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Personalized Pricing in the Presence of Privacy Concerns[*]

By

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Abstract

We study firms' incentives to adopt a tracking technology to collect personal data that enable personalized pricing in an online market where some consumers have innate desires for privacy. In a model where two differentiated goods are sold under two different market structures (monopoly and duopoly), we find that the presence of these privacy-sensitive consumers alters the firms' incentive to adopt personalized pricing. In particular, no firm uses personalized pricing in equilibrium if the proportion of privacy-sensitive consumers in the market is high. Competition, however, leads to wider use of personalized pricing. Privacy regulation that gives consumers control over whether a firm can track their online activities has the intended impact of protecting consumer privacy only if the proportion of privacy-sensitive consumers is low. Otherwise, the regulation makes the use of tracking technology more widespread. A key force that drives these results is the inability of a monopolist to commit to personalized prices that will give privacy-sensitive consumers a non-negative net surplus. This deters these consumers from purchasing from the firm. If the proportion of privacy-sensitive consumers is high, the risk of losing these consumers induces the monopolist to adopt uniform pricing. Under duopoly, on the other hand, competition between firms alleviates the commitment problem as the rivals undercut each other's prices. Privacy regulation also mitigates the commitment problem because a firm can credibly commit to offer a uniform price to those consumers who reject tracking. Consequently, both competition and privacy regulation lead to increased use of tracking technology.

Key Words: Privacy, Personalized Pricing, First-Degree Price Discrimination, Consumer Control of Personal Data

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1. Introduction

Advances in digital technologies have given firms unprecedented capability to collect and process data related to individual consumers (Belleflamme and Vergote 2016). Increasingly, these personal data are used to facilitate sales of goods and services. For example, technological advances have enabled firms in some industries to adopt first-degree price discrimination, also known as personalized pricing (Choe et al. 2018, Chen et al. 2020, Rhodes and Zhou 2024). Indeed, a study by OECD (2018) has documented the use of personalized pricing in some industries, such as retailing, travel, and personal finance. Examples of firms that adopted personalized pricing include popular e-commerce sites such as Priceline, Orbitz and Home Depot (Hannak et al. 2014). More recently, a study by the Federal Trade Commission (2025) also finds evidence of personalized pricing by some consumer-facing businesses.

The tremendous growth in the collection and monetization of personal data over the Internet has caused significant concerns among the public over consumer privacy (Sokol and Comerford 2016). For instance, a survey conducted by Pew Research Center in 2023 found that 81 percent of American adults were concerned about how companies use their personal data (McClain et al. 2023). More broadly, a study of consumers in 16 countries shows online privacy remains a concern for a clear majority of global consumers, with an average of 71 percent of consumers across the 16 countries indicating a degree of concern with the issue (GDMA & Acxiom 2022).

In response to public concerns over privacy, governments in many jurisdictions have enacted or updated laws and regulations that lay down rules for, among other things, the collection, use, and disclosure of personal data by organizations. A common element among these rules is the requirement that organizations obtain individuals' consent before processing their personal data. For example, one of the two key pillars of the European Union's General Data Protection Regulation (GDPR) is privacy rights, which include (among other things) individuals' right to explicit opt-in consent (Das Chaudhury and Choe 2023). The same approach is adopted in the data privacy statutes of various states in the U.S. (Bellamy 2023). Similarly, the privacy laws in Canada generally require organizations to obtain meaningful consent for the collection and use of personal information (Office of the Privacy Commissioner of Canada 2021).

The objective of this paper is to study firms' incentives to collect personal data and adopt personalized pricing in an online market where some consumers have innate desires for privacy. We analyze a model where two differentiated goods are sold under two different market structures: monopoly and duopoly. A firm may sell its good using a uniform price (i.e., the same price for all consumers). Alternatively, it may adopt a tracking technology that enables it to learn each customer's preference for its good(s). This, in turn, allows the firm to engage in personalized pricing. On the other side of the market, consumers may have privacy concerns over the use of tracking technology by firms. To be more specific, we assume that there are two types of consumers: privacy-insensitive consumers and privacy-sensitive consumers. While privacy-insensitive consumers are indifferent to a firm's use of tracking technology, privacy-sensitive consumers would suffer a small utility loss if their online activities were tracked by a firm.

Using this model, we analyze the firms' pricing strategies and identify the conditions under which a firm uses the tracking technology to engage in personalized pricing. Moreover, we examine the impact of a privacy regulation that gives consumers control over whether a firm is allowed to track their online activities. By comparing the equilibria under monopoly and duopoly, we discover the effects of competition on consumer privacy and firms' pricing strategies.

Our analysis produces three interesting findings. First, it demonstrates the importance of incorporating privacy-sensitive consumers in the analysis of personalized pricing. As elaborated below, a common assumption in the literature on the interactions between personalized pricing and consumer privacy is that privacy has no intrinsic value to consumers: a loss of privacy matters to consumers only to the extent that it has an impact on the prices they pay for a good. Under this assumption, it is always more profitable for a monopoly to adopt personalized pricing than uniform pricing (Armstrong 2006). Similarly, in an oligopoly market it is often a dominant strategy for firms to adopt personalized pricing (e.g., Thisse and Vives 1988, Houba et al. 2023). Indeed, these results hold in our model as well if we remove the privacy-sensitive consumers from the market. However, by incorporating privacy-sensitive consumers into the analysis, we find that a firm does not always adopt personalized pricing in equilibrium. In fact, no firm adopts personalized pricing if the proportion of privacy-sensitive consumers in the market exceeds a certain threshold. Therefore, a major contribution of this paper is that it shows how the presence of privacy-sensitive consumers in a market alters firms' incentive to adopt (or not to adopt) personalized pricing.

Second, our analysis sheds light on the role of markets and competition in protecting consumer privacy. It is widely accepted among economists that markets and competition will often supply adequate consumer protection without the need for extra policy intervention (Armstrong 2008). Our analysis confirms that market forces will drive firms to respect consumer privacy, if there are enough consumers concerned about privacy. This is true even if the market is served by a monopoly. More interestingly, competition does not necessarily lead to a higher degree of consumer privacy. In our model, firms in the duopoly market adopt the tracking technology for a wider range of parameters than a monopolist does, implying that (loosely speaking) oligopolists are more likely to adopt personalized pricing than a monopolist. Consequently, while competition benefits consumers through lower prices, it does not help consumers in the protection of consumer privacy.

Third and finally, our analysis shows that the privacy regulation has an unintended consequence of reducing consumer privacy. Specifically, the regulation makes the use of tracking technology more widespread in the sense that more firm(s) will use it and the same firm will use it in a wider set of circumstances. In a monopoly market, the expanded use of tracking technology caused by the regulation reduces consumer welfare and social welfare. In a duopoly market, however, its impact on consumer and social welfare is more nuanced. While competition in a duopoly market ensures that the regulation raises the overall consumer welfare under many circumstances, the wider use of the tracking technology harms some or all consumers and causes welfare losses in some instances.

Intuitively, a key driving force in our model is a commitment problem with personalized pricing. In the case of monopoly, it means that the firm cannot credibly commit to personalized prices that will give privacy-sensitive consumers a non-negative net surplus.[1] This is because the privacy losses incurred by these consumers are sunk costs and hence are disregarded by the firm when setting personalized prices. Consequently, it sets the personalized price at each consumer's valuation of the good. At this price, a privacy-sensitive consumer would obtain a negative net surplus because of her privacy cost, which deters her from visiting the firm. If the proportion of privacy-sensitive consumers is high, the foregone profit due to the loss of these consumers outweighs the gain from personalized pricing, inducing the firm to adopt uniform pricing instead.

Under duopoly, on the other hand, competition between firms alleviates the commitment problem with personalized pricing, as a rival firm would undercut a personalized price that would leave a negative net surplus for privacy-sensitive consumers. Consequently, duopolists are more likely to adopt personalized pricing than a monopolist. However, competition does not eliminate the commitment problem; it still plays a role in a situation where a firm's rival uses uniform pricing. Because of the sunk privacy cost, a firm that adopts personalized pricing in this situation is not able to commit to personalized prices that would give privacy-sensitive consumers a net surplus no lower than what they will obtain by purchasing from the rival at the uniform price. As a result, this firm is unable to attract any privacy-sensitive consumers with personalized pricing. When the proportion of privacy-sensitive consumers is high, the commitment problem causes both firms to adopt uniform pricing.

The privacy regulation also alleviates the commitment problem with personalized pricing: a firm can now credibly commit to offer a uniform price to those consumers who reject tracking. This increased ability to commit allows the firm to adopt personalized pricing without the risk of driving away all privacy-sensitive consumers. This leads to the surprising result that the privacy regulation actually encourages the adoption of tracking technology.

In addition to a baseline model, we also examine extensions that incorporate (i) asymmetric firms, (ii) incomplete market coverage caused by low consumer valuation, and (iii) alternative timing of price revelation, respectively. These extensions demonstrate the robustness of the results discussed above.

This paper is organized as follows. We discuss the related literature in section 2 and present the

[1] We are grateful to an anonymous reviewer for pointing out the role of this commitment problem in our model.

baseline model in section 3. We examine a monopoly market in section 4 and a duopoly market in section 5, followed by discussions of the three extensions in section 6. We conclude in section 7.

2. Related literature

Our paper is closely related to the growing literature on the interactions between personalized pricing and consumer privacy. This literature examines situations where firms may adopt personalized pricing subject to certain constraints on their ability to collect personal data. These constraints may be imposed by a government regulation that grants consumers control over whether and how much of their personal information is disclosed to firms (Ichihashi 2020, Ali et al. 2023, Loertscher and Marx 2020), or by consumers' ability to hide information about their identities and preferences from firms (Belleflamme and Vergote 2016, de Cornière and Montes 2017, Montes et al. 2019, Chen et al. 2020). One interesting insight from this literature is that the impact of privacy on consumer welfare depends on, among other things, the way privacy is protected. When consumers have control over the amount of personal information disclosed to a firm, they may use it strategically to induce the firm to set lower prices (Ichihashi 2020, Ali et al. 2023). On the other hand, in situations where consumers have an option to hide their personal information ("identity management") in response to personalized pricing, the availability of such option may lead to higher prices (Belleflamme and Vergote 2016, Chen et al. 2020) and reduce total welfare (Montes et al. 2019). These results imply that granting consumer control may be more effective in enhancing consumer welfare than relying on identity management.

In another strand of literature on personalized pricing, consumers are assumed to be passive, that is, consumers take no action beyond their purchase decisions. The focus of this literature is on the effects of personalized pricing on prices and profits when the firms know (or can learn) consumers' valuation for their products. For example, the seminal work by Thisse and Vives (1988) demonstrates that personalized pricing reduces the price paid by every consumer and the profit earned by each firm in a symmetric duopoly model. Since then, this result has been extended to other settings involving asymmetries on the demand or cost side (Shaffer and Zhang 2002, Matsumura and Matsushima 2015, Houba et al. 2023). However, several other studies have uncovered circumstances under which this result does not hold (Ghose and Huang 2009, Esteves 2022, Foros et al. 2024, Lu and Matsushima 2024, Rhodes and Zhou 2024). In particular, Rhodes and Zhou (2024) study personalized pricing in a general oligopoly model and find that given the market structure, the impact of personalized pricing depends on the degree of market coverage. Specifically, personalized pricing intensifies competition if coverage is high, whereas the opposite is true if coverage is low. More related to privacy protection, Taylor and Wagman (2014) investigate the effects of prohibiting personalized pricing in various oligopoly models and find that the winners and losers of such privacy enforcement depend largely on the specific economic setting under consideration. More recently, Choe et al. (2024) study how a data-rich firm can benefit by unilaterally sharing its customer data with a data-poor competitor when the data can be used for price discrimination.

A common assumption in the two strands of literature cited above is that consumers' valuation of privacy is based entirely on pecuniary gains and losses.[2] For example, a consumer suffers a loss from a firm's use of tracking technology not because of its monitoring of her online activities per se, but because the firm uses the data obtained from tracking to charge her higher prices. In these models, privacy has no intrinsic value to consumers. As noted by Loertscher and Marx (2020), "From a consumer surplus perspective, the central issue is not the protection of privacy but rather the protection of information rents."

In reality, however, a consumer's concerns over privacy may go beyond information rents. Many consumers have innate desires for privacy regardless of the associated economic benefits or lack thereof (Acquisti et al. 2016). To these consumers, privacy has an intrinsic value. Moreover, a consumer's concerns over privacy may stem not just from the use of personal data for personalized pricing in a single transaction, but also from potential damages caused by possible data breach or misuse in the future (Romanosky and Acquisti 2009). For these reasons, it is important to incorporate consumers with innate desires for privacy in an analysis of personalized pricing and privacy. This is where our paper contributes to this literature.

As we have noted earlier, one of our interesting findings is that in the absence of privacy regulation, firms will voluntarily refrain from adopting personalized pricing if the proportion of privacy-sensitive consumers exceeds a certain threshold. This finding is reminiscent of de Cornière and Montes (2017) and Ichihashi (2020), in which a monopoly seller has an incentive to commit not to use personal data for price discrimination. In their models, personal data have an additional use, which is to provide product customization (de Cornière and Montes 2017) or product recommendation (Ichihashi 2020). By committing not to price-discriminate, the monopolist is able to induce consumers to disclose personal information, which can then be used to provide better product customization or better product recommendation. This allows the firm to extract the additional value through a higher uniform price. Note, however, these results in de Cornière and Montes (2017) and Ichihashi (2020) depend crucially on the (implicit) assumption that privacy has no intrinsic value to consumers. For consumers who do have intrinsic preferences for privacy, a commitment not to price-discriminate will not be sufficient to induce them to disclose personal information if it is collected and used by the firm for other purposes. In contrast, our analysis incorporates such consumers and uncovers a different mechanism that drives a firm not to engage in price discrimination.

Another distinction of our paper from the aforementioned literature is the broader scope of our analysis. As noted above, many studies of personalized pricing assume that consumers are passive about privacy. Conversely, some of the papers that examine consumers' privacy choices (e.g., Chen et al. 2020, Ali et al.

[2] Lin (2022) calls this type of privacy concerns "instrumental preferences for privacy," which comes from a consumer's anticipated economic loss from revealing her private information to the firm and arises endogenously from a firm's usage of consumer data.

2023) do not consider a firm's choice between personalized pricing and uniform pricing; instead, they assume that a firm will adopt personalized pricing whenever it has the information to do so. Other papers that do consider endogenous choice of personalized pricing (Belleflamme and Vergote 2016, de Cornière and Montes 2017, and Ichihashi 2020) do not incorporate consumers with intrinsic preferences for privacy. In contrast, our model allows firms to choose whether to adopt personalized pricing, on the one hand, and consumers to accept or reject tracking, on the other hand. Moreover, our paper is the first to study endogenous choice of personalized pricing in a model where consumers have innate desires for privacy.

Of course, our paper is not the first one to recognize that consumers have innate desires for privacy. A growing number of papers, including Casadesus-Masanell and Hervas-Drane (2015), Campbell et al. (2015), Choi et al. (2019), Acemoglu et al. (2022), Chen (2022), Miklós-Thal et al. (2024), and Choe et al. (2025), have studied models in which consumers incur a cost of privacy loss. But none of them have examined firms' adoption of personalized pricing. Indeed, a major contribution of our paper is that it sheds light on how consumers' intrinsic preferences for privacy affect firms' incentives to adopt personalized pricing.

Nevertheless, there is a partial overlap in the issues analyzed in this paper and the literature cited in the preceding paragraph, namely, the effects of privacy regulation and the role of competition in the protection of privacy. In this regard, this paper complements the literature by addressing these issues in a context where firms may use personal data to engage in personalized pricing. This is different from the context of this literature, which typically studies models of online platforms that may earn revenue from two sources, one from supplying an online service (e.g., online search) and the other from monetizing the consumer data it collects (e.g., targeted advertising). In these studies, the effects of competition (respectively, privacy regulation) depend to a large extent on how competition (privacy regulation) affects these two sources of revenues.[3] In contrast, the firms in our model have only one revenue source, and the effects of competition (respectively, privacy regulation) depend mostly on how competition (privacy regulation) affects the firms' choices between uniform pricing and personalized pricing. Therefore, these effects are driven by a different mechanism than those in this literature.

Also related to our paper are Anderson et al. (2023) and Rhodes and Zhou (2025). Anderson et al. (2023) study list price competition in an oligopoly market where firms can send personalized discounts afterwards, while Rhodes and Zhou (2025) analyze various strategies of personalization, one of which is

[3] For example, Choe et al. (2025) show that privacy regulation in the form of the GDPR's opt-in requirement boosts demand for a platform's online service by allowing consumers with high privacy costs to buy the service without sharing data. This raises the platform's revenue from the service but lowers its revenue from monetarization of data. Consequently, the effects of privacy regulation depend on which of the two revenue sources is more important to the platform. On the role of competition, Casadesus-Masanell and Hervas-Drane (2015) show that competition drives the firms to focus on a single revenue source and compete at the extensive margin by attracting a larger customer base.

personalized discounting.[4] These studies are different from ours in a couple of important ways. While personalized discounting may appear similar to personalized pricing, they are not the same. As noted in Anderson et al. (2023), the combination of a list price and personalized discount offers creates "a quasi-monopoly trade-off" between marginal and inframarginal customers in an oligopoly setting. A more significant difference from our paper is that both studies take the strategy of personalized discounting as given, that is, they assume firms always engage in personalized discounting for those consumers who disclose personal data. Consequently, they do not consider the possibility that, in response to consumers' privacy concerns, a firm may find it profitable to commit not to engage in personalized discounting. In contrast, an important finding from our analysis is that, even in the absence of privacy regulation, a firm may refrain from personalized pricing due to consumer privacy concerns.

3. Baseline Model

3.1 Consumer Preferences

Our model is built on the Hotelling framework. Consider an online market where consumers' preferences for goods are represented by their locations on a line of unit length. Consumers are uniformly distributed on the line, and each of them buys at most one unit of a good. If a consumer located at x purchases a good located at x_i, her utility from consuming the good is $V - t|x - x_i|$, where $V > 0$ and $t > 0$. Here, $t|x - x_i|$ represents the consumer's mismatch cost due to purchasing a good that is not her most preferred.

In this baseline model, we assume that each consumer's valuation for a good is sufficiently large that $V > 2t$. This assumption ensures that, if there were no privacy considerations, the market would be fully covered (i.e., all consumers would purchase a unit in equilibrium).[5] In other words, any instance of incomplete market coverage that occurs in the baseline model is caused by consumer privacy concerns.

One novel aspect of our model is that consumers may have innate desires for privacy. We assume that consumers are heterogeneous in terms of their sensitivity to privacy. To be more specific, there are two types of consumers: privacy-sensitive and privacy-insensitive. A privacy-sensitive consumer incurs a utility loss, denoted by $D > 0$, if her online activities are tracked by a firm. A privacy-insensitive consumer, on the other hand, does not incur any utility loss from being tracked by a firm. We use $\theta \in (0, 1)$ to denote the fraction of consumers who are privacy-sensitive. We assume that consumers' privacy types are independent of their product preferences/locations.[6]

To be clear, a privacy-sensitive consumer will incur the privacy cost D once she visits the online store of a firm that tracks her online activities. In this scenario, the privacy cost is sunk, independent of whether

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[4] Rhodes and Zhou (2025) call this strategy "personalized pricing". But as they acknowledge in the paper, the personalized prices in their model are "targeted discounts off the list price" (Rhodes and Zhou 2025, p.24).

[5] We extend our analysis to consider a case of smaller V (and hence incomplete market coverage) in section 6.2.

[6] Other papers on digital privacy, e.g., Miklós-Thal et al. (2024) and Choe et al. (2025), have made similar assumptions.

she eventually purchases the good from the firm.[7] The consumer will take this into consideration when deciding which online store she will visit.

We assume the size of privacy cost is modest in the sense that $D < t$. Note that t is the amount of mismatch cost if a consumer at one end of the Hotelling line purchases a unit of the good located at the opposite end; in other words, it is the maximum mismatch cost a consumer could incur in this model. Hence, this assumption means that a consumer's privacy cost is no higher than the maximum mismatch cost.

The assumption of a modest privacy cost is consistent with anecdotal and empirical evidence that individuals are willing to trade their personal information for relatively small rewards. For example, a study by Carrascal et al. (2013) finds that Internet users in Spain valued their online browsing history for about €7, approximately the cost of a Big Mac meal at the time of the study. In a more recent study, Lin (2022) conducted an experiment to estimate intrinsic and instrumental preferences for privacy separately, and her estimated mean value of intrinsic preferences ranges from \$0.14 to \$2.37.

Note that $V > 2t$ and $D < t$ together imply $V > t + D$. The latter ensures that, if the price of a good is 0, both privacy-sensitive and privacy-insensitive consumers will obtain a positive utility from purchasing the good no matter where it is located on the Hotelling line. Another way to interpret these assumptions is that for privacy-sensitive consumers, the size of their privacy costs is smaller than their valuation of any good sold in this market.

Suppose two goods are sold in this market. We name the two goods "A" and "B", with good A located at the left end and good B at the right end of the line. Let p_i denote the price of good i $(= A, B)$. The preceding discussion implies that a consumer's utility from consuming good i is $V - t|x - x_i| - p_i$ if the firm does not use tracking technology, and her utility is $V - t|x - x_i| - p_i - \alpha D$ if the firm uses the technology, where $\alpha = 1$ for a privacy-sensitive consumer and $\alpha = 0$ for a privacy-insensitive consumer.

If the two goods are sold at the same price, a consumer obtains a higher utility from purchasing the good closer to her. We will refer to this good as her "preferred brand". In other words, good A is the preferred brand of consumers located on the left half of the Hotelling line, while good B is the preferred brand of consumers on the right half of the line.

3.2 Firms

On the firm side, we will analyze two types of market structure: monopoly and duopoly. To make the analysis of these two situations comparable, we suppose that the two goods are sold in both instances. In the case of monopoly, the single firm sells both goods, while in the case of duopoly, each firm sells one good. The unit cost of producing each good is constant and is normalized to 0.

[7] Note that the sunk cost property is an implication of a privacy-sensitive consumer's intrinsic preferences for privacy: she incurs a utility loss whenever her online activities are being tracked by a firm, irrespective of her purchase decisions. For empirical evidence on intrinsic preferences for privacy, see Lin (2022).

Each firm sells its good(s) through its online store. Related to consumer privacy, each firm has access to a tracking technology that will enable it to learn the location of any consumer who visits its online store. If a firm chooses to use the tracking technology, it can use the data collected about consumers to offer them personalized prices. Otherwise, the firm charges a uniform price. To focus on the strategic factors that drive a firm's decision regarding the tracking technology, we assume that the cost of the technology is minimal so that it serves only to break a tie when the firm is indifferent between adopting and not adopting it.

We assume that a firm does not observe a consumer's attitude towards privacy, with or without the tracking technology. But it knows the probability distribution of the two types of consumers.

3.3 Privacy Regulation

The privacy regulation to be studied in this paper takes the form of a requirement that a firm should obtain a consumer's consent before it can track her online activities. We choose to study this particular form of privacy regulation because, as noted in Introduction, such a requirement is a common element of the privacy rules in the European Union (Das Chaudhury and Choe 2023), the United States (Bellamy 2023), and Canada (Office of the Privacy Commissioner of Canada 2021). As this requirement gives consumers the right to control whether a firm can collect their personal data, we will refer to it as "consumer control".

In our model, the privacy regulation restricts a firm's ability to apply personalized pricing to all consumers. In the case where a consumer rejects tracking, the firm does not have the data needed to set a personalized price for her. Instead, the firm will offer such consumers a uniform price.

3.4 The Game

The firm(s) and consumers in this market play a three-stage game. The specifics of the game differ somewhat depending on the market structure and the privacy regulation (or the lack of). Here we first describe the game for the case of monopoly under laissez-faire (i.e., without privacy regulation). At stage 1, the monopolist chooses and announces its pricing strategy: personalized pricing (denoted by P) or uniform pricing (denoted by U). Since tracking technology is needed to implement personalized pricing, the selection of this pricing strategy entails the adoption of the tracking technology by the firm. At stage 2, each consumer observes the pricing strategy chosen by the firm and determines whether to visit its online store. At stage 3, the firm chooses its prices, and the consumers make purchase decisions.

In the case where the monopolist is subject to the privacy regulation, stage 3 of the game is modified as follows. At the beginning of stage 3, the firm presents to each consumer who visits its online store a request for permission to track her activities, along with a uniform price for each good. The consumer then answers "yes" or "no" to the request. The firm can track her activities and offer her a personalized price only if she has answered "yes". If the consumer says "no" to the request, she may purchase a good at the uniform price.

In a duopoly market, the three-stage game is modified in a straightforward way. We will present details about the duopoly game in section 5 where this game is analyzed.

In our analysis of the baseline model, we use subgame perfect equilibrium as the solution concept. Incidentally, we assume the following tiebreakers in situations where a consumer is different between two options. If a consumer is indifferent between purchasing and not purchasing a good, she will choose the former.[8] If she is indifferent between a firm that adopts tracking technology and a firm that does not, she chooses to buy from the latter. If a consumer is indifferent between two goods sold under the same pricing strategy, she chooses one of them randomly with equal probability. Under the privacy regulation, if a consumer is indifferent between accepting and rejecting tracking, she accepts it if she is privacy-insensitive but rejects it if she is privacy-sensitive.

Because the technical derivations are generally simple but tedious, we relegate most of them to an online appendix. The proofs of all propositions are also presented in the appendix.

4. Monopoly

In this section, we study the pricing strategy of a monopoly, named firm M. The game between firm M and the consumers is as described in section 3. We will first examine the scenario of no privacy regulation. Then we will investigate the effects of consumer control on the firm and consumers.

4.1 No Privacy Regulation

We use backward induction to find the subgame perfect equilibrium. Depending on the pricing strategy chosen by the firm, there are two subgames following stage 1, one associated with uniform pricing and the other associated with personalized pricing.

First, we examine the subgame following firm M's choice of uniform pricing. Since uniform pricing does not involve the use of personal data, the firm does not adopt the tracking technology. In this subgame, a privacy-sensitive consumer does not incur a privacy cost when visiting the online store. Given the uniform prices of goods A and B (denoted by p_A^U and p_B^U), a consumer's choice of good at stage 3 is determined by a comparison of $U_A^U(x) = V - tx - p_A^U$ with $U_B^U(x) = V - t(1-x) - p_B^U$. The consumer will purchase a unit as long as one of these two utility levels is non-negative.

Using standard procedures, it is straightforward to find that the monopolist's profit-maximizing prices are $p_A^U = p_B^U = V - t/2$. At these prices, the market is fully covered; consumers in $[0, 1/2]$ purchase good A while those in $[1/2\,, 1]$ purchase good B.[9] Then firm M's profit is $\Pi_M^U = V - t/2$.

[8] This assumption is relevant to a situation where a firm sets a price that extracts the entire surplus from a consumer. It can be justified by that the firm in this situation has an incentive to lower the price by an infinitesimal amount to induce purchase by the consumer.

[9] Consumers at $x = 1/2$ are indifferent between good A and good B. The tiebreakers we have assumed in section 3.4 imply that half of these consumers purchase A and the other half purchase B. The same observation applies below whenever consumers at a location are indifferent between the two goods sold under the same pricing strategy.

Second, we analyze the subgame associated with personalized pricing. At stage 3, the profit-maximizing choice of the monopolist is to offer each consumer who visits its store a personalized price that makes her indifferent between purchasing and not purchasing a unit of her preferred brand, that is, $p_i^P(x) = V - tx$ $(i = A, B)$ for a consumer located at $x \in [0, 1/2]$ and $p_i^P(x) = V - t(1-x)$ $(i = A, B)$ for a consumer located at $x \in (1/2, 1]$. Since both goods are offered at the same personalized price, each consumer will purchase her preferred brand. Note that because of the use of tracking technology, a privacy-sensitive consumer incurs the privacy cost D once she visits the online store. Consequently, the privacy cost is a sunk cost at the time when she is offered a personalized price. Therefore, conditional on her visit to the store, she will purchase a unit at the personalized price offered by the monopolist.

However, at stage 2, a privacy-sensitive consumer foresees that if she visits the store, she will incur the privacy cost D and be charged a price that leaves her zero surplus from consuming the good, resulting in a negative net utility ($-D$). Thus, a privacy-sensitive consumer will choose not to visit the store. Privacy-insensitive consumers, on the other hand, will visit the store as they do not incur a privacy cost.

Therefore, in the subgame following the monopolist's choice of personalized pricing, privacy-insensitive consumers visit the store while privacy-sensitive consumers do not. Then the monopolist offers the personalized prices to those consumers who visit its store, with consumers whose $x \leq 1/2$ purchasing good A at price $p_A^P(x) = V - tx$ and consumers whose $x \geq 1/2$ purchasing good B at price $p_B^P(x) = V - t(1-x)$. Since only privacy-insensitive consumers purchase a good in the equilibrium of this subgame, firm M's profit is

$$\Pi_M^P = (1-\theta)\left[\int_0^{1/2}(V - tx)\,dx + \int_{1/2}^{1}\big(V - t(1-x)\big)\,dx\right] = (1-\theta)\left(V - \frac{t}{4}\right). \quad (1)$$

Note that the market is not fully covered in this subgame because privacy-sensitive consumers do not purchase the products.

At stage 1, the monopolist chooses between personalized pricing and uniform pricing by comparing Π_M^P and Π_M^U. A simple comparison of these profits leads to Proposition 1.

Proposition 1: Without privacy regulation, the equilibrium pricing strategy of firm M depends on the proportion of privacy-sensitive consumers. Specifically,

(i) if $\theta < t/(4V - t)$, the firm adopts personalized pricing, and its products are purchased by privacy-insensitive consumers only; and

(ii) if $\theta \geq t/(4V - t)$, it adopts uniform pricing, and its products are purchased by all consumers.

Proposition 1 shows that, even though personalized pricing enables the monopolist to extract the entire surplus from a consumer, it will adopt the tracking technology only when the proportion of privacy-sensitive consumers is small. In a standard textbook model of monopoly, personalized pricing (i.e., first-degree price discrimination) always enables the firm to earn a larger profit than uniform pricing (Armstrong 2006), and hence the firm will adopt personalized pricing whenever it is feasible to do so. Indeed, if we remove privacy-sensitive consumers from our model (i.e., letting $\theta = 0$), our model converges to the textbook model, in which case the firm adopts personalized pricing in equilibrium. However, the inclusion of privacy-sensitive consumers in our model alters the firm's incentive to adopt personalized pricing; it may refrain from using this pricing strategy if a large proportion of consumers are privacy-sensitive.

Proposition 1 confirms a simple but important point about protection of consumer privacy. That is, markets may provide protection for consumer privacy without policy intervention. The condition for this to occur is that there must be a sufficiently large number of consumers who care about privacy. Intuitively, this result is driven by a commitment problem faced by the monopoly firm, that is, it cannot credibly commit to setting personalized prices that will give privacy-sensitive consumers a non-negative net surplus. Because privacy costs are sunk once the privacy-sensitive consumers visit its online store, the monopolist has an incentive to take advantage of this situation by charging them high personalized prices that will extract their entire surplus from consumption, leaving them with a negative net surplus due to the privacy costs. Foreseeing this holdup situation, all privacy-sensitive consumers choose not to visit the firm. When the proportion of privacy-sensitive consumers is large, the foregone profit caused by the loss of these consumers outweighs the gain from personalized pricing, which forces the firm to choose uniform pricing.

It is interesting to note that in the extreme case where all consumers are privacy-sensitive (i.e., $\theta = 1$), the adoption of personalized pricing by the monopolist would eliminate all transactions. This result is reminiscent of a finding in Anderson and Renault (2006) that if a monopolist's advertising allows consumers to fully infer their reservation price, no consumer will buy from the firm. In their model, every consumer incurs a search cost to visit the firm. Similar to the privacy cost in our model, the search cost causes a holdup situation that deters consumers from visiting the firm. Consequently, the monopolist never uses such advertising. In our model, the counterpart to this result is that, if all consumers are privacy-sensitive, a monopolist will never adopt personalized pricing.

4.2 Consumer Control of Privacy

Now we examine the case where privacy regulation gives consumers control over the collection of personal data. Specifically, suppose the firm must obtain a consumer's consent before it can track her online activities. Since uniform pricing does not involve the collection of personal data, this regulatory requirement is not binding if the firm adopts this pricing strategy. In other words, the equilibrium in the subgame following the firm's choice of uniform pricing is the same as that derived in section 4.1.

However, the regulatory requirement is binding if the firm chooses to adopt personalized pricing. For those consumers who reject tracking, the firm will not be able to collect the personal data needed to offer personalized prices tailored to their locations. To attract those consumers, the firm can only offer uniform prices, denoted by $p_A^{P'}$ and $p_B^{P'}$. To the other consumers (i.e., those consumers who accept tracking), the monopolist offers personalized prices based on their locations. To maximize its profit from these consumers, the firm will offer a personalized price that extracts the entire surplus from a consumer, that is, $p_i^P(x) = V - tx$ $(i = A, B)$ for a consumer located at $x \in [0, 1/2]$ and $p_i^P(x) = V - t(1-x)$ $(i = A, B)$ for a consumer located at $x \in (1/2, 1]$.

Anticipating that she will be offered such a personalized price if she accepts tracking, each consumer decides on whether to accept tracking at the beginning of her visit to the online store. A privacy-sensitive consumer will not accept tracking because doing so will result in a negative utility level $(-D)$. On the other hand, a privacy-insensitive consumer may or may not accept tracking depending on how the uniform price of a good compares with the personalized price. Specifically, she will reject tracking if the uniform price is below the personalized price she expects to be offered after agreeing to be tracked.

Note that among all consumers who agree to be tracked, consumers located at the center $(x = 1/2)$ are offered the lowest personalized price, equalling $V - t/2$. This implies that if the uniform prices of goods A and B are below $V - t/2$, every consumer is better off purchasing a unit at the uniform prices and hence none of them will accept tracking.

On the other hand, consumers located at the endpoints $(x = 0$ and $1)$ are offered the highest personalized price, equalling V, among all consumers who accept tracking. Hence, if the uniform prices of goods A and B are above V, all privacy-insensitive consumers will agree to be tracked.

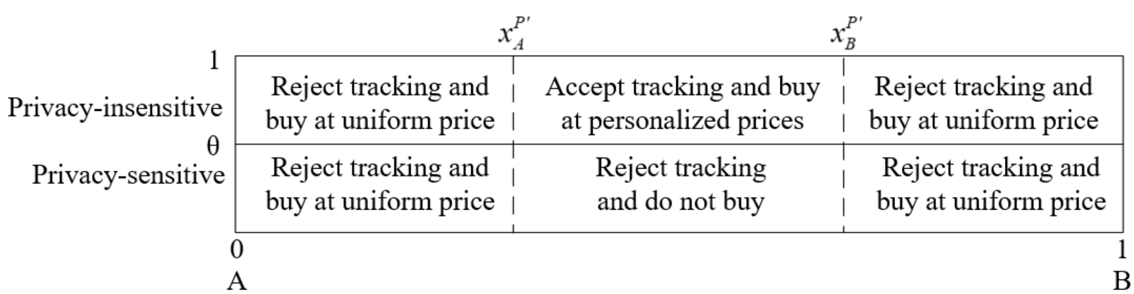

Figure 1: Consumer Decisions when the Uniform Prices Are between $V - t/2$ and V

If the uniform prices of the two goods are between $V - t/2$ and V, some privacy-insensitive consumers will accept tracking while others will not. In particular, consumers who are located around the center may agree to be tracked because their personalized prices are lower than the uniform prices. Let $x_A^{P'}$ and $x_B^{P'}$ denote the locations of the marginal consumers who are indifferent between accepting and rejecting tracking. Then privacy-insensitive consumers in the interval $[x_A^{P'}, x_B^{P'}]$ will accept tracking, while those in $[0, x_A^{P'})$ and $(x_B^{P'}, 1]$ will reject tracking. As noted above, no privacy-sensitive consumers will agree to be tracked. When the uniform prices are in this range, privacy-sensitive consumers in the interval $(x_A^{P'}, x_B^{P'})$

will choose not to purchase the goods because these prices are higher than the utility they would obtain from consuming a unit. Figure 1 illustrates the consumers' decisions in this case.

The preceding analysis implies that, if firm M sets the uniform prices $p_A^{P'}$ and $p_B^{P'}$ at levels that fall in the interval $[V - t/2\,, V]$, its profit can be expressed as

$$\Pi_M^{P'} = p_A^{P'}\, x_A^{P'} + p_B^{P'}\left(1 - x_B^{P'}\right) + (1-\theta)\left[\int_{x_A^{P'}}^{1/2} (V - tx)\, dx + \int_{1/2}^{x_B^{P'}} \left(V - t(1-x)\right) dx\right]. \quad (2)$$

The first two terms on the right-hand side of (2) represent the profit generated by consumers who purchase the goods at uniform prices. The remaining terms denote the profit derived from sales to consumers who purchase the goods at personalized prices, i.e., those consumers who are privacy-insensitive and are located in the interval $[x_A^{P'}, x_B^{P'}]$.

Solving firm M's profit-maximization problem, we find that $p_A^{P'} = p_B^{P'} = V/(1+\theta)$, which falls in the interval $(V - t/2\,, V)$ for $\theta \in (0, t/(2V - t))$.[10] If $\theta \geq t/(2V - t)$, on the other hand, firm M's profit-maximizing prices for consumers who reject tracking are $p_A^{P'} = p_B^{P'} = V - t/2$, which is the same as the uniform prices under laissez-faire. At these prices, no consumer has an incentive to accept tracking.

Proposition 2: Under the privacy regulation, the equilibrium pricing strategy of firm M depends on the proportion of privacy-sensitive consumers. Specifically,

(i) if $\theta < t/(2V - t)$, the firm offers personalized prices to consumers who accept tracking, and its products are purchased by all privacy-insensitive consumers and by those privacy-sensitive consumers located in $[0, x_A^{P'}]$ and $[x_B^{P'}, 1]$;

(ii) if $\theta \geq t/(2V - t)$, it adopts uniform pricing, and its products are purchased by all consumers.

Comparing Proposition 2 with Proposition 1, we see both predict that the firm adopts the tracking technology and personalized pricing if the proportion of privacy-sensitive consumers is below a certain threshold. But they differ in the threshold value. Since $t/(2V - t) > t/(4V - t)$, the firm adopts personalized pricing for a wider range of θ with the privacy regulation than without it. This is an interesting result because it suggests that privacy regulation in the form of consumer control leads to wider use of the tracking technology than without the regulation.

