Kinematic Calibration of Industrial Manipulators

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1. Motivation

As a first step in the validation of a relative measurement concept, several kinematic calibration simulations were performed. The developed programs are essentially learning tools involving the derivation of the Denavit-Hartenberg (DH) parameters, calculation of the Jacobian matrix and application of Singular Value Decomposition (SVD). A similar analysis, involving these steps, will then take place regarding the validation of an error model developed for a KUKA 15/2 industrial manipulator that will incorporate the relative measurement concept [1]. Once accomplished, an error model will then be generated for a Stäubli RX 90B type manipulator, after which, the feasibility of a low-cost kinematic calibration system, employing this technique, will be studied.

2. Method

In order to calibrate a manipulator, measurements, in terms of position and orientation, must be taken of the end effector. For the development of the programs considered here, only the position of the end effector is taken into account. The true end effector position could either be directly supplied to the program with the corresponding joint angles or computed given the joint parameter deviations. Both cases were accounted for, however for ease of simulation, the latter case was pursued. Standard DH parameterization reveals the joint parameters, and to these sufficiently small synthetic deviations are introduced. The parameters considered were the joint angles, ϑ, the link lengths, *a*, and the link offsets, *d*. The joint twist angles were assumed constant in the analysis. Thus, the actual position of the end effector can be calculated as a function of the joint angles, link lengths, link offsets and joint twists along with their corresponding deviations:

$$p_{ACTUAL} = f(\vartheta + \Delta\vartheta, a + \Delta a, d + \Delta d, \alpha).$$

Whereas the position calculated by the controller involves only the nominal parameters:

$$p_{CONTROLLER} = f(\vartheta, a, d, \alpha).$$

The difference is computed vectorially and in three-dimensional space:

$$\{\Delta p_{xyz}\} = \{p_{ACTUAL}\} - \{p_{CONTROLLER}\}.$$

The forward kinematic equations and the elements of the Jacobian can be derived analytically through use of the DH parameters. The Jacobian is determined as follows:

$$J = \begin{bmatrix} \frac{\partial p_x}{\partial \theta_1} & \frac{\partial p_x}{\partial \theta_2} & \cdots & \frac{\partial p_x}{\partial \theta_n} \\ \frac{\partial p_y}{\partial \theta_1} & \frac{\partial p_y}{\partial \theta_2} & \cdots & \frac{\partial p_y}{\partial \theta_n} \\ \frac{\partial p_z}{\partial \theta_1} & \frac{\partial p_z}{\partial \theta_2} & \cdots & \frac{\partial p_z}{\partial \theta_n} \end{bmatrix}.$$

This overall procedure was followed for each of the robot poses and then the data was concatenated into a final Jacobian matrix and a vector of end point deviations. The deviations of the joint parameters can be determined by use of the inverse of the Jacobian matrix:

$$\{\Delta\theta\} = [J]^{-1}\{\Delta p_{xyz}\}.$$

As the Jacobian matrix is not invertible, it must be suitably decomposed. We employed SVD. Thus, the equation relating the joint parameter deviations to the position deviations becomes:

$$\{\Delta\theta\} = [V]\cdot[S]\cdot[U]^{-1}\{\Delta p_{xyz}\},$$

where the *S* matrix is diagonal and comprised of the inverses of the singular values. If the singular value was infinitesimally small, smaller than the floating-point precision of the machine, it was set to zero [2]. The joint parameter deviations found could then be added to the nominal parameters used in the controller.

3. Modelled Manipulators

As previously stated, the motivation behind developing these programs was to use them primarily as a learning tool, thus, different robots employing both revolute and prismatic joints were examined. Each revolute joint was rotated simultaneously in equal increments of 1º from 0º to the number of poses, while prismatic joints were extended similarly to their full extension. There were five manipulators modelled, each progressively more complex, and each modelled using standard DH parameterization. Three simple planar manipulators were modelled, an R, an RR and an RRR, as seen in Figure 1. Also, two more complex manipulators were modelled, each involving

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prismatic joints: a SCARA arm and a Scorbot robot, seen in Figure 2. The SCARA arm has four degrees of freedom while the Scorbot robot has five degrees of freedom and each have three-dimensional workspaces. General representations of each of the five manipulators can be seen below.

