t\{\nu_{t+1}\Pi_{t+1}^{-1} \tilde{z}_{3,t+1}\} + U_{C_t} Y_t$$

Log-linearizing these equations also using $\hat{p}^*_t = \frac{\theta}{1-\theta}\hat{\Pi}_t$, $\widehat{\Delta}_{2,t} = 0$, and $mc = -\Lambda_p \frac{\eta-\epsilon+1}{\epsilon}$gives $\Lambda_{t,p} = 1 + \tau_{t,p}$

$$\hat{z}_{1,t} = \beta\theta E_t\{\hat{\nu}_{t+1} + (\epsilon - 1)\,\hat{\Pi}_{t+1} + \hat{z}_{1,t+1}\} + (1-\beta\theta)\left(\hat{U}_{C_t} + \hat{Y}_t\right)$$
$$\widehat{\tilde{z}}_{2,t} = \beta\theta E_t\{\hat{\nu}_{t+1} + \epsilon\hat{\Pi}_{t+1} + \widehat{\tilde{z}}_{2,t+1}\} + (1-\beta\theta)\left(\widehat{mc}_t + \hat{U}_{C_t} + \hat{Y}_t\right)$$
$$\widehat{\tilde{z}}_{3,t} = \beta\theta E_t\{\hat{\nu}_{t+1} - \hat{\Pi}_{t+1} + \widehat{\tilde{z}}_{3,t+1}\} + (1-\beta\theta)\left(\hat{U}_{C_t} + \hat{Y}_t\right)$$

$$\frac{\eta-\epsilon+\epsilon\eta+1}{\eta-\epsilon+1}\frac{\theta}{1-\theta}\hat{\Pi}_t + \hat{\Lambda}_{t,p} = (\widehat{\tilde{z}}_{2,t} - \hat{z}_{1,t}) + \frac{\eta}{\epsilon-1-\eta}\left(\widehat{\tilde{z}}_{3,t} - \hat{z}_{1,t}\right)(**)$$

Rewrite the first three equations as

$$\frac{\eta}{\epsilon-1-\eta}\left(\widehat{\tilde{z}}_{3,t} - \hat{z}_{1,t}\right) = \beta\theta\frac{\eta}{\epsilon-1-\eta}E_t\{\widehat{\tilde{z}}_{3,t+1} - \hat{z}_{1,t+1} - \epsilon\hat{\Pi}_{t+1}\}$$

$$\widehat{\tilde{z}}_{2,t} - \hat{z}_{1,t} = \beta\theta E_t\{\widehat{\tilde{z}}_{2,t+1} - \hat{z}_{1,t+1} + \hat{\Pi}_{t+1}\} + (1-\beta\theta)\widehat{mc}_t$$

Adding both equations

$$\begin{aligned}\widehat{\tilde{z}}_{2,t} - \hat{z}_{1,t} + \frac{\eta}{\epsilon-1-\eta}\left(\widehat{\tilde{z}}_{3,t} - \hat{z}_{1,t}\right) &= \beta\theta\left(E_t\{\widehat{\tilde{z}}_{2,t+1} - \hat{z}_{1,t+1}\} + \frac{\eta}{\epsilon-1-\eta}E_t\{\widehat{\tilde{z}}_{3,t+1} - \hat{z}_{1,t+1}\}\right)\\ &\quad + \left(1 - \frac{\eta\epsilon}{\epsilon-1-\eta}\right)\beta\theta E_t\hat{\Pi}_{t+1} + (1-\beta\theta)\widehat{mc}_t\end{aligned}$$

Using equation (**) gives

$$\begin{aligned}\frac{\eta-\epsilon+\epsilon\eta+1}{\eta-\epsilon+1}\frac{\theta}{1-\theta}\hat{\Pi}_t &= \frac{\eta-\epsilon+\epsilon\eta+1}{\eta-\epsilon+1}\frac{\theta}{1-\theta}\beta E_t\hat{\Pi}_{t+1}\\ &\quad +(1-\beta\theta)\widehat{mc}_t + \beta\theta\hat{\Lambda}_{t+1,p} - \hat{\Lambda}_{t,p}\end{aligned}$$

Rewriting:

$$\begin{aligned}\hat{\Pi}_t &= \beta E_t\hat{\Pi}_{t+1} + \frac{\eta-\epsilon+1}{\eta-\epsilon+\epsilon\eta+1}\frac{(1-\theta)(1-\beta\theta)}{\theta}\widehat{mc}_t\\ &\quad + \frac{\beta\theta\hat{\Lambda}_{t+1,p} - \hat{\Lambda}_{t,p}}{\frac{\eta-\epsilon+\epsilon\eta+1}{\eta-\epsilon+1}\frac{\theta}{1-\theta}}\end{aligned}$$

Using $\epsilon = \tilde{\epsilon}(1+\eta) = \frac{\mu}{\mu-1}(1+\eta)$ where μ denotes the gross markup:

$$\begin{aligned}\hat{\Pi}_t &= \beta E_t\hat{\Pi}_{t+1} + \frac{(1-\theta)(1-\beta\theta)}{\theta}\frac{1}{1-\mu\eta}\widehat{mc}_t\\ &\quad + \frac{\beta\theta E_t\hat{\Lambda}_{t+1,p} - \hat{\Lambda}_{t,p}}{(1-\mu\eta)\frac{\theta}{1-\theta}}\end{aligned}$$

