TENSEGRITY PLATFORMS AND GEOMETRIC FORM FINDING

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Abstract

Tensegrity systems are made up of extremely lightweight, adaptable structures that can maintain their stiffness while being deployed. These characteristics make them ideal for space applications, but they continue to have a low technology readiness level because they are more difficult to design than conventional structures. These design difficulties manifest themselves during the form finding process, where some geometric parameters for the tensegrity system are used to determine the stable con figuration of the resulting tensegrity system. Conventional form finding methods have been largely based on trigonometry and algebra, which is arguably more abstract than a purely geometric approach. The geometric intersection form finding method presented in this paper can be used to simplify the design and optimization of tensegrity systems, by using a purely geometric approach. The geometric intersection method uses the lengths of the tensegrity elements as the parameters for three geometric shapes. The coordinates of the intersection point between these shapes is the solution to the form finding problem. Certain combinations of element lengths will result in a non-stable tensegrity system. The geometric intersection method is unique, because it provides an intuitive rational for why these parameters result in non-real configuration and which element lengths should be altered to correct the problem. The geometric intersection method can also be more easily adapted to more complex tensegrity configurations then conventional form finding methods.

Introduction

Canada has a long history of improving the quality of life on Earth by developing unique systems for space exploration and development. For example, the Synthetic Aperture Radar (SAR) on RADARSAT-1 and 2 have helped ensure the safety and security of Canadians by providing detailed images of Canada's boarders. The sensors aboard the Alouette, ISIS, and SCISAT satellites have provided Canadians with important data on the ionosphere and atmosphere, which paved the way for commercial satellites, and have informed environmental policy. The success of these missions and payloads have helped bolster Canada's economy by creating high tech jobs, and encouraging Canadian companies to advance Canada's lead in niche space markets. One way to further Canada's position in space exploration and utilization is to develop a novel mounting platform for the sensors and payloads Canada currently provides to the world.

The limitations of current launch systems require many payloads and support structure to be lightweight and deployable. Most payloads (such as Neptec's TriDAR system and MacDonald Dettwiler and Associates' SAR systems) require precise pointing and vibration mitigation, necessitating high stiffness (and usually heavy) support structures. A Tensegrity based adaptive structure has the potential to increase pointing accuracy over the lifetime of the spacecraft, while realizing higher strength to weight ratios than conventional structures.

Tensegrity

Tensegrity systems are made up of compression elements (struts) held in place by a network of tension elements (cables). The simplest 2D tensegrity is the 2 strut 4 cable kite (**Figure 1**).

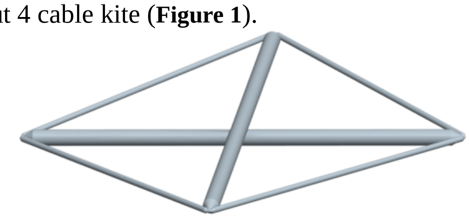


Figure 1. *A tensegrity kite where the 2 struts touch, but are not connected.*

Tensegrity systems include both tensegrity structures and tensegrity mechanisms (or adaptable structures). A tensegrity structure can become a tensegrity mechanism if the cable and strut lengths are controlled. Tensegrity systems have several advantages over conventional deployable structures including high strength to weight ratios, low storage volume, good stability throughout deployment, excellent scalability, and precise control throughout the service life of the system (Zhang & Ohsak, 2006).

Potential Satellite Application

Both Tibert (2002) and Sultan et al. (1999) proposed using tensegrity systems in place of conventional space booms, antennas, and telescope bodies. One way to test these tensegrity based adaptive structures, and add new capabilities to future Canadian spacecraft, is to use them as an intermediate platform between a conventional spacecraft bus and a payload. Such a tensegrity platform could deploy the payload once in orbit, and allow ground controllers to fine tune and calibrate the orientation of the payload with minimal use of the Reaction Control System (RCS). An example spacecraft with a 3 strut tensegrity payload platform is illustrated in **Figure 2**.

