MAAE 3004
Dynamics of Machinery

Lecture Slide Set 9

Balancing of Mechanisms

Department of Mechanical and Aerospace Engineering
Carleton University

Outline

Introduction

Inertia forces and moments exist whenever parts having finite mass / mass moment of inertia are accelerated. Recall that:

$$\vec{F}_I = \text{-}m\vec{a}_G, \text{ and } \vec{M}_I = \text{-}I_G\vec{\alpha}$$

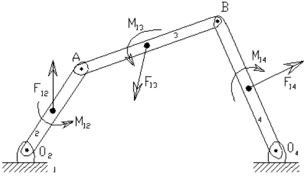
These are important for two reasons:

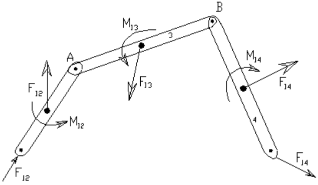
- components must be designed with sufficient internal strength such that they can perform satisfactorily when subjected to these forces
- these inertia forces & moments are transmitted to the support or frame of the mechanism. This can give rise to serious vibrations of the machine parts or the whole machine

Common examples:

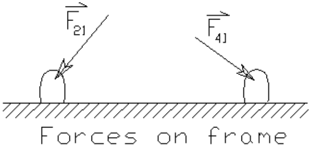
- a reciprocating piston driven by a crank in an RRRP-mechanism.
- Automotive, aircraft, compressors, pumps.
- Machining, manufacturing, assembly, packaging.

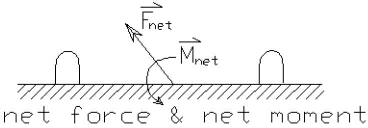
- four-bar linkage in motion





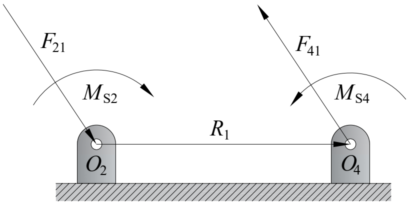
- F_{I2}, F_{I3}, and F_{I4} are inertia forces.
- Inertia forces typically vary over time in magnitude and/or direction causing visible shaking motions of the support.
- $\vec{F}_{net}$ and $\vec{M}_{net}$ are usually referred to as shaking forces and moments.
- Shaking motion is undesirable.
- Requires balancing of machinery.





Shaking Forces and Moments

- Even when counterweights are placed on links 2 and 4, the shaking forces do not disappear entirely.
- It can be shown that the reaction forces acting on the input and output links are equal in magnitude, but oppositely directed.
- The equal and opposite pairs of forces acting on the frame-fixed *R*-pairs at each instant of time as the mechanism moves create a time-varying shaking couple that rocks the frame.
- The trade-off is that the resulting bearing forces can be larger due to the balance weights, thereby increasing the shaking couple compared to the unbalanced linkage.
- The stresses in the links and joints may also increase as a result of force balancing.



Shaking Forces and Moments

- The shaking moment $\mathbf{M}_{S2}$ acting about the input link R-pair centre O_2 is the sum of the reaction torque $\mathbf{T}_{21}$ and the shaking couple, ignoring externally applied loads

$$\mathbf{M}_{S2} = \mathbf{T}_{21} + (\mathbf{R}_1 \times \mathbf{F}_{41})$$

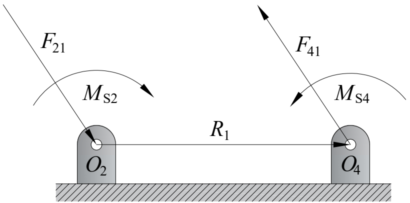
where

$\mathbf{T}_{21}$ is the negative of the motor driving torque $\mathbf{T}_{12}$,

$\mathbf{R}_1$ is the position vector from O_2 to O_4, i.e. link 1,

and $\mathbf{F}_{41}$ is the force of the output link acting on the frame.

- The magnitude of the shaking moment can be reduced but not eliminated by redistribution of mass in the mechanism links.