Using

$$\frac{1}{1+\tau_p}\breve{\tau}_{p,t} = \hat{\Lambda}_{t,p}$$

where $\breve{\tau}_{p,t} = \tau_{p,t} - \tau_p$ and also assuming that $\breve{\tau}_{p,t}$ follows an AR(1) process with persistence ρ_{τ_p} we get:

$$\begin{aligned} \hat{\Pi}_t &= \beta E_t \hat{\Pi}_{t+1} + \frac{(1-\theta)(1-\beta\theta)}{\theta}\frac{1}{1-\mu\eta}\widehat{mc}_t \\ & -\frac{(1-\theta)\left(1-\beta\theta\rho_{\tau_p}\right)}{\theta}\frac{1}{1-\mu\eta}\frac{1}{1+\tau_p}\breve{\tau}_{p,t} \end{aligned}$$

The remaining log-linear equations are:

$$mc_t = \hat{\phi}_t - \hat{A}_t + \sigma\hat{C}_t + \chi\hat{N}_t - \frac{1}{1+\tau}\breve{\tau}_t$$

$$\hat{C}_t = \hat{Y}_t - \frac{1}{1-g}\breve{g}_t$$

where $\breve{g}_t = g_t - g$.

$$\hat{Y}_t = \hat{A}_t + \hat{N}_t$$

$$\hat{\Delta}_t = 0$$

$$\hat{p}_t^* = \frac{\theta}{1-\theta}\hat{\Pi}_t$$

Recall the flexible price economy equations

$$\tilde{Y}_t = \left[\frac{\tilde{\epsilon}-1}{\tilde{\epsilon}}\frac{(1+\tau_t)(1+\tau_{p,t})A_t^{1+\chi}}{\phi_t^{\iota}(1-g_t)^{\sigma}}\right]^{\frac{1}{\sigma+\chi}}$$

$$1 = \beta E_t \nu_{t+1}\left(\frac{1-g_t}{1-g_{t+1}}\right)^{\sigma}\left(\frac{\tilde{Y}_t}{\tilde{Y}_{t+1}}\right)^{\sigma}\widetilde{rr}_t$$

Defining:

$$\begin{aligned} \Lambda_t &= 1+\tau_t \\ \Lambda_{p,t} &= 1+\tau_{p,t} \\ \Gamma_t &= 1-g_t \end{aligned}$$

So that

$$\tilde{Y}_t = \left[\frac{\tilde{\epsilon}-1}{\tilde{\epsilon}}\frac{\Lambda_t \Lambda_{p,t} A_t^{1+\chi}}{\phi_t^{\iota}\Gamma_t^{\sigma}}\right]^{\frac{1}{\sigma+\chi}}$$

$$1 = \beta E_t \nu_{t+1}\left(\frac{\Gamma_t}{\Gamma_{t+1}}\right)^{\sigma}\left(\frac{\tilde{Y}_t}{\tilde{Y}_{t+1}}\right)^{\sigma}\tilde{rr}_t$$

Log-linearizing gives:

$$(\sigma+\chi)\,\hat{\tilde{Y}}_t = (1+\chi)\,\hat{A}_t + \frac{\sigma}{1-g}\breve{g}_t - \iota\hat{\phi}_t + \frac{1}{1+\tau}\breve{\tau}_t + \frac{1}{1+\tau_p}\breve{\tau}_{p,t}$$

$$\hat{\tilde{rr}}_t = \beta E_t\left[\sigma\left(\hat{\tilde{Y}}_{t+1} - \hat{\tilde{Y}}_t\right) - \frac{\sigma}{1-g}\Delta\breve{g}_{t+1} - \hat{\nu}_{t+1}\right]$$

Next, we express the above equations in terms of inflation, the output gap, the policy rate and the potential real rate.

Substituting terms in to the equation for marginal cost gives:

$$mc_t = (\sigma+\chi)\,\hat{Y}_t + \hat{\phi}_t - (1+\chi)\,\hat{A}_t - \frac{\sigma}{1-g}\breve{g}_t - \frac{1}{1+\tau}\breve{\tau}_t$$

Or

$$mc_t = (\sigma+\chi)\left(\hat{Y}_t - \hat{\tilde{Y}}_t\right) + \frac{1}{1+\tau_p}\breve{\tau}_{p,t} + (1-\iota)\hat{\phi}_t$$

Substituting into the Phillips curve gives

$$\hat{\Pi}_t = \beta E_t\hat{\Pi}_{t+1} + \kappa_p\gamma\,(\sigma+\chi)\left(\hat{Y}_t - \hat{\tilde{Y}}_t\right) + \kappa_p\gamma(1-\iota)\hat{\phi}_t - \left(\kappa_{\breve{\tau}_p} - \kappa_p\right)\gamma\breve{\tau}_{p,t}$$

where

$$\begin{aligned}
\kappa_p &= \frac{(1-\theta)\,(1-\beta\theta)}{\theta} \\
\kappa_{\breve{\tau}_p} &= \frac{(1-\theta)\left(1-\beta\theta\rho_{\tau_p}\right)}{\theta}\frac{1}{1+\tau_p} \\
\gamma &= \frac{1}{1-\mu\eta}
\end{aligned}$$

