A Graphical Depiction of Hicksian Partial-Equilibrium Welfare Analysis*

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Abstract

An inescapable conclusion to be drawn from the literature on the measurement of welfare is that the use of consumer's surplus is a bad idea. This is especially true in the light of the fact that modern computing power facilitates the straightforward calculation of equivalent variation, the operational welfare indicator that is most strongly justified by economic theory. Consequently, a correct welfare analysis of a price-wealth change of the sort discussed in virtually every cost-benefit text should be principally in terms of Hicksian demand curves, not ordinary (Marshallian) ones. The present paper explains how to construct a graphical depiction of such an analysis, in partial equilibrium, which may be adopted in teaching the principles of cost-benefit analysis to graduate and advanced undergraduate students. This is done by way of several detailed examples covering the scope of applicability of the technique. In addition, a new formula for approximating equivalent variation is developed and analyzed.

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1. Introduction

More than two decades have now passed since the lack of validity of consumer's surplus as a welfare measure was demonstrated rigorously and reasonable alternatives were proposed. Chipman and Moore (1976, 1980) showed that restrictive and empirically untenable assumptions on preferences are necessary in order to make consumer's surplus a valid measure. McKenzie and Pearce (1976) and Vartia (1983) provided procedures that facilitate the calculation of a generalized version of Hick's (1942) equivalent variation—an operational welfare indicator that "resolves the conceptual problem of [individual] welfare measurement" (Slesnick, 1998, p. 2112)—without the imposition of restrictions on the form of the ordinary demand function other than the standard integrability conditions. Despite these advances, and despite the fact that modern computational power is more than up to the task of handling their inherent complexity, consumer's surplus continues to be "the overwhelming choice as a welfare indicator [in] empirical cost-benefit analyses" (Slesnick, 1998, p. 2110).

Part of the responsibility for this state of affairs must be assigned to the texts used to teach cost-benefit analysis (CBA). While most of these texts acknowledge the problems with using consumer's surplus as a welfare measure, they go ahead and do so anyway after either minimizing the attendant problems or claiming that appropriate adjustments can be made when necessary. For example, Boardman et al. (2001, p. 64) states that

> the biased estimate of [equivalent variation] that results from using Marshallian rather than Hicksian demand schedules to measure [welfare change] depends on the size of the income effect associated with a price change. *Usually this income effect and, hence, the bias are small and can be safely ignored in CBA*. [Emphasis in the original.]

Gramlich (1990, p. 57), on the other hand, asserts that

> when measuring the utility gain to consumers from [a] price reduction … the income effect must be excluded. … To measure the utility gain exactly, an experimenter might ask the consumer [how much compensation he would be willing to accept in lieu of] the price change.

While most texts discuss the concept of equivalent variation, none to my knowledge actually use it as the basis for welfare analysis. In some cases, the stated rationale for this omission is that a full understanding of equivalent variation requires "significant prerequisites in microeconomic theory [beyond the level that most potential practitioners and decision-makers] can grasp given their limited exposure to economics" (Townley, 1998, p. xiii). While such a justification may be fine for a text that "has been written in such a way that only an introductory-level course in microeconomics is required as a prerequisite" (Townley, 1998, p. iv), it is clearly not appropriate for a text aimed at graduate and advanced undergraduate students.

A useful first step towards updating the analytical basis of CBA texts might be the facility to construct a graphical depiction of a welfare analysis of a price-wealth change that is principally in terms of Hicksian demand curves as opposed to ordinary (Marshallian) ones. In the present paper, I provide such a facility in the context of a two-good partial-equilibrium framework. I do so by way of several detailed examples, beginning with a "base case" example in Section 2, and then proceeding to modifications of that example along particular qualitative dimensions in Section 3. The resulting cases cover the scope of applicability of the technique.

In Section 4, I develop a new formula for approximating aggregate equivalent variation in the maintained two-good framework, and then provide an analysis of this formula in relation to its Marshallian counterpart. Concluding remarks are provided in Section 5.

2. An Example

Following Sugden and Williams (1978, pp. 137–42), consider a government-operated rail service between a city and one of its suburbs together with a competitive market for suburban rental housing. For the sake of simplicity, assume that the total demands for these goods are generated by a representative (utility-maximizing) consumer.[1] A project is proposed that would reduce the price of rail trips by some amount. Assuming that rail trips and housing are normal goods and complements in consumption,[2] such a price reduction would shift the ordinary demand curve for housing to the right. This would induce an increase in the price of housing that would, in turn,

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[1] As will be made clear below, the need for the existence of a representative consumer is easily expunged.

[2] And therefore, by the Slutsky equation, *gross* complements as well.

cause the ordinary demand curve for rail trips to shift to the left. Sugden and Williams (1978) argue that the consumer's surplus in the rail market should be calculated as the area between the initial and final rail prices and to the left of the "observed" demand curve obtained by considering a price path that keeps the housing market in equilibrium as the price of rail trips is reduced in a monotonic sequence of very small steps. Since the consumer's and producers' surpluses in the housing market offset one another exactly along this price path, the overall (Marshallian) welfare measure is simply the consumer's surplus in the rail market.

A correct welfare analysis of the rail-price-reduction project would be in terms of equivalent variation—areas to the left of Hicksian (or compensated) demand curves for rail trips and housing at the after-project utility level. Since compensated price effects are symmetric, the line integral that defines such a Hicksian surplus measure is path independent. This means that the analyst is free to choose any price path connecting the initial and final prices of the two goods as the basis for determining the overall welfare effect. The simplest choices involve changing one price completely and then the other.