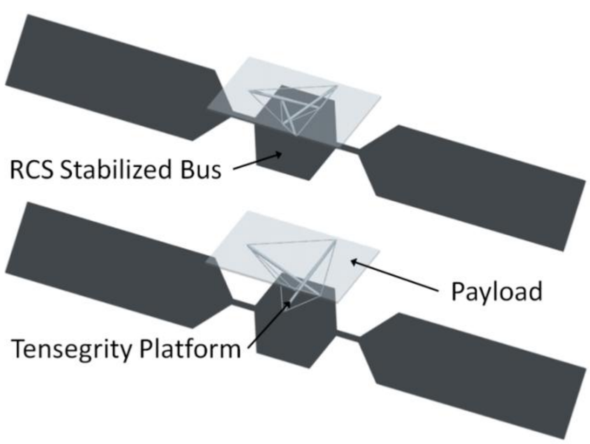


Figure 2. *The tensegrity platform in a stowed and deployed configuration.*

Once the tensegrity platform in **Figure 2** is deployed the strut and cable lengths can be manipulated to roll and pitch the payload until an exact pointing axis is achieved. The tensegrity platform can then be stiffened by increasing the preload in the tensegrity cables and struts. The demand on the spacecrafts RCS is reduced by using the tensegrity platform to gimbal the relatively light payload instead of the entire spacecraft. The RCS must still be used to react the momentum of the tensegrity platform and payload, but the amount of momentum the RCS must compensate for is much less then it would be to maneuver the entire spacecraft.

Control

The tensegrity platform in **Figure 2** can best be thought of as a Stuart 6 Degree Of Freedom (DOF) platform, where 3 of the heavy linear actuators have been replaced by lightweight cables. In this control scheme the 3 remaining struts are linear actuators and the lengths of the 3 side cables are controlled by motors that wind, unwind, and tension the cables. Sultan et al. (1999) proposed using fiber optics as the cable material so that the cables could report their own length (they also proposed a control methodology for a tensegrity space telescope). By eliminating an additional sensor system the tensegrity platform becomes simpler and lighter.

The tensegrity platform can be further simplified by replacing the cables with springs or the struts with leaf springs. While this configuration is not ideal for transient vibration, it does simplify the platform, and still allows the payload pointing axis to be controlled. The additional complexity incurred by adding 3 or even 6 additional actuators is compensated for by the increase in fault tolerance and high degree of control over the stiffness and pointing of the payload. The tensegrity platform could additionally allow the pointing axis to be refined as spacecrafts' systems deteriorate in the space environment. For example, it can be used to compensate for a partial failure of the RCS.

Tensegrity Systems and Form Finding

The advantages of tensegrity systems come at the cost of a slight weight and design complexity penalty. Unfortunately, tensegrity structures are also difficult to design and optimize because it is not always easy to determine how they maintain their stability and equilibrium as the element lengths are altered. The process of determining the stable configuration for a set of tensegrity element lengths is known as the form finding problem. Several authors have developed various form finding methods, but very few of them allow the element lengths in the tensegrity system to be directly controlled, and none provide much insight into the form finding problem. Insight into the form finding problem is defined as providing a clear rational for how the lengths of the tensegrity elements affect the overall shape of the tensegrity system.

One of the only methods that allows the tensegrity elements to be directly controlled, is a method developed by Connelly and Terrell (1995). Connelly's method was formulated using the height and radius of the tensegrity system as input parameters, and used trigonometry to calculate the coordinates of the nodes between elements. Connelly's method will always yield a solution to the form finding problem, but it does not allow the element lengths to be directly controlled.

Later this method was altered by Tibert and Pellegrino (2003) to determine the height and radius of the tensegrity system, based on a set of element lengths. Unfortunately, Tibert's method will fail unexpectedly for certain combinations of element lengths. Neither of these two methods provides much insight into the form finding problem because of their fundamentally algebraic nature. A novel geometric form finding method needs to be developed to improve the ease with which a tensegrity system can be designed and optimized.

The geometric intersection method presented in this paper reduces the complexity of the form finding problem into the simpler problem of determining the intersection point between three geometric surfaces. This simplification allows the relationship between the element lengths and the final shape of the tensegrity system to be immediately apparent. It also provides a new rational for why some configurations fail using Tibert's method, and allows the elements lengths that cause the failure to be identified.

Next, a brief summary of Connelly's method and Tibert's method will be presented. Then they will be compared to the geometric intersection method presented in this paper. Finally the applicability of this method to a space tensegrity platform will be discussed.

Previous Form Finding Methods

Connelly and Terrell (1995) developed a form finding method for a very specific type of tensegrity system, known as a prismatic tensegrity system. Prismatic tensegrity systems are highly symmetric structures composed of three element groups: lateral struts; lateral cables; and horizontal or end cables. Each element in a group has the same length, and this arrangement results in a structure with a circular cylindrical volume with two parallel end planes (**Figure 3**).