Balancing of Machinery

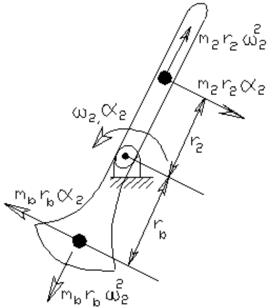
- In the four-bar linkage the inertia forces for links 2 and 4 (the input and output) can be eliminated quite easily, since they are undergoing rotation about a fixed axis and the centre of mass can be moved to the axis of rotation
- e.g., consider link 2
- add counterweight as shown, then

$$m_2 r_2 {\omega_2}^2 = m_b r_b {\omega_2}^2$$

or

$$m_2 r_2 = m_b r_b$$

- centre of mass of link 3 (the coupler, or connecting rod) has a more complicated motion and hence more difficult to balance
- try to replace link 3 with a dynamically equivalent link



- **Dynamic equivalence:** Two systems are said to be dynamically equivalent if under the action of a given system of forces both systems have the same accelerations.
- to continue with this approach, replace link 3 with two concentrated masses attached with a rigid massless rod

- to determine dynamically equivalent link for link 3, introduce two masses m_A and m_B at the two ends of the massless rod AB such that,

$$m = m_A + m_B \tag{1}$$

and the same position for the centre of mass, i.e.,

$$m_A\overline{AG} - m_B\overline{BG} = 0 \tag{2}$$

and the same mass moment of inertia about the centre of mass, i.e.,

$$I_G = m_A(\overline{AG})^2 + m_B(\overline{BG})^2 \tag{3}$$

then from Eqs. (1) and (2), we can get:

$$m_A = \left(\frac{\overline{BG}}{\overline{BA}}\right) m, \qquad m_B = \left(\frac{\overline{AG}}{\overline{AB}}\right) m \tag{4}$$

Note:

By constraining the concentrated masses to be located at points A & B, Eqs. (1–3) become an over-determined set of equations (i.e., 3 equations and 2 unknowns m_A and m_B). In many cases, particularly for connecting rods used in automotive and aircraft piston engines, a good approximation results from satisfying only equations 1 and 2. As the result, an error generally results in the mass moment of inertia of the "equivalent member"

Example 9.2

Consider a connecting rod with weight = 15 N; length = 24 cm and mass moment of inertia: $I_G = 0.016$ kgm^2 and CG at 6 cm from end A.
Design a dynamically equivalent balanced rod and determine the % error in the mass moment of inertia of the equivalent rod.

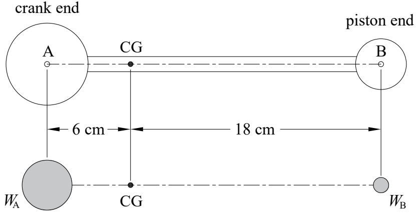
Solution:

m = (15 N)/(9.81 m/s^2) = 1.53 kg

From Equation (4):

$$m_A = \left(\frac{18}{6+18}\right) m = 0.75m = 1.15 \text{ kg}$$

$$m_B = \left(\frac{6}{6+18}\right) m = 0.25m = 0.38 \text{ kg}$$



From Equation (3), we can estimate error in mass-moment of inertia

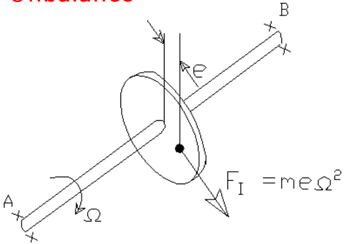
$$I_G = m_A(0.06 \text{ m})^2 + m_B(0.18 \text{ m})^2 = 0.00414 \text{ kgm}^2 + 0.01231 \text{ kgm}^2 = 0.01645 \text{ kgm}^2$$

$$\text{Error in } I_G = \frac{0.01645 - 0.016}{0.01645} = 0.02825 = 2.825\%$$

Balancing of Rotating Shafts

Static (or "single plane" or "single force") Unbalance

In some rotating systems the unbalanced forces exist in only a single transverse plane, such as in fans, gears, disks, propellers, automobile wheels, etc.



- The eccentricity of the centre of mass (CM) is e.
- This class of rotating unbalance is easy to detect. If the shaft is placed on knife-edged bearings, the shaft / disk will always come to rest at a position with the CM directly below the bearing centre line. This is a static test (hence the name is "static unbalance")
- Without balancing, the inertia force is transmitted to bearings A & B. We want to avoid this!
- Therefore, we add or subtract a "balance" mass or counter weight (cw) in the plane of the disk such that $\vec{F}_I + \vec{F}_{cw} = 0$, where $\vec{F}_{cw}$ is the force generated by the balancing mass.

