# The Hybrid Model of Arithmetic Problem Solution:

# The Whole is more than the Sum of its Parts[1]

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## Abstract

Arithmetic ability is a fundamental skill within our society. In the current research proposal I address the phenomenon of the solution of basic arithmetic problems in adults. First, I review current models of arithmetic problem solution and critically evaluate them based on empirical research. Then, I introduce a new model – the Hybrid model of arithmetic problem solution. Finally, I outline the first step and future directions of a research plan designed to evaluate the Hybrid model. Implications of this research to the area of arithmetic cognition, and to cognitive science, are discussed.

The Hybrid Model of Arithmetic Problem Solution:

The Whole is more than the Sum of its Parts

Solve 5 + 6. How did you do it? What representations did you make use of? What solution algorithm did you use? Skill in basic arithmetic is very important in our daily lives. We use arithmetic to figure out if we can afford to buy the drink and the cookie, to decide our portion of the bill, and in many other daily activities. How people solve arithmetic problems is an interesting question for cognitive science because our ability (or inability) in arithmetic points to the limitations of the human mind (Anderson, 1998). Unlike visual processing tasks, which have been elusive to artificial intelligence, arithmetic is difficult for us but easy for computers. Thus, research in how people solve arithmetic problems has the potential to illuminate how the mind works and the nature of cognition. In the current proposal I critically evaluate current models, on the basis of empirical findings. I then propose a new model – the Hybrid model - that is consistent with empirical findings. Finally, I outline the first step in a research plan designed to evaluate the Hybrid model.

Many models have been proposed for arithmetic problem solution. The most notable models of adult performance are MATHNET (McCloskey & Lindemann, 1992), the Triple-code model (Dehaene, 1992; Dehaene & Cohen, 1995), and the Network interference model (Campbell, 1995). MATHNET and the Triple-code model are similar at the level of problem solution, though both model a broader range of arithmetic tasks than simply problem solution. Within these models the problem is encoded from the presentation format into a single mental representation format, which is used to calculate the answer. The two models differ in terms of what the representational format is. MATHNET posits that abstract-semantic (i.e., magnitude) representations are used to access and store problems. In contrast, the Triple-code model posits that phonological representations are used. The only calculation mechanism used for single-digit problem solution within the two models is direct retrieval of the answer from memory. Once retrieved, the answer is then converted from the mental representation to the appropriate output format.

There are two serious problems with MATHNET and the Triple-code model. First, the only solution method within the two models is direct memory retrieval. This is a serious shortcoming because a wealth of evidence shows that adults use a variety of *procedures* (non-retrieval solution methods) including: transformation (e.g., 5 + 6 = [5 + 5 = 10] + 1 = 11), where a known problem is used to aid solution of the given problem, and counting-based strategies (e.g., 5 + 6 = 6, 7, 8, 9, 10, 11), which include counting by ones, twos, and so on (Campbell & Timm, 2000; Campbell & Xue, 2001; Geary, Frensch, & Wiley, 1993; Geary & Wiley, 1991, Hecht, 1999; LeFevre, Sadesky, & Bisanz, 1996; LeFevre, Bisanz, et al., 1996). Campbell and Xue (2001) found that procedures accounted for 12 – 36 % of reported solution methods for single-digit addition problems, depending on the size of the operands. These results are in line with those of many other studies. Thus, any complete model of the solution of arithmetic problems in adults must include multiple solution methods.

Second, the stages of encoding and calculation in MATHNET and the Triple-code model are additive, because regardless of presentation format all calculations are performed on the same mental representation. One important implication of this is that presentation format should not influence calculation. Campbell (1994, 1999; Campbell & Clark, 1992) has found that format does influence calculation stages by showing that the problem-size effect, a calculation effect wherein larger problems take longer to solve than smaller problems, is greater for problems presented in word format. Thus, models that maintain that presentation format cannot affect calculation are unable to account for current data.

