ATLAS MOTION PLATFORM MECANUM WHEEL JACOBIAN IN THE VELOCITY AND STATIC FORCE DOMAINS

Jonathan J. Plumpton, M. John D. Hayes, Robert G. Langlois and Bruce V. Burlton
Department of Mechanical and Aerospace Engineering, Carleton University, Ottawa, ON, Canada
E-mail: jonathanplumpton@cmail.carleton.ca; jhayes@mae.carleton.ca;
rlangloi@mae.carleton.ca; bburlton@mae.carleton.ca

ABSTRACT

Conventional training simulators commonly use the hexapod configuration to provide motion cues. While widely used, studies have shown that hexapods are incapable of producing the range of motion required to achieve *high fidelity* simulation required in many applications. Atlas is a six degree of freedom vehicle operating training simulator motion platform where orienting is decoupled from positioning, and unbounded rotation is possible about any axis. Angular displacements are achieved by manipulating the cockpit contained in a 2.9 metre (9.5 foot) diameter sphere with three Mecanum wheel actuators. The angular velocity Jacobian, $\mathbf{J}_\omega$, maps the desired angular velocity of the sphere to the required speeds of the three Mecanum wheels, while the static force Jacobian, $\mathbf{J}_\tau$, maps the static moment vector required to statically orient the sphere to the static torques required by the three Mecanum wheels. In this paper, the two Jacobians are derived independently, and it is confirmed that $\mathbf{J}_\omega = \mathbf{J}_\tau^T$, as it must. The implications on the required normal forces at the interface between the sphere and three Mecanum wheel contact patches are discussed.

Keywords: unbounded angular displacement; velocity and static force Jacobians; normal forces.

MATRICE JACOBIENNE DES ROUES MECANUM SIMULATOUR DE MOUVEMENT ATLAS DANS LES DOMAINES DE FORCE DE VITESSE ET STATIQUE

RÉSUMÉ

Les simulateurs d'entraînement conventionnels utilisent souvent la configuration hexapode pour générer une perception de mouvement. Bien que largement utilisés, des études montrent que les robots hexapodes sont incapables de produire l'amplitude de mouvement nécessaire pour atteindre le niveau de fidélité requis dans plusieurs applications de simulation. Atlas est un véhicule à six degrés de liberté qui utilise une plateforme de mouvement d'entraînement simulé où l'orientation de la plateforme est découplée de son positionnement et où des rotations illimitées sont possibles sur tous les axes. Les déplacements angulaires sont obtenus en manipulant la cabine de pilotage, qui est contenue dans une sphère de 2.9 mètres (9.5 pieds) de diamètre, à l'aide de trois actionneurs en forme de roues mecanum. La matrice jacobienne associée à la vitesse angulaire, $\mathbf{J}_\omega$, transforme la vitesse angulaire désirée de la sphère aux vitesses requises des trois roues mecanum. La matrice jacobienne associée aux efforts statiques, $\mathbf{J}_\tau$, pour sa part, transforme le vecteur du couple statique requis pour maintenir la sphère dans une orientation donnée aux couples statiques correspondants fournis par les trois roues mecanum. Dans cet article, les deux matrices jacobiennes sont développées indépendamment l'une de l'autre et il est démontré que, tel que prévu, $\mathbf{J}_\omega = \mathbf{J}_\tau^T$. Les répercussions sur les forces normales requises à l'interface entre la sphère et les surfaces de contact des trois roues mecanum sont discutées.

Mots-clés : déplacement angulaire illimité; matrice jacobienne de vitesse et de forces statique; forces normales.

1. INTRODUCTION

Research has been conducted suggesting appropriate minimum levels of motion required for achieving different levels of training fidelity [1]. However, it is widely accepted that the availability of larger ranges of motion, over what is commonly available with conventional hexapod-based simulators, may provide opportunities for improvements in the immersivity of resulting simulations through less aggressive washout filtering. The challenge with conventional Gough–Stewart platform hexapods, that are most often used for land, sea, and air vehicle simulation, is that the six actuator motions and resulting six degree-of-freedom platform motions are all very tightly coupled. As a result, the platform workspaces are generally small relative to the overall platform size, have intricate shape, and are subject to numerous singularities. Further, greater range of angular motion than the typical 20-50 degrees afforded by Gough–Stewart platforms provides improved capability for continuity of motion in directions (such as yaw) in which washout is known to be marginally effective.

