MAAE 3004
Dynamics of Machinery

Lecture Slide Set 4

Machines and Mechanisms

Department of Mechanical and Aerospace Engineering
Carleton University

Outline

Definitions

a) Machine:

- "A combination of resistant **bodies** so arranged that by their means the mechanical **forces** of nature can be compelled to do **work** accompanied by certain determinate **motions**."
  F. Reuleaux (1829-1905)-grand father of systematic treatment of kinematics.
- "an arrangement of parts for doing **work**, a device for applying **power** or changing its direction."
  Shigley and Uicker; The theory of machines and mechanisms; 5th *edition, Oxford, 2017.*

Key concepts:	power (work,energy and force)
	bodies (physical parts)
	motion (kinematics)
Conclusion:	**Machines** are closely associated with **Kinetics.**

b) Mechanism:

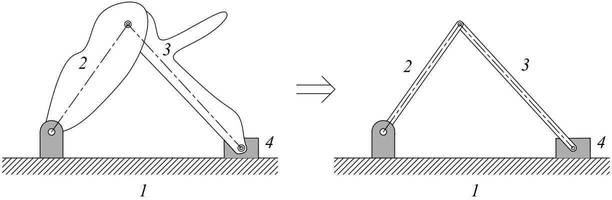
- "An assemblage of resistant **bodies**, connected by moveable **joints** to form a closed kinematic chain with one link fixed and having the purpose of transforming **motion**."
  F. Reuleaux
- "The predominant idea in the mind of the designer is one of achieving a desired **motion**."
  Shigley and Uicker; The theory of machines and mechanisms; 3rd edition, Oxford, 2017.

Key concept:	motion
Conclusion:	**Mechanisms** are closely associated with **Kinematics**

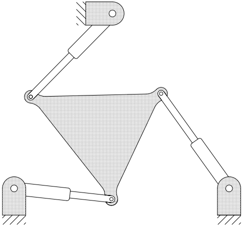
All machines are mechanisms, but not all mechanisms are machines.

c) Link:

A single rigid body linked to two or more rigid bodies by kinematic pairs (joints) maintaining a fixed geometric relationship between the rigid bodies

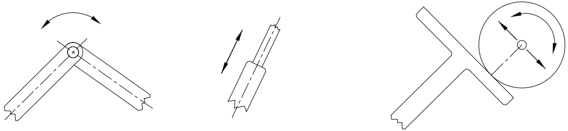


- Chains and belts are links when in tension
- **Binary link:** a single link paired with two others
- **Ternary link:** a single link paired with three others



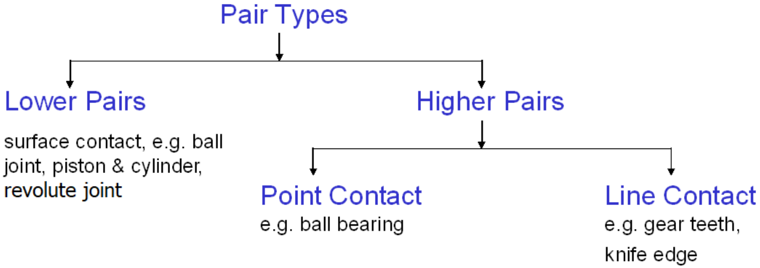
d) Kinematic Pairs (Joints):

Form the attachment between two bodies. The purpose is to constrain the allowable relative motion between connected links.



Lower pairs   Higher pair

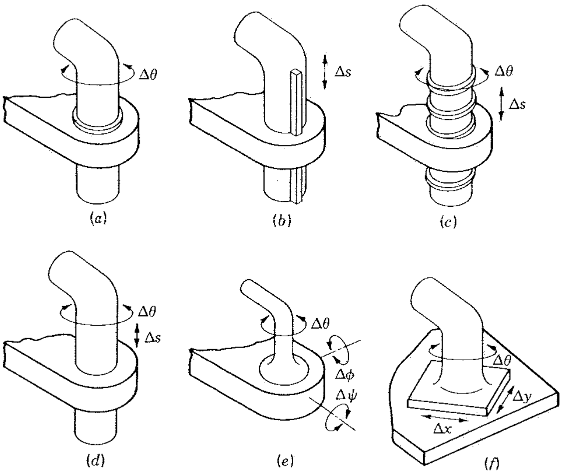
Pair variables: The variables used to measure relative motions.



Six lower pairs

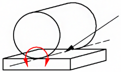
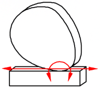
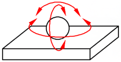
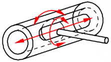
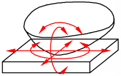
Name	Description	Number of pair variables (ndof)
Revolute / Turning / Pin R-pair	permits rotation about 1 axis	1 Rotary
Prismatic / Sliding P-pair	permits translation in 1 direction	1 Prismatic
Helical / Screw H-pair	permits 1 coupled trans. and rotation; related by helix angle	1 Rotary <u>OR</u> 1 Prismatic
Cylindrical C-pair	permits 1 rotation and translation along 1 rotation axis	1 Rotary <u>AND</u> 1 Prismatic
Spherical S-pair	permits general rotation about a fixed point	3 Rotary
Planar E-pair	permits general planar translation and rotation	1 Rotary 2 Prismatic

Six lower pairs



(a) R-pair (revolute); (b) P-pair (prismatic); (c) H-pair (helical screw); (d)C-pair (cylindrical); (e) S-pair (spherical); and (f) E-pair (planar)

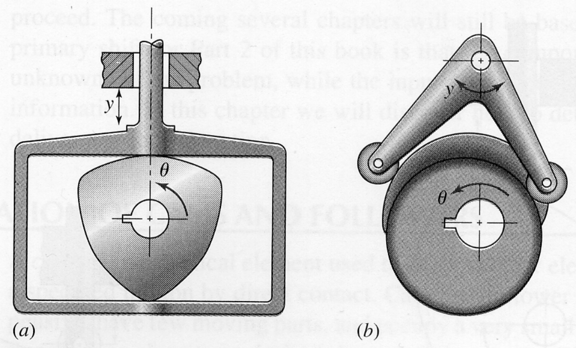
TABLE 1.2 Examples of Higher-Pair Joints

Connectivity (Number of Degrees of Freedom)	Names	Typical Form	Comments
1	Cylindrical roller		Roller rotates about this line at this instant in its motion. Roller does not slip on the surface on which it rolls.
2	Cam pair		Cam rolls and slides on follower
3	Rolling ball		Ball rolls without slipping
4	Ball in cylinder		Ball can rotate about any axis through its center and slide along the cylinder axis
5	Spatial point contact		Body can rotate about any axis through the contact point and slide in any direction in the tangent plane

K.J.Waldron.*Kinematics Dynamics and Design of Machinery*. John Wiley and Sons Inc., United States, 2th edition, 2004.

