Advancements in Kinematic Calibration

Andrew Fratpietro
Carleton University
Dept. of Mech. & Aero. Eng.
1125 Colonel By Drive
Ottawa, ON, K1S 5B6

Nicholas W. Simpson
Carleton University
Dept. of Mech. & Aero. Eng.
1125 Colonel By Drive
Ottawa, ON, K1S 5B6

M. John D. Hayes
Carleton University
Dept. of Mech. & Aero. Eng.
1125 Colonel By Drive
Ottawa, ON, K1S 5B6

Abstract

A vast number of diverse and innovative approaches to the various aspects of kinematic calibration have surfaced in the last several years. New technologies, measurement systems, numerical techniques and error models have been devised to eliminate the geometric discrepancies that undeniably exist in all types of robots. The purpose of this report is to outline the findings of the numerous researchers of this area. Calibration techniques have been developed to compensate for kinematic errors in serial and parallel robots as well as algorithms and procedures to determine optimal robot configurations for measurements and to minimize the time required for calibration. Methods based on absolute measurements, closed-loop analysis and imaging devices are reviewed and discussed. The hand-eye problem has come to the forefront of kinematic calibration as the introduction of cameras to robotic systems can greatly increase their capabilities. A thorough examination of this issue is offered with a summary of recent methodologies employed in camera calibration.

Table of Contents

List of Figures

1.0 Introduction

Kinematic calibration encompasses a myriad of research topics ranging from the algebraic formulation of numerical solutions to hardware implementation. These topics and the research that has taken place within these areas is well documented in the library of conference and journal papers that are made available each year. With such a large number of papers, a summary of the recently completed calibration research becomes a valuable tool for both academia and industry representatives with interest in this area.

In general, kinematic calibration can be broken down and discussed as several basic topics. The first topic, non-camera based calibration, deals with using either an external measurement system or multiple constraints to close the robots kinematic chain. A supplementary topic, camera-based calibration, outlines the situations where visual input is used to close the kinematic chain. The section called parameterization discusses the implementation of a geometric model in representing a physical system such as a manipulator. The fourth topic presented is called multiple configurations. It points out certain instances whereby moving the joints of a manipulator into several different poses can be used to produce a number of equations that can be solved for the kinematic parameters of a robot. Workspace issues are also discussed. These issues deal with the calibration of robotic manipulators throughout the entire available workspace and how this can be achieved. A discussion regarding error models outlines the methods for properly identifying and eliminating calibration error. The final topic presented is tool calibration. This topic outlines the methods for determining the geometric transformation between the robotic manipulator and the tip of the tool that it is using. Most of these topics have applications to both serial and parallel manipulators and are discussed separately in the following sections.

2.0 Serial Robot Calibration

The area of kinematic calibration for serial robots has seen many advancements over the past few years. Along with the utilization of new equipment, measurement devices, and software packages, a multitude of calibration methods have surfaced to address the issue. The development of effective calibration systems that do not rely on expensive external measurement devices has been the focus of many researchers. This has yielded many innovative techniques, which will be discussed in Section 2.1. Recently, camera-aided calibration has received much attention as it has the potential of becoming a relatively inexpensive but proficient solution to the calibration issue and will be discussed in Section 2.2. The approaches discussed in this section revolve around a variety of concepts, including the actuation of individual or multiple joints, reference objects, end-effector position measurement, and different constraints to eliminate parameters. These techniques have been applied to several widespread manipulators, including Puma, SCARA, and Stanford type robots as well as more specialized manipulators used in the medical and construction fields. Efforts have also been taken to produce continuous geometric models of robots and to determine optimal robot configurations for calibration that encompass the entire work volume.

2.1 Non-Calibration-Based Calibration

The need for kinematic calibration arises from geometric discrepancies between the kinematic model stored in the robot controller and the physical machine. The error in the model is determined by performing a comparison between absolute measurement data obtained in numerous robot configurations and positional data derived from the nominal parameters. Absolute measurement data is acquired through use of laser tracking systems or theodolites or by physical constraints used as reference points. Lasers can track the three-dimensional position of some target object and the calibration system can use this data in the minimization of some cost function. Until recently, all calibration schemes used either an external measurement system or closed the kinematic loop. Camera-based calibration systems offered a low cost alternative to systems of the current time and thus garnered much interest. The more recent kinematic calibration methods that do not involve cameras are innovative approaches and firmly reinforce the traditional methods. The methods discussed are based on the actuation of joints, the use of reference objects and constraints, and the use of measurement systems.

2.1.1 Actuation of Individual or Multiple Joints

Several methods are based on the actuation of individual or multiple joints causing the end-effector motion to follow some advantageous path. Absolute measurements of a target or targets mounted on the manipulator links or simply the end-effector are obtained. A procedure proposed by Abderrahim and Whittaker [1] utilizes the fact that revolute joints, when actuated, should ideally move the link, and any subsequent links, in a circular path in a plane. Sultan and Wager [2] outlined a similar procedure but identified the axes of rotation of the joints and related this information to the individual transformation matrices of the robot. Omodei *et al*. [3] employed a unique measurement system that incorporated mirrors and a laser tracking system while the robot end-effector was commanded to move in a straight line. A comparison study was conducted by Khalil *et al*. [4] regarding different methods of calibration, of which one relied on the placement of the end-effector within a plane. The general non-linear form of the equation of a plane was used where the coefficients were determined by commanding the manipulator's end-effector to a sufficient number of points on an imaginary plane in space.

Abderrahim and Whittaker [1] identified the DH model parameters using planes of rotation for individual joints. The method can cope with parallel axes and was validated with simulated and experimental data from a Puma 560 robot. Three joint features were determined: the plane of rotation, centre of rotation and radius of rotation. A laser tracking system, through use of a retro-reflective target, measured the position of the end-effector, relative to the base frame. Retro-reflective targets were appropriately mounted on the end-effector so that a laser tracking system could determine the position of the end-effector relative to the base frame. The measured points form a circle in space where they should theoretically lie in the same plane. This identified the three joint features. A linear least-squares approach is used to estimate the coefficients of the equation representing the plane. The normal vector of this plane, emanating from the centre of rotation yields a coordinate system. This is then translated to the base frame. By using the nominal transformation matrices, the error can be calculated between the nominal parameters and the estimated parameters. An inward sequence is followed where the outermost link is calibrated and then the subsequent inner links. The system increased the accuracy of the robot by approximately 80%. The system suffers from a few drawbacks: a large number of measurements were needed to estimate the plane coefficients and an expensive and sophisticated measurement device was required.

The method described by Sultan and Wager [2] is similar to the method above as it actuates a robot's joints individually in the same manner. However, the joints are activated in an inward sequence, starting with the outermost joint, to determine their respective spatial position and orientation vectors relative to the world coordinate system, which identify the joint axes of the robot. Thus, after all the independent axes are obtained the geometric parameters of the robot can be calculated. For revolute joints, a theodolite measures a target, whose motion is a circular path, mounted on the link. For prismatic joints, the theodolite measures a target, whose motion is linear, on the link. To calibrate these joints, a cost function specific to the type of joint, was minimized through use of a least-squares regression technique. A parameterization modification was employed to compensate for nearly parallel axes. Compensation for kinematic dependencies was also offered as well as comments on joint transmission. A six-axis ASEA IRB6/Lwas calibrated using two theodolites and the procedure presented. The end-effector position error residual was reduced by an order of magnitude.