Intuitively, the privacy regulation alleviates the commitment problem that the firm faces with personalized pricing: it can now credibly commit to offer a uniform price to those consumers who reject tracking. This increased ability to commit allows the firm to adopt personalized pricing without the risk of driving away all privacy-sensitive consumers because it can sell the goods to this group of consumers

[10] Note that $p_A^{P'}$ and $p_B^{P'}$ in this case are higher than the uniform prices under laissez-faire, which are equal to $V - t/2$.

through uniform pricing. The additional profit from these consumers under the privacy regulation makes it profitable for the firm to adopt personalized pricing for a wider range of θ.

However, despite the additional profit the firm earns under the privacy regulation, personalized pricing (supplemented by uniform pricing for consumers who reject tracking) is not always more profitable than uniform pricing. This is because some privacy-insensitive consumers may also reject tracking and purchase the goods at the uniform prices. To discourage privacy-insensitive consumers from doing so, the firm raises the uniform prices to the point that consumers with low valuations (i.e., those consumers near the center of the Hotelling line) do not want to buy at these prices. The high uniform prices drive the low-valuation consumers who are privacy-insensitive to accept personalized prices, but they also push the low-valuation consumers who are privacy-sensitive out of the market. When the proportion of privacy-sensitive consumers is high, the foregone profit due to the exit of the privacy-sensitive consumers becomes large, in which case the firm finds it more profitable to offer uniform prices for both types of consumers.

Further analysis of the equilibrium under consumer control sheds light on the effects of the privacy regulation on profit, consumer welfare, and social welfare, with the latter two measured by consumer surplus and total surplus. Using the laissez-faire equilibrium as the benchmark, we summarize the effects of the privacy regulation in Proposition 3.

Proposition 3: The effects of the privacy regulation in the monopoly market depends on the proportion of privacy-sensitive consumers, as follows.

(i) If $0 < \theta < t/(4V - t)$ (in which case the firm adopts personalized pricing with and without the regulation), consumer control raises profit, consumer welfare and social welfare. For individual consumers, it increases the utility of both privacy-sensitive and privacy-insensitive consumers located in $[0, x_A^{P'})$ and $(x_B^{P'}, 1]$ but has no impact on the utility of the remaining consumers.

(ii) If $t/(4V - t) \leq \theta < t/(2V - t)$ (in which case the firm adopts uniform pricing without the regulation but personalized pricing with the regulation), consumer control raises profit but lowers consumer welfare and social welfare. For individual consumers, it lowers the utility of consumers located in $[0, 1/2)$ and $(1/2, 1]$ but has no impact on the utility of consumers at $x = 1/2$.

(iii) If $\theta \geq t/(2V - t)$ (in which case the firm adopts uniform pricing with and without the regulation), consumer control has no impact on the profit, the utility of every consumer, and social welfare.

From part (i) of Proposition 3, we see that if the proportion of privacy-sensitive consumers is small, the monopolist adopts personalized pricing with and without the privacy regulation. In this scenario, the privacy regulation increases the monopolist's profit because it allows the firm to attract some privacy-sensitive consumers who would have stayed away from its store otherwise. This expands the market coverage and improves the welfare of these consumers as they now obtain a positive surplus from the consumption of the goods. More interestingly, the privacy regulation also increases the welfare of those privacy-insensitive consumers who have high valuations for the goods (i.e., those consumers near the endpoints). By reject tracking, these consumers purchase the goods at uniform prices that are lower than their (expected) personalized prices and thus obtain a positive surplus. Social welfare also increases because the expansion in the market coverage of the privacy-sensitive consumers brings a net gain in total surplus.

However, the privacy regulation will reduce consumer welfare and social welfare if the proportion of privacy-sensitive consumers is in an intermediate range (part (ii) in Proposition 3). In this scenario, the privacy regulation causes the monopolist to switch from uniform pricing to personalized pricing. Consumer welfare decreases because (nearly) all consumers are harmed by this switch. Under uniform pricing, all consumers (except those at $x = 1/2$) obtain a positive surplus from consumption. But the switch to personalized pricing under the privacy regulation enables the monopolist to extract the entire surplus from those consumers who purchase at their personalized prices. The other consumers who purchase the goods at the uniform prices are also worse off because these prices are higher than their counterparts under laissez-faire (i.e., $p_i^{P'} > p_i^U, i = A, B$). Moreover, these higher uniform prices drive the privacy-sensitive consumers in $(x_A^{P'}, x_B^{P'})$ out of the market, narrowing the market coverage of this type of consumers. This, in turn, causes deadweight losses and reduces social welfare. In contrast, the firm earns a larger profit in this scenario because of the higher uniform prices and its ability to extract the entire surplus from those privacy-insensitive consumers who purchase the products at personalized prices.

The preceding discussions paint a nuanced picture about the effects of the privacy regulation in a monopoly market. While consumer control is intended to protect consumers, it has the desired impact only when the proportion of privacy-sensitive consumers is so small that the firm would have adopted tracking technology without the regulation. Otherwise, it has either no impact or harmful impact on consumers.

Finally, we end this section by noting how the presence of privacy-sensitive consumers alters the impact of the privacy regulation in a monopoly market. If there were no privacy-sensitive consumers in the market (i.e., $\theta = 0$), the privacy regulation would have no real impact on the equilibrium. We have noted in section 4.1 that in the absence of privacy-sensitive consumers and without the regulation, the monopolist would adopt personalized pricing. With the regulation, the monopolist would still adopt personalized pricing but give consumers an option to purchase the goods at uniform prices if they reject tracking. We show in section A1 of the online appendix that at $\theta = 0$, the uniform prices are $p_A^{P'} = p_B^{P'} = V$, which is equal to the highest personalized price offered by the firm. At these uniform prices, the consumers (who are all privacy-insensitive) have no incentive to reject tracking; all of them will accept tracking and purchase at personalized prices. The intuition for this result is that, with no privacy-sensitive consumers in the market, it is costless for the monopolist to raise the uniform prices to the level that induces all consumers to accept tracking. This enables the monopolist to achieve the same equilibrium as that under laissez-faire.

5. Duopoly

In this section, we consider a duopoly model where goods A and B are produced by two different firms, named by the goods they produce, that is, firm A and firm B. The three-stage game is modified in a straightforward way, with firm M being replaced by firms A and B. Specifically, at stage 1, the firms choose and announce their pricing strategies simultaneously. At stage 2, each consumer observes the pricing strategies chosen by the firms and determines whether to visit the online stores of these firms. At stage 3, the firms choose their prices, after which the consumers make purchase decisions. Following the seminal work of Thisse and Vives (1988), we assume that, in the case where the two firms adopt different pricing strategies (i.e., one firm adopts uniform pricing and the other firm uses personalized pricing), a uniform price is chosen before personalized prices.

As in the monopoly model, we will analyze and compare two scenarios: no regulation vs. consumer control. In the latter scenario, a consumer decides on whether to accept tracking by each firm separately. On the other side of the market, a firm that adopts the tracking technology will offer a uniform price at the time when it seeks consumers' permission for tracking their online activities.

5.1 No Privacy Regulation

Consider the duopoly market without privacy regulation. The choices of pricing strategies by the two firms give rise to four subgames after stage 1. Specifically, they are associated with the following four combinations of pricing strategies: (i) both firms adopt uniform pricing, denoted by (U, U); (ii) both firms adopt personalized pricing, denoted by (P, P); (iii) firm A adopts personalized pricing while firm B adopts uniform pricing, denoted by (P, U); and (iv) firm A adopts uniform pricing while firm B adopts personalized pricing, denoted by (U, P). Below we first analyze each of these four subgames and then determine the equilibrium in the entire game.

5.1.1 The $(\boldsymbol{U}, \boldsymbol{U})$ Subgame

The (U, U) subgame is a standard Hotelling model where two firms compete in (uniform) prices. Since neither firm adopts the tracking technology, a consumer's utility from purchasing a unit from firm A is $U_A^{UU}(x) = V - tx - p_A^{UU}$, and from firm B is $U_B^{UU}(x) = V - t(1-x) - p_B^{UU}$, where p_A^{UU} and p_B^{UU} denote the uniform prices set by firm A and firm B in this subgame. Using standard procedure, we find that the equilibrium prices in this subgame are $p_A^{UU} = t$ and $p_B^{UU} = t$. The firms share the market equally and each firm earns a profit $\Pi_A^{UU} = \Pi_B^{UU} = t/2$. Note that the market is fully covered because the assumption $V > 2t$ ensures that any consumer who purchases a unit at the equilibrium price earns a positive surplus.

5.1.2 The (P, P) Subgame

In the (P, P) subgame, we use $p_A^{PP}(x)$ and $p_B^{PP}(x)$ to denote the personalized prices that firm A and firm B offer to a consumer at location x. Since both firms use the tracking technology in this subgame, a privacy-sensitive consumer incurs the privacy cost D if she visits any online store. Recall, however, the privacy cost is sunk at the time when she is offered a personalized price. Consequently, conditional on her decision to visit the stores, her purchase decision will depend on the surplus she receives from consuming the good, specifically $U_A^{PP}(x) = V - tx - p_A^{PP}(x)$ from good A, and $U_B^{PP}(x) = V - t(1-x) - p_B^{PP}(x)$ from good B. She will purchase a unit from firm A if $U_A^{PP}(x) \geq U_B^{PP}(x)$, or equivalent if $p_A^{PP}(x) \leq t(1-2x) + p_B^{PP}(x)$. This decision rule is also applicable to privacy-insensitive consumers since they do not incur any privacy cost in the first place. Facing such demand conditions, the two firms will attempt to undercut each other's personalized price for every consumer. Such price competition at stage 3 leads to $p_A^{PP}(x) = t(1-2x)$ and $p_B^{PP}(x) = 0$ for $x \in [0, 1/2]$, while $p_A^{PP}(x) = 0$ and $p_B^{PP}(x) = t(2x-1)$ for $x \in [1/2, 1]$.

At stage 2 of the game, a consumer will choose to visit an online store only if she expects that she will receive a non-negative utility from purchasing a unit at stage 3. At the personalized prices offered by the firms at stage 3, a consumer located at $x < 1/2$ receives a higher utility from good A than from good B, and her utility from purchasing a unit of good A is equal to $V - (1-x)t - \alpha D$, where $\alpha = 1$ for a privacy-sensitive consumer and $\alpha = 0$ for a privacy-insensitive consumer. Given the assumptions $V > 2t$ and $D < t$, the utility level is positive for both privacy-sensitive ($\alpha = 1$) and privacy-insensitive ($\alpha = 0$) consumers at these locations. In other words, these consumers will indeed visit firm A's store and purchase a unit in equilibrium. By symmetry, both privacy-sensitive and privacy-insensitive consumers located at $x > 1/2$ will visit firm B's store and purchase a unit. Therefore, in the equilibrium of this subgame, every consumer purchases a unit, and the firms' profits are $\Pi_A^{PP} = \int_0^{1/2} p_A^{PP}(x)\,dx = t/4$ and $\Pi_B^{PP} = \int_{1/2}^{1} p_B^{PP}(x)\,dx = t/4$. It is easy to verify that $\Pi_i^{PP} < \Pi_i^{UU}$ $(i = A, B)$, that is, each firm earns a smaller profit when both firms adopt personalized pricing than when they adopt uniform pricing.

Note that in this subgame, the personalized price paid by a consumer is lower the closer she is to the center of the Hotelling line, with the consumers at the center paying the lowest price of 0. Moreover, for all consumers except those at the two ends ($x = 0, 1$), their personalized prices are less than t. In other words, these consumers pay lower prices in this subgame than in the subgame (U, U). Therefore, our analysis of the (U, U) and (P, P) subgames reproduces the well-known result in Thisse and Vives (1988) that adoption of personalized pricing in a duopoly model intensifies competition among the firms and lowers their profits.

5.1.3 The (P, U) and (U, P) Subgames

Now we consider the (P, U) subgame, where firm A offers personalize prices, denoted by $p_A^{PU}(x)$, and firm

B charges a uniform price, denoted by p_B^{PU}.[11] The analysis of this subgame is more complex than the previous two because the two firms here adopt different pricing strategies. We relegate the technical details to section A2 of the online appendix, and here we summarize the findings from this analysis.

In this subgame, firm A competes for a consumer by offering her a non-negative personalized price that (when feasible) will give her the same surplus from consumption as what she will obtain from firm B at the latter's uniform price. For a privacy-sensitive consumer, however, this personalized price leaves her with a net surplus below what she can obtain from firm B because she will incur a privacy cost if she purchases from firm A. Foreseeing this personalized price, a privacy-sensitive consumer will choose not to visit firm A's store. Consequently, firm A is unable to attract any privacy-sensitive consumers, and it serves only the other type of consumers (i.e., privacy-insensitive consumers) in the equilibrium of this subgame.

Firm B, on the other hand, attracts privacy-sensitive consumers because of its uniform price. Moreover, it may also serve some privacy-insensitive consumers if its uniform price is sufficiently low. The latter, however, depends on θ. If θ exceeds a certain threshold, firm B will find it profitable to charge a high uniform price and serve privacy-sensitive consumers only. Specifically, define

$$\bar{\theta} \equiv \frac{4V - 5t - 4\sqrt{(V-t)(V-2t)}}{8V - 7t}. \qquad (3)$$

It can be shown that $0 < \bar{\theta} < 1/3$. Firm B will serve both privacy-sensitive and privacy-insensitive consumers if $\theta \in (0, \bar{\theta}]$, but it will serve only privacy-sensitive consumers if $\theta \in (\bar{\theta}, 1)$.

Accordingly, the firms' equilibrium profits in this subgame also depends on θ. Specifically, for $\theta \in (0, \bar{\theta}]$, their profits are

$$\Pi_A^{PU} = \frac{t(3-\theta)^2}{16(1-\theta)}, \quad \Pi_B^{PU} = \frac{t(1+\theta)^2}{8(1-\theta)}. \qquad (4)$$

Conversely, if $\theta \in (\bar{\theta}, 1)$, the firms' profits are represented by

$$\Pi_A^{PU} = (1-\theta)(V-t), \ \ \Pi_B^{PU} = \theta(V-t). \qquad (5)$$

The fourth subgame we need to consider is (U, P), where firm A adopts uniform pricing and firm B uses personalized pricing. Since the analysis of this subgame is symmetric to that of (P, U), it is omitted for brevity. We only need to note that the firms' profits in this subgame can be represented by (4) – (5) by interchanging the subscripts A and B.

5.1.4 Equilibrium without Privacy Regulation

Based on the analysis of the four subgames, we can now determine the firms' choices of pricing strategies

[11] This subgame is similar to the situation studied in Choe et al. (2024), where one firm has data on the locations of all consumers while the other firm does not have any data. They show that the data-rich firm can benefit by unilaterally sharing part of the consumer data with the data-poor competitor. Our paper, however, does not consider data sharing between firms. Instead, our focus is on each firm's endogenous decision regarding the adoption of tracking technology and personalized pricing.

at stage 1 of the game. Table 1 summarizes the firms' profits associated with different combinations of pricing strategies. By comparing each firm's profits associated with different pricing strategies, we determine the equilibrium in this game.

Table 1: Firms' Profits without Privacy Regulation

A \ B	U	P
U	$(\frac{t}{2}, \frac{t}{2})$	(Π_A^{UP}, Π_B^{UP})
P	(Π_A^{PU}, Π_B^{PU})	$(\frac{t}{4}, \frac{t}{4})$

Note: (Π_A^{PU}, Π_B^{PU}) is represented by (4) in the case $\theta \in (0, \bar{\theta}]$, and by (5) in the case $\theta \in (\bar{\theta}, 1)$. The payoffs in (Π_A^{UP}, Π_B^{UP}) are symmetric to those in (Π_A^{PU}, Π_B^{PU}).

To facilitate presentation of the equilibrium, we define two more critical values of θ:

$$\hat{\theta} \equiv \frac{t}{4(V-t)}; \quad \tilde{\theta} \equiv \frac{2V-3t}{2(V-t)}. \qquad (6)$$

It is easy to verify that $V > 2t$ implies $\hat{\theta} < 1/4$ and $\tilde{\theta} > 1/2$. This, in turn, implies that $\hat{\theta} < \tilde{\theta}$. It turns out that these two critical values of θ delineate the boundaries for different combinations of equilibrium pricing strategies, as explained in the following proposition.

Proposition 4: Without privacy regulation, the equilibrium pricing strategies of the duopoly depend on the proportion of privacy-sensitive consumers. Specifically,

(i) both firms adopt personalized pricing if $\theta < \min\{\hat{\theta}, \sqrt{5}-2\}$;

(ii) one firm adopts personalized pricing and the other firm adopts uniform pricing if $\min\{\hat{\theta}, \sqrt{5}-2\} \leq \theta < \tilde{\theta}$;

(iii) both firms adopt uniform pricing if $\theta \geq \tilde{\theta}$.

Figure 2 illustrates the three scenarios in Proposition 4. It shows that both firms adopt personalized pricing (the (P, P) equilibrium) when the proportion of privacy-sensitive consumers is small, and both firms adopt uniform pricing (the (U, U) equilibrium) when the proportion is large. One firm adopts personalized pricing while the other firm adopts uniform pricing (the (P, U) and (U, P) equilibria) when the proportion of privacy-sensitive consumers is in the intermediate range.

It is instructive to compare the duopoly equilibrium in Proposition 4 with the monopoly equilibrium in Proposition 1. One similarity between the two is that no firm adopts personalized pricing if the proportion of privacy-sensitive consumers is large. However, comparing the thresholds of θ in Propositions 1 and 4, we find that $\tilde{\theta} > \min\{\hat{\theta}, \sqrt{5}-2\} > t/(4V-t)$. This implies that the range of parameters for which no firm adopts personalized pricing under oligopoly is narrower than that under monopoly. Loosely speaking, this suggests that increased competition makes uniform pricing less likely in equilibrium.

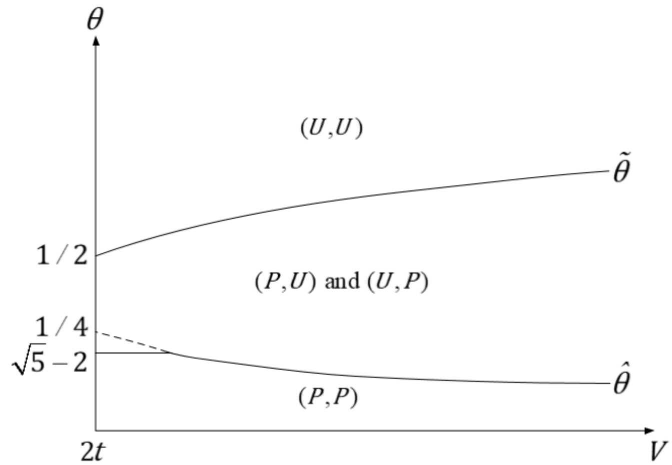

Figure 2: Duopoly Equilibrium without Privacy Regulation

An apparent difference between Propositions 1 and 4 is the possibility of an asymmetric equilibrium under duopoly, where one firm adopts personalized pricing while the other firm uses uniform pricing. On the surface, this result appears to demonstrate a benefit of competition, that is, competition offers consumers more choices of privacy protection. When the proportion of privacy-sensitive consumers is in the intermediate range, competition between the two firms drives one of them to offer uniform pricing to attract these consumers.

However, a close examination of the condition under which the asymmetric duopoly equilibrium arises indicates that this additional choice of privacy protection is an illusion. To understand this, note that the asymmetric equilibrium prevails under duopoly if $\min\{\hat{\theta}, \sqrt{5}-2\} \leq \theta < \tilde{\theta}$, while the monopolist adopts uniform pricing if $\theta \geq t/(4V-t)$. Since $t/(4V-t) < \min\{\hat{\theta}, \sqrt{5}-2\} < \tilde{\theta}$, the asymmetric duopoly equilibrium prevails in a range of parameters over which the tracking technology would not have been used under monopoly. Moreover, $t/(4V-t) < \min\{\hat{\theta}, \sqrt{5}-2\}$ also implies that both firms adopt the tracking technology under duopoly over a wider range of parameters than the monopolist does. Therefore, while competition brings lower prices to consumers, it does not provide more protection for consumer privacy.

The intuition behind these results can be explained using the commitment problem with personalized pricing. Under duopoly, competition between firms alleviates the commitment problem, as a rival firm would undercut any personalized price that would leave a negative net surplus for privacy-sensitive consumers. For example, in the (P, P) subgame where both firms adopt personalized pricing, the highest personalized price is t, paid by consumers located at the two ends of the Hotelling line. At this price, the privacy-sensitive consumers at these locations still obtain a positive net surplus (because $V - t - D > 0$).

Therefore, a duopolist does not have to be concerned about losing privacy-sensitive consumers as much as a monopolist. Consequently, a duopolist is more likely to adopt personalized pricing than a monopolist.

While competition alleviates the commitment problem with personalized pricing, it does not eliminate it. As we have noted in the discussion of the (P, U) subgame in section 5.1.3, the firm that adopts personalized pricing in this subgame is not able to commit to offer privacy-sensitive consumers a price that will give them a net surplus no lower than what they can receive from its rival (who does not use tracking technology). As a result, it is unable to attract any privacy-sensitive consumers. When the proportion of privacy-sensitive consumers is sufficiently large, this commitment problem drives both firms to adopt uniform pricing.

Finally, note that if we remove the privacy-sensitive consumers from the market (i.e., setting $\theta = 0$), our duopoly model converges to the classic Thisse and Vives (1988) model of first-degree price discrimination, in which case personalized pricing is a dominant strategy for every firm. Our analysis shows that the presence of privacy-sensitive consumers alters a firm's incentive to adopt personalized pricing. Parts (ii) and (iii) of Proposition 4 tell us that if the proportion of these consumers exceeds $\min\{\hat{\theta}, \sqrt{5} - 2\}$, at least one firm will find it profitable to switch from personalized pricing to uniform pricing.

5.2 Duopoly and Consumer Control

Now we study duopoly in the scenario where privacy regulation gives consumers control over the collection of personal data. Consequently, a firm must obtain a consumer's consent before it can track her online activities. As in the monopoly model, each firm presents this request for consent, along with a uniform price for its product, to every consumer who visits its online store at the beginning of stage 3. The firm may track a consumer's activities and offer her a personalized price only if she agrees to be tracked. Accordingly, the personalized prices are offered after the firm has set its uniform price.

Recall that a consumer's privacy cost is sunk after her online activities have been tracked by a firm. To simplify the analysis of the subgame where both firms adopt the tracking technology, we assume that the magnitude of privacy cost (D) is independent of whether her activities are tracked by one firm or by both firms. This assumption implies that a privacy-sensitive consumer will either reject tracking by any firm or accept tracking by both firms (if both firms adopt the tracking technology).

To distinguish from the case of no privacy regulation, we add "′" to indicate the pricing strategies under the privacy regulation. That is, we use U' to denote uniform pricing and P' to denote personalized pricing under the privacy regulation. Accordingly, the four subgames after stage 1 are (U', U'), (P', P'), (P', U') and (U', P'). Due to space constraint, we relegate the technical details to section A3 of the online appendix, and here we summarize the findings from this analysis.

In the (U', U') subgame, neither firm uses the tracking technology. Thus, the firms' pricing decisions are not constrained by the privacy regulation. Consequently, the equilibrium prices and profits are the same as those in the case of (U, U), that is, $p_A^{U'U'} = p_B^{U'U'} = t$ and $\Pi_A^{U'U'} = \Pi_B^{U'U'} = t/2$.

In the (P', P') subgame, both firms adopt the tracking technology, but they must obtain a consumer's consent before they can track her online activities. Because of their privacy costs, privacy-sensitive consumers are willing to pay a higher uniform price to avoid being tracked by firms. Consequently, in the equilibrium of this subgame, each firm sets its uniform price at such a high level that all privacy-insensitive consumers accept tracking and only some privacy-sensitive consumers find it beneficial to reject tracking. Specifically, in the equilibrium of this subgame, $p_A^{P'P'} = p_B^{P'P'} = t$, and privacy-sensitive consumers in the intervals $[0, D/2t]$ and $[1 - D/2t, 1]$ reject tracking and purchase at the uniform prices. However, privacy-sensitive consumers in $(D/2t, 1 - D/2t)$ accept tracking because for them the benefit of lower personalized prices outweighs their privacy costs. The firms' equilibrium profits in this subgame are

$$\Pi_A^{P'P'} = \Pi_B^{P'P'} = \frac{t}{4} + \frac{\theta D^2}{4t}. \qquad (7)$$

It can be seen from (7) that each firm's profit increases with the privacy cost D. This is because the size of privacy cost affects the number of privacy-sensitive consumers who reject tracking and purchase at the uniform prices; larger privacy cost causes more of them to purchase at the uniform prices. Since the uniform prices are higher than the personalized prices for these consumers, a firm earns a larger profit when more consumers buy its product at the uniform price.

In the (P', U') subgame, only firm A uses the tracking technology. In contrast to the (P, U) subgame under laissez-faire, firm A can now attract privacy-sensitive consumers by offering a uniform price to those who reject tracking. As a result, firm B no longer has an advantage over firm A in attracting privacy-sensitive consumers. It then charges a uniform price low enough to serve both privacy-sensitive and privacy-insensitive consumers. The firms' equilibrium profits in this subgame are represented by

$$\Pi_A^{P'U'} = \frac{9t(1+\theta)}{4(2+\theta)^2}, \qquad \Pi_B^{P'U'} = \frac{t(1+2\theta)^2}{2(2+\theta)^2}. \qquad (8)$$

Finally, the analysis of the (U', P') subgame is symmetric to that of (P', U'). We can obtain the equilibrium profits by interchanging the subscripts A and B in (8).

Table 2: Firms' Profits with the Privacy Regulation

A \ B	U'	P'
U'	$(\frac{t}{2}, \frac{t}{2})$	$(\frac{t(1+2\theta)^2}{2(2+\theta)^2}, \frac{9t(1+\theta)}{4(2+\theta)^2})$
P'	$(\frac{9t(1+\theta)}{4(2+\theta)^2}, \frac{t(1+2\theta)^2}{2(2+\theta)^2})$	$(\frac{t}{4} + \frac{\theta D^2}{4t}, \frac{t}{4} + \frac{\theta D^2}{4t})$

Using the findings from the analysis of the four subgames, we present in Table 2 the firms' profits associated with different combinations of pricing strategies. By comparing each firm's profits associated with different strategy profiles, we determine the firms' equilibrium pricing strategies at stage 1 of the game.

It turns out that the equilibrium in this game depends, in part, on the magnitude of D. Define

$$D^* \equiv \frac{t}{2+\theta}\sqrt{\frac{7\theta^2+4\theta-2}{\theta}}. \qquad (9)$$

It can be verified that $D^* \in (0,t)$ for θ in the open interval between $(3\sqrt{2}-2)/7$ (≈ 0.32) and 1.

Proposition 5: Under the privacy regulation, the equilibrium pricing strategies of the duopoly depend on the proportion of privacy-sensitive consumers and the size of privacy cost. Specifically,

(i) if either (a) $\theta \leq (3\sqrt{2}-2)/7$, or (b) $\theta > (3\sqrt{2}-2)/7$ and $D \in (D^*,t)$, both firms adopt the tracking technology and offer personalized prices to consumers who accept tracking;

(ii) if $\theta > (3\sqrt{2}-2)/7$ and $D \leq D^*$, one firm adopts the tracking technology and offers personalized prices to consumers who accept tracking while the other firm sells its product at a uniform price and does not use the tracking technology.

An interesting observation from Proposition 5 is the absence of an equilibrium where no firm adopts the tracking technology. This contrasts with the monopoly case in Proposition 2, where the monopolist under the privacy regulation chooses not to use the tracking technology when the proportion of privacy-sensitive consumers is sufficiently high. It shows that in a market with competition, the privacy regulation causes one or both firms to use the tracking technology even if the proportion of privacy-sensitive consumers is high. This reinforces an earlier observation in section 5.1 that competition does not provide more protection for consumer privacy.

Another interesting observation from Proposition 5 is that a larger privacy cost leads to wider adoption of the tracking technology under the privacy regulation. Specifically, in the case where $\theta > (3\sqrt{2}-2)/7$, only one firm adopts the tracking technology when D is small (i.e., when $D \leq D^*$), but both firms adopt the tracking technology when D is larger. This result is counter-intuitive because one might have thought that as the privacy cost becomes larger, competition should drive a firm to adopt uniform pricing as a way to attract more privacy-sensitive consumers. But what actually happens is that in the (P', P') subgame, a larger privacy cost D drives more consumers to reject tracking and purchase at the uniform prices. This increases each firm's profit in this subgame (see equation (7)) because the uniform price is higher than the personalized price. Consequently, starting from the (P', U') or (U', P') subgame in which one of the firms adopts uniform pricing, a larger privacy cost strengthens this firm's incentive to switch from uniform pricing to personalized pricing, thus leading to wider adoption of the tracking technology.

Note also that if we remove the privacy-sensitive consumers from the model (i.e., setting $\theta = 0$), both firms would adopt the tracking technology and, for those who reject tracking, set the uniform prices at t. Given the high uniform prices, all consumers would accept tracking and purchase at personalized prices. In the absence of the privacy-sensitive consumers, therefore, the duopoly equilibrium with the privacy regulation is effectively the same as that without the regulation.

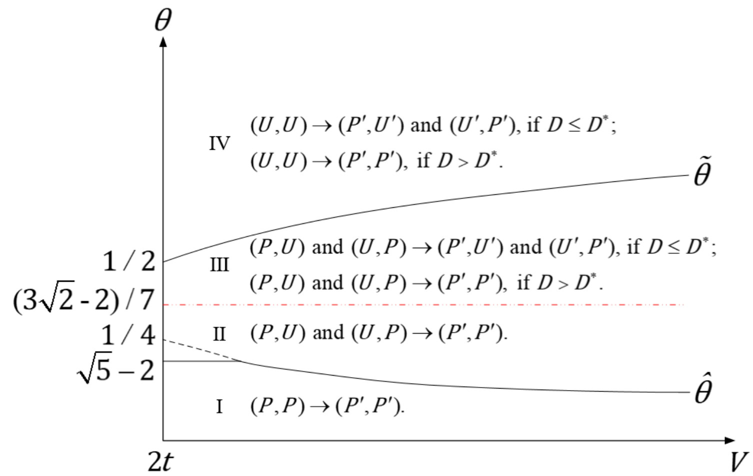

Figure 3: Pricing Strategies with and without the Privacy Regulation

5.3 Effects of the Privacy Regulation

Returning to the case $\theta > 0$, we now examine the effects of the privacy regulation under oligopoly. We start by considering how consumer control over data collection affects the firms' choices of pricing strategies. Figure 3, constructed using the results in Propositions 4 and 5, illustrates the difference in pricing strategies with and without the regulation. The figure is divided into four regions.

- In region I, where $\theta < \min\{\hat{\theta}, \sqrt{5} - 2\}$, both firms adopt personalized pricing with and without the privacy regulation. In this scenario, the regulation does not change the personalized prices offered by the two firms, but it enables some consumers to reject tracking and purchase at uniform prices.
- In region II, where $\min\{\hat{\theta}, \sqrt{5} - 2\} \leq \theta \leq (3\sqrt{2} - 2)/7$, only one firm adopts personalized pricing without the privacy regulation, but the regulation causes both firms to do so.
- In region III, where $(3\sqrt{2} - 2)/7 < \theta < \tilde{\theta}$, only one firm adopts the tracking technology and personalized pricing without the privacy regulation, but the impact of the regulation on the firms' pricing strategies depends on the size of privacy cost D. If the privacy cost is small ($D \leq D^*$), the privacy regulation has no impact on the firms' pricing strategies. But with a larger privacy cost ($D^* < D < t$), the privacy regulation causes both firms to adopt personalized pricing.
- In region IV, where $\theta \geq \tilde{\theta}$, both firms adopt uniform pricing without the privacy regulation. But the regulation causes at least one firm to adopt personalized pricing. In particular, if $D^* < D < t$, both firms switch from uniform pricing to personalized pricing as a result of the regulation.

An important observation from the preceding discussions is that the privacy regulation significantly expands the range of parameters for which both firms adopt the tracking technology and personalized pricing. As we see from Figure 3, while the (P, P) equilibrium occurs in region I only, the (P', P') equilibrium prevails in regions I and II, as well as in regions III and IV if the privacy cost is larger than D^*.

Another notable observation is that the privacy regulation eliminates the equilibrium in which both firms adopt uniform pricing. As a result, at least one firm adopts personalized pricing in equilibrium. Therefore, the privacy regulation makes the use of the tracking technology more widespread in the sense that more firm(s) will use it and the same firm will use it in a wider set of circumstances.

The intuition behind these results is similar to that in the case of monopoly, that is, the privacy regulation alleviates the commitment problem with personalized pricing. To be more specific, recall from section 5.1 that under laissez-faire, the commitment problem plays in role in the subgame where only one firm adopts personalized pricing. Because of the commitment problem, this firm is unable to attract any privacy-sensitive consumers. Under the privacy regulation, however, this firm is able to attract some of these consumers because of its uniform price for consumers who reject tracking. This improves the profitability of adopting personalized pricing unilaterally. Consequently, the privacy regulation expands the adoption of personalized pricing and eliminates the equilibrium where both firms adopt uniform pricing.

Not surprisingly, the change in prices and pricing strategies induced by the privacy regulation has effects on industry profit (i.e., the sum of two firms' profits), consumer welfare and social welfare. These effects are summarized in the following proposition.

Proposition 6: The effects of the privacy regulation in the duopoly market depend on the proportion of privacy-sensitive consumers, as follows.

(i) If $\theta < \min\{\hat{\theta}, \sqrt{5} - 2\}$, consumer control raises industry profit, consumer welfare and social welfare. For individual consumers, it increases the utility of consumers who choose to reject tracking, but it has no impact on the utility of the remaining consumers.

(ii) If $\theta \geq \min\{\hat{\theta}, \sqrt{5} - 2\}$, consumer control reduces industry profit. While it raises the overall consumer welfare, it lowers the utility of some consumers for certain range of parameters. Moreover, consumer control increases social welfare if $\min\{\hat{\theta}, \sqrt{5} - 2\} \leq \theta < \tilde{\theta}$. But it reduces social welfare if $\theta \geq \tilde{\theta}$.

Part (i) of Proposition 6 describes the effects of the privacy regulation in region I of Figure 3, where the proportion of privacy-sensitive consumers is so small that both firms adopt the tracking technology with and without the regulation. In this case, the privacy regulation does not change the firms' personalized prices because competition under laissez-faire already drives them to offer the lowest personalized price for each consumer. As a result, the regulation has no impact on the utility of those consumers who continue to pay personalized prices under the regulation. But the privacy regulation creates an additional option for those consumers who care about privacy: it enables them to reject tracking and purchase the goods at uniform prices. While the uniform prices are higher than the personalized prices that these consumers would have paid if they had accepted tracking, for them the elimination of privacy costs outweighs the higher prices and hence they are better off. Because of these higher uniform prices, each firm earns a larger profit. Therefore, in this scenario the privacy regulation benefits both consumers and firms, thus increasing social welfare.

Part (ii) of Proposition 6 summarizes the effects of the privacy regulation in regions II, III and IV of Figure 3. In these regions, at most one firm adopt personalized pricing under laissez-faire. But under the privacy regulation, one or both firms adopt personalized pricing. Moreover, a firm that adopts personalized pricing offers two types of prices (personalized prices and a uniform price). The wider adoption of personalized pricing and the additional price offers under the regulation intensify price competition and lead to lower prices for at least some consumers. These lower prices, in turn, reduce industry profit but raise consumer welfare. These consequences for industry profit and consumer welfare imply that the impact of the regulation on social welfare may go in either direction. Indeed, Proposition 6 states that the impact on social welfare is positive in regions II and III but negative in region IV of Figure 3.

Another takeaway from Proposition 6 is that while the privacy regulation raises the overall consumer welfare, it lowers the utility of some consumers under certain circumstances. Details about the latter can be found in section A3.5 of the online appendix, where we present the precise conditions under which some consumers are made worse off by the regulation and identify the locations of these consumers on the Hotelling line.

An interesting observation from Propositions 3 and 6 is that consumer control increases a firm's profit under some circumstances. This begs the question: Will a firm voluntarily allow consumer control even without regulation? Our answer to this question is, it depends on whether there are any impediments to voluntary adoption of consumer control. In this regard, one possible impediment is a commitment problem akin to the one discussed earlier: once a firm adopts tracking technology, it can increase its profit by collecting and using the personal data from every customer, regardless of whether she accepts tracking. This may cast doubts among consumers about the credibility of the firm's promise of not collecting data from those consumers who refuse tracking. Under privacy regulation, the credibility of the firm's promise is supported by legal sanctions for violations. But a voluntary promise to allow consumer control may not carry the same degree of credibility. Therefore, a voluntary commitment to privacy protection by a firm will not work if consumers do not believe in the credibility of such a commitment.[12]

[12] In our model, we could adopt the alternative assumption that a firm can voluntarily allow consumer control whenever it is profitable to do so. Under this assumption, the (mandatory) privacy regulation will have no impact in situations where consumer

6. Extensions

In this section, we extend our analysis by relaxing some of the assumptions in the baseline model. Specifically, we will modify our model to allow, respectively, (i) asymmetric firms, (ii) incomplete market coverage caused by low consumer valuation, and (iii) alternative timing of price revelation.[13] The purpose of these extensions is to demonstrate the robustness of our main results and, in one instance, to reinforce them with an additional proposition. Due to space constraint, we will present here only an overview of these three extensions. Interested readers can find details of these analyses in part B of the online appendix.

6.1 Asymmetric Firms

In the baseline model, we have assumed that the firms in the duopoly market are symmetric ex ante. In this section, we relax this assumption and explore the implications of asymmetric firms. Specifically, we consider a situation where the two goods have different quality levels so that a consumer's utility from consuming good i $(= A, B)$ is $V_i - t|x - x_i|$. We assume that $V_A > V_B$, in other words, good A is of higher quality than good B. Let $\Delta V \equiv V_A - V_B$, which represents the quality difference between the two goods. Intuitively, it is clear that if the quality difference is sufficiently large, firm B may be driven out of the market. To ensure that both firms have positive market shares in equilibrium, we assume that $\Delta V < t - D$. All other aspects of the model remain the same as those of the baseline model.

In the literature, Houba et al. (2023) study endogenous adoption of personalized pricing in a Hotelling model with asymmetric firms. They demonstrate that adopting personalized pricing is the dominant strategy for both firms. But with privacy-sensitive consumers in the model, our analysis in section B1 of the online appendix shows that adopting personalized pricing is no longer a dominant strategy. If the proportion of privacy-sensitive consumers is sufficiently high, both firms adopt uniform pricing instead of personalized pricing. But the privacy regulation leads to wider adoption of personalized pricing. Therefore, the main takeaways from this model with asymmetric firms are the same as those from the baseline model.