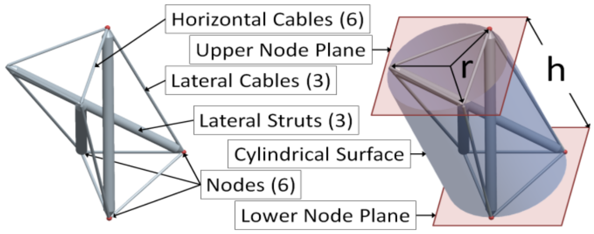


Figure 3. A three strut prismatic tensegrity structure.

Prismatic tensegrity systems are some of the most useful tensegrity systems because they are very stable and they maintain a specific angle of twist between their upper and lower nodes regardless of the element lengths (**Figure 4**). This property creates a stable platform, which does not rotate as the height and radius of the tensegrity system are altered by changing the element lengths.

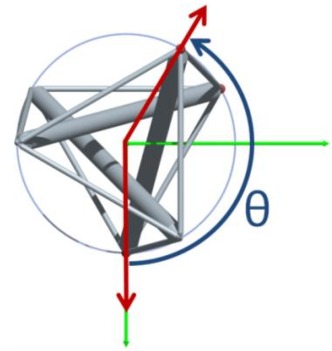


Figure 4. The angle of twist (θ) is the angle between the end nodes of a strut projected into the upper or lower node plane.

Connelly proved that the angle of twist is constant if the prismatic tensegrity system is in equilibrium, and is given by:

$$\theta = \pi\left(\frac{1}{2} - \frac{2j}{n_s}\right). \tag{1.1}$$

Connelly used the angle of twist to calculate the coordinates of all the nodes in a prismatic tensegrity system. The symmetry of a prismatic tensegrity system means that only the coordinates of the nodes connected to a single node (p_i) by the elements must be calculated. Then the rest are determined by reflecting these coordinates about the symmetry planes of the system (**Figure 5**).

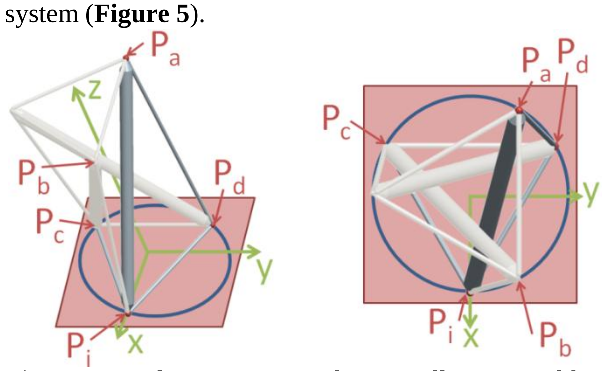


Figure 5. Nodes p_a, p_b, p_c, and p_d are all connected by elements to p_i, and can be reflected about symmetry planes passing through the z-axis to represent all other nodes.

Connelly proved that the coordinates of the nodes connected to p_i are given by:

$$\begin{aligned}
p_i &= [0, r, 0],\\
p_a &= [r\sin\theta, r\cos\theta, h],\\
p_b &= \left[r\sin\left(\theta + \frac{4\pi j}{n_s}\right), r\cos\left(\theta + \frac{4\pi j}{n_s}\right), h\right],\\
p_c &= \left[r\sin\left(\frac{4\pi k}{n_s}\right), r\cos\left(\frac{4\pi k}{n_s}\right), 0\right],\\
p_d &= \left[-r\sin\left(\frac{4\pi k}{n_s}\right), r\cos\left(\frac{4\pi k}{n_s}\right), 0\right],
\end{aligned} \tag{1.2}$$

where r is the radius and h is the height of the tensegrity system; j and k are node connectivity terms; and n_s is the number of struts in the system.

Tibert and Pellegrino (2003) used Equation (1.2) to determine the lengths of the elements by taking the Euclidean norm of the node coordinates at each end of the desired element. The resulting equations for the lateral strut length (l_s), lateral cable length (l_c), and horizontal cable (or end cable) length (l_e) are:

$$l_s = \sqrt{2r^2(1-\cos\theta)+h^2}, \qquad (1.3)$$

$$l_c = \sqrt{2r^2[1-\cos(\theta-\beta)]+h^2}, \qquad (1.4)$$

where:

$$\beta = \frac{2\pi k}{n_s}. \qquad (1.5)$$

Geometric Intersection Method

The geometric intersection method is based in part on the work done by Connelly (1995) and Tibert (2003), but represents a completely new rational for how the structural elements in a prismatic tensegrity are related, and why certain combinations of element lengths result in a non-real configuration. In the geometric intersection method, each set of element lengths define a geometric surface. The lateral strut length defines a sphere with a radius of l_s whose center is located at p_i. The lateral cable length defines a sphere with a radius of l_c whose center is located at p_d. Finally the end cable lengths define the radius of a circular cylinder (**Figure 6**).