The work of Omodei *et al*. [3] is based on the linear motion of the end-effector of a SCARA type robot. Three different kinematic calibration algorithms were devised and implemented. The first minimizes the average error based on the Euclidean norm of the error measurements and joint angles. The second forms a set of linear equations to iteratively solve, which involved the determination of the Jacobian for each pose. It is an iterative procedure. The third is based on an Extended Kalman Filter. The actual measurement system uses triangulation principles and employs a single laser, a plane mirror and a convex mirror. The laser is oriented in one particular direction while the end-effector traverses along a path parallel to this line of action. The laser is reflected onto a screen where a camera records the position of the contact point. The difference between where the measured spot struck the screen and the predicted point of incidence is used in the three procedures to calibrate the manipulator. Accuracy near the repeatability of the robot was achieved after calibration. The Extended Kalman Filter was recommended as the most suitable algorithm in terms of processing time and parameter identification as it was efficient and identified all the parameters. The linearization method failed to identify the parameters relating to the convex mirror but was still able to efficiently achieve reasonable results. This method seems to require an unusual and elaborate set-up, but is an original approach.

2.1.2 Reference Objects and Constraints

Reference objects are dominantly custom built and fabricated to suit one particular robot, calibration situation or scheme. They can take the form of an immovable plate with insertion holes as in Omodei *et al*. [5] or a precisely manufactured external sensing linkage as in Khoshzaban *et al*. [6]. Unlike those constructed for use with cameras, these objects are meant for physical interaction. They are manufactured with the highest accuracy possible, sufficient to calibrate a robot to the same degree. Reference objects impose a constraint on the end-effector to some known value, which can be used for comparison. In the case of Ikits and Hollerbach [7], this took the form of a plane. Khalil *et al*. [4], also studied a constraint method in which the position of the end-effector was fixed. The orientation of the end-effector could also be constricted. One achieved, multiple configurations of the robot would be used to yield the appropriate equations.

In Omodei *et al*. [5], three methodologies were implemented in the kinematic calibration of a 5 DOF Puma type manipulator. The robot in question is employed in the shoe manufacturing industry. A pose matching approach was used, as opposed to a pose measuring approach, where the robot was commanded to a multitude of positions according to a custom built reference block constructed with a CNC machine. The "dime", as to which the authors referred, contained 46 insertion holes that fit the end-effector of the robot. However, several of these holes could be reached with different robot configurations, thus, 81 independent measurements could be taken. Three algorithms were developed for calibration and they are the same as mentioned in Omodei *et al*. [3]. The calibration process was first simulated with arbitrary structural parameter errors and was able to produce good estimations based on whether or not measurement noise was introduced. With the experimental set-up, the average error was 4.702 mm before calibration. The residual error after calibration was approximately 0.125 of that value. Experimental results were displayed for all three methodologies, in the form of computing time, average error, standard deviation and maximum error. Another procedure was written to determine which of the structural parameters primarily contributed to the overall error. Essential structural parameters were chosen by the authors to form the reduced set, and then the procedure would select additional parameters to reduce the error. Overall, the result of the experiment was that the error in the robot's accuracy was reduced to approximately the value of its repeatability.

Most calibration procedures are expressly designed for industrial robots and not heavy machinery. In the case of large hydraulic manipulators, as reported by Khoshzaban *et al*. [6], joint sensing and location must also be taken into consideration as most do not have internal sensors incorporated into their design. In their work, a calibration technique for large, powerful and heavy hydraulically powered manipulators, such as excavators, was developed. As hydraulically powered joints present some difficulty in measuring joint angles, a novel "calibrator" design was proposed. As these machines operate in unsuitable areas, conventional external measurement machines were considered prohibited. The calibrator is simply a mechanism that contains spherical joints, each with a corresponding joint sensor, with a certain number of links, but built to be reliable and durable. The calibrator's joints are all sensed and one of the links is fixed, as required by a closed kinematic chain to achieve scale. The method is based on a closed kinematic chain, where the calibrator is fixed to the end-point of the manipulator. The end-effector grasps a spherical joint on the calibrator and the other end is fixed. Each joint is actuated individually starting with the distal link. The joints are actuated in an inward sequence until all the kinematic parameters are identified. The Jacobian of manipulator is calculated based on 50-100 poses generated by the actuation of one joint and a non-linear least squares problem is solved. Overall, accuracy close to the machines repeatability was achieved. However, this was a marginal improvement. This method does show promise and further study is warranted, however, the calibrator must be precisely manufactured to the strictest tolerances.

Two procedures for the kinematic calibration of a Puma 560 robot were discussed in Ikits and Hollerbach [7]. A 3D motion tracking system was used in conjunction with a non-linear least squares solution scheme to set the performance bar for their plane constraint method. The plane constraint was made possible through use of a touch probe and a flat surface. The Implicit Loop Method was followed for the procedure. Out of the total number of measurements that were taken, most were for parameter estimation but some were for verification. The authors concluded that this approach was surprisingly difficult to develop and there are many factors in which one must pay attention. The plane constraint procedure reduced the RMS error of the endpoint distances to the place to 0.2515 mm whereas the external measurement system reduced it to 0.156 mm. The authors also performed a statistical analysis on the procedures and made some interesting comments.

2.1.3 End-Effector Position Measurement

This type of calibration remains the most expensive due to the need of an external measurement device, but is simple to apply and is widely used and accepted. The measurement device records the three-dimensional coordinates of the end-effector as the manipulator is commanded to different configurations. In some situations it is also possible to measure the orientation vector of the end-effector. The following work reported by Gong *et al*. [8] and Drouet *et al*. [9], use this traditional form of calibration in the areas of temperature compensation and elastic error compensation. Again, from Khalil *et al*. [4], several methods based on end-effector measurement were compared. The first employs an external sensor that measures the position and orientation of the end-effector. The second uses two different poses with measurements of the end-effector in terms of orientation and position. A transformation between the two poses is then identified. The third also uses two distinct poses but utilizes the distance between the end-effector positions for calibration. There exists a multitude of approaches using external measurement systems, making this category of calibration well established.

Thermal effects are experienced as slight expansion of the links and thus classify as geometric errors as a function of temperature. Gong *et al*. [8] developed a kinematic calibration system that is comprised of a laser tracker that measures Cartesian coordinates of the end-effector, a temperature acquisition system, and a PC to perform the identification algorithm. Over the warm-up and subsequent cool-down phases, a series of measurements are taken and from these the thermally induced parameter errors are identified. A number of thermistor sensors are mounted on the robot and they record temperatures specific to individual joints. The maximum temperature variation was recorded as 10 °C. Overall, the robot accuracy was improved by an order of magnitude after calibration.

Drouet *et al*. [9] proposed a method that calibrates a large manipulator in terms of geometric errors and elastic errors. The manipulator in question is used to position cancer patients for radiation therapy. The proton radiation beam must be accurately placed within 0.5 mm of the target location or risk damaging healthy tissue. Since this is a large manipulator that caters to payloads ranging from 25 kg to 200 kg, elastic effects are quite prominent. Typical end-point deviations before calibration are 7 – 8 mm, which is unacceptable for medical purposes. The measured end-point error is decomposed into the geometric error and elastic error as a function of manipulator configuration and payload weight. Geometric errors are a sole function of the joint variables while elastic errors, derived from beam theory, also depend on the payload weight. The procedure was first simulated and then tested experimentally. A 3D laser tracking system, which reportedly had a resolution of 0.04 mm, measured the errors. Three targets are mounted to the patient bed in a triangular configuration, centred on the revolute axis of the bed. A large number of measurements were required but after calibration, the robot placement accuracy was increased to +/- 0.4 mm.