Moreover, the model with asymmetric firms reveals a new possibility about the impact of the privacy regulation on consumers. In the baseline model, we have found that the regulation always raises the overall consumer welfare. However, this is not necessarily true when firms are asymmetric.

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control increases the firms' profits (because the firms will allow consumer control even without the regulation), but the regulation will have an impact otherwise. The latter is applicable to the situations in part (ii) of Proposition 6, where the regulation reduces the profits of at least one firm. In those situations where the regulation has an impact, it leads to wider adoption of the tracking technology. Therefore, this assumption of voluntary privacy protection will not change our conclusion about the unintended consequence of the regulation, that is, the regulation leads to increased encroachment on consumer privacy.

[13] In addition to these three extensions, we have also considered a variant of the baseline model where the size of privacy cost D is continuously distributed over an interval. We find that this alternative specification of D has no impact on the monopoly market because the size of D does not affect the equilibrium with or without the privacy regulation. In the duopoly market, this alternative specification has an impact on only one subgame, namely the (P', P') subgame under the regulation. But it does not qualitatively change our conclusions from the analysis.

Proposition 7: In the duopoly model with asymmetric firms, suppose $D > D^*$ and

$$\theta > \max\left\{\frac{t-\Delta V}{t+\Delta V}, 1 - \frac{(3t-\Delta V)^2}{18(V_B - t)t}\right\}. \quad (10)$$

Consumer control reduces the utility of every consumer and consequently lowers consumer welfare.

Proposition 7 presents a case where the privacy regulation harms all consumers. This happens in a range of parameters where the regulation causes firm A (the higher-quality firm) to switch to personalized pricing while firm B continues with uniform pricing. Intuitively, this result is primarily driven by firm B's higher uniform price under the regulation. Because of its lower quality, firm B is at a disadvantage in competing for privacy-insensitive consumers who are willing to accept firm A's tracking and personalized prices. When the proportion of privacy-sensitive consumers is high, firm B finds it profitable to give up privacy-insensitive consumers and sell only to privacy-sensitive consumers. As a result, it charges a higher uniform price than under laissez-faire. Since prices are strategic complements, the increase in firm B's price induces firm A to raise its prices. Consequently, the privacy regulation reduces the utility of all consumers.

6.2 Incomplete Market Coverage Due to Low Valuation

In the baseline model, we have assumed that $V > 2t$. This assumption ensures that in the absence of privacy costs, the market would always be fully covered. In this section, we examine a case where the value of V is so low that the market is incompletely covered even if the firms adopt uniform pricing.

The case of incomplete market coverage is worth investigating because Rhodes and Zhou (2024) show that the impact of personalized pricing relative to uniform pricing in an oligopoly market depends on the degree of market coverage. If the market coverage is low, personalized pricing increases the firms' profits. Since consumers do not have privacy costs in Rhodes and Zhou (2024), it would be interesting to explore whether their result will change if there are privacy-sensitive consumers in the market. Conversely, it would also be interesting to determine whether our results regarding equilibrium pricing strategies in the baseline model will continue to hold if the market coverage is incomplete.

Now we reconsider our duopoly model under an assumption of small V. Specifically, suppose $V \in (t/2, 2t/3]$. To make the analysis meaningful, we assume $D < V - t/2$ to ensure that privacy-sensitive consumers can earn a positive surplus if they are offered a personalized price of 0. All other aspects of the model are the same as those of the baseline model.

We start by considering the special case where there is no privacy-sensitive consumer in the market, i.e., $\theta = 0$. We use this case to verify that our model can replicate the result in Rhodes and Zhou (2024). As detailed in section B2 of the appendix, we find that, in this case, the market is not fully covered when both firms adopt uniform pricing. Consumers around the center of the Hotelling line are not served in equilibrium because the uniform prices are too high relative to their valuation. Consistent with Rhodes and

Zhou (2024), we find that, in the absence of privacy regulation, each firm earns a larger profit when both firms adopt personalized pricing than when they adopt uniform pricing.

Next, we put privacy-sensitive consumers back into the market and suppose $\theta > 0$. As shown in the appendix, the equilibrium pricing strategies in the absence of privacy regulation are qualitatively the same as those in the baseline model. That is, both firms adopt personalized pricing if θ is small, one firm adopts personalized pricing and the other firm adopts uniform pricing if θ is in the intermediate range, and both firms adopt uniform pricing if θ is large. With the privacy regulation, on the other hand, both firms adopt personalized pricing for any $\theta \in (0, 1)$.

From these results, we make two observations. First, our main results from the baseline model are robust. In this model of incomplete market coverage, we reach the same conclusions as in the baseline model, that is, under laissez-faire the firms will adopt uniform pricing if a large proportion of consumers are privacy-sensitive, and the privacy regulation leads to wider adoption of personalized pricing.

Second, the incorporation of privacy-sensitive consumers into the model changes Rhodes and Zhou's (2024) result in interesting ways. Despite the increased profitability of personalized pricing arising from incomplete market coverage, the firms in our model will refrain from adopting this pricing strategy under laissez-faire if θ is large. But under the privacy regulation, the increased profitability of personalized pricing is enough to induce both firms to adopt this pricing strategy for any $\theta \in (0, 1)$.

6.3 Alternative Timing of Price Revelation

In the baseline model studied in sections 4 and 5, we have assumed that under the privacy regulation, a firm's uniform price is revealed to consumers before they respond to the firm's request for consent to tracking. Here we present an analysis based on an alternative assumption about the timing of price revelation, that is, a firm's uniform price is revealed to a consumer *after* she rejects tracking.

But before proceeding with the analysis, we make a comment about the empirical relevance of these timing assumptions. One way for online firms to collect personal information is through the use of cookies. Our informal sampling of e-commerce websites for Canadian consumers shows that while some websites do not allow a user to see any of its contents until she has selected her cookie settings, others do. In the latter case, a website displays the contents (e.g., product description and price) along with a request for the user to choose her cookie preferences.[14] These observations suggest that both assumptions about the timing of price revelation may be empirically relevant.

We now consider a variant of the baseline model with the second timing assumption. To be more specific, we modify stage 3 of the game by assuming that in the case where a firm adopts the tracking technology, its uniform price is not revealed to a consumer when it asks her for permission to track her

[14] Section B3 of the online appendix includes screenshots taken from the websites of two retailers, Walmart and Home Depot, that illustrate this case.

online activities. Instead, the firm reveals and offers her the uniform price only if she rejects tracking. All other aspects of the model remain the same. In particular, a firm's uniform price is set (but not revealed) at the time when it asks consumers for consent to tracking.

In this revised model, consumers do not know a firm's uniform price at the time when they respond to its request for consent to tracking. This information structure is similar to that in Ichihashi (2020), where a consumer observes price only after she has chosen the amount of personal information to disclose. We follow Ichihashi's (2020) approach and use perfect Bayesian equilibrium (PBE) as our solution concept.[15] As in Ichihashi (2020), this revised model has multiple equilibria (for some ranges of parameters). In such cases, we also follow Ichihashi (2020) by focusing on the equilibrium that maximizes every firm's profit.

The full analysis of this model is presented in section B3 of the online appendix. Here we focus our discussion on the comparison of the equilibria in the revised model and the baseline model. We will say that the two models have the same equilibrium outcome if the PBE in the revised model leads to the same pricing strategies, the same prices, and the same purchase decisions as in the subgame perfect equilibrium in the baseline model.

Our analysis shows that, in the monopoly market, this model has a unique PBE that leads to the same equilibrium outcome as the baseline model if $\theta \geq t/(2V-t)$. If $\theta < t/(2V-t)$, on the other hand, this model has multiple PBEs, but one of these PBEs leads to the same equilibrium outcome as the baseline model. Moreover, this PBE yields the highest profit for the firm among all the PBEs.

In the case of duopoly, we find that there are multiple equilibria in the subgame (P', P') (where both firms adopt personalized pricing) and in the subgames (P', U') and (U', P') (where one firm adopts personalized pricing while the other firm uses uniform pricing). This leads to a proliferation of PBEs in the entire game. But, as in the case of monopoly, one of these PBEs leads to the same equilibrium outcome as in the baseline model. Among all the PBEs in the duopoly case, this PBE yields the highest profit for both firms in every subgame where multiple equilibria exist.

Therefore, the analysis of this revised model confirms the robustness of our results. It shows that the equilibrium in the baseline model remains a possible outcome in the model with the alternative timing assumption. Moreover, this equilibrium outcome generates the highest profit for the firm(s) in every subgame where multiple equilibria exist. Therefore, if we consider a more general setup where firms can determine endogenously when to reveal their uniform prices, they have an incentive to reveal these prices to consumers before the latter respond to the request for consent to tracking. This provides a theoretical justification for the timing assumption in our baseline model.

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[15] In games of asymmetric information, the solution concept of subgame perfect equilibrium is not very effective in eliminating irrational strategies (Mas-Colell et al. 1995, Chapter 9). Hence, we use a stronger solution concept in this revised model.

7. Conclusions

Using a model that incorporates privacy-sensitive consumers, we have analyzed firms' pricing strategies and identified the conditions under which a firm engages in personalized pricing. We have demonstrated how the proportion of privacy-sensitive consumers in a market affects the firms' incentive to adopt personalized pricing. In particular, no firm uses personalized pricing in equilibrium if the proportion of privacy-sensitive consumers is sufficiently high. Competition, however, leads to wider use of personalized pricing.

Moreover, we have examined the impact of a privacy regulation that gives consumers control over whether a firm is allowed to track their online activities. We have found that the privacy regulation has the intended effect of protecting consumer privacy in a market where the proportion of privacy-sensitive consumers is sufficiently small that the firm(s) would have adopted the tracking technology in the absence of the regulation. If the proportion of privacy-sensitive consumers is not so small, however, the privacy regulation leads to more widespread use of the tracking technology. Therefore, a policy implication of our results is that a regulation designed to protect consumer privacy may have an unintended consequence of increasing encroachment on consumer privacy.

The findings from this study also have implications for business managers who contemplate the adoption of personalized pricing. Our analysis suggests that when evaluating the potential costs and benefits of this pricing strategy, managers should be mindful of the privacy sensitivity of their customers. While the magnitude of customers' privacy costs matter, our analysis shows that the proportion of privacy-sensitive customers may play an even bigger role in how this pricing strategy will affect sales. In markets where a large proportion of potential customers are privacy-sensitive, managers should proceed cautiously even if there is evidence indicating that each customer's privacy cost is small.

Finally, it is important to note a limitation in the scope of our analysis, that is, the collection of personal data in our model is driven by the need for personalized pricing. In reality, however, personal data may be collected and used by firms for other reasons, such as personalized services and targeted advertising. When personal data are monetized in other ways, our conclusion about privacy regulation causing increased encroachment on consumer privacy may not necessarily hold. For example, in a model of a platform that earns revenue from an online service and from monetization of consumer data, Choe et al. (2025) show that privacy regulation reduces the quantity of data collected by the platform. Therefore, while the findings from our analysis provides a novel perspective on the consequences of privacy regulation, their policy implications should be interpreted in a broader context of the literature.

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Supplement to
"Personalized Pricing in the Presence of Privacy Concerns"

Online Appendix – Part A

In part A of this appendix, we present the technical details of the baseline model examined in sections 3 and 4 of the paper.

A1. Monopoly Market

Proof of Proposition 1

First, we demonstrate that $p_A^U = p_B^U = V - t/2$. Because of the symmetric locations of goods A and B, it is optimal for firm M to set $p_A^U = p_B^U$. The market will be fully covered and the monopolist will earn a profit equal to p_A^U as long as $p_A^U \leq V - t/2$. Hence, $p_A^U = p_B^U = V - t/2$ are the profit-maximizing prices provided that the market is fully covered. On the other hand, the market will be incompletely covered if $p_A^U = p_B^U > V - t/2$. In that the case, the demand for good $i = A, B$ will be represented by $q_i = (V - p_i^U)/t$, and hence the firm's profit will be

$$\Pi_M^U = \frac{p_A^U(V - p_A^U)}{t} + \frac{p_B^U(V - p_B^U)}{t}. \quad (A1)$$

Using (A1), we find that $\partial\Pi_M^U/\partial p_i^U = (V - 2p_i^U)/t < 0$ for $p_i^U > V - t/2$. In other words, the monopolist's profit decreases if it raises its prices above $V - t/2$. This confirms that the monopolist will not want to raise the prices to a level at which the market is incompletely covered. Therefore, $p_A^U = p_B^U = V - t/2$ are indeed the profit-maximizing prices in this subgame, and the corresponding profit is $\Pi_M^U = V - t/2$.

Second, we compare Π_M^U with Π_M^P given in (1). Simple calculations reveal that $\Pi_M^U \geq \Pi_M^P$ if and only if $\theta \geq t/(4V - t)$. Hence, we have the results in Proposition 1.

Proof of Proposition 2

In the main text, we have defined $x_A^{P'}$ and $x_B^{P'}$ as the location of the marginal consumer who is indifferent between accepting and rejecting tracking by firm A and firm B, respectively. By definition, $x_A^{P'}$ and $x_B^{P'}$ must satisfy the following conditions:

$$V - t x_A^{P'} - p_A^{P'} = 0; \quad (A2)$$
$$V - t\left(1 - x_B^{P'}\right) - p_B^{P'} = 0. \quad (A3)$$

To understand (A2) and (A3), note that 0 on the right-hand sides represents both the utility of a privacy-insensitive consumer from purchasing a unit at her personalized price, and the utility of a privacy-sensitive consumer who does not purchase anything. Using these two equations, we find

$$x_A^{P'} = \frac{V - p_A^{P'}}{t}, \qquad x_B^{P'} = \frac{t - V + p_B^{P'}}{t}. \quad (A4)$$

At the beginning of stage 3, the monopolist sets $(p_A^{P'}, p_B^{P'})$ to maximize (2) subject to (A4). Substituting (A4) for $x_A^{P'}$ and $x_B^{P'}$ into (2), we obtain

$$\Pi_M^{P'} = \frac{p_A^{P'}\,(V - p_A^{P'})}{t} + \frac{p_B^{P'}(V - p_B^{P'})}{t} + (1-\theta)\left[\int_{(V-p_A^{P'})/t}^{1/2} (V - tx)\,dx + \int_{1/2}^{(t-V+p_B^{P'})/t} \big(V - t(1-x)\big)\,dx\right]. \quad (A5)$$

Differentiating (A5) with respect to $p_A^{P'}$ and $p_B^{P'}$, we find the following first-order conditions of the profit-maximization problem:

$$\frac{\partial \Pi_M^{P'}}{\partial p_A^{P'}} = \frac{V - (1+\theta)p_A^{P'}}{t} = 0, \qquad (A6)$$

$$\frac{\partial \Pi_M^{P'}}{\partial p_B^{P'}} = \frac{V - (1+\theta)p_B^{P'}}{t} = 0. \qquad (A7)$$

Solving (A6) and (A7), we obtain

$$p_A^{P'} = p_B^{P'} = \frac{V}{1+\theta}. \quad (A8)$$

Substituting (A8) into (A4) and (A5), we obtain

$$x_A^{P'} = \frac{\theta V}{(1+\theta)t}, \quad x_B^{P'} = \frac{(1+\theta)t - \theta V}{(1+\theta)t}, \qquad (A9)$$

$$\Pi_M^{P'} = \frac{t(4V - t) + \theta^2(2V - t)^2}{4t(1+\theta)}. \quad (A10)$$

Recall that (2) is applicable only if $p_A^{P'}$ and $p_B^{P'}$ fall in the interval $[V - t/2, V]$. Applying this restriction to (A8), we obtain $0 \le \theta \le t/(2V - t)$. In other words, (A9) and (A10) are applicable only if θ satisfies this condition.

Using (A9), we verify that for $\theta \in (0, t/(2V - t))$, we have $x_A^{P'} \in (0, 1/2)$ and $x_B^{P'} \in (1/2, 1)$. In this case, the consumers' purchase decisions are as illustrated in Figure 1.

If $\theta \ge t/(2V - t)$, on the other hand, the constraint $p_i^{P'} \ge V - t/2$ $(i = A, B)$ becomes binding because (A8) implies that $p_i^{P'} \le V - t/2$. With the uniform prices at the lower bound of this constraint, i.e., $p_A^{P'} = p_B^{P'} = V - t/2$, all consumers will reject tracking and purchase the goods at the uniform prices. Lowering the uniform prices to below $V - t/2$ would reduce the firm's profit because it would earn a smaller profit margin on each unit sold but with no increase in quantity. Hence, $p_A^{P'} = p_B^{P'} = V - t/2$ are the firm's profit-maximizing prices for consumers who reject tracking in this subgame. Since no consumer will have an incentive to accept tracking at these prices, the firm's profit in this case is the same as that in the subgame following the firm's choice of uniform pricing. Taking into account the (small) cost of adopting the tracking technology, the firm chooses uniform pricing and all consumers are served in equilibrium.

Incidentally, note in (A8) that if $\theta = 0$, $p_A^{P'} = p_B^{P'} = V$. This is the level of uniform prices under the privacy regulation if there were no privacy-sensitive consumers in the market.

Proof of Proposition 3

Propositions 1 and 2 show that firm M's pricing strategies with and without the regulation depends on the value of θ relative to the two threshold values: $t/(4V - t)$ and $t/(2V - t)$. In Proposition 3, the pricing strategies in scenarios (i), (ii) and (iii) are determined by combining the findings from Propositions 1 and 2.

Next, we analyze the effects of the regulation on profit, consumer welfare, and social welfare. The monopolist's profit associated with different pricing strategies in the two policy contexts has been presented in the main text as Π_M^U, Π_M^P and $\Pi_M^{P'}$, respectively. Noting that $p_A^U = p_B^U = V - t/2$, we obtain

$$CS^U = \int_0^{1/2} (V - tx - p_A^U)dx + \int_{1/2}^{1} (V - t(1-x) - p_B^U)dx = \frac{t}{4}. \quad (A11)$$

Using $p_A^P(x) = V - tx$ and $p_B^P(x) = V - (1-t)x$, we find

$$CS^P = (1-\theta)\left[\int_0^{1/2} \left(V - tx - p_A^P(x)\right)dx + \int_{1/2}^{1} \left(V - t(1-x) - p_B^P(x)\right)dx\right] = 0. \quad (A12)$$

In the case where firm M adopts personalized pricing under the privacy regulation, consumers in $[0, x_A^{P'})$ and $(x_B^{P'}, 1]$ buy the goods at uniform prices and obtain positive surpluses. Observing that the surplus of all remaining consumers is 0, we use (A8) and (A9) to calculate the consumer surplus in this case:

$$CS^{P'} = \int_0^{x_A^{P'}} (V - tx - p_A^{P'})dx + \int_{x_B^{P'}}^{1} (V - t(1-x) - p_B^{P'})dx = \frac{\theta^2 V^2}{t(1+\theta)^2}. \quad (A13)$$

Then we obtain the following expressions of social welfare:

$$W^U = CS^U + \Pi_M^U = V - \frac{t}{4}, \qquad (A14)$$

$$W^P = CS^P + \Pi_M^P = (1-\theta)\left(V - \frac{t}{4}\right), \qquad (A15)$$

$$W^{P'} = CS^{P'} + \Pi_M^{P'} = (1-\theta)\left(V - \frac{t}{4}\right) + \frac{\theta^2 V^2 (2+\theta)}{t(1+\theta)^2}. \qquad (A16)$$

In scenario (i), the firm adopts personalized pricing with and without the regulation. Here we compare $\Pi_M^{P'}$ in (A10) with Π_M^P in (1) to find $\Pi_M^{P'} > \Pi_M^P$. It is clear from (A12) and (A13) that $CS^{P'} > CS^P$. Noting that the last term of the right-hand side of (A16) is positive, we conclude that $W^{P'} > W^P$.

To determine the impact of the regulation on individual consumers in scenario (i), note that the personalized price for consumers at each location is the same with and without the regulation. But with the regulation, consumers have the option of purchasing the goods at the uniform price given in (A8). For consumers located in $[0, x_A^{P'})$ and $(x_B^{P'}, 1]$, this uniform price is lower than the personalized prices they pay without the regulation. Hence, the regulation increases the utility of privacy-insensitive consumers at these locations. The regulation also raises the utility of privacy-sensitive consumers at these locations because they can now purchase the goods at the uniform price and obtain a positive surplus while they would have stayed away from the firm under laissez-faire. On the other hand, the regulation has no impact on the consumers located in $(x_A^{P'}, x_B^{P'})$ either because they are privacy-insensitive and purchase the goods at the same personalized prices with and without the regulation, or because they are privacy-sensitive and do not purchase any unit with and without the regulation. Finally, the utility level of consumers at $x_A^{P'}$ and $x_B^{P'}$ is the same with and without the regulation because they receive 0 surplus in either case.

In scenario (ii), the monopolist's equilibrium strategy changes from uniform pricing to personalized pricing as a result of the privacy regulation. We compare $\Pi_M^{P'}$ in (A10) with $\Pi_M^U = V - t/2$ to find $\Pi_M^{P'} > \Pi_M^U$ for θ in the interval $[t/(4V-t), t/(2V-t))$. Similarly, a comparison of (A13) with (A11), and (A16) with (A14), reveals that $CS^{P'} < CS^U$ and $W^{P'} < W^U$ for θ in this interval.

To determine the impact of the regulation on individual consumers in scenario (ii), note that the uniform price under laissez-faire ($p_i^U = V - t/2$) is lower than the uniform price (for those consumers who reject tracking) under the regulation ($p_i^{P'} = V/(1+\theta)$). Consequently, the regulation reduces the utility of those consumers who purchase the goods at the uniform price under the regulation. Moreover, the regulation reduces the surplus earned by the remaining consumers to 0, either because they are privacy-insensitive consumers who purchase at the personalized prices or because they are privacy-sensitive consumers who stop buying under the regulation. Under laissez-faire, all consumers earn a positive surplus except those consumers located at $x = 1/2$, who earn 0 surplus. Therefore, the regulation reduces the utility of all consumers except those at $x = 1/2$, and the utility of the latter is the same with and without the regulation.

In scenario (iii), the monopolist adopts the uniform pricing with and without the regulation. The regulation has no real effect, as the profit, the utility of every consumer, and social welfare remain the same with and without the regulation.

A2. Duopoly Market without Privacy Regulation

A2.1 Analysis of the Four Subgames

In the subgame (U, U), we set $U_A^{UU}(x) = U_B^{UU}(x)$ to obtain the location of the marginal consumer who is different between firm A and firm B, $x^{UU} = (p_B^{UU} - p_A^{UU} + t)/2t$. Then the profit earned by each firm can be expressed as

$$\Pi_A^{UU} = p_A^{UU} x^{UU} = \frac{p_A^{UU}(p_B^{UU} - p_A^{UU} + t)}{2t}, \quad (A17)$$

$$\Pi_B^{UU} = p_B^{UU}(1 - x^{UU}) = \frac{p_B^{UU}(p_A^{UU} - p_B^{UU} + t)}{2t}. \quad (A18)$$

Solving the firms' profit maximization problems associated with (A17) and (A18), we obtain each firm's best-response function:

$$p_i^{UU} = \frac{p_j^{UU} + t}{2}. \qquad (i, j = A, B, i \neq j). \quad (A19)$$

Solving the system of equations formed by the best-response functions, we find the equilibrium prices in this subgame: $p_A^{UU} = p_B^{UU} = t$. Substituting these prices into (A17) and (A18), we obtain the equilibrium levels of profits: $\Pi_A^{UU} = \Pi_B^{UU} = t/2$.

The derivations of the equilibrium prices and profits in the (P, P) subgame are presented in the main text. The conclusion that each firm earns a smaller profit in the (P, P) subgame than in the (U, U) subgame is based on the results that $\Pi_i^{PP} = t/4$ and $\Pi_i^{UU} = t/2$ $(i = A, B)$.

Turning to the subgame (P, U) subgame, recall that personalized prices are set after the uniform price. Using backward induction, we start with an analysis of firm A's choice of personalized prices. Given firm B's uniform price p_B^{PU}, firm A will undercut firm B's price by setting $p_A^{PU}(x) = t(1-2x) + p_B^{PU}$ as long as $p_A^{PU}(x) \geq 0$. The latter implies $x \leq (t + p_B^{PU})/2t$. Define

$$x_0^{PU} \equiv \min\left\{\frac{t + p_B^{PU}}{2t}, 1\right\}. \qquad (A20)$$

Then, among the consumers who visit firm A's store at stage 3, those located in $[0, x_0^{PU})$ will purchase good A at personalized prices. From (A20), we find that $x_0^{PU} < 1$ if $p_B^{PU} < t$, and $x_0^{PU} = 1$ otherwise.

Moreover, note that at the personalized price $p_A^{PU}(x) = t(1-2x) + p_B^{PU}$, a consumer receives the same surplus from consuming good A as from consuming good B. This implies that a privacy-sensitive consumer will obtain a lower utility from purchasing good A at the personalized price because doing so will involve a loss of privacy. Thus, anticipating that $p_A^{PU}(x)$ is the personalized price she would receive from firm A, a privacy-sensitive consumer will choose not to visit firm A's store. Instead, she will purchase a unit from firm B at the uniform price p_B^{PU} as long as her utility from doing so is non-negative, i.e., $V - t(1-x) - p_B^{PU} \geq 0$. The latter implies $x \geq (t + p_B^{PU} - V)/t$. Define

$$x_1^{PU} \equiv \max\left\{\frac{t + p_B^{PU} - V}{t}, 0\right\}. \qquad (A21)$$

Thus, privacy-sensitive consumers located in $[x_1^{PU}, 1]$ will purchase good B. From (A21), we find that $x_1^{PU} > 0$ if $p_B^{PU} > V - t$, and $x_1^{PU} = 0$ otherwise.

Figure A1 illustrates consumers' purchase decisions for a situation where both x_0^{PU} and x_1^{PU} are strictly between 0 and 1. To recap the preceding discussion using Figure A1, privacy-insensitive consumers located in $[0, x_0^{PU})$ will purchase good A at personalized prices while those in $[x_0^{PU}, 1]$ will purchase good B at the uniform price. On the other hand, privacy-sensitive consumers located in $[0, x_1^{PU})$ do not purchase any good while those $[x_1^{PU}, 1]$ will purchase good B at the uniform price.

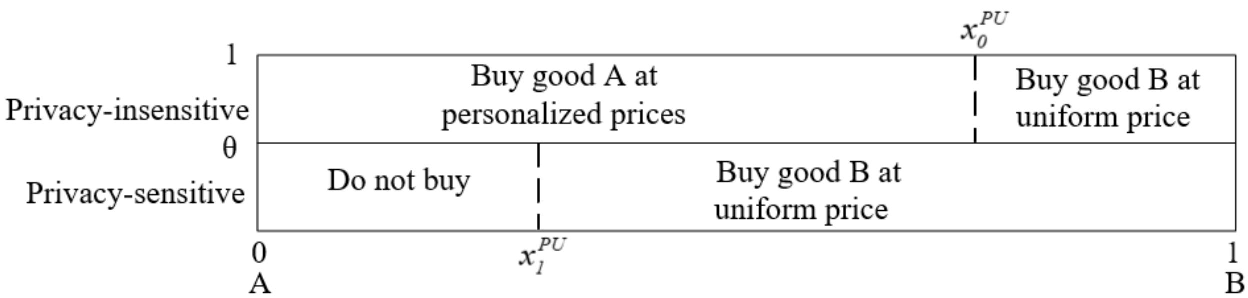

Figure A1: Consumer Decisions under the Assumption that $x_0^{PU} < 1$ and $x_1^{PU} > 0$

As can be seen from Figure A1, only privacy-insensitive consumers located in $[0, x_0^{PU})$ will purchase good A. The remaining consumers will either purchase good B or stay away from the market. To be more specific, note that $V > 2t$ implies $V - t > t$. If $p_B^{PU} < t$ (and thus $x_0^{PU} < 1$ and $x_1^{PU} = 0$ by (A20) and (A21)), privacy-insensitive consumers located in $[x_0^{PU}, 1]$ and all privacy-sensitive consumers will purchase good B. Conversely, if $p_B^{PU} \geq t$ (and thus $x_0^{PU} = 1$), no privacy-insensitive consumers will purchase good B but privacy-sensitive consumers located in $[x_1^{PU}, 1]$ will. Moreover, if p_B^{PU} is so high that it exceeds $V - t$, then $x_1^{PU} > 0$ and privacy-sensitive consumers located in $[0, x_1^{PU})$ will purchase neither good A nor good B.

Therefore, the profit of firm B has two different expressions depending on the price it charges. If $p_B^{PU} < t$, firm B's profit can be expressed as

$$\Pi_B^{PU} = p_B^{PU}[(1-\theta)(1 - x_0^{PU}) + \theta]. \qquad (A22)$$

But if $p_B^{PU} \geq t$, firm B's profit becomes

$$\Pi_B^{PU} = p_B^{PU}\theta(1 - x_1^{PU}). \qquad (A23)$$

Next we analyze firm B's profit-maximization problem with respect to p_B^{PU} in these two intervals separately and then combine the findings to determine the global maximum of firm B's profit in this subgame.

Suppose $p_B^{PU} < t$. Then $V > 2t$ and (A21) imply $x_1^{PU} = 0$. In this case, privacy-insensitive consumers located in $[x_0^{PU}, 1]$ and all privacy-sensitive consumers purchase good B at the uniform price p_B^{PU}, while privacy-insensitive consumers in $[0, x_0^{PU})$ buy good A at personalized prices. From (A20), we have $x_0^{PU} = (t + p_B^{PU})/2t$. Substituting it into (A22), we express firm B's profit-maximization problem in this case as

$$\max_{p_B^{PU}} \Pi_B^{PU} = p_B^{PU}\left[(1-\theta)\left(1 - \frac{t + p_B^{PU}}{2t}\right) + \theta\right]. \quad (A24)$$

Solving (A24), we obtain

$$p_B^{PU} = \frac{t(1+\theta)}{2(1-\theta)}. \qquad (A25)$$

Applying the constraint $p_B^{PU} < t$ to (A21), we find that $\theta < 1/3$. In other words, (A25) is a local max of (A24) if $\theta < 1/3$.

Now suppose $p_B^{PU} \geq t$. In this case, firm B's profit is represented by (A23). We will demonstrate that $p_B^{PU} = V - t$ is a local max of (A23). Consider first $p_B^{PU} \in [t, V-t]$. By (A20) and (A21), we have $x_0^{PU} = 1$ and $x_1^{PU} = 0$. These observations imply that, in this case, all privacy-insensitive consumers purchase good A at personalized prices and all privacy-sensitive consumers buy good B at the uniform price. Firm B's profit in (A23) now becomes: $\Pi_B^{PU} = p_B^{PU}\theta$, which increases with p_B^{PU}. On the other hand, if firm B raises its price above $V - t$, i.e., if $p_B^{PU} > V - t$, then (A21) implies $x_1^{PU} = (t + p_B^{PU} - V)/t > 0$. Substituting this expression for x_1^{PU} into (A23), we obtain

$$\Pi_B^{PU} = \frac{\theta p_B^{PU}(V - p_B^{PU})}{t}. \qquad (A26)$$

From (A26), we find that

$$\frac{\partial \Pi_B^{PU}}{\partial p_B^{PU}} = \frac{\theta(V - 2p_B^{PU})}{t} < 0 \text{ if } p_B^{PU} > \frac{V}{2}. \qquad (A27)$$

Since $V > 2t$ implies $V - t > V/2$, (A27) implies that Π_B^{PU} is decreasing in p_B^{PU} for $p_B^{PU} > V - t$. Combining these observations, we conclude that $p_B^{PU} = V - t$ is a local max of Π_B^{PU} in (A23).

Next, we combine the findings from the two cases to determine the global maximum of firm B's profit in this subgame. Substituting (A25) into (A24), we obtain firm B's local maximum profit for the case $p_B^{PU} < t$, which is,

$$\Pi_B^{PU} = \frac{t(1+\theta)^2}{8(1-\theta)}. \qquad (A28)$$

On the other hand, substituting $p_B^{PU} = V - t$ into (A23), we find firm B's local maximum profit for the case $p_B^{PU} \geq t$, which is,

$$\Pi_B^{PU} = p_B^{PU}\theta(1 - x_1^{PU}) = \theta(V - t). \qquad (A29)$$

In equilibrium, firm B compares (A28) and (A29) and chooses a price that yields the global maximum profit. To be more specific, firm B will choose the price in (A25) if the profit given by (A28) is no lower than that given by (A29), i.e., if

$$\frac{t(1+\theta)^2}{8(1-\theta)} \geq \theta(V - t). \qquad (A30)$$

Noting that $\theta \in (0,1)$, we solve (A30) to find $\theta \leq \bar{\theta}$, where $\bar{\theta}$ is defined by (3) in the paper.

Before we declare that (A25) is the solution to firm B's profit-maximization problem for $\theta \leq \bar{\theta}$, we need to verify that that it is consistent with the earlier finding that for (A25) to be a local max of (A24), θ must satisfy $\theta < 1/3$. Using (3), we verify that $V > 2t$ implies $\bar{\theta} \in (0, 1/3)$. Therefore, firm B's profit-maximization price in this subgame is

$$p_B^{PU} = \begin{cases} \dfrac{t(1+\theta)}{2(1-\theta)} & \text{if } \theta \in (0, \bar{\theta}]; \\ V - t & \text{if } \theta \in (\bar{\theta}, 1). \end{cases} \qquad (A31)$$

Using (A31), we can verify that $p_B^{PU} < t$ and $x_0^{PU} = \frac{3-\theta}{4(1-\theta)}$ (< 1) for $\theta \in (0, \bar{\theta}]$. In this case, firm A sells its product to privacy-insensitive consumers in $[0, x_0^{PU})$ and its profit is

$$\Pi_A^{PU} = (1-\theta)\int_0^{x_0^{PU}} (t(1-2x) + p_B^{PU})\, dx = \frac{t(3-\theta)^2}{16(1-\theta)}. \qquad (A32)$$

On the other hand, firm B sells its product to privacy-insensitive consumers in $[x_0^{PU}, 1]$ and to all privacy-sensitive consumers. Its profit is represented by (A28). Using (A20) and (A31), we can verify that $x_0^{PU} > 3/4$ in this case, implying that firm A serves more than three quarters, while firm B sells to less than one quarter, of privacy-insensitive consumers.

Conversely, if $\theta \in (\bar{\theta}, 1)$, firm B charges a high price, $p_B^{PU} = V - t$, and sells to privacy-sensitive consumers only. The assumption $V > 2t$ ensures that every privacy-sensitive consumer will buy a unit of good B at this price, and firm B's profit represented by (A29). Firm A serves all privacy-insensitive consumers, and its profit is

$$\Pi_A^{PU} = (1-\theta)\int_0^{1} (t(1-2x) + p_B^{PU})\, dx = (1-\theta)(V-t). \quad (A33)$$

Note from (A29) and (A33) that the two firms earn the same average profit per customer, $V - t$.

To summarize the analysis of the (P, U) subgame, the firm that adopts personalized pricing (firm A) specializes in serving privacy-insensitive consumers while the firm that offers a uniform price serves mainly privacy-sensitive consumers. The profits represented by (4) and (5) in the main text are duplicated from (A28)-(A29) and (A32)-(A33).

Finally, the analysis of the subgame (U, P) is symmetric to that of (P, U). This can be done by interchanging the subscripts A and B in the variables.

A2.2 Proof of Proposition 4

As indicated in Table 1, each of Π_i^{PU} and Π_i^{UP} is represented by different expressions depending on whether the value of θ is in $(0, \bar{\theta}]$ or $(\bar{\theta}, 1)$. Accordingly, we start with an analysis of the firms' equilibrium strategies for each of these two ranges of θ and then combine the results from these two cases to determine the equilibrium. In our analysis, the cost of adopting tracking technology is used as a tiebreaker, that is, we assume that a firm chooses uniform pricing whenever it is indifferent between personalized pricing and uniform pricing.

Suppose $0 < \theta \le \bar{\theta}$. In this case, $\Pi_A^{PU} = \Pi_B^{UP} = \frac{t(3-\theta)^2}{16(1-\theta)}$ and $\Pi_A^{UP} = \Pi_B^{PU} = \frac{t(1+\theta)^2}{8(1-\theta)}$. Substituting these profit expressions into Table 1, we obtain the payoff matrix shown in Table A1.

Table A1: Firms' Profits without Privacy Regulation when $0 < \theta \le \bar{\theta}$

A \ B	U	P
U	$(\frac{t}{2}, \frac{t}{2})$	$(\frac{t(1+\theta)^2}{8(1-\theta)}, \frac{t(3-\theta)^2}{16(1-\theta)})$
P	$(\frac{t(3-\theta)^2}{16(1-\theta)}, \frac{t(1+\theta)^2}{8(1-\theta)})$	$(\frac{t}{4}, \frac{t}{4})$

First, we rule out (U, U) as a possible equilibrium in the game illustrated in Table A1. Starting from a situation where both firms choose U, each firm has an incentive to deviate to P because

$$\frac{t(3-\theta)^2}{16(1-\theta)} > \frac{t}{2} \qquad (A34)$$

for any $\theta \in (0, 1)$.

Second, we derive the conditions under which (P, U) and (U, P) are equilibriums. Suppose the firms choose (P, U) as their pricing strategies. Firm A has no incentive to deviate from P because of (A34). Firm B will not want to deviate from U to P if

$$\frac{t(1+\theta)^2}{8(1-\theta)} \geq \frac{t}{4}, \qquad (A35)$$

which is true if $\theta \geq \sqrt{5} - 2$. But this is admissible only if $\bar{\theta} \geq \sqrt{5} - 2$. Using the definition of $\bar{\theta}$ in (3), we find that this condition holds if $2t < V \leq \left(\frac{3}{2} + \frac{\sqrt{5}}{4}\right)t$. Therefore, (P, U), and by extension (U, P), are equilibriums if $\sqrt{5} - 2 \leq \theta \leq \bar{\theta}$ and $2t < V \leq \left(\frac{3}{2} + \frac{\sqrt{5}}{4}\right)t$.

Third, we consider the condition under which (P, P) is an equilibrium. Given that its rival chooses P, a firm has no incentive to deviate from P to U if (A35) is violated, in other words, if $\theta < \sqrt{5} - 2$. In the case where $V > \left(\frac{3}{2} + \frac{\sqrt{5}}{4}\right)t$, we have $\bar{\theta} < \sqrt{5} - 2$, and thus $\theta \leq \bar{\theta}$ implies $\theta < \sqrt{5} - 2$. In the case where $2t < V \leq \left(\frac{3}{2} + \frac{\sqrt{5}}{4}\right)t$, on the other hand, the condition $\theta < \sqrt{5} - 2$ is needed for (P, P) to be an equilibrium.