Higher Pair Joint Examples

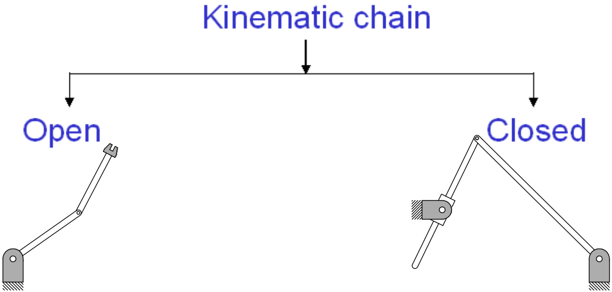
The cam in (a) is bounded by a curve of constant width.
See for example https://www.youtube.com/watch?v=g7SYvLCR3Rk
and https://www.youtube.com/watch?v=quuw4HC96bE



J J.Uicker. *Theory of Machines and Mechanisms*. Oxford University Press Inc., New York, New York, United States, 4th edition, 2011.

e) Kinematic chain:

Kinematic chains can be open (serial) or closed (parallel)



Recall Reuleaux's definition:
When one body in the closed kinematic chain is fixed such that motion of any link will result in definite predictable motion of all other links, the kinematic chain is considered a mechanism.

The fixed link is called 'the ground link', 'base link', or 'frame'.

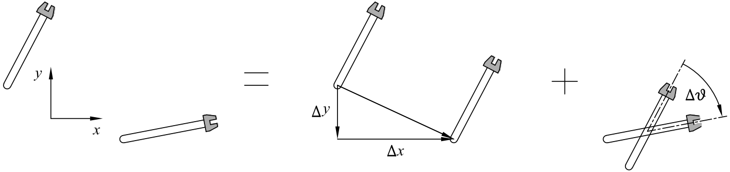
Geometric Categorisation

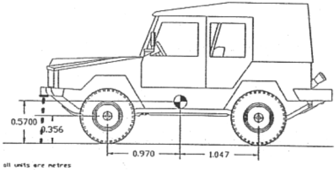
- Based on range of possible relative motions within the mechanism.

a) Planar:

All particles describe plane curves in space and all these curves lie in parallel planes

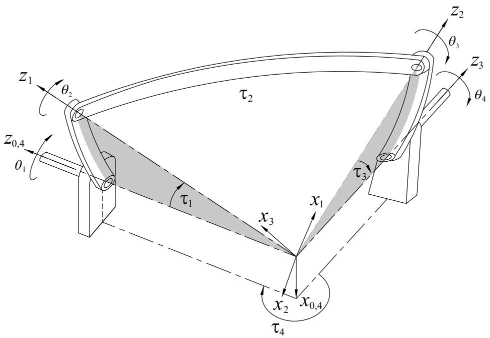
Plane motion ⇒ Translation + Rotation





R.G. Langlois. Preview control algorithms for the active suspension of an off-road vehicle. Master's thesis, Queen's University at Kingston, Kingston,Ontario, Canada K7L 3N6, August 1991.

b) Spherical:



The R-pair axes on each link all intersect at the centre of the sphere on which the circular arc links move.

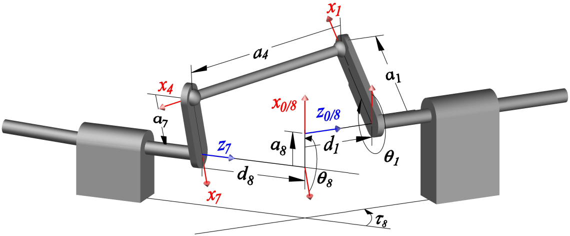
The sphere centre is the mutual stationary point of each link.

c) Spatial:

A spatial mechanism has one link that moves with 6-DOF in the sense that six parameters are needed to specify its position and orientation, such as the RSSR mechanism.

The vast majority of in-service mechanisms are planer because:

- All links can be represented in true size and shape on paper;
- Well developed graphical and algebraic analysis can be applied;
- Operation can be readily visualised;
- Design requirements can be satisfied using straightforward methods;
- Significantly more sophisticated methods are needed for spherical and spacial mechanism design and manufacture.



Mobility (m):

The number of degrees of freedom (DOF) in a mechanism,
i.e; the number of kinematic pair variables that must be specified to determine the configuration of a mechanism

Kutzbach criteria for planer mobility:

$$m = 3(n-1) - 2j_1 - j_2 - \zeta \tag{1}$$

where,

n:	number of links (including the ground link)
$3n$:	DOF of mechanism (before connecting links)
$3(n-1)$:	-1 is to account for the ground link
j_1:	number of pairs with 1 DOF (2 constraints)
j_2:	number of pairs with 2 DOF (1 constraint)
ζ:	represents the number of idle DOF of the kinematic chain (always occurs in SS dyads)

m > 0 $\Rightarrow$ m is # of DOF for the mechanism = number of independent inputs
m $= 0$ $\Rightarrow$ system is a determinate structure with no rigid body motion
m < 0 $\Rightarrow$ structure is statically indeterminate

Chebyshev-Grübler-Kutzbach (CGK) Mobility Formula

Another way to determine the mobility of a mechanical system of rigid bodies is as follows.

- Clearly, n unconstrained rigid links have $d(n-1)$ relative DOF, given that one of the links is designated as a non-moving reference link, where d can be described as the dimension of the space of the motion.
- In general for planar mechanisms $d = 3$ and for spatial mechanisms $d = 6$, but as we will see this mobility model leads sometimes to inconsistencies mostly because distances between the joints is not considered.
- Any joint connecting two neighboring rigid bodies removes at least one relative DOF.
- If the joint removes no DOF then the bodies are not connected.
- If the joint removes 3 DOF in the plane, or 6 DOF in E_3 the two bodies are a rigid structure.