Relatively few motion simulators are available that are designed to provide large ranges of both translational and rotational motion. Three existing large motion simulator facilities are Desdemona [2], Eclipse II [3], and CyberMotion [4]. An alternative novel motion platform concept called Atlas [5] has also been developed within the Carleton University Simulator Project (CUSP) [6]. CUSP is one of several capstone design projects run within the Department of Mechanical and Aerospace Engineering at Carleton University. It began in 2002, and since that time has been incrementally developing the Atlas simulator. Figure 1 illustrates the current detail design, and production of a full scale prototype is well underway. Atlas consists of a spherical capsule in which the trainee or equipment under test is placed. The capsule partially rests on a series of three omni-wheels (alternatively referred to as omni-directional wheels) or Mecanum wheels[1] symmetrically arranged on one side of the equator of the spherical capsule. Actively controlling the angular speed of each of the three wheels allows the sphere to be rotated in an unbounded manner about a continuously-variable axis of rotation at continuously-variable angular speed. Alternatively stated, the sphere (capsule) has an unbounded orientation workspace. This rotational stage is mounted on a hexapod, used as a translational stage using only its three linear degrees of freedom.

The benefits of this arrangement are that it provides unbounded rotational motion of the spherical capsule, translational motion is bounded only by the designer-controlled limitations of the translational stage, and the entire workspace is singularity free and dexterous meaning that any configuration within the bounds of the motion envelope can be easily achieved. Also, rotational motions are decoupled from translational motions thereby not limiting motions due to coupling during operation.

1.1. Atlas Translational Actuation
Due to the prohibitive cost of an adequately sized gantry system to provide the translations, a Moog MB-EP-6DOF 2800KG Gough–Stewart hexapod has been acquired for the purpose. The platform has a payload capacity of 2800 kilograms, and the estimated total weight of the upper Atlas platform is less than 1400 kilograms. This component has been purchased and installed in the Atlas lab. While Gough–Stewart hexapods are capable of providing both translational and rotational actuation, the Atlas platform will decouple the translational and rotational actuation by using the hexapod for purely translational displacements.

1.2. Atlas Rotational Actuation
Three active Mecanum wheels are used to change the sphere orientation. These wheels offer suitable load-carrying capacity and can provide omnidirectional rotation of the sphere while introducing negligible vibration. Developing Mecanum wheels in-house allowed the weight to be reduced by half and the cost to

---

[1]Mecanum wheels are similar to omni-wheels except that the castor axles are rotated 45 degrees relative to the circumferential direction of the wheels.