2.2 Camera-Based Calibration

Great potential lies in the utilization of cameras for automation purposes. With high-resolution cameras supported by advanced image processing and recognition software, a multitude of tasks in the manufacturing industry could be programmed and carried out autonomously. Calibration is an essential part of this vision and is being pursued by many researchers. Camera-based systems offer many advantages including low cost, the ability to perform multiple functions besides calibration, and interchangeability if widespread. Almost all camera-based systems required some reference object, which is used to determine the extrinsic and intrinsic properties of the camera. The "hand-eye" problem is then solved and the remaining calibration procedure pursued. Two camera-based approaches have emerged, one based on self-calibration and the other based on external measurements in conjunction with image data.

2.2.1 Reference Objects

All camera-based calibration procedures use a reference object of which an image is taken from multiple configurations. Spheres, dots, or other simple geometric shapes are arranged on the object so the camera can extrapolate the rotational and translational parameters of the robot. In Meng and Zhuang [10], a non-planar surface with an array of black dots was used as a reference object. The dots were not exactly positioned on the surface but their location were identified through determination of their individual centroids. A reference plate with a number of precisely drilled holes was used in a self-calibration method generated by Gong *et al*. [11] and a calibration board containing 100 evenly spaced dots was used by Zhuang *et al*. [12]. An interesting object that used 16 spheres placed around the apparatus centre was used in Rousseau et al. [13]. This arrangement resulted in uniqueness between the images taken from different angles.

2.2.2 Self-Calibration

This approach to the calibration problem requires no external system to provide absolute measurement data. Instead, the images are used to identify the hand-eye transformation, and then calibrate the robot. In Meng and Zhuang [10], a system was built to calibrate the camera up to a scale factor without absolute measurement data. The system requires a minimum amount of absolute measurement data, which appears in the form of a yardstick or an otherwise known length, to kinematically calibrate the robot. The scale factor is related to the camera but does not change significantly given that the measurement configurations follow some predetermined path. An optimal robot configuration algorithm was designed to plot a trajectory for the procurement of images. The system then takes images at the trajectory points and calibrates the rotational and translation parameters of the robot. This intrinsic camera property can later be identified given some known reference object, which in the author's case was a yardstick. A Puma 560 robot was equipped with a CCD camera, a colour frame grabber and an image-processing system. The calibration object consisted of a non-planar object with an array of black dots fixed to its surface. The calibration procedure increased the accuracy of the robot by 80%.

Gong *et al*. [11] derived a novel kinematic calibration method, which is based on taking measurements of an artifact and using the distance between elements of the plate to calibrate the manipulator. The system employs a camera and laser diode combination attached to the end-effector to measure the XY coordinates of a series of holes in a reference plate. The artifact is a 0.8 m × 0.8 m × 0.2 m aluminum plate with 7 holes. The distance between hole centres is measured by a Computer Measurement Machine (CMM) and this absolute data is compared with that found by the manipulator. Taking multiple measurements of the same target and solving a constrained optimization problem yields the hand frame transformation of the camera. Multiple distance measurements were taken with respect to the 7 holes and three robot configurations were used to triple the data. Three separate plates were constructed and positioned to the left, front and right of the manipulator with the left one raised in height by 0.2 m. To test the calibration procedure, calibration of the robot was performed by concentrating on each plate individually, and then using any two of them and then the data from all three. In all three cases the parameter estimations were very close. Overall the procedure reduced the mean residual error by an order of magnitude.

2.2.3 Known Object Data

The reference objects employed in camera-based calibration schemes are normally constructed to high standards. Elements of the object are fixed distances apart and are of known dimensions. This establishes a known scale to the object, which can then be used for absolute measurement data. As discussed previously, a variety of reference objects can be used but are all based on simple geometric shapes. Motta *et al*. [14] used a calibration board consisting of an array of dots. A single camera was mounted to the end-effector and it took images of this board a relatively large distances. A similar calibration board was used in Zhuang *et al*. [12] in their calibration procedure for

SCARA arms and in Zhuang *et al*. [15] in their simultaneous calibration method for the camera and robot. An unique apparatus was used by Rousseau *et al*. [13] in that it consisted of a number of spheres placed is specific positions around the centre of the object. A series of images were taken of the apparatus at varying angles. In

A kinematic calibration system that utilized a CCD camera and several reference objects was proposed by Motta *et al*. [14]. The CCD camera is an off-the-shelf model and the reference object simply contains an array of black dots of known diameter for camera calibration. The camera acquires images of the reference object at distances of 0.6 m to 1.0 m where the object is positioned in different locations around the robot. First the intrinsic parameters of the camera were calculated and then the extrinsic parameters that describe the hand-eye transformation. 25 different camera positions were required to calibrate the camera. The images where the distance between the camera and reference object was minimal were used for camera calibration. Overall, a robot accuracy of less than 1 mm was achieved. The authors acknowledged that the calculation of the focal length was the key factor in the procedure as it could not be computed exactly and an external measurement system was needed to calibrate the camera. However, overall, the system was quite proficient and it should be noted that the distance between camera and reference object was relatively large for this kind of operation.

The work described by Zhuang *et al*. [12] presents a practical calibration technique for a SCARA type robot. SCARA robots are often used in pick-and-place operations and do not necessarily require absolute accuracy in the vertical direction. The proposed technique used a CCD camera with a 25 mm lens. A calibration board contains 100 evenly spaced dots of 2 mm diameter in a square array 10 mm apart. The intrinsic parameters of the camera, the focal length and the scale factor, are determined through a technique of the author's design that copes with near-parallel axes. The extrinsic parameters are then computed using these estimated intrinsic parameters. Singular Value Decomposition was then used to determine the manipulator parameters.

Rousseau *et al*. [13] developed a novel calibration technique that employs a CCD camera mounted to the end-effector of a robot. The camera produces images of a reference object and has a 35 mm lens. The reference object consists of 16 spheres of varying diameter and with known distances for the object centre. A closed kinematic chain is formed with the camera and the reference object and a system of non-linear equations is solved to reveal the robot kinematic parameters. An iterative Gauss-Newton algorithm was utilized to solve the over-determined system of equations in a least-squares sense. The intrinsic parameters of the camera (focal length, scale factor, centre coordinates of the image and the radial distortion factor) were obtained through standard camera calibration routines. The position and orientation of the target was determined from the centres of the 16 circles in the image. This was determined through use of the quaternion representation of the finite rotation formula. Through simulation, an accuracy of 0.1 mm was achieved for a Fanuc S-10, which is close to the robot's repeatability. A number of well-chosen robot configurations were required to achieve this result. The procedure is primarily sensitive to the calibration of the camera and the images it produces.

A method in which the calibration of a robot and a hand-mounted camera can be performed simultaneously to eliminate propagation errors was developed by Zhuang *et al*. [15]. The usual procedure for this type of system is to first calibrate the camera by identifying its intrinsic parameters, then deriving the transformation from the robot's end-effector to the camera's optical centre and finally the kinematic parameters of the robot. The method outlines in this report suggests a new method that simultaneously performs all the steps. The method was first simulated using the geometric parameters of the Puma 560 robot. Three levels of measurement noise were included and the single stage method was compared against a similar two-stage method. It was found that 10 robot configurations were sufficient to adequately calibrate the robot and that the single stage version out-performed the multiple stage version. An experimental set-up was constructed to test the system. A single CCD camera (510x492 pixels) produced images of a reference plate that contained a 10 x 10 array of 2 mm diameter dots arranged 10 mm apart. The camera calibration algorithms were coded in Microsoft C. 52 measurement configurations produced 4 sets of data with 1 verification configuration each. About 5 of the dots on the reference object were used out of every image. A coordinate measuring machine controlled the position of the calibration board and also produced world coordinates for the dots. The error residual after calibration was approximately 0.2 mm. Similar results to that of the simulated case were obtained in the experimental case.