In order to depict an equivalent variation analysis in graphical terms, it is necessary to be able to show the positions of particular Hicksian demand curves in price-quantity space. Since such curves are not directly observable, this requirement boils down to showing how the Hicksian demand curves are related to their ordinary counterparts, which are observable in principle. Critical means to this end are provided by the Slutsky equation.

Mas-Colell et al. (1995, p. 71) states the Slutsky equation in relation to "a continuous utility function representing a locally nonsatiated and strictly convex preference relation defined on the [*m*-dimensional Euclidean space]" as, for all commodity-price vectors *p* and total expenditures (or wealth levels) *w*,

$$\frac{\partial h_\ell(p,u)}{\partial p_k} = \frac{\partial x_\ell(p,w)}{\partial p_k} + \frac{\partial x_\ell(p,w)}{\partial w} x_k(p,w) \quad \forall \ell, k \in \{1, \ldots, m\},$$

where $u = v(p, w)$, $v(\bullet)$ denotes the associated indirect utility function, and $h_\ell(\bullet)$ and $x_\ell(\bullet)$ denote the associated Hicksian and ordinary demand functions for commodity ℓ. Since both housing and rail trips are normal goods, this equation (with $m = 2$) tells us that, at the point of intersection between the ordinary and Hicksian demand curves for one of the goods, holding the price of the other good fixed, the latter curve is steeper than the former.[3] Since utility is a decreasing function of price along any ordinary demand curve, other things being equal, the corresponding family of Hicksian demand curves for a normal good is such that utility decreases from right to left in the relevant graph.

If the chosen price path moves the price of housing from its initial value to its final value *after* the price of rail trips has been so moved, then the correct welfare analysis of the Sugden and Williams (1978, pp. 137–42) example is illustrated by Figure 1.[4] Respectively, subscripts 1 and 2 denote rail trips and housing, and superscripts 0 and 1 denote before- and after-project values. On the price path that moves p_1^0 to p_1^1 ($< p_1^0$) and then p_2^0 to p_2^1 ($> p_2^0$), the aggregate (consumer's plus producers') equivalent variation with respect to the housing market is the negative of the triangular area *BCH* bounded by the supply curve $y_2(p_2)$, the Hicksian demand curve $h_2(p_1^1, p_2, u^1)$ that passes through the after-project equilibrium point (given by the intersection of the supply curve and the shifted ordinary demand curve $x_2(p_1^1, p_2; w)$), and a horizontal line drawn at the level of the initial housing price p_2^0.

The relevant Hicksian demand curve in the market for rail trips is the one given by the initial price of housing and the after-project utility level. This curve must lie to right of the one given by the *final* price of housing and the after-project utility level since housing and rail trips are complements, and to the left of the one given by the initial price of housing and the (indirect) utility level at that price and the final price of rail trips since this utility level must be higher than the after-project utility level (on account of the fact that indirect utility functions are non-increasing in each commodity price). In algebraic terms, $h_1(p_1, p_2^0, u^1)$ must lie to the right of $h_1(p_1, p_2^1, u^1)$ and to the left of $h_1(p_1, p_2^0, u^*)$, where $u^* := v(p_1^1, p_2^0) > v(p_1^1, p_2^1) =: u^1$. Both of these "reference" Hicksian demand curves are easy to locate in the graph of the market for rail trips: the former passes

[3] This result is at least noted, if not explained intuitively, in several current intermediate-level microeconomics texts. See, for example, Frank and Parker (2004, pp. 599–600) and Nicholson (2002, pp. 128–30).
[4] This diagram extends Sudgen and Williams (1978, p. 139, Fig. 10) by incorporating Hicksian demand curves *à la* standard *one*-good partial-equilibrium welfare analyses such as Nicholson (2002, p. 142, Fig. 5.10), Varian (1992, p. 168, Fig. 10.2), and Mas-Colell et al. (1995, Figs. 3.I.3, 3.I.4, 3.I.6 and 3.I.8).

through the after-project equilibrium point (D); the latter intersects the initial demand curve at the final price of rail trips (point E). The equivalent variation with respect to the market for rail trips is the area $p_1^0 FG p_1^1$ to the left of the relevant Hicksian demand curve between the initial and final prices of rail trips. Note that because this curve lies to the right of Sugden and Williams' (1978) observed demand curve (through points A and D), it is unclear from a qualitative graphical illustration such as Figure 1 whether the overall aggregate equivalent variation $p_1^0 FG p_1^1 - BCH$ is larger or smaller than its Marshallian counterpart ($p_1^0 AD p_1^1$).

If, instead of being competitive, the supply of housing were controlled by a single producer with a constant marginal cost of production c_2, the equivalent variation with respect to the market for rail trips would be (qualitatively) the same, but the aggregate equivalent variation with respect to the housing market would be the rectangular area $BIJK$ bounded by the marginal cost curve, a horizontal line drawn at the level of the initial housing price, and vertical lines drawn at the levels of the before- and after-project quantities of housing exchanged (x_2^0 and x_2^1), less the triangular area CHI bounded by the Hicksian demand curve $h_2(p_1^1, p_2, u^1)$ that passes through the after-project equilibrium point (given by the intersection of the monopoly expansion path $y_2^*(p_2)$ and the shifted ordinary demand curve $x_2(p_1^1, p_2; w)$), the horizontal line drawn at the level of the initial housing price, and the vertical line drawn at the level of the after-project quantity of housing exchanged. This variant of the Sugden and Williams (1978, pp. 137–42) example is illustrated by Figure 2.

