Kinematics of an Atlas Platform with Redundant Contact Points

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Abstract: *This paper presents the kinematics of an Atlas platform actuated via redundant actuation contact points, such as those associated with three-race omnidirectional wheels. The six degree of freedom (DOF) Atlas motion platform architecture comprises a three DOF spherical orienting platform, which is a sphere actuated by three omnidirectional wheels, affixed to an $x-y-z$ linear platform. The kinematics for the three DOF spherical orienting platform are introduced in their most general form for n actuation contact points. The conditions for no-slip are presented, and lead to the conclusion that a three-race solution cannot accomplish a combination of both slip-free and singularity-free conditions simultaneously.*

Keywords: *Kinematics; Singularity-free Architecture; Omnidirectional Wheels; Redundant Actuation.*

1 Introduction

The Atlas platform [1] illustrated in Figure 1 consists of a spherical platform, which is a sphere actuated by three omnidirectional wheels, affixed to an $x-y-z$ linear platform. For a set of idealized (perfectly round) omnidirectional wheels, there exist designs such that it is possible to obtain slip-free and singularity-free conditions [2]. However, in reality it is impossible to have a perfectly round omnidirectional wheel due to its very nature, as illustrated in Figure 2. Still, it is common in the mobile robotics literature to model omnidirectional wheels as ideal [3], or try to add slip factors to compensate for the resulting error [4]. Solutions to the discontinuity problem caused by the basic design of omnidirectional wheels are proposed in [5] by means of minimizing the gaps between the rollers in. While an intermediate sphere is added between the omnidirectional wheel and the actuated surface in [6]. An alternative approach [7] is to use a dual-race omnidirectional wheel as illustrated in Figure 3. This solves the problem of obtaining continuous contact with the sphere; however, it introduces a shift in the location of the contact points on the sphere, thereby inducing significant undesired vibrations, and associated control issues [8].

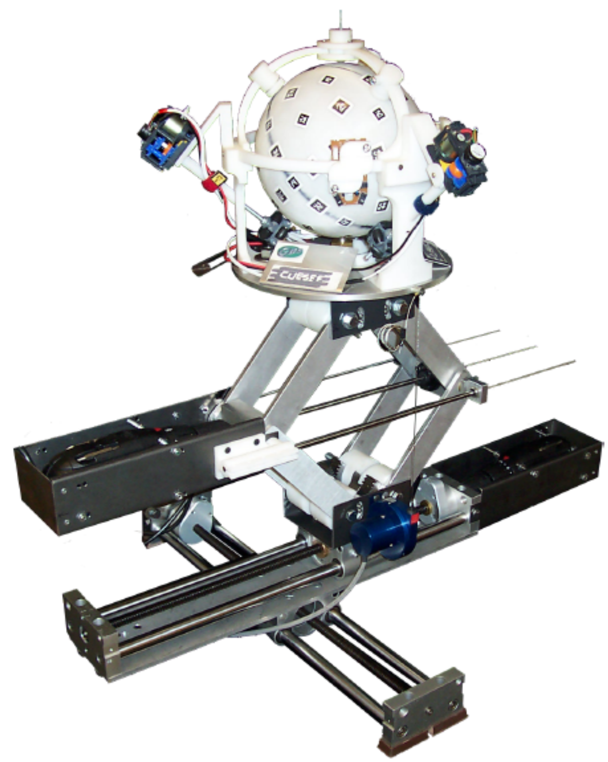

Figure 1: The Atlas table-top 6-DOF demonstrator highlighting the omnidirectional wheel actuation concept.

An attempt to combine the benefits of the two designs (single-race, dual-race) driven by intuition, is to have triple-race omnidirectional wheels as illustreted in Figure 4. The two external races touch the sphere at the same time, alternating with the centre race, yielding an equivalent or effective contact point that is exactly in between them. This generates continuous contact with a continuous effective contact point, thereby eliminating the step function induced vibration associates with dual-race

Figure 2: A single-race omnidirectional wheel.

