Velocity-Level Kinematics of the Atlas Spherical Orienting Device Using Omni-Wheels

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Abstract

Using a novel actuation concept employing omni-directional wheels (or simply omni-wheels), the Atlas simulator motion platform provides unlimited angular displacement about any axis. The Atlas concept completely decouples the orienting and positioning degrees-of-freedom and further decouples each of the positioning degrees-of-freedom. It consists of an omni-wheel driven sphere for orientation that has its geometric centre positioned by an XYZ-table. The Jacobian of the orienting device is independent of time and dependent only on the mechanism architecture, meaning that it is always invertible for any configuration provided that the initial design parameters do not result in architecture singularities. An examination of the Atlas Jacobian and its determinant identifies architecture singular design conditions. It is found that these are not design limiting. Discussion highlights the uniqueness of the Atlas concept and its associated kinematic advantages.

1 Introduction

The Atlas motion platform was developed within the Carleton University Simulator Project (CUSP) as a possible means of expanding the motion envelope of simulator motion bases. It is a six degree-of-freedom (DOF) platform possessing a kinematic architecture that effectively decouples positioning DOF from orienting DOF and further has positioning DOF that are linearly independent from each other, see Figure 1. This is accomplished by mounting a spherical three DOF orienting platform, capable of unlimited angular displacement about any axis, on a three-axis gantry (XYZ-table). Both the orienting and positioning interior workspaces are potentially nonsingular. As it is clear that the three-axis gantry platform is configurationally singular only at its reachable workspace boundary, attention is focused in this paper exclusively on the orienting platform.

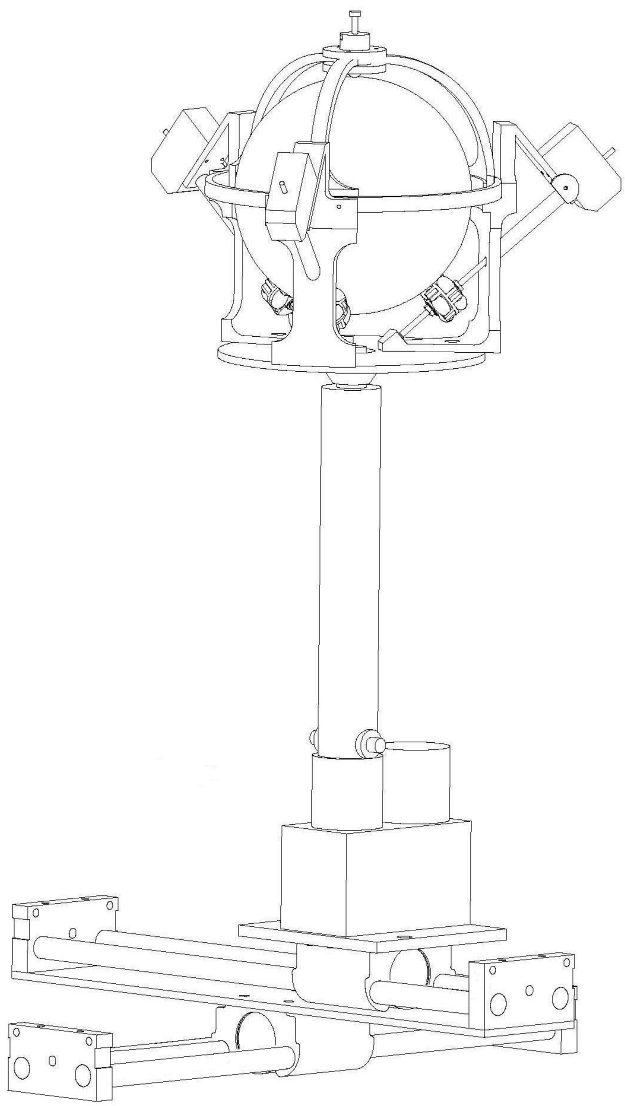
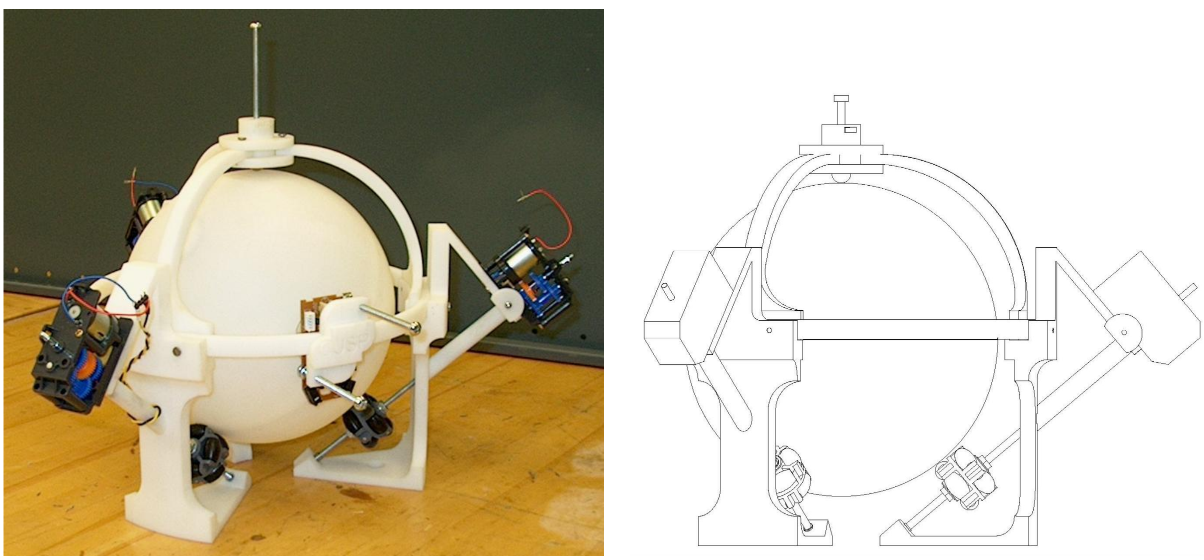


