1. Mastering Math - High School Math Review

This math review is intended to prepare students for the first-year math used in the core engineering courses. This material is accompanied by a corresponding review video posted on the First Year Engineering YouTube channel. This worksheet will cover a review of dot product, an engineering application, and a practice problem set. This review was developed by John O'Keefe along with the Elsie MacGill Learning Centre.

1.1 Dot Product

The dot product takes two vectors and produces a scalar quantity. The dot product determines the projected value of one vector on to a second vector. The figure below shows the projection of vector $\vec{A}$ on vector $\vec{B}$ with the red dotted line.

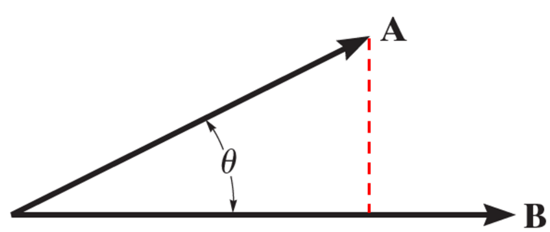

Figure 1.1: Projection of vector A on B [2]

The dot product can be evaluated in 2-D as well as 3-D. If two 2-D vectors are considered: $\vec{A} = A_x\hat{i} + A_y\hat{j}$ and $\vec{B} = B_x\hat{i} + B_y\hat{j}$. The 2-D dot product can be evaluated as follows:

$$\vec{A} \cdot \vec{B} = A_xB_x + A_yB_y \tag{1.1}$$

Where $\vec{A} \cdot \vec{B}$ is the dot product operator between vector $\vec{A}$ and $\vec{B}$. Similarly if two 3-D vectors are considered: $\vec{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}$ and $\vec{B} = B_x\hat{i} + B_y\hat{j} + B_z\hat{k}$. The 3-D dot product can be calculated as follows:

$$\vec{A} \cdot \vec{B} = A_x B_x + A_y B_y + A_z B_z \tag{1.2}$$

The order of operator does not matter such that $\vec{A} \cdot \vec{B} = \vec{B} \cdot \vec{A}$. The dot product is a function of the vector magnitudes and the angle between the two vectors as follows:

$$\vec{A} \cdot \vec{B} = |\vec{A}||\vec{B}|cos(\theta) \tag{1.3}$$

Where $|\vec{A}|$ and $|\vec{B}|$ represents the magnitude of each vector and θ is the angle between the two vectors. This relation holds for 2-D and 3-D problems. This shows that the dot product is a maximum when $\theta = 0°$ as $\cos(\theta) = 1$. The dot product is equivalent to zero when the two vectors are perpendicular because at $\theta = 90°$, $\cos(\theta) = 0$.

1.1.1 Examples

1. What is the dot product between $\vec{A} = 2\hat{i} + 3\hat{j} - \hat{k}$ and $\vec{B} = 4\hat{i} - 2\hat{j} - 5\hat{k}$.

Solution: We apply Eq. 1.2 for a 3-D vector.

$$\vec{A} \cdot \vec{B} = (2)(3) + (3)(-2) + (-1)(-5)$$
$$\vec{A} \cdot \vec{B} = 5$$

2. What is the projection of $\vec{C} = 5\hat{i} + \hat{j} - 3\hat{k}$ on $\vec{F} = -2\hat{i} - 4\hat{j} - \hat{k}$.

Solution: We apply Eq. 1.2 for the projection of a 3-D vector.

$$\vec{C} \cdot \vec{F} = (5)(-2) + (1)(-4) + (-3)(-1)$$
$$\vec{C} \cdot \vec{F} = -11$$

3. What is the dot product between $|\vec{F}| = 500$ and $|\vec{D}| = 5$ where $\theta = 30°$.

Solution: We apply Eq. 1.3 given the magnitudes and angle.

$$\vec{F} \cdot \vec{D} = (500)(5)\cos(30)$$
$$\vec{F} \cdot \vec{D} = 2165$$

4. What is the angle between $\vec{A} = 2\hat{\imath} + \hat{\jmath} + 4\hat{k}$ and $\vec{B} = 3\hat{\imath} - \hat{\jmath} + \hat{k}$.

Solution: We start by applying Eq. 1.2 to determine the dot product of the two vectors.

$$\vec{A} \cdot \vec{B} = (2)(3) + (1)(-1) + (4)(1)$$
$$\vec{A} \cdot \vec{B} = 9$$

Next we calculate the magnitudes of each vector.

$$|\vec{A}| = \sqrt{2^2 + 1^2 + 4^2}$$
$$|\vec{A}| = 4.58$$
$$|\vec{B}| = \sqrt{3^2 + (-1)^2 + 4^2}$$
$$|\vec{B}| = 5.10$$

Finally we apply Eq. 1.3 to determine the angle between the two vectors.

$$9 = (4.58)(5.10)\cos(\theta)$$
$$\theta = 67.3°$$

1.1.2 Engineering Examples

The dot product is used in Statics (ECOR 1045) for determining the projection of a force or in Dynamics (ECOR 1048) to evaluate the work done by a force.