Next, we derive the new-IS equation:

$$0 = E_t\left[\hat{\nu}_{t+1} + \sigma\hat{C}_t - \sigma\hat{C}_{t+1} + \hat{i}_t - \hat{\Pi}_{t+1}\right]$$

Or

$$\left(\hat{Y}_t - \hat{\tilde{Y}}_t\right) = E_t\left(\hat{Y}_{t+1} - \hat{\tilde{Y}}_{t+1}\right) - \frac{1}{\sigma}E_t\left[\hat{i}_t - \hat{\Pi}_{t+1} - \sigma\left(\hat{\tilde{Y}}_{t+1} - \hat{\tilde{Y}}_t\right) + \frac{\sigma}{1-g}E_t\Delta\breve{g}_{t+1} + \hat{\nu}_{t+1}\right]$$

Or

$$\left(\hat{Y}_t - \hat{\hat{Y}}_t\right) = E_t\left(\hat{Y}_{t+1} - \hat{\hat{Y}}_{t+1}\right) - \frac{1}{\sigma}E_t\left[\hat{i}_t - \hat{\Pi}_{t+1} - \hat{\hat{rr}}_t\right]$$

Appendix B Comparing nonlinear pricing equations against LLSY (2007, CEPR) and HLT (2022, 2023)

Nonlinear optimal price setting and dispersion equations. Set $\nu_{t+1} = 1$. Then,

Notes

$$\begin{aligned}
&p_t^* = \tfrac{z_{2,t}}{z_{1,t}} + (p_t^*)^{1+\tilde{\epsilon}(1+\eta)}\,\tfrac{z_{3,t}}{z_{1,t}}\\
&z_{1,t} = \beta\theta E_t\{\Pi_{t+1}^{\tilde{\epsilon}(1+\eta)-1} z_{1,t+1}\} + Y_t^{1-\sigma}\Delta_{2,t}^{\frac{\tilde{\epsilon}(1+\eta)}{1-\tilde{\epsilon}(1+\eta)}}\\
&z_{2,t} = \beta\theta E_t\{\Pi_{t+1}^{\tilde{\epsilon}(1+\eta)} z_{2,t+1}\} + \tfrac{\tilde{\epsilon}(1+\eta)}{\tilde{\epsilon}(1+\eta)-1} Y_t^{1-\sigma} mc_t \Delta_{2,t}^{\frac{\tilde{\epsilon}(1+\eta)}{1-\tilde{\epsilon}(1+\eta)}}\\
&z_{3,t} = \beta\theta E_t\{\Pi_{t+1}^{-1} z_{3,t+1}\} + \eta Y_t^{1-\sigma}\\
&1 = \tfrac{1}{1+\eta}\Delta_{2,t}^{\frac{1}{1-\tilde{\epsilon}(1+\eta)}} + \tfrac{\eta}{1+\eta}\Delta_{3,t}\\
&\Delta_{1,t} = (1-\theta)\,(p_t^*)^{-\tilde{\epsilon}(1+\eta)} + \theta {\pi_t}^{\tilde{\epsilon}(1+\eta)}\Delta_{1,t-1}\\
&\Delta_{2,t} = (1-\theta)\ \,(p_t^*)^{1-\tilde{\epsilon}(1+\eta)} + \theta {\pi_t}^{\tilde{\epsilon}(1+\eta)-1}\Delta_{2,t-1}\\
&\Delta_{3,t} = (1-\theta)\ p_t^* + \theta {\pi_t}^{-1}\Delta_{3,t-1}\\
&\Delta_t \equiv \tfrac{1}{1+\eta}\Delta_{2,t}^{\frac{\tilde{\epsilon}(1+\eta)}{1-\tilde{\epsilon}(1+\eta)}}\Delta_{1,t} + \tfrac{\eta}{1+\eta}
\end{aligned}$$

Levin, Lopez-Salido and Yun (2007, CEPR)