Upper ring of
12 passive wheels

Sphere

Two antipodal hatches

Driven wheels,
three in total

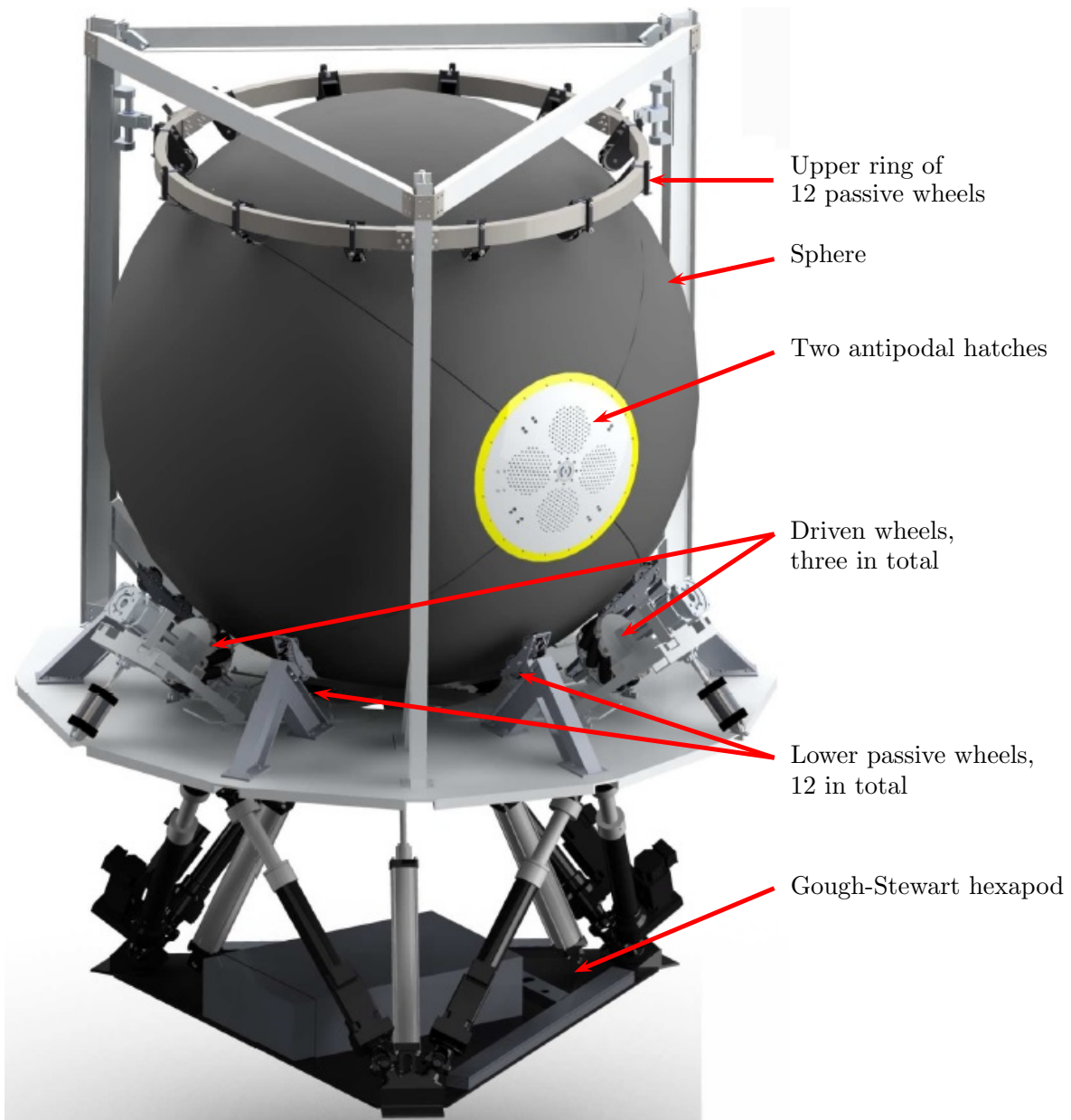Lower passive wheels,
12 in total

Gough-Stewart hexapod

Fig. 1. Atlas motion platform.

be reduced by two thirds compared to commercially-available wheels. It also allowed for control over the characteristics of the interface between the Mecanum wheels and the sphere surface. Urethane roller material and finish has been selected to provide a contact patch of 1290 square millimetres (2 square inches), approximately,[2] and a minimum coefficient of friction of 0.6. Moreover, the roller profiles are elliptical thereby enhancing the transition between rollers as the Mecanum wheel rotates, further reducing vibration.

In addition to the active Mecanum wheels, two rings of smaller passive Mecanum wheels, each containing 12 passive wheels, will be used to help constrain translation of the sphere relative to the support structure, and to ensure sufficient normal force at the contact patch of the driven wheels to prevent slip in the driving direction, see Fig. 1. Production of both passive and active wheels is currently ongoing.

2. OBJECTIVES

The main objectives of the work presented in this paper are to establish the relations between desired sphere angular velocities and the corresponding Mecanum wheel speeds required, as well as the relations between a desired sphere orientation and the corresponding static torques required. These relations are conveniently expressed in the form of a Jacobian. It is, by definition, a mapping between time rates of change. By convention, for velocity-level serial robot kinematics it is the mapping between the time rates of change of the joint variables to the time rates of change of the position and orientation of the end effector [7, 8]. The transpose of the velocity-level Jacobian is the same as the mapping of the static forces acting at the end effector into equivalent joint torques [7, 8].

At first glance, the Atlas motion platform kinematic architecture appears to be parallel. However, the motion control strategy decouples orientation from translation of the sphere centre: orientation of the sphere is controlled by three linearly independent Mecanum wheel actuators; translation is controlled using only the three linear degrees of freedom of the hexapod. Hence, the angular velocity Jacobian maps the linearly-independent angular rates of the three active wheels to the three linearly-independent components of the sphere angular velocity vector. The Jacobian formulation for specific parallel kinematic architectures presented in [8], for example, is not required. It is additionally important to note that both active and passive Mecanum wheel design has included attention to rolling resistance and friction. It is expected that the friction forces will be negligible compared to the orientation actuation forces, and therefore are not modelled. The degree to which this is true will be investigated empirically when the first full-scale prototype is in operation. This is the case for a half-scale prototype already in operation for technology development and demonstration.

The velocity level Jacobian for spheres manipulated using omni-wheels was presented in [9]. However, the geometry for the rollers on Mecanum wheels is different, and additional kinematic parameters must be accounted for. Hence, in this paper the Jacobian will be derived in two independent ways. First, the Jacobian used to compute the magnitude and direction of the torque created in rotating the sphere based on the torque applied to the Mecanum wheels, $\mathbf{J}_\tau$, will be derived. Next, a Jacobian will be derived for mapping the desired angular velocity of the sphere to the required angular rates of the mecanum wheels, $\mathbf{J}_\omega$. This derivation is intended to serve as a verification of the previous one, since it must be that $\mathbf{J}_\omega = \mathbf{J}_\tau^T$, thereby providing concomitant evidence that the Jacobian is indeed valid.

3. STATIC FORCE–TORQUE JACOBIAN

In this section the Jacobian to be used to determine the magnitude and direction of the torque created by rotating the sphere, based on the torque applied to the Mecanum wheels, is derived. Required variables are