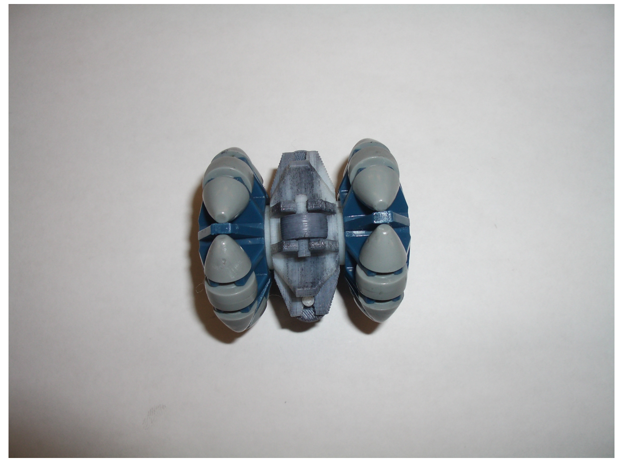

Figure 4: A triple-race omnidirectional wheel.

omnidirectional wheels.

This paper generalizes the kinematics for n contact points, develops the kinematics of the suggested solution, and establishes conditions on the design to attain the desired slip-free and singularity-free traits. The kinematics presented herein are completely general, and are valid for any over-constrained case, i.e. where $n > 3$.

2 Kinematics

To develop the kinematics of the platform, a general configuration is assumed, and an inertial coordinate frame is positioned at the geometric centre of the sphere as illustrated in Figure 5.

The kinematics for the ideal case have been developed in [2]. The underlying concept there was to obtain a relationship between $\vec{\Omega}$, the angular velocity vector of the sphere, and ω_i, the angular speeds of the three omnidirectional wheels, that would account for zero kinematic slip between the sphere and the omnidirectional wheels. The no-slip condition for a single contact point i is for the velocity of the omnidirectional wheel at the contact point, in the actuation direction, to be the same as the velocity component in the same direction at the same contact point on the sphere. That is, we require:

$$\left(\vec{\Omega} \times \vec{R}_i\right) \cdot (\vec{\omega}_i \times \vec{r}_i) = (\omega_i r_i)^2 \tag{1}$$

where $\vec{R}_i$ is the relative position vector directed from the sphere centre of rotation to the sphere contact point with omnidirectional wheel i and $\vec{r}_i$ is the relative position vector directed from the omnidirectional wheel centre of rotation to the sphere contact point with omnidirectional wheel i.

The same approach presented in [2] will be used, but instead of having three contact points, we now have n

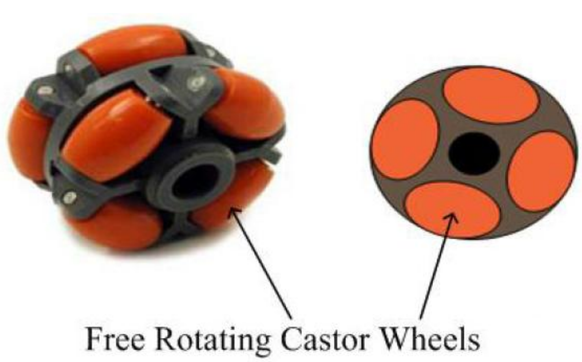

Figure 3: A dual-race omnidirectional wheel.

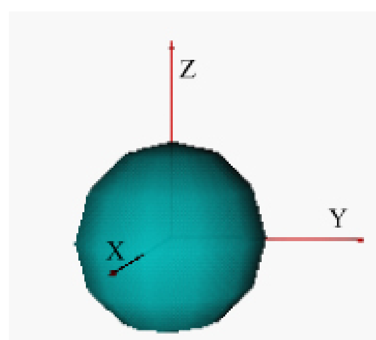

Figure 5: Inertial coordinate frame with origin at the geometric centre of the sphere.

contact points. For the no-slip condition we require:

$$(\hat{R}_i \times \hat{v}_i) \cdot \vec{\Omega} = \frac{r_i}{R}\omega_i \quad i = 1, 2, ..., n \tag{2}$$

where R is the radius of the sphere, $\hat{R}_i$ is a unit vector in the direction of the contact point from the sphere centre, $\hat{v}_i$ is a unit vector in the actuation direction of omnidirectional wheel i, and r_i is its radius. Defining the unit induced angular velocities, $\hat{\Omega}_i$ as

$$\hat{\Omega}_i = \hat{R}_i \times \hat{v}_i \tag{3}$$

yields

$$\hat{\Omega}_i \cdot \vec{\Omega} = \frac{r_i}{R}\omega_i. \tag{4}$$

In the case where $n = 3$, this results in a set of three equations with three unknowns, yielding slip-free singularity-free conditions, presented in [2].