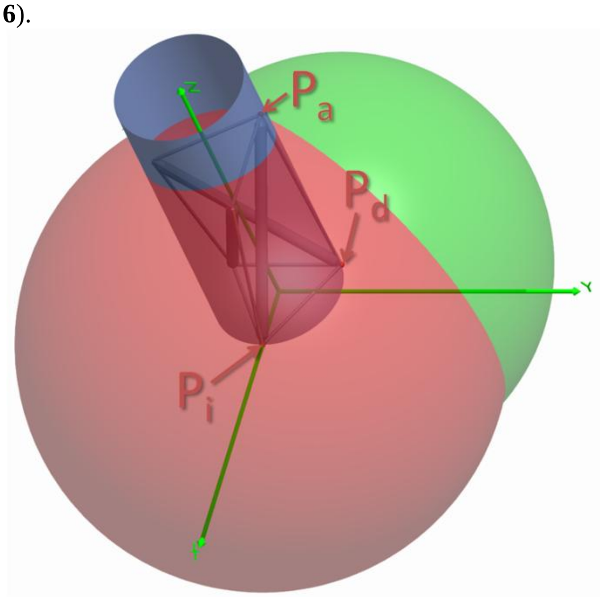


Figure 6. The three surfaces of the geometric intersection method all intersect at point $p_{\boldsymbol{a}}$.

The intersection point between these three surfaces coincides with the location of node $p_{\boldsymbol{a}}$ and the height of the prismatic tensegrity structure. The Cartesian equations for these three surfaces are:

$$\left(x-p_{i_0}\right)^2+\left(y-p_{i_1}\right)^2+\left(z-p_{i_2}\right)^2 = l_s^2, \qquad (1.6)$$

$$\left(x-p_{d_0}\right)^2+\left(y-p_{d_1}\right)^2+\left(z-p_{d_2}\right)^2 = l_c^2, \qquad (1.7)$$

$$x^2+y^2 = r^2, \qquad (1.8)$$

where:

$$x = r\cos\theta. \qquad (1.9)$$

Equations (1.6), (1.7), and (1.8) can be solved simultaneously for the intersection coordinates x, y, and z as long as the element lengths are known. However there are four possible intersection points between the three surfaces and only one can have the correct angle of twist and a positive z value. Equation (1.9) reduces the number of possible solutions from four to two by forcing the x coordinate of the intersection point $p_{\boldsymbol{a}}$ to correspond to the correct angle of twist θ. Equations (1.6), (1.7), and (1.8) can be rearranged to solve for the output parameters of the tensegrity system r and h, as well as either l_s or l_c. The corresponding input parameters will be l_c or l_s, and the end cable length l_e. Next Equations (1.6), (1.7), and (1.8) will be rearranged in terms of the desired output parameters.

Substituting Equation (1.9) into Equations (1.8) and (1.6):

$$\left(r\cos\theta-p_{i_0}\right)^2+\left[\sqrt{r^2-(r\cos\theta)^2}-p_{i_1}\right]^2 + \left(z-p_{i_2}\right)^2 = l_s^2. \qquad (1.10)$$

Substituting the p_i coordinates from Equation (1.2) into Equation (1.10) gives:

$$(r\cos\theta-r)^2+\left[\sqrt{r^2-(r\cos\theta)^2}-0\right]^2 + (z-0)^2 = l_s^2, \qquad (1.11)$$

which simplifies to:

$$r^2(\cos^2\theta-2\cos\theta+2-\cos^2\theta)+z^2 = l_s^2, \qquad (1.12)$$

and finally:

$$2r^2(1-\cos\theta)+z^2 = l_s^2. \qquad (1.13)$$

Similarly, if Equation (1.9) is substituted into Equations (1.8) and (1.7):

$$\left(r\cos\theta-p_{d_0}\right)^2+\left[\sqrt{r^2-(r\cos\theta)^2}-p_{d_1}\right]^2 + \left(z-p_{d_2}\right)^2 = l_c^2. \qquad (1.14)$$