---

[2]Please note that the use of dual metric and Imperial dimensioning reflects the reality of design in Canada: the standard is metric; however, many stock components are sized in Imperial units.
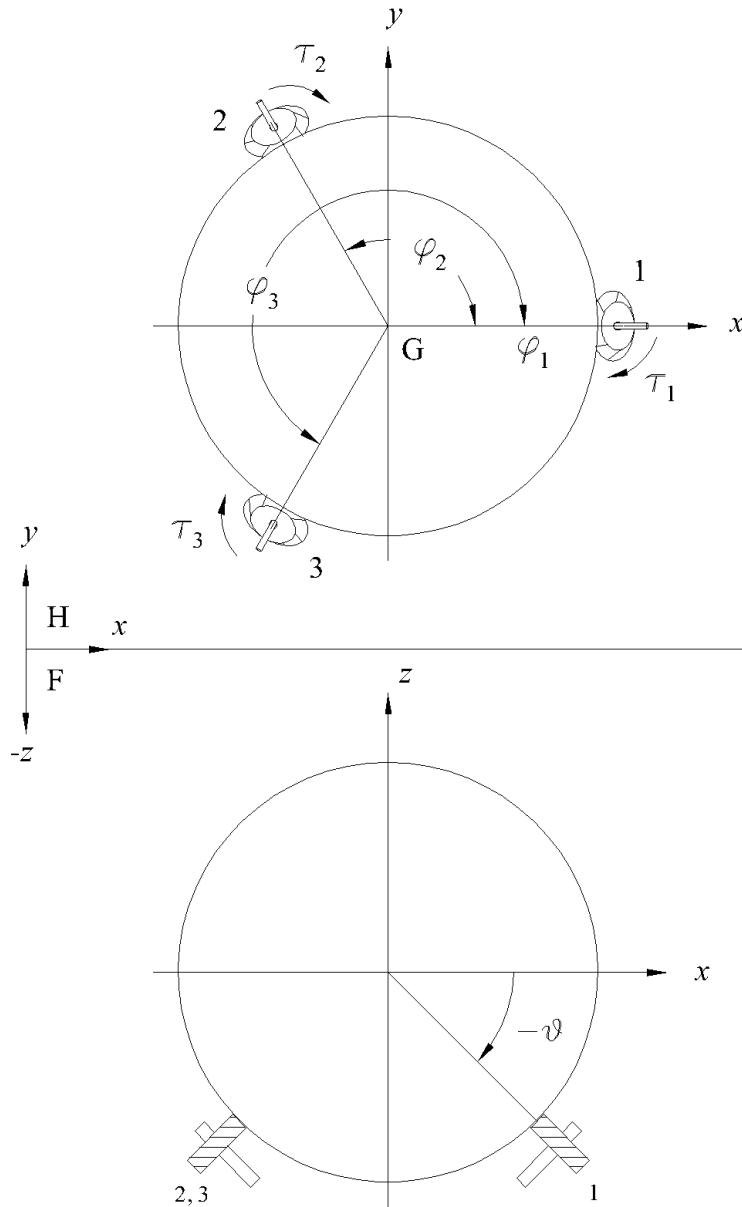
Fig. 2. Configuration of three driven wheels for Atlas.

defined in Fig. 2. The total torque **M** created by the Mecanum wheels acting on the sphere is given by

$$\mathbf{M} = \begin{bmatrix} M_X & M_Y & M_Z \end{bmatrix}^T, \tag{1}$$

where M_i are the torque components expressed in the inertial xyz coordinate system. Position vectors of the idealized Mecanum wheel contact points A_i relative to the centre of the sphere G are given by

$$\mathbf{R}_{A_i/G} = R \begin{bmatrix} C\phi_i C\theta_i & S\phi_i C\theta_i & S\theta_i \end{bmatrix}^T. \tag{2}$$

where R is the sphere radius, ϕ_i is the counterclockwise rotation of wheel i about the Z axis measured relative to the X axis, and S and C are abbreviations for sine and cosine respectively. The torque vector
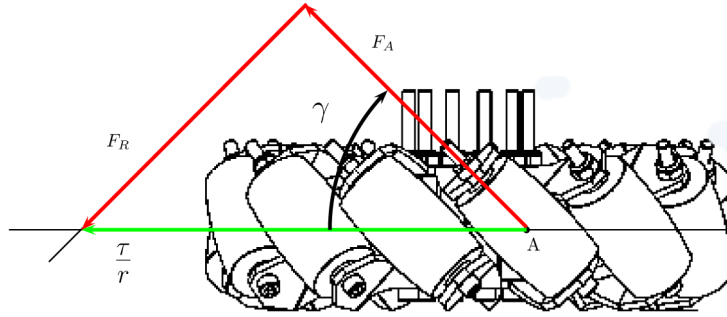
Fig. 3. Mecanum wheel roller forces.