Now suppose $\bar{\theta} < \theta < 1$. In this case, $\Pi_A^{PU} = \Pi_B^{UP} = (1-\theta)(V-t)$ and $\Pi_A^{UP} = \Pi_B^{PU} = \theta(V-t)$. Substituting these profit expressions into Table 1, we obtain the payoff matrix shown in Table A2.

Table A2: Firms' Profits without Privacy Regulation when $\bar{\theta} < \theta < 1$

A \ B	U	P
U	$(\frac{t}{2}, \frac{t}{2})$	$(\theta(V-t), (1-\theta)(V-t))$
P	$((1-\theta)(V-t), \theta(V-t))$	$(\frac{t}{4}, \frac{t}{4})$

First, we derive the condition under which (U, U) is an equilibrium. Given that its rival chooses U, a firm has no incentive to deviate from U to P if $t/2 \geq (1-\theta)(V-t)$. The latter implies $\theta \geq \tilde{\theta}$, where $\tilde{\theta}$ is defined in (6). Using (6) and the assumption $V > 2t$, we can show that $\tilde{\theta} > 1/2$. Recalling that $\bar{\theta} < 1/3$, we know $\tilde{\theta} > \bar{\theta}$. Therefore, $\theta \geq \tilde{\theta}$ is the condition for (U, U) to be an equilibrium.

Next, we examine the conditions under which (U, P) and (P, U) are equilibriums. Consider the case (U, P). Given that firm A chooses U, firm B has no incentive to deviate from P to U if $(1-\theta)(V-t) > t/2$, i.e., if $\theta < \tilde{\theta}$. Given that firm B chooses P, firm A has no incentive to deviate from U to P if $\theta(V-t) \geq t/4$. The latter implies $\theta \geq \hat{\theta}$, where $\hat{\theta}$ is defined in (6). Using this definition, we can verify that $\hat{\theta}$ decreases in V and $\hat{\theta} = \bar{\theta} = \sqrt{5} - 2$ at $V = \left(\frac{3}{2} + \frac{\sqrt{5}}{4}\right)t$. Furthermore, we find that $\hat{\theta} < \bar{\theta}$ for $2t < V < \left(\frac{3}{2} + \frac{\sqrt{5}}{4}\right)t$ and $\hat{\theta} > \bar{\theta}$ for $V > \left(\frac{3}{2} + \frac{\sqrt{5}}{4}\right)t$. Combining these observations with the condition $\theta > \bar{\theta}$, we conclude that (U, P) and (P, U) are equilibriums if (a) $2t < V \leq \left(\frac{3}{2} + \frac{\sqrt{5}}{4}\right)t$ and $\bar{\theta} < \theta < \tilde{\theta}$, or (b) $V > \left(\frac{3}{2} + \frac{\sqrt{5}}{4}\right)t$ and $\hat{\theta} \leq \theta < \tilde{\theta}$.

Third, we consider the conditions under which (P, P) is an equilibrium. Given its rival's choice of P, a firm has no incentive to deviate from P to U if $t/4 > \theta(V-t)$, i.e., if $\theta < \hat{\theta}$. Note, however, that this condition is admissible only if $\hat{\theta} > \bar{\theta}$, which is true when $V > \left(\frac{3}{2} + \frac{\sqrt{5}}{4}\right)t$. Therefore, (P, P) is an equilibrium if $V > \left(\frac{3}{2}+\frac{\sqrt{5}}{4}\right)t$ and $\bar{\theta} < \theta < \hat{\theta}$.

Combining the above results about equilibriums for θ in $(0, \bar{\theta}]$ and $(\bar{\theta}, 1)$, we find that

(a) (P, P) is the equilibrium if $V \le \left(\frac{3}{2}+\frac{\sqrt{5}}{4}\right)t$ and $\theta < \sqrt{5}-2$, or if $V > \left(\frac{3}{2}+\frac{\sqrt{5}}{4}\right)t$ and $\theta < \hat{\theta}$;

(b) (P, U) and (U, P) are the equilibriums if $V \le \left(\frac{3}{2}+\frac{\sqrt{5}}{4}\right)t$ and $\sqrt{5}-2 \le \theta < \tilde{\theta}$, or if $V > \left(\frac{3}{2}+\frac{\sqrt{5}}{4}\right)t$ and $\hat{\theta} \le \theta < \tilde{\theta}$;

(c) (U, U) is the equilibrium if $\theta \ge \tilde{\theta}$.

Noting that $\hat{\theta} > \sqrt{5}-2$ when $V < \left(\frac{3}{2}+\frac{\sqrt{5}}{4}\right)t$ and $\hat{\theta} < \sqrt{5}-2$ when $V > \left(\frac{3}{2}+\frac{\sqrt{5}}{4}\right)t$, we rewrite the conditions for the (P, P) equilibrium in (a) as $\theta < \min\{\hat{\theta}, \sqrt{5}-2\}$ and those for the (P, U) and (U, P) equilibriums in (b) as $\min\{\hat{\theta}, \sqrt{5}-2\} \le \theta < \tilde{\theta}$. Hence, we have parts (i)-(iii) in Proposition 4.

A2.3 Proof that $t/(4V-t) < \min\{\hat{\theta}, \sqrt{5}-2\} < \tilde{\theta}$

Using the definition of $\hat{\theta}$ in (6) and noting that $4V - t > 4(V-t)$, we conclude that $t/(4V-t) < \hat{\theta}$. Since $V > 2t$ implies $t/(4V-t) < 1/7$, we have $t/(4V-t) < \sqrt{5}-2$. Note that $V > 2t$ also implies $\tilde{\theta} > 1/2$. Hence, we obtain $\tilde{\theta} > \sqrt{5}-2 \ge \min\{\hat{\theta}, \sqrt{5}-2\}$.

A3 Results from the Duopoly Market with Privacy Regulation

A3.1 Analysis of the (P', P') Subgame

In this subgame, each firm sets a uniform price and offers personalized prices to those consumers who accept tracking. We use $p_A^{P'P'}$ and $p_B^{P'P'}$ to denote the uniform prices, and $p_A^{P'P'}(x)$ and $p_B^{P'P'}(x)$ to denote the personalized prices of the two firms.

Note that consumers located close to the two ends of the Hotelling line have stronger incentives to reject tracking because they would be charged higher personalized prices than those consumers near the center. Based on this observation, we define $x_{A\alpha}^{P'P'}$ and $x_{B\alpha}^{P'P'}$ as the locations of type-α consumers who are indifferent between accepting and rejecting tracking, where $\alpha = 0$ denotes privacy-insensitive consumers and $\alpha = 1$ denotes privacy-sensitive consumers. As illustrated in Figure A2, type-α consumers in the interval $(x_{A\alpha}^{P'P'}, x_{B\alpha}^{P'P'})$ accept tracking, while those in the intervals $[0, x_{A\alpha}^{P'P'}]$ and $[x_{B\alpha}^{P'P'}, 1]$ reject tracking. Note that the definitions of $x_{A\alpha}^{P'P'}$ and $x_{B\alpha}^{P'P'}$ are without loss of generality. If all type-α consumers accept tracking, we have $x_{A\alpha}^{P'P'} = 0$ and $x_{B\alpha}^{P'P'} = 1$. Conversely, if none of type-α consumers accept tracking, $x_{A\alpha}^{P'P'} = 1$ and $x_{B\alpha}^{P'P'} = 0$.

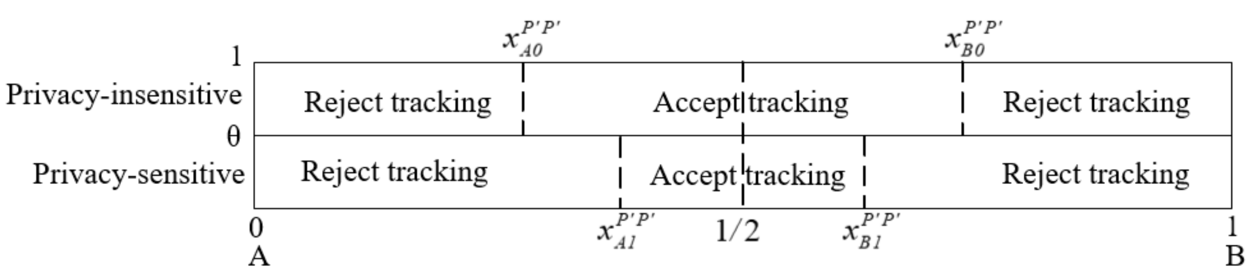

Figure A2: Consumers' Choices about Tracking in the Subgame (P', P')

Based on the above observations, we write firm A's and firm B's profit at stage 3 as

$$\Pi_A^{P'P'} = p_A^{P'P'}\left[\theta x_{A1}^{P'P'} + (1-\theta)x_{A0}^{P'P'}\right] + (1-\theta)\int_{x_{A0}^{P'P'}}^{\frac{1}{2}} t(1-2x)\,dx + \theta\int_{x_{A1}^{P'P'}}^{\frac{1}{2}} t(1-2x)\,dx, \quad (A36)$$

$$\Pi_B^{P'P'} = p_B^{P'P'}[\theta(1-x_{B1}^{P'P'}) + (1-\theta)(1-x_{B0}^{P'P'})] + (1-\theta)\int_{\frac{1}{2}}^{x_{B0}^{P'P'}} t(2x-1)\,dx$$

$$+\theta\int_{\frac{1}{2}}^{x_{B1}^{P'P'}} t(2x-1)\,dx. \quad (A37)$$

In each of (A36) and (A37), the first term on the right-hand side represents the firm's profit from sales at its uniform price, while the second and third term represent its profit from sales at personalized prices to privacy-insensitive and privacy-sensitive consumers.

When a firm offers a personalized price to a consumer who has accepted tracking, it believes (correctly) that this consumer either will accept or has accepted tracking by the other firm. Thus, competition for such a consumer drives the personalized prices down to $p_A^{P'P'}(x) = \max\{t(1-2x), 0\}$ and $p_B^{P'P'}(x) = \max\{t(2x-1), 0\}$. These personalized prices are in the range $[0, t]$, with the highest price (t) charged to consumers at the endpoints of the Hotelling line.

Consider a privacy-insensitive consumer located at $x_{A0}^{P'P'}$. When she determines whether to accept tracking by firm A, she foresees that she will be offered a personalized price $p_A^{P'P'}\left(x_{A0}^{P'P'}\right) = t\left(1-2x_{A0}^{P'P'}\right)$ if she says "yes". On the other hand, if she rejects tracking, she may purchase good A at the uniform price $p_A^{P'P'}$. Since this consumer is indifferent between accepting and rejecting tracking by firm A, her utility from these two options must be equal; in other words, the following condition must hold:

$$V - tx_{A0}^{P'P'} - t\left(1-2x_{A0}^{P'P'}\right) = V - tx_{A0}^{P'P'} - p_A^{P'P'}. \quad (A38)$$

From (A38), we obtain

$$x_{A0}^{P'P'} = \frac{-p_A^{P'P'} + t}{2t}. \quad (A39)$$

Using the same logic, we find the location of a privacy-insensitive consumer who is indifferent between accepting and rejecting tracking by firm B:

$$x_{B0}^{P'P'} = \frac{p_B^{P'P'} + t}{2t}. \quad (A40)$$

Recall that a privacy-sensitive consumer will incur a privacy cost D if her online activities are tracked. Therefore, for a privacy-sensitive consumer who is indifferent between accepting and rejecting tracking by firm A, the following condition must hold:

$$V - tx_{A1}^{P'P'} - t\left(1-2x_{A1}^{P'P'}\right) - D = V - tx_{A1}^{P'P'} - p_A^{P'P'}. \quad (A41)$$

From (A41), we find the location of this privacy-sensitive consumer:

$$x_{A1}^{P'P'} = \frac{-p_A^{P'P'} + t + D}{2t}. \quad (A42)$$

Using the same procedure, we derive the location of a privacy-sensitive consumer who is indifferent between accepting and rejecting tracking by firm B:

$$x_{B1}^{P'P'} = \frac{p_B^{P'P'} + t - D}{2t}. \quad (A43)$$

Substituting (A39) – (A40) and (A42) – (A43) for $x_{A0}^{P'P'}$, $x_{A1}^{P'P'}$, $x_{B0}^{P'P'}$ and $x_{B1}^{P'P'}$ into the firms' profits in (A36) and (A37), we express their profit-maximization problems as

$$\max_{p_A^{P'P'}} \Pi_A^{P'P'} = \frac{-p_A^{P'P'^2} + 2tp_A^{P'P'} + \theta D^2}{4t}, \qquad (A44)$$

$$\max_{p_B^{P'P'}} \Pi_B^{P'P'} = \frac{-p_B^{P'P'^2} + 2tp_B^{P'P'} + \theta D^2}{4t}. \qquad (A45)$$

Using the first-order conditions of (A44) and (A45), we find the equilibrium uniform prices to be

$$p_A^{P'P'} = p_B^{P'P'} = t. \qquad (A46)$$

Recall that the highest personalized price charged by a firm is t, which is offered to consumers at the two ends. Hence, (A46) implies that consumers located in $(0,1)$ will obtain a lower price than the uniform price if they accept tracking. This suggests that all privacy-insensitive consumers will accept tracking, that is, $x_{A0}^{P'P'} = 0$ and $x_{B0}^{P'P'} = 1$.

Substituting (A46) into (A42) and (A43), we obtain $x_{A1}^{P'P'} = D/2t$ and $x_{B1}^{P'P'} = 1 - D/2t$. Hence, privacy-sensitive consumers in the intervals $[0, D/2t]$ and $[1 - D/2t, 1]$ purchase the goods at the uniform prices.

Substituting (A46) for $p_i^{P'P'}$ $(i = A, B)$ into (A44) and (A45), we obtain the firms' profits in this subgame, $\Pi_A^{P'P'}$ and $\Pi_B^{P'P'}$, presented in (7). Comparing them with the firms' profits in the (U', U') subgame, we find that $\Pi_A^{P'P'} < \Pi_A^{U'U'}$.

A3.2 Analysis of the (P', U') Subgame

In the (P', U') subgame, only firm A uses the tracking technology. We use $p_A^{P'U'}(x)$ to denote the personalized price that it offers a consumer who accepts tracking and $p_A^{P'U'}$ to denote the uniform price that it charges a consumer who rejects tracking. Firm B, on the other hand, does not use the tracking technology and charges all consumers a uniform price, denoted by $p_B^{P'U'}$.

Consider firm A's choice of personalized prices. Given firm B's uniform price $p_B^{P'U'}$, firm A has an incentive to undercut firm B's price by setting $p_A^{P'U'}(x) = t(1-2x) + p_B^{P'U'}$, provided that $p_A^{P'U'}(x) \geq 0$. The latter implies $x \leq (t + p_B^{P'U'})/2t$. Similar to (A20) in the (P, U) subgame, we define

$$x_0^{P'U'} \equiv \min\left\{\frac{t + p_B^{P'U'}}{2t}, 1\right\}. \qquad (A47)$$

Then firm A is able to attract the privacy-insensitive consumers located in $[0, x_0^{P'U'})$ by slightly undercutting firm B's uniform price $p_B^{P'U'}$.

However, not all these consumers will purchase good A at personalized prices because they may find it cheaper to buy the good at the uniform price offered by firm A. Since consumers located close to the left end $(x = 0)$ pay the highest personalized prices for good A, these are the consumers who may prefer the uniform price over their personalized prices. Let $x_{A0}^{P'U'}$ denote the location of a privacy-insensitive consumer who is indifferent between the uniform price and the personalized price offered by firm A. Then $x_{A0}^{P'U'}$ must satisfy the condition

$$V - tx_{A0}^{P'U'} - p_A^{P'U'} = V - tx_{A0}^{P'U'} - \left[t\left(1 - 2x_{A0}^{P'U'}\right) + p_B^{P'U'}\right]. \quad (A48)$$

From (A48), we obtain

$$x_{A0}^{P'U'} = \frac{p_B^{P'U'} - p_A^{P'U'} + t}{2t}. \qquad (A49)$$

By the definition of $x_{A0}^{P'U'}$, privacy-insensitive consumers in $[0, x_{A0}^{P'U'})$ will reject tracking and purchase good A at the uniform price, while privacy-insensitive consumers in $[x_{A0}^{P'U'}, x_0^{P'U'}]$ will accept tracking and buy good A at the personalized prices. These choices of privacy-insensitive consumers are illustrated in the top portion of Figure A3.

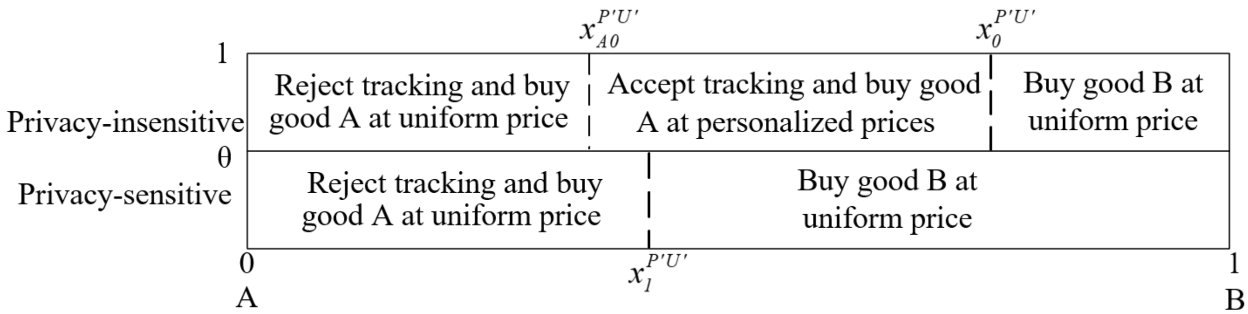

Figure A3: Consumers' Choices in the Subgame (P', U')

Because of the commitment problem noted in the discussion of the (P, U) subgame, privacy-sensitive consumers will not purchase good A at personalized prices. Hence, if they purchase good A in the (P', U') subgame, it will be at firm A's uniform price. Let $x_1^{P'U'}$ be the location of a privacy-sensitive consumer who is indifferent between purchasing good A and good B at their respective uniform prices. Then privacy-sensitive consumers in $[0, x_1^{P'U'}]$ purchase good A at its uniform price while privacy-sensitive consumers in $[x_1^{P'U'}, 1]$ buy good B which is available only at uniform price $p_B^{P'U'}$. These choices of privacy-sensitive consumers are illustrated in the bottom portion of Figure A3.

By its definition, $x_1^{P'U'}$ must satisfy the following condition:

$$V - tx_1^{P'U'} - p_A^{P'U'} = V - t\left(1 - x_1^{P'U'}\right) - p_B^{P'U'}. \quad (A50)$$

From (A50), we find that $x_1^{P'U'}$ has the same expression as $x_{A0}^{P'U'}$ in (A49), that is, $x_1^{P'U'} = x_{A0}^{P'U'}$.

At the beginning of stage 3, firms A and B set their uniform prices to maximize their profits,

$$\Pi_A^{P'U'} = p_A^{P'U'}\left((1-\theta)x_{A0}^{P'U'} + \theta x_1^{P'U'}\right) + (1-\theta)\int_{x_{A0}^{P'U'}}^{x_0^{P'U'}} [t(1-2x) + p_B^{P'U'}]\, dx, \quad (A51)$$

$$\Pi_B^{P'U'} = p_B^{P'U'}\left[(1-\theta)\left(1 - x_0^{P'U'}\right) + \theta\left(1 - x_1^{P'U'}\right)\right]. \quad (A52)$$

The two terms on the right-hand side of (A51) represent firm A's profit from sales at the uniform price and at personalized prices, respectively. On the other hand, firm B earns its entire profit from sales at its uniform price, represented by the right-hand side of (A52).

Assuming that $x_0^{P'U'} < 1$, we use (A47), (A49) and $x_1^{P'U'} = x_{A0}^{P'U'}$ to rewrite the firms' profit-maximization problems as

$$\max_{p_A^{P'U'}} \Pi_A^{P'U'} = \frac{p_A^{P'U'}\left(2\left(p_B^{P'U'} + t\right) - p_A^{P'U'}(1+\theta)\right)}{4t}. \quad (A53)$$

$$\max_{p_B^{P'U'}} \Pi_B^{P'U'} = \frac{p_B^{P'U'}\left(\theta p_A^{P'U'} + t - p_B^{P'U'}\right)}{2t}. \quad (A54)$$

Solving the first-order conditions of (A53) and (A54) simultaneously, we find

$$p_A^{P'U'} = \frac{3t}{2+\theta}, \quad p_B^{P'U'} = \frac{t(1+2\theta)}{2+\theta}. \quad (A55)$$

Substituting (A55) into (A47) and (A49), we obtain

$$x_0^{P'U'} = \frac{3(1+\theta)}{2(2+\theta)}, \quad x_{A0}^{P'U'} = x_1^{P'U'} = \frac{3\theta}{2(2+\theta)}. \quad (A56)$$

From (A56) we confirm that $x_0^{P'U'} < 1$ indeed holds in equilibrium. Therefore, in the subgame (P', U'), both privacy-insensitive and privacy-sensitive consumers in $[0, \frac{3\theta}{2(2+\theta)}]$ purchase good A at the uniform price while only privacy-insensitive consumers in $(\frac{3\theta}{2(2+\theta)}, \frac{3(1+\theta)}{2(2+\theta)}]$ buy good A at personalized prices. The remaining consumers purchase good B at its uniform price.

Substituting (A55) into (A53) and (A54), we obtain the firms' profits presented in (8). Incidentally, using (8), we find

$$\Pi_A^{P'U'} - \Pi_B^{P'U'} = \frac{t(7+\theta-8\theta^2)}{4(2+\theta)^2} > 0. \quad (A57)$$

In other words, in this subgame the firm that uses the tracking technology earns a larger profit than the firm that does not. To see the intuition behind this result, we use (A56) to calculate the proportion of consumers served by firm A:

$$\theta x_1^{P'U'} + (1-\theta) x_0^{P'U'} = \frac{3}{2(2+\theta)} > \frac{1}{2}. \quad (A58)$$

This suggests that the use of tracking technology and personalized pricing enables firm A to capture more than half of the market, hence the larger profit.

A3.3 Proof of Proposition 5

Proposition 5 is derived using the payoff matrix in Table 2. First, we establish that (U', U') cannot be an equilibrium. Starting with a situation where both firms choose U', each firm has an incentive to deviate to P' because

$$\frac{t}{2} < \frac{9t(1+\theta)}{4(2+\theta)^2} \qquad (A59)$$

for any $\theta \in (0, 1)$.

Second, we derive the conditions under which (P', U') and (U', P') are equilibriums. Consider the case (P', U'). Firm A has no incentive to deviate from P' to U' because of (A59). Firm B will have no incentive to deviate from U' to P' if

$$\frac{t(1+2\theta)^2}{2(2+\theta)^2} \geq \frac{t}{4} + \frac{\theta D^2}{4t}. \qquad (A60)$$

Rewriting (A60), we obtain

$$D \leq \frac{t}{2+\theta}\sqrt{\frac{7\theta^2+4\theta-2}{\theta}}. \quad (A61)$$

To satisfy (A61), it is necessary to have $7\theta^2 + 4\theta - 2 > 0$. The latter entails

$$\theta > \frac{3\sqrt{2}-2}{7}. \qquad (A62)$$

Therefore, (P, U), and by extension (U, P), are equilibriums if (A61) and (A62) hold. This proves part (ii) of Proposition 5.

In the main text, the right-hand side of (A61) is defined as D^* in (9). From (A61) and (A62) we know that D^* is real and positive for $\theta > (3\sqrt{2}-2)/7$. Using (9), we find

$$\frac{\partial D^{*2}}{\partial \theta} = \frac{t^2(4+6\theta+6\theta^2-7\theta^3)}{\theta^2(2+\theta)^3} > 0 \quad \text{for } \theta \in (0,1]. \quad (A63)$$

This implies that $\partial D^*/\partial\theta > 0$ for $\theta \in [(3\sqrt{2}-2)/7, 1]$. Setting $\theta = 1$ in (9), we find $D^* = t$. The last two observations imply that $D^* < t$ for $\theta \in ((3\sqrt{2}-2)/7, 1)$.

Third, we examine the conditions under which (P', P') is an equilibrium. Given that its rival chooses P', a firm will not want to deviate from P' to U' if (A60) is violated. The latter is true if $\theta \le (3\sqrt{2}-2)/7$, or if $\theta > (3\sqrt{2}-2)/7$ and $D \in (D^*, t)$. This proves part (i) of Proposition 5.

A3.4 Proof of Proposition 6

Consistent with earlier notations, we use Π^i to denote the industry profit, CS_α^i to denote the surplus of α-type consumers, with $\alpha = 0$ representing privacy-insensitive consumers and $\alpha = 1$ representing privacy-sensitive consumers. Moreover, we use CS^i to denote the combined consumer surplus of both types of consumers, and W^i to denote social welfare. Superscript i in these notations denote the combinations of pricing strategies. That is, $i = (U, U)$, (U, P), (P, U), or (P, P) when there is no privacy regulation, and $i = (U', U')$, (U', P'), (P', U'), or (P', P') when there is privacy regulation. For example, Π^{UU} represents the industry profit when both firms adopt uniform pricing without regulation.

Using the results in Tables A1 and A2, we sum Π_A^i and Π_B^i to obtain the industry profit in different scenarios. Specifically, in the case where there is no privacy regulation, we have

$$\Pi^{UU} = t, \qquad \Pi^{PP} = \frac{t}{2}, \qquad (A64)$$

$$\Pi^{UP} = \Pi^{PU} = \begin{cases} \dfrac{t(3-\theta)^2}{16(1-\theta)} + \dfrac{t(1+\theta)^2}{8(1-\theta)} = \dfrac{t(11-2\theta+3\theta^2)}{16(1-\theta)} & \text{if } \theta \le \bar{\theta}, \\ V - t \quad \text{if } \theta > \bar{\theta}. \end{cases} \qquad (A65)$$

With the privacy regulation, on the other hand, industry profits are represented by

$$\Pi^{U'U'} = t, \qquad \Pi^{P'P'} = \frac{t}{2} + \frac{\theta D^2}{2t}, \qquad \Pi^{U'P'} = \Pi^{P'U'} = \frac{t(11+17\theta+8\theta^2)}{4(2+\theta)^2}. \qquad (A66)$$

Regarding consumer welfare, we use $p_A^{UU} = p_B^{UU} = t$ to obtain

$$CS^{UU} = \int_0^{\frac{1}{2}} (V - tx - p_A^{UU})dx + \int_{\frac{1}{2}}^1 (V - t(1-x) - p_B^{UU})dx = V - \frac{5t}{4}. \quad (A67)$$

In the (P, P) subgame, we use $p_A^{PP}(x) = t(1-2x)$ and $p_B^{PP}(x) = t(2x-1)$ to find

$$CS_0^{PP} = (1-\theta)\left(\int_0^{\frac{1}{2}} \left(V - tx - p_A^{PP}(x)\right)dx + \int_{\frac{1}{2}}^1 \left(V - t(1-x) - p_B^{PP}(x)\right)dx\right)$$
$$= (1-\theta)\left(V - \frac{3t}{4}\right). \qquad (A68)$$

$$CS_1^{PP} = \theta\left(\int_0^{\frac{1}{2}} (V - tx - p_A^{PP}(x) - D)dx + \int_{\frac{1}{2}}^1 (V - t(1-x) - p_B^{PP}(x) - D)dx\right)$$
$$= \theta\left(V - \frac{3t}{4} - D\right). \qquad (A69)$$

Then the combined welfare of both types of consumers is

$$CS^{PP} = CS_0^{PP} + CS_1^{PP} = V - \frac{3t}{4} - \theta D. \qquad (A70)$$

In the (P,U) subgame, the expression of consumer surplus depends on the value of θ. In section A2.1, we have shown that for $\theta \in (0,\bar{\theta}]$, p_B^{PU} in (A25) is firm B's profit-maximizing price, in which case privacy-insensitive consumers located in $[x_0^{PU},1]$ and all privacy-sensitive consumers purchase good B at the uniform price p_B^{PU} while privacy-insensitive consumers in $[0,x_0^{PU})$ buy good A at personalized prices. Accordingly, the welfare levels of privacy-insensitive consumers and privacy-sensitive consumers are

$$CS_0^{PU} = (1-\theta)(\int_0^{x_0^{PU}} \left(V - tx - p_A^{PU}(x)\right) dx + \int_{x_0^{PU}}^1 (V - t(1-x) - p_B^{PU}) dx)$$
$$= (1-\theta)V - t. \qquad (A71)$$
$$CS_1^{PU} = \theta \int_0^1 (V - t(1-x) - P_B^{PU}) dx = \theta\left(V - \frac{t}{1-\theta}\right). \quad (A72)$$

Summing (A71) and (A72), we obtain

$$CS^{PU} = V - \frac{t}{1-\theta} \text{ if } \theta \le \bar{\theta}. \qquad (A73)$$

For $\theta \in (\bar{\theta},1)$, on the other hand, $p_B^{PU} = V - t$ is firm B's profit-maximizing price, in which case all privacy-sensitive consumers purchase good B at this price and all privacy-insensitive consumers buy good A at personalized prices. Then

$$CS_0^{PU} = (1-\theta)(\int_0^1 \left(V - tx - p_A^{PU}(x)\right) dx) = \frac{t(1-\theta)}{2}, \quad (A74)$$
$$CS_1^{PU} = \theta \int_0^1 (V - t(1-x) - p_B^{PU}) dx = \frac{\theta t}{2}. \qquad (A75)$$

Adding (A74) and (A75), we find

$$CS^{PU} = \frac{t}{2} \text{ if } \theta > \bar{\theta}. \qquad (A76)$$

By symmetry, we know that CS^{UP} has the same value as CS^{PU} in (A73) if $\theta \in (0,\bar{\theta}]$ and in (A76) if $\theta \in (\bar{\theta},1)$.

The privacy regulation has no impact on consumer welfare in the (U',U') subgame because no firm adopts the tracking technology in this case. But the regulation changes consumer welfare in the other three subgames. Specifically, it has been shown in section A3.1 that in the equilibrium of the (P',P') subgame, the two firms set their uniform prices at $p_A^{P'P'} = p_B^{P'P'} = t$, and privacy-sensitive consumers in the intervals $[0,D/2t]$ and $[1-D/2t,1]$ purchase the goods at these uniform prices while all remaining consumers buy the goods at personalized prices. Accordingly,

$$CS_0^{P'P'} = (1-\theta)\left(\int_0^{\frac{1}{2}} \left(V - tx - p_A^{P'P'}(x)\right) dx + \int_{\frac{1}{2}}^1 \left(V - t(1-x) - p_B^{P'P'}(x)\right) dx\right)$$
$$= (1-\theta)\left(V - \frac{3t}{4}\right). \qquad (A77)$$
$$CS_1^{P'P'} = \theta\left(\int_0^{D/2t} \left(V - tx - p_A^{P'P'}\right) dx + \int_{D/2t}^{\frac{1}{2}} \left(V - tx - p_A^{P'P'}(x) - D\right) dx\right.$$
$$\left. + \int_{\frac{1}{2}}^{1-D/2t} \left(V - t(1-x) - p_B^{P'P'}(x) - D\right) dx + \int_{1-D/2t}^1 \left(V - t(1-x) - p_B^{P'P'}\right) dx\right)$$
$$= \theta\left(V - \frac{3t}{4} - D + \frac{D^2}{2t}\right). \quad (A78)$$

Using (A77) and (A78), we obtain

$$CS^{P'P'} = V - \frac{3t}{4} - \theta D + \frac{\theta D^2}{2t}. \qquad (A79)$$

In the (P', U') subgame, consumers choices are illustrated in Figure A3 and the equilibrium uniform prices are given in (A55). With the aid of Figure A3 and (A55)-(A56), we derive the welfare of privacy-insensitive and privacy-sensitive consumers as follows:

$$CS_0^{P'U'} = (1-\theta)\left(\int_0^{x_{A0}^{P'U'}} \left(V - tx - p_A^{P'U'}\right) dx + \int_{x_{A0}^{P'U'}}^{x_0^{P'U'}} \left(V - tx - p_A^{P'U'}(x)\right) dx + \int_{x_0^{P'U'}}^{1} \left(V - tx - p_B^{P'U'}\right) dx\right) = (1-\theta)\left(V - \frac{t(16 + 28\theta + \theta^2)}{4(2+\theta)^2}\right). \quad (A80)$$

$$CS_1^{P'U'} = \theta\left(\int_0^{x_1^{P'U'}} \left(V - tx - p_A^{P'U'}\right) dx + \int_{x_1^{P'U'}}^{1} \left(V - tx - p_B^{P'U'}\right) dx\right) = \theta\left(V - \frac{t(16 + 28\theta + \theta^2)}{4(2+\theta)^2}\right). \qquad (A81)$$

Adding (A80) and (A81), we obtain

$$CS^{P'U'} = V - \frac{t(16 + 28\theta + \theta^2)}{4(2+\theta)^2}. \quad (A82)$$

By symmetry, we know that $CS^{U'P'}$ has the same expression as (A82).

Regarding social welfare, we have $W^i = \Pi^i + CS^i$. Using (A64)-(A65) and (A67)-(A76), we find the levels of social welfare in the four subgames without the privacy regulation. They are

$$W^{UU} = V - \frac{t}{4}, \qquad W^{PP} = V - \frac{t}{4} - \theta D, \quad (A83)$$

$$W^{PU} = W^{UP} = \begin{cases} V - \dfrac{t(5 + 2\theta - 3\theta^2)}{16(1-\theta)} & \text{if } \theta \le \bar{\theta}, \\ V - \dfrac{t}{2} & \text{if } \theta > \bar{\theta}. \end{cases} \qquad (A84)$$

Similarly, using (A66) and (A77)-(A82), we obtain the levels of social welfare in the four subgames with the privacy regulation:

$$W^{U'U'} = V - \frac{t}{4}, \qquad W^{P'P'} = V - \frac{t}{4} - \theta D + \frac{\theta D^2}{t}, \qquad (A85)$$

$$W^{P'U'} = W^{U'P'} = V + \frac{t(7\theta^2 - 5 - 11\theta)}{4(2+\theta)^2}. \qquad (A86)$$

In scenario (i) of Proposition 6, the value of θ is in a range where both firms adopt personalized pricing with and without the privacy regulation, i.e., in region I of Figure 3. Comparing Π^{PP} in (A64) with $\Pi^{P'P'}$ in (A66), we see that $\Pi^{P'P'} > \Pi^{PP}$ because $\theta D^2 > 0$. For the same reason, we find $CS^{P'P'} > CS^{PP}$ and $W^{P'P'} > W^{PP}$ from comparisons of (A70) with (A79), and of (A83) with (A85). To determine the impact on individual consumers, note that the personalized prices remain the same with and without the regulation. Consequently, the regulation has no impact on the utility of those consumers who accept tracking because they buy the same goods at the same personalized prices. For individual consumers who reject tracking, specifically, those privacy-sensitive consumers located in $[0, D/2t)$ and $(1 - D/2t, 1]$, (A41) implies that they achieve a higher level of utility by being able to purchase at the uniform prices under the regulation.

Scenario (ii) of Proposition 6 encompasses regions II, III and IV of Figure 3. In section A3.5 below, we present in Proposition A1 the conditions under which the privacy regulation reduces the utility of some individual consumers. Here we demonstrate the impact of the regulation on industry profit, consumer welfare, and social welfare.

For θ in regions II and III of Figure 3, i.e., for $\theta \in [\min\{\hat{\theta}, \sqrt{5}-2\}, \tilde{\theta})$, the two firms adopt asymmetric pricing strategies under laissez-faire. Under the privacy regulation, for parameters in the ranges of $\min\{\hat{\theta}, \sqrt{5}-2\} \leq \theta \leq (3\sqrt{2}-2)/7$, or $(3\sqrt{2}-2)/7 < \theta < \tilde{\theta}$ and $D \in (D^*, t)$, both firms adopt personalized pricing, i.e., (P', P') is an equilibrium. In the range $(3\sqrt{2}-2)/7 < \theta < \tilde{\theta}$ and $D \leq D^*$, the firms still adopt asymmetric pricing strategies, i.e., (P', U') and (U', P') are equilibriums. Below we examine these two situations, named as scenarios (ii-a) and (ii-b), separately.

Scenarios (ii-a)

Consider the situation in which the firms' equilibrium strategies change from (P, U) and (U, P) to (P', P') as a result of the privacy regulation. The industry profits without and with the regulation are expressed in (A65) and (A66), respectively. To show that the industry profit is lower with the regulation than without it, we note from (A66) that $\Pi^{P'P'} = t$ at $\theta = 1$ and $D = t$. Moreover, it is easy to verify using (A66) that $\partial\Pi^{P'P'}/\partial\theta > 0$ and $\partial\Pi^{P'P'}/\partial D > 0$, implying that $\Pi^{P'P'}$ increases in θ and D. Then $\Pi^{P'P'} < t$ for $\theta < 1$ and $D < t$. Note from (A65) that if $\theta > \bar{\theta}$, $\Pi^{UP} = \Pi^{PU} = V - t > t$. Hence, $\Pi^{UP} = \Pi^{PU} > \Pi^{P'P'}$ for $\theta > \bar{\theta}$.

If $\theta \leq \bar{\theta}$, on the other hand, we use (A65) and (A66) to obtain

$$\Pi^{PU} - \Pi^{P'P'} = \frac{t(11 - 2\theta + 3\theta^2)}{16(1-\theta)} - \left[\frac{t}{2} + \frac{\theta D^2}{2t}\right] = \frac{3t(1+\theta)^2}{16(1-\theta)} - \frac{\theta D^2}{2t}. \quad (A87)$$

Differentiating (A87), we find

$$\frac{\partial(\Pi^{PU} - \Pi^{P'P'})}{\partial\theta} = \frac{3t(3 + 2\theta - \theta^2)}{16(1-\theta)^2} - \frac{D^2}{2t}, \quad (A88)$$

and

$$\frac{\partial^2(\Pi^{PU} - \Pi^{P'P'})}{\partial\theta^2} = \frac{3t}{2(1-\theta)^3} > 0. \quad (A89)$$

Recalling that $D < t$, we observe from (A88) that

$$\left.\frac{\partial(\Pi^{PU} - \Pi^{P'P'})}{\partial\theta}\right|_{\theta=0} = \frac{9t}{16} - \frac{D^2}{2t} > 0. \quad (A90)$$

Then (A89) and (A90) imply that the sign of (A88) is positive for $\theta > 0$. This result, along with the observation from (A87) that

$$(\Pi^{PU} - \Pi^{P'P'})\big|_{\theta=0} = \frac{3t}{16} > 0, \quad (A91)$$

entails $\Pi^{PU} - \Pi^{P'P'} > 0$ for $\theta \in (0, \bar{\theta}]$. Hence, we conclude that $\Pi^{PU} = \Pi^{UP} > \Pi^{P'P'}$ if $\theta \in (0, \bar{\theta}]$.