Figure 1: Planar Manipulators

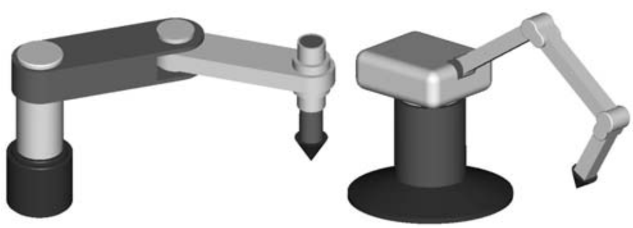

Figure 2: SCARA Arm and Scorbot Robot

4. Results

With reference to Table 1, the calibration procedure identified nearly all of the joint parameter deviations with high accuracy for the Scorbot robot. Similar results were obtained with the other manipulators as well. The only requirement of the procedure was the joint parameter deviations be sufficiently small, approximately four orders of magnitude smaller than the dimensions of the link lengths. For the simple planar manipulators, R, and RR, their respective programs were able to observe and identify all of the parameter deviations with errors of less than 0.01% with 10 poses.

For the SCARA arm, all of the link offsets, *d*, in the vertical direction of the manipulator were not observable. The program was able to successfully identify the deviations in terms of the link lengths and the first two joint angles. As the position of the end effector is not dependent on the angular position of the final joint, the elements of the Jacobian relating to this joint parameter were zero. Similarly, the derivatives of the position vector in the z-direction with respect to all the DH parameters were constants, namely –1, 0, or 1.

In the case of the Scorbot robot with the following set of parameter deviations, 100 poses, and one application of SVD, the error of greatest magnitude was 0.1813 %. Even if the number of poses was reduced to 25, the magnitude of the greatest error only grew to approximately 2%. This indicates that an iterative procedure could be implemented using a smaller number of measurements to save time. Increasing the number of poses reduces the magnitude of the error accordingly. All of the deviations were observable and identified.

Table 1: Results of Kinematic Calibration of the Scorbot Robot

Deviation	Synthetic	Identified	Error (%)
$\Delta\vartheta_1$	0.010000 º	0.010001 º	0.0980
$\Delta\vartheta_2$	- 0.005000 º	-0.005007 º	0.1454
$\Delta\vartheta_3$	0.007500 º	0.007514 º	0.1813
$\Delta\vartheta_4$	- 0.008500 º	-0.008510 º	0.1167
Δa_1	0.1500 mm	0.1500 mm	- 0.0279
Δa_2	- 0.2000 mm	-0.2000 mm	0.0194
Δa_3	0.1250 mm	0.1250 mm	0.0095
Δd_1	- 0.1750 mm	-0.1749 mm	- 0.0477
Δd_5	0.2500 mm	0.2501 mm	0.0208

5. Conclusions

The results indicate that kinematic calibration is an efficient approach to the elimination of parameter errors and further study is justified. In terms of the manipulators analyzed in this paper, almost all of the joint parameter deviations were observable and identified with little error. However, this calibration procedure was followed under the assumption that there was no measurement noise inherent in the data. The absolute position of the end effector was calculated directly knowing the parameter deviations. The effectiveness of the error model generated for the KUKA 15/2 will be tested on actual measurement data and this will reveal the effect of measurement noise.

Modifications will also be realized such that the programs follow a different approach, where a particular threshold value for sufficient calibration is designated. Thus, the same general procedure of determining the spatial deviations of the end effector, calculation of the Jacobian matrix and application of SVD, will be iteratively performed while a minimum number of pose measurements will be required.

References

[1] English, K., Hayes, M.J.D., Leitner, M., Sallinger, C., "Kinematic Calibration of Six-Axis Robots", *Proc. CSME Forum*, 2002.

[2] Press, William H., Teukolsky, Saul A., Vetterling, William T., Flannery, Brian P., *Numerical Recipes in C: The Art of Scientific Computing,* 2nd Edition, Cambridge University Press, 1992.