

1. Mastering Math - High School Math Review

This math review is intended to prepare students for the first-year math used in the core engineering courses. This material is accompanied by a corresponding review video posted on the [First Year Engineering YouTube channel](#). This worksheet will cover a review of rate of change, an engineering application, and a practice problem set. This review was developed by John O’Keefe along with the Elsie MacGill Learning Centre.

1.1 Rate of Change

To determine the average rate of change between two points x_1 and x_2 on a function $f(x)$ a secant line can be drawn between the two points as shown below:

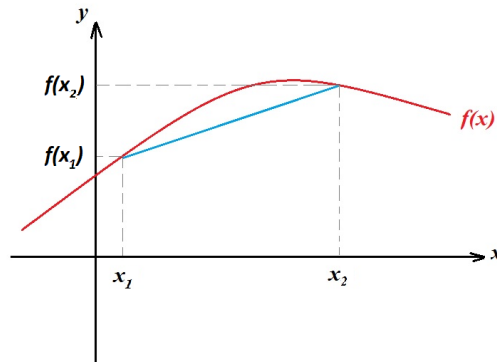


Figure 1.1: Average rate of change with a secant line [1]

The rate of change (RoC) can be calculated as the slope of the line:

$$RoC = \frac{\text{rise}}{\text{run}} = \frac{f(x_2) - f(x_1)}{x_2 - x_1} \quad (1.1)$$

To determine the instantaneous rate of change the distance between the two points is decreased until $\Delta x \approx 0$. We will consider a single point, x , and a second point which is infinitely close,

separated by a distance, h . The line that forms between these two points is tangent to the curve as shown with the blue line in Fig. 1.2.

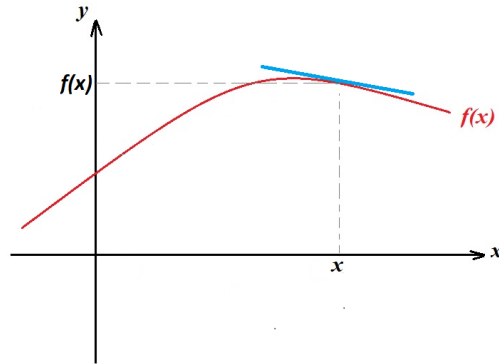


Figure 1.2: Instantaneous rate of change with a tangent line [1]

The slope of the tangent line will produce the instantaneous rate of change as defined by:

$$RoC = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \quad (1.2)$$

This approach does not require a scalar value of x , and can be evaluated over the whole function $f(x)$. This is known as the derivative of $f(x)$ by first principles.

1.1.1 Examples

1. Calculate the average rate of change of $f(x) = x^2 + 5x$ between $x = 2$ and $x = 5$.

Solution: The slope of the secant line can be calculated according to Eq. 1.1:

$$\begin{aligned} RoC &= \frac{f(5) - f(2)}{5 - 2} \\ RoC &= \frac{(5^2 + 5(5)) - (2^2 + 5(2))}{5 - 2} \\ RoC &= 12 \end{aligned}$$

2. Calculate the instantaneous rate of change of $f(x) = x^2 + 5x$ at $x = 5$.

Solution: The slope of the tangent line can be calculated according to Eq. 1.2 and simplifying:

$$\begin{aligned}
 RoC &= \lim_{h \rightarrow 0} \frac{f(5+h) - f(5)}{h} \\
 RoC &= \lim_{h \rightarrow 0} \frac{(5+h)^2 + 5(5+h) - (5^2 + 5(5))}{h} \\
 RoC &= \lim_{h \rightarrow 0} \frac{(h^2 + 10h + 25 + 25 + 5h) - 50}{h} \\
 RoC &= \lim_{h \rightarrow 0} \frac{h^2 + 15h}{h} \\
 RoC &= \lim_{h \rightarrow 0} h + 15
 \end{aligned}$$

Finally evaluate the limit:

$$RoC = 15$$

3. Calculate the derivative by first principles of $f(x) = 3x^2 + 2x + 9$.

Solution: The derivative by first principles can be calculated according to Eq. 1.2 and simplifying.

$$\begin{aligned}
 RoC &= \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \\
 RoC &= \lim_{h \rightarrow 0} \frac{(3(x+h)^2 + 2(x+h) + 9) - (3x^2 + 2x + 9)}{h} \\
 RoC &= \lim_{h \rightarrow 0} \frac{(3x^2 + 6xh + 3h^2 + 2x + 2h + 9) - (3x^2 + 2x + 9)}{h} \\
 RoC &= \lim_{h \rightarrow 0} \frac{6xh + 2h + 3h^2}{h} \\
 RoC &= \lim_{h \rightarrow 0} 6x + 2 + 3h
 \end{aligned}$$

Finally evaluate the limit:

$$RoC = 6x + 2$$

1.1.2 Engineering Examples

Instantaneous rate of change is applied in Data Management (ECOR 1042) through the finite difference method. Computers are not able to calculate the analytical derivative efficiently but it can approximate the derivative at a point effectively. The rate of change at a point is approximated through the finite difference method:

$$RoC = \frac{f(x + \Delta x) - f(x)}{\Delta x} \quad (1.3)$$

Where Δx is selected as a small number to obtain accurate results. This can vary depending on the application.

1. Calculate the rate of change of $f(x) = x^2$ at $x = 5$.

Solution: Applying the finite difference method with a $\Delta x = 0.01$.

$$\begin{aligned} RoC &= \frac{f(5 + 0.01) - f(5)}{0.01} \\ RoC &= \frac{5.01^2 - 5^2}{0.01} \\ RoC &= 10.01 \end{aligned}$$

This provides a relatively close result to the actual answer of 10. To obtain a more accurate solution a smaller Δx can be applied. The table below summarizes how decreasing the Δx converges the value to 10.

Δx	Result
0.1	10.1
0.01	10.01
0.001	10.001
0.0001	10.0001

2. Calculate the rate of change of $f(x) = \ln(x^3 + \sin(x))$ at $x = 1$.

Solution: Applying the finite difference method with a $\Delta x = 0.01$.

$$\begin{aligned} RoC &= \frac{f(1 + 0.01) - f(1)}{0.01} \\ RoC &= \frac{\ln((1.01)^3 + \sin(1.01)) - \ln(1^3 + \sin(1))}{0.01} \\ RoC &= 1.92 \end{aligned}$$

When calculating the rate of change make sure to use radians when evaluating trigonometric expressions.

1.1.3 Practice Problems

1. Calculate the average rate of change of $6x^2 + 9$ between $x = 1$ and $x = 3$.
2. Calculate the average rate of change of $\ln(3x)$ between $x = 2$ and $x = 5$.
3. Calculate the instantaneous rate of change of $-6x + 12$ at $x = 3$.
4. Calculate the instantaneous rate of change of $5x^2 - 6x$ at $x = 1$.
5. Calculate the derivative by first principles of $-8x^2 + 3x - 4$.
6. Calculate the instantaneous rate of change of $e^{2x}\sin(x)$ using finite different method at $x = 3$ using a $\Delta x = 0.01$.
7. Calculate the instantaneous rate of change of $\sin(e^{6x})$ using finite different method at $x = 2$ using a $\Delta x = 0.01$.

1.1.4 Answers

1. 24

2. 0.31

3. -6

4. 4

5. $-16x + 3$

6. 49.8

7. -101

Bibliography

- [1] Socratic. What does average rate of change mean? <https://socratic.org>, Jul 2015.
Accessed on 2020-06-12.