A The Overdetermined Equation Approach

In the case of $n > 3$, the result is an overdetermined set of equations:

$$\begin{bmatrix} \hat{\Omega}_1^T \\ \hat{\Omega}_2^T \\ \hat{\Omega}_3^T \\ \cdot \\ \cdot \\ \hat{\Omega}_n^T \end{bmatrix} \vec{\Omega} = \frac{1}{R} \begin{bmatrix} r_1 & 0 & 0 & \cdot & \cdot & \cdot \\ 0 & r_2 & 0 & \cdot & \cdot & \cdot \\ 0 & 0 & r_3 & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & r_n \end{bmatrix} \begin{Bmatrix} \omega_1 \\ \omega_2 \\ \omega_3 \\ \cdot \\ \cdot \\ \omega_n \end{Bmatrix}. \tag{5}$$

This case usually calls for an approximate solution, typically a least squares approach.

3 Application to Triple-Race Omnidirectional Wheels

In the specific case of a sphere actuated by three triple-race omnidirectional wheels, there are up to six contact points. To illustrate the difference between triple-race omnidirectional wheels and single or dual race wheels, the extreme case of six contact points will be used. In the triple-race case, there are additional constraints, since for each pair of contact points on the two outer races that belong to the same omnidirectional wheel, r, ω, and $\hat{v}$ are the same. This allows for the simplification of the equations:

$$\begin{bmatrix} \hat{\Omega}_{11}^T \\ \hat{\Omega}_{12}^T \\ \hat{\Omega}_{21}^T \\ \hat{\Omega}_{22}^T \\ \hat{\Omega}_{31}^T \\ \hat{\Omega}_{32}^T \end{bmatrix} \vec{\Omega} = \frac{1}{R} \begin{bmatrix} r_1 & 0 & 0 & 0 & 0 & 0 \\ 0 & r_1 & 0 & 0 & 0 & 0 \\ 0 & 0 & r_2 & 0 & 0 & 0 \\ 0 & 0 & 0 & r_2 & 0 & 0 \\ 0 & 0 & 0 & 0 & r_3 & 0 \\ 0 & 0 & 0 & 0 & 0 & r_3 \end{bmatrix} \begin{Bmatrix} \omega_1 \\ \omega_1 \\ \omega_2 \\ \omega_2 \\ \omega_3 \\ \omega_3 \end{Bmatrix} \tag{6}$$

where

$$\hat{\Omega}_{ij} = \hat{R}_{ij} \times \hat{v}_i \tag{7}$$

and $\hat{R}_{ij}$ is a unit vector in the direction of the contact point j of omnidirectional wheel i.

As mentioned above, this kind of over-determined set of equations is usually solved using an approximation method. However, using such an approach leads to a solution that is missing the point of the design. The aim of the design is to achieve motion that is equivalent to that induced by the contact point of the centre race; i.e., one should expect any set of two equations belonging to the same omnidirectional wheel to yield a result in the same direction as the equivalent result for the single-race case. While this may be accomplished using the results from the single-race analysis, this assumption, as well as any other approximation technique, would still yield an approximation, which implies that the slip-free condition is compromised. Thus, without being able to accomplish the slip-free conditions, the kinetics of the system must be taken into consideration, and the true motion cannot be determined using kinematics alone.

A Revisiting the No-Slip Condition

Rearranging the terms in Equation (6) yields another way to look at the problem:

$$\begin{bmatrix} \hat{\Omega}_{11}^T & -\frac{r_1}{R} & 0 & 0 \\ \hat{\Omega}_{12}^T & -\frac{r_1}{R} & 0 & 0 \\ \hat{\Omega}_{21}^T & 0 & -\frac{r_2}{R} & 0 \\ \hat{\Omega}_{22}^T & 0 & -\frac{r_2}{R} & 0 \\ \hat{\Omega}_{31}^T & 0 & 0 & -\frac{r_3}{R} \\ \hat{\Omega}_{32}^T & 0 & 0 & -\frac{r_3}{R} \end{bmatrix} \begin{Bmatrix} \Omega_x \\ \Omega_y \\ \Omega_z \\ \omega_1 \\ \omega_2 \\ \omega_3 \end{Bmatrix} = \vec{0}. \tag{8}$$

This representation enables examination of a given design. In order for one to obtain a nontrivial solution, it is required that the determinant of the matrix in Equation (8) be zero. Once this is established, then sets of allowable solutions $\{\vec{\Omega}, \vec{\omega}\}$ that yield slip-free conditions would exist.