Chebyshev-Grübler-Kutzbach (CGK) Mobility Formula

Summarising this discussion, the DOF, or mobility of a kinematic chain, relative to one fixed link in the chain, can be expressed as:

$$m = d(n-1) - \sum_{i=1}^{p} \mu_i - \zeta \qquad (2)$$

where,

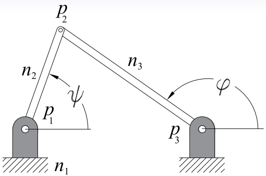
n:	number of links (including the ground link)
d:	dimension of the motion space
(n-1):	-1 is to account for the ground link
μ_i:	is the number of constraints imposed by the i^{th} joint (pair)
p:	is the number of joints (pairs)
ζ	represents the number of idle DOF of the kinematic chain

m $>$ 0 $\Rightarrow$ m is # of DOF for the mechanism = number of independent inputs
m = 0 $\Rightarrow$ system is a determinate structure with no rigid body motion
m $<$ 0 $\Rightarrow$ structure is over constrained and statically indeterminate

Example 1.1

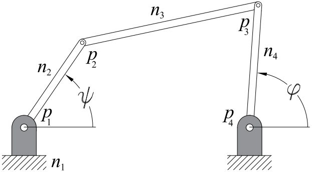
a) n = 3, $j_1 = 3, \zeta = 0$,
m= 3(3-1)-2(3) = 0.

$n = 3,\ p = 3,\ u_i = 2,\ \zeta = 0,$
$m = 3(3\text{-}1)\text{-}2(3) = 0.$



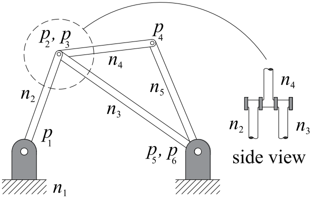
b) n = 4, $j_1 = 4, \zeta = 0$,
m = 3(4-1)-2(4) = 1.

$n = 4,\ p = 4,\ u_i = 2,\ \zeta = 0,$
$m = 3(4\text{-}1)\text{-}2(4) = 1.$



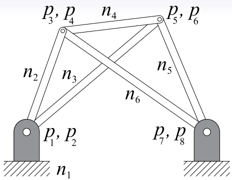
c) n = 5, $j_1 = 6, \zeta = 0$,
m = 3(5-1)-2(6) = 0.

$n = 5,\ p = 6,\ u_i = 2,\ \zeta = 0,$
$m = 3(5\text{-}1)\text{-}2(6) = 0.$



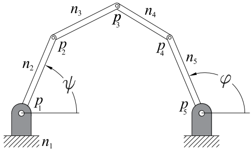
d) $n = 6,\ j_1 = 8, \zeta = 0,$
$m = 3(6\text{-}1)\text{-}2(8) = \text{-}1.$

$n = 6,\ p = 8,\ u_i = 2,\ \zeta = 0,$
$m = 3(6\text{-}1)\text{-}2(8) = \text{-}1.$



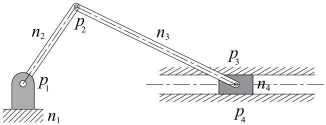
e) $n = 5,\ j_1 = 5, \zeta = 0,$
$m = 3(5\text{-}1)\text{-}2(5) = 2.$

$n = 5,\ p = 5,\ u_i = 2,\ \zeta = 0,$
$m = 3(5\text{-}1)\text{-}2(5) = 2.$



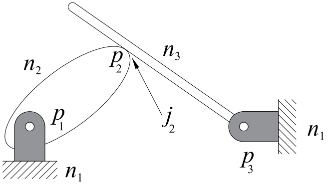
f) $n = 4,\ j_1 = 4, \zeta = 0,$
$m = 3(4\text{-}1)\text{-}2(4) = 1.$

$n = 4,\ p = 4,\ u_i = 2,\ \zeta = 0,$
$m = 3(4\text{-}1)\text{-}2(4) = 1.$



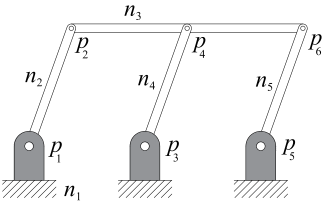
g) n $= 3$, $j_1 = 2$, $j_2 = 1, \zeta = 0$,
m $= 3(3\text{-}1)\text{-}2(2)\text{-}1 = 1$.

$n = 3,\ p = 3,\ u_{1,3} = 2,\ u_2 = 1,\ \zeta = 0,$
$m = 3(3\text{-}1)\text{-}2(2)\text{-}1(1) = 1.$



h) n $= 5$, $j_1 = 6, \zeta = 0$,
m $= 3(5\text{-}1)\text{-}2(6) = 0$.

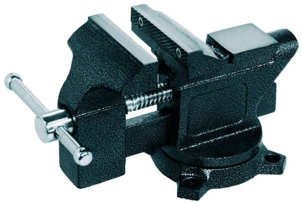
$n = 5,\ p = 6,\ u_i = 2,\ \zeta = 0,$
$m = 3(5\text{-}1)\text{-}2(6) = 0.$

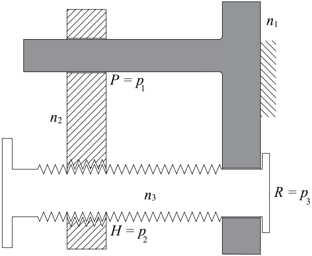


m is actually 1!

Some geometric exceptions cause Kutzbach's criterion to produce erroneous results - some caution required

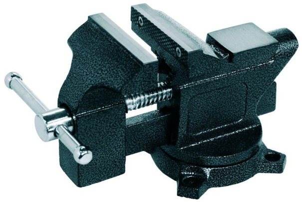
i) Consider a typical bench vice:

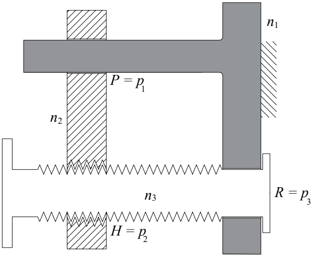




At first glance the bench vice is planar, so we immediately set d = 3:

$$d = 3, \ n = 3, \mu_i = 2, p = 3, \zeta = 0,$$
$$m = 3(3-1) - 3(2) - 0 = 0.$$





A bench vice has 1 DOF! What gives?