generated by each wheel is defined as τ_i, with the Mecanum wheel having the following radial vector of point A_i relative to point B_i, the centre of the respective Mecanum wheel, opposite in direction to $\mathbf{R}_{A_i/G}$, or equivalent to

$$\mathbf{r}_{A_i/B_i} = r \left[-C\phi_i C\theta_i \quad -S\phi_i C\theta_i \quad -S\theta_i \right]^T, \tag{3}$$

where r is the radius of the mecanum wheel. The Mecanum wheels have tractive force vectors along the roller axes that are given by

$$\mathbf{F}_{A_i} = F_{A_i} \left[-C\phi_i S\gamma_i S\theta_i - S\phi_i C\gamma_i \quad -S\phi_i S\gamma_i S\theta_i + C\phi_i C\gamma_i \quad S\gamma_i C\theta_i \right]^T, \tag{4}$$

where γ_i is the angle of the Mecanum wheel roller axis relative to the plane of the wheel, and

$$F_{A_i} = \frac{\tau_i}{rC\gamma_i}, \tag{5}$$

as shown in Fig. 3. The contribution of the force $\mathbf{F}_{A_i}$ to the overall torque applied to the sphere is then calculated as follows:

$$\mathbf{M}_i = \mathbf{R}_{A_i/G} \times \mathbf{F}_{A_i} = \frac{\tau_i R}{rC\gamma_i} \left[-C\phi_i C\gamma_i S\theta_i + S\phi_i S\gamma_i \quad -S\phi_i C\gamma_i S\theta_i - C\phi_i S\gamma_i \quad C\gamma_i C\theta_i \right]^T. \tag{6}$$

Summing the three sphere torque vectors together results in the total torque applied to the sphere

$$\mathbf{M} = \sum \mathbf{M}_i = \mathbf{M}_1 + \mathbf{M}_2 + \mathbf{M}_3. \tag{7}$$

where

$$\mathbf{M} = \frac{R}{r} \begin{bmatrix} -C\phi_1 S\theta_1 + S\phi_1 T\gamma_1 & -C\phi_2 S\theta_2 + S\phi_2 T\gamma_2 & -C\phi_3 S\theta_3 + S\phi_3 T\gamma_3 \\ -S\phi_1 S\theta_1 - C\phi_1 T\gamma_1 & -S\phi_2 S\theta_2 - C\phi_2 T\gamma_2 & -S\phi_3 S\theta_3 - C\phi_3 T\gamma_3 \\ C\theta_1 & C\theta_2 & C\theta_3 \end{bmatrix} \tau, \tag{8}$$

τ represents the magnitudes for the three Mecanum wheel torques, and T is the abbreviation for tangent.[3] This expression defines the Jacobian $\mathbf{J}_\tau$ between driven wheel torque and sphere activation torque

$$\mathbf{M} = \mathbf{J}_\tau \tau, \tag{9}$$

---

[3]Note that in this case T^{-1} is referring to the cotangent as opposed to the arctangent.
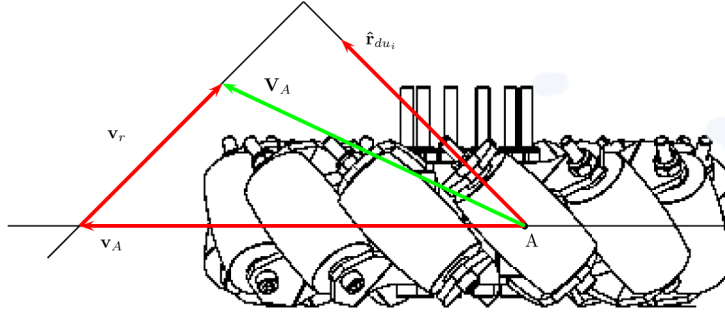
Fig. 4. Schematic of Mecanum wheel velocity vectors.

where the Jacobian is given by

$$\mathbf{J}_\tau = \frac{R}{r} \begin{bmatrix} -C\phi_1 S\theta_1 + S\phi_1 T\gamma_1 & -C\phi_2 S\theta_2 + S\phi_2 T\gamma_2 & -C\phi_3 S\theta_3 + S\phi_3 T\gamma_3 \\ -S\phi_1 S\theta_1 - C\phi_1 T\gamma_1 & -S\phi_2 S\theta_2 - C\phi_2 T\gamma_2 & -S\phi_3 S\theta_3 - C\phi_3 T\gamma_3 \\ C\theta_1 & C\theta_2 & C\theta_3 \end{bmatrix}. \tag{10}$$

4. ANGULAR VELOCITY JACOBIAN

As a means of verifying the results of Section 3, a Jacobian was derived for mapping the angular rates of the wheels to the angular velocity of the Atlas sphere. First, let the sphere have the following angular velocity about the centre of the sphere G,

$$\mathbf{\Omega} = \begin{bmatrix} \Omega_X & \Omega_Y & \Omega_Z \end{bmatrix}^T, \tag{11}$$

each Mecanum wheel having an angular velocity given by

$$\boldsymbol{\omega}_i = \begin{bmatrix} \omega_{iX} \\ \omega_{iY} \\ \omega_{iZ} \end{bmatrix} = \omega_i \begin{bmatrix} C\phi_i S\theta_i \\ S\phi_i S\theta_i \\ -C\theta_i \end{bmatrix}. \tag{12}$$

Each Mecanum wheel will have a unit vector along the roller axis, shown in Fig. 4, that has the following form:

$$\hat{\mathbf{r}}_{du_i} = \begin{bmatrix} -C\phi_i S\gamma_i S\theta_i - S\phi_i C\gamma_i \\ -S\phi_i S\gamma_i S\theta_i + C\phi_i C\gamma_i \\ S\gamma_i C\theta_i \end{bmatrix}. \tag{13}$$