In the Network interference model (Campbell, 1995) multiple internal codes are posited, including at least magnitude codes, visual codes (i.e., digits), and verbal codes. Magnitude codes comprise the semantic aspect of numbers. In the model, larger magnitudes become increasingly less discriminable. Visual codes capture the visual features of the number, whereas verbal codes capture the sound of the number names. The encoding and calculation stages are interactive in the Network interference model, thus it is able to account for the finding that large problems in word format take longer to solve. The model, however, does not have any non-retrieval solution methods. As a result, it cannot account for the effects of format on solution methods. Thus, the Network interference model is also incapable of accounting for the non-retrieval data.

Indeed, no model of adult problem solution has a role for procedure use. Thus, researchers interested in interpreting their data on solution methods have turned to the Adaptive strategy choice model (ASCM), a model of children's problem solution (Siegler & Shipley, 1995). In ASCM, associations are formed between problems and answers based on experience. These associations may be strong – the problem may be associated with a single answer, or they may be weak – the problem may be associated with multiple answers. Given a problem to solve, a choice is made as to which strategy to use. The decision is based on the speed and accuracy of the available strategies on previous experiences with the specific problem presented (e.g., 5 + 6), problems that share a common feature (e.g., 5 as an operand), and for arithmetic problems in general. A novelty index is included that functions to develop new strategies by boosting the speed and accuracy scores of novel strategies.

In ASCM two strategy-choice pathways exist: one for procedures and one for retrieval. If a procedure is chosen, the procedure is implemented and the resulting answer is produced. If retrieval is chosen, a retrieval attempt is made. The success of the attempt is determined by the strength of the association between the problem and an answer. The stronger the association, the more successful a retrieval attempt will be. Thus, retrieval is often chosen for problems with strong associations because it is efficient. Unlike procedural answers, however, confidence in the retrieved answer must exceed a criterion before it is produced. If confidence in the correctness of the answer does not exceed the criterion, another attempt to retrieve an answer is made. If still unsuccessful at retrieving an answer with sufficient confidence after a number of trials, a procedure is implemented.

The ASCM model is useful because it does incorporate multiple solution methods. Another advantage of the model is the many ways individual differences arise within it. Empirical research has found a vast range of individual differences in arithmetic in terms of skill, solution methods, and confidence. Within ASCM individual differences can be accounted for by considering the strength of problem-answer associations, the relative speed and efficacy of strategies, and the confidence criterion. The model's shortcomings are that it does not make explicit the representations used to store and access arithmetic facts and that it is not equipped to explain the robust effects of presentation format.

The lack of a complete model hurts the field of arithmetic cognition. Although the models described can account for many aspects of arithmetic cognition, no one model is able to account for a full range of data. Models are important as they provide a framework for interpreting results and drive empirical work by facilitating predictions. Therefore, a comprehensive model of arithmetic cognition is crucial because it will allow researchers to integrate and unify the wealth of empirical results pouring in form this relatively new field. Thus, the goal of the proposed research is to develop and empirically test a new model of arithmetic problem solution in adults – the Hybrid model.

*The Hybrid model of adult problem solution*

Instead of throwing the baby out with the bathwater, I propose taking the most successful elements of previous models and combining them to achieve an improved and more complete model of problem solution. The proposed Hybrid model uses ASCM as a starting point because it has roles for both for procedure use and individual differences. The notion of multiple representational forms, visual and verbal, is incorporated from the Network interference model to account for format effects. The Hybrid model differs from ASCM in a number of ways. First, the novelty index is dropped because novelty does not seem to influence the strategy choices of adults and the need to develop new arithmetic solution procedures is not a common feature in adults' performance. Second, the procedures used by adults vary from those used by children, thus the Hybrid model will make use of common adult procedures including transformation as well as counting based procedures. Third, the Hybrid model makes explicit the representations used to store and access arithmetic facts, allowing format to influence the relative speed and accuracy of different strategies and thus, strategy choice. Along with other features of the Hybrid model, these adaptations of ASCM are expected to result in an improved model of arithmetic problem solution that is greater than making use of the parts in isolation, as is currently being done by researchers. For example, Campbell and Timm (2000) use the Network interference model to explain the effects of interference on retrieval of addition problems, but use ASCM to explain the effects on procedure use.