- The measure of static unbalance is "*me*" with units of (kg-mm) or (oz-in)
- Large shaking forces can be transferred to structures due to relatively small values of *me*, as we have seen in the vibration lectures (see Slide Set 2).
- As a result, it becomes necessary that these forces are balanced.

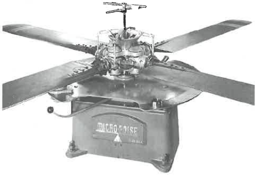


Figure 15.4 Aircraft propeller assembly balancer. (Courtesy of Schenck Ro Tec Corporation, Auburn Hills, MI.)

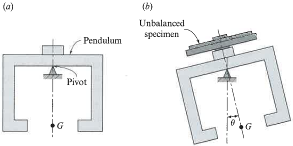


Figure 15.5 Operation of a static balancing machine.

Example 9.3

The rotor in an aircraft gas turbine, having a mass of m = 180 kg operates at 16000 rev/min. If the CM has an eccentricity of 0.025mm from the axis of rotation, determine the unbalance force.

Solution:

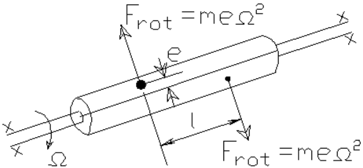
This results in an unbalance of
$me = (180 \text{ kg})(0.025\text{mm}) = 4.5 \text{ kg-mm}$
that would in turn a cause a centrifugal force of
$F_{rot} = me\omega^2 = (180\text{kg})(0.025\times10^{-3}\text{m})(16000 \times \frac{2\pi}{60})^2 = 12630 \text{ N} \approx 2800 \text{ lbf}$

In this case centrifugal force is the unbalance force. Such a force could potentially damage the machine. It would be difficult and very time consuming to manufacture the rotor of a machine, so that the centre of mass would lie within 0.025mm from the axis of rotation. Therefore, the rotor in this case would be balanced after manufacture. This balancing is done experimentally.

Dynamic (or "moment" or "two plane") Unbalance:

- Dynamic unbalance occurs in equipment which has mass distributed over large axial lengths. For example, automobile crankshafts and multistage turbine rotors.
- Dynamic unbalance occurs when an unbalance moment (i.e., a moment caused by an unbalanced inertia force) exists in a rotating body.
- This can occur even when the rotor is statically balanced, i.e.,

$\sum \vec{F}_I = \vec{0}$ (statically balanced),
but there is a net moment of
$\sum \vec{M}_I = me\omega^2 l$



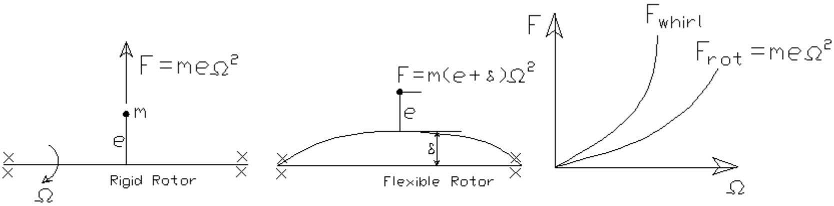
- A static balancing procedure would not correct this type of unbalance, and a statically balanced rotor may perform very poorly under actual operating conditions.
- Dynamic unbalance can be corrected with two (or more) balance weights to create a moment equal and opposite to $me\omega^2 l$.
  Note that if the rotor is sufficiently rigid, this need not to be in the plane defined by F_{rot}.
- The measure of this unbalance is "mel" in (kg-mm^2) or (lb-in^2)

General Unbalance:

A rotor is both statically and dynamically unbalanced, that is $\sum \vec{F} \neq \vec{0}$ and $\sum \vec{M} \neq \vec{0}$, i.e., (both shaking force and shaking couple exist).

Consequences of Unbalance

- Large bearing forces lead to bearing fatigue.
- Large structural forces in turn lead to structural fatigue.
- "Whirling" can occur if rotor stiffness is too low, see Example 2.2 in Lecture Slide Set 2.
- This leads to even larger displacements and therefore larger bearing forces.