Substituting the p_d coordinates from Equation (1.2) into Equation (1.14) gives:

$$(r\cos\theta-r\cos\beta)^2+\cdots + \left[\sqrt{r^2-(r\cos\theta)^2}-r\sin\beta\right]^2 + z^2 = l_c^2. \qquad (1.15)$$

Trigonometric relationships can be used to simplify Equation (1.15):

$$r^2\left[-2\sin\left(\frac{\theta+\beta}{2}\right)\sin\left(\frac{\theta-\beta}{2}\right)\right]^2 + \left[2\cos\left(\frac{\theta+\beta}{2}\right)\sin\left(\frac{\theta-\beta}{2}\right)\right]^2 + z^2 = l_c^2, \tag{1.16}$$

which can be further simplified to:

$$4r^2\left\{\sin^2\left(\frac{\theta-\beta}{2}\right)\left[\sin^2\left(\frac{\theta+\beta}{2}\right)+\cdots + \cos^2\left(\frac{\theta+\beta}{2}\right)\right\} + z^2 = l_c^2, \tag{1.17}$$

and finally:

$$2r^2[1-\cos(\theta-\beta)] + z^2 = l_c^2. \tag{1.18}$$

Equation (1.13) or Equation (1.18) can be used to solve for the height of the tensegrity system depending on whether l_s or l_c is used as the input variable. $r$can be calculated using the following equation:

$$r = \frac{l_e}{2\sin\left(\frac{\beta}{2}\right)} \tag{1.19}$$

Equations (1.13) and (1.18) are identical to Equations (1.3) and (1.4) (because $z = h$), but they have been derived using completely different methodologies. Connelly and Tibert's method was derived using algebra and the Euclidean norm of the end point nodes. The geometric intersection method was derived using a geometric approach, which is arguably much more intuitive. One of the reasons the geometric intersection method is more intuitive is that it gives a physical explanation for why Tibert's method will fail for certain combinations of element lengths. These boundary conditions will be examined next using some examples of the geometric form finding method.

Examples and Boundary Conditions

The inputs for the geometric form finding method are the end cable length, which determines the radius of the tensegrity system and circular cylinder; and either the lateral strut length, which determines the radius of the sphere centered about point p_i; or the lateral cable length, which determines the radius of the sphere centered about point p_d. With these inputs, the geometric intersection method will output the radius and height of the tensegrity system, as well as the missing lateral element length. **Table 1** illustrates the beginning and end configurations of a 7 and 8 strut tensegrity. l_s is the input variable and $l_c$is the output variable. The output variables ($h, r, and\ l_c$) are calculated using both Connelly's method (CON) and the geometric intersection method (GEO). The output data further confirms the mathematical equivalence of the two methods. Both methods fail to find a real configuration for the 8 strut system when $l_s = 2.0$, indicating that a boundary condition has been reached.

Connelly's method calculates imaginary values for the height of the system, because Equation (1.13) rearranged for z is $z^2 = l_s^2 - 2r^2(1-\cos\theta)$, which becomes imaginary when $l_s^2 < 2r^2(1-\cos\theta)$. While this condition is straightforward, it is not very intuitive and it does not give any feedback about which element lengths should be changed to correct the problem. The geometric intersection method does indicate which elements should be altered in this situation, because each element is represented by a unique surface, and must be selected to maintain a unique intersection point.

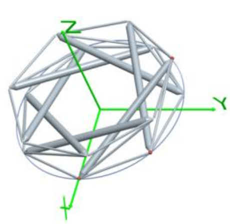
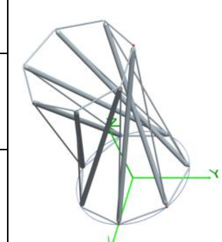
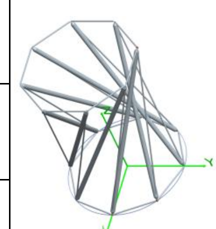
Initial	Parameters		$l_s = 2.0$	$l_s = 2.5$	$l_s = 3.0$	$l_s = 3.5$	$l_s = 4.0$	Final
		n_s	7	7	7	7	7	
		θ	115.7°	115.7°	115.7°	115.7°	115.7°	
		r	1.15238	1.15238	1.15238	1.15238	1.15238	
	GEO	h	0.43778	1.56258	2.27852	2.90545	3.49165	
		l_c	1.30201	1.98626	2.58752	3.15361	3.70071	
		l_e	1.00000	1.00000	1.00000	1.00000	1.00000	
	CON	h	0.43778	1.56258	2.27852	2.90545	3.49165	
		l_c	1.30201	1.98626	2.58752	3.15361	3.70071	
		l_e	1.00000	1.00000	1.00000	1.00000	1.00000	
Undefined		n_s	8	8	8	8	8	
		θ	112.5°	112.5°	112.5°	112.5°	112.5°	
		r	1.30656	1.30656	1.30656	1.30656	1.30656	
	GEO	h	undefined	1.23662	2.06863	2.74394	3.35846	
		l_c	undefined	1.90706	2.52723	3.10433	3.65881	
		l_e	1.00000	1.00000	1.00000	1.00000	1.00000	
	CON	h	0.84899i	1.23662	2.06863	2.74394	3.35846	
		l_c	1.17766	1.90706	2.52723	3.10433	3.65881	
		l_e	1.00000	1.00000	1.00000	1.00000	1.00000	