Turning to consumer welfare, we use (A76) and (A79) to find that in the case $\theta > \bar{\theta}$,

$$\mathrm{CS}^{P'P'} - \mathrm{CS}^{PU} = V - \frac{5t}{4} - \theta D + \frac{\theta D^2}{2t}. \quad (A92)$$

Differentiating (A92), we obtain

$$\frac{\partial(\mathrm{CS}^{P'P'} - \mathrm{CS}^{PU})}{\partial D} = \theta\left(\frac{D}{t} - 1\right) \leq 0 \quad (A93)$$

for $D \leq t$. Note that

$$(\mathrm{CS}^{P'P'}-\mathrm{CS}^{PU})\big|_{D=t}=V-\frac{5t}{4}-\frac{\theta t}{2}>V-\frac{7t}{4}>0. \qquad (A94)$$

We conclude from (A93) and (A94) that $\mathrm{CS}^{P'P'}-\mathrm{CS}^{PU}>0$ for $\theta\in(\bar{\theta},1)$.

In the case where $\theta\leq\bar{\theta}$, we use (A73) and (A79) to obtain

$$\mathrm{CS}^{P'P'}-\mathrm{CS}^{PU}=\frac{t(1+3\theta)}{4(1-\theta)}-\theta D+\frac{\theta D^2}{2t}. \qquad (A95)$$

Similar to the case $\theta>\bar{\theta}$, we use (A95) to find $\partial(\mathrm{CS}^{P'P'}-\mathrm{CS}^{PU})/\partial D\leq 0$ and $(\mathrm{CS}^{P'P'}-\mathrm{CS}^{PU})\big|_{D=t}>0$. Thus, $\mathrm{CS}^{P'P'}-\mathrm{CS}^{PU}>0$ for $\theta\leq\bar{\theta}$.

Now we consider the impact on social welfare. Using (A84) and (A85), we find that if $\theta>\bar{\theta}$,

$$\mathrm{W}^{P'P'}-\mathrm{W}^{PU}=\frac{t}{4}-\theta D+\frac{\theta D^2}{t}. \qquad (A96)$$

Differentiating (A96), we obtain

$$\frac{\partial(\mathrm{W}^{P'P'}-\mathrm{W}^{PU})}{\partial D}=\theta\left(\frac{2D}{t}-1\right)\begin{cases}<0 \text{ if } D<\dfrac{t}{2},\\ >0 \text{ if } D>\dfrac{t}{2}.\end{cases} \qquad (A97)$$

From (A97), we conclude that $\mathrm{W}^{P'P'}-\mathrm{W}^{PU}$ reaches a local minimum at $D=t/2$. Since

$$(\mathrm{W}^{P'P'}-\mathrm{W}^{PU})\big|_{D=t/2}=\frac{t}{4}(1-\theta)>0, \qquad (A98)$$

we have $\mathrm{W}^{P'P'}-\mathrm{W}^{PU}>0$ in the case $\theta>\bar{\theta}$.

If $\theta\leq\bar{\theta}$, we use (A84) and (A85) to find

$$\mathrm{W}^{P'P'}-\mathrm{W}^{PU}=\frac{t(1+6\theta-3\theta^2)}{16(1-\theta)}-\theta D+\frac{\theta D^2}{t}. \qquad (A99)$$

Similar to the case $\theta>\bar{\theta}$, we use (A99) to find that $\mathrm{W}^{P'P'}-\mathrm{W}^{PU}$ reaches a local minimum at $D=t/2$ and that

$$(\mathrm{W}^{P'P'}-\mathrm{W}^{PU})\big|_{D=t/2}=\frac{t(1+\theta)^2}{16(1-\theta)}>0. \qquad (A100)$$

Hence, we have $\mathrm{W}^{P'P'}-\mathrm{W}^{PU}>0$ in the case $\theta\leq\bar{\theta}$.

Scenario (ii-b)

We now consider the situation in which the firms adopt asymmetric pricing strategies both with and without the privacy regulation. For ease of exposition, we present an analysis based on the assumption that firm A adopts personalized pricing and firm B adopts uniform pricing, i.e., (P,U) and (P',U'). This is without loss of generality because the levels of industry profit, consumer welfare, and social welfare in the (P,U) equilibrium (respectively, (P',U') equilibrium) are the same as those in the (U,P) equilibrium (respectively, (U',P') equilibrium).

First, we consider the impact on industry profit. For $\theta>\bar{\theta}$, we use (A65) and (A66) to find

$$\Pi^{PU}-\Pi^{P'U'}=V-\frac{3t(9+11\theta+4\theta^2)}{4(2+\theta)^2}. \qquad (A101)$$

Differentiating (A101), we obtain

$$\frac{\partial(\Pi^{PU}-\Pi^{P'U'})}{\partial\theta}=-\frac{3t(4+5\theta)}{4(2+\theta)^3}<0. \qquad (A102)$$

From (A101), we also find $\left(\Pi^{PU}-\Pi^{P'U'}\right)\big|_{\theta=1}=V-2t>0$. Hence, we have $\Pi^{PU}-\Pi^{P'U'}>0$ for $\theta\in(\bar{\theta},1)$. If $\theta\leq\bar{\theta}$, the difference in industry profit is represented by

$$\Pi^{PU} - \Pi^{P'U'} = \frac{3\theta t(4 + 17\theta + 14\theta^2 + \theta^3)}{16(1-\theta)(2+\theta)^2}. \quad (A103)$$

Differentiating (A103), we obtain

$$\frac{\partial(\Pi^{PU} - \Pi^{P'U'})}{\partial\theta} = \frac{3t(8 + 64\theta + 58\theta^2 - 17\theta^3 - 4\theta^4 - \theta^5)}{16(1-\theta)^2(2+\theta)^3}, \quad (A104)$$

and

$$\frac{\partial^2(\Pi^{PU} - \Pi^{P'U'})}{\partial\theta^2} = \frac{3t(17 + 34\theta + 12\theta^2 + 22\theta^3 - 4\theta^4)}{2(1-\theta)^3(2+\theta)^4} > 0. \quad (A105)$$

Note from (A104) that $\partial(\Pi^{PU} - \Pi^{P'U'})/\partial\theta > 0$ at $\theta = 0$. This and (A105) imply $\partial(\Pi^{PU} - \Pi^{P'U'})/\partial\theta > 0$ for $\theta \geq 0$; in other words, $\Pi^{PU} - \Pi^{P'U'}$ increases in θ. Noting from (A103) that $\Pi^{PU} - \Pi^{P'U'} = 0$ at $\theta = 0$, we conclude that $\Pi^{PU} - \Pi^{P'U'} > 0$ for $\theta \in (0, \bar{\theta}]$.

Next, we examine the impact on consumer welfare. For $\theta > \bar{\theta}$, we use (A76) and (A82) to find

$$\mathrm{CS}^{P'U'} - \mathrm{CS}^{PU} = V - \frac{3t(8 + 12\theta + \theta^2)}{4(2+\theta)^2}. \quad (A106)$$

Using (A106), we find that $\partial(\mathrm{CS}^{P'U'} - \mathrm{CS}^{PU})/\partial\theta < 0$ and $(\mathrm{CS}^{P'U'} - \mathrm{CS}^{PU})\big|_{\theta=1} > 0$. These findings imply that $\mathrm{CS}^{P'U'} - \mathrm{CS}^{PU} > 0$ for $\theta \in (\bar{\theta}, 1)$. Similarly, in the case $\theta \leq \bar{\theta}$, we use (A73) and (A82) to obtain

$$\mathrm{CS}^{P'U'} - \mathrm{CS}^{PU} = \frac{\theta t(4 + 31\theta + \theta^2)}{4(1-\theta)(2+\theta)^2}. \quad (A107)$$

From (A107), we find that $\partial(\mathrm{CS}^{P'U'} - \mathrm{CS}^{PU})/\partial\theta > 0$ and $(\mathrm{CS}^{P'U'} - \mathrm{CS}^{PU})\big|_{\theta=0} = 0$. They imply that $\mathrm{CS}^{P'U'} - \mathrm{CS}^{PU} > 0$ for $\theta \in (0, \bar{\theta}]$.

Turning to social welfare, we use (A84) and (A86) to find the difference in social welfare. In the case $\theta > \bar{\theta}$, this is represented by

$$\mathrm{W}^{P'U'} - \mathrm{W}^{PU} = \frac{3t(1 - \theta + 3\theta^2)}{4(2+\theta)^2}. \quad (A108)$$

Differentiating (A108), we find

$$\frac{\partial(\mathrm{W}^{P'U'} - \mathrm{W}^{PU})}{\partial\theta} = \frac{3t(13\theta - 4)}{4(2+\theta)^3}\begin{cases} < 0 \text{ if } \theta < \frac{4}{13}, \\ > 0 \text{ if } \theta > \frac{4}{13}. \end{cases} \quad (A109)$$

From (A109), we conclude that $\mathrm{W}^{P'U'} - \mathrm{W}^{PU}$ reaches a local minimum at $\theta = 4/13$. Using (A108), we observe that

$$(\mathrm{W}^{P'U'} - \mathrm{W}^{PU})\big|_{\theta=4/13} = \frac{11t}{80} > 0. \quad (A110)$$

Then (A109) and (A110) imply that $\mathrm{W}^{P'U'} - \mathrm{W}^{PU} > 0$. On the other hand, if $\theta \leq \bar{\theta}$, the difference in social welfare is represented by

$$\mathrm{W}^{P'U'} - \mathrm{W}^{PU} = \frac{\theta t(4 + 73\theta - 38\theta^2 - 3\theta^3)}{16(1-\theta)(2+\theta)^2}, \quad (A111)$$

which is positive for any $\theta \in (0, 1)$.

For θ in region IV of Figure 3, i.e., for $\theta \geq \tilde{\theta}$, the equilibrium under the privacy regulation depends on the value of D. Specifically, given that $\theta \geq \tilde{\theta}$, (P', P') is an equilibrium if $D > D^*$, but (P', U') and

(U', P') are equilibria if $D \leq D^*$. Accordingly, this part of the proof involves the consideration of these two subcases.

First, we examine the subcase where the firms' equilibrium strategies change from (U, U) to (P', P') as a result of the privacy regulation. Recall from the analysis in scenario (ii-a) that $\Pi^{P'P'} < t$ for $\theta < 1$ and $D < t$. Observing from (A64) that $\Pi^{UU} = t$, we conclude $\Pi^{UU} > \Pi^{P'P'}$. Turing to consumer welfare, we use (A67) and (A79) to obtain

$$\mathrm{CS}^{P'P'} - \mathrm{CS}^{UU} = \frac{t}{2} - \theta D + \frac{\theta D^2}{2t}. \qquad (A112)$$

From (A112), we find that $\partial(\mathrm{CS}^{P'P'} - \mathrm{CS}^{UU})/\partial\theta < 0$ and $(\mathrm{CS}^{P'P'} - \mathrm{CS}^{UU})\big|_{\theta=1} > 0$. These findings imply that $\mathrm{CS}^{P'P'} - \mathrm{CS}^{UU} > 0$. Regarding social welfare, we use (A83) and (A85) to find

$$\mathrm{W}^{UU} - \mathrm{W}^{P'P'} = \frac{\theta D(t-D)}{t} > 0. \qquad (A113)$$

Next, we consider the subcase where the firms' equilibrium strategies change from (U, U) to (P', U') or (U', P') as a result of the regulation. For ease of exposition, we use (P', U') as the representative of the post-regulation equilibriums. From (A64) and (A66). We obtain

$$\Pi^{UU} - \Pi^{P'U'} = \frac{t(5 - \theta - 4\theta^2)}{4(2+\theta)^2}. \qquad (A114)$$

Using (A114), we find that $\partial(\Pi^{UU} - \Pi^{P'U'})/\partial\theta < 0$ and $(\Pi^{UU} - \Pi^{P'U'})\big|_{\theta=1} = 0$. These findings imply that $\Pi^{UU} - \Pi^{P'U'} > 0$ for $\theta \in [\tilde{\theta}, 1)$. Using (A67) and (A82), we calculate the change consumer welfare:

$$\mathrm{CS}^{P'U'} - \mathrm{CS}^{UU} = \frac{t(1-\theta)^2}{(2+\theta)^2} > 0. \quad (A115)$$

Regarding social welfare, we use (A83) and (A86) to obtain

$$\mathrm{W}^{UU} - \mathrm{W}^{P'U'} = \frac{t(1 + 7\theta - 8\theta^2)}{4(2+\theta)^2}. \quad (A116)$$

Differentiating (A116), we find

$$\frac{\partial\left(\mathrm{W}^{UU} - \mathrm{W}^{P'U'}\right)}{\partial\theta} = \frac{3t(13\theta - 4)}{4(2+\theta)^3}\begin{cases} < 0 \text{ if } \theta < \dfrac{4}{13}, \\ > 0 \text{ if } \theta > \dfrac{4}{13}. \end{cases} \qquad (A117)$$

From (A117), we conclude that $\mathrm{W}^{UU} - \mathrm{W}^{P'U'}$ reaches a local minimum at $\theta = 4/13$. In (A116), we observe that

$$(\mathrm{W}^{UU} - \mathrm{W}^{P'U'})\big|_{\theta=4/13} = \frac{9t}{80} > 0. \quad (A118)$$

Then (A117) and (A118) imply that $\mathrm{W}^{UU} - \mathrm{W}^{P'U'} > 0$. This completes the proof of Proposition 6.

A3.5 Cases where Privacy Regulation Reduces the Utility of Individual Consumers

As detailed in Proposition A1 below, there are two cases where some consumers are made worse off by the regulation. Both cases involve asymmetric pricing strategies either with or without the privacy regulation. For ease of presentation, we assume (without loss of generality) that in an equilibrium with asymmetric pricing strategies, firm A adopts personalized pricing while firm B uses uniform pricing. Define

$$x_1^* \equiv \begin{cases} \dfrac{3-\theta}{4(1-\theta)} - \dfrac{D}{2t} & \text{if } D \leq \dfrac{(1+\theta)t}{2(1-\theta)}; \\ \dfrac{1+\theta}{4(1-\theta)} & \text{if } D > \dfrac{(1+\theta)t}{2(1-\theta)}. \end{cases} \qquad (A119)$$

Proposition A1: The privacy regulation reduces the utility of some consumers in the following two cases.

(i) If $\theta \geq \tilde{\theta}$ and $D \leq D^*$, the regulation reduces the utility of privacy-sensitive and privacy-insensitive consumers located in $[0, \frac{1+2\theta}{2(2+\theta)})$.

(ii) If $V < \left(3/2 + \sqrt{5}/4\right)t$ and $\min\{\hat{\theta}, \sqrt{5}-2\} \leq \theta < \bar{\theta}$, the regulation reduces the utility of privacy-sensitive consumers located in $(x_1^*,\ 1]$ and privacy-insensitive consumers located in $(x_0^{PU}, 1]$.

For other ranges of parameter values, the regulation increases the utility of some (or all) consumers without harming any other consumers.

In part (i) of Proposition A1, both firms use uniform pricing under laissez-faire but they switch to asymmetric pricing strategies under the privacy regulation. The firm that adopts personalized pricing under the regulation (firm A) is able to capture more than half of privacy-insensitive consumers because of its ability to engage in price discrimination. For those consumers who rejects tracking, firm A offers a uniform price that is higher than what it offers under laissez-faire. This reduces the utility of some (privacy-sensitive and privacy-insensitive) consumers who buy good A under laissez-faire.

In part (ii) of Proposition A1, only one firm uses personalized pricing under laissez-faire but both firms adopt personalized pricing under the privacy regulation. Under laissez-faire, the firm that adopts uniform pricing (firm B) sets a low uniform price to attract all privacy-sensitive consumers and a fraction of privacy-insensitive consumers for whom good B is the preferred brand. Under the regulation, firm B adopts personalized pricing and, for those who reject tracking, sets a uniform price that is higher than what it offers under laisses-faire. The higher uniform price harms some of those consumers who would have purchased good B under laissez-faire.

Proof of Proposition A1.

With the aid of Figure 3, we see that case (i) of the proposition deals with a situation where the privacy regulation causes the firms to change their equilibrium pricing strategies from (U, U) to (P', U'). Recall from section A2.1 that the equilibrium prices in the (U, U) subgame are $p_A^{UU} = p_B^{UU} = t$. In the (P', U') subgame, the uniform price offered by firm A (to its customers who reject tracking), $p_A^{P'U'}$, and the uniform price of firm B, $p_B^{P'U'}$, are in (A55). Comparing these prices, we find that $p_A^{P'U'} > p_A^{UU}$ and $p_B^{P'U'} < p_B^{UU}$. Because $p_A^{P'U'} > p_A^{UU}$, the regulation reduces the utility of consumers who purchase good A both with and without the regulation. From section A3.2, we see that these consumers are located in the interval $[0, \frac{3\theta}{2(2+\theta)}]$

Moreover, the regulation also reduces the utility of those consumers who are to the right of $x = \frac{3\theta}{2(2+\theta)}$ because it causes them either to buy good A at higher personalized prices or to buy their less-preferred brand, good B (albeit at a lower uniform price). To be more specific, the privacy-insensitive consumers at these locations purchase good A at personalized prices, and they are made worse off by the regulation if $t(1-2x) + p_B^{P'U'} > p_A^{UU}$, or equivalently, if $x < \frac{1+2\theta}{2(2+\theta)}$. Similarly, the privacy-sensitive consumers at these locations switch to buy good B, and they are made worse off by the regulation if $p_B^{P'U'} + t(1-x) > p_A^{UU} + tx$, which also implies $x < \frac{1+2\theta}{2(2+\theta)}$. Combining these findings, we conclude that the regulation reduces the utility of privacy-sensitive and privacy-insensitive consumers located in $[0, \frac{1+2\theta}{2(2+\theta)})$.

Turning to case (ii) of the proposition, recall from section A2.2 that $\hat{\theta} < \bar{\theta}$ for $2t < V < \left(\frac{3}{2} + \frac{\sqrt{5}}{4}\right)t$ and $\tilde{\theta} > \bar{\theta}$. Then with the help of Figure 3, we see that case (ii) deals with a situation where the privacy regulation causes the firms to change their equilibrium pricing strategies from (P, U) to (P', P'). Since $\bar{\theta} < 1/3$, we note that $\theta < 1/3$ in this case. Recall also that firm B's uniform price in the (P, U) subgame in this case, p_B^{PU}, is given in (A25), while the uniform price that this firm offers to consumers who reject tracking in the (P', P') subgame is in (A46). Comparing these two uniform prices, we find $p_B^{P'P'} > p_B^{PU}$ for $\theta < 1/3$. In other words, the regulation in this case raises firm B's uniform price. This, in turn, reduces the utility of privacy-sensitive consumers who purchase good B at the uniform price both with and without the regulation. Recall from section A3.1 that these privacy-sensitive consumers are located in $[1 - D/2t, 1]$. Moreover, some privacy-sensitive consumers located to the left of $x = 1 - D/2t$ are also made worse off by the regulation. To be more specific, consider a privacy-sensitive consumer who purchases good B at personalized price under the regulation. This consumer, who is located at $x \in [1/2, 1 - D/2t)$, will be made worse off by the regulation if $V - t(2x-1) - D - t(1-x) < V - p_B^{PU} - t(1-x)$. Substituting (A25) for p_B^{PU} into this inequality, we find

$$x > \frac{3-\theta}{4(1-\theta)} - \frac{D}{2t}. \quad (A120)$$

The right-hand side of (A120) is no less than $1/2$ if $D \leq \frac{(1+\theta)t}{2(1-\theta)}$. Noting the first part of the definition of x_1^* in (A119), we conclude that the regulation reduces the utility of privacy-sensitive consumers in $(x_1^*, 1]$ if $D \leq \frac{(1+\theta)t}{2(1-\theta)}$.

If $D > \frac{(1+\theta)t}{2(1-\theta)}$, on the other hand, the privacy-sensitive consumers who purchase good A at personalized prices, i.e., those consumers located in $(D/2t, 1/2]$, are also made worse off by the regulation because $V - t(1-2x) - D - tx < V - p_B^{PU} - t(1-x)$. Moreover, a privacy-sensitive consumer who buys good A at the uniform price is harmed by the regulation if $V - p_A^{P'P'} - tx < V - p_B^{PU} - t(1-x)$. Substituting (A46) for $p_A^{P'P'}$ and (A25) for p_B^{PU}, we find

$$x > \frac{1+\theta}{4(1-\theta)}. \quad (A121)$$

Noting the second part of the definition of x_1^* in (A119), we conclude that the regulation reduces the utility of privacy-sensitive consumers in $(x_1^*, 1]$ if $D > \frac{(1+\theta)t}{2(1-\theta)}$.

Now consider the impact on privacy-insensitive consumers in case (ii). Note that those consumers located in $[x_0^{PU}, 1]$ purchase good B at the uniform price under laissez-faire, but under the regulation they buy the same good at personalized prices. The regulation reduces the utility of such a consumer if her personalized price is higher, i.e., if $t(2x-1) > p_B^{PU}$. Using (A25), we find that the latter implies $x < x_0^{PU}$. Therefore, the regulation reduces the utility of privacy-insensitive consumers located in $(x_0^{PU}, 1]$.

Finally, we show that for ranges of parameter values outside those of cases (i) and (ii), the regulation improves the utility of some (or all) consumers without harming any other consumers. We separate these ranges into four cases and consider each of them below.

(a) $\theta < \min\{\hat{\theta}, \sqrt{5}-2\}$. In this case, both firms adopt personalized pricing with and without the regulation. Because the personalized price for consumers at each location remains the same, the utility of those consumers who purchase goods at personalized prices under the regulation does not change. With the regulation, moreover, consumers have the additional option of rejecting tracking and purchasing a good at the uniform price. Those consumers who choose this option are better off as a result. Therefore, the regulation increases the utility of privacy-sensitive consumers located in $[0, D/2t)$ and $(1-D/2t, 1]$.

(b) $\max\{\hat{\theta}, \bar{\theta}\} < \theta \leq (3\sqrt{2}-2)/7$. Recall from section A2.2 that $\hat{\theta} < \bar{\theta}$ for $2t < V < \left(\frac{3}{2}+\frac{\sqrt{5}}{4}\right)t$ and $\hat{\theta} > \bar{\theta}$ for $V > \left(\frac{3}{2}+\frac{\sqrt{5}}{4}\right)t$. Hence, the range of parameters in this case can also be stated as $\bar{\theta} < \theta \leq (3\sqrt{2}-2)/7$ for $2t < V < \left(\frac{3}{2}+\frac{\sqrt{5}}{4}\right)t$ and $\hat{\theta} < \theta \leq (3\sqrt{2}-2)/7$ for $V > \left(\frac{3}{2}+\frac{\sqrt{5}}{4}\right)t$. In this case, the regulation changes the equilibrium pricing strategies from (P, U) to (P', P'). With $\theta > \bar{\theta}$, firm B's uniform price before the regulation is $p_B^{PU} = V - t$. Comparing it with firm B's uniform price in the (P', P') subgame, $p_B^{P'P'} = t$, we find that $p_B^{PU} > p_B^{P'P'}$, that is, the regulation lowers the uniform price of firm B. Moreover, the regulation reduces firm A's personalized prices because it has to compete with firm B in personalized prices. As a result of these lower prices, all consumers are better off under the regulation even if they do not change their purchasing behavior. The utility is even higher for those consumers who change their purchasing behavior in response to the additional options available under the regulation.

(c) $(3\sqrt{2}-2)/7 < \theta < \tilde{\theta}$ and $D > D^*$. In this case, the regulation has the same impact on pricing strategies and prices as in case (b) above. Therefore, the regulation improves the utility of all consumers in this case.

(d) $(3\sqrt{2}-2)/7 < \theta < \tilde{\theta}$ and $D \leq D^*$. In this case, the regulation does not change the firms' pricing strategies. That is, one firm adopts personalized pricing while the other uses uniform pricing with and without the regulation. Firm B's uniform price under laissez-faire, p_B^{PU}, is given in (A31) and that under the regulation, $p_B^{P'U'}$, is in (A55). Comparing these prices, we find that $p_B^{P'U'} < p_B^{PU}$. The lower uniform price of firm B under the regulation leads to lower personalized prices of firm A. Consequently, the regulation increases the utility of every consumer in this case.

(e) $\theta \geq \tilde{\theta}$ and $D > D^*$. In this case, the regulation changes the equilibrium pricing strategies from (U, U) to (P', P'). The firms' uniform prices in the (P', P') subgame are $p_A^{P'P'} = p_B^{P'P'} = t$, which are the same as the uniform prices in the (U, U) subgame. Moreover, the personalized prices in the (P', P') subgame are lower than t for consumers located in the interior of the Hotelling line, i.e., for $x \in (0, 1)$. Since all privacy-insensitive consumers purchase the goods at the personalized prices in (P', P') subgame, these observations suggest that the regulation increases the utility of privacy-insensitive consumers except those located at the two ends (i.e., $x = 0$ and 1). On the other hand, the regulation has no impact on those privacy-sensitive consumers who reject tracking and purchase at the uniform prices, i.e., those consumers located in $[0, D/2t]$ and $[1-D/2t, 1]$. The remaining privacy-sensitive consumers, located in $(D/2t, 1-D/2t)$, are better off because they benefit from the option of buying at personalized prices under the regulation.

This concludes the proof of Proposition A1.

Supplement to
"Personalized Pricing in the Presence of Privacy Concerns"

Online Appendix – Part B

Part B of this appendix contains the technical details of the three extensions presented in section 6 of the paper. These extensions deal with (1) asymmetric firms, (2) incomplete market coverage, and (3) alternative timing of price revelation, respectively. Because these extensions and the baseline model share many common elements, we reuse many of the same notations. But the meaning of these notations should be interpreted in the context of each extension.

B1. Asymmetric Firms

We analyze this model of asymmetric firms using the same process as in the baseline model, which consists of examining the subgame games associated with the four combinations of pricing strategies, followed by a determination of the equilibrium in the whole game. Note that condition (10) in Proposition 7 specifies a minimum value of θ. To reduce the number of scenarios we must discuss, our presentation here focuses on the case where θ is large enough to satisfy (10).

Proposition B1: In the duopoly model with asymmetric firms, both firms adopt uniform pricing under laissez-faire if

$$\theta \geq 1 - \frac{(3t - \Delta V)^2}{18(V_B - t)t}. \qquad (B1.1)$$

Proof of Proposition B1

Recall that the four subgames associated with the firms' choices of pricing strategies under laissez-faire are denoted by (U, U), (P, P), (P, U), and (U, P). In the (U, U) subgame, a consumer's utility from purchasing a unit of good A is $U_A^{UU}(x) = V_A - tx - p_A^{UU}$, and her utility from good B is $U_B^{UU}(x) = V_B - t(1 - x) - p_B^{UU}$. Using standard procedure, we find that the equilibrium prices in this subgame are $p_A^{UU} = (3t + \Delta V)/3$ and $p_B^{UU} = (3t - \Delta V)/3$. At these prices, firm A captures a market share of $1/2 + \Delta V/6t$ while firm B has a market share of $1/2 - \Delta V/6t$. The profits of the two firms are $\Pi_A^{UU} = (3t + \Delta V)^2/18t$ and $\Pi_B^{UU} = (3t - \Delta V)^2/18t$.

In the (P, P) subgame, competition in personalized pricing between the two firms leads to $p_A^{PP}(x) = t(1 - 2x) + \Delta V$ and $p_B^{PP}(x) = 0$ for $x \in [0, (t + \Delta V)/2t]$, while $p_A^{PP}(x) = 0$ and $p_B^{PP}(x) = t(2x - 1) - \Delta V$ for $x \in [(t + \Delta V)/2t, 1]$. At these personalized prices, consumers located at $x < (t + \Delta V)/2t$ purchase good A while those located at $x > (t + \Delta V)/2t$ buy good B. The firms' profits are $\Pi_A^{PP} = (t + \Delta V)^2/4t$ and $\Pi_B^{PP} = (t - \Delta V)^2/4t$.

In the (P, U) subgame, firm A offers personalized prices and firm B charges a uniform price. Given firm B's uniform price p_B^{PU}, firm A will undercut it by setting $p_A^{PU}(x) = \Delta V + t(1 - 2x) + p_B^{PU}$ as long as $p_A^{PU}(x) \geq 0$. The latter implies $x \leq (t + p_B^{PU} + \Delta V)/2t$. We modify the definition of x_0^{PU} to be

$$x_0^{PU} \equiv \min\left\{\frac{t + p_B^{PU} + \Delta V}{2t}, 1\right\}. \qquad (B1.2)$$

Then privacy-insensitive consumers located in $[0, x_0^{PU})$ will purchase good A at personalized prices. However, none of the privacy-sensitive consumers will buy from firm A because of their privacy costs. Instead, they will either buy from firm B at uniform prices or do not buy at all.

Note from (B1.2) that $x_0^{PU} = 1$ if $p_B^{PU} \geq t - \Delta V$. Using the same process as in the baseline model, we find that it is profit-maximizing for firm B to choose such a high uniform price if the proportion of privacy-sensitive consumers is sufficiently large. Specifically, define

$$\bar{\bar{\theta}} \equiv \frac{4tV_B + \Delta V^2 - 5t^2 - 4t\sqrt{(V_B - t)(V_A - 2t)}}{2t(V_A + 3V_B) + \Delta V^2 - 7t^2}. \qquad (B1.3)$$

It can be verified that $0 < \bar{\bar{\theta}} < \frac{t-\Delta V}{3t-\Delta V}$ given the assumptions $V_i > 2t$, $t > D$ and $\Delta V \in (0, t - D)$. An analysis of firm B's profit-maximization problem shows that if $\theta > \bar{\bar{\theta}}$, it sets uniform price at $p_B^{PU} = V_B - t$. In this case, firm B sells to all privacy-sensitive consumers while firm A serves all privacy-insensitive consumers. Their profits in this subgame are

$$\Pi_A^{PU} = (1-\theta)(V_A - t), \quad (B1.4)$$
$$\Pi_B^{PU} = \theta(V_B - t). \qquad (B1.5)$$

In the (U, P) subgame, where firm A offers a uniform price, denoted by p_A^{UP}, and firm B charges personalized prices, denoted by $p_B^{UP}(x)$. Given p_A^{UP}, firm B will undercut firm A's price by setting $p_B^{UP}(x) = t(2x-1) - \Delta V + p_A^{UP}$ as long as $p_B^{UP}(x) \geq 0$. The latter implies $x \geq (t - p_A^{UP} + \Delta V)/2t$. Define

$$x_0^{UP} \equiv \max\left\{\frac{t - p_A^{UP} + \Delta V}{2t}, 0\right\}. \qquad (B1.6)$$

Then privacy-insensitive consumers located in $(x_0^{UP}, 1]$ will purchase good B at personalized prices. On the other hand, privacy-sensitive consumers will stay away from firm B and buy from firm A instead.

Observe from (B1.6) that $x_0^{UP} = 0$ if $p_A^{UP} \geq t + \Delta V$. Following the same process as the one for the (P, U) subgame in the baseline model, we find that it is profit-maximizing for firm A to choose such a high uniform price if the proportion of privacy-sensitive consumers is sufficiently large. Specifically, define

$$\bar{\bar{\bar{\theta}}} \equiv \frac{4tV_A + \Delta V^2 - 5t^2 - 4t\sqrt{(V_A - t)(V_B - 2t)}}{2t(3V_A + V_B) + \Delta V^2 - 7t^2}. \qquad (B1.7)$$

It can be verified that $0 < \bar{\bar{\bar{\theta}}} < \frac{t+\Delta V}{3t+\Delta V}$ given the assumption $V_i > 2t$, $t > D$ and $\Delta V \in (0, t - D)$. An analysis of firm A's profit-maximization problem shows that if $\theta > \bar{\bar{\bar{\theta}}}$, firm A sets its uniform price at $p_A^{UP} = V_A - t$ and serves all privacy-sensitive consumers. In this case, firm B sells to all privacy-insensitive consumers. Their profits in this subgame are

$$\Pi_A^{UP} = \theta(V_A - t), \quad (B1.8)$$
$$\Pi_B^{UP} = (1-\theta)(V_B - t). \qquad (B1.9)$$

Table B1.1: Firms' Profits without Privacy Regulation in the Model with Asymmetric Firms

A \ B	U	P
U	$(\frac{(3t+\Delta V)^2}{18t}, \frac{(3t-\Delta V)^2}{18t})$	$(\theta(V_A - t), (1-\theta)(V_B - t))$
P	$((1-\theta)(V_A - t), \theta(V_B - t))$	$(\frac{(t+\Delta V)^2}{4t}, \frac{(t-\Delta V)^2}{4t})$

Comparing the critical values of θ in (B1.1), (B1.3) and (B1.7), we find that given the assumption $V_i > 2t$, $t > D$ and $\Delta V \in (0, t - D)$, we have

$$1-\frac{(3t-\Delta V)^2}{18(V_B-t)t}>\bar{\bar{\bar{\theta}}}>\bar{\bar{\theta}}. \quad (B1.10)$$

Therefore, we can combine the results from the preceding analysis of the four subgames to construct the payoff matrix of this game, presented in Table B1.1. Condition (B1.1) implies that $\Pi_A^{UU}>\Pi_A^{PU}$, $\Pi_B^{UU}>\Pi_B^{UP}$, $\Pi_A^{UP}>\Pi_A^{PP}$, and $\Pi_B^{PU}>\Pi_B^{PP}$. From these rankings of the firms' profits, we conclude that (U,U) is a unique equilibrium in this game.

QED

It is interesting to compare Proposition B1 with Houba et al. (2023), which shows, in a duopoly model with asymmetric firms, that personalized pricing is a dominant strategy and hence both firms adopt personalized pricing in equilibrium. If we remove privacy-sensitive consumers from our model, it converges to Houba et al. (2023). But Proposition B1 shows that in the presence of privacy-sensitive consumers, personalized pricing is no longer a dominant strategy. It confirms the finding from the baseline model that if the proportion of privacy-sensitive consumers is sufficiently large, both firms adopt uniform pricing under laissez-faire.

Proposition B2: In the duopoly model with asymmetric firms, under the privacy regulation firm A adopts personalized pricing while firm B adopts uniform pricing if $D>D^*$ and

$$\theta>\max\left\{\frac{t-\Delta V}{t+\Delta V},1-\frac{(3t-\Delta V)^2}{18(V_B-t)t}\right\}. \quad (B1.11)$$

Proof of Proposition B2

We start with an analysis of the four subgames associated with different pricing strategies, namely, (U',U'), (P',P'), (P',U') and (U',P'). In the (U',U') subgame, neither firm uses the tracking technology. Thus, the firms' pricing decisions are not constrained by the privacy regulation. Consequently, the equilibrium prices and profits are the same as those in the case of (U,U), that is, $p_A^{U'U'}=(3t+\Delta V)/3$, $p_B^{U'U'}=(3t-\Delta V)/3$, $\Pi_A^{U'U'}=(3t+\Delta V)^2/18t$ and $\Pi_B^{U'U'}=(3t-\Delta V)^2/18t$.

In the (P',P') subgame, both firms adopt the tracking technology, but they must obtain a consumer's consent before they can track her online activities. In this subgame, each firm sets a uniform price and offers personalized prices to those consumers who accept tracking. Following the same analytical process as in the baseline model, we find that firms' personalized pricing strategies are $p_A^{P'P'}(x)=\Delta V+t(1-2x)$ and $p_B^{P'P'}(x)=0$ for consumers at $x\in[0,(t+\Delta V)/2t]$ who accept tracking, while $p_A^{P'P'}(x)=0$ and $p_B^{P'P'}(x)=-\Delta V+t(2x-1)$ for consumers at $x\in[(t+\Delta V)/2t,1]$ who accept tracking. Using these observations, we examine the firms' choice of uniform prices and derive the following equilibrium prices:

$$p_A^{P'P'}=t+\Delta V, \quad (B1.12)$$
$$p_B^{P'P'}=t-\Delta V. \quad (B1.13)$$

At these prices, privacy-sensitive consumers in the intervals $[0,D/2t]$ and $[1-D/2t,1]$ reject tracking, while those in $(D/2t,1-D/2t)$ accept tracking. As for privacy-insensitive consumers, all of them accept tracking. The firms' profits in this subgame are

$$\Pi_A^{P'P'}=\frac{(t+\Delta V)^2}{4t}+\frac{\theta D^2}{4t}, \quad (B1.14)$$
$$\Pi_B^{P'P'}=\frac{(t-\Delta V)^2}{4t}+\frac{\theta D^2}{4t}. \quad (B1.15)$$

In the (P', U') subgame, only firm A uses the tracking technology. Given firm B's uniform price $p_B^{P'U'}$, firm A has an incentive to undercut firm B's price by setting $p_A^{P'U'}(x) = \Delta V + t(1-2x) + p_B^{P'U'}$, provided that $p_A^{P'U'}(x) \geq 0$. The latter implies $x \leq (\Delta V + t + p_B^{P'U'})/2t$. We define

$$x_0^{P'U'} \equiv \min\left\{\frac{\Delta V + t + p_B^{P'U'}}{2t}, 1\right\}. \qquad (B1.16)$$

Then firm A is able to attract the privacy-insensitive consumers located in $[0, x_0^{P'U'})$. However, not all these consumers will purchase good A at personalized prices because they may find it cheaper to buy the good at the uniform price offered by firm A. As in the baseline model, let $x_{A0}^{P'U'}$ denote the location of a privacy-insensitive consumer who is indifferent between the uniform price and the personalized price offered by firm A. Then $x_{A0}^{P'U'}$ must satisfy the condition

$$V_A - tx_{A0}^{P'U'} - p_A^{P'U'} = V_A - tx_{A0}^{P'U'} - \left[\Delta V + t\left(1 - 2x_{A0}^{P'U'}\right) + p_B^{P'U'}\right]. \quad (B1.17)$$

From (B1.17), we obtain

$$x_{A0}^{P'U'} = \frac{\Delta V + p_B^{P'U'} - p_A^{P'U'} + t}{2t}. \qquad (B1.18)$$

By the definition of $x_{A0}^{P'U'}$, privacy-insensitive consumers in $[0, x_{A0}^{P'U'})$ will reject tracking and purchase good A at the uniform price, while privacy-insensitive consumers in $[x_{A0}^{P'U'}, x_0^{P'U'})$ will accept tracking and buy good A at the personalized prices.