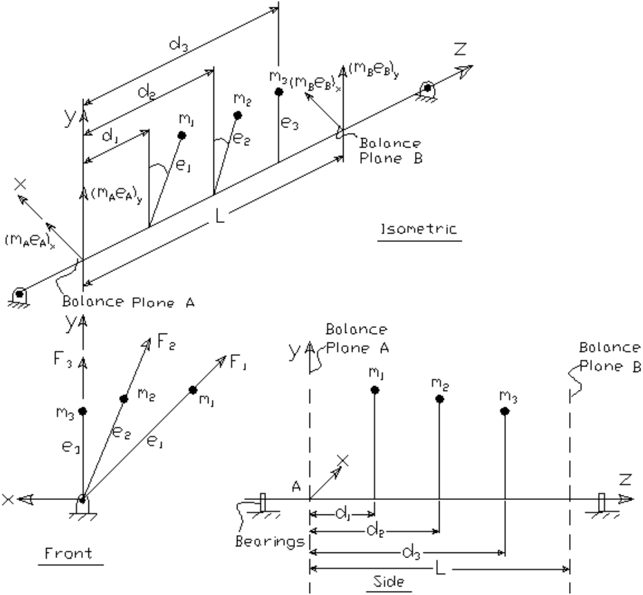
Analysis of Rotor Balancing

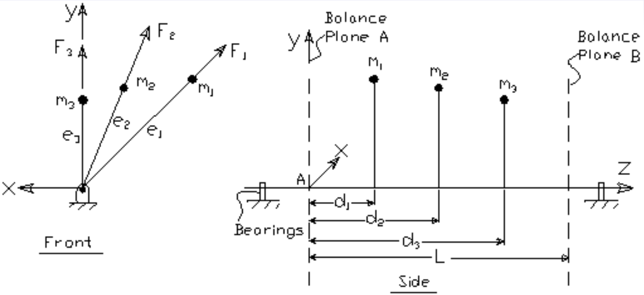
Assumptions:

Rigid shaft $\Rightarrow$ deflections are assumed to be zero $\Rightarrow$ static analysis.

The most general case of distribution of unbalanced rotating masses on a rigid rotor occurs when the masses lie in various transverse and axial planes.

General Unbalance





For complete balance of the rotor,

$$\sum \vec{F} = \vec{0}, \quad \text{and} \quad \sum \vec{M} = \vec{0}$$

$$\vec{F} = \sum(m_i \vec{e}_i \omega^2) = \sum(m_i \vec{e}_i)\omega^2 = \vec{0},$$

but since $\omega \neq \vec{0}$

$$\Rightarrow \qquad \sum(m_i \vec{e}_i) = \vec{0} \tag{5}$$

Also

$$\sum \vec{M} = \sum(\vec{d}_i \times m_i \vec{e}_i)\omega^2 = \vec{0}$$

but since $\omega \neq \vec{0}$

$$\Rightarrow \qquad \sum(\vec{d}_i \times m_i \vec{e}_i) = \vec{0} \tag{6}$$

Note:

- The moment arm of any given inertia force is d_i and it is always along the shaft, so it can be treated as a scalar, i.e., Eq (6) can be written as:

  $\sum \vec{M} = \sum(m_i \vec{e}_i d_i)\omega^2 = \vec{0}$

  Note that $\vec{d}_1 \times (m_1 \vec{e}_1 \omega^2) + \vec{d}_2 \times (m_2 \vec{e}_2 \omega^2) + \ldots = \vec{0}$

  $\Rightarrow d_1 \vec{k} \times (m_1 \vec{e}_1 \omega^2) + d_2 \vec{k} \times (m_2 \vec{e}_2 \omega^2) + \ldots = \vec{0}$

  $\Rightarrow \vec{k} \times (d_1 m_1 \vec{e}_1 + d_2 m_2 \vec{e}_2 + \ldots) = \vec{0}$

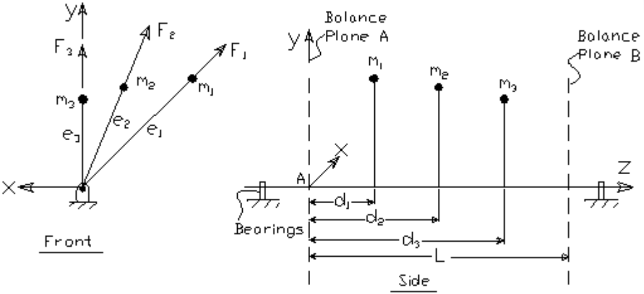
  $\therefore d_1 m_1 \vec{e}_1 + d_2 m_2 \vec{e}_2 + \ldots = \vec{0}$

  i.e.,

  $\sum(m_i d_i)\vec{e}_i = \vec{0}$

- For a rotor with masses which lie in multiple transverse axial planes, in order to satisfy both $\sum F = 0$ and $\sum M = 0$, a minimum of two balancing masses are required.