Figure 1: The 6 DOF conceptual design.

The orienting concept uses omni-wheels to actuate the three rotational DOF motion (roll, pitch, and yaw). Omni-wheels were introduced in 1985 and intended for zero-turn-radius planar automated guided vehicles (AGVs) [1]. Atlas, in turn, is a novel spherical actuation concept (patent pending). Unlimited spherical motion is achieved by placing a sphere on three omni-wheels strategically spaced around the sphere, illustrated in Figure 2. Omni-wheels provide grip tangent to their circumference while allowing passive motion in the bi-normal direction on the free spinning castors mounted on the periphery. The omni-wheel type currently employed is shown in Figure 3. Fisette et al. [2] provide an overview and discussion of the characteristics of different omni-directional wheel types. Different linear combinations of omni-wheel angular displacements produce angular displacement of the sphere. Due to the absence of mechanical constraints and link interference, the spherical motion is unlimited. The orienting workspace is configurational singularity free.

The concept of a spherical actuator is not new. Spherical dc induction motors were introduced in 1959 by Williams, et al. in [3]. Developments continued over the next 30 years leading to designs presented in [4, 5], for example. However, due to physical limitations imposed by the stator and commutator angular displacements are limited. The ability to produce continuous unlimited angular displacements in roll, pitch and yaw puts Atlas in new territory in terms of freedom of motion in mechanical devices. Potential applications include: land, sea, and air vehicle simulators; testing, calibrating, and commissioning satellite attitude acquisition devices; motion platforms for gaming applications.



(a) ABS prototype.

(b) 3D CAD model.

Figure 2: The Atlas table-top demonstrator highlighting the omni-wheel actuation concept.

The aim of this paper is to describe relevant aspects of the Atlas concept; develop velocity-level kinematics; extract the system Jacobian relating actuating omni-wheel angular velocities to the resulting angular velocity of the Atlas sphere; and demonstrate that conditions resulting in configurational singularities, situations where rotation about an axis is not possible, are based solely on the platform kinematic architecture and not on the configuration of the mechanism at