The kinematics at point A_i can now be evaluated with respect to both the sphere and the Mecanum wheel i. The velocity of point A_i on the sphere side of the Mecanum wheel/sphere interface is

$$\mathbf{V}_{A_i} = \mathbf{\Omega} \times \mathbf{R}_{A_i/G} = R \begin{bmatrix} \Omega_Y S\theta_i - \Omega_Z S\phi_i C\theta_i \\ \Omega_Z C\phi_i C\theta_i - \Omega_X S\theta_i \\ \Omega_x S\phi_i C\theta_i - \Omega_Y C\phi_i C\theta_i \end{bmatrix}, \tag{14}$$

while the velocity of point A_i on the Mecanum wheel side of the Mecanum wheel/sphere interface is a result of the wheel rotating about the wheel axis

$$\mathbf{v}_{A_i} = \boldsymbol{\omega}_i \times \mathbf{r}_{A_i/B_i} = \omega_i r \begin{bmatrix} -S\phi_i \\ C\phi_i \\ 0 \end{bmatrix}. \tag{15}$$

From Fig. 4, $\mathbf{V}_{A_i}$ is the vector sum of $\mathbf{v}_{A_i}$ and $\mathbf{v}_{r_i}$, which is the velocity vector of the Mecanum wheel roller. Furthermore, assuming no slip, $\mathbf{V}_{A_i}$ and $\mathbf{v}_{A_i}$ must create the same projection, which gives the following result:

$$\mathbf{V}_{A_i} \cdot \hat{\mathbf{r}}_{du_i} = \mathbf{v}_{A_i} \cdot \hat{\mathbf{r}}_{du_i}, \tag{16}$$

where the left-hand side is

$$\mathbf{V}_{A_i} \cdot \hat{\mathbf{r}}_{du_i} = R \begin{bmatrix} S\phi_i S\gamma_i - C\phi_i S\theta_i C\gamma_i \\ -C\phi_i S\gamma_i - S\phi_i S\theta_i C\gamma_i \\ C\theta_i C\gamma_i \end{bmatrix}^T \begin{bmatrix} \Omega_X \\ \Omega_Y \\ \Omega_Z \end{bmatrix}, \tag{17}$$

and the right-hand side is

$$\mathbf{v}_{A_i} \cdot \hat{\mathbf{r}}_{du_i} = \omega_i r C\gamma_i. \tag{18}$$

Equating these two results and solving for the signed magnitude of the angular velocity for Mecanum wheel i results in

$$\omega_i = \frac{R}{r} \begin{bmatrix} S\phi_i T\gamma_i - C\phi_i S\theta_i \\ -C\phi_i T\gamma_i - S\phi_i S\theta_i \\ C\theta_i \end{bmatrix}^T \begin{bmatrix} \Omega_X \\ \Omega_Y \\ \Omega_Z \end{bmatrix}. \tag{19}$$

Gathering the set of linear equations for all three driven Mecanum wheels on Atlas results in the following equation:

$$\begin{bmatrix} \omega_1 \\ \omega_2 \\ \omega_3 \end{bmatrix} = \frac{R}{r} \begin{bmatrix} S\phi_1 T\gamma_1 - C\phi_1 S\theta_1 & -C\phi_1 T\gamma_1 - S\phi_1 S\theta_1 & C\theta_1 \\ S\phi_2 T\gamma_2 - C\phi_2 S\theta_2 & -C\phi_2 T\gamma_2 - S\phi_2 S\theta_2 & C\theta_2 \\ S\phi_3 T\gamma_3 - C\phi_3 S\theta_3 & -C\phi_3 T\gamma_3 - S\phi_3 S\theta_3 & C\theta_3 \end{bmatrix} \begin{bmatrix} \Omega_X \\ \Omega_Y \\ \Omega_Z \end{bmatrix}. \tag{20}$$

The relationship between the magnitudes of the angular velocities of the Mecanum wheels and the angular velocity of the sphere can be summarized by the following equation:

$$\boldsymbol{\omega} = \mathbf{J}_\omega \boldsymbol{\Omega}, \tag{21}$$

where $\boldsymbol{\omega}$ represents the magnitudes of the angular velocities of the Mecanum wheels, and the corresponding Jacobian is defined as

$$\mathbf{J}_\omega = \frac{R}{r} \begin{bmatrix} S\phi_1 T\gamma_1 - C\phi_1 S\theta_1 & -C\phi_1 T\gamma_1 - S\phi_1 S\theta_1 & C\theta_1 \\ S\phi_2 T\gamma_2 - C\phi_2 S\theta_2 & -C\phi_2 T\gamma_2 - S\phi_2 S\theta_2 & C\theta_2 \\ S\phi_3 T\gamma_3 - C\phi_3 S\theta_3 & -C\phi_3 T\gamma_3 - S\phi_3 S\theta_3 & C\theta_3 \end{bmatrix}. \tag{22}$$

At this point it can be observed that

$$\mathbf{J}_\omega = \mathbf{J}_\tau^T, \tag{23}$$

which is as expected.