The monopoly expansion path $y_2^*(p_2)$ is the locus of output-price combinations chosen by the housing producer as the ordinary demand for housing varies due to changes in the price of rail trips. The relationship between the price of rail trips and the monopoly's output, $p_1 = \varphi(x_2)$, is given by the solution to the first-order necessary condition for profit maximization

$$P_2(x_2, p_1) + \frac{\partial P_2(x_2, p_1)}{\partial x_2} x_2 = c_2 ,$$

where $P_2(x_2, p_1)$ denotes the inverse ordinary demand for housing. Inverting $P_2(x_2, p_1)$ after substituting $\varphi(x_2)$ for p_1 yields $y_2^*(p_2)$. Clearly, this approach can be generalized to accommodate more complex market structures; e.g., Cournot competition among *m* firms with possibly different, possibly non-constant, cost functions.

The maintained assumption that the total demands for the two goods are generated by a representative consumer can be relaxed at the cost of a minor reinterpretation of the supply curve $y_2(p_2)$. To see that this is so, suppose there are *n* consumers with possibly different ordinary demand functions. The total demand for either commodity is the sum of the corresponding individual demands. In particular, the total demand for housing is

$$X_2(p_1, p_2) := \sum_i x_{2(i)}(p_1, p_2) \ .$$

Letting $Y_2(p_2)$ denote the total supply of housing, the market clearing condition

$$X_2(p_1, p_2) = Y_2(p_2)$$

yields $p_1 = \psi(p_2)$, the relationship between the price of rail trips and the equilibrium price of housing. Substituting $\psi(p_2)$ for p_1 in $x_{2(i)}(p_1, p_2)$ yields the consumer expansion path $y_{2(i)}(p_2)$. Thus, under the interpretation of $h_\ell(p_1, p_2, u)$ and $x_\ell(p_1, p_2)$ as the Hicksian and ordinary demands for commodity ℓ of an *individual* consumer and $y_2(p_2)$ as the expansion path along which he adjusts his quantity demanded of housing in response to variations in p_1, the expression $\Delta\mu_{\Sigma EV(i)} := p_1^0 FG p_1^1 - BCH$ is the aggregate equivalent variation with respect to that consumer; i.e., the consumer's equivalent variation $\Delta\mu_{EV(i)} := p_1^0 FG p_1^1 - p_2^1 CH p_2^0$ plus the portion of the producers' equivalent variation due to that consumer $\Delta\pi_{(i)} := p_2^1 CB p_2^0$.

Now assume that the *n* consumers can be partitioned into two groups: one in which each individual owns a share of the housing industry $\theta_i > 0$ *and* has zero demands for rail trips and housing, and one in which $\theta_i \equiv 0$. Assume further that $\Sigma_i\theta_i = 1$. Hence, the equivalent variation of individual *i* in the shareholding group is $\Delta\mu_{EV,i} = \Delta w_i \equiv \theta_i\Sigma_j\Delta\pi_{(j)}$ (since $\Delta\mu_{EV(i)} \equiv 0$), and that of individual *i* in the non-shareholding group is $\Delta\mu_{EV,i} = \Delta\mu_{EV(i)}$ (since $\Delta w_i \equiv 0$).[5]

[5] In general, $\Delta\mu_{EV,i} = \Delta w_i + \Delta\mu_{EV(i)}$, where $\Delta w_i := w_i^1 - w_i^0$ and

$$\Delta\mu_{EV(i)} := -\int_{p^0}^{p^1} \sum_\ell h_{\ell(i)}(p, u^1)\,dp_\ell \ .$$

Simply summing the aggregate equivalent variations with respect to each consumer or, equivalently, summing the individual equivalent variations,[6] "as is common, embodies a version of utilitarianism [and thereby] ignores distributional concerns" (Slesnick, p. 2141). Moreover, Blackorby and Donaldson (1990, Sections III and IV) have shown that the sum of equivalent variations in general equilibrium contexts must be non-positive for any move away from a Walrasian equilibrium and, if preferences and technologies are convex, for any move away from an efficient allocation. The implication of this result for the present context is that $\Sigma_i \Delta\mu_{EV,i}$ might exhibit a downward bias relative to a hypothetical, ideal efficiency measure.

Note that the $\Delta\mu_{EV,i}$s, treated as functions of (p^1, w_i^1), are (indirect) money metrics with reference prices p^0. Money metrics can be aggregated using a social-welfare-type function $\Gamma(\bullet)$, which can then be used to compare alternative price-wealth situations in relation to the status quo. According to Donaldson (1992, p. 92), the social binary relation R defined implicitly by $\Gamma(\bullet)$

> is always an ordering. There are never problems of preference reversals, even with [consumer]-specific prices. Further, the function $\Gamma(\bullet)$ may exhibit inequality aversion, and different degrees of inequality aversion can be incorporated by employing a single-parameter family of functions such as the *S*-Ginis. … The ordering R, in general, does depend on p^0, of course ….

In order for social decision rules based on $\Gamma(\bullet)$ to be "consistent with usual distributional judgments," the $\Delta\mu_{EV,i}$s must be concave in wealth for all price vectors. Otherwise, such rules "may recommend that a fixed total [wealth] should be given entirely to one person, even when $\Gamma(\bullet)$ … is quasiconcave." The $\Delta\mu_{EV,i}$s are concave in wealth for all price vectors if and only if each individual's preferences are quasihomothetic[7] (Blackorby and Donaldson, 1988, Theorem 2), a property that does not characterize the preferences of real consumers.

One possible choice for $\Gamma(\bullet)$ is a weighted sum. Following Pearce and Nash (1981, pp. 32–3), the weights could be chosen to be

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[6] $\Sigma_i \Delta\mu_{\Sigma EV(i)} \equiv \Sigma_i \Delta\mu_{EV,i}$.