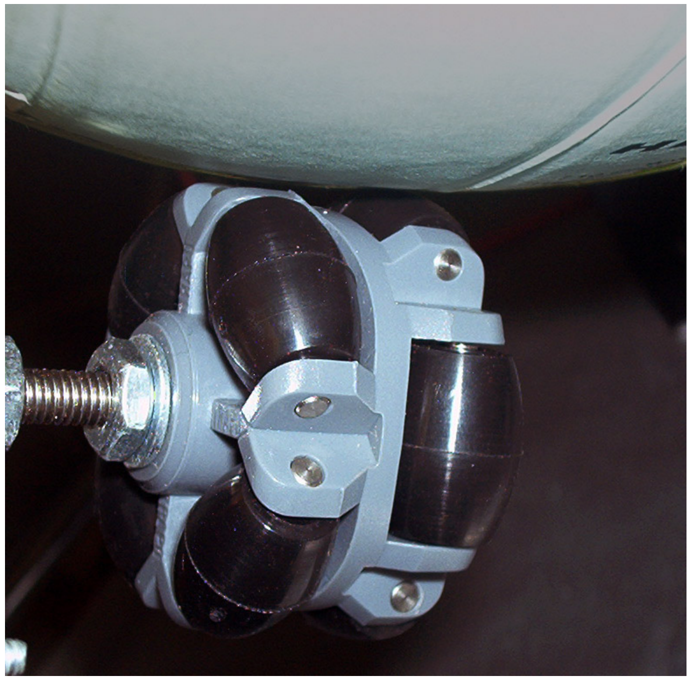


Figure 3: Detail of omni-wheel actuator.

any instant in time. It will be shown that for non-architecturally-singular design parameters the mechanism orienting workspace is entirely free of configurational singularities. While planar AGVs using omni-wheels have been extensively studied [6], it appears that employing omni-wheels for spherical motion is novel.

2 Atlas Velocity-level Kinematics

Derivation of the velocity-level kinematics of the Atlas platform requires expressing the angular velocity of the sphere in terms of the magnitude of the angular velocities of each omni-wheel. The matrix relating these two arrays is the platform Jacobian. The variables required to define the system geometry are the position vectors from the sphere centre to each of the omni-wheel contact point locations, $\mathbf{R}_k$, $k \in \{1, 2, 3\}$, expressed in a cartesian coordinate system $[X, Y, Z]$ fixed at the geometric centre of the sphere; and the omni-wheel radius vectors, $\mathbf{r}$, defining the position vector from the omni-wheel axis of rotation to the point of contact with the sphere expressed in a local coordinate system $[x, y, z]$ defined such that x points outward along the omni-wheel axis of rotation and z is directed toward the point of contact with the sphere. Relevant coordinate systems are shown in Figure 4 for a configuration where the omni-wheels are spaced 120° apart (with angles β_k defining the angle to each contact point in the XY-plane and referenced to the X-axis) and each tilted by an angle α to the horizontal as indicated in Figures 4 and 5.

It is assumed that each of the three omni-wheels is identical, and has a circular profile. Therefore each will have the same dimensions. The radius vector of each omni-wheel, in its local coordinate frame, is referenced relative to its sphere contact point. Since the contact point lies along the z-axis the omni-wheel radius, $\mathbf{r}_k$, is equal to $[0, 0, r_{k_z}]^T$ where r_{k_z} is the distance from the centre of the

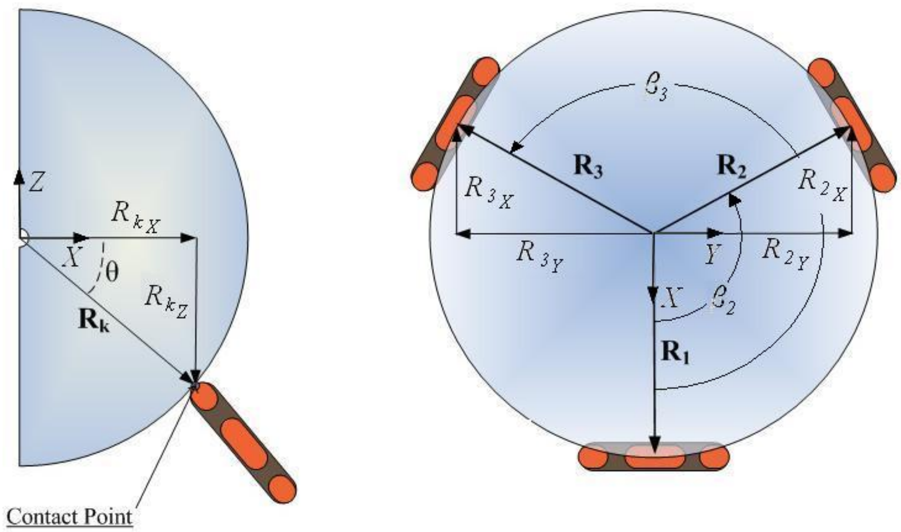


Figure 4: Sphere contact point radial vector components.