A closer look at the equation leads to even more simplifications. Gaussian elimination yields the following:

$$\begin{bmatrix} \hat{\Omega}_{11}^T & -\frac{r_1}{R} & 0 & 0 \\ \hat{\Omega}_{12}^T - \hat{\Omega}_{11}^T & 0 & 0 & 0 \\ \hat{\Omega}_{21}^T & 0 & -\frac{r_2}{R} & 0 \\ \hat{\Omega}_{22}^T - \hat{\Omega}_{21}^T & 0 & 0 & 0 \\ \hat{\Omega}_{31}^T & 0 & 0 & -\frac{r_3}{R} \\ \hat{\Omega}_{32}^T - \hat{\Omega}_{31}^T & 0 & 0 & 0 \end{bmatrix} \begin{Bmatrix} \Omega_x \\ \Omega_y \\ \Omega_z \\ \omega_1 \\ \omega_2 \\ \omega_3 \end{Bmatrix} = \vec{0}. \quad (9)$$

Rows 1, 3, and 5 of the coefficient matrix of Equation (9) are clearly linearly independent with respect to each other, and with respect to rows 2, 4, and 6. Thus, all that remains is to assure that the determinant of the smaller 3×3 matrix

$$\begin{bmatrix} \hat{\Omega}_{12}^T - \hat{\Omega}_{11}^T \\ \hat{\Omega}_{22}^T - \hat{\Omega}_{21}^T \\ \hat{\Omega}_{32}^T - \hat{\Omega}_{31}^T \end{bmatrix} \quad (10)$$

is zero. Remembering that $\hat{\Omega}_{ij} = \hat{R}_{ij} \times \hat{v}_i$, the rows of the matrix above may be rewritten as:

$$\hat{\Omega}_{i2} - \hat{\Omega}_{i1} = (\hat{R}_{i2} - \hat{R}_{i1}) \times \hat{v}_i. \quad (11)$$

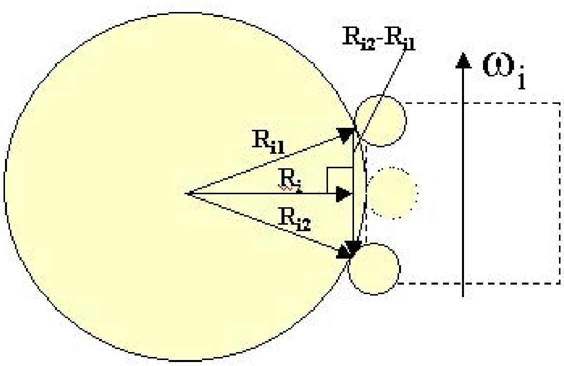

Figure 6: Contact point geometry when two races touch the sphere simultaneously.

From the geometry of the problem, as illustrated in Figure 6, we conclude that the result is a vector in the direction of $\hat{R}_i$. Hence, we may rewrite the matrix above as:

$$\begin{bmatrix} \hat{R}_1^T \\ \hat{R}_2^T \\ \hat{R}_3^T \end{bmatrix}. \quad (12)$$

The no-slip condition now becomes a requirement on the position vectors of the effective contact points to be linearly dependent. Finally, combining this requirement with the no-slip requirement on the centre row combination, which was shown in [2] to be that the matrix

$$\begin{bmatrix} \hat{\Omega}_1^T \\ \hat{\Omega}_2^T \\ \hat{\Omega}_3^T \end{bmatrix} \quad (13)$$

be non-singular, we need $\hat{R}_i$ to be linearly dependent, while the $\hat{\Omega}_i$ are linearly independent.

4 Examples

The following examples re-examine two architectures that complied with the conditions for the single-row and dual-row omnidirectional wheels for the added criterion posed by the triple-race analysis. A third architecture that emerges from the analysis is then presented. For all the examples, the sphere has a radius R and each of the omnidirectional wheels has a radius r.

A The orthogonal case

This example considers the architecture illustrated in Figure 7.