[7] Note that this property holds (trivially) for each individual in the shareholding group of the present example.

$$\beta_i := \left(\frac{\overline{w}}{w_i}\right)^{\gamma}, \; i = 1, \ldots, n,$$

where $\overline{w} := \frac{1}{n}\Sigma_j w_j$ and $\gamma > 0$. Suppose that, for the sake of simplicity, there are n_R identical individuals in the shareholding group and n_P identical individuals in the non-shareholding group ($n_R + n_P \equiv n$), and $w_R > w_P$; i.e., the n_R shareholders constitute "the rich" and the n_P non-shareholders constitute "the poor." In this case, the social welfare measure of the rail-price-reduction project would be

$$\begin{aligned}\Gamma(\Delta\mu_{EV,1}, \ldots, \Delta\mu_{EV,n}) &= \sum_{i=1}^{n} \beta_i \Delta\mu_{EV,i} \\ &= n_R \beta_R \Delta\mu_{EV,R} + n_P \beta_P \mu_{EV,P} \\ &= \left[1 - \tfrac{n_P}{n}\left(1 - \tfrac{w_P}{w_R}\right)\right]^{\gamma} \left[\Delta\pi + \left(\tfrac{w_R}{w_P}\right)^{\gamma} \Delta\mu_{EV}\right].\end{aligned}$$

Thus, the wider is the gap between the rich and the poor (w_R / w_P), *ceteris paribus*, the heavier is the weight placed on the net benefits accruing to the latter group ($\Delta\mu_{EV}$) relative to those accruing to the former ($\Delta\pi$). Furthermore, the higher is the fraction of poor individuals (n_P / n), *ceteris paribus*, the lower is the weight placed on the overall net benefit. Note that both of these effects vary directly with the magnitude of the γ parameter, the degree of inequality aversion.[8]

While problematic with respect to the construction of a social ordering, especially in general equilibrium contexts, a social decision rule based on some Γ(•) is arguably sufficient for the purposes of CBA. The usual objective of a CBA is to provide an assessment of the social desirability of one or more alternative projects in relation to one another and the status quo. The projects under consideration are usually variations on a fairly narrowly defined proposal such as, for example, the construction of a highway between locations A and B. Alternative routes or the possibility of levying a toll may be considered, but

> [r]arely does the analyst compare a highway project more broadly to completely different types of projects, such as health care, antipoverty, or national defense projects. As a practical matter, full optimization is impossible (Boardman et al., 2001, p. 9).

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[8] Note also that if γ = 0, then Γ(•) reduces to the overall aggregate equivalent variation.

Cognitive capacity limitations, budgetary and/or political constraints, and limited data availability are the major factors that work against welfare measurement in practice. Incremental improvements to social welfare sanctioned by generalized-aggregate-equivalent-variation-based cost-benefit analyses is most probably the best that can be hoped for.

3. Modifications

The two-good partial-equilibrium model analyzed above can be modified to some extent with respect to the qualitative characteristics of the goods without changing its essential structure. The Slutsky equation provides four rules that govern the relationships among these characteristics: a good that is both a complement and normal must also be a gross complement (C $\wedge$ N $\Rightarrow$ GC); a good that is both a gross complement and inferior must also be a complement (GC $\wedge$ I $\Rightarrow$ C); a good that is both a substitute and inferior must also be a gross substitute (S $\wedge$ I $\Rightarrow$ GS); and a good that is both a gross substitute and normal must also be a substitute (GS $\wedge$ N $\Rightarrow$ S). The symmetry property of compensated price effects provides a fifth rule: the two goods are either both substitutes or both complements. These five rules imply eighteen distinct cases:

(i)	C $\wedge$ (N, N) $\Rightarrow$ (GC, GC)
(ii), (iii)	C $\wedge$ (N, I) $\Rightarrow$ (GC, GC) $\vee$ (GC, GS)
(iv), (v)	C $\wedge$ (I, N) $\Rightarrow$ (GC, GC) $\vee$ (GS, GC)
(vi)–(ix)	C $\wedge$ (I, I) $\Rightarrow$ (GC, GC) $\vee$ (GC, GS) $\vee$ (GS, GC) $\vee$ (GS, GS)
(x)	S $\wedge$ (I, I) $\Rightarrow$ (GS, GS)
(xi), (xii)	S $\wedge$ (I, N) $\Rightarrow$ (GS, GS) $\vee$ (GS, GC)
(xiii), (xiv)	S $\wedge$ (N, I) $\Rightarrow$ (GS, GS) $\vee$ (GC, GS)
(xv)–(xviii)	S $\wedge$ (N, N) $\Rightarrow$ (GS, GS) $\vee$ (GS, GC) $\vee$ (GC, GS) $\vee$ (GC, GC)

Each of these cases can be characterized by a combination of one of eight possible graphs for the first good (“rail trips”) together with one of four possible graphs for the second good (“housing”). The eight possible graphs for the first good differ with respect to the direction of the shift of the initial ordinary demand curve (L or R), whether the ordinary demand curves are flatter than their Hicksian counterparts (F) or vice versa (V), and the position of the relevant Hicksian demand curve (either to the left or to the right of one of the two reference Hicksian demand curves—hereinafter called D and E after the names of the critical points they pass through). The four possible graphs for the second good differ with respect to the direction of the shift of the initial ordinary demand curve and whether or not the ordinary demand curves are flatter than their Hicksian counterparts. More specifically, the twelve graphs and the cases they help to characterize are summarized as follows:

L-F-LE $\rightarrow$ (i), (ii)

L-F-RE $\rightarrow$ (xiii), (xv), (xviii)

L-V-LE $\rightarrow$ (x), (xi)

L-V-RE $\rightarrow$ (iv), (vi), (ix)

R-F-LD $\rightarrow$ (iii)

R-F-RD $\rightarrow$ (xiv), (xvi), (xvii)

R-V-LD $\rightarrow$ (xii)

R-V-RD $\rightarrow$ (v), (vii), (viii)

R-F $\rightarrow$ (i), (iv), (v), (xii), (xvi), (xviii)

R-V $\rightarrow$ (ii), (vi), (viii)

L-F $\rightarrow$ (xi), (xv), (xvii)

L-V $\rightarrow$ (iii), (vii), (ix), (x), (xiii), (xiv)

In the preceding list, the notation that the relevant Hicksian demand curve lies to the left of one of the reference curves (LD or LE) means that it also lies to the right of the other (RE or RD), and the notation that it lies to the right of *D* or *E* means that it is situated even further to the right of *E* or *D*.