A graphical description of the Hybrid model is given in Figure 1. Given a presented problem to solve, the problem is encoded into a mental representation (a visual or verbal representation, depending on the presentation format, with a representation of the associated magnitude). This mental representation may then, optionally, be transcoded into a preferred representational format, which is expected to vary across individuals and tasks (Noël & Seron, 1993). This option is included based on self-reports, although evidence shows that all individuals do not simply transcode all of the time with no other effect of presentation format (Campbell, 1999). In the Hybrid model, a strategy is chosen based its previous speed and accuracy: (1) overall (i.e., for solving arithmetic problems in general); (2) for a specific problem (e.g., for solving 6 x 5); (3) for a class of problems (e.g., for solving problems with five as an operand, or for solving multiplication problems); and (4) for problems of a specific representational type (i.e., for problems represented visually or verbally). The strategy choices include direct *retrieval* from memory, *transformation* (using another known fact to solve a given problem), and *counting*, as these are the most common strategies reported by adults.

The Hybrid model, like ASCM, has two strategy-choice pathways. If a procedure is chosen, it is implemented and the answer is produced. For transformation, implementation consists of retrieving the answer to a related problem, one that is strongly associated with a single answer. This is generally a tie problem in addition or multiplication (a problem where both operands are the same, e.g. 4 + 4), a fives problem in multiplication (e.g., 5 x 4), or a problem where the operands sum to 10 in addition (e.g., 6 + 4). Then, an addition or subtraction is made to the answer of this problem to arrive at the answer to the presented problem. For 6 + 5, likely transformations include 5 + 5 + 1, 6 + 6 – 1, and 6 + 4 + 1. For counting in addition, the larger operand is selected and incremented by one the number of times of the smaller operand (6 + 5 = 6, 7, 8, 9, 10, 11). In multiplication, counting is done in increments of one of the operands (e.g., 6 x 5 = 5, 10, 15, 20, 25, 30). The operand chosen to increment by is made on the basis of experience. Two and five are common choices, likely because counting by twos and fives is part of the elementary mathematics curriculum.

It should become apparent that the speed and accuracy in implementing a transformation or counting based procedure are affected by the presented problem. For example, 9 + 8 = 9, 10, 11, 12, 13, 14, 15, 16, 17 would take a long time to count because of the number of increments required. Counting also requires keeping track of the number of increments. When solutions require a large number of increments, errors are more likely to be made. Counting is a slow and error prone method for the solution of problems like 9 + 8 and thus, another strategy choice would likely be made. Consider the use of a transformation in the case of 9 + 8. Possibilities include 9 + 8 = [10 + 8 = 18] –1 = 17 or 9 + 8 = [9 + 9 = 18] –1 = 17. Both options can be executed rather quickly, in relation to counting, assuming that the related facts are strongly associated. Also, the transformations require fewer steps and, thus, fewer chances to make an error. Direct retrieval of the answer from memory would, of course, be the fastest option if the problem were strongly associated with an answer. Hence, for a given problem it is easy to see how the features of the problem will logically affect the speed and efficacy of any given strategy.

How does format affect problem solution? Campbell (1994) found that solution times were 30 % slower for problems presented as words than for problems presented as digits. Problem-answer associations are weaker for less familiar formats (including words). Hence, representational format influences the relative speed and efficacy of strategies. The weaker associations would be expected to lead to a greater use of procedures, and slower *retrieval* times, due to an increased number of attempts. Moreover, the effect of format is expected to be greater for larger problems, for which associations are weaker to start with. The problem-size effect has been found to be greater for problems presented as words than for problems presented as digits (Campbell, 1994, 1999; Campbell & Clark, 1992). In the proposed research, I will use the Hybrid model as a framework for interpreting results of adult problem solution and to test predictions based on the model.

The primary goal of this first study is to test the predictions that presentation format will affect problem solution in two ways. First, that word format will increase procedure use compared to digit format. And second, that word format will increase solution times compared to digit format when trials on which participants reported retrieval are considered alone. These effects are predicted to be greater for large problems than small problems. A secondary goal is to explore whether some individuals do just transcode unfamiliar formats into a preferred format. To this end, participants will solve arithmetic problems and their response times will be recorded along with their reported solution methods. Participants will complete a large number of trials in order to allow examination of response time distributions.