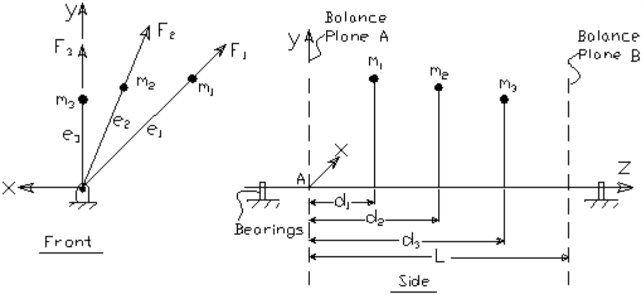
Analytical Method



- Determine the planes A and B as balance planes.
- Consider the equilibrium of moments about plane A in terms of the x and y components:

$$\sum M_{Ax} = 0 \;\Rightarrow\; \underset{\text{known}}{\underset{\Downarrow}{(m_1e_1)_y d_1}} + \underset{\text{known}}{\underset{\Downarrow}{(m_2e_2)_y d_2}} + \ldots + \underset{\text{solve for } m_B e_{B_y}}{\underset{\Downarrow}{(m_Be_B)_y L}} = 0$$

$$\sum M_{Ay} = 0 \;\Rightarrow\; \underset{\text{known}}{\underset{\Downarrow}{(m_1e_1)_x d_1}} + \underset{\text{known}}{\underset{\Downarrow}{(m_2e_2)_x d_2}} + \ldots + \underset{\text{solve for } m_B e_{B_x}}{\underset{\Downarrow}{(m_Be_B)_x L}} = 0$$



- Next, consider the equilibrium of forces:

$$\sum F_x = 0 \;\Rightarrow\; \underset{\text{known}}{(m_1 e_1)_x} + \underset{\text{known}}{(m_2 e_2)_x} + \ldots + \underset{\text{found}}{(m_B e_B)_x} + \underset{\text{solve for } m_A e_{A_x}}{(m_A e_A)_x} = 0$$

$$\sum F_y = 0 \;\Rightarrow\; \underset{\text{known}}{(m_1 e_1)_y} + \underset{\text{known}}{(m_2 e_2)_y} + \ldots + \underset{\text{found}}{(m_B e_B)_y} + \underset{\text{solve for } m_A e_{A_y}}{(m_A e_A)_y} = 0$$

Note:

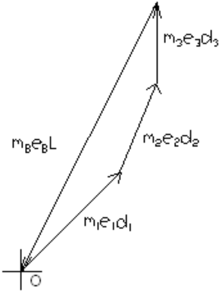
- Distances d_i and e_i can be positive or negative values depending on their directions relative to the positive coordinate directions.
- When we add the mass m_A to balance the forces, it must be added in plane A, otherwise it would destroy the balance of moments about plane A which was previously produced.
- Once broken into components, the analytical method only uses scalar equations

Graphical Method

- Consider equilibrium of moments we have:

$$\sum \vec{M}_A = \overrightarrow{m_1 d_1 e_1} + \overrightarrow{m_2 d_2 e_2} + \dots + \overrightarrow{m_B e_B L} = \vec{0}$$

$$\Downarrow \text{ known} \qquad \Downarrow \text{ known} \qquad \Downarrow \text{ unknown}$$



This is a vector equation; hence solve the vector polygon for $m_B e_B L$ and thus finally for $m_B e_B$.

- In the graphical method of balancing analysis, it is common to represent the moment vectors in the direction of inertia forces, i.e., if $d_i > 0$, then moment vector is drawn radially outward, and if $d_i < 0$, then moment vector is drawn radially inward
- The graphical method uses vector equations.