Because of the commitment problem discussed in the paper, privacy-sensitive consumers will not purchase good A at personalized prices. Hence, if they purchase good A in the (P', U') subgame, it will be at firm A's uniform price. As in the baseline model, let $x_1^{P'U'}$ be the location of a privacy-sensitive consumer who is indifferent between purchasing good A and good B at their respective uniform prices. Then privacy-sensitive consumers in $[0, x_1^{P'U'}]$ purchase good A at its uniform price while privacy-sensitive consumers in $[x_1^{P'U'}, 1]$ buy good B which is available only at uniform price $p_B^{P'U'}$. By definition, $x_1^{P'U'}$ must satisfy the following condition:

$$V_A - tx_1^{P'U'} - p_A^{P'U'} = V_B - t\left(1 - x_1^{P'U'}\right) - p_B^{P'U'}. \qquad (B1.19)$$

From (B1.19), we find that $x_1^{P'U'}$ has the same expression as $x_{A0}^{P'U'}$ in (B1.18), that is, $x_1^{P'U'} = x_{A0}^{P'U'}$.

At the beginning of stage 3, firms A and B set their uniform prices to maximize their profits,

$$\Pi_A^{P'U'} = p_A^{P'U'}\left((1-\theta)x_{A0}^{P'U'} + \theta x_1^{P'U'}\right) + (1-\theta)\int_{x_{A0}^{P'U'}}^{x_0^{P'U'}} \left[\Delta V + t(1-2x) + p_B^{P'U'}\right] dx, \quad (B1.20)$$

$$\Pi_B^{P'U'} = p_B^{P'U'}\left[(1-\theta)\left(1 - x_0^{P'U'}\right) + \theta\left(1 - x_1^{P'U'}\right)\right]. \qquad (B1.21)$$

We solve the first-order conditions of the firms' profit-maximization problems in (B1.20) and (B1.21) to obtain the equilibrium uniform prices:

$$p_A^{P'U'} = \frac{3t + \Delta V}{1 + 2\theta}, \qquad (B1.21)$$

$$p_B^{P'U'} = \frac{t(2+\theta) - \theta\Delta V}{1 + 2\theta}. \qquad (B1.22)$$

Substituting (B1.22) into (B1.16), we find that that $x_0^{P'U'} \equiv \min\left\{\frac{\Delta V + t + p_B^{P'U'}}{2t}, 1\right\} = 1$ if

$$\theta > \frac{t - \Delta V}{t + \Delta V}. \qquad (B1.23)$$

Substituting (B1.21) and (B1.22) into (B1.18), we obtain $x_{A0}^{P'U'} = x_1^{P'U'} = \frac{\theta(3t+\Delta V)}{2t(2+\theta)}$. For θ that satisfies (B1.23), the firms' profits in this subgame are

$$\Pi_A^{P'U'} = \frac{(\Delta V)^2\theta^2(1+\theta) + t^2(8+12\theta-3\theta^2+\theta^3) + 2t\Delta V(2+4\theta+\theta^2-\theta^3)}{4t(1+2\theta)^2}, \qquad (B1.24)$$

$$\Pi_B^{P'U'} = \frac{\theta(t(2+\theta)-\theta\Delta V)^2}{2t(1+2\theta)^2}. \qquad (B1.25)$$

In the (U',P') subgame, only firm B uses the tracking technology. Given firm A's uniform price $p_A^{U'P'}$, firm B has an incentive to undercut firm A's price by setting $p_B^{U'P'}(x) = -\Delta V + t(2x-1) + p_A^{U'P'}$, provided that $p_B^{U'P'}(x) \geq 0$. The latter implies $x \geq (\Delta V + t - p_A^{U'P'})/2t$. We define

$$x_0^{U'P'} \equiv \max\left\{0, \frac{\Delta V + t - p_A^{U'P'}}{2t}\right\}. \qquad (B1.26)$$

Then firm B is able to attract the privacy-insensitive consumers located in $(x_0^{U'P'}, 1]$.

Following the same analytical process as in the (P',U') subgame above, we derive the equilibrium uniform prices in this subgame:

$$p_A^{U'P'} = \frac{t(1+2\theta)+\Delta V}{2+\theta}, \qquad (B1.27)$$

$$p_B^{U'P'} = \frac{3t-\Delta V}{2+\theta}. \qquad (B1.28)$$

Substituting (B1.27) into (B1.26), we find that $x_0^{U'P'} > 0$, implying that firm A always serves some privacy-insensitive consumers in equilibrium (due to its higher quality). The firms' profits in this subgame are

$$\Pi_A^{U'P'} = \frac{(t(1+2\theta)+\Delta V)^2}{2t(2+\theta)^2}, \qquad (B1.29)$$

$$\Pi_B^{U'P'} = \frac{(1+\theta)(3t-\Delta V)^2}{4t(2+\theta)^2}. \qquad (B1.30)$$

Using the findings from the above analysis of the four subgames, we present in Table B1.2 the firms' profits associated with different combinations of pricing strategies under the assumption that θ satisfies (B1.23). By comparing these profits, we find that (P',U') is an equilibrium because $\Pi_A^{P'U'} > \Pi_A^{U'U'}$ and $\Pi_B^{P'U'} > \Pi_B^{P'P'}$.

Table B1.2: Firms' Profits with the Privacy Regulation in the Model with Asymmetric Firms

A \ B	U'	P'
U'	$(\frac{(3t+\Delta V)^2}{18t}, \frac{(3t-\Delta V)^2}{18t})$	$(\Pi_A^{U'P'}, \Pi_B^{U'P'})$
P'	$(\Pi_A^{P'U'}, \Pi_B^{P'U'})$	$(\frac{(t+\Delta V)^2}{4t} + \frac{\theta D^2}{4t}, \frac{(t-\Delta V)^2}{4t} + \frac{\theta D^2}{4t})$

Note: $(\Pi_A^{P'U'}, \Pi_B^{P'U'})$ is represented by (B1.24)-(B1.25), and $(\Pi_A^{U'P'}, \Pi_B^{U'P'})$ is represented by (B1.29)-(B1.30).

Moreover, (P', U') is a unique equilibrium if $D > D^*$ and θ satisfies (B1.11). To prove this, we need to rule out (U', P') as an equilibrium. The latter is true if $\Pi_A^{U'P'} < \Pi_A^{P'P'}$. Using (B1.14) and (B1.29), we find

$$\frac{\partial(\Pi_A^{P'P'} - \Pi_A^{U'P'})}{\partial \Delta V} = \frac{t(2+\theta^2) + \Delta V(2+4\theta+\theta^2)}{2t(2+\theta)^2} > 0, \quad (B1.31)$$

$$\left(\Pi_A^{P'P'} - \Pi_A^{U'P'}\right)\Big|_{\Delta V=0} = \frac{t}{4} + \frac{\theta D^2}{4t} - \frac{t(1+2\theta)^2}{2(2+\theta)^2}. \quad (B1.32)$$

The sign of (B1.32) is positive if $D > D^*$. Recalling its definition in (9), we note that D^* is a real number if $\theta \geq (3\sqrt{2}-2)/7$. The latter is assured by (B1.11). Hence, (B1.31) and (B1.32) imply that $\Pi_A^{P'P'} > \Pi_A^{U'P'}$ if $D > D^*$ and θ satisfies (B1.11).

QED

Combining Propositions B1 and B2, we see that the privacy regulation leads to wider adoption of personalized pricing. In a market with a sufficiently large proportion of privacy-sensitive consumers, no firm use personalized pricing under laissez-faire, but the regulation causes firm A to switch to personalized pricing. Proposition 7 in the paper shows the impact of this change in pricing strategies on individual consumers. Below is the proof.

Proof of Proposition 7

Comparing $p_A^{P'U'}$ and $p_B^{P'U'}$ in (B1.21) and (B1.22) with $p_A^{UU} = (3t + \Delta V)/3$ and $p_B^{UU} = (3t - \Delta V)/3$, we find that $p_i^{P'U'} > p_i^{UU}$ for $i = A, B$. In other words, both firms charge higher uniform prices with the regulation than without the regulation. Consequently, those consumers who reject tracking and purchase at the uniform prices from either firm are made worse off by the regulation. The utility of those consumers who accept tracking and purchase at firm A's personalized prices is the same as the utility from buying good B at the uniform price. The increase in the uniform price of good B implies that the utility of these consumers is reduced by the regulation as well.

QED

B2. Incomplete Market Coverage Due to Low Valuation

In this section, we consider the duopoly model under the assumption that consumers have low valuation for the two goods. Specifically, we consider $V \in (t/2, 2t/3]$. We will first demonstrate that for V in this range, this model reproduces Rhodes and Zhou's (2024) result that personalized pricing increases the firms' profits. Then we will derive the equilibrium in our model with privacy-sensitive consumers under laissez-faire and under the privacy regulation, respectively.

Proposition B3: In the duopoly model with low consumer valuation, suppose $\theta = 0$. The market is not fully covered when the firms adopt uniform pricing. Moreover, each firm earns a larger profit when both firms adopt personalized pricing than when they adopt uniform pricing.

Proof of Proposition B3

When the market is incompletely covered at uniform prices, the two firms do not directly compete for customers and hence each firm acts like a monopolist. Firm A's demand is determined by $p_A + tx_A = V$, which yields the demand function: $x_A = (V - p_A)/t$. Firm A chooses its uniform price to maximize its profit $p_A(V - p_A)/t$, from which we find $p_A^{UU} = V/2$, $x_A^{UU} = V/2t$, and $\Pi_A^{UU} = V^2/4t$, where x_A^{UU}

denotes the location of consumers who are indifferent between purchasing from firm A and not purchasing. Firm B's price and profit are symmetric to those of firm A, specifically, $p_B^{UU} = V/2$, $x_B^{UU} = 1 - V/2t$, and $\Pi_B^{UU} = V^2/4t$, where x_B^{UU} denotes the location of consumers who are indifferent between purchasing from firm B and not purchasing. Using $V \leq 2t/3$, we can verify that $x_A^{UU} \leq 1/3$ and $x_B^{UU} \geq 2/3$. This confirms that the market is incompletely covered when both firms adopt uniform pricing.

In the case where both firms adopt personalized pricing (with $\theta = 0$), each firm has a segment of "captive" customers who will never buy from its competitor. Specifically, those consumers whose locations x satisfy the condition $V - t(1-x) < 0$ will not buy from firm B even if its price $p_B = 0$. Solving the inequality, we find $x < 1 - V/t$. Note that $V > t/2$ implies $1 - V/t < 1/2$. Therefore, for those consumers with $x \in [0, (1 - V/t))$, firm A acts like a monopolist and charges a personalized price that extracts the entire surplus from a consumer, i.e., $p_A(x) = V - tx$. For those consumers with $x \in [(1 - V/t), 1/2]$, firm A faces competition from firm B and charges personalized prices $p_A(x) = t(1 - 2x)$. Therefore, firm A's profit in this case is

$$\Pi_A^{PP} = \int_0^{1-V/t} (V - tx)dx + \int_{1-V/t}^{1/2} t(1-2x)dx = V - \frac{V^2}{2t} - \frac{t}{4}. \quad (B2.1)$$

By the same reasoning, firm B acts like a monopolist for consumers with $x \in (V/t, 1]$, and it charges each of them a personalized price $p_B(x) = V - t(1-x)$. For those consumers with $x \in [1/2, V/t]$, firm B faces competition from firm A and charges personalized prices $p_B(x) = t(2x - 1)$. Therefore, firm B's profit in this case is

$$\Pi_B^{PP} = \int_{V/t}^{1} (V - t(1-x))dx + \int_{1/2}^{V/t} t(2x-1)dx = V - \frac{V^2}{2t} - \frac{t}{4}. \quad (B2.2)$$

To show that $\Pi_i^{PP} > \Pi_i^{UU}$ $(i = A, B)$, we note that

$$(\Pi_i^{PP} - \Pi_i^{UU})\big|_{V=t/2} = \frac{t}{16} > 0, \quad (B2.3)$$

$$\frac{\partial(\Pi_i^{PP} - \Pi_i^{UU})}{\partial V} = 1 - \frac{3V}{2t} \geq 0 \text{ for } V \leq \frac{2t}{3}. \quad (B2.4)$$

From (B2.3) and (B2.4), we conclude that $\Pi_i^{PP} > \Pi_i^{UU}$ $(i = A, B)$ for $V \in (t/2, 2t/3]$.

QED

We now analyze our model with privacy-sensitive consumers, i.e., $\theta > 0$. For this analysis, we define another critical value of θ:

$$\theta_1 \equiv \frac{(3V - t)(t - V)}{2[(t + D)(4V - 2D) - 3V^2 - t^2]}. \quad (B2.5)$$

It can be verified that $\theta_1 \in (0, 1/2)$ given the assumptions that $V \in (t/2, 2t/3]$ and $D < V - t/2$.

Proposition B4: In the duopoly model with low consumer valuation and privacy-sensitive consumers, the equilibrium pricing strategies without privacy regulation depend on the proportion of privacy-sensitive consumers. Specifically,

(i) both firms adopt personalized pricing if $\theta < \theta_1$;

(ii) one firm adopts personalized pricing and the other firm adopts uniform pricing if $\theta_1 \leq \theta < 1/2$;

(iii) both firms adopt uniform pricing if $\theta \geq 1/2$.

Proof of Proposition B4

The process of deriving the equilibrium pricing strategies in this model is the same as that in the baseline model. The main difference here is that each firm has a segment of captive consumers who will never buy from its rival. Consequently, the personalized price offered by a firm follows two different schedules, one for its captive consumers, and the other for contestable consumers.

The equilibrium in the (U, U) subgame is not affected by the presence of privacy-sensitive consumers because neither firm use the tracking technology. Hence, the results from the proof of Proposition B3 are still applicable, that is, the firms offer prices $p_A^{UU} = p_B^{UU} = V/2$ and gain profits $\Pi_A^{UU} = \Pi_B^{UU} = V^2/4t$.

In the (P, P) subgame, consumers located in $[0, 1 - V/t)$ are captive to firm A, while those located in $(V/t, 1]$ are captive to firm B. For its captive consumers, a firm charges a personalized price that extracts their entire surplus from consumption. But at these prices, privacy-sensitive consumers would earn a negative net surplus because of their privacy costs. As a result, captive consumers who are privacy-sensitive stay away from both firms. For those privacy-insensitive and privacy-sensitive consumers with $x \in [1 - V/t, 1/2]$, firm A faces competition from firm B and charges personalized prices $p_A^{PP}(x) = t(1 - 2x)$. Similarly, for consumers with $x \in [1/2, V/t]$, firm B faces competition from firm A and charges personalized prices $p_B^{PP}(x) = t(2x - 1)$.

Therefore, as illustrated in Figure B2.1, privacy-insensitive consumers in the interval $[0, 1 - V/t)$ will buy good A at personalized prices $p_A^{PP}(x) = V - tx$ and those in the interval $[1 - V/t, 1/2]$ buy the same good at personalized prices $p_A^{PP}(x) = t(1 - 2x)$. Privacy-insensitive consumers in the interval $[1/2, V/t]$ buy good B at personalized prices $p_B^{PP}(x) = t(2x - 1)$ and those in the interval $(V/t, 1]$ buy the same good at personalized prices $p_B^{PP}(x) = V - t(1 - x)$. As for privacy-sensitive consumers, those in the interval $[1 - (V - D)/t, 1/2]$ will buy good A at personalized prices $p_A^{PP}(x) = t(1 - 2x)$ and those in the interval $[1/2, (V - D)/t]$ will buy good B at personalized prices $p_B^{PP}(x) = t(2x - 1)$, but the remaining privacy-sensitive consumers will not make a purchase.

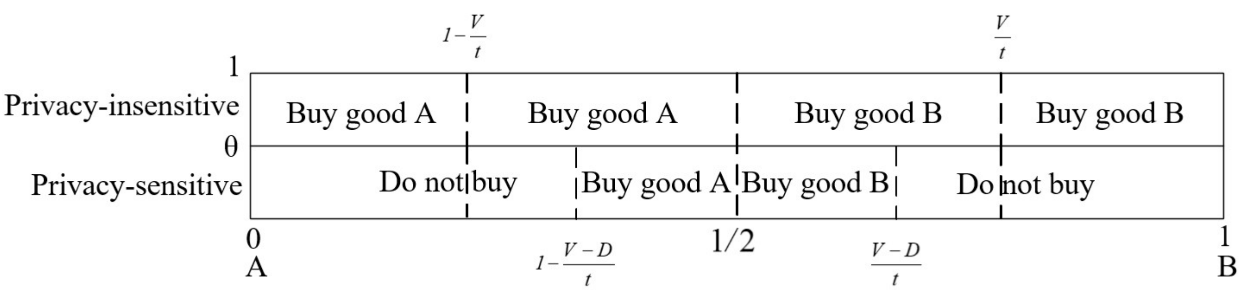

Figure B2.1: Consumers' Choices in the (P, P) Subgame

Using these observations, we derive the profits of firm A and B in this subgame:

$$\Pi_A^{PP} = (1-\theta)\left(\int_0^{1-\frac{V}{t}} (V - tx)dx + \int_{1-\frac{V}{t}}^{\frac{1}{2}} t(1-2x)dx\right) + \theta \int_{1-\frac{V-D}{t}}^{\frac{1}{2}} t(1-2x)dx$$

$$= V - \frac{V^2}{2t} - \frac{t}{4} + \frac{\theta[3V^2 + t^2 - (t+D)(4V - 2D)]}{2t}. \qquad (B2.6)$$

$$\Pi_B^{PP} = (1-\theta)\left(\int_{\frac{1}{2}}^{\frac{V}{t}} t(2x-1)dx + \int_{\frac{V}{t}}^{1} (V - t(1-x))dx\right) + \theta \int_{\frac{1}{2}}^{\frac{V-D}{t}} t(2x-1)dx$$
$$= V - \frac{V^2}{2t} - \frac{t}{4} + \frac{\theta[3V^2 + t^2 - (t+D)(4V-2D)]}{2t}. \quad (B2.7)$$

Now we analyze the (P, U) subgame, where firm A offers personalized prices and firm B charges a uniform price. As we have noted, firm A and firm B have monopoly power in the interval $[0, 1 - V/t)$ and $(V/t, 1]$, respectively. For the contestable consumers in the interval $[1 - V/t, V/t]$, firm A uses personalized prices to undercut firm B's uniform price. In this situation, firm B have two options: it could either charge a low uniform price to compete for these contestable consumers or give up the contestable consumers and charge its captive customers the monopoly price. Our analysis of these two options reveals that, given $V \in (t/2, 2t/3]$, the second option is more profitable for firm B.

To be more specific, we find that firm B's equilibrium price in this subgame is $p_B^{PU} = V/2$. At this price, firm B serves privacy-sensitive and privacy-insensitive consumers in the interval $[1 - V/2t, 1]$. Given p_B^{PU}, firm A does not compete directly with firm B. Hence it charges the maximum personalized price $p_A^{PU}(x) = V - tx$ and sells to those privacy-insensitive consumers in the interval $[0, V/t]$. The remaining consumers do not purchase because they will earn a negative surplus if they do. The profits of firm A and firm B are

$$\Pi_A^{PU} = \frac{V^2(1-\theta)}{2t}, \quad (B2.8)$$
$$\Pi_B^{PU} = \frac{V^2}{4t}. \quad (B2.9)$$

Figure B2.2 illustrates the consumers' purchase decisions in the equilibrium of this subgame.

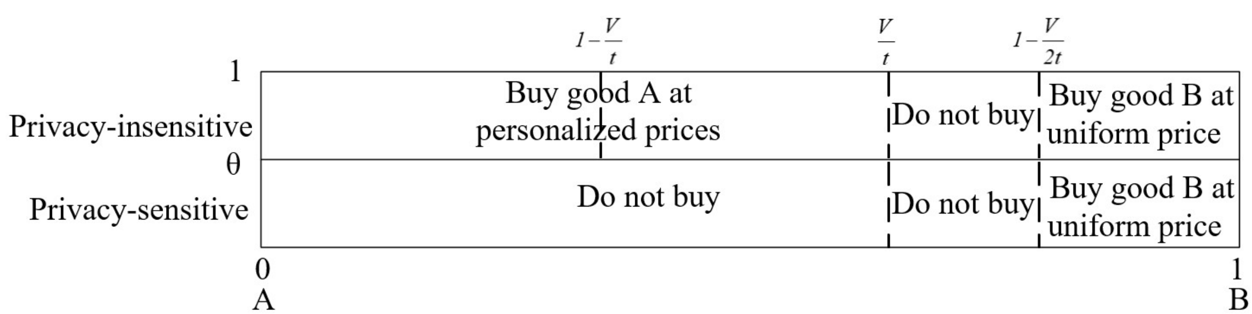

Figure B2.2: Consumer Decisions in the (P, U) Subgame

The (U, P) subgame is symmetric to the (P, U) subgame. Hence, we can obtain the firms' equilibrium profit in this subgame by interchanging the subscripts A and B in (B2.8) and (B2.9).

Table B2.1 summarizes the firms' profits associated with different combinations of pricing strategies. By comparing each firm's profits associated with different pricing strategies, we find that $\Pi_A^{UU} \geq \Pi_A^{PU}$ and $\Pi_B^{UU} \geq \Pi_B^{UP}$ if and only if $\theta \geq 1/2$. Moreover, $\Pi_A^{PP} < \Pi_A^{UP}$ and $\Pi_B^{PP} < \Pi_B^{PU}$ if and only if $\theta > \theta_1$. Noting that $\theta_1 < 1/2$, we conclude that (U, U) is the unique equilibrium if $\theta \geq 1/2$, and (P, P) is the unique equilibrium if $\theta < \theta_1$. For $\theta \in [\theta_1, 1/2)$, we have $\Pi_A^{UU} < \Pi_A^{PU}$, $\Pi_B^{UU} < \Pi_B^{UP}$, $\Pi_A^{PP} \leq \Pi_A^{UP}$ and $\Pi_B^{PP} \leq \Pi_B^{PU}$, which imply that (P, U) and (U, P) are equilibria in this case.

Table B2.1: Firms' Profits without Privacy Regulation in the Model with Low Consumer Valuation

A \ B	U	P
U	$(\frac{V^2}{4t},\frac{V^2}{4t})$	$(\frac{V^2}{4t},\frac{V^2(1-\theta)}{2t})$
P	$(\frac{V^2(1-\theta)}{2t},\frac{V^2}{4t})$	(Π_A^{PP},Π_B^{PP})

Note: (Π_A^{PP},Π_B^{PP}) is represented by (B2.6)-(B2.7).

QED

Proposition B5: In the duopoly model with low consumer valuation and privacy-sensitive consumers, both firms adopt personalized pricing in the equilibrium under the privacy regulation.

Proof of Proposition B5

We start with an analysis of the four subgames, (U',U'), (P',P'), (P',U') and (U',P'), in this model with low consumer valuation. The equilibrium in the (U',U') subgame is not affected by the privacy regulation because neither firm uses the tracking technology. Thus, the equilibrium prices and profits are the same as those in the case of (U,U), that is, $p_A^{U'U'}=p_B^{U'U'}=V/2$ and $\Pi_A^{U'U'}=\Pi_B^{U'U'}=V^2/4t$.

For the (P',P') subgame, we know from the earlier analysis of the (P,P) subgame that the firms will set the following personalized prices for consumer who accept tracking: $p_A^{P'P'}(x)=V-tx$ and $p_B^{P'P'}(x)=0$ for $x\in[0,1-V/t)$, $p_A^{P'P'}(x)=t(1-2x)$ and $p_B^{P'P'}(x)=0$ for $x\in[1-V/t,1/2]$, while $p_A^{P'P'}(x)=0$ and $p_B^{P'P'}(x)=t(2x-1)$ for $x\in[1/2,V/t]$, and $p_A^{P'P'}(x)=0$ and $p_B^{P'P'}(x)=V-t(1-x)$ for $x\in(V/t,1]$.

As we have noted in the baseline model, consumers located close to the two ends of the Hotelling line have stronger incentives to reject tracking because they would be charged higher prices under personalized pricing than those consumers near the center. Define $[0,x_{A\alpha}^{P'P'}]$ and $[x_{B\alpha}^{P'P'},1]$ as the intervals that type-α consumers who reject tracking and buy a good, where $\alpha=0$ denotes privacy-insensitive consumers and $\alpha=1$ denotes privacy-sensitive consumers. To determine the firms' uniform prices for consumers who reject tracking, we consider the following three cases based on the position of $x_{A\alpha}^{P'P'}$ and $x_{B\alpha}^{P'P'}$ relative to the intervals of the firms' captive consumers, which is $[0,1-V/t)$ for firm A and $(V/t,1]$ for firm B:

(1) $x_{A0}^{P'P'}\in[1-\frac{V}{t},\frac{1}{2}]$, $x_{A1}^{P'P'}\in[1-\frac{V-D}{t},\frac{1}{2}]$, $x_{B0}^{P'P'}\in(\frac{1}{2},\frac{V}{t}]$ and $x_{B1}^{P'P'}\in(\frac{1}{2},\frac{V-D}{t}]$;

(2) $x_{A0}^{P'P'}\in[0,1-\frac{V}{t})$, $x_{A1}^{P'P'}\in[1-\frac{V-D}{t},\frac{1}{2}]$, $x_{B0}^{P'P'}\in(\frac{V}{t},1]$ and $x_{B1}^{P'P'}\in(\frac{1}{2},\frac{V-D}{t}]$;

(3) $x_{A0}^{P'P'}\in[0,1-\frac{V}{t})$, $x_{A1}^{P'P'}\in[0,1-\frac{V-D}{t})$, $x_{B0}^{P'P'}\in(\frac{V}{t},1]$ and $x_{B1}^{P'P'}\in(\frac{V-D}{t},1]$.

Our analysis of cases (1) and (2) reveals that the values $x_{A\alpha}^{P'P'}$ and $x_{B\alpha}^{P'P'}$ in these two cases are incompatible with an equilibrium. Hence, we present here the analysis of case (3) only. The assumptions in case (3) imply that $x_{A0}^{P'P'}$, $x_{A1}^{P'P'}$, $x_{B0}^{P'P'}$ and $x_{B1}^{P'P'}$ satisfy

$$V-tx_{A0}^{P'P'}-p_A^{P'P'}=0, \qquad (B2.10)$$

$$V-tx_{A1}^{P'P'}-p_A^{P'P'}=0, \qquad (B2.11)$$

$$V-t\left(1-x_{B0}^{P'P'}\right)-p_B^{P'P'}=0, \qquad (B2.12)$$

$$V-t\left(1-x_{B1}^{P'P'}\right)-p_B^{P'P'}=0. \qquad (B2.13)$$

From (B2.10)-(B2.13), we obtain $x_{A0}^{P'P'}=x_{A1}^{P'P'}=(V-p_A^{P'P'})/t$ and $x_{B0}^{P'P'}=x_{B1}^{P'P'}=(p_B^{P'P'}+t-V)/t$.

Consumers' choices in this situation are illustrated in Figure B2.3. Privacy-insensitive consumers in the interval $[0, x_{A0}^{P'P'})$ purchase good A at the uniform price, those in the interval $[x_{A0}^{P'P'}, 1 - V/t)$ purchase good A at personalized price $p_A^{P'P'}(x) = V - tx$, and those in the interval $[1 - V/t, 1/2]$ purchase good A at personalized price $p_A^{P'P'}(x) = t(1 - 2x)$, while privacy-insensitive consumers in the interval $[1/2, V/t]$ purchase good B at personalized price $p_B^{P'P'}(x) = t(2x - 1)$, those in the interval $(V/t, x_{B0}^{P'P'}]$ purchase good B at personalized price $p_B^{P'P'}(x) = V - t(1 - x)$, and those in the interval $(x_{B0}^{P'P'}, 1]$ purchase good B at the uniform price. On the other hand, privacy-sensitive consumers in the interval $[0, x_{A1}^{P'P'}]$ purchase good A at the uniform price, those in the interval $[1 - (V - D)/t, 1/2]$ purchase good A at personalized price $p_A^{P'P'}(x) = t(1 - 2x)$; while privacy-sensitive consumers in the interval $[1/2, (V - D)/t]$ purchase good B at personalized price $p_B^{P'P'}(x) = t(2x - 1)$, those in the interval $[x_{B1}^{P'P'}, 1]$ purchase good B at the uniform price, and those privacy-sensitive consumers in the interval $(x_{A1}^{P'P'}, 1 - (V - D)/t)$ and $((V - D)/t, x_{B1}^{P'P'})$ do not purchase.

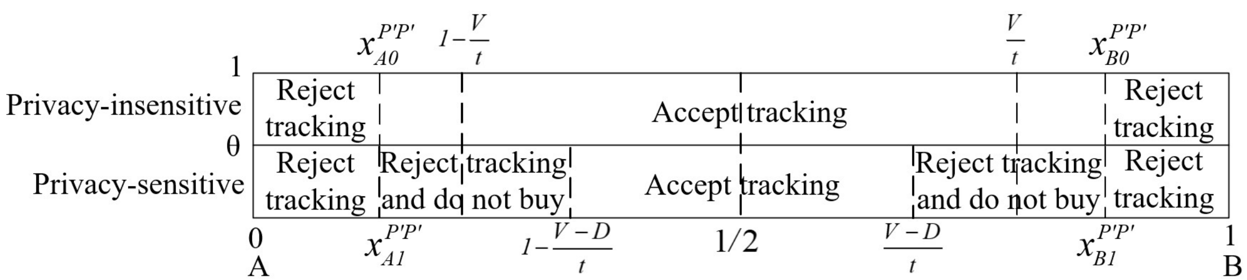

Figure B2.3: Consumers' Choices about Tracking in the Subgame (P', P')

Based on the above observations, we express the firms' profits as

$$\Pi_A^{P'P'} = p_A^{P'P'}\left[\theta x_{A1}^{P'P'} + (1-\theta)x_{A0}^{P'P'}\right] + (1-\theta)\left(\int_{x_{A0}^{P'P'}}^{1-\frac{V}{t}} (V - tx)\, dx + \int_{1-\frac{V}{t}}^{\frac{1}{2}} t(1-2x)\, dx\right)$$

$$+ \theta \int_{1-\frac{V-D}{t}}^{\frac{1}{2}} t(1-2x)\, dx, \qquad (B2.14)$$

$$\Pi_B^{P'P'} = p_B^{P'P'}[\theta(1 - x_{B1}^{P'P'}) + (1-\theta)(1 - x_{B0}^{P'P'})] + (1-\theta)\left(\int_{\frac{1}{2}}^{\frac{V}{t}} t(2x-1)\, dx\right.$$

$$\left. + \int_{\frac{V}{t}}^{x_{B0}^{P'P'}} \big(V - t(1-x)\big)\, dx\right) + \theta \int_{\frac{1}{2}}^{\frac{V-D}{t}} t(2x-1)\, dx. \qquad (B2.15)$$

Using (B2.14) and (B2.15), we examine the firms' profit-maximization problems and find the equilibrium uniform prices to be

$$p_A^{P'P'} = p_B^{P'P'} = \frac{V}{1+\theta}. \qquad (B2.16)$$

Substituting (B2.16) into $x_{A0}^{P'P'}$, $x_{A1}^{P'P'}$, $x_{B0}^{P'P'}$ and $x_{B1}^{P'P'}$, we have $x_{A0}^{P'P'} = x_{A1}^{P'P'} = \frac{\theta V}{t(1+\theta)}$ and $x_{B0}^{P'P'} = x_{B1}^{P'P'} = 1 - \frac{\theta V}{t(1+\theta)}$, which are consistent with the initial assumption in case (3). Therefore, we have found an equilibrium in this subgame. Substituting these results into (B2.14) and (B2.15), we obtain the equilibrium profits of firm A and firm B:

$$\Pi_A^{P'P'} = \Pi_B^{P'P'} = V - \theta(2V - D) + \frac{\theta D^2 - 2\theta DV - V^2(1-2\theta)}{t} - \frac{t(1-2\theta)}{4} + \frac{V^2}{2t(1+\theta)}, \qquad (B2.17)$$

Comparing $\Pi_i^{P'P'}$ $(i = A, B)$ in (B2.17) with the firms' profits in the (P, P) and (U', U') subgames, we find that $\Pi_i^{P'P'} > \Pi_i^{PP}$ and $\Pi_i^{P'P'} > \Pi_i^{U'U'}$ for $\theta \in (0,1)$.

In the (P', U') subgame, firm A uses personalized prices and a uniform price for different groups of consumers. This puts firm B at a disadvantage because it offers a uniform price only. Similar to the (P, U) subgame, we find that in equilibrium, firm B gives up competing for contestable consumers and sells only to its captive customers at the monopoly price, $p_B^{P'U'} = V/2$. At this price, privacy-sensitive consumers and privacy-insensitive consumers in the interval $[1 - V/2t, 1]$ will purchase good B.

Since there is no direct competition between the two firms in this case, firm A also sets a monopoly price for those consumers who reject tracking. Define $[0, x_A^{P'U'}]$ as the interval that consumers who reject tracking and buy good A. Then $x_A^{P'U'}$ satisfies $V - tx_A^{P'U'} - p_A^{P'U'} = 0$, from which we find $x_A^{P'U'} = (V - p_A^{P'U'})/t$.

Figure B2.4 illustrates the consumers' choices in this subgame. Privacy-insensitive consumers in the interval $[0, x_A^{P'U'})$ purchase good A at the uniform price, those in the interval $[x_A^{P'U'}, V/t]$ purchase good A at personalized price $p_A^{P'U'}(x) = V - tx$; while privacy-insensitive consumers in the interval $(V/t, 1 - V/2t)$ exit the market, and those in the interval $[1 - V/2t, 1]$ purchase good B at the uniform price. On the other hand, privacy-sensitive consumers in the interval $[0, x_A^{P'U'}]$ purchase good A at the uniform price, those in the interval $(x_A^{P'U'}, 1 - V/2t)$ exit the market; while privacy-sensitive consumers in the interval $[1 - V/2t, 1]$ purchase good B at the uniform price.

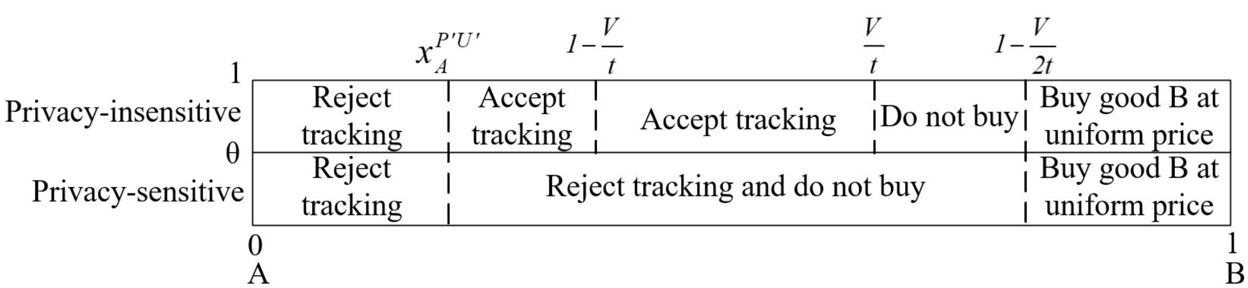

Figure B2.4: Consumers' Choices in the (P', U') Subgame

Based on these observations, we express firm A's profit as

$$\Pi_A^{P'U'} = p_A^{P'U'} x_A^{P'U'} + (1-\theta)\int_{x_A^{P'U'}}^{\frac{V}{t}} (V - tx)\, dx, \qquad (B2.18)$$

Using (B2.18), we examine firm A's profit-maximization problem and find the equilibrium uniform price to be

$$p_A^{P'U'} = \frac{V}{1+\theta}. \qquad (B2.19)$$

Substituting (B2.19) into $x_A^{P'U'}$, we obtain $x_A^{P'U'} = \frac{\theta V}{t(1+\theta)}$. It can be verified that $0 < x_A^{P'U'} < 1 - V/t$. Substituting these results into (B2.18), we find the equilibrium profits of firm A and firm B:

$$\Pi_A^{P'U'} = \frac{V^2}{2t(1+\theta)}, \quad (B2.20)$$

$$\Pi_B^{P'U'} = \frac{V^2}{4t}. \quad (B2.21)$$

Comparing $\Pi_A^{P'U'}$ in (B2.20) with firm A's profit in the (U', U') subgames, we find that $\Pi_A^{P'U'} > \Pi_A^{U'U'}$.

Finally, the analysis of the fourth subgame (U', P') is symmetric to that of (P', U'). We can obtain the equilibrium profits in this subgame by interchanging the subscripts A and B in (B2.20) and (B2.21).

Table B2.2: Firms' Profits with the Privacy Regulation in the Model with Low Consumer Valuation

A \ B	U'	P'
U'	$(\frac{V^2}{4t}, \frac{V^2}{4t})$	$(\frac{V^2}{4t}, \frac{V^2}{2t(1+\theta)})$
P'	$(\frac{V^2}{2t(1+\theta)}, \frac{V^2}{4t})$	$(\Pi_A^{P'P'}, \Pi_B^{P'P'})$

Note: $(\Pi_A^{P'P'}, \Pi_B^{P'P'})$ is represented by (B2.17).

Using the findings from the analysis of the four subgames, we present in Table B2.2 the firms' profits associated with different combinations of pricing strategies. By comparing each firm's profits associated with different strategy profiles, we find that $\Pi_A^{U'P'} < \Pi_A^{P'P'}$ and $\Pi_B^{P'U'} < \Pi_B^{P'P'}$, which implies that (P', P') is an equilibrium. Moreover, $\Pi_A^{U'U'} < \Pi_A^{P'U'}$ and $\Pi_B^{U'U'} < \Pi_B^{U'P'}$ ensure that (P', P') is a unique equilibrium in this model. Note that the above profit rankings are true for any $\theta \in (0, 1)$.

QED

A comparison of Propositions B2.2 and B2.3 shows that the privacy regulation leads to wider use of tracking technology and personalized pricing. More remarkable is that under the regulation, both firms adopt personalized pricing independent of the proportion of privacy-sensitive consumers.