$$\begin{aligned}
&P_t^* = \left(\tfrac{1}{1+\tau_p}\tfrac{\tilde{\epsilon}(1+\eta)}{\tilde{\epsilon}(1+\eta)-1}\right)\tfrac{Z_{2,t}}{Z_{1,t}} + \left(\tfrac{\eta}{\tilde{\epsilon}(1+\eta)}\tfrac{\tilde{\epsilon}(1+\eta)}{\tilde{\epsilon}(1+\eta)-1}\right)(P_t^*)^{1+\tilde{\epsilon}(1+\eta)}\ \tfrac{Z_{3,t}}{Z_{1,t}}\\
&Z_{1,t} = \beta\theta E_t\{\Pi_{t+1}^{\tilde{\epsilon}(1+\eta)-1} Z_{1,t+1}\} + Y_t^{1-\sigma}\Delta_{2,t}^{\frac{\tilde{\epsilon}(1+\eta)}{1-\tilde{\epsilon}(1+\eta)}}\\
&Z_{2,t} = \beta\theta E_t\{\Pi_{t+1}^{\tilde{\epsilon}(1+\eta)} Z_{2,t+1}\} + mc_t Y_t^{1-\sigma}\Delta_{2,t}^{\frac{\tilde{\epsilon}(1+\eta)}{1-\tilde{\epsilon}(1+\eta)}}\\
&Z_{3,t} = \beta\theta E_t\{\Pi_{t+1}^{-1} Z_{3,t+1}\} + Y_t^{1-\sigma}\\
&1 = \tfrac{1}{1+\eta}\Delta_{2,t}^{\frac{1}{1-\tilde{\epsilon}(1+\eta)}} + \tfrac{\eta}{1+\eta}\Delta_{3,t}\\
&\Delta_{1,t} = (1-\theta)\,(p_t^*)^{-\tilde{\epsilon}(1+\eta)} + \theta {\pi_t}^{\tilde{\epsilon}(1+\eta)}\Delta_{1,t-1}\\
&\Delta_{2,t} = (1-\theta)\ \,(p_t^*)^{1-\tilde{\epsilon}(1+\eta)} + \theta {\pi_t}^{\tilde{\epsilon}(1+\eta)-1}\Delta_{2,t-1}\\
&\Delta_{3,t} = (1-\theta)\ p_t^* + \theta {\pi_t}^{-1}\Delta_{3,t-1}\\
&\Delta_t \equiv \tfrac{1}{1+\eta}\Delta_{2,t}^{\frac{\tilde{\epsilon}(1+\eta)}{1-\tilde{\epsilon}(1+\eta)}}\Delta_{1,t} + \tfrac{\eta}{1+\eta}
\end{aligned}$$

Rewriting the first four equations in the second column:

Notes

$$\begin{aligned}
&p_t^* = \left(\tfrac{1}{1+\tau_p}\right)\tfrac{z_{2,t}}{z_{1,t}} + \tfrac{1}{\tilde{\epsilon}(1+\eta)-1}\,(p_t^*)^{1+\tilde{\epsilon}(1+\eta)}\ \tfrac{z_{3,t}}{z_{1,t}}\\
&z_{1,t} = \beta\theta E_t\{\Pi_{t+1}^{\tilde{\epsilon}(1+\eta)-1} z_{1,t+1}\} + Y_t^{1-\sigma}\Delta_{2,t}^{\frac{\tilde{\epsilon}(1+\eta)}{1-\tilde{\epsilon}(1+\eta)}}\\
&z_{2,t} = \beta\theta E_t\{\Pi_{t+1}^{\tilde{\epsilon}(1+\eta)} z_{2,t+1}\} + \tfrac{\tilde{\epsilon}(1+\eta)}{\tilde{\epsilon}(1+\eta)-1} Y_t^{1-\sigma} mc_t \Delta_{2,t}^{\frac{\tilde{\epsilon}(1+\eta)}{1-\tilde{\epsilon}(1+\eta)}}\\
&z_{3,t} = \beta\theta E_t\{\Pi_{t+1}^{-1} z_{3,t+1}\} + \eta Y_t^{1-\sigma}\\
&1 = \tfrac{1}{1+\eta}\Delta_{2,t}^{\frac{1}{1-\tilde{\epsilon}(1+\eta)}} + \tfrac{\eta}{1+\eta}\Delta_{3,t}\\
&\Delta_{1,t} = (1-\theta)\,(p_t^*)^{-\tilde{\epsilon}(1+\eta)} + \theta {\pi_t}^{\tilde{\epsilon}(1+\eta)}\Delta_{1,t-1}\\
&\Delta_{2,t} = (1-\theta)\ \,(p_t^*)^{1-\tilde{\epsilon}(1+\eta)} + \theta {\pi_t}^{\tilde{\epsilon}(1+\eta)-1}\Delta_{2,t-1}\\
&\Delta_{3,t} = (1-\theta)\ p_t^* + \theta {\pi_t}^{-1}\Delta_{3,t-1}\\
&\Delta_t \equiv \tfrac{1}{1+\eta}\Delta_{2,t}^{\frac{\tilde{\epsilon}(1+\eta)}{1-\tilde{\epsilon}(1+\eta)}}\Delta_{1,t} + \tfrac{\eta}{1+\eta}
\end{aligned}$$

Levin, Lopez-Salido and Yun (2007, CEPR)