The twelve graphs are shown in Figures 1 and 3 through 11 and are ordered by the cases they characterize: Figure 1 shows the two case-(i) graphs (discussed in the preceding section); Figure 3 shows the graph of good 2 required to characterize case (ii) (along with the graph of good 1 in Figure 1); Figure 4 shows the two case-(iii) graphs; Figure 5 shows the graph of good 1 required to characterize case (iv) (along with the graph of good 2 in Figure 1); etc.

4. Approximations

It is common practice in standard CBA to obtain rough estimates of Marshallian surpluses by assuming that the relevant demand and supply curves are locally linear. In the Sugden and Williams (1978, pp. 137–42) example, the assumption that the observed demand curve for rail service is linear on the subdomain $[p_1^{1},\ p_1^{0}]$ allows the associated Marshallian surplus $p_1^{0}ADp_1^{1}$ to be calculated using the mathematical formula for the area of a trapezoid as

$$\Delta s_M = -\tfrac{1}{2}\Delta p_1\left[x_1^0 + x_1^1\right] = -\tfrac{1}{2}\Delta p_1\left[2 - \varepsilon_{11}^{*}\frac{\Delta p_1}{p_1^1}\right]x_1^1 \ ,$$

where $\Delta p_1 := p_1^{1} - p_1^{0}$ and ε_{11}^{*} denotes the price elasticity of observed demand (at point D). Similar assumptions with respect to the relevant Hicksian demand curves enable the calculation of an approximation to the aggregate equivalent variation. The present section develops a formula along these lines that is applicable in the context of the maintained two-good partial equilibrium model.

The slope of the relevant Hicksian demand curve in the housing market at point C can be written as

$$\frac{\partial h_2\left(p_1^1, p_2^1, u^1\right)}{\partial p_2} = \xi_{22}^1 \frac{x_2^1}{p_2^1} \ ,$$

where ξ_{22}^{1} denotes the price elasticity of compensated demand for housing. Assuming that $h_2(p_1^{1}, p_2, u^{1})$ is linear on the subdomain $[\min\{p_2^{0},\ p_2^{1}\}, \max\{p_2^{0},\ p_2^{1}\}]$, the x_2-coordinate of point H is given by the straight-line equation

$$x_2 = \left(1 - \xi_{22}^1 \frac{p_2^1 - p_2}{p_2^1}\right)x_2^1$$

and the area of the triangle BCH may be calculated as

$$\tfrac{1}{2}\Delta p_2\left[\Delta x_2 - \xi_{22}^1\frac{\Delta p_2}{p_2^1}x_2^1\right] \ ,$$

where $\Delta p_2 := p_2^{\,1} - p_2^{\,0}$ and $\Delta x_2 := x_2^{\,1} - x_2^{\,0}$.

The rate of change of the compensated demand for rail service at point *D* with respect to a change in the price of housing is

$$\frac{\partial h_1\left(p_1^1, p_2^1, u^1\right)}{\partial p_2} = \xi_{12}^1 \frac{x_1^1}{p_2^1} \ ,$$

where $\xi_{12}^{\,1}$ denotes the housing-price elasticity of compensated demand for rail service. Assuming that $h_1(p_1^{\,1}, p_2, u^1)$ is linear on the subdomain $[\min\{p_2^{\,0},\ p_2^{\,1}\}, \max\{p_2^{\,0},\ p_2^{\,1}\}]$, the x_1-coordinate of point *G* is given by the straight-line equation

$$x_1 = \left(1 - \xi_{12}^1 \frac{p_2^1 - p_2}{p_2^1}\right) x_1^1 \ .$$

Also, the slope of the relevant Hicksian demand curve for rail service at point *G* can be written as

$$\frac{\partial h_1\left(p_1^1, p_2^0, u^1\right)}{\partial p_1} = \frac{\xi_{11}^*}{p_1^1}\left[1 - \xi_{12}^1 \frac{\Delta p_2}{p_2^1}\right] x_1^1 \ ,$$

where ξ_{11}^* denotes the price elasticity of compensated demand for rail service. Now, assuming that $h_1(p_1, p_2^{\,0}, u^1)$ is linear on the subdomain $[p_1^{\,1},\ p_1^{\,0}]$, the x_1-coordinate of point *F* is given by the straight-line equation

$$x_1 = \left[1 - \xi_{12}^1 \frac{\Delta p_2}{p_2^1}\right]\left[1 - \xi_{11}^* \frac{p_1^1 - p_1}{p_1^1}\right] x_1^1$$

and the area of the trapezoid $p_1^{\,0}FGp_1^{\,1}$ may be calculated as

$$-\tfrac{1}{2}\Delta p_1 \left[2 - \xi_{11}^* \frac{\Delta p_1}{p_1^1}\right]\left[1 - \xi_{12}^1 \frac{\Delta p_2}{p_2^1}\right] x_1^1 \ .$$