k^{th} omni-wheel to the contact point on the castor wheel (see Figure 5). Hence

$$\mathbf{r}_1 = \mathbf{r}_2 = \mathbf{r}_3.$$

Without loss in generality, but assuming geometric symmetry, we can impose the condition that

$$r_{1_z} = r_{2_z} = r_{3_z} = r_z.$$

In order to achieve unconstrained angular displacement, each of the three omni-wheels is controlled independently. Therefore, the sphere can be given an angular velocity about any desired axis. The angular velocity of each omni-wheel is denoted $\boldsymbol{\omega}_k$, $k \in \{1, 2, 3\}$. The omni-wheel axis of rotation is fixed relative to its reference frame where the x-axis points along this axis; therefore the omni-wheel angular velocity vector is always in the direction of the x-axis:

$$\boldsymbol{\omega}_k \quad = \quad [\omega_{k_x}, 0, 0]^T.$$

Using the specified variables $\mathbf{R}_1$, $\boldsymbol{\omega}_1$, $\boldsymbol{\omega}_2$, $\boldsymbol{\omega}_3$, $\mathbf{r}$, and the geometry of the Atlas simulator it is possible to determine the Jacobian.

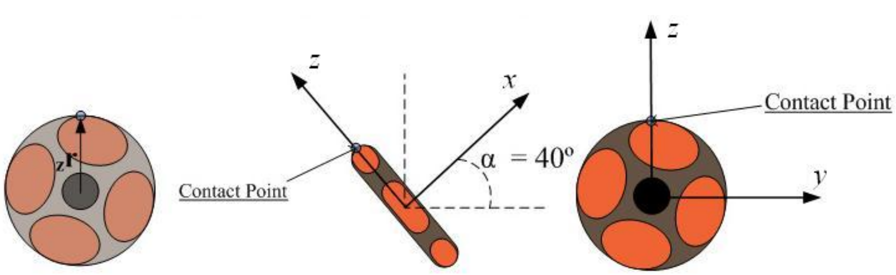


Figure 5: Omni-wheel coordinate reference frames.

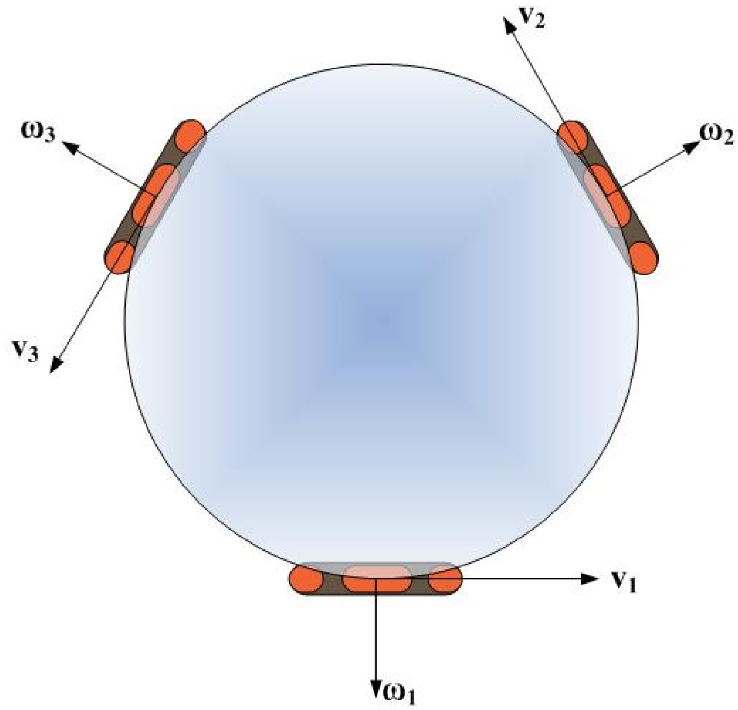


Figure 6: Omni-wheel angular, and sphere contact point tangential linear velocities.