B3. Alternative Timing of Price Revelation

B3.1 Examples of Request for Selecting Cookie Settings

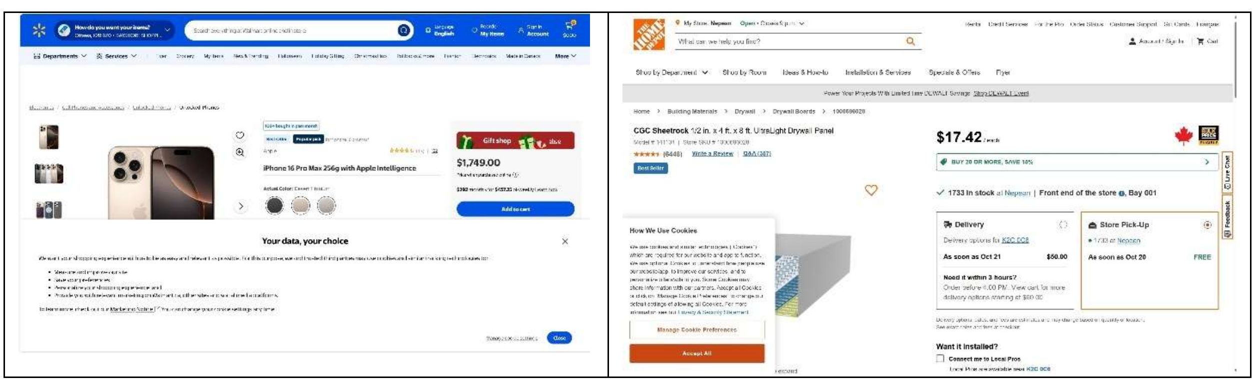

Figure B3.1: Screenshots of the Canadian websites of Walmart and Home Depot, taken in October 2025

In Figure B3.1 are screenshots from the Canadian websites of Walmart (walmart.ca) and Home Depot (homedepot.ca). These pages show product information (including prices) along with a request for the user to choose her cookie preferences. Note that these examples are intended to demonstrate the timing of retailers' requests for consent. We have no information on whether these two specific retailers use personalized pricing or not.

B3.2 Revised Model with Alternative Timing: Monopoly

In this extension, we modify stage 3 of the game by assuming that in the case where the monopolist adopts the tracking technology under the privacy regulation, it reveals the uniform price to a consumer only if she rejects tracking. All other aspects of the model remain the same. In particular, the firm's uniform price is set (but not revealed) at the time when it asks consumers for consent to tracking.

We use the concept of perfect Bayesian equilibrium (PBE) to determine the equilibrium in this model. An important element in a PBE is players' beliefs about variables that they do not observe. In this revised model, a consumer's utility after rejecting a firm's request for tracking depends on the firm's uniform price, yet she does not observe the uniform price when she has to respond to the request. In such a scenario, she chooses her response based on her belief about the uniform price. In a PBE, her belief must be consistent with the equilibrium price. In a nutshell, the process of finding a PBE in the model consists of identifying a set of player strategies and beliefs such that that (a) the strategies are sequentially rationally given the beliefs and (b) the beliefs are consistent with the strategies.

Note that this modification affects only the stage-3 subgame after the firm chooses to adopt the tracking technology and personalized pricing. We use the following procedure to determine the equilibria in this subgame. We start by postulating a belief about the uniform prices that the firm will offer to consumers who reject tracking, denoted by $p_i^{eP'}$ $(i = A, B)$. Given this belief, we determine every consumer's rational response to the firm's request for tracking. Then we use backward induction to find the firm's profit-maximizing uniform price $p_i^{P'}$ $(i = A, B)$. If this price is consistent with the belief we have postulated, i.e., if $p_i^{P'} = p_i^{eP'}$ $(i = A, B)$, we conclude that $p_i^{P'}$ is part of an equilibrium in this subgame. Otherwise, we conclude that the belief we have postulated cannot be part of an equilibrium in this subgame. We iterate this process for all possible beliefs. After we determine the equilibria in this subgame, we integrate them with the analysis on the rest of the game to obtain the equilibria in the whole game.

Proposition B6: In the monopoly model with the alternative timing of price revelation, the equilibrium pricing strategy of firm M under the privacy regulation depends on the proportion of privacy-sensitive consumers as follows.

(i) If $\theta < t/(2V - t)$, there are multiple equilibria. In one equilibrium, the firm adopts uniform pricing and set them at $p_i^{P'} = V - t/2$. In the other equilibria, the firm adopts personalized pricing and offers uniform prices to those who reject tracking at $p_i^{P'} \in (V - t/2, V/(1 + \theta)]$.

(ii) If $\theta \geq t/(2V - t)$, the firm offers the uniform price $p_i^{P'} = V - t/2$ to all consumers.

Proof of Proposition B6

When a consumer chooses her response to the firm's request for tracking, she foresees the personalized price she would be offered if she accepts tracking. That is, the firm will offer a personalized price that extracts the entire surplus from the consumer: $p_A^P(x) = V - tx$ for a consumer located at $x \in [0, 1/2]$ and $p_B^P(x) = V - t(1 - x)$ for a consumer located at $x \in [1/2\,, 1]$. Given these anticipated personalized prices, a privacy-sensitive consumer will not accept tracking because doing so will result in a negative utility level $(-D)$. On the other hand, a privacy-insensitive consumer may or may not accept tracking depending on her belief about the uniform price. Specifically, she will reject tracking if she believes that the uniform price is below the personalized price she will be offered after agreeing to be tracked.

Since the belief about uniform prices must be consistent with the actual prices chosen by the firm, all consumers must have the same belief in equilibrium. Hence, we can ignore the possibility of heterogeneous beliefs among consumers. In our analysis, we divide the range of possible beliefs about $p_i^{P'}$ into three cases: (i) $p_i^{eP'} \leq V - t/2$, (ii) $p_i^{eP'} > V$, and (iii) $p_i^{eP'} \in (V - t/2\,, V]$. We now analyze each case separately.

To be clear, those consumers who accept tracking will be offered personalized prices and hence they do not have the option to purchase at uniform prices. Consequently, in all three cases we consider below, the firm's profit from selling to consumers who accept tracking is independent of $p_i^{P'}$, and it is equal to

$$(1-\theta)\left[\int_{x_A^{eP'}}^{1/2} (V - tx)\,dx + \int_{1/2}^{x_B^{eP'}} \big(V - t(1-x)\big)\,dx\right]. \quad (B3.1)$$

To determine the equilibrium uniform prices, we only need to consider the firm's profit from selling to consumers who reject tracking.

Case (i): $p_i^{eP'} \leq V - t/2$ $(i = A, B)$

Note that among all consumers who agree to be tracked, those located at the center ($x = 1/2$) are offered the lowest personalized price, equalling $V - t/2$. This implies that if consumers believe that the uniform prices of goods A and B are below $V - t/2$, every consumer is better off purchasing a unit at the uniform prices and hence none of them will accept tracking. In this case, the firm will not have an opportunity to offer a personalized price to any consumer (because all consumers reject tracking), and it will sell the products at uniform prices only. As shown in Appendix A1, the firm's profit-maximizing uniform price in this situation is $p_i^{P'} = V - t/2$ $(i = A, B)$. Hence, Stage 3 of this revised model has an equilibrium in which $p_i^{eP'} = p_i^{P'} = V - t/2$ $(i = A, B)$.

Case (ii): $p_i^{eP'} > V$ $(i = A, B)$

Since consumers located at the endpoints ($x = 0$ and 1) are offered the highest personalized price V among all consumers who accept tracking, all privacy-insensitive consumers agree to be tracked in this case. But none of the privacy-sensitive consumers will accept tracking because of their privacy cost. Now consider the firm's choice of uniform prices to these consumers who reject tracking. If the firm offers $p_i^{P'} > V$, it will sell 0 units at the uniform prices and earn no profit from these consumers. But the firm can earn a positive profit from these consumers if it lowers the uniform prices to below V. Note that consumers who accept tracking cannot purchase at these lower uniform prices because they are offered personalized prices only. Therefore, by reducing the uniform prices to below V, the firm can earn a larger profit from those consumers who refuse tracking while earning the same profit from consumers who accept tracking. This implies that $p_i^{P'} < V < p_i^{eP'}$. Consequently, the belief $p_i^{eP'} > V$ cannot be part an equilibrium.

Case (iii): $p_i^{eP'} \in (V - t/2\,, V]$ $(i = A, B)$

In this case, some privacy-insensitive consumers will accept tracking while others will not. Specifically, consumers who are located around the center may agree to be tracked because their personalized prices are lower than $p_i^{eP'}$. Let $x_A^{eP'}$ and $x_B^{eP'}$ denote the locations of the marginal consumers who are indifferent between accepting and rejecting tracking. Then privacy-insensitive consumers in the interval $[x_A^{eP'}, x_B^{eP'}]$ will accept tracking, while those in $[0, x_A^{eP'})$ and $(x_B^{eP'}, 1]$ will reject tracking. But no privacy-sensitive consumers will agree to be tracked. Figure B3.2 illustrates the consumers' responses to the request for tracking in this scenario.

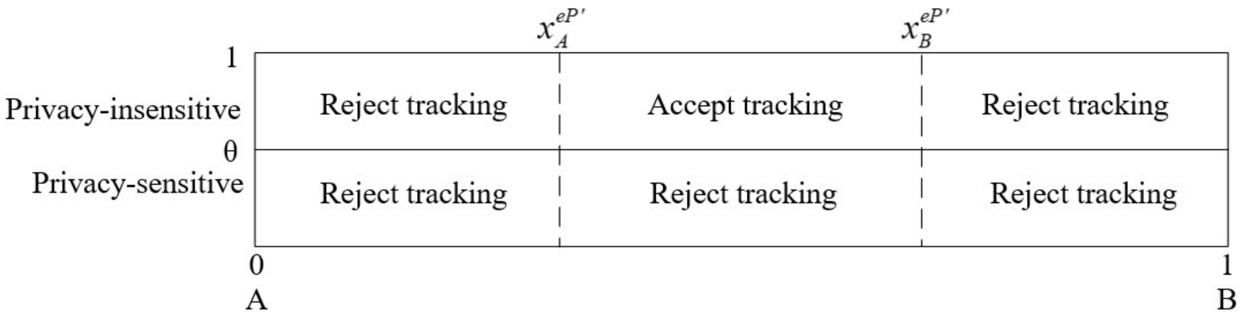

Figure B3.2: Consumer Response to Request for Tracking when $p_i^{eP'} \in (V - t/2, V]$

By definition, $x_A^{eP'}$ and $x_B^{eP'}$ must satisfy the following conditions:

$$V - t x_A^{eP'} - p_A^{eP'} = 0; \quad (B3.2)$$

$$V - t\left(1 - x_B^{eP'}\right) - p_B^{eP'} = 0. \quad (B3.3)$$

Using these two equations, we find

$$x_A^{eP'} = \frac{V - p_A^{eP'}}{t}, \qquad x_B^{eP'} = \frac{t - V + p_B^{eP'}}{t}. \quad (B3.4)$$

Those privacy-insensitive consumers who accept tracking will receive offers of personalized prices and will purchase at these prices. On the other hand, the purchase decisions of those consumers who reject tracking will depend on the actual uniform prices offered by the firm, $p_i^{P'}$. As in the baseline model, we use $x_A^{P'}$ and $x_B^{P'}$ to denote the locations of the privacy-insensitive consumers who are indifferent between purchasing at the uniform price $p_i^{P'}$ and purchasing at their respective personalized prices (if they were offered these prices). The expressions of $x_A^{P'}$ and $x_B^{P'}$ are given in (A4) of Appendix A.

Anticipating these purchases decisions by consumers, the firm chooses the uniform prices to maximize its profit. There are three possibilities for the uniform prices: $p_i^{P'} = p_i^{eP'}$, $p_i^{P'} < p_i^{eP'}$, and $p_i^{P'} > p_i^{eP'}$. Noting that $p_i^{P'}$ is an equilibrium price if $p_i^{P'} = p_i^{eP'}$ yields the highest profit among the three possibilities, we examine the firm's profit under each possibility to determine the profit-maximizing price.

Sub-case (iii-a): The firm sets $p_i^{P'} = p_i^{eP'}$ $(i = A, B)$.

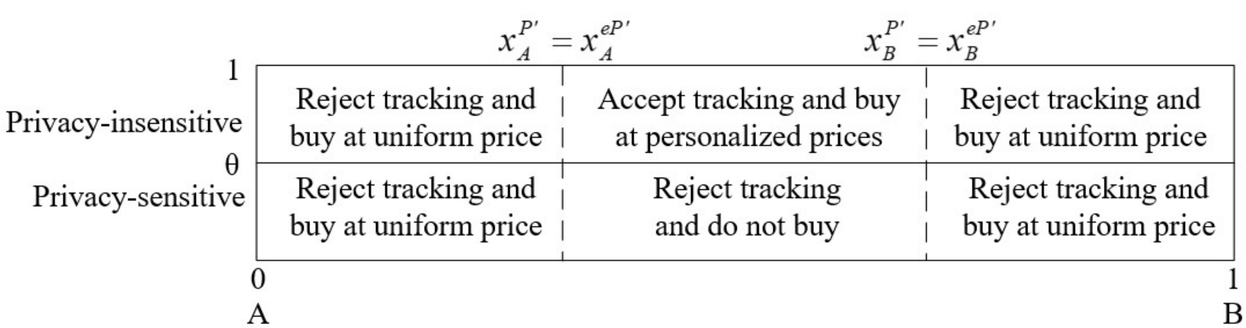

Figure B3.3: Sub-case (iii-a) Consumer Decisions when $p_i^{P'} = p_i^{eP'}$ and $p_i^{eP'} \in (V - t/2, V]$

In this sub-case, $x_i^{P'} = x_i^{eP'}$. Both privacy-insensitive and privacy-sensitive consumers in the intervals $[0, x_A^{eP'}]$ and $[x_B^{eP'}, 1]$ purchase at the uniform prices, while privacy-sensitive consumers in $(x_A^{eP'}, x_B^{eP'})$ do not purchase. Figure B3.3 illustrates the consumers' decisions in this case.

The profit that the firm earns from those consumers who reject tracking is equal to

$$\Pi_{Ma}^{P'} = p_A^{eP'} x_A^{eP'} + p_B^{eP'}\left(1 - x_B^{eP'}\right). \quad (B3.5)$$

Substituting (B3.4) for $x_A^{eP'}$ and $x_B^{eP'}$ into (B3.5), we obtain

$$\Pi_{Ma}^{P'} = \frac{p_A^{eP'}\left(V - p_A^{eP'}\right)}{t} + \frac{p_B^{eP'}\left(V - p_B^{eP'}\right)}{t}. \quad (B3.6)$$

Sub-case (iii-b): The firm sets $p_i^{P'} < p_i^{eP'}$ $(i = A, B)$.

In this case $x_A^{P'} > x_A^{eP'}$ and $x_B^{P'} < x_B^{eP'}$. Since all privacy-sensitive consumers are offered the uniform prices, those in the interval $[0, x_A^{P'}]$ and $[x_B^{P'}, 1]$ will purchase at these prices. However, privacy-insensitive consumers in the intervals $[x_A^{eP'}, x_A^{P'}]$ and $[x_B^{P'}, x_B^{eP'}]$ will not have the option of purchasing at the uniform prices because they are offered the personalized prices instead. Hence, among the privacy-insensitive consumers, only those in $[0, x_A^{eP'})$ and $(x_B^{eP'}, 1]$ will purchase at the uniform prices. Figure B3.4 illustrates the consumers' decisions in this case.

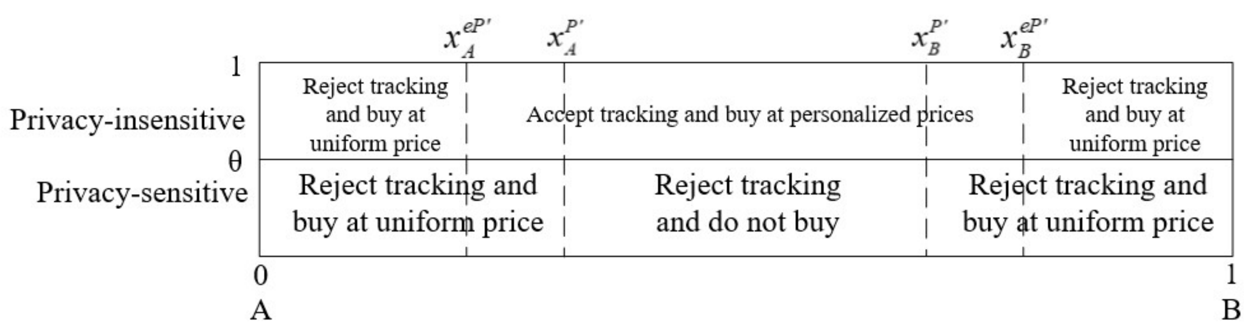

Figure B3.4: Sub-case (iii-b) Consumer Decisions when $p_i^{P'} < p_i^{eP'}$ and $p_i^{eP'} \in (V - t/2, V]$

Consequently, the firm's profit from selling to those consumers who reject tracking is

$$\Pi_{Mb}^{P'} = \theta\left[p_A^{P'} x_A^{P'} + p_B^{P'}\left(1 - x_B^{P'}\right)\right] + (1-\theta)\left[p_A^{P'} x_A^{eP'} + p_B^{P'}\left(1 - x_B^{eP'}\right)\right]. \quad (B3.7)$$

Note in (B3.7) that an adjustment in the uniform price $p_i^{P'}$ changes $x_i^{P'}$ but it does not affect $x_i^{eP'}$. The monopolist sets $(p_A^{P'}, p_B^{P'})$ to maximize (B3.7). Substituting (A4) for $x_A^{P'}$ and $x_B^{P'}$ and (B3.4) for $x_A^{eP'}$ and $x_B^{eP'}$ into (B3.7), we obtain

$$\Pi_{Mb}^{P'} = \frac{p_A^{P'}\left(V - (1-\theta)p_A^{eP'} - \theta p_A^{P'}\right)}{t} + \frac{p_B^{P'}\left(V - (1-\theta)p_B^{eP'} - \theta p_B^{P'}\right)}{t}. \quad (B3.8)$$

Differentiating (B3.8) with respect to $p_i^{P'} (i = A, B)$, we find

$$\frac{\partial \Pi_{Mb}^{P'}}{\partial p_i^{P'}} = \frac{V - (1-\theta)p_i^{eP'} - 2\theta p_i^{P'}}{t}. \quad (B3.9)$$

Using (B3.9), we obtain the interior solution to the firm's profit-maximization problem:

$$p_A^{P'} = \frac{V - (1-\theta)p_A^{eP'}}{2\theta}, \qquad p_B^{P'} = \frac{V - (1-\theta)p_B^{eP'}}{2\theta}. \quad (B3.10)$$

Substituting (B3.10) into (B3.9), we obtain

$$\Pi_{Mb}^{P'} = \frac{2V^2 - 2V(1-\theta)(p_A^{eP'} + p_B^{eP'}) + (1-\theta)^2({p_A^{eP'}}^2 + {p_B^{eP'}}^2)}{4t\theta}. \quad (B3.11)$$

Note, however, (B3.11) is relevant only if $p_i^{P'} < p_i^{eP'}$. Using (B3.10), we verify that this condition holds if and only if $p_i^{eP'} > V/(1+\theta)$, in which case we find that $\Pi_{Mb}^{P'}$ in (B3.11) is greater than $\Pi_{Ma}^{P'}$ in (B3.6). This means that if $p_i^{eP'} > V/(1+\theta)$, the firm earns a larger profit by setting $p_i^{P'} < p_i^{eP'}$ and, therefore, none of the $p_i^{eP'}$ in is ranges can be part of an equilibrium.

If $p_i^{eP'} \leq V/(1+\theta)$, we have $p_i^{P'} \geq p_i^{eP'}$. Using (B3.9), we find

$$\frac{\partial \Pi_{Mb}^{P'}}{\partial p_i^{P'}} \geq \frac{V - (1-\theta)p_i^{eP'} - 2\theta p_i^{eP'}}{t} \geq 0, \quad (B3.12)$$

which implies that the firm has no incentive to reduce $p_i^{P'}$ in this case. But for such $p_i^{eP'}$ to be part of an equilibrium, it must be in the range $(V - t/2, V]$. To satisfy the latter, it is necessary that $V - t/2 < V/(1+\theta)$, which entails $\theta < t/(2V-t)$. Therefore, if $\theta < t/(2V-t)$ and $p_i^{eP'} \in (V - t/2, V/(1+\theta)]$, the firm has no incentive to set $p_i^{P'}$ below $p_i^{eP'}$.

Sub-case (iii-c): The firm sets $p_i^{P'} > p_i^{eP'}$ $(i = A, B)$.

In this case $x_A^{P'} < x_A^{eP'}$ and $x_B^{P'} > x_B^{eP'}$. Both privacy-insensitive and privacy-sensitive consumers in the intervals $[0, x_A^{P'}]$ and $[x_B^{P'}, 1]$ will purchase at the uniform prices. Privacy-sensitive consumers in $(x_A^{P'}, x_B^{P'})$ will not purchase. Neither will privacy-insensitive consumers in the intervals $(x_A^{P'}, x_A^{eP'})$ and $(x_B^{eP'}, x_B^{P'})$ purchase because the sum of price and transport cost for these consumers exceed their maximum willingness to pay. Figure B3.5 illustrates the consumers' decisions in this case.

The firm's profit from selling to consumers who reject tracking is

$$\Pi_{Mc}^{P'} = p_A^{P'} x_A^{P'} + p_B^{P'}\left(1 - x_B^{P'}\right). \quad (B3.13)$$

Substituting (A4) for $x_A^{P'}$ and $x_B^{P'}$ into (B3.13), we obtain

$$\Pi_{Mc}^{P'} = \frac{p_A^{P'}\,(V - p_A^{P'})}{t} + \frac{p_B^{P'}\left(V - p_B^{P'}\right)}{t}. \quad (B3.14)$$

Comparing (B3.14) with (B3.6), we find that $\Pi_{Mc}^{P'} < \Pi_{Ma}^{P'}$ when $p_i^{P'} > p_i^{eP'}$ and $p_i^{eP'} \in (V - t/2, V]$. Therefore, the firm will not earn a larger profit if it sets the uniform prices above $p_i^{eP'}$.

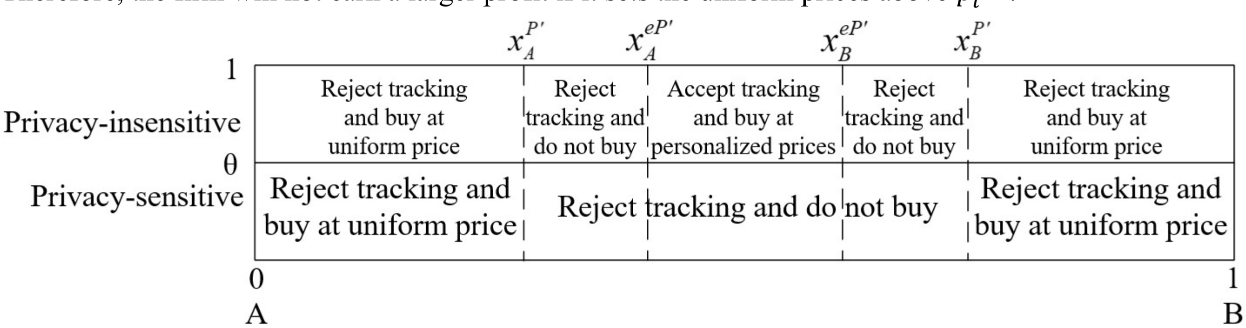

Figure B3.5: Sub-case (iii-c) Consumer Decisions when $p_i^{P'} > p_i^{eP'}$ and $p_i^{eP'} \in (V - t/2, V]$

To summarize the above analysis of case (iii), we conclude that if $\theta < t/(2V-t)$ and $p_i^{eP'} \in (V - t/2, V/(1+\theta)]$, the firm will not earn more profit through setting higher or lower uniform prices than $p_i^{eP'}$, and hence $p_i^{P'} = p_i^{eP'} \in (V - t/2, V/(1+\theta)]$ is part an equilibrium.

Proposition B6 is obtained by combining the findings from cases (i), (ii) and (iii). The equilibrium where the firm adopts uniform price in both parts of the proposition is from case (i), while the other equilibria in part (i) of the proposition comes from case (iii).

QED

Comparing Proposition B6 with Proposition 2 in the baseline model, we can verify that the unique PBE in the case $\theta \geq t/(2V-t)$ leads to the same equilibrium outcome as the baseline model, i.e., firm M adopts uniform pricing.

If $\theta < t/(2V-t)$, on the other hand, the PBE with the firm adopting personalized pricing and setting the uniform price at $p_i^{P'} = V/(1+\theta)$ leads to the same equilibrium outcome as the baseline model. Next, we prove that this PBE yields the highest profit for the firm among all the PBEs in the case $\theta < t/(2V-t)$. If firm M adopts uniform pricing with $p_i^{P'} = V - t/2$, it earns a profit of $\Pi_M^{P'} = V - t/2$. On the other hand, if it adopts personalized pricing and offers prices $p_i^{P'} \in (V - t/2, V/(1+\theta)]$, its profit is given by the sum of (B3.1) and (B3.6) with $p_i^{eP'} = p_i^{P'}$, which is equal to

$$\Pi_M^{P'} = \frac{1}{4t}\left[4V\left(p_A^{P'} + p_B^{P'}\right) - 2(1+\theta)\left({p_A^{P'}}^2 + {p_B^{P'}}^2\right) - (2V-t)^2(1-\theta)\right]. \quad (B3.15)$$

Evaluating (B3.15) at $p_i^{P'} = V - t/2$, we obtain $\Pi_M^{P'} = V - t/2$. Differentiating (B3.15), we find

$$\frac{\partial \Pi_M^{P'}}{\partial p_i^{P'}} = \frac{V - p_i^{P'}(1+\theta)}{t}\begin{cases} > 0 \text{ if } p_i^{P'} < V/(1+\theta), \\ < 0 \text{ if } p_i^{P'} > V/(1+\theta). \end{cases} \quad (B3.16)$$

Therefore, we conclude that firm M's profit is the highest at $p_i^{P'} = V/(1+\theta)$.

B3.3 Revised Model with Alternative Timing: Duopoly

Among the four subgames after the firms' choices of pricing strategies, the (U', U') subgame is not affected by the change in the timing of price revelation because the firms in this case do not use tracking technology and hence do not need to ask for consumers' consent. However, the other three subgames, namely, (P', P'), (P', U') and (U', P'), is affected by this change because one or both firms adopt the tracking technology in these cases. As in the analysis of the monopoly market in section B3.2, we use the following procedure to determine the equilibrium(s) in each of these three subgames. We start by postulating a belief about the uniform prices that the firm(s) will offer to consumers who reject tracking. Given this belief, we determine every consumer's rational response to a firm's request for tracking. Then we use backward induction to find the firm's profit-maximizing uniform price. If this price is consistent with the belief we have postulated, we conclude that it is part of an equilibrium in this subgame. Otherwise, we conclude that the belief we have postulated cannot be part of an equilibrium in this subgame. We iterate this process for all possible beliefs. After we determine the equilibria in the three subgames, we integrate them with the analysis on the rest of the game to obtain the equilibria in the whole game. Because of the need to consider different possible beliefs, the analysis is more complex than in the baseline model.

Proposition B7: In the (P', P') subgame of the duopoly model with the alternative timing of price revelation, there are multiple equilibria, with the uniform prices of both firms $p_i^{P'P'} \in [\max\{D, (t + \theta D)/2\}, t]$. Among these equilibria, the one with $p_i^{P'P'} = t$ yields the highest profit for both firms and leads to the same equilibrium outcome as in the baseline model.

Proof of Proposition B7

Let $p_i^{ep'p'}$ $(i = A, B)$ denote the beliefs about the uniform prices, and $p_i^{P'P'}$ $(i = A, B)$ denote the actual uniform prices that firms offer to those consumers who reject tracking. In equilibrium, we must have $p_i^{eP'P'} = p_i^{P'P'}$ $(i = A, B)$. Because the two firms adopt the same pricing strategy in this subgame, we focus on symmetric equilibria where $p_A^{P'P'} = p_B^{P'P'}$. To find the equilibria in this subgame, we consider all possible values of $p_i^{ePP'}$. Specifically, we consider the following three cases: (1) $p_i^{eP'P'} \in [0, D)$, (2) $p_i^{eP'P'} \in [D, t]$, and (3) $p_i^{eP'P'} > t$. Note that the equilibrium prices in this proposition falls in the interval in case (2). Hence, we will present below the analysis of this case, followed by a brief discussion of the other two cases.

Now we analyze the case $p_i^{eP'P'} \in [D, t]$ and show that a belief in this interval may be compatible with an equilibrium. Recall that the personalized prices offered by the two firms in this subgame are: $p_A^{P'P'}(x) = t(1 - 2x)$ for a consumer located at $x \in [0, 1/2]$ and $p_B^{P'P'}(x) = t(2x - 1)$ for a consumer located at $x \in [1/2, 1]$, with the highest personalized being t paid by the consumers at the two ends of the Hotelling line. Given the belief that $p_i^{eP'P'} \leq t$, some consumers located close to the ends of the line will reject tracking. Let $x_{A\alpha}^{eP'P'}$ and $x_{B\alpha}^{eP'P'}$ denote the locations of type-α consumers who are indifferent between accepting and rejecting tracking, where $\alpha = 0$ denotes privacy-insensitive consumers and $\alpha = 1$ denotes privacy-sensitive consumers. Figure B3.6 illustrates the consumers' responses to the request for tracking in this scenario.

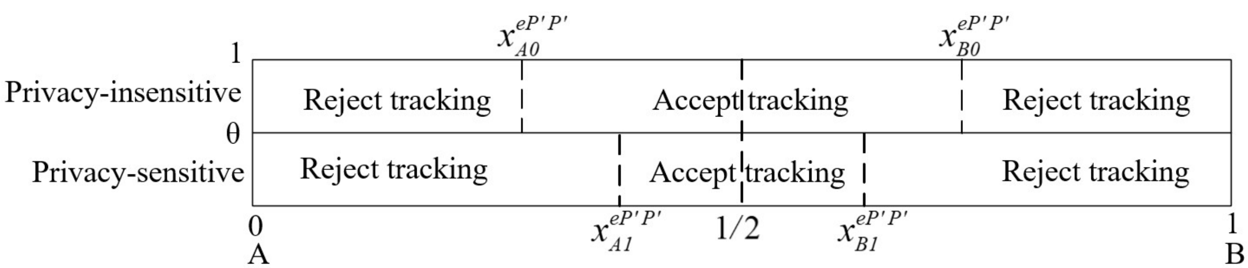

Figure B3.6: Consumer Decisions when the Expected Uniform Prices Are between D and t

For the privacy-sensitive consumers located at $x_{A1}^{eP'P'}$ and $x_{B1}^{eP'P'}$, the following conditions must hold.

$$V - tx_{A1}^{eP'P'} - p_A^{eP'P'} = V - tx_{A1}^{eP'P'} - t\left(1 - 2x_{A1}^{eP'P'}\right) - D. \qquad (B3.17)$$

$$V - t\left(1 - x_{B1}^{eP'P'}\right) - p_B^{eP'P'} = V - t\left(1 - x_{B1}^{eP'P'}\right) - t\left(2x_{B1}^{eP'P'} - 1\right) - D. \qquad (B3.18)$$

The left-hand sides of (B3.17) and (B3.18) represent the utilities of the privacy-sensitive consumers at these locations from purchasing at their expected uniform prices, while the right-hand sides represent the utilities of these consumers from purchasing at their personalized prices. From (B3.17) and (B3.18), we find

$$x_{A1}^{eP'P'} = \frac{t + D - p_A^{eP'P'}}{2t}. \qquad (B3.19)$$

$$x_{B1}^{eP'P'} = \frac{t - D + p_B^{eP'P'}}{2t}. \qquad (B3.20)$$

Meanwhile, the following conditions must hold for the privacy-insensitive consumers located at $x_{A0}^{eP'P'}$ and $x_{B0}^{eP'P'}$:

$$V - tx_{A0}^{eP'P'} - p_A^{eP'P'} = V - tx_{A0}^{eP'P'} - t\left(1 - 2x_{A0}^{eP'P'}\right). \qquad (B3.21)$$

$$V - t(1 - x_{B0}^{eP'P'}) - p_B^{eP'P'} = V - t(1 - x_{B0}^{eP'P'}) - t(2x_{B0}^{eP'P'} - 1). \quad (B3.22)$$

From these two conditions, we obtain

$$x_{A0}^{eP'P'} = \frac{t - p_A^{eP'P'}}{2t}. \quad (B3.23)$$

$$x_{B0}^{eP'P'} = \frac{t + p_B^{eP'P'}}{2t}. \quad (B3.24)$$

Depending on whether a consumer accepts tracking, she will be offered either a personalized price or a uniform price. Note that a consumer's purchase decision will depend on the actual price offered to her, not her beliefs about the prices. But in equilibrium, the belief about prices must be consistent with the actual prices set by the firms.

To determine whether a belief is compatible with an equilibrium, we must find out the firms' profit-maximizing prices given the consumers' decisions about consent to tracking. The resulting uniform price of a firm may turn out to be higher than, lower than, or equal to the price belief $p_i^{eP'P'}$. Accordingly, we need to consider three scenarios.

Scenario (a): $p_i^{P'P'} = p_i^{eP'P'}$ $(i = A, B)$.

In this scenario, type-α consumers in the intervals $[0, x_{A\alpha}^{eP'P'}]$ and $[x_{B\alpha}^{eP'P'}, 1]$ will purchase at the uniform prices, while others in the intervals $(x_{A\alpha}^{eP'P'}, x_{B\alpha}^{eP'P'})$ will purchase at the personalized prices. The firms' profits are

$$\Pi_{Aa}^{P'P'} = \theta \int_{x_{A1}^{eP'P'}}^{1/2} t(1 - 2x)\, dx + (1 - \theta) \int_{x_{A0}^{eP'P'}}^{1/2} t(1 - 2x)\, dx + p_A^{eP'P'} [\theta x_{A1}^{eP'P'} + (1 - \theta) x_{A0}^{eP'P'}]. \quad (B3.25)$$

$$\Pi_{Ba}^{P'P'} = \theta \int_{1/2}^{x_{B1}^{eP'}} t(2x - 1)\, dx + (1 - \theta) \int_{1/2}^{x_{B0}^{eP'}} t(2x - 1)\, dx + p_B^{eP'P'} [\theta(1 - x_{B1}^{eP'P'}) + (1 - \theta)(1 - x_{B0}^{eP'P'})]. \quad (B3.26)$$

In each of (B3.25) and (B3.26), the first term and the second term on the right-hand side represent the firm's profit from sales at personalized prices to privacy-sensitive and privacy-insensitive consumers, and the third term represents the firm's profit from sales at its uniform price.

Scenario (b): $p_i^{P'P'} < p_i^{eP'P'}$ and $p_j^{P'P'} = p_j^{eP'P'}$ $(i \neq j, i, j = A, B)$.

Without loss of generality, we consider $i = A$ and suppose $p_A^{P'P'} < p_A^{eP'P'}$ and $p_B^{P'P'} = p_B^{eP'P'}$. By settling a uniform price lower than the expected price, firm A may be able to poach some consumers who would otherwise purchase from firm B. But we need to determine whether such a move will increase its profit.

We first exam whether firm A can attract the privacy-insensitive consumers who initially choose to visit firm B by setting a uniform price below the expected price. Recall that consumers who choose to buy good B at its uniform price earn a surplus at least as high as accepting firm B's tracking and purchasing the good at the personalized price. Hence, privacy-insensitive consumers in the interval $[1/2, 1]$ obtain utility $U_{Bb}^{P'P'} = V - t(1 - x) - t(2x - 1)$ (or higher) when purchasing from firm B. In comparison, they will obtain utility $U_{Ab}^{P'P'} = V - tx - p_A^{P'P'}$ if they switch to purchase from firm A at a uniform price. Only if $U_{Ab}^{P'P'} > U_{Bb}^{P'P'}$ can firm A attract some of the privacy-insensitive consumers in the interval $[1/2, 1]$. But that requires $p_A^{P'P'} < 0$, which is not profitable for firm A. Consequently, firm A cannot profitably poach privacy-insensitive consumers from firm B with $p_A^{P'P'} < p_A^{eP'P'}$.

Next, we investigate whether firm A can attract the privacy-sensitive consumers who initially choose to visit firm B by setting $p_A^{P'P'} < p_A^{eP'P'}$. Privacy-sensitive consumers in the interval $[1/2, 1]$ obtain utility $U_{Bb}^{P'P'} = V - t(1-x) - t(2x-1) - D$ (or higher) when purchasing from firm B and utility $U_{Ab}^{P'P'} = V - tx - p_A^{P'P'}$ when purchasing from firm A at a uniform price. Note that $U_{Ab}^{P'P'} > U_{Bb}^{P'P'}$ entails $p_A^{P'P'} < D$. Therefore, it is possible for firm A to poach some customers from firm B if it sets a uniform price below D.

To determine whether it is profitable for firm A to set $p_A^{P'P'} < D$, we use $V - t\left(1 - x_{A2}^{P'P'}\right) - p_B^{eP'P'} = V - tx_{A2}^{P'P'} - p_A^{P'P'}$ to find that $x_{A2}^{P'P'} = \frac{t + p_B^{eP'P'} - p_A^{P'P'}}{2t} < x_{B0}^{eP'P'}$. This implies that, if firm A sets a uniform price $p_A^{P'P'} < D$, privacy-sensitive consumers in the interval $[1/2, x_{A2}^{P'P'}]$ will purchase from firm A at the uniform price, while privacy-sensitive consumers in the interval $[x_{A2}^{P'P'}, 1]$ will purchase from firm B. Figure B3.7 illustrates the consumers' responses in this scenario.

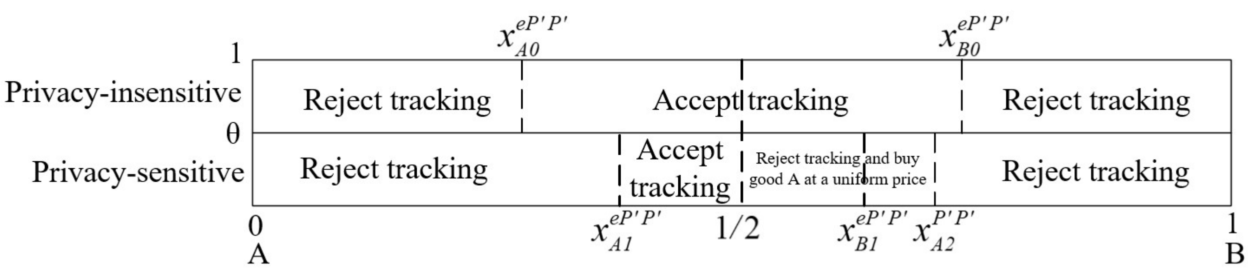

Figure B3.7: Consumer Decisions when the Expected Uniform Prices Are between D and t – Scenario (b)

With $p_A^{P'P'} < D$, firm A's profit is

$$\Pi_{Ab}^{P'P'} = \theta \int_{x_{A1}^{eP'P'}}^{1/2} t(1-2x)\,dx + (1-\theta)\int_{x_{A0}^{eP'P'}}^{1/2} t(1-2x)\,dx + p_A^{P'P'}\,[\theta x_{A1}^{eP'P'} + (1-\theta)x_{A0}^{eP'P'} + \theta(x_{A2}^{P'P'} - 1/2)]. \qquad (B3.27)$$

By comparing firm A's profits in scenarios (a) and (b), i.e., (B3.25) and (B3.27), we find that when $p_i^{eP'P'} \in [D, t]$ $(i = A, B)$, firm A cannot earn higher profit by setting a uniform price lower than D. Therefore, given the actions of its rival, a firm has no incentive to unilaterally set a uniform price below the price belief.