Thus, the aggregate equivalent variation ($p_1^{\,0}FGp_1^{\,1} - BCH$) may be estimated as

$$\Delta\mu_{\Sigma EV} = -\tfrac{1}{2}\Delta p_1 \left[2 - \xi_{11}^* \frac{\Delta p_1}{p_1^1}\right]\left[1 - \xi_{12}^1 \frac{\Delta p_2}{p_2^1}\right] x_1^1 - \tfrac{1}{2}\Delta p_2 \left[\Delta x_2 - \xi_{22}^1 \frac{\Delta p_2}{p_2^1} x_2^1\right] \ .$$

The apparent difficulty of the compensated demand elasticities in this formula not being directly observable is easily dealt with by means of the elasticity form of the Slutsky equation:

$$\xi_{\ell k} = \varepsilon_{\ell k} + \eta_\ell \alpha_k \quad \forall \ell, k \in \{1, \ldots, m\},$$

where $\xi_{\ell k} := \partial \ln h_\ell(p, u) / \partial \ln p_k$ and $\varepsilon_{\ell k} := \partial \ln x_\ell(p, w) / \partial \ln p_k$ are the price elasticities of compensated and ordinary demand, respectively, $\eta_\ell := \partial \ln x_\ell(p, w) / \partial \ln w$ is the wealth elasticity of demand, and $\alpha_k := p_k x_k(p, w) / w$ is the expenditure share of good k.

The area BCH is zero if and only if either the housing supply is perfectly elastic (so that $\Delta p_2 = 0$) or both the supply and the compensated demand for housing are perfectly inelastic and coincident (so that $\Delta x_2 = \xi_{22}{}^1 = 0$). Otherwise, this area offsets, to some extent, the welfare gain measured by the area $p_1{}^0 FG p_1{}^1$ in the determination of $\Delta\mu_{\Sigma EV}$ (since $\Delta p_2 \Delta x_2 > 0$ and $\xi_{22}{}^1 < 0$).

Under the assumption that $\eta_1\alpha_1 = 0$ at point G, $\xi_{11}{}^* = \varepsilon_{11}{}^*$. The areas $p_1{}^0 FG p_1{}^1$ and $p_1{}^0 AD p_1{}^1$ are then equal if and only if $\xi_{12}{}^1 \Delta p_2 = 0$; i.e., if and only if the Hicksian demands $h_1(p_1, p_2{}^0, u^1)$ and $h_1(p_1, p_2{}^1, u^1)$ are both equal to the observed demand $x_1{}^*(p_1)$ on the subdomain $[p_1{}^1,\ p_1{}^0]$ so that point F coincides with point A and point G coincides with point D.[9]

Inspection of Figures 4, 5, 6 and 7 reveals the possibility of several different situations in which the equivalent variation $p_1{}^0 FG p_1{}^1$ is equal to the Marshallian surplus $p_1{}^0 AD p_1{}^1$. The prevalence of such situations—and close approximations to them—in relevant applied contexts is an empirical question that is beyond the scope of this paper. It would seem reasonable to claim, however, that such situations are comparatively rare.

A simple numerical example may be a useful means of consolidating the foregoing analysis. Following Sugden and Williams (1978, pp. 141–2) once more, suppose that $(p_1{}^0, x_1{}^0) = (1.2, 9000)$, $(p_1{}^1, x_1{}^1) = (1, 10000)$, $(p_2{}^0, x_2{}^0) = (9.5, 950)$, and $(p_2{}^1, x_2{}^1) = (10, 1000)$. Then $\Delta p_1 = -.2$, $\Delta x_1 = 1000$, $\Delta p_2 = .5$, $\Delta x_2 = 50$ and

$$\varepsilon_{11}^* = \frac{\Delta x_1}{\Delta p_1}\frac{p_1^1}{x_1^1} = \frac{1000}{-.2}\frac{1}{10000} = -.5\,,$$

which implies that

$$\Delta s_M = -\tfrac{1}{2}(-.2)\left[2-(-.5)\frac{(-.2)}{1}\right]10000 = 1900\,.$$

[9] If $\Delta p_2 = 0$, then point E coincides with point D as well.

Suppose further that $\varepsilon_{11} = -.6$, $\varepsilon_{12} = -.9$, $\varepsilon_{22} = -.4$, $\eta_1 = \eta_2 = .8$, and $\alpha_1 = \alpha_2 = .25$.[10] Then

$$\xi_{11} = -.6 + (.8)(.25) = -.4\ ,$$

$$\xi_{12} = -.9 + (.8)(.25) = -.7\ ,$$

$$\xi_{22} = -.4 + (.8)(.25) = -.2\ ,$$

and

$$\begin{aligned}\Delta\mu_{\Sigma EV} &= -\tfrac{1}{2}(-.2)\left[2-(-.4)\frac{(-.2)}{1}\right]\left[1-(-.7)\frac{.5}{10}\right]10000 - \tfrac{1}{2}(.5)\left[50-(-.2)\frac{.5}{10}1000\right] \\ &= 1987.2 - 15 \\ &= 1972.2\ .\end{aligned}$$

The percentage difference between $\Delta\mu_{\Sigma EV}$ and Δs_M is 3.8.

5. Concluding Remarks

The figures included in this paper provide a basis for a wide range of straightforward examples of correct welfare analyses of price-income changes. While the microeconomic theory underpinning these figures is certainly beyond that of a standard introductory-level course, it is assuredly not beyond that of relatively non-technical intermediate-level microeconomics texts. For example, Hicksian demand and equivalent variation are both discussed intuitively and fairly thoroughly in Eaton et al. (2005, pp. 122–24, 133–38) and Varian (2003, pp. 140–56, 254–58).[11] Since any such discussion can serve as an adequate prelude to the understanding of the figures herein, most of the associated analysis should be accessible to advanced undergraduates.