$$\begin{aligned}
&p_t^* = \left(\tfrac{1}{1+\tau_p}\right)\tfrac{z_{2,t}}{z_{1,t}} + \tfrac{1}{\tilde{\epsilon}(1+\eta)-1}\,(p_t^*)^{1+\tilde{\epsilon}(1+\eta)}\ \tfrac{z_{3,t}}{z_{1,t}}\\
&z_{1,t} = \beta\theta E_t\{\Pi_{t+1}^{\tilde{\epsilon}(1+\eta)-1} z_{1,t+1}\} + Y_t^{1-\sigma}\Delta_{2,t}^{\frac{\tilde{\epsilon}(1+\eta)}{1-\tilde{\epsilon}(1+\eta)}}\\
&z_{2,t} = \beta\theta E_t\{\Pi_{t+1}^{\tilde{\epsilon}(1+\eta)} z_{2,t+1}\} + \tfrac{\tilde{\epsilon}(1+\eta)}{\tilde{\epsilon}(1+\eta)-1} mc_t Y_t^{1-\sigma}\Delta_{2,t}^{\frac{\tilde{\epsilon}(1+\eta)}{1-\tilde{\epsilon}(1+\eta)}}\\
&z_{3,t} = \beta\theta E_t\{\Pi_{t+1}^{-1} z_{3,t+1}\} + \eta Y_t^{1-\sigma}\\
&1 = \tfrac{1}{1+\eta}\Delta_{2,t}^{\frac{1}{1-\tilde{\epsilon}(1+\eta)}} + \tfrac{\eta}{1+\eta}\Delta_{3,t}\\
&\Delta_{1,t} = (1-\theta)\,(p_t^*)^{-\tilde{\epsilon}(1+\eta)} + \theta {\pi_t}^{\tilde{\epsilon}(1+\eta)}\Delta_{1,t-1}\\
&\Delta_{2,t} = (1-\theta)\ \,(p_t^*)^{1-\tilde{\epsilon}(1+\eta)} + \theta {\pi_t}^{\tilde{\epsilon}(1+\eta)-1}\Delta_{2,t-1}\\
&\Delta_{3,t} = (1-\theta)\ p_t^* + \theta {\pi_t}^{-1}\Delta_{3,t-1}\\
&\Delta_t \equiv \tfrac{1}{1+\eta}\Delta_{2,t}^{\frac{\tilde{\epsilon}(1+\eta)}{1-\tilde{\epsilon}(1+\eta)}}\Delta_{1,t} + \tfrac{\eta}{1+\eta}
\end{aligned}$$

Steady State Markup at Zero Inflation

$$\frac{P}{MC} = \frac{1}{1+\tau_p}\frac{\tilde{\epsilon}}{\tilde{\epsilon}-1}$$

$$\frac{P}{MC} = \frac{1}{1+\tau_p}\frac{\tilde{\epsilon}}{\tilde{\epsilon}-1}$$

Which shows that the nonlinear equations in our notes can be written to be identical to those in LLSY (2007, CEPR).

Consider the nonlinear optimal price setting equations in HLT (2022, 2023), set $\delta_{t+1} = \tilde{\Pi}_{t+1} = 1$ and using the same notation for parameters as in these notes, to get

Levin, Lopez-Salido and Yun (2007, CEPR)
(setting $\sigma = 1$ and $\tau_p = 0$)

$$p_t^* = \frac{z_{2,t}}{z_{1,t}} + \frac{1}{\tilde{\epsilon}(1+\eta)-1} (p_t^*)^{1+\tilde{\epsilon}(1+\eta)} \frac{z_{3,t}}{z_{1,t}}$$

$$z_{1,t} = \beta\theta E_t\{\Pi_{t+1}^{\tilde{\epsilon}(1+\eta)-1} z_{1,t+1}\} + \Delta_{2,t}^{\frac{\tilde{\epsilon}(1+\eta)}{1-\tilde{\epsilon}(1+\eta)}}$$

$$z_{2,t} = \beta\theta E_t\{\Pi_{t+1}^{\tilde{\epsilon}(1+\eta)} z_{2,t+1}\} + \frac{\tilde{\epsilon}(1+\eta)}{\tilde{\epsilon}(1+\eta)-1} mc_t \Delta_{2,t}^{\frac{\tilde{\epsilon}(1+\eta)}{1-\tilde{\epsilon}(1+\eta)}}$$

$$z_{3,t} = \beta\theta E_t\{\Pi_{t+1}^{-1} z_{3,t+1}\} + \eta$$

$$1 = \frac{1}{1+\eta}\Delta_{2,t}^{\frac{1}{1-\tilde{\epsilon}(1+\eta)}} + \frac{\eta}{1+\eta}\Delta_{3,t}$$

$$\Delta_{1,t} = (1-\theta)\,(p_t^*)^{-\tilde{\epsilon}(1+\eta)} + \theta\pi_t^{\tilde{\epsilon}(1+\eta)}\Delta_{1,t-1}$$

$$\Delta_{2,t} = (1-\theta)\,(p_t^*)^{1-\tilde{\epsilon}(1+\eta)} + \theta\pi_t^{\tilde{\epsilon}(1+\eta)-1}\Delta_{2,t-1}$$

$$\Delta_{3,t} = (1-\theta)\,p_t^* + \theta\pi_t^{-1}\Delta_{3,t-1}$$

$$\Delta_t \equiv \frac{1}{1+\eta}\Delta_{2,t}^{\frac{\tilde{\epsilon}(1+\eta)}{1-\tilde{\epsilon}(1+\eta)}}\Delta_{1,t} + \frac{\eta}{1+\eta}$$