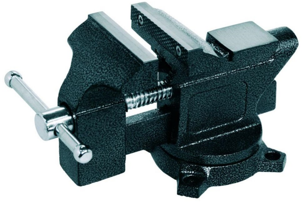
This seeming anomaly is an artifact of representation.

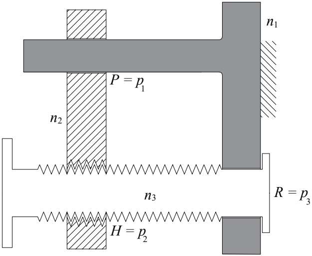
The *R*- and *H*-axes are parallel while the translation direction of the *P*-pair is also parallel.

The *P*- and *R*-pairs are kinematically equivalent to a single *C*-pair.

Moreover, the axis of the *H*-pair is parallel to the axis of the *C*-pair.

Therefore, the dimension of the motion sub-group represented by the common bench vice is $d = 2$, **not** $d = 3$.



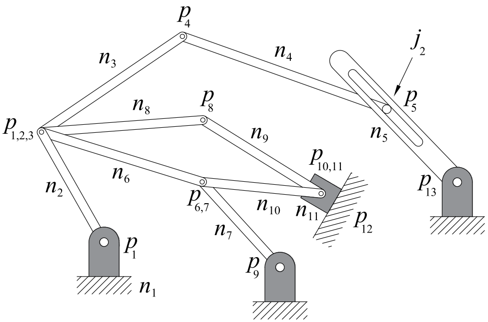


$$d = 2,\ n = 3, \mu_i = 1, p = 3, \zeta = 0,$$
$$m = 2(3-1) - 3(1) - 0 = 1.$$

The Kutzbach mobility criterion must used very carefully and the results

considered with healthy suspicion!

j) $n = 11$
$\zeta = 0$
$j_1 = 13$
$j_2 = 1$
$m = 3(11\text{-}1)\text{-}2(13)\text{-}1$
$= 3$



Example 4.2

Determine the simplest possible single DOF mechanisms that use only single DOF kinematic pairs

Solution: $j_2 = 0$ & m = 1

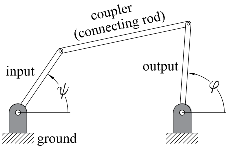
m = 3(n-1)- 2j_1 - j_2
1 = 3n - 3 - 2j_1
3n - 2j_1 = 4
solve for n;

$n = \frac{4+2j_1}{3}$

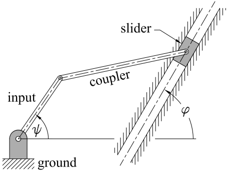
j_1	n	
1	2	Not a mechanism(Kinematic chain not closed)
2	8/3	Not possible (# of link must be integer)
3	10/3	Not possible (# of link must be integer)
4	4	First possible condition

Therefore the simplest mechanism must have 4 links and 4 joints
Three configurations possible such that the coupler translates and rotates:

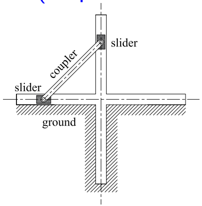
Planar 4R



RRRP (Slider-crank)



PRRP (Elliptical trammel)



A similar form of the Kutzbach criterion is available for spatial linkages.

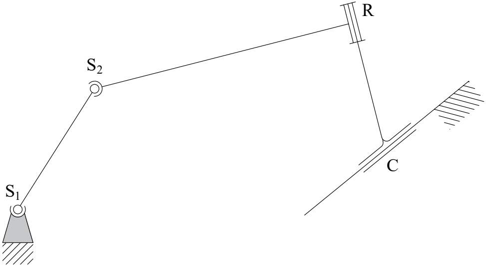
$$m = 6(n-1) - 5j_1 - 4j_2 - 3j_3 - 2j_4 - j_5 - \zeta \tag{3}$$

where,

n:	number of links
j_1:	number of pairs with 1 DOF
j_2:	number of pairs with 2 DOF
j_3:	number of pairs with 3 DOF
j_4:	number of pairs with 4 DOF
j_5:	number of pairs with 5 DOF
ζ:	number of idle DOF

Consider a spatial SSRC linkage. The bar connecting the two S-pairs can rotate along its axis without affecting the relation between input and output and is therefore an idle DOF. Here we will use the generalised Kutzbach criterion.

k)
$$\begin{aligned} d &= 6 \\ n &= 4 \\ j_1 &= 1 \\ j_2 &= 1 \\ j_3 &= 2 \\ \zeta &= 1 \\ m &= 6(4-1) - 5(1) - 4(1) - 3(2) - 1 \\ &= 2 \end{aligned}$$

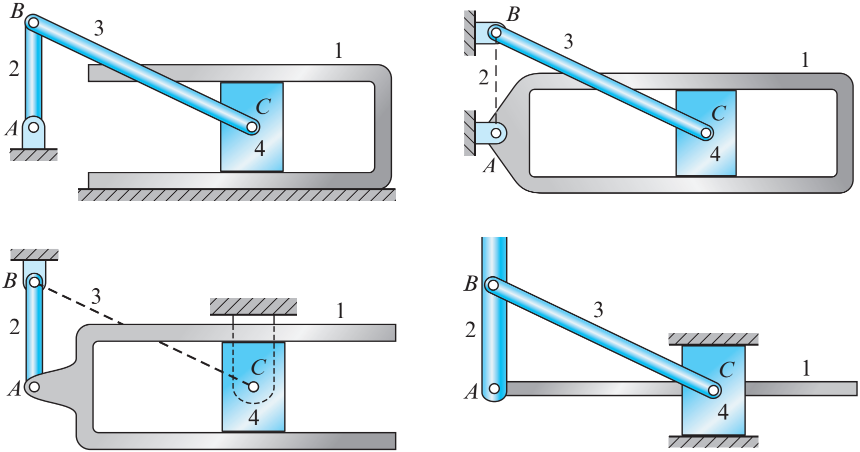


4.5 Kinematic Inversion

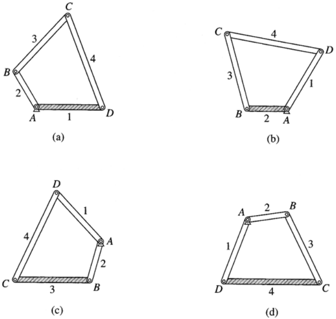
A simple closed kinematic chain becomes a mechanism when one link is fixed to the ground.