The approximation formula for aggregate equivalent variation developed in the preceding section should be of some assistance in easing practitioners of CBA out of their reliance on consumer surplus measures. Ultimately, of course, it is to be hoped that the use of distributionally sensitive equivalent-variation aggregates will become the norm in cases where sufficient data are available.

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[10] Note that the chosen expenditure-share values are consistent with the fact that $p_1^1 x_1^1 = p_2^1 x_2^1 =$ 10000.

[11] Note that neither of these texts, unlike those mentioned above (in fn. 3), discusses explicitly the relationship between ordinary and Hicksian demand curves.

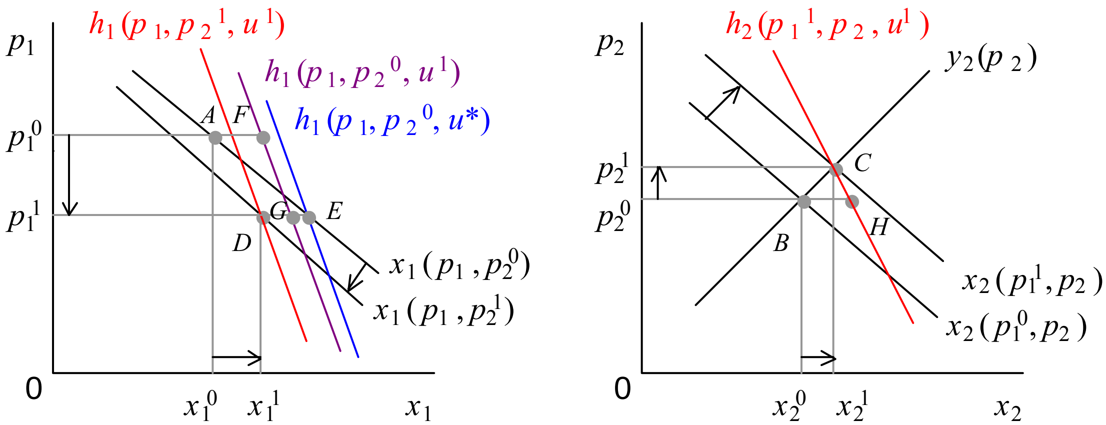

Figure 1. Case (i) $C \wedge (N, N) \Rightarrow (GC, GC)$ [Graphs L-F-LE and R-F]

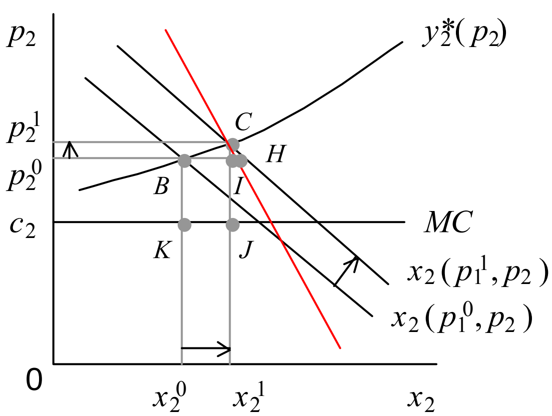

Figure 2. Case (i*)

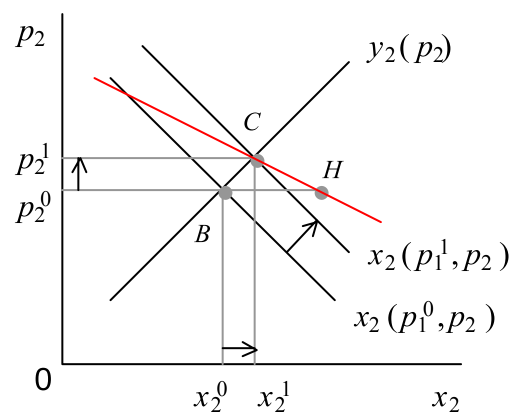

Figure 3. Graph R-V

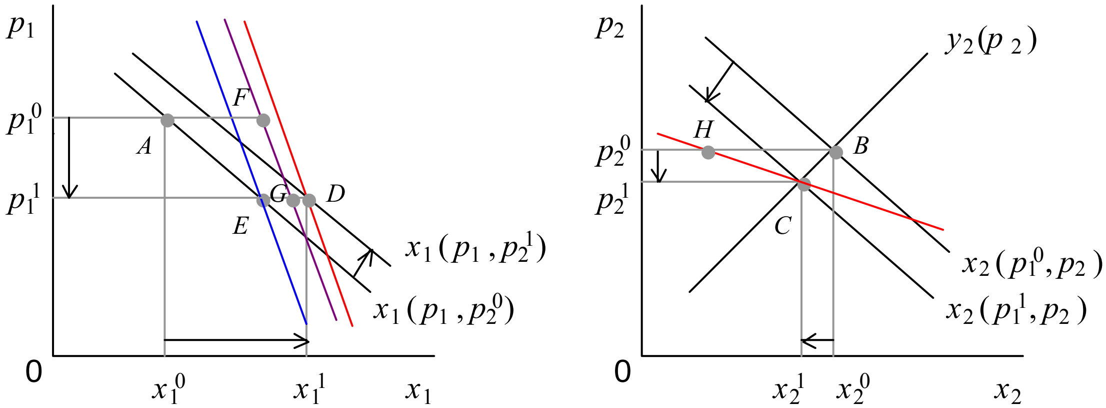

Figure 4. Case (iii) C ∧ (N, I) ∧ (GC, GS) [Graphs R-F-LD and L-V]