HLT (2022, 2023)
(noting that $y_t\lambda_t = 1$; redefining variables; $\frac{1+\theta_p}{\theta_p} = \tilde{\epsilon}$ etc)

$$p_t^* = \frac{s_t}{f_t} + (p_t^*)^{1+\tilde{\epsilon}(1+\eta)} \frac{\alpha_t}{f_t}$$

$$f_t = \beta\theta E_t\left\{\Pi_{t+1}^{\tilde{\epsilon}(1+\eta)-1} f_{t+1}\right\} + \Delta_{t,2}^{\tilde{\epsilon}(1+\eta)}$$

$$s_t = \beta\theta E_t\left\{\Pi_{t+1}^{\tilde{\epsilon}(1+\eta)} s_{t+1}\right\} + \frac{\tilde{\epsilon}(1+\eta)}{\tilde{\epsilon}(1+\eta)-1} mc_t \Delta_{t,2}^{\tilde{\epsilon}(1+\eta)}$$

$$(\tilde{\epsilon}(1+\eta)-1)\,\alpha_t = \beta\theta E_t\left\{\Pi_{t+1}^{-1}(\tilde{\epsilon}(1+\eta)-1)\,\alpha_{t+1}\right\} + \eta$$

$$1 = \frac{1}{1+\eta}\Delta_{t,2} + \frac{\eta}{1+\eta}\Delta_{t,3}$$

$$\Delta_{t,1}^{-\tilde{\epsilon}(1+\eta)} = (1-\theta)\,(p_t^*)^{-\tilde{\epsilon}(1+\eta)} + \theta\Pi_t^{\tilde{\epsilon}(1+\eta)}\Delta_{t-1,1}^{-\tilde{\epsilon}(1+\eta)}$$

$$\Delta_{t,2}^{1-\tilde{\epsilon}(1+\eta)} = (1-\theta)\,(p_t^*)^{1-\tilde{\epsilon}(1+\eta)} + \theta\Pi_t^{\tilde{\epsilon}(1+\eta)-1}\Delta_{t-1,2}^{1-\tilde{\epsilon}(1+\eta)}$$

$$\Delta_{t,3} = (1-\theta)\,(p_t^*) + \theta\Pi_t^{-1}\Delta_{t-1,3}$$

$$\Delta_t = \frac{1}{1+\eta}\Delta_{t,2}^{\tilde{\epsilon}(1+\eta)}\Delta_{t,1}^{-\tilde{\epsilon}(1+\eta)} + \frac{\eta}{1+\eta}$$

Redefining dispersion variables etc gives:

Levin, Lopez-Salido and Yun (2007, CEPR)
(setting $\sigma = 1$ and $\tau_p = 0$)

$$p_t^* = \frac{z_{2,t}}{z_{1,t}} + \frac{1}{\tilde{\epsilon}(1+\eta)-1} (p_t^*)^{1+\tilde{\epsilon}(1+\eta)} \frac{z_{3,t}}{z_{1,t}}$$

$$z_{1,t} = \beta\theta E_t\{\Pi_{t+1}^{\tilde{\epsilon}(1+\eta)-1} z_{1,t+1}\} + \Delta_{2,t}^{\frac{\tilde{\epsilon}(1+\eta)}{1-\tilde{\epsilon}(1+\eta)}}$$

$$z_{2,t} = \beta\theta E_t\{\Pi_{t+1}^{\tilde{\epsilon}(1+\eta)} z_{2,t+1}\} + \frac{\tilde{\epsilon}(1+\eta)}{\tilde{\epsilon}(1+\eta)-1} mc_t \Delta_{2,t}^{\frac{\tilde{\epsilon}(1+\eta)}{1-\tilde{\epsilon}(1+\eta)}}$$

$$z_{3,t} = \beta\theta E_t\{\Pi_{t+1}^{-1} z_{3,t+1}\} + \eta$$

$$1 = \frac{1}{1+\eta}\Delta_{2,t}^{\frac{1}{1-\tilde{\epsilon}(1+\eta)}} + \frac{\eta}{1+\eta}\Delta_{3,t}$$

$$\Delta_{1,t} = (1-\theta)\,(p_t^*)^{-\tilde{\epsilon}(1+\eta)} + \theta\pi_t^{\tilde{\epsilon}(1+\eta)}\Delta_{1,t-1}$$

$$\Delta_{2,t} = (1-\theta)\,(p_t^*)^{1-\tilde{\epsilon}(1+\eta)} + \theta\pi_t^{\tilde{\epsilon}(1+\eta)-1}\Delta_{2,t-1}$$

$$\Delta_{3,t} = (1-\theta)\,p_t^* + \theta\pi_t^{-1}\Delta_{3,t-1}$$

$$\Delta_t \equiv \frac{1}{1+\eta}\Delta_{2,t}^{\frac{\tilde{\epsilon}(1+\eta)}{1-\tilde{\epsilon}(1+\eta)}}\Delta_{1,t} + \frac{\eta}{1+\eta}$$

HLT (2022, 2023)
(noting that $y_t\lambda_t = 1$; redefining variables; $\frac{1+\theta_p}{\theta_p} = \tilde{\epsilon}$ etc)

$$p_t^* = \frac{z_{2,t}}{z_{1,t}} + \frac{1}{\tilde{\epsilon}(1+\eta)-1} (p_t^*)^{1+\tilde{\epsilon}(1+\eta)} \frac{z_{3,t}}{z_{1,t}}$$

$$z_{1,t} = \beta\theta E_t\left\{\Pi_{t+1}^{\tilde{\epsilon}(1+\eta)-1} z_{1,t+1}\right\} + \Delta_{t,2}^{\frac{\tilde{\epsilon}(1+\eta)}{1-\tilde{\epsilon}(1+\eta)}}$$