The proposed research is unique in that it combines the following elements: word- and digit-format problems, addition and multiplication problems, trial-by-trial self-reports, and large numbers of trials. Previous studies have looked at the effect of format, but examining effects across operations and collecting participants' self-reports will allow me to test the Hybrid model and determine if the findings generalize across operations. Without self-reports I would be unable to test the predictions generated by the Hybrid model, that procedure use increases and that response times for *retrieval* trials increase for problems in word format, because I would not have a measure of procedure use or be able to separate procedure trials from retrieval trials. The large number of trials collected will allow me to make use of distributional analyses of response times, which can be used to confirm the overall pattern of each participants' self-reports, to test more specific hypotheses including examination of the transcoding option, and to and refine the model (Penner-Wilger, Leth-Steensen, & LeFevre, 2002).

## Method

### *Participants*

Sixty Carleton University students will be recruited to participate for course credit or payment. Participants will be required to have normal or corrected-to-normal vision.

### *Apparatus, Stimuli, and Design*

A 2 (operation: addition, multiplication) x 2 (format: digits, words) x 2 (problem size: small, large) repeated measures design will be used. Participants will solve single-digit addition and multiplication problems. Problem operands will appear as Arabic digits or as lower-case English words, displayed horizontally using white characters against a dark background. Small problems will be defined as problems with operands whose product is less than or equal to 25; large problems will be defined as those whose product is greater than 25. The two operands will be separated by the operation sign (+ or x), with three spaces on each side of the operation sign for digit format, and one space on each side for word format. Therefore, digit problems will occupy eight character spaces and the length of word problems will range from 8 to 13 spaces.

Participants will receive two sessions (one addition, one multiplication, with order randomized across participants) of eight blocks of 72 trials with format (i.e., Arabic digits or lower-case English words) alternating across trials. There are 36 possible addition combinations involving the operands 2 through 9 when commuted pairs (e.g., 5 + 8 and 8 + 5) are counted as one problem. Within each block, participants will receive all 36 problems once in word format and once in digit format. Word format will be used for odd-numbered trials and digit format for even-numbered trials. The set of 36 problems includes 8 "ties" (e.g., 5 + 5) and 28 "non-ties". Approximately half of the non-ties will be selected randomly to be tested in the first block with the smaller operand on the left. The operand order of non-tie problems will then be alternated across the blocks for each operation. Problem order in each block will be pseudo-random with the constraint that word and digit versions of the same problem be separated by at least 18 trials.

*Procedure*

Testing will occur in a quiet room with an experimenter present and will require approximately 45 minutes per session for a total of two sessions. The problem solution task will be preceded by a 20-trial naming task that alternates Arabic digits and English numbers words for naming. This will allow participants to find a comfortable viewing distance, accustom them to rapid responding, and permit adjustments to the sensitivity of the voiced-activated relay. For the problem solution task, participants will be asked to respond accurately and quickly, but told that occasional errors are normal. The experimenter will press a key to initiate each block. Prior to the first block of trials the following instructions will appear on the monitor and be read out loud by the experimenter: "After each problem please indicate how you solved the problem by choosing from among the following strategies: Transform, Count, Remember, Other. Say

TRANSFORM if you used knowledge of a related problem. Say COUNT if you used a strategy based on counting. Say REMEMBER if the answer seemed to come to you without any intermediate steps, inferences, or calculations. Choose OTHER if you used some other strategy or are uncertain." For reference during arithmetic trials, participants will also receive a sheet of strategy descriptions as follows:

Transform: You solve the problem by referring to a related problem in the same or another operation. For example, you might solve 17-9=? by remembering that 17 - 10 = 7 so 17 - 9 must equal 8.

Count: You solve the problem by counting a certain number of times to get the answer.

Remember: You solve the problem by just remembering or knowing the answer directly from memory without any intervening steps.

Other: You may solve the problem by a strategy unlisted here, or you may be uncertain how you solved the problem.