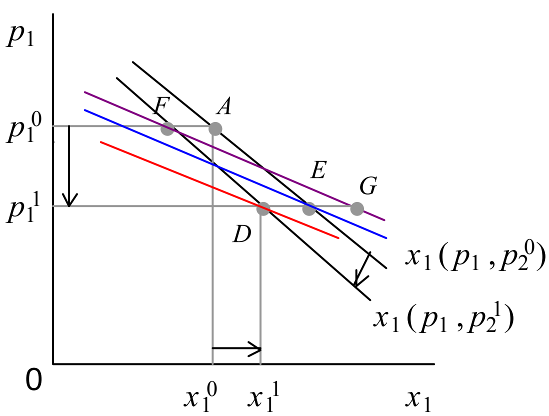

Figure 5. Graph L-V-RE

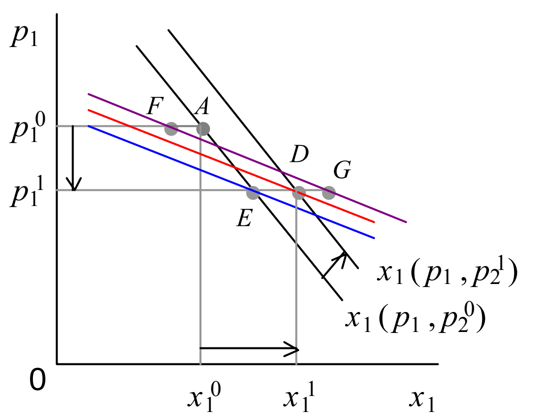

Figure 6. Graph R-V-RD

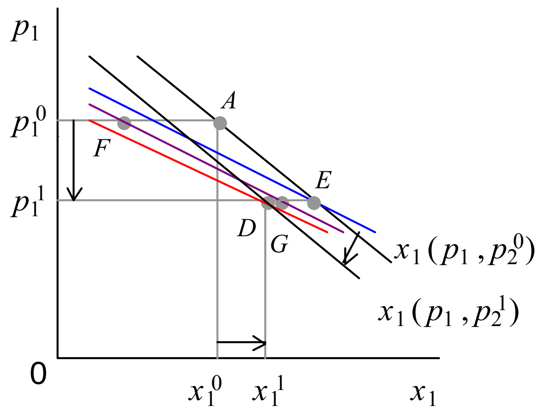

Figure 7. Graph L-V-LE

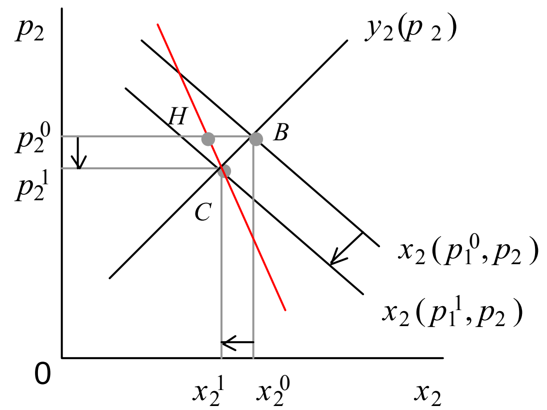

Figure 8. Graph L-F

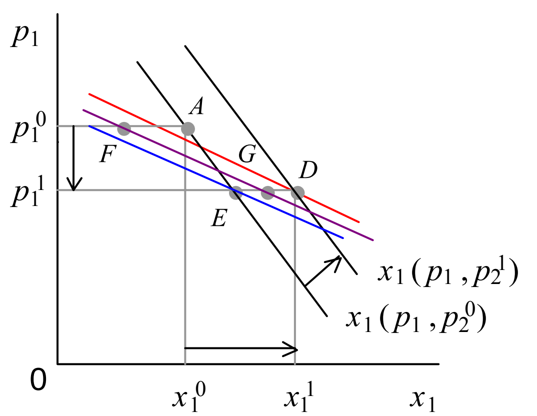

Figure 9. Graph R-V-LD

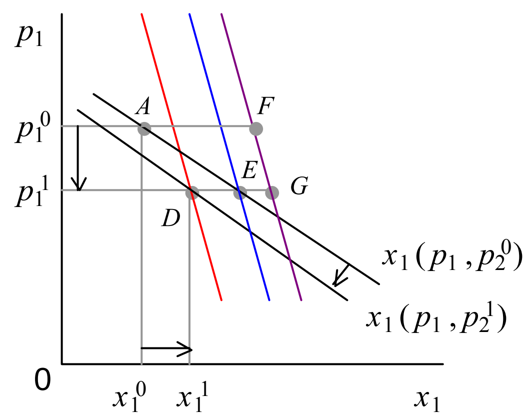

Figure 10. Graph L-F-RE

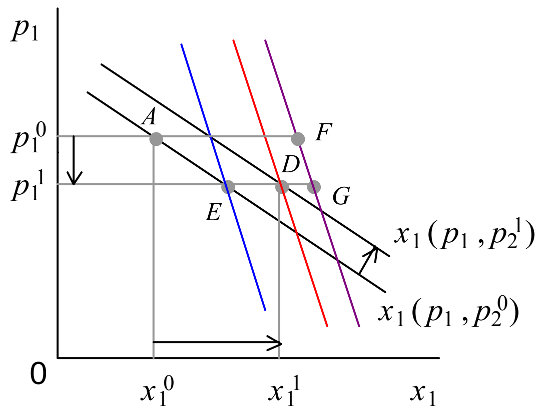

Figure 11. Graph R-F-RD

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