5. ATLAS TORQUE ANALYSIS

Having now derived a Jacobian for torques and another Jacobian for angular velocities, the following sections describe how the Atlas dynamic model can be used in sizing the motors for driving the Mecanum wheels as well as determining the normal force that the driven wheels must apply to the sphere in order to maintain sufficient traction.

5.1. Sphere Torque

The first step in determining the maximum wheel torque is to determine the maximum torque that must be applied to the sphere during operation. The sphere torque is found using the generalized moment balance equation

$$\mathbf{M} = \mathbf{I_s}\boldsymbol{\alpha} + \boldsymbol{\Omega} \times \mathbf{I_s}\boldsymbol{\Omega}. \tag{24}$$

where $\mathbf{I_s}$ is the mass moment of inertia matrix of the Atlas sphere with respect to its geometric centre, and the vectors $\boldsymbol{\alpha}$ and $\boldsymbol{\Omega}$ are found using user-defined magnitudes as follows:

$$\boldsymbol{\alpha} = |\boldsymbol{\alpha}| \begin{bmatrix} S\theta_\alpha C\phi_\alpha & S\theta_\alpha S\phi_\alpha & C\theta_\alpha \end{bmatrix}^T. \tag{25}$$

$$\boldsymbol{\Omega} = |\boldsymbol{\Omega}| \begin{bmatrix} S\theta_\Omega C\phi_\Omega & S\theta_\Omega S\phi_\Omega & C\theta_\Omega \end{bmatrix}^T. \tag{26}$$

where θ_i and ϕ_i represent similar angles to those representing wheel position in Fig. 2, and are all independent. The process involves iterating Eq. (24) through different values of θ_α, ϕ_α, θ_Ω and ϕ_Ω between -180 degrees and $+180$ degrees. The maximum norm of $\mathbf{M}$ is taken to be the maximum torque experienced by the sphere.

5.2. Wheel Torque

The process to determine the maximum wheel torque is similar to the process for finding the sphere torque. Iteration is performed for different directions for the sphere torque vector

$$\mathbf{M} = |\mathbf{M}| \begin{bmatrix} S\theta_M C\phi_M & S\theta_M S\phi_M & C\theta_M \end{bmatrix}^T. \tag{27}$$

The torque applied in the driving direction of each wheel is then found using Eq. (8) found in Section 3. Iteration of Eq. (8) is performed for different values of θ_M and ϕ_M between -180 degrees and +180 degrees and the maximum wheel torque τ is taken to be the maximum component of $\boldsymbol{\tau}$ encountered after all the iterations, because each element of $\boldsymbol{\tau}$ is the signed magnitude of the input torque supplied by each of the three Mecanum wheels.

5.3. Normal Force

The offset angle of the Mecanum wheel roller results in a similar offset between the force caused by the wheel torque and the resultant force along the roller axis. The maximum tangential force will be the result of the maximum wheel torque, and can be calculated as follows:

$$F_t = \frac{\tau}{rC\gamma}. \tag{28}$$

which was previously defined in Section 3. To effectively turn the sphere, this tangential force must be less than the force required to overcome friction. This gives the following relation for the normal force:

$$N \geq \frac{F_t}{\mu} = \frac{\tau}{r\mu C\gamma}. \tag{29}$$

where μ is the friction coefficient.

6. APPLICATION

The Atlas motion platform will be constructed with a 2.9 metre (9.5 foot) external diameter sphere and three 381 millimetre (15 inch) diameter Mecanum wheels. The design calls for the wheels to be positioned 45 degrees below the sphere equator, and the wheels are to be separated by 120 degrees around the z axis,
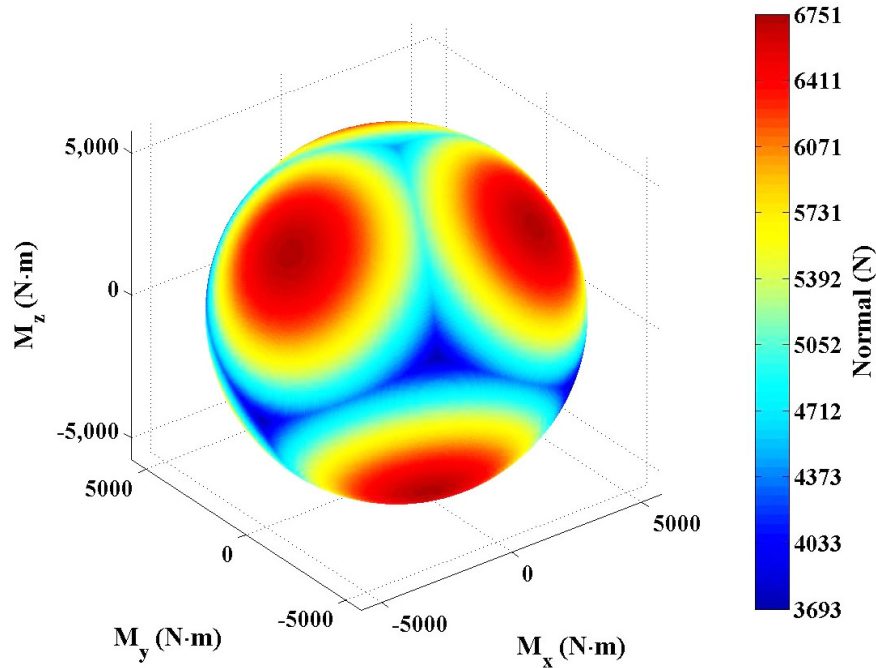
Fig. 5. Required applied wheel normal force for different directions of maximum sphere torque.