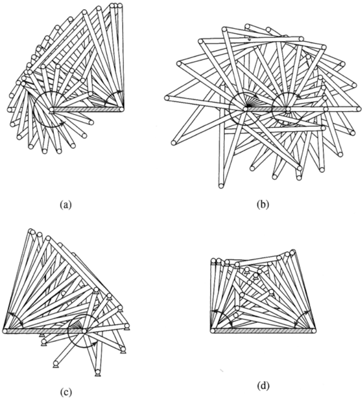
- Changing the link that is fixed creates kinematic inversions of the mechanism.
- The number of possible kinematic inversions equals the number of links in the mechanism.
- Kinematic inversions of a mechanism have the same relative motions between links.
- But, the absolute motions , i.e. those relative to the fixed link, can be distinctly different.

Four inversions of the slider-crank mechanism

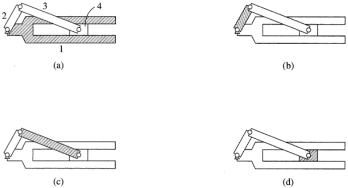


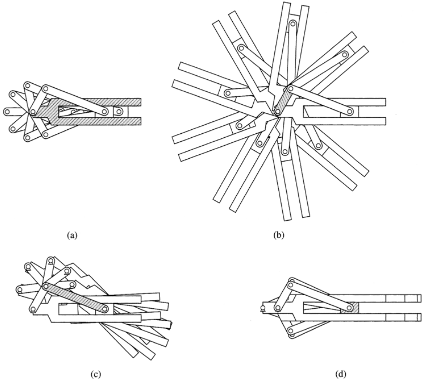
Evolution Diagrams





Evolution Diagrams

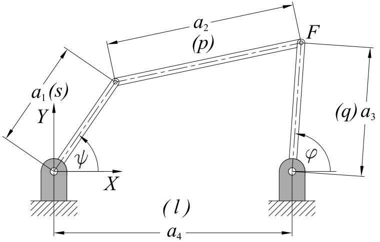




4.6 The Grashof Condition

- Important design consideration: condition on link lengths in a planar 4R (4-bar) to have at least one link with the ability to fully rotate through 360°, making it a crank
- To test if continuous relative rotation between two members of a 4R (4-bar) mechanism exists, Grashof's inequality can be used:

$$s + l < p + q \qquad (4)$$

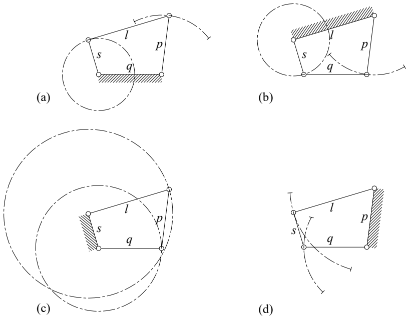


where

s = length of the shortest link
l = length of the longest link
p,q = lengths of intermediate-length links

It is important to satisfy this condition if the mechanism is to be driven by a unidirectional rotary actuator (i.e. non-reversing motor)

4 possible inversions of the 4-bar linkage satisfying Grashof's inequality:



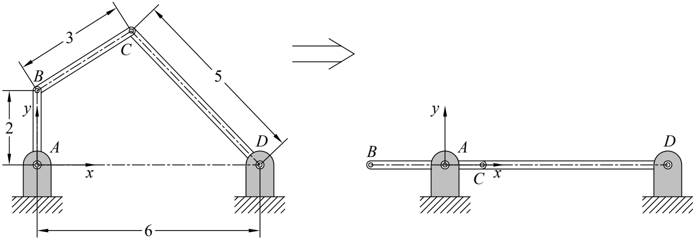
Four inversions of a Grashof chain: (a,b) crank-rocker mechanisms; (c) drag-link mechanism; (d) double-rocker mechanism.

Naming Convention:

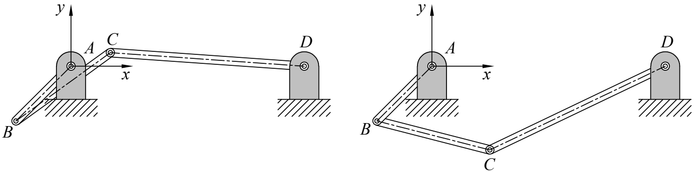
Name	Condition
Crank-Rocker(a,b)	s jointed to base link
Drag link / double crank (c)	s is the base link
Double rocker (d)	s is the coupler link

In all of the above s has a continuous relative rotation

- If $s + l = p + q$ then it can assume a flattened (singular folding) configuration, mechanism is called "Grashof's neutral linkage", "transition linkage", or "folding linkage"

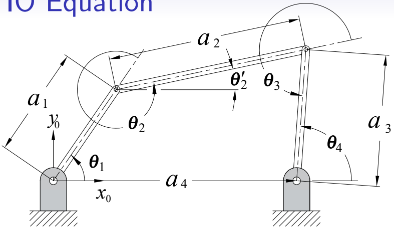


- From the flattened configuration (ignoring inertia) the coupler and output links can randomly switch between either of two modes (generally undesirable in mechanism design)



The Freudenstein IO Equation

- A trigonometric equation that relates the input and output link angles θ_1 and θ_4 in terms of the link lengths a_1, a_2, a_3, and a_4 is



$$k_1 + k_2 \cos\theta_4 - k_3 \cos\theta_1 = \cos(\theta_4 - \theta_1), \quad (5)$$