2.2.4 Hand-Eye Calibration

The use of digital cameras in robotic applications has explosively increased over the past two decades. It is simple to see the importance of such devices when one considers that the behaviours that robots are trying to emulate are based on human behaviours, and that one of the more important human senses used in the completion of these tasks is vision. For a human, a task such as picking up an apple seems trivial. One locates the apple with his vision and then proceeds to move one's hand into a position to grab the apple. Little consideration is given to the location of the hand with respect to the eyes, but only because such a routine has likely been accomplished thousands and thousands of times over the course of one's lifespan. The repetition of this motion has provided the human with a strong idea of where the hand is located with respect to the eyes. For a robotic vision system, including a camera and a robotic manipulator, the transformation from camera to robot hand must be calculated and included in the robot controller. The calculation of this transformation is referred to as the hand-eye calibration problem and has been well documented in robotic literature since the late 1980s.

In its simplest form, the problem can be stated as the solution of the homogeneous transformation equation of the form AX = XB, where the matrix X is the relative position and orientation of the camera coordinate system with respect to the robot-hand coordinate system and is referred to as the hand-eye transformation. X is the unknown matrix in the equation. The matrix B represents the transformation between robot-hand poses. The coefficients of this matrix can be calculated from the robot joint encoder readings before and after an arm movement if the robot arm is properly calibrated. The matrix A represents the transformation between camera positions before and after a robot movement. Assuming that the intrinsic parameters of the camera are already calibrated, the coefficients of this matrix can be calculated using 3-D geometry and a recorded camera image recorded before and after each robot movement. In order to increase the accuracy in the solution of X, several different robot arm poses are required resulting in the equation of the form $A_iX = XB_i$. Equations of this form can be solved in several ways, including both closed form and non-linear optimization. Figure 1 shows the visual relationship between these three parameters. Considering that the variables i and j represent the initial and final pose of the robot, the matrix A can be written as:

$$A = {}^{R}T_{Hj}\ {}^{R}T_{Hi}^{-1}$$

The matrix B can be written as:

$$B = {}^{B}T_{Cj}\ {}^{B}T_{Ci}^{-1}$$

The matrix X can be written as:

$$X = {}^{H}T_{C}$$

For each robot movement, one A and one B matrix can be produced for the solution of X.

Between the years 1996 and 2002, several innovations were made on this approach to robotic hand-eye calibration. Outlines of the journal and conference papers discussing these innovations will be presented in the following paragraphs.

The classical procedure presented above provides the hand-eye transformation matrix. Remy *et al.* [16] advanced this approach by simultaneously considering the determination of the hand-eye transformation as well as the location of a so-called calibration object in the robot world coordinate system. The calibration object being implemented is a tetrahedron with each of its vertices illuminated by a single light emitting diode (LED). These LEDs make the vertices more visible in the eye of the camera. A fifth LED is located beside the tetrahedron to allow the distinction between similar looking points of view. With this method, the robot is moved to *n* different positions and at each position an image of the calibration object is acquired. The data from these images is used to formulate equations solving for the best hand-eye transformation and the most likely location of the calibration object in the robot world coordinate system. The number of unknowns in these equations remains constant at twelve: three Euler angles and three translation components for each of the two transformations. The solution is still consistent with the entire set of calibration data, regardless of the number of images acquired. This solution is based on the non-linear minimization of a sum of scalar products and is described in the paper as being both quick and easy. Intrinsic calibration of the camera is assumed to have been already been performed and is not discussed further in this paper.

In Ma [17] presents a new formulation of the hand-eye calibration problem as well as a technique for the intrinsic calibration of a camera. The latter technique is referred to as camera self-calibration since it requires no reference calibration object and uses images

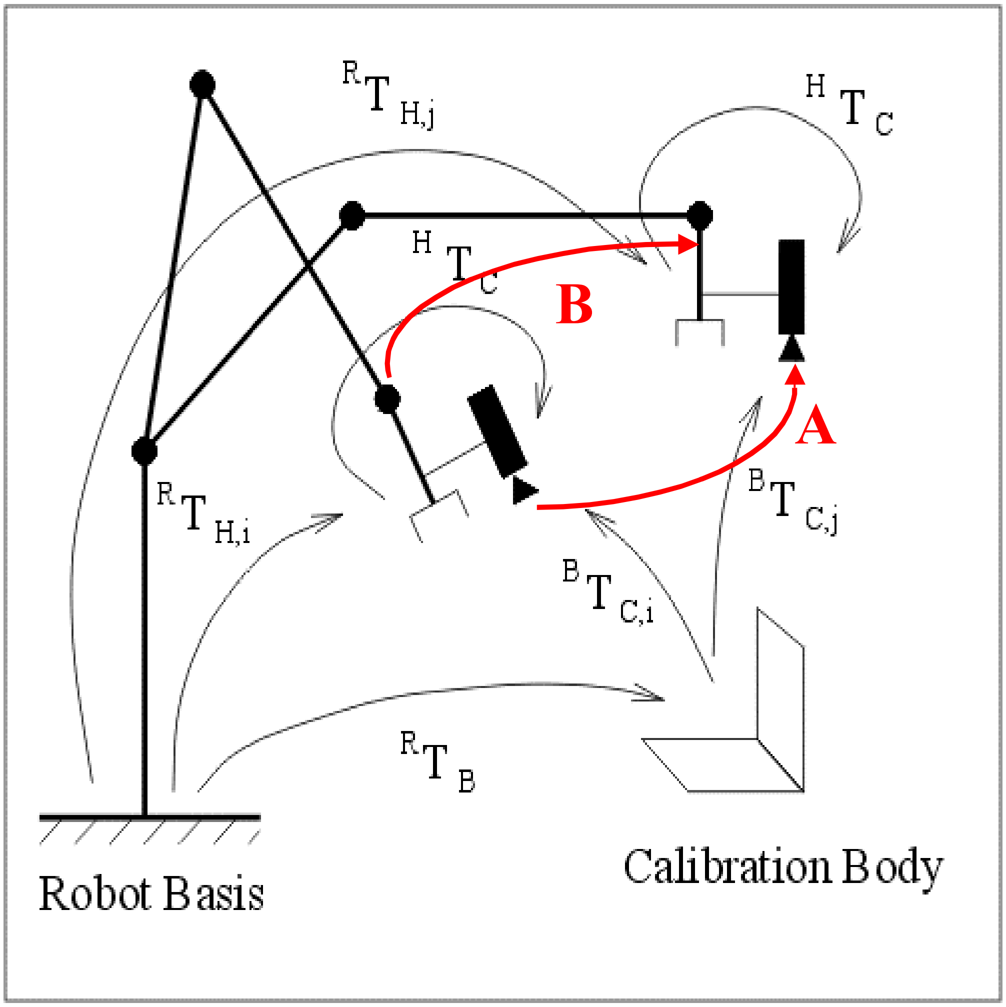

Figure 1: Image of robotic transformations (http://www.cs.tu-bs.de/rob/forschung/projekte/kamera_kalib/kamera_kalib_en.htm)

of the environment. This method takes advantage of the fact that the vision system is fully active by using specially designed robot motions in producing calibration data. Two sequences of three pure orthogonal translations are required to calibration both the camera orientation and the intrinsic camera parameters. A seventh arbitrary non-translational motion is then used to determine the translation between the robot gripper coordinate system and the camera coordinate system. The extrinsic parameters of the camera are not required for this step since a calibration object of known location, size, and orientation in the world coordinate system is not used. Instead, a series of pure translational motions help to produce images from where 3-D parallel displacement vectors may be extracted. These displacement vectors are used to compute the focus of expansion for each camera motion and, subsequently, the induced camera translations. The translations allow the forming of four linear equations that can be solved for the intrinsic parameters of the camera. The orientation of the platform is determined using these acquired parameters and several more translational camera motions. A stereovision procedure is used to produce the hand-eye translation vector. The results obtained through experimentation with this procedure are described as "rather faithful and stable".