$$z_{2,t} = \beta\theta E_t\left\{\Pi_{t+1}^{\tilde{\epsilon}(1+\eta)} z_{2,t+1}\right\} + \frac{\tilde{\epsilon}(1+\eta)}{\tilde{\epsilon}(1+\eta)-1} mc_t \Delta_{t,2}^{\frac{\tilde{\epsilon}(1+\eta)}{1-\tilde{\epsilon}(1+\eta)}}$$

$$z_{3,t} = \beta\theta E_t\left\{\Pi_{t+1}^{-1} z_{3,t+1}\right\} + \eta$$

$$1 = \frac{1}{1+\eta}\Delta_{2,t}^{\frac{1}{1-\tilde{\epsilon}(1+\eta)}} + \frac{\eta}{1+\eta}\Delta_{t,3}$$

$$\Delta_{t,1} = (1-\theta)\,(p_t^*)^{-\tilde{\epsilon}(1+\eta)} + \theta\Pi_t^{\tilde{\epsilon}(1+\eta)}\Delta_{t-1,1}$$

$$\Delta_{2,t} = (1-\theta)\,(p_t^*)^{1-\tilde{\epsilon}(1+\eta)} + \theta\Pi_t^{\tilde{\epsilon}(1+\eta)-1}\Delta_{2,t-1}$$

$$\Delta_{t,3} = (1-\theta)\,(p_t^*) + \theta\Pi_t^{-1}\Delta_{t-1,3}$$

$$\Delta_t = \frac{1}{1+\eta}\Delta_{t,2}^{\frac{\tilde{\epsilon}(1+\eta)}{1-\tilde{\epsilon}(1+\eta)}}\Delta_{t,1} + \frac{\eta}{1+\eta}$$

Steady State Markup at Zero Inflation

$$\frac{P}{MC} = \frac{\tilde{\epsilon}}{\tilde{\epsilon}-1} \qquad\qquad \frac{P}{MC} = \frac{\tilde{\epsilon}}{\tilde{\epsilon}-1}$$

Which shows that the HLT optimality equations can be rewritten in exactly the same ways as the LLSY (2007, CEPR) equations.

B.1 Solution and Implementation

We use the nonlinear ('simul') solver in Dynare to solve the model. Specifically, we will use the two-point boundary value solver that is implemented in dynare. In stylized form, the model can be written as follows:

$$\begin{aligned}
f(y_{-1}, y_0, y_1; a_0) &= 0 \text{ in period } t = 0 \\
f(y_0, y_1, y_2; a_1) &= 0 \text{ in period } t = 1 \\
f(y_1, y_2, y_3; a_2) &= 0 \text{ in period } t = 2 \\
&\ .... \\
f(y_{T-1}, y_T, y_{T+1}; a_T) &= 0 \text{ in period } t = T
\end{aligned}$$

Where y_0 denotes the vector of endogenous variables of the model. Dynare's <simul> command solves this set of equations for periods $t = 0, ..., T$ using a Newton algorithm.

y_{-1} and y_{T+1} are given, and most often equal the steady state of the model.

To simulate the model stochastically, we apply the solution method sequentially. That is, for each realization of shocks from their stochastic processes, we solve the above system of equations. More precisely, say in period $t = 0$ a shock is observed. Then, we solve the system of equations from $t = 0$ to $t = T$. Then, we move one period forward, i.e. $t = 1$. There, a new shock is realized. We take the state y_0 from the previous simulation as an initial state value and solve the system of equations from $t = 1$ to $t = T$. And so on until no new shocks are realized.