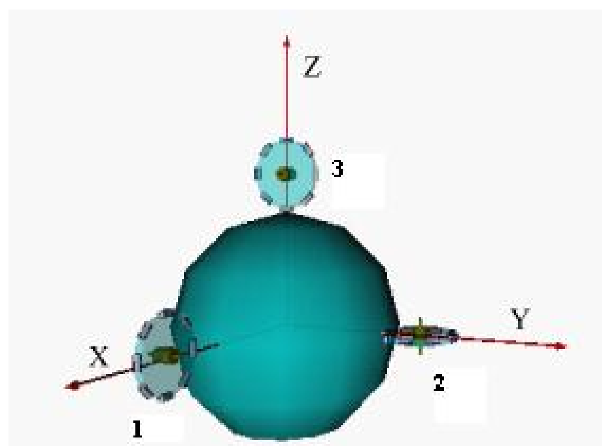

Figure 7: Kinematic architecture for the orthogonal case.

The position vectors of the three contact points are:

$$\vec{R}_1 = R\hat{i};\ \vec{R}_2 = R\hat{j};\ \vec{R}_3 = R\hat{k}. \quad (14)$$

The unit induced angular velocities are:

$$\hat{\Omega}_1 = \hat{i} \times \hat{k} = -\hat{j}; \hat{\Omega}_2 = \hat{j} \times \hat{i} = -\hat{k}; \hat{\Omega}_3 = \hat{k} \times \hat{j} = -\hat{i}. \quad (15)$$

It is clear that these are mutually orthogonal, since:

$$\hat{\Omega}_i \cdot \hat{\Omega}_j = 0 ... i \neq j. \quad (16)$$

It is also clear that the position vectors $\vec{R}_i$ are mutually orthogonal, thus the added condition for the triple-race case is not satisfied. Hence this architecture, that was shown to be slip-free for the single-race and dual-race cases, fails in the triple-race case.

B The Atlas Sphere

The current configuration of the Atlas motion platform has the three omnidirectional wheels arranged on the edges of an equilateral triangle with an elevation angle of 40°. The reason for the equilateral configuration is to accomplish as even force and torque distribution on the omnidirectional wheels as possible. However, the elevation angle of 40° was chosen for the demonstrator solely due to ease of manufacturing and assembly. To generalize the equilateral configuration an arbitrary elevation angle θ will be used. The configuration is presented in Figure 8.

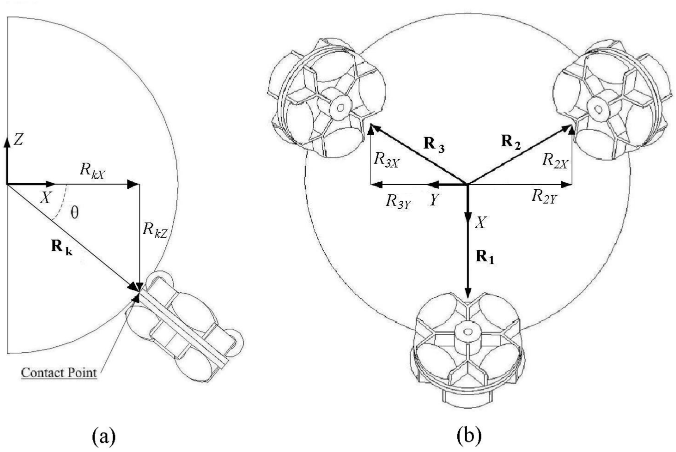

Figure 8: Kinematic architecture for the Atlas sphere case [2].

In this case, the position vectors of the three contact points are:

$$\begin{aligned}\vec{R}_1 &= R(\cos\theta\hat{i} - \sin\theta\hat{k}),\\ \vec{R}_2 &= R(-\frac{1}{2}\cos\theta\hat{i} + \frac{\sqrt{3}}{2}\cos\theta\hat{j} - \sin\theta\hat{k}),\\ \vec{R}_3 &= R(-\frac{1}{2}\cos\theta\hat{i} - \frac{\sqrt{3}}{2}\cos\theta\hat{j} - \sin\theta\hat{k}),\end{aligned} \quad (17)$$

and the unit induced angular velocities, in matrix form, are:

$$\begin{bmatrix}\hat{\Omega}_1^T\\ \hat{\Omega}_2^T\\ \hat{\Omega}_3^T\end{bmatrix} = \begin{bmatrix}\sin\theta & 0 & \cos\theta\\ -\frac{1}{2}\sin\theta & \frac{\sqrt{3}}{2}\sin\theta & \cos\theta\\ -\frac{1}{2}\sin\theta & -\frac{\sqrt{3}}{2}\sin\theta & \cos\theta\end{bmatrix}. \quad (18)$$