Dongmin [18] applies a duel quaternion representation to the translational and rotational displacements associated with the hand-eye transformation. The use of a duel quaternion representation results in a computationally simpler closed form solution when compared to some previously performed work using a quaternion representation and screw-axis theory. This representation also allows the computation of unique solutions in situations where previous attempts have failed. The kinematic parameters are computed using an offline non-linear regression method (gradient descent) and both translation and rotation are considered simultaneously.

Zhuang and Wu [19] present a modification of Tsai's radial alignment constraint method (RAC) to be used for camera calibration. This modification addresses the issue that arises when the plane of the camera being calibrated is near parallel to the plane of the calibration board from which data is being extracted. This is referred to as a singular point of the system. Such a configuration can be observed by the application of Tsai's RAC method to the calibration of a selectively compliant robot arm. With the modification, the RAC method can be applied to the SCARA arm resulting in quick and accurate calibration. This method assumes that the pre-calibration of the cameras intrinsic parameters has already occurred. The resulting calibrated data includes the six parameters describing the rotation and translation in the hand-eye transformation as well as the distortion coefficient of the camera. Also discussed are the conditions that facilitate the switch from the general RAC method to the modified RAC method and vice versa.

In 1995, Horaud and Dornaika wrote a paper presenting a modified approach to the hand-eye calibration problem involving the separation of the camera extrinsic and intrinsic parameters and the resulting errors. Zhuang [20] builds on the concepts discussed in Horaud and Dornaika (1995) revealing the merits and limitations of the concepts in that paper. One such merit is the explanation that intrinsic and extrinsic camera parameters need not be made explicit due to the representation errors that this separation may cause. Another concept from Horaud and Dornaika (1995) is the use of the pinhole camera model in decomposing the camera perspective matrices into intrinsic and extrinsic parameters. Zhuang discusses that a complicated camera model should be used in this case in order to avoid compromising the accuracy of the system.

Wei *et al.* [21] describe a method of self-calibration for robotic cameras using active motion. This method is based on the tracking of a set of arbitrary world coordinate points. The location of these points is not required and can be estimated by the procedure. This method is iterative, but all initial conditions are determined through the solution of a set of closed form non-linear equations. As an extension of Tsai's RAC method, it is stated that the following modifications provide more accurate results then the original method. The image centre is assumed to be located at the apparent image centre. The image centre, focal lengths, the radial distortion parameter and external parameters are estimated simultaneously using global optimization.

Zhuang [22] presents a new linear solution method for determining the hand-eye robotic transformation. It is assumed that both the robot and sensor have been calibrated. The focus of the paper resides in the solution of the hand-eye transformation for a robotic arm whose hand contains three degrees of translation and one degree of rotation. Previous solutions have failed in the case where the robotic arm contained only one degree of rotation, or have required specialized movement patterns of the robotic manipulator. This method solves for three rotation parameters and two position parameters of this transformation without restricting the motion of the manipulator.

Dornaika and Horaud [23] presented two new solutions to the homogeneous matrix of the form AX=XB. The first solution is a closed-form method that uses quaternion algebra. Two quaternion are directly solved for and used to constrain the resolution method. This method also remains feasible for special configurations where previous work performed by Zhuang *at al.* (1994) is not feasible. The second solution solves for both rotations and translations simultaneously based on non-linear constrained minimization. This simultaneous solution results in less propagation of error throughout the solution and the confidence of the solution. Upon comparison to the method proposed by Zhuang *et al.* (1994), the closed form solution exhibits similar performance and the non-linear solution exhibits better performance.

Batista *et al.* [24] developed an explicit three-dimensional hand-eye calibration solution using monoplane calibration points. The iterative solution requires a first guess of the cameras intrinsic parameters, which are updated and improved as the procedure iterates. The results of this process are the calculation of camera's extrinsic parameters as well as the first coefficient of radial distortion and skew angle. There are no restrictions placed on the pose geometry of the camera. This procedure generally has an improvement over Tsai's RAC method due to the inclusion of the misalignment angles between the frame and image coordinate systems.

Malm and Heyden [25] performed the hand-eye calibration procedure using the normal flow field and information regarding the motion of the robotic hand. It is assumed that the robot-hand calibration is already known, as well as the intrinsic parameters of the camera. The normal flow field is obtained from camera images. It is a projection of a motion field along the normal direction of an image. This normal flow field is used to recover the motion of the robotic hand and is used to resolve the hand-eye transformation. Small translational motions are performed in order to estimate the rotational parameters of the transformation and then small rotational motions are made in order to estimate the translational parameters involved in this transformation. The results for this procedure showed that robotic hand-eye calibration could be performed using image derivatives, but the precision of the results has yet to be optimized.

Angeles *et al.* [26] presented a solution to the hand-eye calibration problem based on recursive linear least squares and applicable online. This solution involves the use of a dodecahedron as a calibration reference object in refining the calibration parameters as well as a laser range finder and a CCD mounted to the robot gripper. The feasibility of this procedure is demonstrated through experimental results, although the procedure has trouble providing results to greater than two digits of precision.

Andreff *et al.* [27] described a new flexible method for hand-eye calibration combining structure-from-motion with known robot motions to obtain a solution in linear form. The camera produces images of an unknown environment, tracks several points within that environment and then the hand-eye transform is calculated. No calibration rig is required and the manipulator need not perform large calibration motions. Contributions from this paper include the reformulation of hand-eye calibration to take into account structure-from-motion algorithms, a linear formulation that enables small calibration motions, and an algebraic study of this linear formulation.

Shen and Menq [28] present an automatic camera calibration scheme using a Coordinate Measuring Machine (CMM). This scheme involves the implementation of a novel calibration algorithm designed for use with a CMM. The CMM generates high-precision target points and flexible target calibration patterns. A tip-locating algorithm is used to provide the touch probe tip centre to sub-pixel accuracy within the camera images. The intrinsic and extrinsic parameters of the camera are coupled together. This procedure requires no initial guess of parameters and no non-linear minimization process, unlike most multiple stage methods.

2.3 Parameterizations

Different techniques in which to calibrate a robot are based on traditional forms of parameterization, such as the common Denavit-Hartenberg (DH) parameters. Numerous modifications have been developed to eliminate singularities with nearly parallel neighbouring axes or cope with continuity. A new form of representing manipulator geometry with exponentials for the purpose of calibration was proposed by Okamura and Park [29]. Using the product-of-exponentials (POE) formula, Chen *et al*. [30] developed a method in which to calibrate open-chain modular robots.

Okamura and Park [29] described a method based on the product of exponentials for open chain mechanisms. The POE parameterization model has several advantages over classical DH parameterization and its subsequent modifications proposed by Hayati, Stone and others. The POE model does not experience difficulties with nearly parallel joint axes and closed-form analytical expressions for the derivative of the forward kinematics with respect to the kinematic parameters are easily obtained. The POE formula is a type of zero reference position description of the kinematics. Two coordinate frames are required, one at the tool tip and one at the base. The forward kinematics are based on a set of constant matrices in the zero position. Revolute and prismatic joints are represented by two matrices each, one that describes the zero position and one that relates the joint parameter. The advantages of the POE model over other commonplace parameterization models ensures that calibration is a straightforward procedure.