Table 1. Initial and final configurations for a 7 and 8 strut tensegrity system with l_s as the input variable.

Figure 7 is an illustration of the 8 strut tensegrity system approaching a boundary condition. The geometric shapes show that as the boundary condition is reached, the l_c sphere becomes completely enveloped by the l_s sphere and no unique intersection point is possible. This results in an entirely new way to view the boundary conditions of the form finding problem, and can be expressed as:

$$l_s \geq l_c + l_e. \qquad (1.20)$$

Equation (1.20) is a much more useful boundary condition, because it directly indicates which element length(s) should be changed to restore the intersection point between the three surfaces.

These geometric shapes also make the geometric intersection method easier to adapt to more complex tensegrity systems. For example the Cartesian equation for a cone can be substituted for the circular cylinder to find the shape of a prismatic tensegrity structure with two different upper and lower radii. While Connelly's method can also be adapted to suit these tapered structures it requires the equations for the node coordinates to be re-derived.

Conclusion

There are many space applications that could benefit from the application of tensegrity theory. One way to quantify their usefulness while adding value to future missions would be to use them as intermediate adaptable structures between the spacecraft bus and the payload. While tensegrity structures have many advantages they are arguably more difficult to design then conventional structures. The geometric intersection form finding method presented in this paper is a much more intuitive method for designing and optimizing prismatic tensegrity structures. All form finding methods can fail when certain combinations of element lengths are specified, but the geometric intersection method provides a direct indication of which element lengths should be altered to restore the system to a stable geometry. The geometric intersection method is also easier to adapt to new tensegrity configurations because the surfaces used in this method can be exchanged for other surfaces that are more closely related to the desired final shape. Tensegrity systems could provide unique capabilities to future Canadian mission contributions, and further strengthen Canada's position in niche global space markets for the direct and indirect benefit of all Canadians.

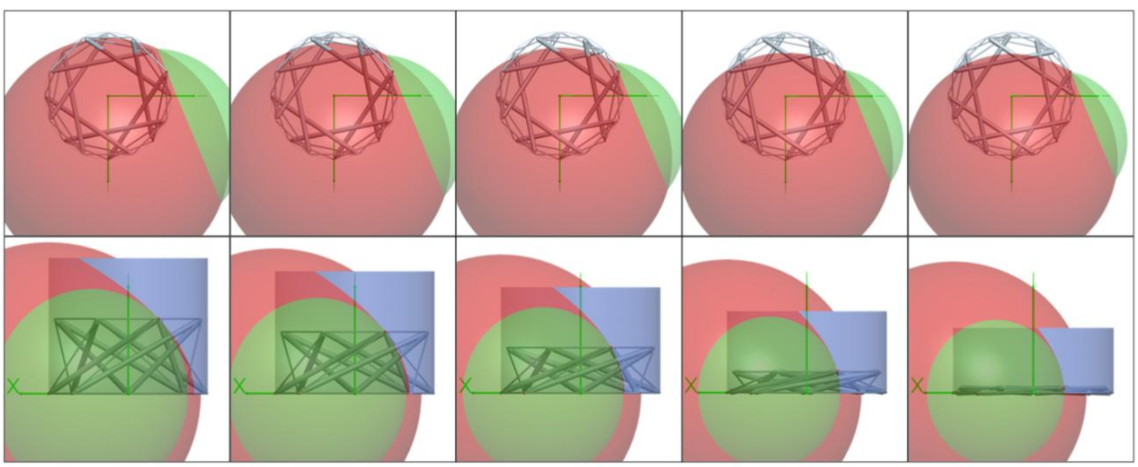


Figure 7. Top and side views of the 8 strut tensegrity system approaching a feasible geometry boundary.

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