Appendix C Additional results

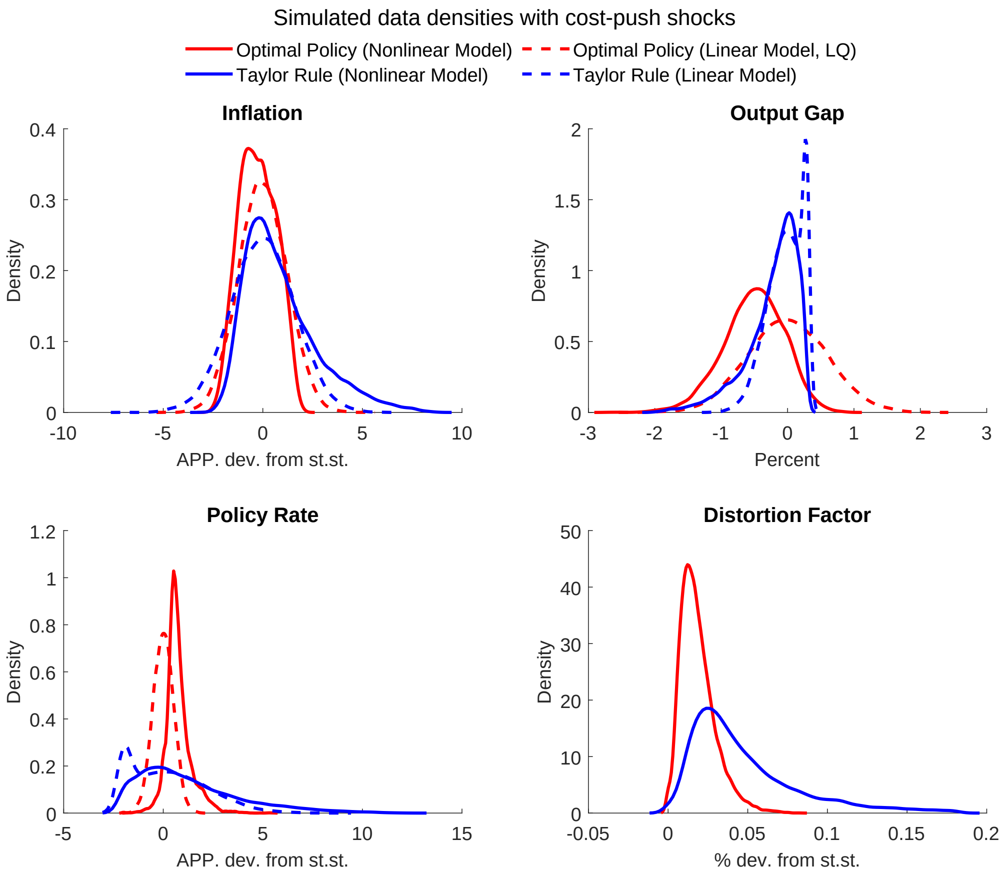

Figure C.1: ZLB: optimal policy in nonlinear model and LQ setup. Densities after long simulation sample with cost-push shocks.

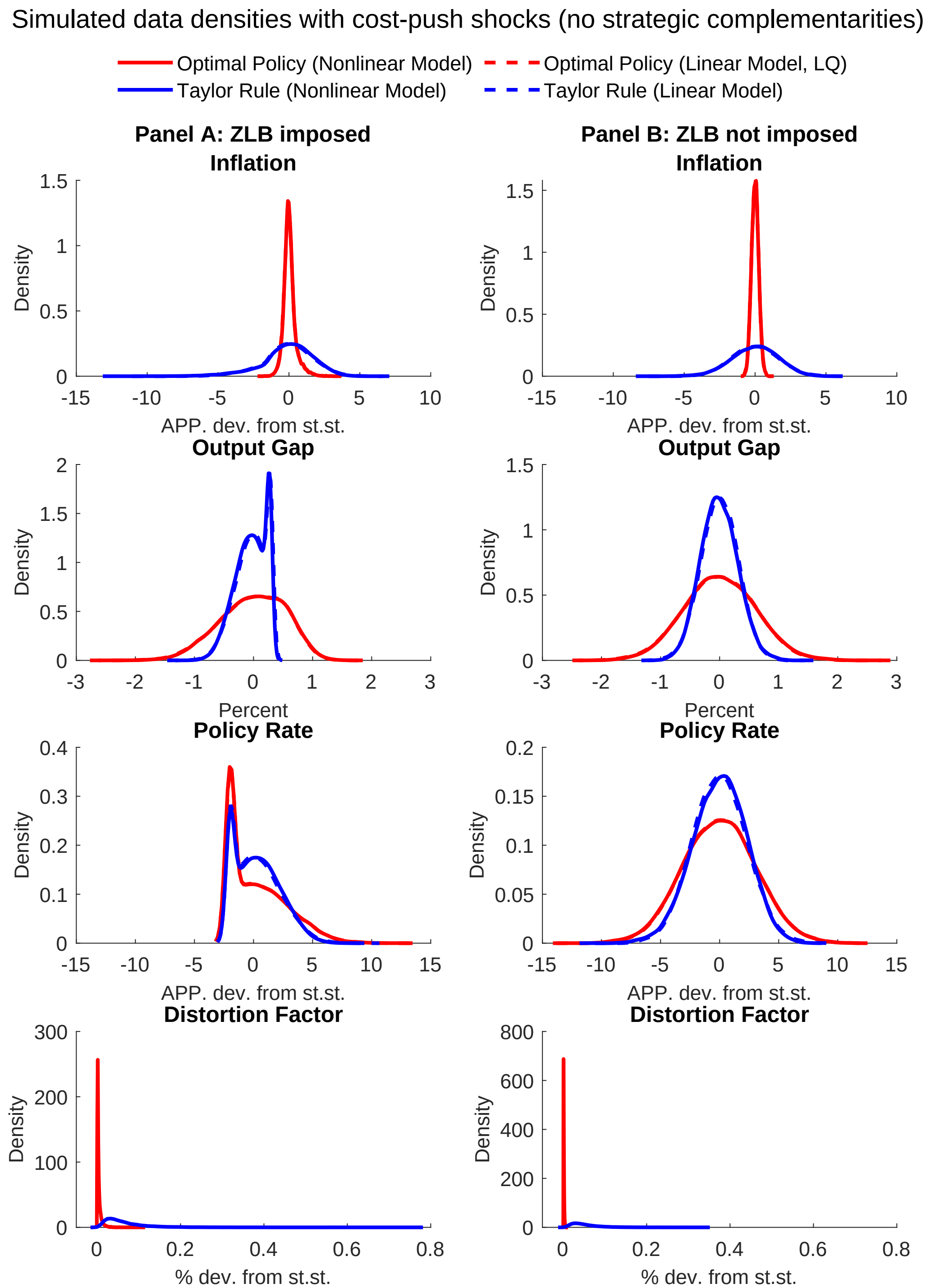

Figure C.2: Optimal policy in nonlinear model and LQ setup without strategic complementarities in price setting (Dixit-Stiglitz model). Densities after long simulation sample with cost-push shocks.