The Atlas sphere acquires angular velocity from the tangential velocities at the contact points of the omni-wheels (see Figure 6). The tangential velocity, $\mathbf{v}_k$, at each contact point can be obtained from the cross-product of the omni-wheel angular velocity vector, $\boldsymbol{\omega}_k$ and omni-wheel radial vector, $\mathbf{r}_k$:

$$\mathbf{v}_k = \boldsymbol{\omega}_k \times \mathbf{r}_k. \tag{1}$$

All omni-wheel tangential velocities at their respective sphere contact points will lie along the local y-axis direction because the cross product of $\boldsymbol{\omega}_k = [\omega_{k_x}, 0, 0]^T$ and $\mathbf{r}_k = [0, 0, r_z]^T$ yields only a y component of linear tangential velocity at each omni-wheel contact point:

$$\mathbf{v}_k = [0, v_{k_y}, 0]^T = \boldsymbol{\omega}_k \times \mathbf{r}_k. \tag{2}$$

Hence, the tangential velocities at the omni-wheel contact points, expressed in each omni-wheel reference coordinate system are:

$$\begin{aligned} \mathbf{v}_1 &= [0, -\omega_{1_x} r_z, 0]^T \\ \mathbf{v}_2 &= [0, -\omega_{2_x} r_z, 0]^T \\ \mathbf{v}_3 &= [0, -\omega_{3_x} r_z, 0]^T \end{aligned} \tag{3}$$

The tangential velocities $\mathbf{v}_k$ are each expressed in their respective omni-wheel reference coordinate system $[x_k, y_k, z_k]$. These velocity vectors can be transformed into the inertial reference coordinate system $[X, Y, Z]$ with the following geometric transformation: first, rotate $\mathbf{v}_k$ about y_k

by α, which is effectively the identity transformation as $\mathbf{v}_k$ is directed along the y_k-axis; then rotate the transformed vector about the Z-axis by β_k. This geometric transformation takes the form:

$$\mathbf{T}_k = \begin{bmatrix} c_{\beta_k} & s_{\beta_k} & 0 \\ -s_{\beta_k} & c_{\beta_k} & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad (4)$$

where c_{β_k}, and s_{β_k} respectively denote $\cos\beta_k$ and $\sin\beta_k$, $k \in \{1, 2, 3\}$.

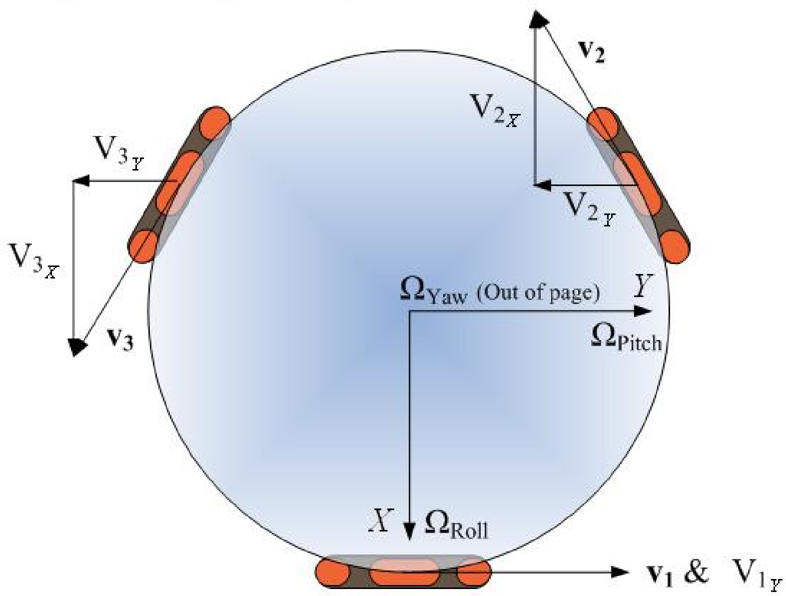


Figure 7: Omni-wheel tangential linear velocity components in the inertial $[X, Y, Z]$ coordinate system.

Using $\mathbf{T}_k$, the tangential velocities at each contact point are transformed to the inertial frame $[X, Y, Z]$, with $\beta_1 = 0°$, $\beta_2 = 120°$, and $\beta_3 = 240°$, as illustrated in Figure 7:

$$\mathbf{V}_1 = \mathbf{T}_1\mathbf{v}_1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ -\omega_1 r_z \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ -\omega_1 r_z \\ 0 \end{bmatrix}, \quad (5)$$

$$\mathbf{V}_2 = \mathbf{T}_2\mathbf{v}_2 = \begin{bmatrix} c_{\beta_2} & s_{\beta_2} & 0 \\ -s_{\beta_2} & c_{\beta_2} & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ -\omega_2 r_z \\ 0 \end{bmatrix} = \begin{bmatrix} -\omega_2 r_z s_{\beta_2} \\ -\omega_2 r_z c_{\beta_2} \\ 0 \end{bmatrix}, \quad (6)$$

$$\mathbf{V}_3 = \mathbf{T}_3\mathbf{v}_3 = \begin{bmatrix} c_{\beta_3} & s_{\beta_3} & 0 \\ -s_{\beta_3} & c_{\beta_3} & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ -\omega_3 r_z \\ 0 \end{bmatrix} = \begin{bmatrix} -\omega_3 r_z s_{\beta_3} \\ -\omega_3 r_z c_{\beta_3} \\ 0 \end{bmatrix}. \quad (7)$$