[1]

where:

$$k_1 = \frac{a_1^2 - a_2^2 + a_3^2 + a_4^2}{2a_1 a_3};$$

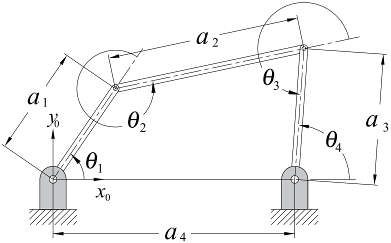
$$k_2 = \frac{a_4}{a_1};$$

$$k_3 = \frac{a_4}{a_3}.$$

[1] F. Freudenstein, "An Analytical Approach to the Design of Four-link Mechanisms", *Trans. ASME*, vol. 77, pp. 483-492, 1954

The Six v_i-v_j IO Equations

- Using relative angles, it is possible to derive an algebraic equation that relates the input and output link angle parameters $v_1 = \tan\left(\frac{\theta_1}{2}\right)$ and $v_4 = \tan\left(\frac{\theta_4}{2}\right)$ in terms of the link lengths a_1, a_2, a_3, and a_4, which is



$$Av_1^2v_4^2 + Bv_1^2 + Cv_4^2 - 8a_1a_3v_1v_4 + D = 0, \quad (6)$$ [2]

where:

$$\begin{aligned}
A &= (a_1 - a_2 - a_3 + a_4)(a_1 + a_2 - a_3 + a_4) = A_1A_2;\\
B &= (a_1 - a_2 + a_3 + a_4)(a_1 + a_2 + a_3 + a_4) = B_1B_2;\\
C &= (a_1 - a_2 + a_3 - a_4)(a_1 + a_2 + a_3 - a_4) = C_1C_2;\\
D &= (a_1 + a_2 - a_3 - a_4)(a_1 - a_2 - a_3 - a_4) = D_1D_2;\\
v_1 &= \tan\left(\frac{\theta_1}{2}\right); \qquad v_4 = \tan\left(\frac{\theta_4}{2}\right).
\end{aligned}$$

[2] M.J.D. Hayes, M. Rotzoll, Q. Bucciol, Z.A. Copeland, 2023, "Planar and Spherical Four-bar Linkage vi–vj Algebraic Input-output Equations", Mechanism and Machine Theory, Vol. 182, April 2023

The Six v_i-v_j IO Equations

Since there are six ways to distinctly pair four relative angles in a planar quadrilateral $4R$ linkage ($4R$ means the four links are connected with four R-pairs), there are five additional IO equations:

$$A_1B_1v_1^2v_2^2 + A_2B_2v_1^2 + C_1D_2v_2^2 + 8a_2a_4v_1v_2 + C_2D_1 = 0, \tag{7}$$

$$A_2B_1v_1^2v_3^2 + A_1B_2v_1^2 + C_1D_1v_3^2 + C_2D_2 = 0, \tag{8}$$

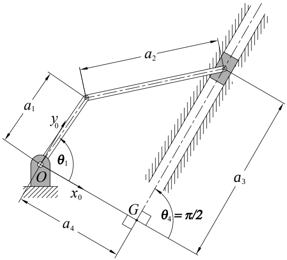
$$B_1C_1v_2^2v_3^2 + A_1D_2v_2^2 + A_2D_1v_3^2 - 8a_1a_3v_2v_3 + B_2C_2 = 0, \tag{9}$$

$$A_1C_1v_2^2v_4^2 + B_1D_2v_2^2 + A_2C_2v_4^2 + B_2D_1 = 0, \tag{10}$$

$$A_2C_1v_3^2v_4^2 + B_1D_1v_3^2 + A_1C_2v_4^2 - 8a_2a_4v_3v_4 + B_2D_2 = 0. \tag{11}$$

Planar RRRP Algebraic IO Equation

- Replacing the 4^{th} R-pair in a planar 4R with a P-pair gives a *crank-slider*, an RRRP mechanism, but the *crank* isn't always a crank.
- The input link can instead rock somewhere in the range $\pi < \theta_1 > 0$, or it can cross the x_0-axis through π or 0.
- These are *rockers*, π-*rockers*, and 0-*rockers*.



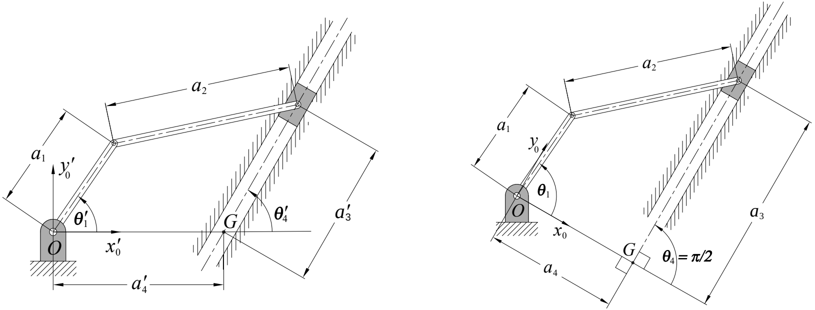
  - The IO equation of an RRRP mechanism can be obtained by simply recollecting the terms in Equation (6) of the planar 4R exchanging the now constant variable v_4 with the now variable slider displacement a_3 which gives:

$$v_1^2 a_3^2 + R v_1^2 + a_3^2 - 4 a_1 v_1 a_3 + S = 0, \tag{12}$$

where

$$\begin{aligned} R &= R_1 R_2 = (a_1 + a_2 + a_4)(a_1 - a_2 + a_4), \\ S &= S_1 S_2 = (a_1 + a_2 - a_4)(a_1 - a_2 - a_4), \\ v_1 &= \tan\left(\frac{\theta_1}{2}\right); \quad v_4 = \tan\left(\frac{\theta_4}{2}\right) = \tan\left(\frac{\pi/2}{2}\right) = 1. \end{aligned}$$

Planar RRRP Trigonometric IO Equation



- If the RRRP linkage is not given with the reference frame x-axis (or y-axis) perpendicular to the slider direction of travel, we can always rotate the coordinate system so that it is, but it changes some dimensions.
- Then we can derive a trigonometric $a_3 = f(\theta_1)$ IO equation (we will in another lecture):

$$a_3 \quad = \quad a_1 \sin(\theta_1) \pm \sqrt{a_2^2 - (a_4 - a_1 \cos(\theta_1))^2}. \qquad (13)$$