A procedure for the kinematic description of modular robots, which consist of revolute and prismatic joints in the form of modules, and two methods in which to calibrate these robots is outlined by Chen *et al*. [30]. The authors pursued the product of exponentials formulation in order to describe the robot kinematics. Module configurations are represented through AIM, an Assembly Incidence Matrix. Its purpose is to separate the assembly sequence and spatial relationship of the assembled modules. Dyad kinematics was proposed to derive the forward kinematics. This relates the motion of two connected modules under some joint displacement. A linear superposition of all the dyads in the robot was computed to calculate the total error of the end-effector. Two calibration methods were derived, one based on 6 parameters and one on 7. The forward kinematics equations are linearized and then solved in a least-squares sense. Two computer simulations were performed to calibrate a simple 2 DOF modular robot and a 4 DOF SCARA robot that was constructed using the modules. Overall, the magnitude of the error of the robots was reduced by two orders of magnitude.

2.4 Multiple Configurations

In many kinematic calibration schemes, multiple configurations resulting in the same end-effector position are used to provide additional data. In Omodei *et al*. [5], this was achieved through use of a custom-built reference object where the end-effector was physically inserted into holes. However, several of the holes could be reached with different joint parameters. This allowed for an additional 35 measurements, and hence an additional 35 equations. Hypothetically, this could allow for smaller reference objects at lower cost. In the case that multiple reference objects were to be mounted in several locations of the task-specific workspace of the robot, benefits could be realized due to the size of the objects and their constraint to the machine and the cost of their manufacture. Another means in which to effect multiple configurations was discussed and evaluated by Khalil *et al*. [4]. Some sort of port or interface would close the kinematic chain by commanding the end-effect to grasp it. This could either restrain the end-effector in terms of position and orientation or just position. This is a highly effective way to obtain measurement data in that one would simply have to interface the robot with the port and then actuate the joints. In Gong *et al*. [11], multiple configurations were used to identify the hand-eye transformation. Measuring the same spot with a laser and camera system allowed for the solution of the matrix equation representing the transformation.

2.5 Workspace Issues

Many kinematic calibration procedures address the issue of robot accuracy within the reachable volume of the robot. Other schemes were designed to account for one task-specific area for calibration. Different production and manufacturing applications would agree with different methods. For a pick-and-place operation utilizing a SCARA type robot, two locations would have to be calibrated, but it is possible that one zone would have to be calibrated for a robot used in a welding application. Several researchers have addressed this issue by constructing additional reference objects and mounting them throughout the workspace. This was done in Gong *et al*. [11], where three separate objects were placed in the workspace. Others, such as Abderrahim and Whittaker [1] and

Sultan and Wager [2], focussed on open loop methods based on planes of rotation, which allow the joints to be actuated in whichever part of the workspace as needed. The capability of calibrating the entire workspace is simply inherent to the method. Methods based on external measurement system enjoy the same advantage as well as those based on constraints such as the scheme proposed by Ikits and Hollerbach [7]. Theoretically, the surface that the probe touches could be anywhere, as long as it is sufficiently flat.

Zhuang *et al*. [31] proposed a process in which an optimal set of manipulator configurations for kinematic calibration can be determined. A novel genetic algorithm is employed to determine the optimal measurement configurations given the workspace volume. Essentially, a population of initial measurement configurations is supplied to an algorithm that breeds new measurement configurations based on an evolutionary response. The stronger survive while the weak are annihilated. The fitness function, reproduction operators, crossover operators, and mutation operators, which define the process, have to be determined experimentally or through simulation. The authors tested this algorithm through simulation with a SCARA type robot. Although it did yield successful results, the performance of the genetic algorithm was only a marginal improvement over conventional techniques based on complete randomness. However, if multiple optimal solutions are required, this technique could be quite useful. Therefore, given the entire workspace volume, this process could determine the ideal place for a reference object and quite possibly reduce calibration time by identifying the most favourable robot configurations.

3.0 Parallel Robot Calibration

Parallel mechanisms have a favourable reputation with respect to accuracy and stiffness. Thus, calibration of such mechanisms is supremely important as they will be used in many more applications in the near future. This is driven by the fact that manufacturers face the demand of quality products in shorter and shorter time frames. Unlike serial manipulators, where the links and joints form one kinematic chain, a parallel robot consists of a number of joint-link trains that coalesce at one point. Through activation of the legs, comprised of prismatic joints, the end-effector can achieve different orientations and positions rather accurately. The mechanism is also quite stiff and can handle a variety of loads. Error models for parallel robots can involve up to 132 independent parameters. By making different assumptions regarding joints or the workspace, this can be reduced significantly. Methods of calibration for parallel robots involve external measurement systems or for self-calibration, the internal joint sensors.

3.1 Error Models

Calibration of any kind of robot requires the use of an error model. This usually takes the form of the minimization of some measurement residual which then leads to the robot parameters. With serial robots, measurements are usually taken of the end-effector and a comparison is made between the calculated position and the measured position using the nominal parameters. In parallel robot calibration, the leg lengths are used. Masory *et al*. [32], derived an error model for a Stewart platform that takes into account the manufacturing imperfections of the spherical joints and U-joints. A simplified model is also presented which is later employed in the calibration of a Stewart platform by minimizing inverse kinematic residuals in Zhaung *et al*. [33]. An external measurement system was used to identify the position and orientation of the plate so that the leg lengths could be computed. Vischer and Clavel [34], derived models for the calibration of the Delta robot and more recently, the Argos mechanism in Vischer and Clavel [35]. In both of these works, the virtual centre of rotation is used along with an external measurement system instead of using leg length deviations. Multiple models are used depending on the number of parameters to be calibrated and the assumptions used.

Kinematic modelling and calibration of a Stewart platform is derived and implemented in Masory *et al*. [32]. All the possible sources of error in the platform are addressed in a 132- parameter model that accounts for imperfections in the spherical joints and U-joints. The nominal kinematic model with 42 parameters, which assumes ideal joints, was presented first followed by the actual kinematic model. The definition of a joint-link train was provided as well as the resultant equation of the transformation matrix from the robot base to the robot plate. An effective algorithm was presented to estimate the kinematic parameters. This involved the calculation of the Jacobian and the solution of a linear least-squares problem. Solution of the inverse kinematic equations is performed numerically through use of the Newton-Raphson method. As such, an initial estimate of the joint variable vector must be made using the nominal parameters. In the simulations with the two models, measurement noise was first disregarded and then incorporated to see its effect. Relatively large joint errors were inputted into the simulation. For the first case with 7 poses and no measurement noise, 4 iterations of the procedure were required to reduce the error to essentially zero. Overall, the pose error was reduced by an order of magnitude through use of this method in the actual model.

3.2 Self-Calibration

Self-calibration of parallel manipulators is achieved through redundant sensing of joints or using the internal joint sensors in a situation where one of the legs is held in a fixed position and orientation. As the name implies, no external measurement system is required to obtain positional of the end-effector. In Zhuang [36], redundant sensors were installed on the joints of the mechanism to produce a measurement residual that could be minimized. In Khalil and Besnard [37], one of the U-joints or one of the spherical joints was locked into position and the manipulator was commanded to a set of robot configurations. Both of these methods would require little additional expense to the cost of the robot in comparison to purchasing an external measurement system.

3.2.1 Redundant Sensing

Zhuang [36] described a thorough derivation of the self-calibration problem for parallel manipulators. As no external measurement device is used to provide position and orientation data of the end-effector, redundant sensors are installed on the U-joints and spherical joints. Calculating the forward and inverse kinematics through multiple paths based on these redundant sensors provides a measurement residual in which to calibrate the mechanism. The calculation of the Identification Jacobian matrix revealed the number of kinematic parameters that could be identified. Three different cost functions were presented through error-model analysis. A simulation study was performed on the Stewart platform with 6 redundant sensors installed. It found that the accuracy could be significantly increased after just 12 pose configurations.