Prior to each arithmetic trial, a fixation dot will appear and flash twice over a 1 s interval at the center of the screen. The problem will appear (synchronized with the monitor's raster scan) on what would have been the third flash with the operation sign (+) at the fixation point. Timing will begin when the problem appears and end when the sound-activated relay is triggered. Triggering the relay will cause the problem to disappear immediately. This will allow the experimenter to mark response times spoiled because the microphone failed to detect the onset of the response. Immediately after the response, the prompt "Strategy Choice" will appear at the center of the screen with the words

Transform, Count, Remember, and Other centered immediately below. The four words will always appear in the same order separated by six spaces. The experimenter will record the strategy, reported verbally by the participant, by pressing one of four buttons on the computer keyboard. Once the strategy is recorded, and the experimenter has entered the stated arithmetic answer, the screen will be cleared and display the fixation dot for the next trial. No feedback about speed or accuracy will be provided during the experiment.

## Expected Results

A 2 (operation: addition, multiplication) x 2 (format: digits, words) x 2 (problem size: small, large) repeated measures analysis of variance (ANOVA) will be performed for each of the dependent variables: response times, error rates, and reported solution methods. Also, response time distributions for each treatment combination will be examined using the ex-Gaussian distributional model. Expected patterns of results are outlined.

### *Response times*

Response times are expected to show main effects of all independent variables. Participants are expected to solve multiplication problems more slowly than addition problems, consistent with previous research (Campbell & Xue, 2001). Using the Hybrid model, this finding would be expected because stronger problem-answer associations are formed for addition as a result of greater experience with addition. Thus, problem solution is expected to be slower for multiplication because the weaker associations lead to multiple retrieval attempts and/or greater use of procedures.

Participants are expected to solve problems in word format more slowly than problems in digit format, again consistent with previous research (Campbell, 1999). Using the Hybrid model, this finding would be expected because stronger problem-answer associations are formed for digit format as a result of greater experience with digit format. In comparison, the less familiar word format will lead to weaker associations. Thus, problem solution is expected to be slower for word format because the weaker associations lead to multiple retrieval attempts and/or greater use of procedures.

Participants are expected to solve large problems more slowly than small problems, showing the robust problem-size effect. Using the Hybrid model, this finding would be expected because stronger problem-answer associations are formed for smaller problems as a result of greater experience with small problems. Thus, problem solution is expected to be slower for large problems because the weaker associations lead to multiple retrieval attempts and/or greater use of procedures.

In terms of two-way interactions, as format is the independent variable of most interest in this experiment, predictions will be made for the Operation x Format interaction and the Format x Problem size interaction. Participants are expected to show a greater effect of format in multiplication than in addition because the weaker problem-answer associations in multiplication are expected to be more vulnerable to the unfamiliar word format. A similar outcome is expected for large problems such that large problems are more affected by the unfamiliar word format. Thus, participants should be much slower to solve multiplication problems and large problems in word format due to multiple retrieval attempts and/or greater use of procedures. The expected pattern of results for the Format x Problem size interaction is shown in Figure 2.

The Hybrid model predicts that operation, format, and problem size affect the strength of associations and as a result, solution times. This effect on solution times may occur through increased number of retrieval attempts, a greater use of procedures, or both. To determine where the effects are happening, analysis of procedure usage will be done. As well, the trials on which retrieval was reported will be analyzed separately in a 2 (operation: addition, multiplication) x 2 (format: digits, words) x 2 (problem size: small, large) repeated measures ANOVA. This will allow use to determine if *retrieval* times are indeed slower for multiplication, word format, and large problems as well as combinations such as multiplication problems in word format and large problems in word format. Slower response times would support the hypothesis that more retrieval attempts are needed to arrive at an answer that exceeds the confidence criterion. I predict that increases in retrieval response times will be found for these categories.

*Error rates*

Error rates are expected to be low and thus, will not be discussed further here.