with the x axis oriented such that it is aligned with the first wheel. Given these parameters, the static force-torque Jacobian is evaluated as

$$
\mathbf{J}_\tau = \begin{bmatrix} 5.374 & 3.895 & -9.269 \\ -7.600 & 8.454 & -0.854 \\ 5.374 & 5.374 & 5.374 \end{bmatrix}.
\tag{30}
$$

In the final stages of development, the fully-loaded Atlas sphere had inertia estimated as

$$
\mathbf{I}_s = \begin{bmatrix} 941.129 & 0.477 & 0.061 \\ 0.477 & 864.456 & 43.457 \\ 0.061 & 43.457 & 918.303 \end{bmatrix} \mathrm{kg} \cdot \mathrm{m}^2,
\tag{31}
$$

with intentions of achieving a maximum angular acceleration of $\alpha = 350$ degrees per second squared and a maximum angular velocity of $\omega = 35$ degrees per second. Applying the techniques from Section 5 to these parameters, it was determined that the sphere would be expected to require a maximum torque of close to 5200 Newton metres, with the wheels required to apply a maximum torque of roughly 540 Newton metres under an applied normal force of approximately 6700 Newtons. The results of the analysis are further shown in Fig. 5 which emphasizes the requirement for applying the analysis to different directions, as application of the maximum required sphere torque in the wrong direction would result in underestimating the maximum required wheel torques and forces by nearly 50 percent.

7. CONCLUSIONS

In this paper, novel generalized kinematic and static force models for the Atlas spherical platform, actuated with Mecanum wheels, has been presented. The model was first formulated at the static force level leading

to an expression for $\mathbf{J}_\tau$, and verified by the model formulated at the velocity level, leading to $\mathbf{J}_\omega$. The results confirm that $\mathbf{J}_\omega = \mathbf{J}_\tau^T$, as it must. This paper further described how the kinematic and static force models are required for accurate control of the rotational actuation for the Atlas platform, as well as being essential in determining the mechanical requirements of the actuation system. This analysis was applied to the design specifications for the platform, with the results demonstrating the need for the rigorous steps that were suggested for the analysis.

REFERENCES

1. Gawron, V.J., Bailey, R. and Lehman, E., "Lessons learned in applying simulators to crewstation evaluation", *International Journal of Aviation Psychology*, Vol. 5, No. 2, pp. 277–290, 1995.
2. Bles, W. and Groen, E., "The desdemona motion facility: Applications for space research", *Microgravity Science and Technology*, Vol. 21, No. 4, pp. 281–286, 2009.
3. Kim, J., Jwang, J., Jim, J., Iurascu, C., Park, F. and Cho, Y., "Eclipse II: A new parallel mechanism enabling continuous 360-degree spinning plus three-axis translational motions", *IEEE Transactions on Robotics and Automation*, Vol. 18, No. 3, pp. 367–373, 2002.
4. Giordano, P.R., Masone, C., Tesch, J., Breidt, M., Pollini, L. and Bulthoff, H.H., "A novel framework for closed-loop robotic motion simulation – Part I: Inverse kinematics design", *IEEE International Conference on Robotics and Automation*, pp. 3876–3883, 2010.
5. Hayes, M. and Langlois, R., "Atlas: A novel kinematic architecture for six DOF motion platforms", *Canadian Society for Mechanical Engineering (CSME) Transactions*, Vol. 29, No. 4, pp. 701–709, 2005.
6. Hayes, M., Langlois, R., Pearce, T., Tan, C.L. and Gaydos, J., "Carleton University Simulator Project (CUSP)", in *Proceedings of the CSME Forum 2004,* University of Western Ontario, London, ON, Canada, June 1-4, 2004.
7. Craig, J., *Introduction to Robotics* (2nd ed.), Addison Wesley, 1989.
8. Tsai, L.W., *Robot Analysis*, Wiley-Interscience, 1999.
9. Hayes, M.J.D., Langlois, R.G. and Weiss, A., "Atlas motion platform generalized kinematic model", *Meccanica*, Vol. 46, No. 1, pp. 17–25, 2011.