The components of the sphere radial vectors are similarly found to be:

$$\mathbf{R}_k = \mathbf{T}_k\mathbf{R}_1 = \begin{bmatrix} c_{\beta_k} & s_{\beta_k} & 0 \\ -s_{\beta_k} & c_{\beta_k} & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} R_{1_X} \\ 0 \\ R_{1_Z} \end{bmatrix} = \begin{bmatrix} c_{\beta_k} R_{1_X} \\ -s_{\beta_k} R_{1_X} \\ R_{1_Z} \end{bmatrix}. \quad (8)$$

While the current design parameters are $\beta_1 = 0°$, $\beta_1 = 120°$, and $\beta_3 = 240°$, we leave them general and obtain:

$$\mathbf{R}_1 = \begin{bmatrix} R_{1_X} \\ 0 \\ R_{1_Z} \end{bmatrix}; \ \mathbf{R}_2 = \begin{bmatrix} \mathrm{c}_{\beta_2} R_{1_X} \\ -\mathrm{s}_{\beta_2} R_{1_X} \\ R_{1_Z} \end{bmatrix}; \ \mathbf{R}_3 = \begin{bmatrix} \mathrm{c}_{\beta_3} R_{1_X} \\ -\mathrm{s}_{\beta_3} R_{1_X} \\ R_{1_Z} \end{bmatrix}. \tag{9}$$

With the radial vectors, $\mathbf{R}_k$, and the tangential linear velocity components, $\mathbf{V}_k$, we can determine the angular velocity of the Atlas sphere. For this, we require an inverse cross-product. That is, we know that $\mathbf{v} = \boldsymbol{\omega} \times \mathbf{r}$. If instead we know $\mathbf{v}$ and $\mathbf{r}$, but wish to compute $\boldsymbol{\omega}$, the following relation can be used [7]:

$$\boldsymbol{\omega} = \frac{\mathbf{r} \times \mathbf{v}}{\|\mathbf{r}\|^2}.$$

Hence, we can express three sphere angular velocities, $\boldsymbol{\omega}_k$, each one due to the contribution of the k^{th} omni-wheel $k \in \{1, 2, 3\}$:

$$\mathbf{\Omega}_k = \frac{\mathbf{R}_k \times \mathbf{V}_k}{\|\mathbf{R}\|^2}. \tag{10}$$

Note that since the norm of each sphere radial vector is the same, the denominator in Equation 10 does not require a subscript. The three vector components are summed to give the angular velocity of the sphere, $\boldsymbol{\omega}$ given the three omni-wheel angular velocities:

$$\mathbf{\Omega} = \sum_{k=1}^{3} \mathbf{\Omega}_k. \tag{11}$$

Extracting the omni-wheel angular velocities, the Jacobian, $\mathbf{J}$, is obtained as:

$$\mathbf{\Omega} = \mathbf{J}\boldsymbol{\omega} = \frac{r_z}{\|\mathbf{R}\|^2} \begin{bmatrix} R_{1_Z} & R_{1_Z}\mathrm{c}_{\beta_2} & R_{1_Z}\mathrm{c}_{\beta_3} \\ 0 & -R_{1_Z}\mathrm{s}_{\beta_2} & -R_{1_Z}\mathrm{s}_{\beta_3} \\ -R_{1_X} & -R_{1_X} & -R_{1_X} \end{bmatrix} \begin{bmatrix} \omega_1 \\ \omega_2 \\ \omega_3 \end{bmatrix}. \tag{12}$$

This Jacobian maps the omni-wheel angular velocities onto the $[X, Y, Z]$ components of the angular velocity of the sphere.