*Solution methods*

Solution methods are expected to show main effects of all independent variables. Participants are expected to use procedures more often with addition problems than with multiplication problems, consistent with previous research (Campbell & Xue, 2001). This prediction may seem counterintuitive, as I have already posited that stronger problem-answer associations are formed for addition as a result of greater experience with addition. In North America, however, multiplication problems are generally learned by drilling and rote memory leading to greater reliance on retrieval (in contrast to addition). Thus, greater use of procedures is expected in addition.

Participants are expected to use more procedures for problems in word format than problems in digit format. Using the Hybrid model, this finding would be expected because stronger problem-answer associations are formed for digit format as a result of greater experience with digit format. In comparison, the less familiar word format will lead to weaker associations. Thus, word format is expected to lead to greater use of procedures.

Participants are expected to use more procedures to solve large problems than small problems, consistent with previous findings (LeFevre, Sadesky, & Bisanz, 1996). Using the Hybrid model, this finding would be expected because stronger problem-answer associations are formed for smaller problems as a result of greater experience with small problems. Thus, the weaker associations are expected to lead to greater use of procedures.

In terms of two-way interactions, it is unclear whether participants will show a greater effect of format in multiplication or in addition. The weaker problem-answer associations in multiplication may be more vulnerable to the unfamiliar word format and result in an increase in procedures; however, this increase is not expected to exceed the levels of procedure use in addition. Solution of large problems is expected to be more affected by the unfamiliar word format. Thus, participants should use more procedures to solve large problems in word format. The expected pattern of results for the Format x Problem size interaction is shown in Figure 3.

Examining participant’s self reports will allow me to determine if strategy choice is affected by operation, format or problem size. If so, this would be strong evidence in support of the Hybrid model. Indeed, the finding that format affects solution method would not be explainable within any other current model.

## Discussion

In summary, the results of most interest are the affects of format on: (1) the response times for *retrieval* trials; and (2) the amount and type of procedure use. Despite the prediction that both response times and procedure use will be affected, this need not happen. So long as either pathway is affected, retrieval or procedural, the findings will still be consistent with the Hybrid model. The model would allow for both pathways to be affected but does not necessitate that this is so.

If the overall increase in response times for problems in word versus digit format is not the result of an increased use in procedures or an increase in time to retrieve, two options remain. The first alternative is that participants may simply be using the optional recoding for problems presented in word format and then operating on the same representation regardless of format, similar to the position taken in MATHNET and the Triple-code model. By looking at the shapes of the response time distributions for each participant in each condition this possibility of simply transcoding can be evaluated. Although the model supports this possibility, it is not expected to be the case that all participants will simply recode for all problems. Thus, although it would not be inconsistent with the model, the finding that recoding is the effect of interest would render the model unparsimonious and unnecessary. The second alternative is that participants may simply take longer to implement procedures for problems presented in word format. This result could be explained within the Hybrid model such that repeated retrieval attempts fail and a procedure is implemented. The procedure would be reported but the solution time would reflect both the time to implement the procedure along with the time spent attempting retrieval beforehand.

## Conclusion

So how do people solve arithmetic problems? If the findings of the proposed research support the Hybrid model, then we will be closer to answering this important question. The Hybrid model has the potential to provide a framework for the study of arithmetic cognition with which to interpret new and existing data, and to spark new predictions. Further, such a model would have the power to unite various theories/perspectives in the field; enabling researchers to further advance our understanding of arithmetic cognition. Thus, if supported, the Hybrid model is expected to be influential both in the field of arithmetic cognition and in the broader goals of cognitive science. As arithmetic performance makes use of more general cognitive abilities, understanding how people solve arithmetic problems should inform other cognitive domains. Components of our arithmetic ability may also be domain specific, however, and an understanding of what makes arithmetic difficult for humans compared to computers would provide valuable insight into our cognitive capacity.

Future directions include implementation of the Hybrid model. This process will include many challenges including: the need for more explicit descriptions of the representations, determining how representations are connected to one another, and more detailed solution algorithms. The issue of representations is central to cognitive science; information on the representations used to store and access arithmetic facts is expected to be valuable to other areas of cognition. Also, the Hybrid model includes many possible ways in which individual differences may arise. Further investigation of these possibilities will lead to a greater understanding of how we are different from one another in our arithmetic abilities. These insights are expected to be of value to researchers in other areas of cognitive science, as well, because many areas of cognition are embracing the idea that individual cognitive systems do not function in identical ways. Therefore, the design and testing of the Hybrid model is a worthy endeavor, one that should be undertaken.