The determinant of this matrix will be zero for:

$$\frac{3\sqrt{3}}{2}\sin^2\theta\cos\theta = 0 \quad (19)$$

thus, with the exception of $\theta = 0$ and $\theta = \pm 90°$, this matrix is non-singular. Applying the same process to the position vector, we have the matrix:

$$\begin{bmatrix}\hat{R}_1^T\\ \hat{R}_2^T\\ \hat{R}_3^T\end{bmatrix} = \begin{bmatrix}\cos\theta & 0 & -\sin\theta\\ -\frac{1}{2}\cos\theta & \frac{\sqrt{3}}{2}\cos\theta & -\sin\theta\\ -\frac{1}{2}\cos\theta & -\frac{\sqrt{3}}{2}\cos\theta & -\sin\theta\end{bmatrix}. \quad (20)$$

The determinant of this matrix will be zero for:

$$\frac{3\sqrt{3}}{2}\cos^2\theta\sin\theta = 0. \quad (21)$$

With the same exceptions as the last case, this matrix is also non-singular. Thus this architecture, like the previous one, fails for the triple-race case.

C The Collinear Case

An example that conforms to both conditions could easily be constructed.

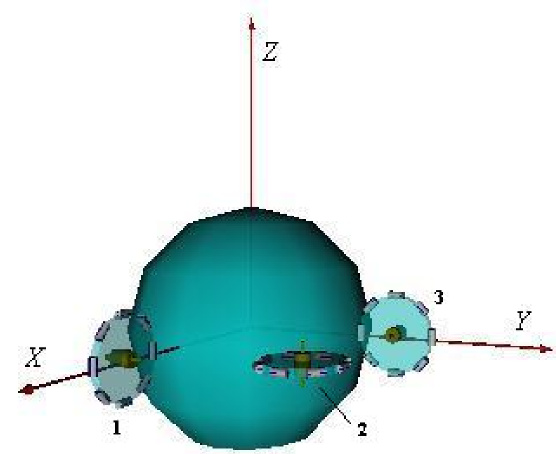

Figure 9: A configuration that allows for slip-free conditions.

In the configuration shown in Figure 9 we have:

$$\hat{R}_1 = \hat{i};\ \hat{R}_2 = \frac{\sqrt{2}}{2}\hat{i} + \frac{\sqrt{2}}{2}\hat{j};\ \hat{R}_3 = \hat{j}$$
$$\hat{\Omega}_1 = \hat{j};\ \hat{\Omega}_2 = \hat{k};\ \hat{\Omega}_3 = \hat{i} \quad (22)$$

and it is clear that the three induced angular velocities are linearly independent, while all three position unit vectors lay in the XY plane, thus being linearly dependent, as any three vectors contained in a plane.

Two main examples have been demonstrated so far in previous papers [2, 8]. Both have been shown to conform with the latter condition ($\hat{\Omega}_i$ are linearly independent). However, both fail to conform with the new requirement that $\hat{R}_i$ be linearly dependent. On the other hand, the third collinear case presented above, conforms to our added requirement. However, the result is that the determinant of the coefficient matrix in Equation (9) vanishes, implying that there are sets of combinations of allowable solutions $\{\vec{\Omega}, \vec{\omega}\}$ that yield slip-free conditions. That, in turn, means that not any set of input angular speeds $\vec{\omega}$ yields slip free conditions, thereby compromising the singularity-free workspace.

5 Conclusion

Although designs that meet the no-slip conditions exist, as presented above, clearly none of them could be singularity-free while maintaining the no-slip conditions. Any three-race omnidirectional wheel solution would be kinematically inferior to single-race [2] and double-race [8] solutions, that were shown to have Jacobians that represent one-to-one mappings from the input angular speeds of the omnidirectional wheels to the output angular velocity of the sphere. It seems that while intuition suggests that the triple-race omnidirectional wheels may give us the best of both worlds, this simple analysis suggests otherwise. This deception of intuition may be easily explained by exploring the single degree-of-freedom case, where a single omnidirectional wheel would drive a sphere or a cylinder. In this simple case the desired effect is accomplished, however, once more degrees of freedom are introduced, kinematic slip is imminent.

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