Conversely, provided the Jacobian is non-singular, it may be inverted yielding

$$\boldsymbol{\omega} = \mathbf{J}^{-1}\mathbf{\Omega}, \tag{13}$$

which is an expression for the angular speeds of each of the three omni-wheels, $\boldsymbol{\omega}$, required to provide a desired angular velocity of the sphere $\mathbf{\Omega}$ such that no tangential slip exists at the interface between the sphere and the omni-wheels. Practically, it must be recognized that some tangential slip occurs at this interface. A related paper by Holland et al [8] explores the issue of tangential and transverse slip in detail.

3 Investigation of the Atlas Jacobian

Inspection of the system Jacobian (Equation 12) reveals that unlike typical manipulator Jacobians, $\mathbf{J}$ is time invariant and depends only on design constants. Hence, these constants can be chosen such that the Jacobian has full rank so that the orienting workspace of the sphere is configurationally singularity free.

Therefore, only singularities arising from the initial design parameters (architecture singularities [9]) will result in a rank deficient Jacobian. The determinant of the Jacobian will be used to identify architecture singular configurations so that they can be avoided in practice.

Computationally, a Jacobian singularity means that $\mathbf{J}^{-1}$ is undefined and Equation 13 cannot be used to solve for the omni-wheel speed vector $\boldsymbol{\omega}$ [10]. Conceptually, a singular Jacobian means that it is not possible to actuate the sphere under the conditions for which the Jacobian was evaluated thereby implying a less than full three DOF motion envelope [9].

Singularities occur when $\det \mathbf{J} = 0$:

$$\det \mathbf{J} \;=\; 0 \;=\; -\left(\frac{r_z^3 R_{1_Z}^2 R_{1_X}}{\|\mathbf{R}\|^6}\right)(-\sin\beta_2 + \sin\beta_3 - \cos\beta_2 \sin\beta_3 + \cos\beta_3 \sin\beta_2). \tag{14}$$

Concerning the design parameters, the determinant will vanish if any, or all of r_z, R_{1_Z}, and R_{1_X} are identically zero, or if $\mathbf{R} \sim \infty$. Alternately, the determinant will vanish if the design parameter $\beta_3 = 0$, or if $\beta_2 = \beta_3$.

Physical interpretations of the conditions that cause architectural singularities reveal that these conditions are legitimate but do not limit the practicality of the Atlas concept. The case where $r_z = 0$ corresponds to omni-wheels having zero radius. This situation would lack a moment arm enabling rotation about the omni-wheel shaft to produce linear velocity along the surface of the Atlas sphere. The condition where $R_{1_Z} = 0$ would result in omni-wheels being placed along the equator of the sphere. This geometry would preclude the possibility of generating roll- or pitch-inducing velocity components (or moments) thereby limiting the motion envelope. The condition where $R_{1_X} = 0$ corresponds to the impractical case where the radius of the Atlas sphere is zero. The case where $\mathbf{R} \sim \infty$ corresponds to the Atlas sphere becoming a plane. The conditions where $\beta_3 = 0$ and $\beta_2 = \beta_3$ correspond to coincident location of two or more omni-wheels which limits the ability to independently prescribe three tangential velocities along the surface of the sphere: this condition leads to an under-actuated motion platform.

4 Discussion and Conclusion

With the incorporation of actuation to include the three linear DOF (surge, sway, and heave) in the motion of Atlas, it is possible to move the axis of rotation of the sphere away from the origin of the global frame, allowing any arbitrary axis of rotation to be achieved. If each DOF is actuated independently of the others it is conceivable that singularity analysis will show that only boundary singularities will exist and the mechanism should not exhibit internal singularities. Boundary singularities exist at the limits of the reachable workspace while internal workspace singularities can be encountered within the work space and are associated with undesirable joint configurations [11]. Boundary singularities can be avoided by not pushing the mechanism to the limits of actuation. Note that there are no theoretical boundaries for the three rotational DOF, and the linear boundaries correspond to the length limits of the linear actuators.

This paper has shown that the novel spherical actuation concept for the Atlas sphere results in a Jacobian that is independent of time and sphere orientation. Using the determinant of the Jacobian matrix, design conditions resulting in singularities were determined. It was found that in all cases, conditions causing singularities are of no practical consequence and therefore do not limit the Atlas design. The important result is that once design parameters have been selected that avoid these architectural singularities, the mechanism can be actuated without restriction in all three rotational DOF without the possibility of encountering any configurational singularities. By incorporating actuation of the three translational DOF it becomes possible to actuate the mechanism to produce rotation about any axis provided the translational travel limits of the platform are not reached.

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