3.2.2 Fixed Links

A method in which to self-calibrate a parallel manipulator using only the prismatic joint sensors of the robot was developed by Khalil and Besnard [37]. The method is based on fixing either one of the spherical joints of the movable platform in one position or mechanically locking one of the U-joints of the base. The calibration procedure was implemented in MatLab using a function representing the least-squares method of Levenberg-Marquardt. A simulation study was performed that introduced between 0 and 5 cm randomly distributed parameter error. This created a relatively large error with respect to the position of the end-effector. Two simulations were performed, one without measurement noise and one with measurement noise representative of that found in reality. 40 to 80 robot configurations were used which corresponded to 10 min or 20 min trials. The two locking methods were studied individually and combined, which yielded the better results. Two data sets taken from the locked spherical case and one from the locked U-joint case reduced the magnitude of the error by an order of magnitude. This method is dependent on being able to mechanically lock the joints and has not been tested experimentally.

3.3 External Measurements Systems

Similar to the serial robot case, to obtain measurement data of the end-effector, an external sensing device can be utilized. This device can measure the position and the orientation of the end-effector relative to the base. This type of system is straightforward to implement but requires the acquisition of an expensive system. In Zhuang *et al*. [33], a theodolite was used to measure three targets on a Stewart platform. In Vischer and Clavel [34, 35], an orthogonal arrangement of linear touch probes was used in conjunction with joint sensors.

In Zhuang *et al*. [33], a kinematic calibration method for Stewart platforms and other parallel manipulators was offered. The method is based on the formulation of a cost function, which is based on the inverse kinematic solution. The function represents the leg length error or measurement residual, which has to be minimized. The error model involves the solution of 42 parameters, where the joints are considered to be ideal. The calculation of the identification Jacobian is necessary to be able to compute the solution. This method was tested on a Stewart platform, which made use of hydraulic manipulators. To determine the plate coordinate system, three targets were mounted on the plate and measured by a theodolite. A similar approach was taken with the base. The orientation of plate could then be determined as the targets were coincident with the plane of the plate. 20 measurements were taken where 12 were used to identify the kinematic parameters and 8 were used for verification. The Gauss-Newton method was employed to solve for the platform parameters. Overall, an accuracy of close to the robot's repeatability was obtained.

Vischer and Clavel [34] discussed in great detail two kinematic calibration techniques for the Delta robot. Two parametric models were pursued, model 24 and model 54. Model 24 did not account for orientation errors of the end-effector by simply keeping the plate parallel with the base. Model 54 incorporated both position and orientation errors. Sets of 74 measurements were taken and were used to estimate the parameters. This was done through implicit calibration by using the implicit closure equations. The second calibration technique is called semiparametric calibration. It involved expansion of the closure equations and the substitution of linear factors. Overall, the position error was reduced by a factor of 12.3 and the orientation error was reduced by a factor of 3.4. A measurement machine recorded the position of the position of the end-effector. The Delta robot was rigidly attached to this machine. The orientation of the end-effector, which is attached to the z-axis of the measuring machine by a spherical joint, was measured by three linear digital probes, which were orthogonally arranged. The position measurement was accurate to +/- 10 μm and the orientation is accurate to +/- 15 arcseconds. The joint angles are measured by high-resolution laser encoders and were accurate up to 25 arcseconds. Thus a high degree of accuracy could be obtained through calibration. For the Argos mechanism, Vischer and Clavel [35] generated error models based on the virtual centre of rotation, where the end-effector was commanded to a single point but in different robot configurations. Two models were derived, Model 9 and Model 27. In Model 9, only the deviations in orientation were considered, which led to the simplification. The measurement system again comprised of a three linear touch probes, orthogonally arranged to measure the position of the end-effector. A gimbal arrangement of three incremental encoders measured the orientation of the end-effector and three encoders measured the joint angles. The touch probes had a resolution of 0.1 μm and an accuracy of +/- 1.9 μm. The encoders for orientation had a resolution of 2.7 arcminutes while those for the joint angles had a resolution of 6.5 arcseconds. The high resolutions of the devices used allowed the authors to improve the accuracy of the mechanism by a factor of 5.3 for the orientation and 3.4 for the position.

3.4 Base and Tool Calibration

Most of the calibration methods discussed in this paper took into account the base and plate of the parallel robot. In Yang *et al*. [38], the calibration of the tool mounted on the plate is addressed. The transformation from the world-coordinate system to the base to the plate and finally to the tool is derived using the same principles behind the product-of-exponentials formula. An iterative least-squares algorithm was employed to solve for the kinematic errors. A simulation was performed using a 3-legged parallel manipulator incorporating noisy data measurements. After just three poses and no measurement noise, the procedure identified the inputted deviations, which were considerable, within 3 to 4 iterations. With measurement noise, two sets of 50 measurements were used to calibrate the robot and then verify the procedure. Extensive study was performed at this stage and it was found that 20 measurement s were sufficient for the deviations to stabilize. The positional error stabilized at about 0.1 mm while the orientation errors stabilized at around 0.0005 radians. These values mirror those inputted as the noise magnitudes. This method was successful however it relies on the self-calibration stage where errors could propagate.

4.0 Conclusion

There have been many advances in the realm of robotic calibration over the past 6 years. Both parallel and serial robots can now be parameterized by equations that reduce the number of points of singularity in the workspace. Algorithms used to solve for robot transformations have been tailored towards increasing processing efficiency and decreasing processing time. Many unique and innovative methods of generating calibration data have been tested and documented. Camera-based calibration has seen enormous improvement as a partial result of innovations in the area of solving the hand-eye calibration problem. This class of calibration is expected to see exponential growth in the near future. The new techniques discussed in this paper have been used for several interesting applications including the construction and health-care industries. A sensing linkage implementing a precisely machined spherical joint has been used in the calibration of an industrial excavator. The treatment of cancer patients has also been affected by improvements in robotic calibration. Exposing cancerous tumours to radiation is a process that requires a high level of accuracy. A recently developed technique involving the resolution of both linear geometric parameters as well as the contribution of patient payload weight to the configuration of cancer treatment beds has provided that level of accuracy.

In conclusion, research in the area of kinematic calibration has come a long way in such a short time, and looks to advance even further in the years to come. As the degree of automation in our society grows, the need for devices that are both accurate and able to maintain that accuracy will also grow. This growth will facilitate the demand for more innovative techniques in kinematic calibration.

6.0 References

[1] M. Abderrahim and A.R. Whittaker, “Kinematic Model Identification of Industrial Manipulators”, Robotics and Computer Integrated Manufacturing, v. 16, pp. 1-8, 2000.

[2] Ibrahim A. Sultan and John G. Wager, “A Technique for the Independent-Axis Calibration of Robot Manipulators with Experimental Verification”, International Journal of Computer Integrated Manufacturing, v. 14, n. 5, pp. 501-512, 2001.

[3] Alberto Omodei, Giovanni Legnani and Riccardo Adamini, “Calibration of a Measuring Robot: Experimental Results on a 5 DOF Structure, Journal of Robotic Systems, v. 18, n. 5, pp. 237-250, 2001.

[4] W. Khalil, S. Bernard, P. Lemoine, “Comparison Study of the Geometric Parameter Calibration Methods”, International Journal of Robotics and Automation, v. 15, n. 2, pp. 56-68, 2000.