1. Determine the projection of the 800 lb force on the 500 lb force.

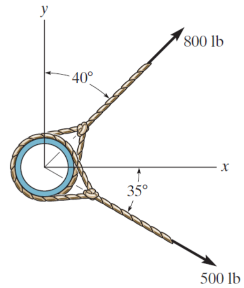

Figure 1.2: Engineering example 1 [2]

Solution: We are given the magnitudes of the two vectors and the angle between them is $\theta = 85°$. We can then apply Eq. 1.3 to determine the value of the projection:

$$\vec{A} \cdot \vec{B} = (800)(500)\cos(85)$$
$$\vec{A} \cdot \vec{B} = 34,862\ lb$$

2. Determine the projection of the 800 N force on the X axis.

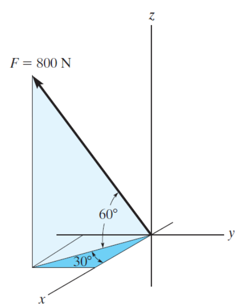

Figure 1.3: Engineering example 2 [2]

Solution: From the 3-D vectors section the force can be written in vector notation by:

$$\vec{F} = 800(\cos(60°)\cos(30°)\hat{\imath} - \cos(60°)\sin(30°)\hat{\jmath} + \sin(60°)\hat{k})$$
$$\vec{F} = 346\hat{\imath} - 200\hat{\jmath} + 692\hat{k}$$

The vector that defines the X axis is $\vec{X} = \hat{\imath}$, where it is directed along the X axis with a magnitude of 1. Eq. 1.2 is then used to evaluate the projection:

$$\vec{F}\cdot\vec{X} = (346)(1) + (-200)(0) + (692)(0)$$
$$\vec{F}\cdot\vec{X} = 346N$$

3. If the block is pulled forward 15 m using a force of 300 N, determine the work done by the force ($W = \vec{F}\cdot\vec{d}$).

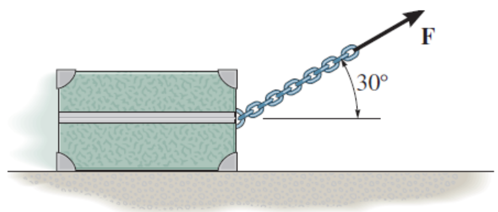

Figure 1.4: Engineering example 3 [1]

Solution: The magnitudes of the force ($|\vec{F}| = 300\ N$) and the distance ($|\vec{d}| = 15\ m$) have been provided along with the angle between the vectors ($\theta = 30°$). The dot product can be evaluated using Eq. 1.3 as follows:

$$W = \vec{F}\cdot\vec{d} = (300)(15)\cos(30°)$$
$$W = 3897\ Nm$$

1.1.3 Practice Problems

1. Calculate the dot product of $\vec{A} = 2\hat{\imath} + 2\hat{\jmath} + \hat{k}$ and $\vec{B} = 2\hat{\imath} - 1\hat{\jmath} + 1\hat{k}$.

2. Calculate the dot product of $\vec{C} = -3\hat{\imath} + \hat{\jmath} - 10\hat{k}$ and $\vec{D} = 2\hat{\imath} - 6\hat{\jmath} - 2\hat{k}$.

3. Calculate the dot product of $|\vec{F}| = 200$, $|\vec{d}| = 5$ and the angle between the two vectors in 15°.

4. Calculate the angle between $\vec{V} = 2\hat{\imath} + 3\hat{\jmath} + 6\hat{k}$ and $\vec{W} = 7\hat{\imath} + 2\hat{\jmath} + \hat{k}$.

5. Calculate the angle between the vectors $\vec{AO}$ and $\vec{BO}$ below.

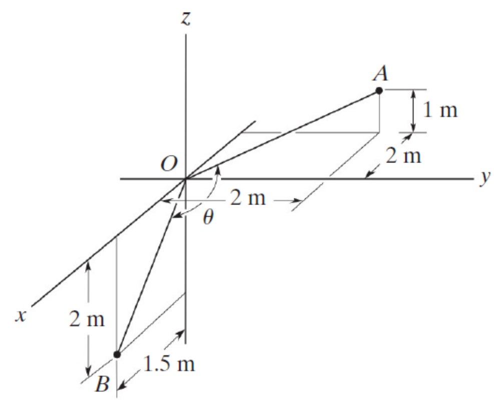

Figure 1.5: Problem 5 [2]

6. Calculate the projection of the force ,$\vec{F}_2$, on the force, $\vec{F}_1$, if $\vec{F}_2 = -20\hat{\imath} + 60\hat{\jmath} - 3\hat{k}\ N$ and $\vec{F}_1 = -22\hat{\imath} + 33\hat{\jmath} + 5\hat{k}\ N$.

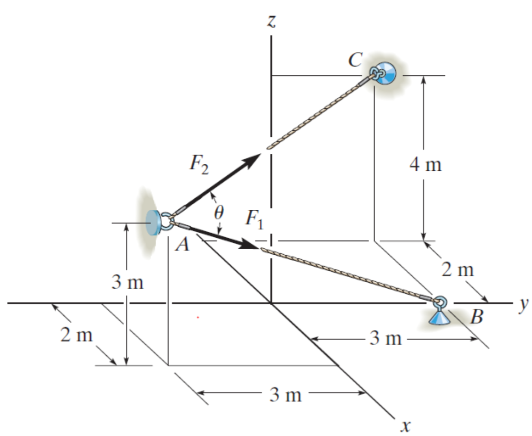

Figure 1.6: Problem 6 [2]

1.1.4 Answers

1. 4

2. 8

3. 966

4. $60°$

5. $132°$

6. 2405 N

Bibliography

[1] R C Hibbeler. *Engineering Mechanics Dynamics 14th Edition*. Pearson, 2016.

[2] R C Hibbeler. *Engineering Mechanics Statics 14th Edition*. Pearson, 2016.