Scenario (c): $p_i^{P'P'} > p_i^{eP'P'}$ and $p_j^{P'P'} = p_j^{eP'P'}$ $(i \neq j, i, j = A, B)$.

Again, we consider $i = A$ and suppose $p_A^{P'P'} > p_A^{eP'P'}$ and $p_B^{P'P'} = p_B^{eP'P'}$. In this scenario, setting a higher uniform price does not enable firm A to attract consumers who would otherwise purchase from firm B. However, it allows firm A to extract more surplus from consumers who eventually purchase from firm A. The question is whether firm A will earn a larger profit from doing so.

A higher uniform price by firm A will drive some consumers away from good A. Starting from the situation illustrated in Figure B3.6, we expect that a higher uniform price by firm A will drive some privacy-sensitive consumers to the left of $x_{A1}^{eP'P'}$ and some privacy-insensitive consumers to the left of $x_{A0}^{eP'P'}$ to purchase good B at personalized prices. If they do so, their utility will be $V - t(1-x) - 0 - \alpha D$, where $\alpha = 1$ for a privacy-sensitive consumers and $\alpha = 0$ for a privacy-insensitive consumer. Comparing this utility level with their utility from purchasing good A at the uniform price, $V - tx - p_A^{P'P'}$, we can determine the location of the privacy-sensitive consumer (and respectively, privacy-insensitive consumer) who is indifferent between these options. These locations, denoted by $x_{A3}^{P'P'}$ and $x_{A4}^{P'P'}$, must satisfy $V - tx_{A3}^{P'P'} - p_A^{P'P'} = V - t(1 - x_{A3}^{P'P'}) - 0 - D$ and $V - tx_{A4}^{P'P'} - p_A^{P'P'} = V - t(1 - x_{A4}^{P'P'}) - 0$. From these conditions, we obtain $x_{A3}^{P'P'} = \frac{t+D-p_A^{P'P'}}{2t} < x_{A1}^{eP'P'}$ and $x_{A4}^{P'P'} = \frac{t-p_A^{P'P'}}{2t} < x_{A0}^{eP'P'}$. Therefore, as illustrated in Figure B3.8, when firm A sets a uniform price $p_A^{P'P'} > p_A^{eP'P'}$, privacy-sensitive consumers in the interval $(x_{A3}^{P'P'}, x_{A1}^{eP'P'})$ and privacy-insensitive consumers in the interval $[x_{A4}^{P'P'}, x_{A0}^{eP'P'}]$ will switch to purchase good B at personalized prices. But privacy-sensitive consumers in the interval $[0, x_{A3}^{P'P'}]$ and privacy-insensitive consumers in the interval $[0, x_{A4}^{P'P'})$ will still purchase good A at the uniform price.

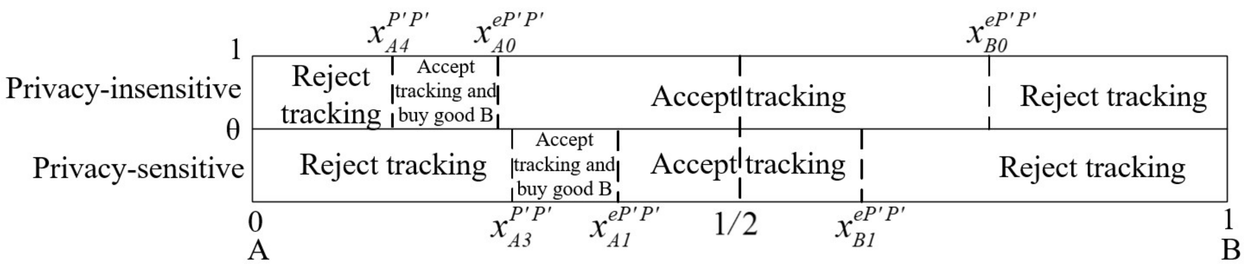

Figure B3.8: Consumer Decisions when the Expected Uniform Prices Are between D and t – Scenario (c)

Therefore, firm A's profit in scenario (c) can be expressed as

$$\Pi_{Ac}^{P'P'} = \theta \int_{x_{A1}^{eP'P'}}^{1/2} t(1-2x)\,dx + (1-\theta) \int_{x_{A0}^{eP'P'}}^{1/2} t(1-2x)\,dx + p_A^{P'P'}(\theta x_{A3}^{P'P'} + (1 - \theta) x_{A4}^{P'P'}). \quad (B3.28)$$

By comparing firm A's profits in scenarios (a) and (c), i.e., (B3.25) and (B3.28), we find that firm A will not earn a larger profit by setting a uniform price $p_A^{P'P'} > p_A^{eP'P'}$ if when $p_i^{eP'P'} > (t + \theta D)/2$ $(i = A, B)$. Noting that $p_i^{eP'P'} \in [D, t]$ in the current case, we conclude that a firm will have no incentive to set a uniform price above $p_i^{eP'P'}$ if $p_i^{eP'P'} \in [\max\{D, (t + \theta D)/2\}, t]$ $(i = A, B)$.

To summarize the analysis of scenarios (a)-(c), we conclude that $p_i^{P'P'} = p_i^{eP'P'} \in [\max\{D, (t + \theta D)/2\}, t]$ is part of an equilibrium in the (P', P') subgame. Note that $D < (t + \theta D)/2$ if and only if $D < t/(2 - \theta)$. In addition, $t/(2 - \theta) < t$. Hence the equilibrium uniform prices are in the range $[(t + \theta D)/2, t]$ if $D < t/(2 - \theta)$, and they are in the range $[D, t]$ if $D > t/(2 - \theta)$.

Using the equilibrium conditions to rewrite (B3.25) and (B3.26), we obtain each firm's equilibrium profit

$$\Pi_{ia}^{P'P'} = \frac{-p_i^{eP'P'2} + 2tp_i^{eP'P'} + \theta D^2}{4t}, (i = A, B), \quad (B3.29)$$

where $p_i^{eP'P'} \in [\frac{t+\theta D}{2}, t]$. Differentiating (B3.29), we find

$$\frac{\partial \Pi_{ia}^{P'P'}}{\partial p_i^{eP'P'}} = \frac{-p_i^{eP'P'} + t}{2t} \geq 0, \quad (B3.30)$$

for $p_i^{eP'P'} \in \left[\frac{t+\theta D}{2}, t\right]$. Hence, $\Pi_{ia}^{P'P'}$ rises with $p_i^{eP'P'}$ and reaches a maximum at $p_i^{eP'P'} = t$.

Our analysis of the remaining two cases, where $p_i^{eP'P'} \in [0, D)$ and $p_i^{eP'P'} > t$, shows that these beliefs cannot be supported as a part of an equilibrium. What happens in these two cases is that for any price belief $p_i^{eP'P'}$ in these intervals, a firm will find it profitable to choose a uniform price that is either higher or lower than $p_i^{eP'P'}$. Consequently, we rule out $p_i^{eP'P'}$ in these intervals as a candidate for an equilibrium.

We conclude from the above analysis that the PBEs of the (P', P') subgame is characterized by uniform prices $p_i^{P'P'} \in [max\{D, (t + \theta D)/2\},\ t]$ $(i = A, B)$. Recall from (A46) that in the baseline model, the equilibrium of the (P', P') subgame is characterized by uniform prices $p_A^{P'P'} = p_B^{P'P'} = t$, which falls in the interval $[max\{D, (t + \theta D)/2\},\ t]$. Hence, one of the PBEs in the revised model leads to the same equilibrium outcome as the equilibrium in the baseline. Moreover, (B3.29) and (B3.30) imply that this PBE yields the highest profit for both firms.

QED

Because the subgame (P', U') and the subgame (U', P') are symmetric, we present here a proposition and proof for the (P', U') subgame only. The results for the (U', P') subgame can be obtained by interchanging the subscripts A and B.

Proposition B8: In the (P', U') subgame of the duopoly model with the alternative timing of price revelation, there are multiple equilibria, with firm A's uniform price being $p_A^{P'U'} \in [3t/(4 - \theta), 3t/(2 + \theta)]$ and firm B's uniform price being $p_B^{P'U'} = (\theta p_A^{P'U'} + t)/2$. Among these equilibria, the one with $p_A^{P'U'} = 3t/(2 + \theta)$ yields the highest profit for both firms and leads to the same equilibrium outcome as in the baseline model.

Proof of Proposition B8

Let $p_A^{eP'U'}$ and $p_B^{eP'U'}$ denote the beliefs about the uniform prices of firms A and B, respectively. Because the two firms adopt different pricing strategies in this subgame, we expect that the two firms will offer different uniform prices in equilibrium and hence $p_A^{eP'U'} \neq p_B^{eP'U'}$. To find the equilibria in this subgame, we consider all possible values of $p_A^{eP'U'}$ relative to $p_B^{eP'U'}$. Specifically, we consider the following two cases: (1) $p_A^{eP'U'} \in [0, t + p_B^{eP'U'})$, and (2) $p_A^{eP'U'} \geq t + p_B^{eP'U'}$.

Now we start with an analysis of the case $p_A^{eP'U'} \in [0, t + p_B^{eP'U'})$. We can immediately rule out $p_A^{eP'U'} = 0$ as part of a possible PBE because with such a belief, all consumers who purchase good A will reject tracking, in which case firm A competes with firm B in uniform price only, in which case the equilibrium uniform prices of both goods are equal to t, which is incompatible with $p_A^{eP'U'} = 0$. Therefore, we consider $p_A^{eP'U'} > 0$ in the ensuing analysis.

Given $p_A^{eP'U'} \in [0, t + p_B^{eP'U'})$, some privacy-insensitive consumers located close to the left endpoint $x = 0$ of the Hotelling line will reject tracking while other privacy-insensitive consumers who choose firm A will accept tracking. Let $x_{A0}^{eP'U'}$ denote the location of privacy-insensitive consumers who are indifferent between accepting and rejecting firm A's tracking, and $x_0^{eP'U'}$ denotes the location of privacy-insensitive consumers who are indifferent between purchasing from firm A at the expected personalized price and purchasing at firm B at the expected uniform price. Then $x_{A0}^{eP'U'}$ must satisfy the condition

$$V - tx_{A0}^{eP'U'} - p_A^{eP'U'} = V - tx_{A0}^{eP'U'} - \left[t\left(1 - 2x_{A0}^{eP'U'}\right) + p_B^{eP'U'}\right]. \quad (B3.31)$$

From (B3.31), we obtain:

$$x_{A0}^{eP'U'} = \frac{p_B^{eP'U'} - p_A^{eP'U'} + t}{2t}. \qquad (B3.32)$$

To find an expression for $x_0^{eP'U'}$, note that consumers anticipate that firm A will set personalized prices $p_A^{eP'U'}(x) = t(1-2x) + p_B^{eP'U'}$ to undercut firm B's price as long as $p_A^{eP'U'}(x) > 0$. The latter implies $x \le (t + p_B^{eP'U'})/2t$. Hence,

$$x_0^{eP'U'} \equiv \min\left\{\frac{t + p_B^{eP'U'}}{2t}, 1\right\}. \qquad \text{(B3.33)}$$

Note that (B3.33) is independent of both $p_A^{eP'U'}$ and $p_A^{P'U'}$.

As in the baseline model, in this subgame privacy-sensitive consumers will not purchase from firm A at personalized prices because of the privacy cost. If privacy-sensitive consumers purchase from firm A, it will be at firm A's uniform price. Let $x_1^{eP'U'}$ be the location of privacy-sensitive consumers who are indifferent between purchasing from firm A at the expected uniform price, $p_A^{eP'U'}$, and from firm B at the expected uniform price, $p_B^{eP'U'}$. By definition, $x_1^{eP'U'}$ must satisfy the following condition

$$V - tx_1^{eP'U'} - p_A^{eP'U'} = V - t\left(1 - x_1^{eP'U'}\right) - p_B^{eP'U'}. \qquad (B3.34)$$

From (B3.34), we find that $x_1^{eP'U'}$ has the same expression as $x_{A0}^{eP'U'}$ in (B3.32), that is, $x_1^{eP'U'} = x_{A0}^{eP'U'}$. Based on these observations, we illustrate the consumers' responses to the request for tracking in Figure B3.9.

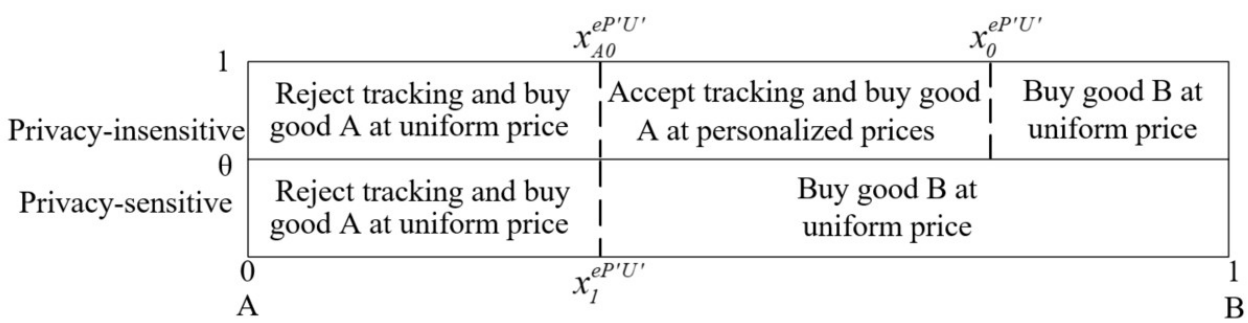

Figure B3.9: Consumer Decisions when the Expected Uniform Price is between 0 and $t + p_B^{eP'U'}$

Privacy-insensitive consumers in $[x_{A0}^{eP'U'}, x_0^{eP'U'}]$ who accept tracking will receive offers of firm A's personalized prices and firm B's uniform price. However, they cannot receive the offer of firm A's uniform price due to their acceptance of the tracking. Hence, their purchase decisions will depend on $p_A^{P'U'}(x)$ and $p_B^{P'U'}$. On the other hand, the purchase decisions of consumers in $[0, x_{A0}^{eP'U'})$ who reject firm A's tracking and consumers who plan to purchase from firm B based on the expected prices will depend on the uniform prices offered by firm A and firm B, i.e., $p_A^{P'U'}$ and $p_B^{P'U'}$.

To find the uniform prices chosen by the two firms in an equilibrium, we need to consider five different scenarios. In the first scenario, both firms choose the uniform prices that are consistent with the expected prices. In the remaining four scenarios, one of the firms sets a uniform price that is either higher or lower uniform price than the belief about the price. We can declare an equilibrium when we find a situation where neither firm has an incentive to deviate from the expected price.

Scenario (a): $p_A^{P'U'} = p_A^{eP'U'}$ and $p_B^{P'U'} = p_B^{eP'U'}$.

In this scenario, consumers' purchase decisions are as shown in Figure B3.9. Specifically, consumers in the interval $[0, x_{A0}^{eP'U'})$ will purchase from firm A at the uniform price, while privacy-insensitive consumers in the interval $[x_{A0}^{eP'U'}, x_0^{eP'U'}]$ will purchase from firm A at personalized prices. Privacy-insensitive consumers in the interval $(x_0^{eP'U'}, 1]$ and privacy-sensitive consumers in the interval $[x_1^{eP'U'}, 1]$ will purchase from firm B. The profits of firm A and firm B can be expressed as

$$\Pi_{Aa}^{P'U'} = p_A^{eP'U'} x_{A0}^{eP'U'} + (1-\theta)\int_{x_{A0}^{eP'U'}}^{x_0^{eP'U'}} \left(t(1-2x) + p_B^{eP'U'}\right) dx. \quad (B3.34)$$

$$\Pi_{Ba}^{P'U'} = p_B^{eP'U'} \left[(1-\theta)\left(1 - x_0^{eP'U'}\right) + \theta\left(1 - x_1^{eP'U'}\right)\right]. \quad (B3.35)$$

Substituting $x_{A0}^{eP'U'}$, $x_0^{eP'U'}$ and $x_1^{eP'U'}$ into (B3.34) and (B3.35), we obtain

$$\Pi_{Aa}^{P'U'} = p_A^{eP'U'} \left(\frac{2\left(t + p_B^{eP'U'}\right) - (1+\theta) p_A^{eP'U'}}{4t}\right). \quad (B3.36)$$

$$\Pi_{Ba}^{P'U'} = p_B^{eP'U'} \left(\frac{t - p_B^{eP'U'} + \theta p_A^{eP'U'}}{2t}\right). \quad (B3.37)$$

Scenario (b): $p_A^{P'U'} = p_A^{eP'U'}$ and $p_B^{P'U'} < p_B^{eP'U'}$.

In this scenario, firm B sets a uniform price is lower than the expected price, which will cause some consumers who initially choose firm A based on the expected prices to purchase from firm B instead. Let $x_1^{P'U'}$ denote the location of the privacy-sensitive consumers who are indifferent between purchasing from firm A and firm B at their respective uniform prices. By definition, $x_1^{P'U'}$ must satisfy the following condition

$$V - t x_1^{P'U'} - p_A^{eP'U'} = V - t\left(1 - x_1^{P'U'}\right) - p_B^{P'U'}. \quad (B3.38)$$

From (B3.38), we obtain:

$$x_1^{P'U'} = \frac{p_B^{P'U'} - p_A^{eP'U'} + t}{2t}. \quad (B3.39)$$

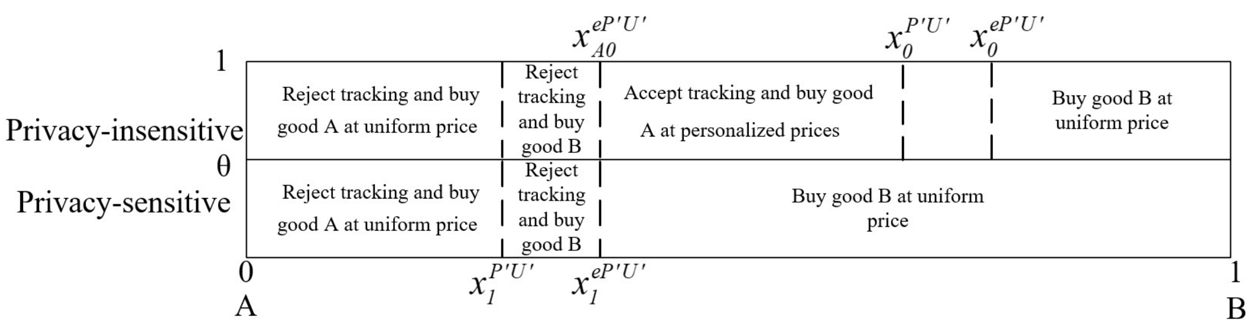

Figure B3.10: Consumer Decisions when the Expected Uniform Price is between 0 and $t + p_B^{eP'U'}$ -- Scenario (b)

Figure B3.10 illustrates the consumers' choices in this scenario. Due to the lower-than-expected uniform price of firm B, consumers in $[x_1^{P'U'}, x_{A0}^{eP'U'})$ who initially choose to reject firm A's tracking will also purchase from firm B. Recall that $x_0^{P'U'}$ denotes the location of privacy-insensitive consumers who are indifferent between purchasing from firm A at the personalized price and purchasing at firm B at its uniform price. Privacy-insensitive consumers in $[x_0^{P'U'}, x_0^{eP'U'}]$ who initially accepted firm A's tracking will still purchase good A at personalized prices despite the lower-than-expected uniform price of firm B because firm A responds by cutting its price to $p_A^{P'U'}(x) = 0$. Based on these observations, we express the profits of firm A and firm B as

$$\Pi_{Ab}^{P'U'} = p_A^{eP'U'} x_1^{P'U'} + (1-\theta)[\int_{x_{A0}^{eP'U'}}^{x_0^{P'U'}} \left(t(1-2x) + p_B^{P'U'}\right) dx + \int_{x_0^{P'U'}}^{x_0^{eP'U'}} 0\, dx]. \quad (B3.40)$$

$$\Pi_{Bb}^{P'U'} = p_B^{P'U'} [(1-\theta)\left(1 - x_0^{eP'U'}\right) + \theta\left(1 - x_1^{P'U'}\right) + (1-\theta)(x_{A0}^{eP'U'} - x_1^{P'U'})]. \quad (B3.41)$$

Substituting $x_{A0}^{eP'U'}$, $x_0^{eP'U'}$ and $x_1^{P'U'}$ into (B3.41), we have

$$\Pi_{Bb}^{P'U'} = p_B^{P'U'} (\frac{\theta p_A^{eP'U'} - p_B^{P'U'} + t}{2t}). \quad (B3.42)$$

Differentiating (R3.42) with respect to $p_B^{P'U'}$ and solving the resulting first-order condition, we obtain the uniform price of firm B:

$$p_B^{P'U'} = \frac{\theta p_A^{eP'U'} + t}{2}. \quad (B3.43)$$

Substituting (B3.43) into (B3.42), we find firm B's profit in this scenario:

$$\Pi_{Bb}^{P'U'} = \frac{(\theta p_A^{eP'U'} + t)^2}{8t}. \quad (B3.44)$$

By comparing firm B's profit in scenarios (a) and (b), i.e., (B3.37) and (B3.44), we find that firm B has no incentive to lower its uniform price if $p_B^{eP'U'} \leq \frac{\theta p_A^{eP'U'} + t}{2}$.

Scenario (c): $p_A^{P'U'} = p_A^{eP'U'}$ and $p_B^{P'U'} > p_B^{eP'U'}$.

In this scenario, firm B's actual uniform price is higher than the expected uniform price, which will cause consumers who originally chose firm B based on their expectations to actually purchase from firm A. Figure B3.11 illustrates the consumers' choices in this scenario. Here $x_1^{P'U'}$ has the same meaning and expression as in scenario (b). Due to firm B's higher-than expected uniform price, privacy-insensitive consumers in $(x_0^{eP'U'}, x_0^{P'U'}]$ who initially choose firm B will purchase from firm A at personalized prices, and privacy-sensitive consumers in $[x_1^{eP'U'}, x_1^{P'U'}]$ will turn to purchase from firm A at the uniform price.

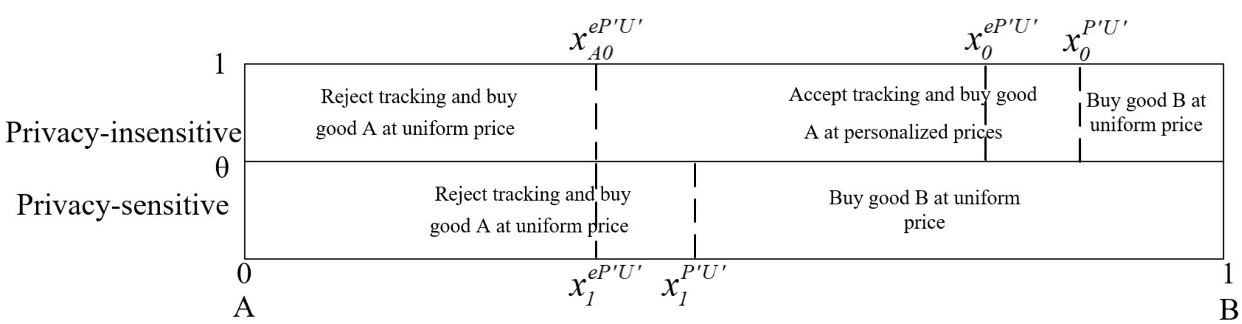

Figure B3.11: Consumer Decisions when the Expected Uniform Price is between 0 and $t + p_B^{eP'U'}$ -- Scenario (c)

Based on the above observations, the profits of firm A and firm B can be expressed as

$$\Pi_{Ac}^{P'U'} = p_A^{eP'U'}[(1-\theta)x_{A0}^{eP'U'} + \theta x_1^{P'U'}] + (1-\theta)\int_{x_{A0}^{eP'U'}}^{x_0^{P'U'}}\left(t(1-2x) + p_B^{P'U'}\right)dx. \quad (B3.45)$$

$$\Pi_{Bc}^{P'U'} = p_B^{P'U'}\ [(1-\theta)\left(1-x_0^{P'U'}\right) + \theta\left(1-x_1^{P'U'}\right)]. \quad (B3.46)$$

Substituting $x_0^{P'U'}$ and $x_1^{P'U'}$ into (B3.46), we find

$$\Pi_{Bc}^{P'U'} = p_B^{P'U'}\left(\frac{\theta p_A^{eP'U'} - p_B^{P'U'} + t}{2t}\right). \quad (B3.47)$$

Differentiating (B3.47) with respect to $p_B^{P'U'}$ and solving the resulting first-order condition, we obtain the uniform price of firm B

$$p_B^{P'U'} = \frac{\theta p_A^{eP'U'} + t}{2}. \quad (B3.48)$$

Using (B3.48) to rewrite (B3.47), we find firm B's profit in this scenario:

$$\Pi_{Bc}^{P'U'} = \frac{(\theta p_A^{eP'U'} + t)^2}{8t}. \quad (B3.49)$$

By comparing firm B's profits in scenarios (a) and (c), i.e., (B3.37) and (B3.49), we find that firm B has no incentive to increase its uniform price if $\frac{\theta p_A^{eP'U'}+t}{2} \leq p_B^{eP'U'} < t$.

Combining the findings from the analysis of scenarios (b) and (c), we conclude that firm B's has no incentive to deviate from its expected uniform price if

$$p_B^{eP'U'} = \frac{\theta p_A^{eP'U'} + t}{2}. \quad (B3.50)$$

Next, in scenarios (d) and (e), we examine firm A's incentive to deviate.

Scenario (d): $p_A^{P'U'} < p_A^{eP'U'}$ and $p_B^{P'U'} = p_B^{eP'U'}$.

In this scenario, firm A's actual uniform price is lower than the expected uniform price, which will cause some consumers who originally choose firm B based on the expected price to actually purchase from firm A. Figure B3.12 illustrates the consumers' choices in this scenario. Note that in this scenario, $x_1^{P'U'} = \frac{p_B^{eP'U'} - p_A^{P'U'} + t}{2t}$, which is strictly smaller than $x_0^{eP'U'}$ due to the non-negativity of $p_A^{P'U'}$, $p_A^{eP'U'}$ and $p_B^{P'U'}$.

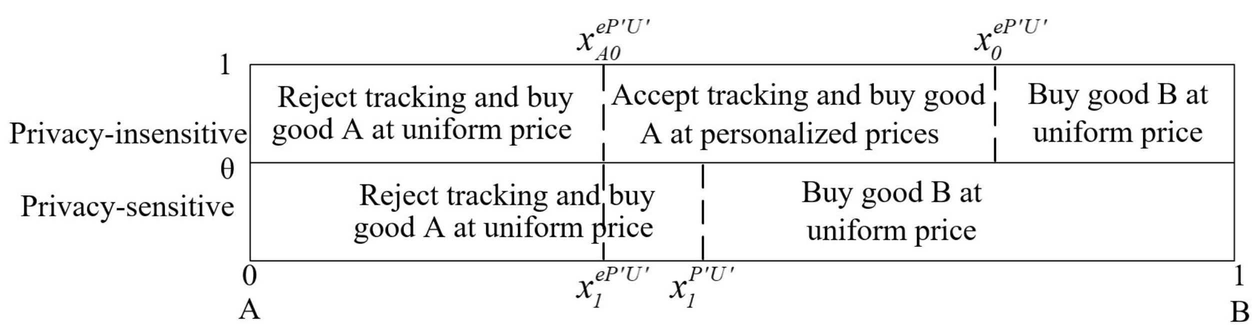

Figure B3.12: Consumer Decisions when the Expected Uniform Price is between 0 and $t + p_B^{eP'U'}$ -- Scenario (d)

The profits of firm A and firm B can be expressed as

$$\Pi_{Ad}^{P'U'} = p_A^{P'U'}[(1-\theta)x_{A0}^{eP'U'} + \theta x_1^{P'U'}] + (1-\theta)\int_{x_{A0}^{eP'U'}}^{x_0^{eP'U'}} \left(t(1-2x) + p_B^{eP'U'}\right) dx. \quad (B3.51)$$

$$\Pi_{Bd}^{P'U'} = p_B^{P'U'}\ [(1-\theta)\left(1 - x_0^{eP'U'}\right) + \theta\left(1 - x_1^{P'U'}\right)]. \quad (B3.52)$$

Substituting $x_{A0}^{eP'U'}$, $x_0^{eP'U'}$ and $x_1^{P'U'}$ into (B3.52), we find

$$\Pi_{Ad}^{P'U'} = \frac{(1-\theta)\left(p_A^{eP'U'} - 2p_A^{P'U'}\right)p_A^{eP'U'} + 2p_A^{P'U'}\left(p_B^{eP'U'} + t - \theta p_A^{P'U'}\right)}{4t}. \quad (B3.53)$$

Differentiating (B3.53) with respect to $p_A^{P'U'}$ and solving the resulting first-order condition, we obtain the uniform price of firm A in this scenario,

$$p_A^{P'U'} = \frac{p_B^{eP'U'} + t - (1-\theta)p_A^{eP'U'}}{2\theta}. \quad (B3.54)$$

Thus, firm A's profit is

$$\Pi_{Ad}^{P'U'} = \frac{\left(p_B^{eP'U'} + t\right)^2 - 2(1-\theta)\left(p_B^{eP'U'} + t\right)p_A^{eP'U'} + p_A^{eP'U'^2}(1-\theta^2)}{8t\theta}. \quad (B3.55)$$

By comparing firm A's profits in scenarios (a) and (d), i.e., (B3.36) and (B3.55), we find that firm A has no incentive to reduce its uniform price if $p_A^{eP'U'} \leq \frac{p_B^{eP'U'}+t}{1+\theta}$, firm A has no incentive to decrease its uniform price.

Scenario (e): $p_A^{P'U'} > p_A^{eP'U'}$ and $p_B^{P'U'} = p_B^{eP'U'}$.

In this scenario, firm A's actual uniform price is higher than the expected uniform price, which will cause some consumers who originally choose firm A based on the expected price to actually purchase from firm B. Figure B3.13 illustrates the consumers' choices in this scenario. Note that $x_1^{P'U'} = \frac{p_B^{eP'U'} - p_A^{P'U'} + t}{2t}$ in this scenario.

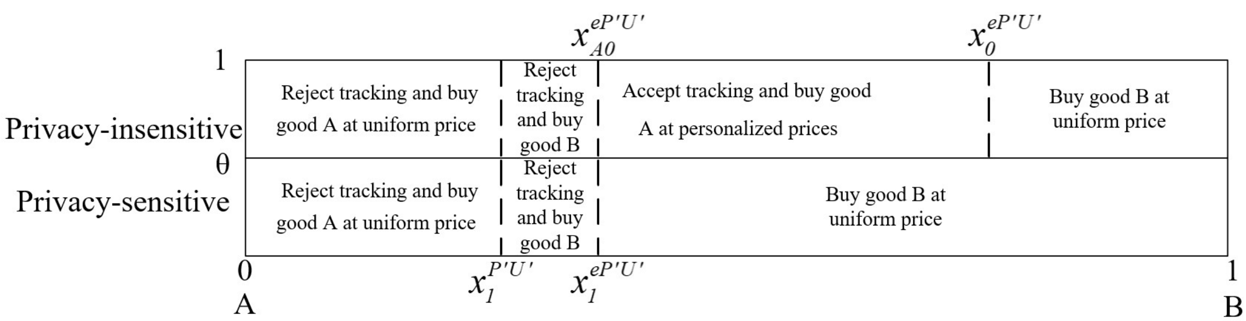

Figure B3.13: Consumer Decisions when the Expected Uniform Price is between 0 and $t + p_B^{eP'U'}$ -- Scenario (e)

The profits of firm A and firm B can be expressed as

$$\Pi_{Ae}^{P'U'} = p_A^{P'U'}x_1^{P'U'} + (1-\theta)\int_{x_{A0}^{eP'U'}}^{x_0^{eP'U'}} \left(t(1-2x) + p_B^{eP'U'}\right) dx, \quad (B3.56)$$

$$\Pi_{Be}^{P'U'} = p_B^{P'U'}\ [(1-\theta)\left(1 - x_0^{eP'U'}\right) + \theta\left(1 - x_1^{P'U'}\right) + (1-\theta)(x_{A0}^{eP'U'} - x_1^{P'U'})]. \quad (B3.57)$$

Substituting $x_{A0}^{eP'U'}$, $x_0^{eP'U'}$ and $x_1^{P'U'}$ into (B3.56), we have

$$\Pi_{Ae}^{P'U'} = \frac{(1-\theta)p_A^{eP'U'^2} + 2p_A^{P'U'}(p_B^{eP'U'} + t - p_A^{P'U'})}{4t}. \quad (B3.58)$$

Differentiating (B3.58) with respect to $p_A^{P'U'}$ and solving the resulting first-order condition, we find

$$p_A^{P'U'} = \frac{p_B^{eP'U'} + t}{2}. \quad (B3.59)$$

Substituting (B3.59) into (B3.58), we obtain firm A's profit in this scenario:

$$\Pi_{Ae}^{P'U'} = \frac{(p_B^{eP'U'} + t)^2 + 2(1-\theta)p_A^{eP'U'^2}}{8t}. \quad (B3.60)$$

By comparing firm A's profits in scenarios (a) and (e), i.e., which in (B3.36) and (B3.60), we find that firm A has no incentive to increase its uniform price if $p_A^{eP'U'} \geq \frac{p_B^{eP'U'}+t}{2}$.

Combining the findings from scenarios (d) and (e), we conclude that firm A has no incentive to deviate from its expected price if

$$\frac{p_B^{eP'U'} + t}{2} \leq p_A^{eP'U'} \leq \frac{p_B^{eP'U'} + t}{1+\theta}. \quad (B3.61)$$

Substituting (B3.50) into (B3.61), we obtain

$$\frac{3t}{4-\theta} \leq p_A^{eP'U'} \leq \frac{3t}{2+\theta}. \quad (B3.62)$$

Therefore, for $p_A^{eP'U'} \in (0, t + p_B^{eP'U'})$, we have found a continuum of PBEs characterized by

$$p_A^{P'U'} \in \left[\frac{3t}{4-\theta}, \frac{3t}{2+\theta}\right] \text{ and } p_B^{P'U'} = \frac{\theta p_A^{eP'U'} + t}{2}. \quad (B3.63)$$

Substituting (B3.50) into (B3.36) and (B3/37), we rewrite the firms' profits as

$$\Pi_{Aa}^{P'U'} = \frac{p_A^{eP'U'}(3t - p_A^{eP'U'})}{4t}, \quad (B3.64)$$

$$\Pi_{Ba}^{P'U'} = \frac{(\theta p_A^{eP'U'} + t)^2}{8t}. \quad (B3.65)$$

From (B3.64) and (B3.65), we find

$$\frac{\partial \Pi_{Aa}^{P'U'}}{\partial p_A^{eP'U'}} = \frac{3t - 2p_A^{eP'U'}}{4t} > 0 \quad (B3.66)$$

for $p_A^{eP'U'}$ in the range defined by (B3.62), and

$$\frac{\partial \Pi_{Ba}^{P'U'}}{\partial p_A^{P'U'}} = \frac{\theta p_A^{eP'U'} + t}{4t} > 0. \quad (B3.67)$$

This implies that the equilibrium profits of firm A and firm B are the highest in this scenario if $p_A^{eP'U'}$ is at the upper end of the range in (B3.62), i.e., if $p_A^{eP'U'} = \frac{3t}{2+\theta}$.

Our analysis of case (2), where $p_A^{eP'U'} \geq t + p_B^{eP'U'}$, shows that these beliefs cannot be supported as a part of an equilibrium. We find that for any price belief that satisfies $p_A^{eP'U'} \geq t + p_B^{eP'U'}$, a firm will find it profitable to choose a uniform price that is either higher or lower than $p_i^{eP'P'}$. Consequently, we rule out the possibility of any equilibrium in case (2).

We conclude from the above analysis that the PBEs of the (P', U') subgame is characterized by uniform prices presented in (B3.63). Recall from (A55) that in the baseline model, the equilibrium of the

(P', U') subgame is characterized by uniform prices $p_A^{P'U'} = \frac{3t}{2+\theta}$ and $p_B^{P'U'} = \frac{t(1+2\theta)}{2+\theta}$, which is at the upper end of the interval defined in (B3.63). Hence, one of the PBEs in the revised model leads to the same equilibrium outcome as the equilibrium in the baseline. Moreover, B3.66) and (B3.67) implies that this PBE yields the highest profit for both firms.

QED

Proposition B9: In the duopoly model with the alternative timing of price revelation, there are multiple equilibria. One of these equilibria leads to the same equilibrium outcome as in the baseline model. Moreover, this equilibrium outcome generates the highest profit for both firms in every subgame where multiple equilibria exist.

Proof of Proposition B9

As we have established in Propositions B7 and B8, there are multiple equilibria in the subgame (P', P') and the subgames (P', U') and (U', P'). This leads to a proliferation of PBEs in the whole game. To prove that one of these PBEs leads to the same equilibrium outcome as in the baseline model, consider the case where the equilibrium in the (P', P') subgame is characterized by the uniform prices $p_i^{P'P'} = t \;\; (i = A, B)$, the equilibrium in the (P', U') subgame is characterized by the uniform prices $p_A^{P'U'} = 3t/(2+\theta)$ and $p_B^{P'U'} = t(1+2\theta)/(2+\theta)$, and the equilibrium in the (U', P') subgame is characterized by the uniform prices symmetric to those in the (P', U') subgame. Using the results in the proof of Propositions B7 and B8, we present the firms' equilibrium profits in these subgames in Table B3.1.

Table B3.1: Firms' Profits with the Privacy Regulation – A Special Case

A \ B	U'	P'
U'	$(\frac{t}{2}, \frac{t}{2})$	$(\frac{t(1+2\theta)^2}{2(2+\theta)^2}, \frac{9t(1+\theta)}{4(2+\theta)^2})$
P'	$(\frac{9t(1+\theta)}{4(2+\theta)^2}, \frac{t(1+2\theta)^2}{2(2+\theta)^2})$	$(\frac{t}{4} + \frac{\theta D^2}{4t}, \frac{t}{4} + \frac{\theta D^2}{4t})$

Note that the profit pairs in Table B3.1 are identical to those in Table 2 in the baseline model. Therefore, the equilibrium outcomes derived from Table B3.1 for various values of θ are identical to those from the baseline model. Moreover, we have shown in the proofs of Propositions B7 and B8 that this equilibrium outcome generates the highest profit for both firms in the subgames (P', P'), (P', U') and (U', P').

QED