For force equilibrium

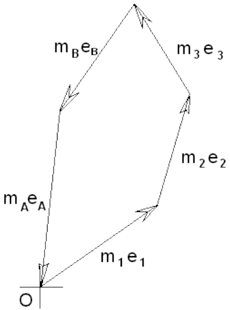
$\sum \vec{F} = 0$

$m_1\vec{e_1} + m_2\vec{e_2} + ... + m_B\vec{e_B} + m_A\vec{e_A} = \vec{0}$

$\Downarrow$ known, $\Downarrow$ known, $\Downarrow$ found, $\Downarrow$ unknown

known known found unknown

The unknown can be solved by using the force polygon



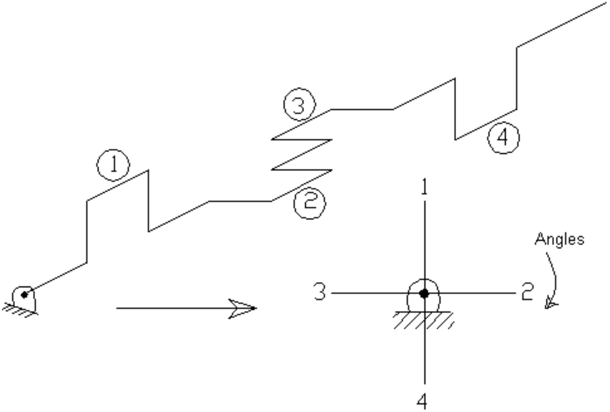
Comments:

- Balance can be achieved by adding masses in any two arbitrary transverse planes A and B.
- However, in practice, these planes are chosen as far apart as possible so as to reduce the amount of balancing mass needed.
- Likewise balancing masses are usually placed at the maximum possible eccentricity, e, such that the amount of mass required is as small as possible.
- Although any rotor can be balanced by adding a mass in each of the two transverse planes, such a method leaves a bending moment in the shaft.
- For this reason, it is usually desirable to balance each unbalance in the plane of unbalance, e.g., in an automobile crankshaft, each crank produces an unbalance, and frequently the crankshaft is balanced by a counter balance opposite each crank.

Example 9.4 Balancing the rotor of a V8 crankshaft

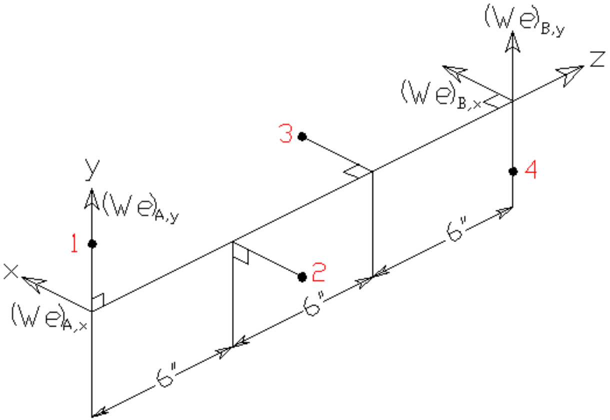
The crankshaft has four out of balance weights W, each representing a crank mass including equivalent connecting rod mass. Orientation of the cranks relative to crank 1 are: crank 1: 0^o; crank 2: 90^o; crank 3: 270^o; crank 4: 180^o. Also given are: $W = 4$ lbf, $e = 1.5$ in (stroke = 3 in).

Balance the rotor by using planes of cranks 1 and 4 as balancing planes.



Solution:

- For each mass: $We = mge = (4 \text{ lbf})(1.5") = 6$ lbf-in
- Assume masses are to be added in planes of cranks 1 & 4
- Determine "weight-eccentricity" product which must be added to crankshaft for complete balance.



Analytical Approach:

$$\circlearrowleft\!+ \quad \sum M_{A,y} = 0 \Rightarrow (We)_{B,x}(18in) + \underbrace{(We)_{4,x}(18in)}_{0} + (We)_{3,x}(12in) - (We)_{2,x}(6in) + \underbrace{(We)_{1,x}(0in)}_{0} = 0$$

$(We)_{B,x}(18in)$ + (6lbf-in)(12in) - (6lbf-in)(6in) = 0

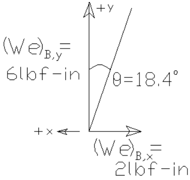
$(We)_{B,x} = -$ 2 lbf-in

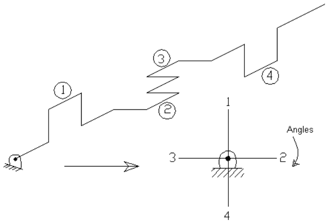
$$\circlearrowleft\!+ \quad \sum M_{A,x} = 0$$

$$\Rightarrow -(We)_{B,y}(18in) + (We)_{4,y}(18in) + \underbrace{(We)_{3,y}(12in)}_{0} + \underbrace{(We)_{2,y}(6in)}_{0} + \underbrace{(We)_{1,y}(0in)}_{0} = 0$$