[5] Alberto Omodei, Giovanni Legnani and Riccardo Adamini, “Three Methodologies for the Calibration of Industrial Manipulators: Experimental Results on a SCARA Robot”, Journal of Robotic Systems, v. 17, n. 6, pp. 291-307, 2000.

[6] M. Khoshzaban, F. Sassani, P.L. Lawrence, “Kinematic Calibration of Industrial Hydraulic Manipulators”, Robotica, v. 14, n. 5, pp. 541-551, 1996.

[7] M. Ikits and John Hollerbach, “Kinematic Calibration Using a Plane Constraint”, Proc. IEEE International Conference on Robotics and Automation, Albuquerque, NM, pp. 3191-3196, 1997.

[8] Chunhe Gong, Jingxia Yuan, Jun Ni, “Non-geometric Error Identification and Compensation for Robotic System by Inverse Calibration”, International Journal of Machine Tools and Manufacture, v. 40, pp. 2119-2137, 2000.

[9] P. Drouet, S. Dubowsky, S. Zeghloul, C. Mavroidis, “Compensation of Geometric and Elastic Errors in Large Manipulators with an Application to a High Accuracy Medical System”, Robotica, v. 20, n. 3, pp. 341-, 2002

[10] Yan Meng and Hanqi Zhuang, “Self-Calibration of Camera-Equipped Robot Manipulators”, International Journal of Robotics Research, v. 20, n. 11, pp. 909-921, 2001.

[11] Chunhe Gong, Jingxia Yuan, Jun Ni, “A Self-Calibration Method for Robotic Measurement System”, Journal of Manufacturing Science and Engineering, v. 122, pp. 174-181, 2000.

[12] Hanqi Zhuang, Wen-Chiang Wu, Zvi S. Roth, “Camera-Assisted Calibration of SCARA Arms”, IEEE Robotics and Automation Magazine, v. 3, n. 4, pp. 46-52, 1996.

[13] P. Rousseau, A. Desrochers, N. Krouglicof, “Machine Vision System for the Automatic Identification of Robot Kinematic Parameters”, IEEE Transactions on Robotics and Automation, v. 17, n. 6, pp. 972-999, 2001.

[14] Jose Mauricio S.T. Motta, Guilherme C. de Carvalho, R.S. McMaster, “Robot Calibration Using a 3D Vision-based Measurement System with a Single Camera”, Robotics and Computer Integrated Manufacturing, v. 17, n. 6, pp. 487-497, 2001.

[15] Hanqi Zhuang, K. Wang, Z.S. Roth, “Simultaneous Calibration of a Robot and a Hand-Mounted Camera”, IEEE Transactions on Robotics ad Automation, v. 11, n. 5, pp. 649-661, 1995.

[16] Remy, S., Dhome, M., Lavest, J., Daucher, N., “Hand-eye calibration”, IEEE International Conference on Intelligent Robots and Systems, v.2 , pp. 1057-1065, 1997.

[17] Song De Ma, “A Self-Calibration Technique for Active Vision Systems”, IEEE Transactions on Robotics and Automation, v. 12, n. 1, pp. 114-121, 1996.

[18] Dongmin Kim, “Dual Quaternion Application to Kinematic Calibration of Wrist-Mounted Camera”, Journal of Robotic Systems, v. 13, n. 3, pp. 153-162, 1996.

[19] Hanqi Zhuang and W-C. Wu, “Camera Calibration with a Near-Parallel (Ill-Conditioned) Calibration Board Configuration”, IEEE Transactions on Robotics and Automation, v. 12, n. 6, pp. 918-922, 1996.

[20] Hanqi Zhuang, “A Note on Hand-Eye Calibration”, International Journal of Robotics Research, v. 16, n. 5, pp. 725-730, 1997.

[21] Guo-Qing Wei, Klaus Arbter, Gerd Hirzinger, “Active Self-Calibration of Robotic Eyes and Hand-Eye Relationships with Model Identification”, IEEE Transactions on Robotics and Automation, v. 14, n. 1, pp. 158-166, 1998.

[22] Hanqi Zhuang, “Hand-Eye Calibration for Electronic Assembly Robots”, IEEE Transactions on Robotics and Automation, v. 14, n. 4, pp. 612-616, 1998.

[23] Fadi Dornaika and Radu Horaud, “Simultaneous Robot-World and Hand-Eye Calibration”, IEEE Transactions on Robotics and Automation, v. 14, n. 4, pp. 617-622, 1998.

[24] J. Batista, H. Araujo, A.T. de Almeida, "Iterative Multistep Explicit Camera Calibration", IEEE Transactions on Robotics and Automation, v. 15, n. 5, p. 897-918, 1999.

[25] H. Malm and A. Heyden, "Hand-Eye Calibration from Image Derivatives", Lecture Notes in Computer Science. 1843: 493-507, 2000.

[26] J. Angeles, G. Soucy, F.P. Ferrie, "The Online Solution of the Hand-Eye Problem", IEEE Transactions on Robotics and Automation, v. 16, n. 6, pp. 720-732, 2000.

[27] Nicolas Andreff, Radu Horaud, Bernard Espiau, "Robot Hand-Eye Calibration Using Structure-from-Motion", International Journal of Robotics Research, v. 20, n. 3, pp. 228-248, 2001.

[28] T-S. Shen and C-H. Menq, "Automatic Camera Calibration for a Multiple-Sensor Integrated Coordinate Measurement System", IEEE Transactions on Robotics and Automation, v. 17, n. 4, pp. 502-507, 2001.

[29] Koichiro Okamura and F.C. Park, "Kinematic Calibration Using the Product of Exponentials Formula", Robotica, v. 14, n. 4, pp. 415-421, 1996.

[30] I-Ming Chen and Guilin Yang, "Kinematic Calibration of Modular Reconfigurable Robots Using Product-of-Exponentials Formula", Journal of Robotic Systems, v. 14, n. 11, pp. 807-821, 1997.

[31] Hanqi Zhuang, Jie Wu, Weizhen Huang, "Optimal Planning of Robot Calibration Experiments by Genetic Algorithms", Journal of Robotic Systems, v. 14, n. 10, pp. 741-752, 1997

[32] O. Masory, L. Wang, H. Zhuang, "Kinematic Modelling and Calibration of a Stewart Platform", Advanced Robotics, v. 11, n. 5, pp. 519-539, 1997.

[33] Hanqi Zhuang, Jiahua Yan, Oren Masory, "Calibration of Stewart Platforms and Other Parallel Manipulators by Minimizing Inverse Kinematic Residuals", Journal of Robotic Systems, v. 15, n. 7, pp. 395-405, 1998.

[34] Peter Vischer and Reymond Clavel, "Kinematic Calibration of the Parallel Delta Robot", v. 16, n. 2, pp. 207-218, 1998.

[35] Peter Vischer and Reymond Clavel, "Kinematic Calibration of the Parallel Argos Mechanism", Robotica, v. 18, n. 6, pp. 589-599, 2000.

[36] Hanqi Zhuang, "Self-Calibration of Parallel Mechanisms with a Case Study on Stewart Platforms", IEEE Transactions on Robotics and Automation, v. 13, n. 3, pp. 387-397, 1997.

[37] W. Khalil and S. Bernard, “Self-Calibration of Stewart-Gough Parallel Robots without Extra Sensors”, IEEE Transactions on Robotics and Automation, v. 15, n. 6, pp. 1116-1121, 1999.

[38] Guilin Yang, I-Ming Chen, Song Huat Yeo, Wee Lim, “Simultaneous Base and Tool Calibration for Self-Calibrated Parallel Robots”, Robotica, v. 20, n.4, pp. 367-375, 2002.