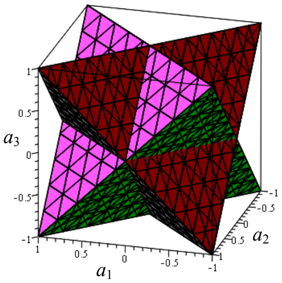
4.7 Function Generation

- A planar 4R mechanism can be designed so that the output parameter is a function of the input parameter using either Equation (5) for functions of the form $\theta_4 = f(\theta_1)$, and Equation (6) for functions of the form $v_4 = f(v_1)$.
- These are called *input-output* (IO) equations: Equation (5) is a trigonometric IO equation; and Equation (6) is an algebraic IO equation.
- Without loss in generality the four link lengths of a 4R function generator can be uniformly scaled up or down in length without changing the function.
- We can consider the four a_i to be the four homogeneous coordinates of a projective 3-D parameter space.
- As long as $a_4 \neq 0$ we can abstractly project the values of a_1, a_2, and a_3 into the special plane $a_4 = 1$ by dividing all four coordinates by the value of a_4 and impose the Euclidean metric on the 3-D space.
- In this design parameter space every distinct point represents a distinct function generating linkage.
- The location of the point determines the function and the mobility limits, if they exist, on the input and output links.

Planar 4R Design Parameter Space

- The eight bilinear factors from the coefficients in the IO equation, projected into the special plane $a_4 = 1$ can be thought of as the equations of eight different planes

$$\begin{aligned}
A &= (a_1 - a_2 - a_3 + 1)(a_1 + a_2 - a_3 + 1) = A_1 A_2;\\
B &= (a_1 - a_2 + a_3 + 1)(a_1 + a_2 + a_3 + 1) = B_1 B_2;\\
C &= (a_1 - a_2 + a_3 - 1)(a_1 + a_2 + a_3 - 1) = C_1 C_2;\\
D &= (a_1 + a_2 - a_3 - 1)(a_1 - a_2 - a_3 - 1) = D_1 D_2;
\end{aligned}$$



Exact Kinematic Synthesis

- Kinematic synthesis of a given type of mechanism involves determining the a_i link lengths required to generate a desired motion.
- We will now consider kinematic synthesis for function generation.
- The functions we can generate will be realised by the relative motion of the input and output links.
- For a planar 4R the function that must be realised by the linkage is

$$\theta_4 = f(\theta_1), \quad \text{or} \quad v_4 = f(\theta_1), \quad \text{or} \quad v_4 = f(v_1)$$

  and the linkage defined by the three ratios of a_i link lengths must satisfy both the function and either Equation (5) or Equation (6) over a specified input range.
- For a planar RRRP the function that must be realised by the linkage is

$$a_3 = f(\theta_1), \quad \text{or} \quad a_3 = f(v_1)$$

  and the linkage defined by the three link lengths a_1, a_2, and a_4 must satisfy both the function and either Equation (12) or Equation (13) over a specified input range $v_1 = \tan(\theta_1/2)$, or θ_1.

Example 4.3: Planar 4R Exact Kinematic Synthesis

- Design a planar 4R mechanism that will generate the function

$$v_4 = 6\sin(\theta_1) - 1$$

over the input angle range $0 \leq \theta_1 \leq \pi/2$

- The IO equation for a planar 4R mechanism is Equation (6)
- It has four unknown link lengths, but only the ratios of the link lengths are needed to identify a mechanism that will approximately generate the desired function
- This is because the function is insensitive to the overall scale of the linkage, so we need only solve for a_1, a_2, and a_3 in terms of a_4
- We can specify three pairs of θ_1 and θ_4 that exactly satisfy the desired function: these are typically called *accuracy points*.
- Let's choose a uniform distribution of accuracy points over the desired input angle range as

$$v_1 = 0, \; \tan\left(\frac{\pi/4}{2}\right), \; \tan\left(\frac{\pi/2}{2}\right)$$

- The values of the output angle parameter, v_4, for each of the specified input angle parameters v_1 as

$$\begin{aligned} v_{4_i} &= 6\sin\left(2\tan^{-1}(v_{1_i})\right) - 1 \\ &= -1,\ 3.242640688,\ 5 \end{aligned}$$

so that

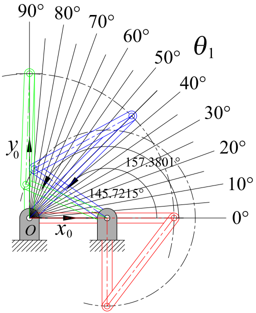
$$\begin{aligned} \theta_{4_i} &= \left(2\tan^{-1}(v_{4_i})\right)^{\circ} \\ &= -90^{\circ},\ 145.7214555^{\circ},\ 157.3801350^{\circ} \end{aligned}$$

- Obtain three equations by substituting the IO pairs of each v_{1_i}, v_{4_i} into Equation (6).
- Solving the three equations for a_1, a_2, and a_3 in Maple yields three distinct solutions where

$$\begin{aligned} &a_1 = (1.861308073)a_4, && a_2 = (1.425334098)a_4, && a_3 = (1.135660907)a_4, \\ &a_1 = 0, && a_2 = -a_4, && a_3 = 0, \\ &a_1 = 0, && a_2 = a_4, && a_3 = 0. \end{aligned}$$

Analysis of Synthesised 4R Function Generator

- The function generator identified exactly generates the desired function at the three specified accuracy points uniformly distributed over the specified input range.