$-(We)_{B,y}(18in)$ + (6lbf)(18in) = 0

$(We)_{B,y}$ = 6 lbf-in



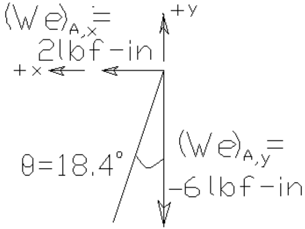


$$\sum F_x = 0 \Rightarrow (We)_{A,x} - 6 + 6 + (We)_{B,x} = 0$$

$$(We)_{A,x} = +\,2 \text{ lbf-in}$$

$$\sum F_y = 0 \Rightarrow (We)_{A,y} + 6 - 6 + (We)_{B,y} = 0$$

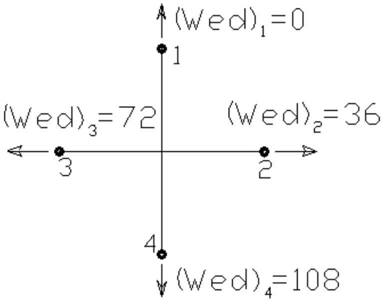
$$(We)_{A,y} = -\,6 \text{ lbf-in}$$

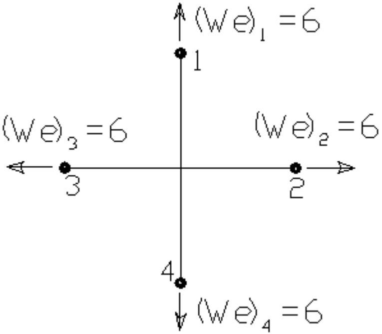


Note that $|\,(\vec{W}e)_B\,| = |\,(\vec{W}e)_A\,|$, i.e., all we did was add a couple, in other words crankshaft was initially balanced statically.

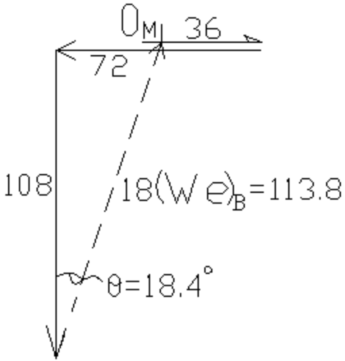
Graphical solution:

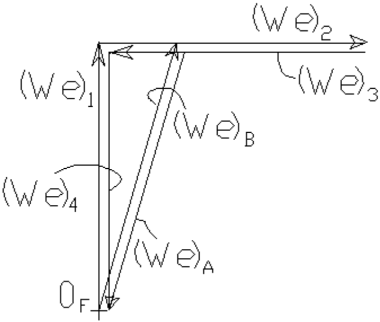
	W (lbf)	e (in)	We (lbf in)	d (in)	Wed (lbf in^2)
A	?	?	$(We)_A$	0	0
1	4	1.5	6	0	0
2	4	1.5	6	6	36
3	4	1.5	6	12	72
4	4	1.5	6	18	108
B	?	?	$(We)_B$	18	$18(We)_B$





	W (lbf)	e (in)	We (lbf in)	d (in)	Wed (lbf in^2)
A	?	?	$(We)_A$	0	0
1	4	1.5	6	0	0
2	4	1.5	6	6	36
3	4	1.5	6	12	72
4	4	1.5	6	18	108
B	?	?	$(We)_B$	18	$18(We)_B$





$18(We)_B = 113.8$

$\therefore (We)_B = 6.32$ lbf-in. @ $\theta = 18.4^o$ from $\sum M = 0$ diagram

Then in $\sum F = 0$ diagram draw $(We)_B = 6.32$ lbf-in. @ $\theta = 18.4^o$ and complete the diagram to obtain $(\vec{W}e)_A =$ -$(\vec{W}e)_B$

At 4000 rpm = 418.9 rad/sec

$F_{rot} = me\omega^2 = \frac{We}{g}\omega^2 = \frac{6.32}{(32.2)(12)}(418.9)^2 = 2870$ lbf

This force is to be provided either by counterweight or by bearings.