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*Figure 1.* The Hybrid model of arithmetic problem solution.

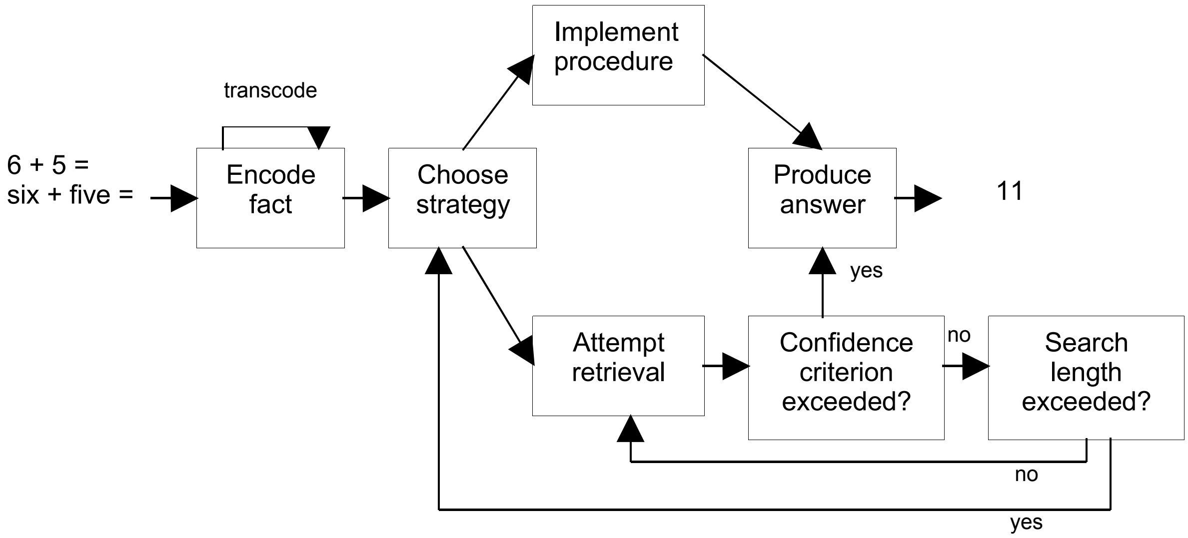

*Figure 2.* Response times for retrieval trials as a function of problem size and presentation format

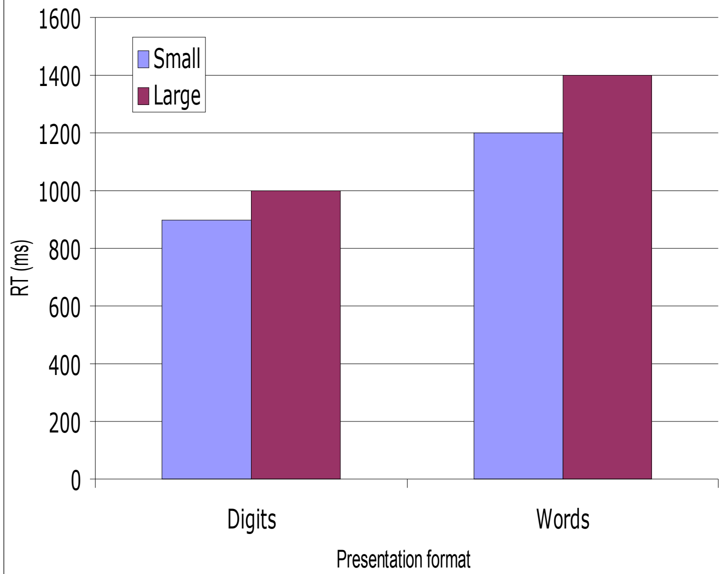

*Figure 3.* Percent procedure use as a function of problem size and presentation format