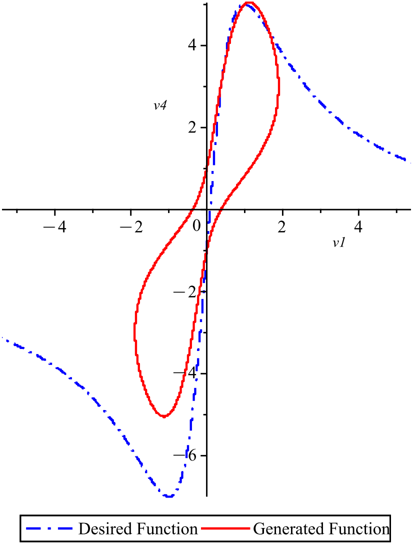


Function: $\theta_4 = 6\,sin(\theta_1)$ -1

Uniform accuracy point distribution

	Input	Output
$a_1 = 1.861308073$		
$a_2 = 1.425334098$	$\theta_1 = 0°$	$\theta_4 = -90°$
$a_3 = 1.135660907$	$\theta_1 = 45°$	$\theta_4 = 145.7215°$
$a_4 = 1.000000000$	$\theta_1 = 90°$	$\theta_4 = 157.3801°$

Analysis of Synthesised 4R Function Generator



- We see that to go from the first IO angle pair $(\theta_1, \theta_4) = (0^\circ, -90^\circ)$, which is $(v_1, v_4) = (0, -1)$, to the IO next IO pair $(\theta_1, \theta_4) = (45^\circ, 145.7515^\circ)$, $(v_1, v_4) = (0.4142, 3.2426)$, the output first has to pass through the final IO pair $(\theta_1, \theta_4) = (90^\circ, 157.3801^\circ)$, $(v_1, v_4) = (1, 5)$.
- The identified mechanism doesn't generate the desired function in the sense that it can't generate the function in the desired order.
- This order problem can often be resolved by increasing the cardinality of the IO data set: making the number of IO pairs that satisfy the function to be a number $>> 3$.
- This process is known as *approximate synthesis* and the solution is obtained with numerical error minimisation.

Example 4.4: Planar RRRP Exact Kinematic Synthesis

- Design a planar RRRP mechanism that will generate the same function as the planar 4R, but now with

$$a_3 = 6\sin(\theta_1) - 1$$

over the same input angle range $0 \leq \theta_1 \leq \pi/2$.

- The IO equation for a planar RRRP mechanism is Equation (12) or (13)
- It has three unknown link lengths, a_1, a_2, and a_4.
- Again, let's choose a uniform distribution of accuracy points over the desired input angle range as

$$v_{1_i} = 0, \ \tan\left(\frac{\pi/4}{2}\right), \ \tan\left(\frac{\pi/2}{2}\right)$$

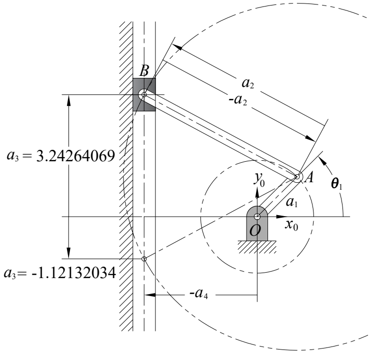
- The values of the output angle parameter, a_3, for each of the specified input angle parameters v_1 are the same as for the previous example

$$\begin{aligned} a_{3_i} &= 6\sin\left(2\tan^{-1}(v_{1_i})\right) - 1 \\ &= -1, \ 3.242640688, \ 5 \end{aligned}$$

- Obtain three equations by substituting the IO pairs of each v_{1_i}, a_{3_i} into Equation (12).
- Solving the three equations for a_1, a_2, and a_4 in Maple yields two distinct solutions where

$$a_1 = 1.500000000, \quad a_2 = 4.609772229, \quad a_4 = -3.000000000, \text{ and}$$
$$a_1 = 1.500000000, \quad a_2 = -4.609772229, \quad a_4 = -3.000000000.$$

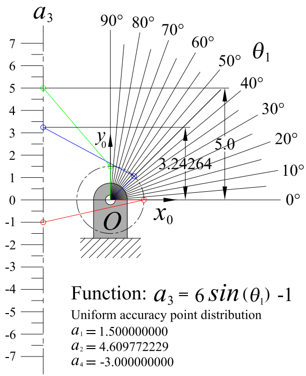
- Let's select the second solution
- Note that $a_2 < 0$ and $a_4 < 0$, what does this mean?



- We see that in our selected coordinate system where the x_0- and y_0-axes are perpendicular and parallel, respectively, to the slider direction of travel
- $a_4 < 0$ simply means that the slider longitudinal centre line is directed -3 units along the x_0-axis
- $a_2 < 0$ means the coupler points from B to A instead of from A to B.

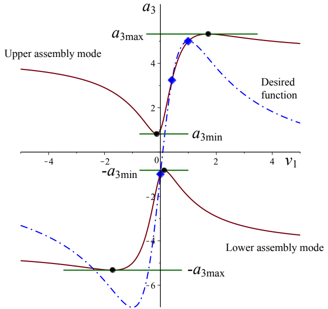
Example 4.4: Analysis of Synthesised RRRP Function Generator

- The function generator identified exactly generates the desired function at the three specified accuracy points uniformly distributed over the specified input range.



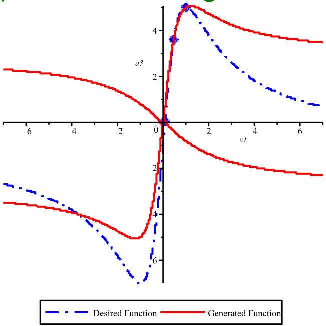
- It can be shown that the input link is a crank.
- But, can the linkage generate all three IO values without needing to be taken apart and reassembled in a different assembly mode?
- The *Circuit Defect* (also known as the *Branch Defect*) refers to a completed linkage design that meets all of the prescribed requirements at all three precision input angles, but cannot be moved continuously between all three prescribed IO values without being taken apart and reassembled in a different assembly mode.

Example 4.4: Circuit Defect



- The synthesised crank-slider has two assembly modes.
- The first precision input angle function value requires $a_3 = -1$, which is in the lower assembly mode, while the remaining two are in the upper assembly mode.
- This linkage suffers from the circuit defect.
- One way to overcome this defect is to decrease the desired input angle range from 0°-90° to 10°-90° and recompute the link lengths to see if the circuit defect is adequately resolved.

Example 4.4: Resolving Circuit Defect



- Here we have reduced the specified input range to 10°-90°.
- Solving the three equations for a_1, a_2, and a_4 in Maple yields two distinct solutions where

$$a_1 = 2.096134490, \quad a_2 = \pm 3.066433626, \quad a_4 = -0.9851803279$$

- It is clear the circuit defect has been resolved at the expense of a smaller IO range.